Complementarity and Substitution Analysis

This document reviews the Claims and Propositions in the paper "Spiffed-Up Channels: The Role of Spiffs in Hierarchical Selling Organizations," adding complementarity and substitution interactions in the sales effort necessary to induce high sales in both products, and identifying the impact of these effects.

Technically, we accomodate complementarity (substitution) trough the assumption that the occurence of high sales in one product implies a decrease (increase) in the selling effort the salesperson needs to exert to get high sales in another product. A simple modification in the additiveness of the salesperson disutility for effort (cost of effort) function is sufficient to accomodate both complementarity and substitution:[1]

$$C\left[e_i\right]+C\left[e_{3-i}\right]=\begin{cases}0 & \text{if } e_i=L \text{ and } e_{3-i}=L\\ c & \text{if } \{e_i=H \text{ and } e_{3-i}=L\}, \text{ or } \{e_i=L \text{ and } e_{3-i}=H\}\\ (2-\theta)\,c & \text{if } e_i=H \text{ and } e_{3-i}=H\end{cases}$$

for $i=1,2$,

where $\theta\in[-1,1]$ is the interaction parameter.

When $\theta<0$ products exhibit substitution effects; when $\theta=0$ there is no interaction; and when $\theta>0$ products exhibit complementarity effects.

Due to the discrete nature of the models, solutions when θ is small (which we call substitution or weak complementarity) are different from those when θ is big (which we call strong complementarity).

For the cases in which the rep firm can set individualized commission rates for each product, the range of θ corresponding to substitution or weak complementarity is defined as $\underline{\Theta}=\left\{\theta:\theta\in\left[-1\ ,\ 1-\frac{\Delta_2^2}{\Delta_1^2}\right]\right\}$, while the range of θ corresponding to strong complementarity is defined as $\overline{\Theta}=\left\{\theta:\theta\in\left[1-\frac{\Delta_2^2}{\Delta_1^2}\ ,\ 1\right]\right\}$. For the cases in which the rep firm can only set a single commission rate for both products, the range of θ corresponding to substitution or weak complementarity is defined as $\underline{\Theta}=\left\{\theta:\theta\in\left[-1\ ,\ 1-\frac{\Delta_2}{\Delta_1}\right]\right\}$, while the range of θ corresponding to strong complementarity is defined as $\overline{\Theta}=\left\{\theta:\theta\in\left[1-\frac{\Delta_2}{\Delta_1}\ ,\ 1\right]\right\}$.

The meaning of these intervals is related to the rep firm's assessment of the cost of implementing high selling effort for both products. When complementarity is strong ($\theta\in\overline{\Theta}$), the savings due to complementarity when selling effort is high on both products (equal to θc) can make the lower selling-effort productivity task appear to be more rewarding than the higer selling-effort productivity task if all complementarity savings are applied towards the former task. As we will see below, this simple phenomenon implies different optimal commission and spiffing behaviors by the manufacturer and the rep firm according to whether $\theta\in\underline{\Theta}$ or $\theta\in\overline{\Theta}$. Furthermore, these optimal behaviors themselves differ with differences in industry structure and ability to set individualized commission rates by products.

Below we proceed with our detailed analysis of complementarity and substitution interactions.[2] The main conclusion is that substitution or weak complementarity does not change the qualitative results regarding spiffs. Strong complementarity, however, can change the qualitative results regarding spiffs, but only in case 2.a.

[1] Notice that it is unecessary to have two interaction parameters: one for the demand and other for the cost of effort, because one of these parameters could be normalized and colapsed into the other (either colapsed in the demand function or in the cost function).

[2] Thruought this document we consider that q_2 is not much greater than q_1. If $q_2>>q_1$, some of the results regarding the manufacturer commissions y_1 and y_2 could be flipped. Nevertheless, results regarding the use of spiffs would not change.

Interactions and the unrestricted monopoly case

Both complementarity and substitution do not change the qualitative results presented in the main body of our paper. The resulting effect of complementarity (substitution) is simply a reduction (increase) in the commissions that have to be paid by the manufacturer to the salesperson.

Claim 1 restated for complementarity and substitution interactions is as follows.

Claim 1C *For the case of joint profit maximization and* ***substitution or weak complementarity****, the maximum outcome possible for the channel members is:*

$$\pi_{syst}^{m\_max} = (q_1 + \Delta_1 + q_2 + \Delta_2) - \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right) - (2-\theta)\,c - m.$$

For the case of joint profit maximization and ***strong complementarity****, the maximum outcome possible for the channel members is:*

$$\pi_{syst}^{m\_max} = (q_1 + \Delta_1 + q_2 + \Delta_2) - \frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2 c^2}{\Delta_1^2 + \Delta_2^2}\right) - (2-\theta)\,c - m.$$

Proof. To obtain the maximum outcome the channel system can get, we consider that the manufacturer and the rep firm vertically integrate and denote these two agents simply as the "firm." Assuming that the firm wants to implement high selling effort for both products, it needs to offer the right contract to the salesperson so that she would prefer to exert high effort on both products. In this case, the firm's problem is:

$$\begin{array}{lll}\left(P_{sys}^{m\_max}\right): & \max\limits_{A\in\mathbb{R},\mathbf{b},\mathbf{z}\in\mathbb{R}_+} \sum_{i=1}^{2}(1-b_i-z_i)\,X_i[e_i] - A & \text{s.t.} \\ SPC: & \sum_{i=1}^{2}\left[(b_i+z_i)\,X_i[e_i] - \frac{r\sigma^2}{2}(b_i+z_i)^2 - C[e_i]\right] + A \geq m & \\ SIC: & \mathbf{e} \in \underset{e_i\in\{L,H\}}{\operatorname{argmax}} \sum_{i=1}^{2}\left[(b_i+z_i)\,X_i[e_i] - \frac{r\sigma^2}{2}(b_i+z_i)^2 - C[e_i]\right] + A. & \end{array}$$

Because the terms b_i and z_i always appear together, spiffs are clearly unnecessary when the firm contracts directly with the sales force; hence, we set $\mathbf{z} = \{0, 0\}$.

The salesperson has four effort options: exerting low selling effort for both products (LL), exerting high selling effort only for the first product (HL), exerting high selling effort only for the second product (LH), or exerting high effort for both products (HH). Her utility for each of this options is respectively:

(U_{LL}^S): $(b_1)(q_1) - \frac{r\sigma^2}{2}(b_1)^2 + (b_2)(q_2) - \frac{r\sigma^2}{2}(b_2)^2 + A$

(U_{HL}^S): $(b_1)(q_1+\Delta_1) - \frac{r\sigma^2}{2}(b_1)^2 + (b_2)(q_2) - \frac{r\sigma^2}{2}(b_2)^2 - c + A$

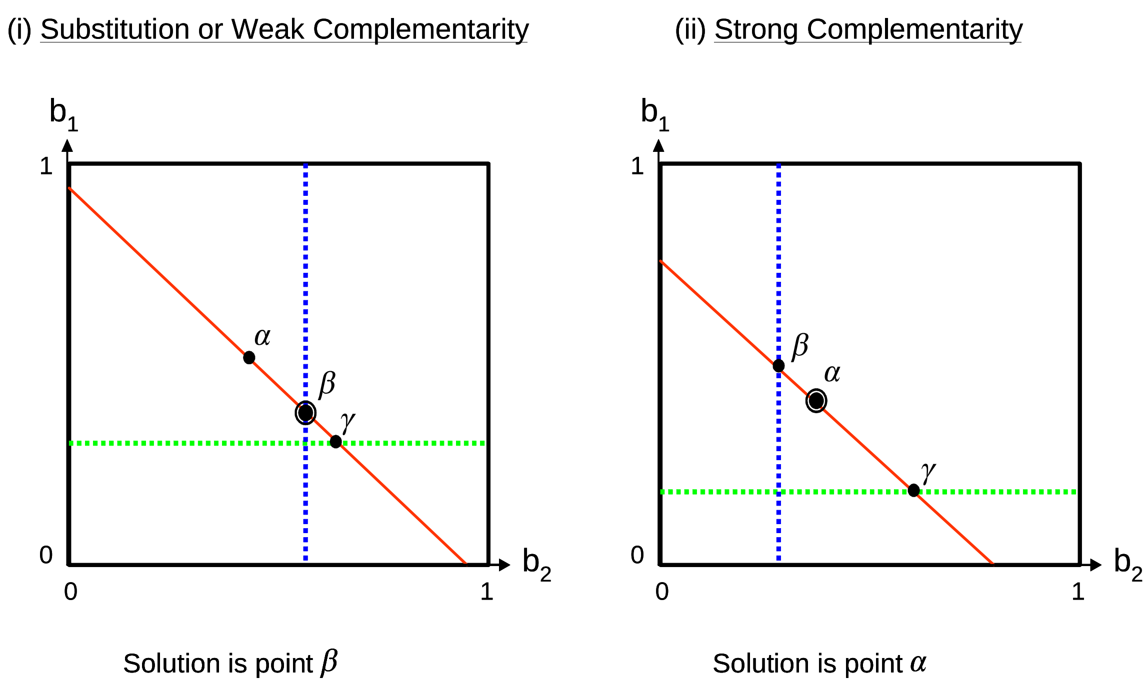
(U_{LH}^S): $(b_1)(q_1) - \frac{r\sigma^2}{2}(b_1)^2 + (b_2)(q_2+\Delta_2) - \frac{r\sigma^2}{2}(b_2)^2 - c + A$

(U_{HH}^S): $(b_1)(q_1+\Delta_1) - \frac{r\sigma^2}{2}(b_1)^2 + (b_2)(q_2+\Delta_2) - \frac{r\sigma^2}{2}(b_2)^2 - (2-\theta)\,c + A$

The salesperson's optimal effort choice is driven by the firm's choice of commissions $\mathbf{b}$. Since effort generates disutility for the salesperson, the firm induces high effort in both tasks (HH) by solving the problem:

$$\begin{array}{llll}\left(P_{systHH}^{m\_max}\right): & \max\limits_{A\in\mathbb{R},\mathbf{b}\in\mathbb{R}_+} (1-b_1)(q_1+\Delta_1) + (1-b_2)(q_2+\Delta_2) - A & & \\ & \text{s.t.} \quad (SPC_{HH}): & U_{HH}^S \geq m & \\ & \quad (SLC_{HHLH}): & U_{HH}^S \geq U_{LH}^S & \\ & \quad (SLC_{HHHL}): & U_{HH}^S \geq U_{HL}^S & . \\ & \quad (SGC_{HHLL}): & U_{HH}^S \geq U_{LL}^S & \end{array}$$

Because the cost of high effort on just one product is c, while the cost of high effort on both products is $(2-\theta)\,c$, the effective incremental cost of high effort on the second product is only $(1-\theta)c$, not c. Clearly, then, the solution depends on the magnitude of θ. Figure 1C below represents the role of the incentive constraints for this problem for both substitution and weak complementarity (panel i) and strong complementarity (panel ii). In the figure, point α represents the optimal solution if only SGC is considered ($U^S_{HH} = U^S_{LL}$;) point β represents a candidate solution in which the interaction effect is allocated to the least productive task (both $U^S_{HH} = U^S_{LL}$, and $U^S_{HH} = U^S_{HL}$ holds;) point γ represents a candidate solution in which the interaction effect is allocated to the most productive task (both $U^S_{HH} = U^S_{LL}$, and $U^S_{HH} = U^S_{LH}$ holds.)



Legend:

dotted horizontal line: $U^S_{HH} = U^S_{LH}$ (SLC_{HHLH} holds with equality)

dotted vertical line: $U^S_{HH} = U^S_{HL}$ (SLC_{HHHL} holds with equality)

solid diagonal line: $U^S_{HH} = U^S_{LL}$ (SGC_{HHLL} holds with equality)

Figure 1C: Sales Force Commission with Complementarity and Substitution

with Complementarity and Substitution

Substitution or Weak Complementarity. For $\theta \in \underline{\Theta}$, it is necessary to provide enough compensation to the most productive task to assure its implementation, and allocate the extra costs (in case of substitution) or savings (in case of weak complementarity) to the least productive task. Therefore, at the optimal solution, the SGC is binding and one of the SLC constraints is binding. In this particular case in which $\Delta_1 > \Delta_2$, SLC_{HHLH} is slack, and thus the solution to the above problem is:

$$\begin{aligned} \mathbf{b}^{m\_max}_{syst} &= \left\{ \frac{c}{\Delta_1}, \frac{(1-\theta)\,c}{\Delta_2} \right\}, \\ A^{m\_max}_{syst} &= m + (2-\theta)\,c - \left(\frac{c}{\Delta_1}\right)(q_1 + \Delta_1) - \left(\frac{(1-\theta)\,c}{\Delta_2}\right)(q_2 + \Delta_2) + \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2\,c^2}{\Delta_2^2}\right). \end{aligned}$$

Strong Complementarity. For $\theta \in \overline{\Theta}$, the complementarity effect is strong, and the more "relaxed" solution, the one that only binds on SGC automatically satisfies both SLCs (loosely speaking, this solution allocates some of the complementarity savings to each task.) Thus, the solution to the above problem is:

$$\begin{aligned}
\mathbf{b}_{syst}^{m_max} &= \left\{ \frac{(2-\theta)\, c\Delta_1}{\Delta_1^2+\Delta_2^2}, \frac{(2-\theta)\, c\Delta_2}{\Delta_1^2+\Delta_2^2} \right\}, \\
A_{syst}^{m_max} &= m + (2-\theta)\, c - \left(\frac{(2-\theta)\, c\Delta_1}{\Delta_1^2+\Delta_2^2} \right)(q_1+\Delta_1) - \left(\frac{(2-\theta)\, c\Delta_2}{\Delta_1^2+\Delta_2^2} \right)(q_2+\Delta_2) + \frac{r\sigma^2}{2} \left(\frac{(2-\theta)^2 c^2}{\Delta_1^2+\Delta_2^2} \right).
\end{aligned}$$

Plugging the above solutions into the manufacturer's profit function (adapted from (2) and (3)), we obtain the channel's optimal profits for both cases ($\theta \in \underline{\Theta}$ and $\theta \in \overline{\Theta}$). ■

Interactions and the baseline unrestricted monopoly model (Case 1.a.)

Both complementarity and substitution do not change the manufacturer's spiffing strategy: spiffs are not profit-enhancing and are not used in equilibrium. The resulting effect of complementarity (substitution) is simply a reduction (increase) in the commissions that have to be paid by the manufacturer to the rep firm and by the rep firm to the salesperson. Evidently this increases (decreases) the resulting total profits in the channel.

Proposition 1 restated for complementarity and substitution interactions is as follows.

Proposition 1C *For the case of joint manufacturer profit maximization, when contracts can accommodate compensation rates for every individual product both at the manufacturer and at the rep firm level, spiffs do not change the outcome for any player, and hence* ***there is no need for spiffs****.*

For the case of ***substitution or weak complementarity****, the optimal rep firm contract to its sales force entails:*

$$\begin{aligned}
b_1^{m_free} &= \frac{c}{\Delta_1},\ b_2^{m_free} = \frac{(1-\theta)\, c}{\Delta_2}, \\
A^{m_free} &= m + (2-\theta)\, c - \left(\frac{c}{\Delta_1} \right)(q_1+\Delta_1) - \left(\frac{(1-\theta)\, c}{\Delta_2} \right)(q_2+\Delta_2) + \frac{r\sigma^2}{2} \left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2} \right).
\end{aligned}$$

The optimal manufacturer contract to the rep firm and sales force entails:

$$\begin{aligned}
y_1^{m_free} &= \frac{c\left(c\frac{r\sigma^2}{2} + \Delta_1^2 \right)}{\Delta_1^3},\ y_2^{m_free} = \frac{(1-\theta)\, c\left[(1-\theta)\, c\frac{r\sigma^2}{2} + \Delta_2^2 \right]}{\Delta_2^3}, \\
z_1^{m_free} &= z_2^{m_free} = 0.
\end{aligned}$$

Manufacturer profits in equilibrium are:

$$\pi_M^{m\_free} = \left[1 - \frac{c\left(c\frac{r\sigma^2}{2} + \Delta_1^2 \right)}{\Delta_1^3} \right](q_1+\Delta_1) + \left[1 - \frac{(1-\theta)\, c\left[(1-\theta)\, c\frac{r\sigma^2}{2} + \Delta_2^2 \right]}{\Delta_2^3} \right](q_2+\Delta_2).$$

Rep firm profits in equilibrium are:

$$\pi_R^{m\_free} = \frac{c\left(c\frac{r\sigma^2}{2}+\Delta_1^2\right)}{\Delta_1^3}(q_1+\Delta_1)+\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}(q_2+\Delta_2)$$
$$-\frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2}+\frac{(1-\theta)^2c^2}{\Delta_2^2}\right)-(2-\theta)c-m.$$

For the case of ***strong complementarity****, the optimal rep firm contract to its sales force entails:*

$$b_1^{m\_free} = \frac{(2-\theta)c\Delta_1}{\Delta_1^2+\Delta_2^2},\ b_2^{m\_free}=\frac{(2-\theta)c\Delta_2}{\Delta_1^2+\Delta_2^2},$$
$$A^{m\_free} = m+(2-\theta)c-\left(\frac{(2-\theta)c\Delta_1}{\Delta_1^2+\Delta_2^2}\right)(q_1+\Delta_1)-\left(\frac{(2-\theta)c\Delta_2}{\Delta_1^2+\Delta_2^2}\right)(q_2+\Delta_2)+\frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2c^2}{\Delta_1^2+\Delta_2^2}\right).$$

The optimal manufacturer contract to the rep firm and sales force entails:

$$y_1^{m\_free} = \frac{c}{\Delta_1}-\frac{c^2r\sigma^2\left[(1-\theta)^2\Delta_1^2-(3-2\theta)\Delta_2^2\right]}{2\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)},\ y_2^{m\_free}=\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3},$$
$$z_1^{m\_free} = z_2^{m\_free}=0.$$

Manufacturer profits in equilibrium are:

$$\pi_M^{m\_free} = \left[1-\left(\frac{c}{\Delta_1}-\frac{c^2r\sigma^2\left[(1-\theta)^2\Delta_1^2-(3-2\theta)\Delta_2^2\right]}{2\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}\right)\right](q_1+\Delta_1)$$
$$+\left[1-\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}\right](q_2+\Delta_2).$$

Rep firm profits in equilibrium are:

$$\pi_R^{m\_free} = \left(\frac{c}{\Delta_1}-\frac{c^2r\sigma^2\left[(1-\theta)^2\Delta_1^2-(3-2\theta)\Delta_2^2\right]}{2\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}\right)(q_1+\Delta_1)+\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}(q_2+\Delta_2)$$
$$-\frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2c^2}{\Delta_1^2+\Delta_2^2}\right)-(2-\theta)c-m.$$

Proof. The manufacturer's goal is to implement high selling effort for both products. Here its problem is the same as in $\left(P^{m\_free}\right)$. The only difference is that now $C[e_1]+C[e_2]$ accommodates the interaction effect. We solve using backwards induction.

The salesperson has four effort options: exerting low selling effort for both products (LL), exerting high selling effort only for the first product (HL), exerting high selling effort only for the second product (LH), or exerting high effort for both products (HH). Her utility for each of this options is respectively:

$\left(U_{LL}^S\right)$: $(b_1+z_1)(q_1)-\frac{r\sigma^2}{2}(b_1+z_1)^2+(b_2+z_2)(q_2)-\frac{r\sigma^2}{2}(b_2+z_2)^2+A$

$\left(U_{HL}^S\right)$: $(b_1+z_1)(q_1+\Delta_1)-\frac{r\sigma^2}{2}(b_1+z_1)^2+(b_2+z_2)(q_2)-\frac{r\sigma^2}{2}(b_2+z_2)^2-c+A$

$\left(U_{LH}^S\right)$: $(b_1+z_1)(q_1)-\frac{r\sigma^2}{2}(b_1+z_1)^2+(b_2+z_2)(q_2+\Delta_2)-\frac{r\sigma^2}{2}(b_2+z_2)^2-c+A$

$\left(U_{HH}^S\right)$: $(b_1+z_1)(q_1+\Delta_1)-\frac{r\sigma^2}{2}(b_1+z_1)^2+(b_2+z_2)(q_2+\Delta_2)-\frac{r\sigma^2}{2}(b_2+z_2)^2-(2-\theta)c+A$

The salesperson's optimal effort choice is driven by the rep firm's choice of the commissions **b** and the manufacturer choice of spiffs incentives **z**. If the rep firm wants to implement low effort for both products (LL), then the SIC_{LL} constraint is slack and the rep firm's problem is:

$$\left(P_{R\_LL}^{m\_free}\right): \quad \begin{array}{ll} \max_{A\in\mathbb{R},b_i\in\mathbb{B}_i}(y_1-b_1)q_1+(y_2-b_2)q_2-A \\ \text{s.t.} \quad (SPC_{LL}): \qquad U_{LL}^S \geq m \end{array} .$$

The solution to this problem is simply to give minimal commission to the salesperson and a salary that is just high enough to guarantee her minimum utility:

$$\mathbf{b}_{LL}^* = \{-z_1, -z_2\}, \quad A_{LL}^* = m. \tag{1C}$$

If the rep firm wants to implement high effort in only one of the tasks (i.e., (HL) or (LH)), then the rep firm faces a standard principal-agent problem:

$$\left(P_{R\_HL}^{m\_free}\right): \quad \begin{array}{lll} \max_{A\in\mathbb{R},b_i\in\mathbb{B}_i}(y_1-b_1)(q_1+\Delta_1)+(y_2-b_2)q_2-A & & \\ \text{s.t.} \quad (SPC_{HL}): & U_{HL}^S \geq m & \\ \qquad (SIC_{HL}): & U_{HL}^S \geq U_{LL}^S & \end{array},$$

$$\left(P_{R\_LH}^{m\_free}\right): \quad \begin{array}{lll} \max_{A\in\mathbb{R},b_i\in\mathbb{B}_i}(y_1-b_1)q_1+(y_2-b_2)(q_2+\Delta_2)-A & & \\ \text{s.t.} \quad (SPC_{LH}): & U_{LH}^S \geq m & \\ \qquad (SIC_{LH}): & U_{LH}^S \geq U_{LL}^S & \end{array},$$

The solutions to the above problems are respectively:

$$\begin{aligned} \mathbf{b}_{HL}^* &= \left\{\frac{c}{\Delta_1}-z_1,-z_2\right\}, \quad A_{HL}^*=m+c-\left(\frac{c}{\Delta_1}\right)(q_1+\Delta_1)+\frac{r\sigma^2}{2}\left(\frac{c}{\Delta_1}\right)^2, \\ \mathbf{b}_{LH}^* &= \left\{-z_1,\frac{c}{\Delta_2}-z_2\right\}, \quad A_{LH}^*=m+c-\left(\frac{c}{\Delta_2}\right)(q_2+\Delta_2)+\frac{r\sigma^2}{2}\left(\frac{c}{\Delta_2}\right)^2. \end{aligned}$$

Lastly, if the rep firm wants to implement high effort in both tasks (HH), then the rep firm will face the problem:

$$\left(P_{R\_HH}^{m\_free}\right): \quad \begin{array}{lll} \max_{A\in\mathbb{R},b_i\in\mathbb{B}_i}(y_1-b_1)(q_1+\Delta_1)+(y_2-b_2)(q_2+\Delta_2)-A & & \\ \text{s.t.} \quad (SPC_{HH}): & U_{HH}^S \geq m & \\ \qquad (SLC_{HHLH}): & U_{HH}^S \geq U_{LH}^S & \\ \qquad (SLC_{HHHL}): & U_{HH}^S \geq U_{HL}^S & \\ \qquad (SGC_{HHLL}): & U_{HH}^S \geq U_{LL}^S & \end{array}.$$

Because the cost of high effort on just one product is c, while the cost of high effort on both products is $(2-\theta)\,c$, the solution depends on the magnitude of θ. The role of the incentive constraints in this problem is similar to the representation in Figure 1C above.

Substitution or Weak Complementarity. For $\theta \in \underline{\Theta}$, it is necessary to provide enough compensation to the high selling-effort productivity task to assure its implementation, and allocate the extra costs (in case of substitution) or savings (in case of weak complementarity) to the low selling-effort productivity task. Therefore, at the optimal solution, the SGC is binding and one of the SLC constraints is binding. In this particular case in which $\Delta_1 > \Delta_2$, SGC_{HHLH} is slack, and thus, assuming that spiffs will not change the order of productivity of selling tasks (which will be further confirmed to be true), the solution to the above problem is:

$$\begin{aligned} \mathbf{b}^*_{HH} &= \left\{ \frac{c}{\Delta_1} - z_1, \frac{(1-\theta)\,c}{\Delta_2} - z_2 \right\}, \\ A^*_{HH} &= m + (2-\theta)\,c - \left(\frac{c}{\Delta_1}\right)(q_1+\Delta_1) - \left(\frac{(1-\theta)\,c}{\Delta_2}\right)(q_2+\Delta_2) + \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right). \end{aligned} \tag{2C}$$

Strong Complementarity. For $\theta \in \overline{\Theta}$, the complementarity effect is strong, and the more "relaxed" solution, the one that only binds on SGC automatically satisfies both SLCs (loosely speaking, this solution allocates some of the complementarity savings to each task.) Thus, the solution to the above problem is:

$$\begin{aligned} \mathbf{b}^*_{HH} &= \left\{ \frac{(2-\theta)\,c\Delta_1}{\Delta_1^2+\Delta_2^2} - z_1, \frac{(2-\theta)\,c\Delta_2}{\Delta_1^2+\Delta_2^2} - z_2 \right\}, \\ A^*_{HH} &= m + (2-\theta)\,c - \left(\frac{(2-\theta)\,c\Delta_1}{\Delta_1^2+\Delta_2^2}\right)(q_1+\Delta_1) - \left(\frac{(2-\theta)\,c\Delta_2}{\Delta_1^2+\Delta_2^2}\right)(q_2+\Delta_2) + \frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2 c^2}{\Delta_1^2+\Delta_2^2}\right). \end{aligned} \tag{3C}$$

Now we are ready to analyze the manufacturer's problem. If the manufacturer wants to implement high effort in both tasks (HH), it has to make sure the rep firm receives enough incentives so that the rep firm will, in turn, provide the incentives for the salesperson to work hard on both tasks.

By plugging the results from expressions (1C) to (3C) into the rep firm's profit function (adapted from (2)), we obtain the rep firm's profits for each of the possible outcomes:

(π_{R_LL}): $\quad (y_1+z_1)(q_1) + (y_2+z_2)(q_2) - m$

(π_{R_HL}): $\quad (y_1+z_1)(q_1+\Delta_1) + (y_2+z_2)(q_2) - m - c - \frac{r\sigma^2}{2}\left(\frac{c}{\Delta_1}\right)^2$

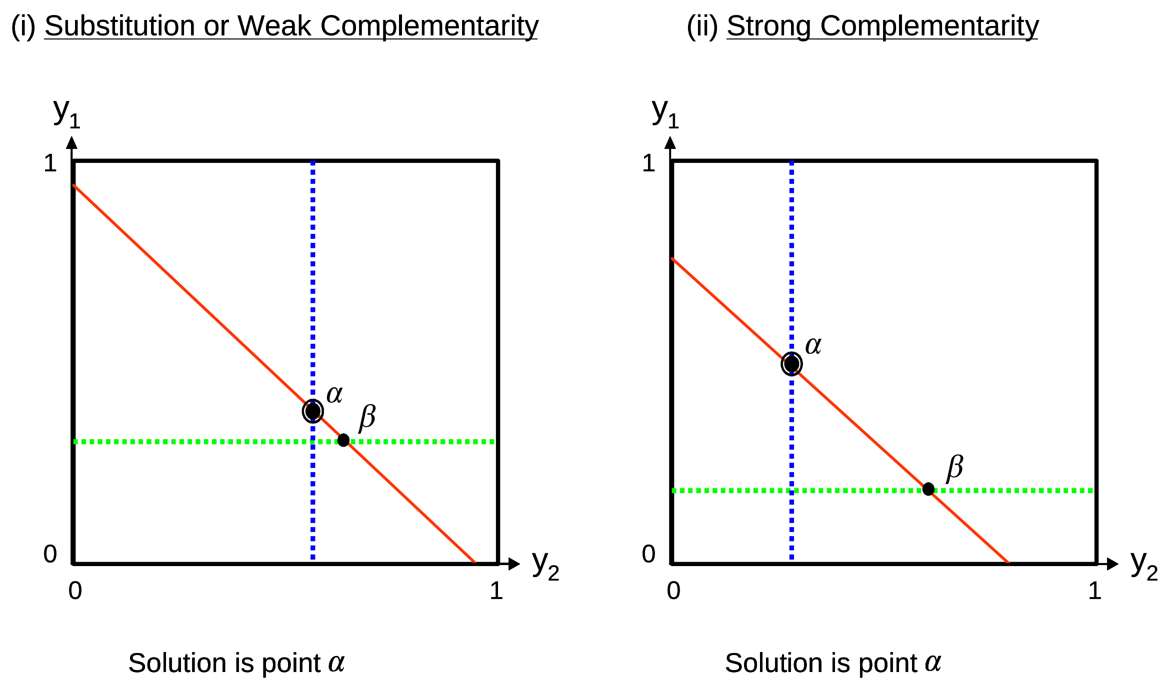
(π_{R_LH}): $\quad (y_1+z_1)(q_1) + (y_2+z_2)(q_2+\Delta_2) - m - c - \frac{r\sigma^2}{2}\left(\frac{c}{\Delta_2}\right)^2$

$$(\pi_{R\_HH}): \quad \begin{cases} (y_1+z_1)(q_1+\Delta_1) + (y_2+z_2)(q_2+\Delta_2) - m - (2-\theta)\,c - \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right) & \text{if} \quad \theta \in \underline{\Theta} \\ (y_1+z_1)(q_1+\Delta_1) + (y_2+z_2)(q_2+\Delta_2) - m - (2-\theta)\,c - \frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2 c^2}{\Delta_1^2+\Delta_2^2}\right) & \text{if} \quad \theta \in \overline{\Theta} \end{cases}.$$

Hence, if the manufacturer wants to implement high selling effort on both products (HH), the manufacturer will face the problem:

$$\begin{array}{lll} \left(P^{m\_free}_{M\_HH}\right): & \max_{\mathbf{y},\mathbf{z}\in\mathbb{R}_+} (1-y_1-z_1)(q_1+\Delta_1) + (1-y_2-z_2)(q_2+\Delta_2) & \\ & \text{s.t.} \quad (RPC_{HH}): & \pi_{R\_HH} \geq 0 \\ & \qquad (RLC_{HHLH}): & \pi_{R\_HH} \geq \pi_{R\_LH} \\ & \qquad (RLC_{HHHL}): & \pi_{R\_HH} \geq \pi_{R\_HL} \\ & \qquad (RGC_{HHLL}): & \pi_{R\_HH} \geq \pi_{R\_LL} \end{array}.$$

Because the terms y_i and z_i always appear together, spiffs are unnecessary; hence, we set $\mathbf{z}^*_{HH} = \{0, 0\}$. As before, the solution depends on the magnitude of θ, but only due to the rep firm's choice of the salesperson's commission $\mathbf{b}$. Figure 2C below represents the role of the incentive constraints for this problem for both substitution or weak complementarity (panel i) and strong complementarity (panel ii). The solution is never at an intermediary point along the RGC. Instead, the solution is at one of the two candidate points. Point α represents a candidate solution for which both $\pi_{R\_HH} = \pi_{R\_LL}$, and $\pi_{R\_HH} = \pi_{R\_HL}$; point β represents a candidate solution for which both $\pi_{R\_HH} = \pi_{R\_LL}$, and $\pi_{R\_HH} = \pi_{R\_LH}$.



Legend:

dotted horizontal line: $\pi_{R\_HH} = \pi_{R\_LH}$ (RLC_{HHLH} holds with equality)

dotted vertical line: $\pi_{R\_HH} = \pi_{R\_HL}$ (RLC_{HHHL} holds with equality)

solid diagonal line: $\pi_{R\_HH} = \pi_{R\_LL}$ (RGC_{HHLL} holds with equality)

Figure 2C: Rep firm Commission with Complementarity and Substitution

Under **substitution or weak complementarity** RLC_{HHLH} is slack and the solution to this problem is obtained by solving the RGC and the RLC_{HHHL} constraints with equality for $\mathbf{y}$, which yields the solution depicted as point α in Figure 2C:

$$\mathbf{y}_{HH}^{weak} = \left\{ \frac{c\left(c\frac{r\sigma^2}{2} + \Delta_1^2\right)}{\Delta_1^3}, \frac{(1-\theta)\, c\left[(1-\theta)\, c\frac{r\sigma^2}{2} + \Delta_2^2\right]}{\Delta_2^3} \right\}.$$

Under **strong complementarity** the solution is the same. However, because the rep firm implements a different sales force commission $\mathbf{b}$, the actual expressions for $\mathbf{y}$ (more specifically for y_1) are different:

$$\mathbf{y}_{HH}^{strong} = \left\{ \frac{c}{\Delta_1} - \frac{c^2 r\sigma^2\left[(1-\theta)^2\,\Delta_1^2 - (3-2\theta)\,\Delta_2^2\right]}{2\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}, \frac{(1-\theta)\, c\left[(1-\theta)\, c\frac{r\sigma^2}{2} + \Delta_2^2\right]}{\Delta_2^3} \right\}.$$

One can verify that these solution satisfy the RPC, unless the minimum utility, m, for the salesperson is very high. If m was indeed very high, then we would have a corner solution, and we would need to solve the problem for the RPC. Since we are interested in interior solutions, we will assume that this is not the case here.

Therefore, the optimal contractual provisions that implement high effort in both tasks are: $\mathbf{b}^{m\_free} = \mathbf{b}_{HH}^*$, $A^{m\_free} = A_{HH}^*$, $\mathbf{y}^{m\_free} = \mathbf{y}_{HH}^*$, and $\mathbf{z}^{m\_free} = \mathbf{z}_{HH}^*$.

Finally, plugging this solution into the manufacturer's profit function (adapted from (3)), and into the rep firm's profit function (adapted from (2)), we obtain the manufacturer's and rep firm's optimal profits.

For the cases of **substitution or weak complementarity**, these expressions are, respectively:

$$\pi_M^{m\_free} = \left[1 - \frac{c\left(c\frac{r\sigma^2}{2} + \Delta_1^2\right)}{\Delta_1^3}\right](q_1 + \Delta_1) + \left[1 - \frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2} + \Delta_2^2\right]}{\Delta_2^3}\right](q_2 + \Delta_2)\,,$$

$$\begin{aligned}\pi_R^{m\_free} &= \frac{c\left(c\frac{r\sigma^2}{2} + \Delta_1^2\right)}{\Delta_1^3}(q_1 + \Delta_1) + \frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2} + \Delta_2^2\right]}{\Delta_2^3}(q_2 + \Delta_2) \\ &\quad - \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right) - (2-\theta)\,c - m.\end{aligned}$$

For the case of **strong complementarity**, these expressions are as follows:

$$\begin{aligned}\pi_M^{m\_free} &= \left[1 - \left(\frac{c}{\Delta_1} - \frac{c^2 r\sigma^2\left[(1-\theta)^2\Delta_1^2 - (3-2\theta)\,\Delta_2^2\right]}{2\Delta_1\Delta_2^2\left(\Delta_1^2 + \Delta_2^2\right)}\right)\right](q_1 + \Delta_1) \\ &\quad + \left[1 - \frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2} + \Delta_2^2\right]}{\Delta_2^3}\right](q_2 + \Delta_2)\,,\end{aligned}$$

$$\begin{aligned}\pi_R^{m\_free} &= \left(\frac{c}{\Delta_1} - \frac{c^2 r\sigma^2\left[(1-\theta)^2\Delta_1^2 - (3-2\theta)\,\Delta_2^2\right]}{2\Delta_1\Delta_2^2\left(\Delta_1^2 + \Delta_2^2\right)}\right)(q_1 + \Delta_1) + \frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2} + \Delta_2^2\right]}{\Delta_2^3}(q_2 + \Delta_2) \\ &\quad - \frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2 c^2}{\Delta_1^2 + \Delta_2^2}\right) - (2-\theta)\,c - m.\end{aligned}$$

■

Interactions and the restricted monopoly model (Case 1.b.)

Both substitution and weak complementarity do not change the manufacturer's spiffing strategy: only the product with lower selling-effort productivity receives spiffs. The resulting effect of complementarity (substitution) is simply a reduction (increase) in the commissions that have to be paid by the manufacturer to the rep firm and by the rep firm to the salesperson. Evidently this increases (decreases) the resulting total profits in the channel.

Strong complementarity, however, does changes the manufacturer's spiffing strategy: in this case, spiffs become unnecessary.

Proposition 2 restated for complementarity and substitution interactions is as follows.

Proposition 2C *For the case of joint manufacturer profit maximization and* ***substitution or weak complementarity****, when neither the manufacturer's nor the rep firm's contracts can accommodate different compensation rates for every individual product,* ***spiffs are used in equilibrium****. Moreover, only the product with* ***lower*** *selling-effort productivity receives spiffs.*

In this case, the optimal rep firm contract to its sales force entails:

$$\begin{aligned} b^{m\_spiff} &= \frac{c}{\Delta_1}, \\ A^{m\_spiff} &= m + 2c - \theta - \left(\frac{c}{\Delta_1}\right)(q_1 + \Delta_1) - \left(\frac{c-\theta}{\Delta_2}\right)(q_2 + \Delta_2) + \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right). \end{aligned}$$

The optimal manufacturer contract to the rep firm and sales force entails:

$$y^{m\_spiff} = \frac{c\left[(1-\theta)\, cr\sigma^2 + \Delta_1\Delta_2 + \Delta_2^2\right]}{\Delta_1\Delta_2\,(\Delta_1+\Delta_2)}, \quad \mathbf{z}^{m\_spiff} = \left\{0\ ,\ \frac{(1-\theta)\, c}{\Delta_2} - \frac{c}{\Delta_1}\right\}.$$

Manufacturer profits in equilibrium are:

$$\pi_M^{m\_spiff} = \left[1 - \frac{c\left[(1-\theta)\, cr\sigma^2 + \Delta_1\Delta_2 + \Delta_2^2\right]}{\Delta_1\Delta_2\,(\Delta_1+\Delta_2)}\right](q_1 + \Delta_1 + q_2 + \Delta_2) - \left(\frac{(1-\theta)\, c}{\Delta_2} - \frac{c}{\Delta_1}\right)(q_2 + \Delta_2).$$

Rep firm profits in equilibrium are:

$$\begin{aligned} \pi_R^{m\_spiff} &= \left[\frac{c\left[(1-\theta)\, cr\sigma^2 + \Delta_1\Delta_2 + \Delta_2^2\right]}{\Delta_1\Delta_2\,(\Delta_1+\Delta_2)}\right](q_1 + \Delta_1 + q_2 + \Delta_2) + \left(\frac{c-\theta}{\Delta_2} - \frac{c}{\Delta_1}\right)(q_2 + \Delta_2) \\ &\quad - \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right) - (2c - \theta) - m. \end{aligned}$$

For the case of joint manufacturer profit maximization and ***strong complementarity****, when neither the manufacturer's nor the rep firm's contracts can accommodate different compensation rates for every individual product, spiffs do not change the outcome for any player, and hence* ***there is no need for spiffs****.*

In this case the optimal rep firm contract to its sales force entails:

$$\begin{aligned} b^{m\_spiff} &= \frac{(2-\theta)\, c}{\Delta_1 + \Delta_2}, \\ A^{m\_spiff} &= m + (2-\theta)\, c - \left(\frac{c}{\Delta_1}\right)(q_1 + \Delta_1) - \left(\frac{c}{\Delta_2}\right)(q_2 + \Delta_2) + \frac{r\sigma^2}{2}\left(\frac{(2-\theta)\, c}{\Delta_1 + \Delta_2}\right)^2. \end{aligned}$$

The optimal manufacturer contract to the rep firm and sales force entails:

$$y^{m\_spiff} = \frac{(2-\theta)\, c}{\Delta_1 + \Delta_2}\left(1 + r\sigma^2 \frac{(2-\theta)\, c}{(\Delta_1 + \Delta_2)^2}\right), \quad \mathbf{z}^{m\_spiff} = \{0\ ,\ 0\}.$$

Manufacturer profits in equilibrium are:

$$\pi_M^{m\_spiff} = \left[1 - \frac{(2-\theta)\, c}{\Delta_1 + \Delta_2}\left(1 + r\sigma^2 \frac{(2-\theta)\, c}{(\Delta_1 + \Delta_2)^2}\right)\right](q_1 + \Delta_1 + q_2 + \Delta_2).$$

Rep firm profits in equilibrium are:

$$\pi_R^{m\_spiff} = \left[\frac{c\,(2-\theta)}{\Delta_1 + \Delta_2}\left(1 + r\sigma^2 \frac{(2-\theta)\, c}{(\Delta_1 + \Delta_2)^2}\right)\right](q_1 + \Delta_1 + q_2 + \Delta_2) - \frac{r\sigma^2}{2}\left(\frac{(2-\theta)\, c}{\Delta_1 + \Delta_2}\right)^2 - (2-\theta)\, c - m.$$

Proof. The manufacturer's goal is to implement high selling effort for both products. Here its problem is the same as in $\left(P^{m\_spiff}\right)$. The only difference is that now the $C[e_1] + C[e_2]$ accommodates the interaction effects. We solve using backwards induction.

The salesperson's utilities for the four effort options $\{(LL), (HL), (LH), (HH)\}$ are respectively:

$\left(U^S_{LL}\right)$: $\quad (b+z_1)(q_1) - \frac{r\sigma^2}{2}(b+z_1)^2 + (b+z_2)(q_2) - \frac{r\sigma^2}{2}(b+z_2)^2 + A$

$\left(U^S_{HL}\right)$: $\quad (b+z_1)(q_1+\Delta_1) - \frac{r\sigma^2}{2}(b+z_1)^2 + (b+z_2)(q_2) - \frac{r\sigma^2}{2}(b+z_2)^2 - c + A$

$\left(U^S_{LH}\right)$: $\quad (b+z_1)(q_1) - \frac{r\sigma^2}{2}(b+z_1)^2 + (b+z_2)(q_2+\Delta_2) - \frac{r\sigma^2}{2}(b+z_2)^2 - c + A$

$\left(U^S_{HH}\right)$: $\quad (b+z_1)(q_1+\Delta_1) - \frac{r\sigma^2}{2}(b+z_1)^2 + (b+z_2)(q_2+\Delta_2) - \frac{r\sigma^2}{2}(b+z_2)^2 - (2-\theta)c + A$

The salesperson's optimal effort choice is driven by the rep firm's choice of the commission b and the manufacturer choice of spiffs incentives $\mathbf{z}$. If the rep firm wants to implement low effort for both products (LL), then the SIC constraint is slack and the rep firm's problem is:

$$\begin{array}{lll}\left(P^{m\_spiff}_{R\_LL}\right): & \max_{A\in\mathbb{R},b\in\mathbb{B}}(y-b)q_1 + (y-b)q_2 - A & \\ & \text{s.t.} \quad (SPC_{LL}): \qquad U^S_{LL} \geq m & ,\end{array}$$

The solution to this problem is simply to give no commission to the salesperson and a salary that is just high enough to guarantee the minimum utility for the salesperson:

$$b^*_{LL} = \max\{-z_1, -z_2\}, \quad A^*_{LL} = m - (b^*_{LL}+z_1)\,q_1 - (b^*_{LL}+z_2)\,q_2 + \frac{r\sigma^2}{2}\left[(b^*_{LL}+z_1)^2 + (b^*_{LL}+z_2)^2\right]. \tag{4C}$$

If the rep firm wants to implement high effort in only one of the tasks (i.e., (HL) or (LH)), then the rep firm faces a standard principal-agent problem:

$$\begin{array}{lll}\left(P^{m\_spiff}_{R\_HL}\right): & \max_{A\in\mathbb{R},b\in\mathbb{B}}(y-b)(q_1+\Delta_1) + (y-b)q_2 - A & \\ & \text{s.t.} \quad (SPC_{HL}): \qquad U^S_{HL} \geq m & \\ & \qquad\quad (SIC_{HL}): \qquad U^S_{HL} \geq U^S_{LL} & ,\end{array}$$

$$\begin{array}{lll}\left(P^{m\_spiff}_{R\_LH}\right): & \max_{A\in\mathbb{R},b\in\mathbb{B}}(y_1-b)q_1 + (y_2-b)(q_2+\Delta_2) - A & \\ & \text{s.t.} \quad (SPC_{LH}): \qquad U^S_{LH} \geq m & \\ & \qquad\quad (SIC_{LH}): \qquad U^S_{LH} \geq U^S_{LL} & ,\end{array}$$

The solutions to the above problems would be respectively:

$$b^*_{HL} = \frac{c}{\Delta_1} - z_1, \quad A^*_{HL} = m + c - \left(\frac{c}{\Delta_1}\right)(q_1+\Delta_1+q_2) + r\sigma^2\left(\frac{c}{\Delta_1}\right)^2, \tag{5C}$$

$$b^*_{LH} = \frac{c}{\Delta_2} - z_2, \quad A^*_{LH} = m + c - \left(\frac{c}{\Delta_2}\right)(q_1+q_2+\Delta_2) + r\sigma^2\left(\frac{c}{\Delta_2}\right)^2. \tag{6C}$$

However, since we only have one control for the commission rate, it is impossible to induce high effort only on the lower-productivity product.[3] In our case in which $\Delta_1 > \Delta_2$, the rep firm cannot implement (LH), because the commission b that makes $U^S_{LH} > U^S_{LL}$ automatically makes $U^S_{HL} > U^S_{LL}$. Hence the rep firm, by its own means, can only implement (LL), (HL), or (HH). This also means that the highest sales force commission rate from (5C) and (6C) already implements high effort in both tasks.

In the cases of **substitution or weak complementarity**, if the manufacturer wants to implement high effort in both tasks (HH), it has to make sure the rep firm receives enough incentives so that the rep firm

[3] Unless $\mathbf{z}$ is such that the perceived rewards on the products are reversed. Endogenously, this will never be the case.

will, in turn, provide the incentives for the salesperson to work hard on both tasks, which would entail a rep firm commission of $b_{HH} = \max\left\{\frac{c}{\Delta_1} - z_1,\ \frac{c-\theta}{\Delta_2} - z_2\right\}$.

In the case of **strong complementarity**, incentives that implement one of the tasks automatically provide enough incentives to implement the other task. Hence, the rep firm can pay a smaller common commission to the salesperson: $b_{HH} = \frac{(2-\theta)c - z_1\Delta_1 - z_2\Delta_2}{\Delta_1+\Delta_2}$.

In order to completely understand the equilibrium of this problem, let us investigate the outcome if the manufacturer **cannot use spiffs** ($\mathbf{z} \equiv \{0,0\}$). In this situation, the equilibrium commission for the cases of **substitution or weak complementarity** would be:

$$b_{HH}^{no} = \frac{c-\theta}{\Delta_2}, \quad A_{HH}^{no} = m + (2-\theta)\,c - \left(\frac{(1-\theta)\,c}{\Delta_2}\right)(q_1+\Delta_1+q_2+\Delta_2) + r\sigma^2\left(\frac{(1-\theta)\,c}{\Delta_2}\right)^2, \tag{7C}$$

since $\frac{(1-\theta)c}{\Delta_2} = \max\left\{\frac{c}{\Delta_1} - 0,\ \frac{(1-\theta)c}{\Delta_2} - 0\right\}$.

And for the case of **strong complementarity** the commission would be:

$$b_{HH}^{no} = \frac{(2-\theta)\,c}{\Delta_1+\Delta_2}, \quad A_{HH}^{no} = m + (2-\theta)\,c - \left(\frac{(2-\theta)\,c}{\Delta_1+\Delta_2}\right)(q_1+\Delta_1+q_2+\Delta_2) + r\sigma^2\left(\frac{(2-\theta)\,c}{\Delta_1+\Delta_2}\right)^2. \tag{8C}$$

By plugging the results from expressions (4C) to (8C) into the rep firm's profit function (adapted from (2)), we obtain the rep firm's profits for each of the possible outcomes:

(π_{R_LL}): $\quad (y+z_1)(q_1) + (y+z_2)(q_2) - m - c - \frac{r\sigma^2}{2}(z_1)^2 - \frac{r\sigma^2}{2}(z_2)^2$

(π_{R_HL}): $\quad (y+z_1)(q_1+\Delta_1) + (y+z_2)(q_2) - m - c - r\sigma^2\left(\frac{c}{\Delta_1}\right)^2$

(π_{R_LH}): "not possible"

$$(\pi_{R\_HH}): \quad \begin{cases} (y+z_1)(q_1+\Delta_1) + (y+z_2)(q_2+\Delta_2) - m - (2-\theta)\,c - r\sigma^2\left(\frac{(1-\theta)c}{\Delta_2}\right)^2 & \text{if} \quad \theta \in \underline{\Theta} \\ (y+z_1)(q_1+\Delta_1) + (y+z_2)(q_2+\Delta_2) - m - (2-\theta)\,c - r\sigma^2\left(\frac{(2-\theta)c}{\Delta_1+\Delta_2}\right)^2 & \text{if} \quad \theta \in \overline{\Theta} \end{cases}$$

Therefore, if the manufacturer wants to implement high selling effort on both products (HH), the manufacturer will face the problem:

$$\left(P_{M\_HH}^{no}\right): \quad \begin{array}{lll} \max_{y,\mathbf{z}\in\mathbb{R}_+} (1-y-z_1)(q_1+\Delta_1) + (1-y-z_2)(q_2+\Delta_2) - A \\ \text{s.t.} \quad (RPC_{HH}): & \pi_{R\_HH} \geq 0 \\ \qquad (RLC_{HHHL}): & \pi_{R\_HH} \geq \pi_{R\_HL} & , \\ \qquad (RGC_{HHLL}): & \pi_{R\_HH} \geq \pi_{R\_LL} \end{array}$$

In the cases of **substitution or weak complementarity**, the rep firm has a natural incentive to implement only the most productive task, and thus RGC is slack and RLC needs to be enforced. Hence, solving RLC with equality we obtain the solution to this problem as $y_{HH}^{no} = \frac{c}{\Delta_2}\left[(1-\theta) + r\sigma^2\left(\frac{(1-\theta)^2 c}{\Delta_2^2} - \frac{c}{\Delta_1^2}\right)\right]$.

In the case of **strong complementarity**, the rep firm has a natural incentive to implement both tasks, and thus RLC is slack and RGC is the only binding constraint. Hence, solving RGC with equality we obtain the solution to this problem as $y_{HH}^{no} = \frac{(2-\theta)c}{\Delta_1+\Delta_2}\left(1 + r\sigma^2\frac{(2-\theta)c}{(\Delta_1+\Delta_2)^2}\right)$.

Therefore, the optimal contractual provisions that implement high effort in both tasks when the manufacturer cannot spiff are: $b^{m\_no} = b_{HH}^{no}$, $A^{m\_no} = A_{HH}^{no}$, $y^{m\_no} = y_{HH}^{no}$, and $\mathbf{z}^{m\_no} = \mathbf{z}_{HH}^{no}$. One can verify that these solutions satisfy the RPC, unless the minimum utility, m, for the salesperson is very high. With these provisions we can compute equilibrium profits for the manufacturer and rep firm.

For the cases of **substitution or weak complementarity**, these solutions are, respectively:

$$\pi_M^{m\_no} = \left\{1 - \frac{c}{\Delta_2}\left[(1-\theta) + r\sigma^2\left(\frac{(1-\theta)^2 c}{\Delta_2^2} - \frac{c}{\Delta_1^2}\right)\right]\right\}(q_1 + \Delta_1 + q_2 + \Delta_2),$$

$$\pi_R^{m\_no} = \frac{c}{\Delta_2}\left[(1-\theta) + r\sigma^2\left(\frac{(1-\theta)^2 c}{\Delta_2^2} - \frac{c}{\Delta_1^2}\right)\right](q_1 + \Delta_1 + q_2 + \Delta_2) - r\sigma^2\left(\frac{(1-\theta)c}{\Delta_2}\right)^2 - (2-\theta)c - m.$$

For the case of **strong complementarity**, these solutions are, respectively:

$$\pi_M^{m\_no} = \left[1 - \frac{c(2-\theta)}{\Delta_1 + \Delta_2}\left(1 + r\sigma^2\frac{(2-\theta)c}{(\Delta_1+\Delta_2)^2}\right)\right](q_1 + \Delta_1 + q_2 + \Delta_2), \tag{9C}$$

$$\pi_R^{m\_no} = \frac{c(2-\theta)}{\Delta_1 + \Delta_2}\left(1 + r\sigma^2\frac{(2-\theta)c}{(\Delta_1+\Delta_2)^2}\right)(q_1 + \Delta_1 + q_2 + \Delta_2) - r\sigma^2\left(\frac{(2-\theta)c}{\Delta_1+\Delta_2}\right)^2 - (2-\theta)c - m. \tag{10C}$$

However, the manufacturer **does have the possibility of using spiffs** ($\mathbf{z} \in \mathbb{R}_+$).

For the cases of **substitution or weak complementarity**, the manufacturer can provide spiffs to the least productive product so as to equate the salesperson's reward for exerting effort in any of the two products. This entails $z_1^{m\_spiff} = 0$, and $z_2^{m\_spiff} = \frac{(1-\theta)c}{\Delta_2} - \frac{c}{\Delta_1}$.

Given this, the rep firm cannot implement high selling effort on only one of the tasks, since the commission that implements the least productive task also implements the most productive task. Therefore, the rep firm contract that makes the salesperson prefer to work hard in both tasks now entails:

$$b_{HH} = \frac{c}{\Delta_1}, \quad A_{HH} = m + (2-\theta)c - \left(\frac{c}{\Delta_1}\right)(q_1 + \Delta_1) - \left(\frac{c}{\Delta_2}\right)(q_2 + \Delta_2) + \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right).$$

The manufacturer wants the salesperson to work hard, so it has to set a commission and spiffs structure so that the rep firm indeed implements this outcome. Now, however, the rep firm cannot make the salesperson work hard in only one of the tasks, since for any commission b it provides to the salesperson, the combination of commission and spiff makes each product equally rewarding. Thus, either the salesperson does not work hard, or she works hard in both tasks.

If the manufacturer wants to implement high selling effort on both products (HH), it will face the modified problem:

$$\begin{array}{lll} \left(P_{M\_HH}^{spiff}\right): & \max_{\mathbf{y},\mathbf{z}\in\mathbb{R}_+}(1-y)(q_1+\Delta_1) + (1 - y - \frac{c-\theta}{\Delta_2} + \frac{c}{\Delta_1})(q_2+\Delta_2) - A & \\ & \text{s.t.} \quad (RPC_{HH}): \qquad \pi_{R\_HH} \geq 0 & \\ & \qquad (RGC_{HHLL}): \quad \pi_{R\_HH} \geq \pi_{R\_LL} & , \end{array}$$

in which only the global constraint RGC will bind. Solving RGC with equality, we obtain the commission rate:

$$y_{HH}^{spiff} = \frac{c\left[(1-\theta)cr\sigma^2 + \Delta_1\Delta_2 + \Delta_2^2\right]}{\Delta_1\Delta_2(\Delta_1+\Delta_2)},$$

which is the rate the manufacturer needs to provide the rep firm so that it will be indifferent between the outcomes of high effort in both tasks or low effort in both tasks.

Therefore, when spiffs are possible, the rep firm optimal contract to the sales force entails:

$$\begin{aligned} b^{m\_spiff} &= \frac{c}{\Delta_1}, \\ A^{m\_spiff} &= m + (2-\theta)\,c - \left(\frac{c}{\Delta_1}\right)(q_1+\Delta_1) - \left(\frac{(1-\theta)\,c}{\Delta_2}\right)(q_2+\Delta_2) + \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right). \end{aligned}$$

The optimal manufacturer contract to the rep firm and sales force entails:

$$y^{m\_spiff} = \frac{c\left[(1-\theta)\,cr\sigma^2 + \Delta_1\Delta_2 + \Delta_2^2\right]}{\Delta_1\Delta_2\,(\Delta_1+\Delta_2)}, \qquad \mathbf{z}^{m\_spiff} = \left\{0,\ \frac{(1-\theta)\,c}{\Delta_2} - \frac{c}{\Delta_1}\right\}.$$

Equilibrium profits for the manufacturer and rep firm are respectively:

$$\pi_M^{m\_spiff} = \left[1 - \frac{c\left[(1-\theta)\,cr\sigma^2 + \Delta_1\Delta_2 + \Delta_2^2\right]}{\Delta_1\Delta_2\,(\Delta_1+\Delta_2)}\right](q_1+\Delta_1+q_2+\Delta_2) - \left(\frac{(1-\theta)\,c}{\Delta_2} - \frac{c}{\Delta_1}\right)(q_2+\Delta_2),$$

$$\begin{aligned} \pi_R^{m\_spiff} &= \frac{c\left[(1-\theta)\,cr\sigma^2 + \Delta_1\Delta_2 + \Delta_2^2\right]}{\Delta_1\Delta_2\,(\Delta_1+\Delta_2)}(q_1+\Delta_1+q_2+\Delta_2) + \left(\frac{(1-\theta)\,c}{\Delta_2} - \frac{c}{\Delta_1}\right)(q_2+\Delta_2) \\ &\quad - \frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2} + \frac{(1-\theta)^2 c^2}{\Delta_2^2}\right) - (2-\theta)\,c - m. \end{aligned}$$

For the case of **strong complementarity**, the manufacturer does not need to provide spiffs, since the complementarity effect more than compensates for the productivity difference between the tasks. Hence, there is no need for spiffs to force the simultaneous implementation of both tasks. Consequently, the salesperson compensations are the same as those in expressions (8C), and the rep firm compensations are the same as those in the situation in which spiffs are absent: $\mathbf{z}^{m\_spiff} = \{0,0\}$, and $y^{m\_spiff} = y^{m\_no}$.

Consequently, the profit expressions for the manufacturer and rep firm are the same as those in expressions (9C) and (10C) above. ■

Interactions and the unrestricted oligopoly model (Case 2.a.)

In this case, both complementarity and substitution do not change the manufacturers' spiffing strategies: spiffs are not profit enhancing and will not be used. However, the interactions do change the manufacturers' commission strategies. Here, interactions cause an instability in the manufacturers' commissions, since in the case of complementarity manufacturers will attempt to appropriate as much of the complementarity savings as possible, while in the case of substitution manufacturers will try to pass the substitution costs to their competitors (the other manufacturer). The result is the possibility of multiple equilibria in commission levels between the manufacturers and the rep firm.

Following the same basic structure in the unconstrained oligopolistic problem $\left(P^{o\_free}\right)$, we construct the oligopolistic problem $\left(P^{o\_spiff}\right)$ by restricting the sales force commission to a single rate:

$$\left(P^{o\_spiff}\right): \max_{y_j,z_j\in\mathbb{R}_+} \left(1-y_j-z_j\right)X_j[e_j], \text{ for } j\in\{1,2\} \quad \text{s.t.}$$
$$RPC: \quad \textstyle\sum_{j=1}^{2}\left(y_j-b\right)X_j[e_j]-A\geq 0$$
$$RIC: \quad A,b\in \underset{A\in\mathbb{R},b\in\mathbb{B}}{\operatorname{argmax}} \textstyle\sum_{j=1}^{2}\left(y_j-b\right)X_j[e_j]-A \quad \text{s.t.}$$
$$SPC: \quad \textstyle\sum_{j=1}^{2}\left[\left(b+z_j\right)X_j[e_j]-\frac{r\sigma^2}{2}\left(b+z_j\right)^2-C\left[e_j\right]\right]+A\geq m$$
$$SIC: \quad \mathbf{e}\in \underset{e_j\in\{L,H\}}{\operatorname{argmax}} \textstyle\sum_{j=1}^{2}\left[\left(b+z_j\right)X_j[e_j]-\frac{r\sigma^2}{2}\left(b+z_j\right)^2-C\left[e_j\right]\right]+A.$$

Up to the rep firm level, the oligopolistic problem $\left(P^{o\_spiff}\right)$ is the same as the monopolistic problem $\left(P^{m\_spiff}\right)$, and yields the same form of results. At the manufacturers' level, however, the restrictions on the rep firm contract cause a strategic interaction that intensifies manufacturer competition.

Proposition 3 restated for complementarity and substitution interactions is as follows.

Proposition 3C *For the case of oligopoly competition, when contracts can accommodate compensation rates for each individual product both at the manufacturer and at the rep firm level, spiffs do not change the outcome for any player, and hence* ***spiffs are not used in equilibrium****. Furthermore, there are multiple equilibria in commissions between the manufacturers and the rep firm.*

For the cases of ***substitution or weak complementarity*** *the optimal rep firm contract to its sales force entails:*

$$\begin{aligned}
\mathbf{b}^{o_free} &= \left\{\frac{c}{\Delta_1}\ ,\ \frac{(1-\theta)\,c}{\Delta_2}\right\},\\
A^{o_free} &= m+(2-\theta)\,c-\left(\frac{c}{\Delta_1}\right)(q_1+\Delta_1)-\left(\frac{(1-\theta)\,c}{\Delta_2}\right)(q_2+\Delta_2)+\frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2}+\frac{(1-\theta)^2c^2}{\Delta_2^2}\right).
\end{aligned}$$

The optimal manufacturers' contracts to the rep firm entail:

$$\begin{aligned}
y_1^{o_free} &= \lambda\frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}+(1-\lambda)\,\frac{c\left[c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3},\\
y_2^{o_free} &= (1-\lambda)\,\frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}+\lambda\frac{c\Delta_2^2\left[(2-\theta)\,cr\sigma^2+\Delta_1^2\right]+c\Delta_1^2\left[(1-\theta)^2\,cr\sigma^2+\Delta_2^2\right]}{2\Delta_1^2\Delta_2^3},\\
\mathbf{z}^{o_free} &= \{0\ ,\ 0\}\,.
\end{aligned}$$

Profits for the manufacturers and rep firm are of the form:

$$\pi_{M1}^{o\_free}=\left[1-\left(\lambda\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}+(1-\lambda)\,\frac{c\left[c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}\right)\right](q_1+\Delta_1)\,,$$

$$\pi_{M2}^{o\_free}=\left[1-\left((1-\lambda)\,\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}+\lambda\frac{c\Delta_2^2\left[(2-\theta)cr\sigma^2+\Delta_1^2\right]+c\Delta_1^2\left[(1-\theta)^2cr\sigma^2+\Delta_2^2\right]}{2\Delta_1^2\Delta_2^3}\right)\right](q_2+\Delta_2)\,,$$

$$\begin{aligned}
\pi_R^{o_free} &= \left(\lambda\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}+(1-\lambda)\,\frac{c\left[c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}\right)(q_1+\Delta_1)\\
&+\left((1-\lambda)\,\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}+\lambda\frac{c\Delta_2^2\left[(2-\theta)cr\sigma^2+\Delta_1^2\right]+c\Delta_1^2\left[(1-\theta)^2cr\sigma^2+\Delta_2^2\right]}{2\Delta_1^2\Delta_2^3}\right)(q_2+\Delta_2)\\
&-\frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2}+\frac{(1-\theta)^2c^2}{\Delta_2^2}\right)-(2-\theta)\,c-m.
\end{aligned}$$

*For the case of **strong complementarity** the optimal rep firm contract to its sales force entails:*

$$
\begin{aligned}
b_1^{m\_free} &= \frac{(2-\theta)\,c\Delta_1}{\Delta_1^2+\Delta_2^2},\ b_2^{m\_free} = \frac{(2-\theta)\,c\Delta_2}{\Delta_1^2+\Delta_2^2},\\
A^{m\_free} &= m+(2-\theta)\,c-\left(\frac{(2-\theta)\,c\Delta_1}{\Delta_1^2+\Delta_2^2}\right)(q_1+\Delta_1)-\left(\frac{(2-\theta)\,c\Delta_2}{\Delta_1^2+\Delta_2^2}\right)(q_2+\Delta_2)+\frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2\,c^2}{\Delta_1^2+\Delta_2^2}\right).
\end{aligned}
$$

The optimal manufacturers' contracts to the rep firm entail:

$$
\begin{aligned}
y_1^{o\_free} &= \lambda\frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}+(1-\lambda)\,\frac{c\Delta_2^2\left[(3-2\theta)\,c\frac{r\sigma^2}{2}+\Delta_2^2\right]-c\Delta_1^2\left[(1-\theta)^2\,c\frac{r\sigma^2}{2}-\Delta_2^2\right]}{\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)},\\
y_2^{o\_free} &= (1-\lambda)\,\frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}+\lambda\frac{c\Delta_1^2\left[(3-2\theta)\,c\frac{r\sigma^2}{2}+\Delta_1^2\right]-c\Delta_2^2\left[(1-\theta)^2\,c\frac{r\sigma^2}{2}-\Delta_1^2\right]}{\Delta_1^2\Delta_2\left(\Delta_1^2+\Delta_2^2\right)},\\
\mathbf{z}^{o\_free} &= \{0\ ,\ 0\}\,.
\end{aligned}
$$

Profits for the manufacturers and rep firm are of the form:

$$
\begin{aligned}
\pi_{M1}^{o\_free} &= \left[1-\left(\lambda\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}+(1-\lambda)\,\frac{c\Delta_2^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]-c\Delta_1^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_2^2\right]}{\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}\right)\right](q_1+\Delta_1)\,,\\
\pi_{M2}^{o\_free} &= \left[1-\left((1-\lambda)\,\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}+\lambda\frac{c\Delta_1^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]-c\Delta_2^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_1^2\right]}{\Delta_1^2\Delta_2\left(\Delta_1^2+\Delta_2^2\right)}\right)\right](q_2+\Delta_2)\,,\\
\pi_{R}^{o\_free} &= \left(\lambda\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}+(1-\lambda)\,\frac{c\Delta_2^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]-c\Delta_1^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_2^2\right]}{\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}\right)(q_1+\Delta_1)\\
&\quad+\left((1-\lambda)\,\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3}+\lambda\frac{c\Delta_1^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]-c\Delta_2^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_1^2\right]}{\Delta_1^2\Delta_2\left(\Delta_1^2+\Delta_2^2\right)}\right)(q_2+\Delta_2)\\
&\quad-\frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2c^2}{\Delta_1^2+\Delta_2^2}\right)-(2-\theta)\,c-m.
\end{aligned}
$$

For both cases, $\lambda\in[0,1]$ determines the manufacturers' "sharing" of savings (extra costs) due to complementarity (substitution) between the products.

Proof. In this scenario, both manufacturers want to implement high selling effort for their products. Hence, each manufacturer needs to offer the right contract and spiffs commission so that its product receives high effort, considering not only the rep firm's and the sales force's optimal behavior, but also the optimal moves of the other manufacturer.

The optimal strategies for the sales force and for the rep firm are mathematically identical to those in the proof of Proposition 1C, so they are not repeated here.

Knowing the optimal behavior of the rep firm and its sales force, we analyze the manufacturers' optimal decisions. Both manufacturers simultaneously solve the problem:

$$
\left(P_{M\_HH}^{o\_free}\right):\quad \max_{y_j,z_j\in\mathbb{R}_+}(1-y_j-z_j)\,(q_j+\Delta_j)\quad \text{for } j\in\{1,2\}
$$

$$
\begin{aligned}
\text{s.t.}\quad &(RPC_{HH}): && \pi_{R\_HH}\geq 0\\
&(RLC_{HHLH}): && \pi_{R\_HH}\geq \pi_{R\_LH}\\
&(RLC_{HHHL}): && \pi_{R\_HH}\geq \pi_{R\_HL}\\
&(RGC_{HHLL}): && \pi_{R\_HH}\geq \pi_{R\_LL}
\end{aligned}\ ,
$$

where the profits for the rep firm come from the proof of Proposition 1C.

Because the terms y_i and z_i always appear together, spiffs are unnecessary; hence, we set $\mathbf{z} = \{0, 0\}$. As before, the solution depends on the magnitude of θ.

For all values of θ there are multiple candidate solutions to this problem. All of them should satisfy RGC and will be bounded by the two RLCs. Because both manufacturers will try, uncooperatively, to accrue the maximum savings from complementarity (or incurr the least extra costs due to substitution), there is no unique equilibrium. In the previous Figure 2C, the line segment that join points α and β contains all the possible equilibria for this problem. Manufacturer 1 will prefer the solution to be as close as possible to point β, while Manufacturer 2 would prefer the solution to be as close as possible to point α.

Under **substitution and weak complementarity**, all solutions are of the form:

$$\begin{aligned} y_{HH1}^{*weak} &= \lambda \frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3} + (1-\lambda)\,\frac{c\left[c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}, \\ y_{HH2}^{*weak} &= (1-\lambda)\,\frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3} + \lambda \frac{c\Delta_2^2\left[(2-\theta)\,cr\sigma^2+\Delta_1^2\right]+c\Delta_1^2\left[(1-\theta)^2\,cr\sigma^2+\Delta_2^2\right]}{2\Delta_1^2\Delta_2^3}. \end{aligned}$$

Under **strong complementarity**, all solutions are of the form:

$$\begin{aligned} y_{HH1}^{*strong} &= \lambda \frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3} + (1-\lambda)\,\frac{c\Delta_2^2\left[(3-2\theta)\,c\frac{r\sigma^2}{2}+\Delta_2^2\right]-c\Delta_1^2\left[(1-\theta)^2\,c\frac{r\sigma^2}{2}-\Delta_2^2\right]}{\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}, \\ y_{HH2}^{*strong} &= (1-\lambda)\,\frac{(1-\theta)\,c\left[(1-\theta)\,c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3} + \lambda \frac{c\Delta_1^2\left[(3-2\theta)\,c\frac{r\sigma^2}{2}+\Delta_1^2\right]-c\Delta_2^2\left[(1-\theta)^2\,c\frac{r\sigma^2}{2}-\Delta_1^2\right]}{\Delta_1^2\Delta_2\left(\Delta_1^2+\Delta_2^2\right)}. \end{aligned}$$

For both cases, $\lambda \in [0, 1]$ is an exogenous parameter that determines the "sharing" of savings (costs) due to complementarity (substitution) between the manufacturers. When λ is close to zero, Manufacturer 1 accrues most of the savings (costs). Conversely, when λ is close to one, Manufacturer 2 does.

Once again one can verify that these solutions satisfy the RPC, unless the minimum utility, m, for the salesperson is very high.

The manufacturers' and the rep firm's profits for both weak and strong complementarity can be found by plugging the contractual terms into their respective profit functions.

For the cases of **substitution or weak complementarity**, profits for the manufacturers and rep firm are of the form:

$$\begin{aligned} \pi_{M1}^{o\_free} &= \left[1-\left(\lambda\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3} + (1-\lambda)\,\frac{c\left[c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}\right)\right](q_1+\Delta_1), \\ \pi_{M2}^{o\_free} &= \left[1-\left((1-\lambda)\,\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3} + \lambda\frac{c\Delta_2^2\left[(2-\theta)cr\sigma^2+\Delta_1^2\right]+c\Delta_1^2\left[(1-\theta)^2cr\sigma^2+\Delta_2^2\right]}{2\Delta_1^2\Delta_2^3}\right)\right](q_2+\Delta_2), \\ \pi_{R}^{o\_free} &= \left(\lambda\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3} + (1-\lambda)\,\frac{c\left[c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3}\right)(q_1+\Delta_1) \\ &\quad+\left((1-\lambda)\,\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3} + \lambda\frac{c\Delta_2^2\left[(2-\theta)cr\sigma^2+\Delta_1^2\right]+c\Delta_1^2\left[(1-\theta)^2cr\sigma^2+\Delta_2^2\right]}{2\Delta_1^2\Delta_2^3}\right)(q_2+\Delta_2) \\ &\quad-\frac{r\sigma^2}{2}\left(\frac{c^2}{\Delta_1^2}+\frac{(1-\theta)^2c^2}{\Delta_2^2}\right)-(2-\theta)\,c-m. \end{aligned}$$

For the case of **strong complementarity**, profits for the manufacturers and rep firm are of the form:

$$\begin{aligned} \pi_{M1}^{o\_free} &= \left[1-\left(\lambda\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3} + (1-\lambda)\,\frac{c\Delta_2^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]-c\Delta_1^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_2^2\right]}{\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}\right)\right](q_1+\Delta_1), \\ \pi_{M2}^{o\_free} &= \left[1-\left((1-\lambda)\,\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3} + \lambda\frac{c\Delta_1^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]-c\Delta_2^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_1^2\right]}{\Delta_1^2\Delta_2\left(\Delta_1^2+\Delta_2^2\right)}\right)\right](q_2+\Delta_2), \end{aligned}$$

$$\pi_R^{o\_free} = \left(\lambda \frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]}{\Delta_1^3} + (1-\lambda)\frac{c\Delta_2^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]-c\Delta_1^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_2^2\right]}{\Delta_1\Delta_2^2\left(\Delta_1^2+\Delta_2^2\right)}\right)(q_1+\Delta_1)$$
$$+\left((1-\lambda)\frac{(1-\theta)c\left[(1-\theta)c\frac{r\sigma^2}{2}+\Delta_2^2\right]}{\Delta_2^3} + \lambda \frac{c\Delta_1^2\left[(3-2\theta)c\frac{r\sigma^2}{2}+\Delta_1^2\right]-c\Delta_2^2\left[(1-\theta)^2c\frac{r\sigma^2}{2}-\Delta_1^2\right]}{\Delta_1^2\Delta_2\left(\Delta_1^2+\Delta_2^2\right)}\right)(q_2+\Delta_2)$$
$$-\frac{r\sigma^2}{2}\left(\frac{(2-\theta)^2c^2}{\Delta_1^2+\Delta_2^2}\right) - (2-\theta)\,c - m. \quad \blacksquare$$

Interactions and the restricted oligopoly model (Case 2.b.)

In this case, both complementarity and substitution do not change the qualitative insights regarding spiffing strategies: only the product with higher selling-effort productivity receives spiffs. The resulting effect of complementarity (substitution) is simply a reduction (increase) in the compensation terms (commission and spiffs). Evidently this increases (decreases) the resulting total profits in the channel.

Proposition 4 restated for complementarity and substitution interactions is as follows.

Proposition 4C *For the case of oligopoly competition, when each competing manufacturer can offer an individualized commission rate for its product to the rep firm, but the rep firm is constrained to offer a common commission rate on both products to its sales force,* ***spiffs are used in equilibrium****. Moreover, only the product with* ***higher*** *selling-effort productivity receives spiffs.*

For the cases of ***substitution or weak complementarity*** *the optimal rep firm contract to its sales force entails:*

$$b^{o\_spiff} = \frac{(1-\theta)\,c}{\Delta_2},$$
$$A^{o\_spiff} = m + 2c - \theta - \left(\frac{c}{\Delta_2} + z_1^{o\_spiff}\right)(q_1+\Delta_1) - \left(\frac{(1-\theta)\,c}{\Delta_2}\right)(q_2+\Delta_2)$$
$$+\frac{r\sigma^2}{2}\left[\left(\frac{c-\theta}{\Delta_2} + z_1^{o\_spiff}\right)^2 + \frac{(1-\theta)^2c^2}{\Delta_2^2}\right].$$

The optimal manufacturers' contracts to the rep firm and sales force entail:

$$y_1^{o\_spiff} = 0, \quad y_2^{o\_spiff} = \frac{c\left\{\Delta_1\left[(1-\theta)\,\Delta_1 + \Delta_2\right] + (1-\theta)^2 cr\sigma^2\right\}}{\Delta_1\left[\Delta_2\left(\Delta_1+\Delta_2\right) - (1-\theta)\,cr\sigma^2\right]},$$
$$z_1^{o\_spiff} = \frac{c\left\{\Delta_2^2\left[\Delta_1 + (1-\theta)\,\Delta_2\right] + (1-\theta)^2 cr\sigma^2\Delta_1\right\}}{\Delta_1\Delta_2\left[\Delta_2\left(\Delta_1+\Delta_2\right) - (1-\theta)\,cr\sigma^2\right]}, \quad z_2^{o\_spiff} = 0.$$

The manufacturers' profits in equilibrium are:

$$\pi_{M1}^{o\_spiff} = \left(1 - z_1^{o\_spiff}\right)(q_1+\Delta_1), \quad \pi_{M2}^{o\_spiff} = \left(1 - y_2^{o\_spiff}\right)(q_2+\Delta_2).$$

Rep firm profits in equilibrium are

$$\pi_R^{o\_spiff} = (q_1+\Delta_1)\,z_1^{o\_spiff} + (q_2+\Delta_2)\,y_2^{o\_spiff} - \frac{r\sigma^2}{2}\left[\left(\frac{(1-\theta)\,c}{\Delta_2} + z_1^{o\_spiff}\right)^2 + \frac{(1-\theta)^2c^2}{\Delta_2^2}\right] - (2-\theta)\,c - m.$$

*For the case of **strong complementarity** the optimal rep firm contract to its sales force entails:*

$$
\begin{aligned}
b^{o\_spiff} &= \frac{(2-\theta)\,c - z_1^{o\_spiff}\Delta_1}{\Delta_1+\Delta_2}, \\
A^{o\_spiff} &= m + (2-\theta)\,c - \left(\frac{(2-\theta)\,c + z_1^{o\_spiff}\Delta_2}{\Delta_1+\Delta_2}\right)(q_1+\Delta_1) - \left(\frac{(2-\theta)\,c - z_1^{o\_spiff}\Delta_1}{\Delta_1+\Delta_2}\right)(q_2+\Delta_2) \\
&\quad + \frac{r\sigma^2}{2}\left[\left(\frac{(2-\theta)\,c + z_1^{o\_spiff}\Delta_2}{\Delta_1+\Delta_2}\right)^2 + \left(\frac{(2-\theta)\,c - z_1^{o\_spiff}\Delta_1}{\Delta_1+\Delta_2}\right)^2\right].
\end{aligned}
$$

The optimal manufacturers' contracts to the rep firm and sales force entail:

$$
y_1^{o\_spiff} = 0, \quad y_2^{o\_spiff} = \frac{(\Delta_1+\Delta_2)\sqrt{[(2-\theta)cr\sigma^2]^2+(\Delta_1+\Delta_2)\left[2\Delta_1(2-\theta)cr\sigma^2+(\Delta_1+\Delta_2)^3\right]}-(\Delta_1+\Delta_2)^3}{2r\sigma^2\Delta_1\Delta_2}
$$

$$
z_1^{o\_spiff} = \frac{(\Delta_1+\Delta_2)\sqrt{[(2-\theta)cr\sigma^2]^2+(\Delta_1+\Delta_2)\left[2\Delta_1(2-\theta)cr\sigma^2+(\Delta_1+\Delta_2)^3\right]}-(\Delta_1+\Delta_2)^3-(2-\theta)cr\sigma^2(\Delta_1-\Delta_2)}{2\Delta_1\Delta_2 r\sigma^2}, \quad z_2^{o\_spiff} = 0.
$$

The manufacturers' profits in equilibrium are:

$$
\pi_{M1}^{o\_spiff} = \left(1 - z_1^{o\_spiff}\right)(q_1+\Delta_1), \quad \pi_{M2}^{o\_spiff} = \left(1 - y_2^{o\_spiff}\right)(q_2+\Delta_2).
$$

Rep firm profits in equilibrium are

$$
\begin{aligned}
\pi_R^{o\_spiff} &= (q_1+\Delta_1)\,z_1^{o\_spiff} + (q_2+\Delta_2)\,y_2^{o\_spiff} \\
&\quad - \frac{r\sigma^2}{2}\left[\left(\frac{(2-\theta)\,c + z_1^{o\_spiff}\Delta_2}{\Delta_1+\Delta_2}\right)^2 + \left(\frac{(2-\theta)\,c - z_1^{o\_spiff}\Delta_1}{\Delta_1+\Delta_2}\right)^2\right] - (2-\theta)\,c - m.
\end{aligned}
$$

Proof. Both manufacturers want to implement high selling effort for their products. The optimal strategies for the sales force and for the rep firm are mathematically identical to those in the proof of Proposition 2C, so they are not repeated here.

Knowing the optimal behavior of the rep firm and its sales force, the manufacturers simultaneously solve the problem:

$$
\left(P_{HH}^{o\_spiff}\right): \quad
\begin{array}{lll}
\max_{y_j,z_j\in\mathbb{R}_+}(1-y_j-z_j)(q_j+\Delta_j) & \text{for } j\in\{1,2\} & \\
\text{s.t.} \quad (RPC_{HH}): & \pi_{R\_HH} \geq 0 & \\
\quad (RLC_{HHLH}): & \pi_{R\_HH} \geq \pi_{R\_LH} & \\
\quad (RLC_{HHHL}): & \pi_{R\_HH} \geq \pi_{R\_HL} & , \\
\quad (RGC_{HHLL}): & \pi_{R\_HH} \geq \pi_{R\_LL}, &
\end{array}
$$

where the profits for the rep firm come from the proof of Proposition 2C. The solution to this problem is more complicated than simply solving for the constraints.

Recall that manufacturer 1 has the product with higher selling-effort productivity. Let us label this manufacturer m1 and the other manufacturer m2. If manufacturers could not offer spiffs, m1 would be inclined to adopt a very small commission ($y_1 \to 0$) yet still could enjoy high effort, since it could free ride on the rep firm's rigid compensation contract. In this case m2 would suffer the burden of paying a commission high enough to support high effort for both m1's and m2's products. Therefore, for both the cases of $\theta \in \underline{\Theta}$ and $\theta \in \overline{\Theta}$, m2 has only two options: implementing low effort on both products, or high effort on both products.

Let us investigate the outcome if the manufacturers **cannot use spiffs** ($\mathbf{z} \equiv \{0,0\}$).

In this situation, under **substitution or weak complementarity** the equilibrium compensation to the salesperson would be

$$b_{HH}^{no} = \frac{(1-\theta)\,c}{\Delta_2}, \quad A_{HH}^{no} = m + (2-\theta)\,c - \left(\frac{(1-\theta)\,c}{\Delta_2}\right)(q_1+\Delta_1+q_2+\Delta_2) + r\sigma^2\left(\frac{(1-\theta)\,c}{\Delta_2}\right)^2,$$

while under **strong complementarity**, equilibrium compensation to the salesperson would be:

$$b_{HH}^{no} = \frac{(2-\theta)\,c}{\Delta_1+\Delta_2}, \quad A_{HH}^{no} = m + (2-\theta)\,c - \left(\frac{(2-\theta)\,c}{\Delta_1+\Delta_2}\right)(q_1+\Delta_1+q_2+\Delta_2) + r\sigma^2\left(\frac{(2-\theta)\,c}{\Delta_1+\Delta_2}\right)^2.$$

In both cases, by plugging $y_1 = 0$, $\mathbf{z} = \{0, 0\}$, and the results from the rep firm's solutions in the proof of Proposition 2C (expressions (4C) to (7C)) into the rep firm's profit function (adapted from (2)), we obtain the rep firm's profits for each of the possible outcomes:

(π_{R_LL}): $(y_2)(q_2) - m$

(π_{R_HL}): $(y_2)(q_2) - m - c - r\sigma^2\left(\frac{c}{\Delta_1}\right)^2$

(π_{R_LH}): "not possible"

$$(\pi_{R\_HH}): \begin{cases} (y_2+z_2)(q_2+\Delta_2) - m - (2-\theta)\,c - r\sigma^2\left(\frac{(1-\theta)c}{\Delta_2}\right)^2 & \text{if} \quad \theta \in \underline{\Theta} \\ (y_2+z_2)(q_2+\Delta_2) - m - (2-\theta)\,c - r\sigma^2\left(\frac{(2-\theta)c}{\Delta_1+\Delta_2}\right)^2 & \text{if} \quad \theta \in \overline{\Theta} \end{cases}.$$

If m2 wants high effort exerted on its product, this automatically implies high effort on product 1 as well; m2 therefore solves the problem:

$$\left(P_{M2\_HH}^{no}\right): \quad \max_{y_2\in\mathbb{R}_+}(1-y_2)(q_2+\Delta_2)$$
$$\text{s.t.} \quad (RPC_{HH}): \quad \pi_{R\_HH} \geq 0$$
$$(RGC_{HHLL}): \quad \pi_{R\_HH} \geq \pi_{R\_LL}.$$

For both weak and strong complementarity, the solution to this problem is simply to solve RGC with equality for y_2. Under **substitution or weak complementarity** m2's optimal commission payment to the rep firm is $y_{HH2}^{o\_no} = \frac{c\left[(1-\theta)^2 cr\sigma^2 + (2-\theta)\Delta_2^2\right]}{\Delta_2^3}$. This leads to optimal manufacturer profits, rep firm profits, and total channel profits of:

$$\begin{aligned}
\pi_{M1}^{o_no} &= q_1 + \Delta_1, \\
\pi_{M2}^{o_no} &= \left[1 - \frac{c\left[(1-\theta)^2 cr\sigma^2 + (2-\theta)\,\Delta_2^2\right]}{\Delta_2^3}\right](q_2+\Delta_2), \\
\pi_{R}^{o_no} &= \frac{c\left[(1-\theta)^2 cr\sigma^2 + (2-\theta)\,\Delta_2^2\right]}{\Delta_2^3}(q_2+\Delta_2) - r\sigma^2\left(\frac{(1-\theta)\,c}{\Delta_2}\right)^2 - (2-\theta)\,c - m, \\
\pi_{syst}^{o_no} &= (q_1+\Delta_1+q_2+\Delta_2) - r\sigma^2\left(\frac{(1-\theta)\,c}{\Delta_2}\right)^2 - (2-\theta)\,c - m.
\end{aligned}$$

Under **strong complementarity**, m2's optimal commission payment to the rep firm is
$y_{HH}^{o\_no} = \frac{(2-\theta)c}{\Delta_2}\left(1 + \frac{(2-\theta)cr\sigma^2}{(\Delta_1+\Delta_2)^2}\right)$. This leads to optimal manufacturer profits, rep firm profits, and total channel

profits of:

$$\begin{aligned}
\pi_{M1}^{o_no} &= q_1 + \Delta_1, \\
\pi_{M2}^{o_no} &= \left[1 - \frac{(2-\theta)\,c}{\Delta_2}\left(1 + \frac{(2-\theta)\,cr\sigma^2}{(\Delta_1+\Delta_2)^2}\right)\right](q_2+\Delta_2), \\
\pi_{R}^{o_no} &= \frac{(2-\theta)\,c}{\Delta_2}\left(1 + \frac{(2-\theta)\,cr\sigma^2}{(\Delta_1+\Delta_2)^2}\right)(q_2+\Delta_2) - r\sigma^2\left(\frac{(2-\theta)\,c}{\Delta_1+\Delta_2}\right)^2 - (2-\theta)\,c - m, \\
\pi_{syst}^{o_no} &= (q_1+\Delta_1+q_2+\Delta_2) - r\sigma^2\left(\frac{(2-\theta)\,c}{\Delta_1+\Delta_2}\right)^2 - (2-\theta)\,c - m.
\end{aligned}$$

However, since **spiffs are indeed possible** ($\mathbf{z} \in \mathbb{R}_+$), m2 can try to provide enough spiffs to the salesperson so that exerting high selling effort on product 2 would become more profitable than exerting high selling effort on product 1, and thus assume the privileged position of being the free-rider manufacturer. Firm m1 would consequently react by also offering spiffs.

To find the equilibrium we need to define the indifference point such that m2 does not find it profitable to engage in this spiffing war. This condition is for m1 to provide enough spiffs such that m2 would be indifferent between also providing spiffs and not providing any spiffs.

Under **substitution or weak complementarity**, when m2 does not provide spiffs, the maximum profit it can earn is obtained by solving the Local Constraint RLC_{HHLL} with equality for y_2, obtaining:

$$y_2^{accommodate} = \frac{(1-\theta)\,c^2r\sigma^2 + (2-\theta)\,c\Delta_2^2 + z_1\,(1-\theta)\,cr\sigma^2\Delta_2 - z_1\Delta_1\Delta_2^2}{\Delta_2^3}.$$

Hence, when m2 uses spiffs, its profit as a function of spiffs z_1 is given by:

$$\pi_2^{accommodate} = \left[1 - \frac{(1-\theta)\,c^2r\sigma^2 + (2-\theta)\,c\Delta_2^2 + z_1\,(1-\theta)\,cr\sigma^2\Delta_2 - z_1\Delta_1\Delta_2^2}{\Delta_2^3}\right](q_2+\Delta_2). \tag{11C}$$

On the other hand, to surpass m1 in productivity of selling effort, m2 has to offer a large enough amount of spiffs so that: $z_2 > \frac{z_1\Delta_1}{\Delta_2} + b\frac{\Delta_1-\Delta_2}{\Delta_2}$. In this case, the rep firm would pay the salesperson a commission of $b = \frac{(1-\theta)c}{\Delta 1} - z_1$, which would be enough to provide incentives to implement high selling effort on m1's product (since high selling effort on m2's would occur naturally).

Therefore, the maximum profit outcome m2 could obtain through a "surpassing" strategy occurs when $y_2 = 0$ and $z_2 = z_1 + (1-\theta)\,c\left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1}\right)$. Its profits are then given by:

$$\pi_2^{surpass} = \left[1 - z_1 - (1-\theta)\,c\left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1}\right)\right](q_2+\Delta_2). \tag{12C}$$

The indifference point occurs when expressions (11C) and (12C) are equated. Since these expressions are independent of y_1, we conclude that it is optimal to set $y_1 = 0$ and adjust the expressions only with z_1. Solving $\pi_2^{accommodate} = \pi_2^{surpass}$ for z_1 we obtain:

$$z_1^{o\_spiffs} = \frac{c\left\{\Delta_2^2\left[\Delta_1 + (1-\theta)\,\Delta_2\right] + (1-\theta)^2\,cr\sigma^2\Delta_1\right\}}{\Delta_1\Delta_2\left[\Delta_2\,(\Delta_1+\Delta_2) - (1-\theta)\,cr\sigma^2\right]}.$$

By plugging this result into m2's manufacturer commission $y_2^{accommodate}$, we obtain:

$$y_2^{o\_spiffs} = \frac{c\left\{\Delta_1\left[(1-\theta)\Delta_1+\Delta_2\right]+(1-\theta)^2 cr\sigma^2\right\}}{\Delta_1\left[\Delta_2\left(\Delta_1+\Delta_2\right)-(1-\theta)cr\sigma^2\right]}.$$

Once again one can verify that this solution satisfies the RPC, unless the minimum utility, m, for the salesperson is very high. Plugging this solution into the manufacturers' profit functions (adapted from (??)), we obtain the manufacturers' optimal profits:

$$\pi_{M1}^{o\_spiff} = \left[1 - z_1^{o\_spiffs}\right](q_1+\Delta_1), \quad \pi_{M2}^{o\_spiff} = \left[1 - y_2^{o\_spiffs}\right](q_2+\Delta_2).$$

The rep firm' equilibrium profits can then be expressed as an implicit function of the optimal manufacturer contractual provisions:

$$\pi_R^{o\_spiff} = (q_1+\Delta_1)\, z_1^{o\_spiff} + (q_2+\Delta_2)\, y_2^{o\_spiff} - \frac{r\sigma^2}{2}\left[\left(\frac{(1-\theta)c}{\Delta_2} + z_1^{o\_spiff}\right)^2 + \left(\frac{(1-\theta)c}{\Delta_2}\right)^2\right] - 2c - m.$$

Under **strong complementarity**, when m2 does not provide spiffs, the maximum profit it can earn is obtained by solving the Local Constraint RGC_{HHLL} with equality for y_2, obtaining:

$$y_2^{accommodate} = \frac{(2-\theta)c - z_1\Delta_1}{\Delta_2}\left(1 + r\sigma^2\frac{(2-\theta)c + z_1\Delta_2}{(\Delta_1+\Delta_2)^2}\right).$$

Hence, when m2 uses spiffs, its profit as a function of spiffs z_1 is given by:

$$\pi_2^{accommodate} = \left[1 - \frac{(2-\theta)c - z_1\Delta_1}{\Delta_2}\left(1 + r\sigma^2\frac{(2-\theta)c + z_1\Delta_2}{(\Delta_1+\Delta_2)^2}\right)\right](q_2+\Delta_2). \tag{13C}$$

On the other hand, to surpass m1 in productivity of selling effort, m2 has to offer a large enough amount of spiffs so that: $z_2 > \frac{z_1\Delta_1}{\Delta_2} + b\frac{\Delta_1-\Delta_2}{\Delta_2}$. In this case, the rep firm would pay the salesperson a commission of $b = \frac{(2-\theta)c - z_1\Delta_1 - z_2\Delta_2}{\Delta_1+\Delta_2}$, which would be enough to provide incentives to implement high selling effort on m1's product (since high selling effort on m2's would occur naturally).

Therefore, the maximum profit outcome m2 could obtain through a "surpassing" strategy occurs when $y_2 = 0$ and $z_2 = z_1 + \frac{(2-\theta)c}{2}\left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1}\right)$. Its profits are then given by:

$$\pi_2^{surpass} = \left[1 - z_1 - \frac{(2-\theta)c}{2}\left(\frac{1}{\Delta_2} - \frac{1}{\Delta_1}\right)\right](q_2+\Delta_2). \tag{14C}$$

The indifference point occurs when expressions (13C) and (14C) are equated. Since these expressions are independent of y_1, we conclude that it is optimal to set $y_1 = 0$ and adjust the expressions only with z_1. Solving $\pi_2^{accommodate} = \pi_2^{surpass}$ for z_1 we obtain:

$$z_1^{o\_spiffs} = \frac{(\Delta_1+\Delta_2)\sqrt{[(2-\theta)cr\sigma^2]^2 + (\Delta_1+\Delta_2)\left[2\Delta_1(2-\theta)cr\sigma^2 + (\Delta_1+\Delta_2)^3\right]} - (\Delta_1+\Delta_2)^3 - (2-\theta)cr\sigma^2(\Delta_1-\Delta_2)}{2\Delta_1\Delta_2 r\sigma^2}.$$

By plugging this result into m2's manufacturer commission $y_2^{accommodate}$, we obtain:

$$y_2^{o\_spiffs} = \frac{(\Delta_1+\Delta_2)\sqrt{[(2-\theta)cr\sigma^2]^2 + (\Delta_1+\Delta_2)\left[2\Delta_1(2-\theta)cr\sigma^2 + (\Delta_1+\Delta_2)^3\right]} - (\Delta_1+\Delta_2)^3}{2r\sigma^2\Delta_1\Delta_2}.$$

Once again one can verify that this solution satisfies the RPC, unless the minimum utility, m, for the salesperson is very high. Plugging this solution into the manufacturers' profit functions (adapted from (**??**)), we obtain the manufacturers' optimal profits:

$$\pi_{M1}^{o\_spiff} = \left[1 - z_1^{o\_spiffs}\right](q_1 + \Delta_1), \quad \pi_{M2}^{o\_spiff} = \left[1 - y_2^{o\_spiffs}\right](q_2 + \Delta_2).$$

The rep firm' equilibrium profits can then be expressed as an implicit function of the optimal manufacturer contractual provisions:

$$\begin{aligned} \pi_R^{o\_spiff} &= (q_1 + \Delta_1) z_1^{o\_spiff} + (q_2 + \Delta_2) y_2^{o\_spiff} \\ &\quad - \frac{r\sigma^2}{2}\left[\left(\frac{(2-\theta)c + z_1^{o\_spiff}\Delta_2}{\Delta_1 + \Delta_2}\right)^2 + \left(\frac{(2-\theta)c - z_1^{o\_spiff}\Delta_1}{\Delta_1 + \Delta_2}\right)^2\right] - (2-\theta)c - m. \end{aligned}$$

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