Cross-market Network Effect with Asymmetric Customer Loyalty: Implications for Competitive Advantage

Technical Appendix (to be made available online)

T1: Relaxation of assumptions in the basic model

In this technical appendix, we show the robustness of our main findings. We relax several simplified assumptions made in our basic model by allowing (a) a positive reservation price for switchers, i.e., $k \geq s > 0$; (b) both firms to have loyal customers, i.e. $\theta_H > \theta_L > 0$; (c) a less restrictive lower bound of firms' prices (p_0 can be negative); and (d) three different competitive structures in the primary product market (i.e., a simultaneous-move game, a *H*-led sequential-move game, and a *L*-led sequential-move game). We will prove the following remark.

Remark 1: *The general results given in Proposition 1 still hold. Mathematically, for any* $0 < \theta_L < \theta_H$ *, there exists* $0 < \underline{h} < h < \overline{h}$ *so that* $\pi_H < \pi_L$ *for* $\theta_L < \underline{\theta} < \theta_H < \overline{\theta}$ *regardless of the sequence of price setting of the game.*

Proof:

Before we prove the remark by examining the simultaneous-move game, the *H*-led sequential-move game, and the *L*-led sequential-move game respectively, we first define two key variables that will be used in the proof.

Define p_{H0} as the lowest price firm *H* might set in the primary product market if p_0 can be ignored. p_{H0} satisfies the condition

(T1) $(\theta_H+\theta_s)(p_{H0}+h-c)=\theta_H(k+h-c)$.

The left hand side of equation (T1) is firm H's profit from selling to both its loyal consumers and switchers at price p_{H0}. The right hand side of equation (T1) is firm H's profit from selling to its loyal consumers only by charging them the reservation price, k. From (T1), we can solve for $p_{H0} = \dfrac{\theta_H(k+h-c)}{\theta_H + \theta s} - (h-c)$.

Similarly, define p_{L0} as the lowest price firm L might set in the primary product market if p_0 can be ignored. p_{L0} satisfies the condition

(T2) $(\theta_L+\theta_s)(p_{L0}+h-c) = \theta_L(s+h-c)$.

From (T2), we can solve for $p_{L0} = \dfrac{\theta_L(s+h-c)}{\theta_L+\theta s} - (h-c)$. It is easy to see that $p_{H0} > p_{L0}$.

<u>When two firms set prices simultaneously</u>

In this game, we will show below that $\pi_H < \pi_L$ can occur under

(T3) $p_{H0} < p_0$,

(T4) $\pi_H(p_{HP}=p_0, p_{LP}=p_0) < \pi_H(p_{HP}=k, p_{LP}=p_0)$ and

(T5) $\pi_L(p_{HP}=p_0, p_{LP}=p_0) > \pi_L(p_{HP}=p_0, p_{LP}=s)$

and will give the precise conditions on when $\pi_H < \pi_L$ occurs.

Under (T3)-(T5), there is no pure strategy equilibrium, because each firm would like to undercut the rival if $p_{iP} > p_0$ as $p_0 > p_{H0} > p_{L0}$, and at least one firm would like to deviate to $p_{HP}=k$ or $p_{LP}=s$ when $p_{HP}=p_{LP}=p_0$. Denote $H_i(p)=Pr(p_{iP} \geq p)$. Following a similar procedure of deriving mixed strategy equilibrium as outlined in Narasimhan (1988), we obtain that firm H's price support is $\{k\} \cup (p_0,s)$ and firm L's price support is $[p_0,s]$. Therefore, we have

(T6) $\pi_H = \theta_H(k+h-c)$ $(p_{HP}=k)$

(T7) $\pi_H = (\theta_H + \theta_s H_L(p_{HP}))(p_{HP}+h-c)$ $(p_0 < p_{HP} < s)$

(T8) $\pi_L = (\theta_L + \theta_s H_H(p_{LP}))(p_{LP}+h-c)$ $(p_0 \leq p_{LP} \leq s)$

in the mixed strategy equilibrium. Solving π_H and π_L from (T6) to (T8) using the boundary condition $H_H(p_0)=1$, we obtain that $\pi_H = \theta_H(k+h-c)$ and $\pi_L = (\theta_L+\theta_s)(p_0+h-c)$. The equilibrium pricing strategies are

$H_H(p_{HP}) = \dfrac{\theta_L+\theta_s}{\theta_s}\dfrac{p_0+h-c}{p_{HP}+h-c} - \dfrac{\theta_L}{\theta_s}$ for $p_0 < p_{HP} < s$, $H_H(k) = \dfrac{\theta_L+\theta_s}{\theta_s}\dfrac{p_0+h-c}{s+h-c} - \dfrac{\theta_L}{\theta_s}$, and

$H_L(p_{LP}) = \dfrac{\theta_H}{\theta_s}\dfrac{k-p_{LP}}{p_{LP}+h-c}$ for $p_0 \leq p_{LP} \leq s$.

Note that an important difference in the equilibrium results here from those in Narasimhan (1988) is that there is a mass point at p_0 for firm L.

Therefore, $\pi_H<\pi_L$ occurs in this case if and only if

(T9) $\theta_H(k+h-c)<(\theta_L+\theta_s)(p_0+h-c)$

and (T3)-(T5) hold. Those conditions can be summarized into

(T10) $\theta_H(k+h-c)/(\theta_L+\theta_s)<(p_0+h-c)<\theta_H(k+h-c)/(\theta_H+0.5\theta_s)$ and
(T11) $(p_0+h-c)>\theta_L(k+h-c)/(\theta_L+0.5\theta_s)$.

Letting $\beta=(p_0+h-c)/(k+h-c)$ and using the fact that $\theta_s=1-\theta_H-\theta_L$, (T10)-(T11) can be rewritten as

(T12) $\beta(1-\theta_L)/(2-\beta)<\theta_H<min[\beta/(1+\beta),1-\theta_L(2-\beta)/\beta]$

<u>When firm H sets price first</u>

In this game, we will show below that $\pi_H<\pi_L$ can occur again under the conditions (T3)-(T5) and will give the precise conditions on when $\pi_H<\pi_L$ occurs.

Given (T3), if firm H sets its price first, firm L always undercuts firm H's price if $p_{HP}>p_0$. Therefore, firm H will not set p_{HP} between (k, p_0) given (T3). If $p_{HP}=p_0$, firm L will set $p_{LP}=p_0$ given (T5). Hence, given (T4), firm H will set $p_{HP}=k$ in equilibrium. Consequently, the equilibrium is of pure strategy with $p_{HP}=k, p_2=s,$ $\pi_H=\theta_H(k+h-c)$ and $\pi_L=(\theta_L+\theta_s)(s+h-c)$.

Therefore, $\pi_H<\pi_L$ occurs in this case if and only if

(T13) $\theta_H(k+h-c)<(\theta_L+\theta_s)(s+h-c)$

and (T3)-(T5) hold. Because (T9) implies (T13), $\pi_H<\pi_L$ occurs in this case if (T9) and (T3)-(T5) hold, i.e. when (T12) holds, as (T12) summaries (T9) and (T3)-(T5).

When firm L sets price first

In this game, we will show below that $\pi_H<\pi_L$ can occur under the conditions (T3)-(T4) and will give the precise conditions on when $\pi_H<\pi_L$ occurs.

Given (T3) and the fact $p_{H0} > p_{L0}$, if firm L sets its price first, firm H always undercuts firm L's price if $p_{LP}>p_0$. Therefore, firm L will not set p_{LP} between $[s, p_0)$ given (T3). If $p_{LP}=p_0$, firm H will set $p_{HP}=k$ given (T4). Hence, the equilibrium is of pure strategy with $p_{HP}=k$, $p_2=p_0$, $\pi_H=\theta_H(k+h-c)$ and $\pi_L=(\theta_L+\theta_s)(p_0+h-c)$.

Therefore, $\pi_H<\pi_L$ occurs in this case if and only if (T9) and (T3)-(T4) hold. Obviously, $\pi_H<\pi_L$ occurs in this case if (T12) holds because (T12) summaries (T9) and (T3)-(T5).

Summary

Summarizing the above derivations, if (T12) holds, there is a window of θ_H within which the firm with a smaller loyal following earns more profit than the firm with a larger loyal following, regardless of the sequence of move of the firms.

For (T12) to hold, we must have $\beta>0$ (i.e. $h>c-p_0$). Otherwise, Firm L's profit will be negative.

We can verify that if $\theta_L=0$ and $p_0=0$, (T12) reduces to the exactly same condition as given in our basic model. If $\theta_L=0$, then $\theta_H<1-\theta_L(2-\beta)/\beta$ always holds. Therefore, for the window of θ_H to exist, $\beta/(2-\beta) < \beta/(1+\beta)$ must hold. This leads to $\beta<0.5$ and implies that $h<k+c-2p_0$. Hence, we find that the window of θ_H exists at $\theta_L=0$ if and only if $c-p_0<h<k+c-2p_0$.

If $\theta_L>0$, $\beta(1-\theta_L)/(2-\beta)<\theta_H<min[\beta/(1+\beta),1-\theta_L(2-\beta)/\beta]$ leads to either

(T14) $\beta(1-\theta_L)/(2-\beta)<\beta/(1+\beta)$ and $\beta/(1+\beta)<1-\theta_L(2-\beta)/\beta$ or
(T15) $\beta(1-\theta_L)/(2-\beta)<1-\theta_L(2-\beta)/\beta$ and $\beta/(1+\beta)\geq 1-\theta_L(2-\beta)/\beta$.

(T14) results in $\frac{\sqrt{(1-\theta_L)^2+8\theta_L{}^2}-(1-\theta_L)}{2\theta_L}<\beta<\frac{1+\theta_L}{2-\theta_L}$ and (T15) results in

$2\theta_L<\beta\le\frac{\sqrt{(1-\theta_L)^2+8\theta_L{}^2}-(1-\theta_L)}{2\theta_L}$. Since $\theta_L<0.5$ as $\theta_H+\theta_L\le 1$ and $\theta_H>\theta_L$, we have

$2\theta_L<\frac{\sqrt{(1-\theta_L)^2+8\theta_L{}^2}-(1-\theta_L)}{2\theta_L}<\frac{1+\theta_L}{2-\theta_L}$ always hold. Therefore, the above condition of β becomes

$2\theta_L<\beta<(1+\theta_L)/(2-\theta_L)$. In addition, $\beta(1-\theta_L)/(2-\beta)>\theta_L$ always holds as $\beta>2\theta_L$, which ensures that $\theta_H>\theta_L$ always holds for β in this range. Hence, the window of θ_H exists for β in this mid-range. From the definition of β, this range of β implies that

(T16) $[2\theta_L(k-c)+(c-p_0)]/(1-2\theta_L)<h<[(1+\theta_L)(k-c)+(2-\theta_L)(c-p_0)]/(1-2\theta_L)$.

Because $[2\theta_L(k-c)+(c-p_0)]/(1-2\theta_L)<[(1+\theta_L)(k-c)+(2-\theta_L)(c-p_0)]/(1-2\theta_L)$ is always true and $[2\theta_L(k-c)+(c-p_0)]/(1-2\theta_L)>0$ always holds, the feasible set of h is non-empty for any given θ_L.

Hence, from the above derivations, we can see that for any $0<\theta_L<\theta_H$, there exists $0<\underline{h}<h<\overline{h}$ so that $\pi_H<\pi_L$ for $\theta_L<\underline{\theta}<\theta_H<\overline{\theta}$ regardless of the sequence of price setting of the game, where

$\underline{h}=[2\theta_L(k-c)+(c-p_0)]/(1-2\theta_L)$,

$\overline{h}=[(1+\theta_L)(k-c)+(2-\theta_L)(c-p_0)]/(1-2\theta_L)$,

$\beta=(p_0+h-c)/(k+h-c)$,

$\underline{\theta}=\beta(1-\theta_L)/(2-\beta)$, *and*

$\overline{\theta}=\min[\beta/(1+\beta),1-\theta_L(2-\beta)/\beta]$.

Q.E.D.

T2. An Alternative Model for the Secondary Market à la Hotelling (1929)

Model Setup and Results

We assume that the secondary product market has m buyers who are uniformly distributed on a unit line with two firms on each end (H at 0 and L at 1). Each customer buys at most one unit of the product. Denote d_i to be a customer's distance to firm i and t to be the disutility per unit distance. Given that both firms have a positive demand for the primary product, the demand in the market of the secondary product from a consumer who locates at d_i to firm i is assumed to be

$$\begin{cases} D_{iS}(d_i) = 1, & \text{if } h(D_{iP} - D_{jP}) - t(d_i - d_j) - (p_{iS} - p_{jS}) > 0 \\ D_{iS}(d_i) = 0.5, & \text{if } h(D_{iP} - D_{jP}) - t(d_i - d_j) - (p_{iS} - p_{jS}) = 0 \\ D_{iS}(d_i) = 0, & \text{if otherwise}, \end{cases} \quad \text{(T17)}$$

where $j \neq i$ and p_{iS} ($i=H, L$) is the price charged by firm i in the secondary product market.[1] Equation (T17) allows preference difference that is not directly linked to the demand in the primary product market to affect firms' demand in the secondary product market. For example, advertisers may have different preferences for *The Wall Street Journal* or *USA Today* depending on the products they market and the client service provided by the newspapers. As another example, file creators can have different preferences for Windows Media Player or RealPlayer depending on the interface designs and features offered by the software, given that the demand for both by document viewers is positive. We focus on the parameter ranges which ensure that (i) each firm earns positive equilibrium demand in the secondary market and (ii) it is optimal for firm H to adopt the "secondary-product-driven" strategy by setting p_{HP}=0 when $\theta = 0$. The main results of our basic model still hold in this extended model. In addition, we obtain the following two Remarks.

> **Remark 2:** *It is more likely for the firm with a disadvantage in loyalty to outperform the firm with loyalty advantage when the market of the secondary product becomes more competitive, i.e.,* $\frac{\partial l(\theta)}{\partial t} < 0$ (*given that* $t > \max[\frac{h}{3}, \frac{mh^2}{3(2mh-3c)}]$).

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[1] In equilibrium, both firms have positive demand from the primary product market. When a firm has no demand from its primary product, its demand from the secondary market is assumed to be zero and the other firm obtains all the customers in the secondary product market if it has positive demand from the primary product. When neither firm has positive demand from the primary product market, neither of them will have demand from the secondary product market.

Remark 3: *In the presence of a loyalty window within which the firm with a loyalty disadvantage can obtain a higher profit than the firm with a loyalty advantage, firms adopt different pricing strategies in the two markets. The firm with a loyalty advantage (firm H) sets a higher price in the primary market, but a lower price in the secondary market than the firm with a loyalty disadvantage (firm L). In addition, firm H's market shares are lower than those of firm L in both markets.*

Remark 2 reveals that competitiveness in the secondary product market affects strategies in the primary product market. The intensified competition reduces the firms' profits from the secondary product market and, hence, *H* is more likely to adopt the primary-product-driven strategy by charging a high price in the primary product market, and *L* is more likely to outperform *H* as the latter becomes less aggressive in the primary product market. Remark 3 further highlights the insight revealed in the basic model as to why a firm may not be able to convert its loyalty advantage to profit advantage in a market with a cross-market network effect. Firm *H* faces conflicting incentives in the two markets because a higher margin and market share in the secondary market are obtained at the cost of profit decrease from its loyal customers. The result of this tradeoff for firm *H* with a mid-level loyal customer base in the primary market is to focus on serving its loyal customers, which gives firm *L* the opportunity to out-profit firm *H*. The different pricing strategies adopted by firm *H* in such a case reflect the dilemma it faces to balance its interests in the two markets connected by a cross-market network effect.

Proofs

Let *d* be the location of the customer who is indifferent between firm H and firm L in the secondary product market. From equation (T17), *d* can be solved as $d = \frac{h\Delta D_P - \Delta p_S + t}{2t}$, where $\Delta D_P = D_{HP} - D_{LP}$ and $\Delta p_S = p_{HS} - p_{LS}$. Therefore, firm H and firm L's profits from selling the secondary product are $\pi_{HS} = mp_{HS} \frac{h\Delta D_P - \Delta p_S + t}{2t}$ and $\pi_{LS} = mp_{LS} \frac{-h\Delta D_P + \Delta p_S + t}{2t}$ respectively. Solving for the firms' optimal prices from $\frac{\partial \pi_{HS}}{\partial p_{HS}} = 0$ and $\frac{\partial \pi_{LS}}{\partial p_{LS}} = 0$ gives that $p^*_{HS} = t + \frac{h\Delta D_P}{3}$ and $p^*_{LS} = t - \frac{h\Delta D_P}{3}$. Thus, the firms' demand and profits from the secondary market under the optimal prices are $D^*_{HS} = \frac{m}{2t}(t + \frac{h\Delta D_P}{3})$, $D^*_{LS} = \frac{m}{2t}(t - \frac{h\Delta D_P}{3})$, $\pi^*_{HS} = \frac{m}{2t}(t + \frac{h\Delta D_P}{3})^2$ and $\pi^*_{LS} = \frac{m}{2t}(t - \frac{h\Delta D_P}{3})^2$. Then the total profits for the two firms can be written as

$$\pi_H = \pi_{HP} + \pi_{HS} = \begin{cases} k\theta + \frac{m}{2t}\left(t + \frac{h(2\theta - 1)}{3}\right)^2 - \theta c & if \quad p_{HP} = k \\ \frac{m}{2t}\left(t + \frac{h\theta}{3}\right)^2 - \frac{1+\theta}{2}c & if \quad p_{HP} = 0 \end{cases}$$

$$\pi_L = \pi_{LP} + \pi_{LS} = \begin{cases} \frac{m}{2t}\left(t - \frac{h(2\theta-1)}{3}\right)^2 - (1-\theta)c & if \quad p_{HP} = k \\ \frac{m}{2t}\left(t - \frac{h\theta}{3}\right)^2 - \frac{1-\theta}{2}c & if \quad p_{HP} = 0 \end{cases}$$

To ensure $D^*_{HS}>0$ and $D^*_{LS}>0$, we have **(i)** $t>h/3$. For $p_{HP}=0$ to be optimal for firm H when $\theta = 0$, we have $mh^2 +3(3c-2mh)t<0$, which implies that **(ii)** $c<2mh/3$ and **(iii)** $t>mh^2/[3(2mh-3c)]$. Following the same steps as in the proof of Lemma 1 and Proposition 1-2, we have that $p^*_{HP} = k$ if $\theta > \underline{\theta}$ and $p^*_{HP} = 0$ otherwise. Here $\underline{\theta}$ solves $f(\theta) = \pi_H|_{p_{HP}=k} - \pi_H|_{p_{HP}=0} = k\theta + \frac{m}{2t}(t + \frac{h(2\theta-1)}{3})^2 - \frac{m}{2t}(t + \frac{h\theta}{3})^2 + \frac{1-\theta}{2}c = 0$. Because $f(\theta)$ is a quadratic function of θ with $f(0)<0$ and $f(1)>0$, the solution exists and is unique, which is

$\underline{\theta} = \dfrac{4h^2 + 9\frac{c}{m}t - 6ht - 18\frac{k}{m}t + \sqrt{-12h^2(h^2 + 9\frac{c}{m}t - 6ht) + (4h^2 - 6ht + 9\frac{c}{m}t - 18\frac{k}{m}t)^2}}{6h^2}$. The condition of $\pi_H < \pi_L$ now is $\theta > \underline{\theta}$ and $\theta < \overline{\theta}$, where $\overline{\theta} = \dfrac{2hm-3c}{4hm+3k-6c}$ solves

$g(\theta) = \pi_H|_{p_{HP}=k} - \pi_L|_{p_{HP}=k} = k\theta + \frac{m}{2t}(t + \frac{h(2\theta-1)}{3})^2 - \frac{m}{2t}(t - \frac{h(2\theta-1)}{3})^2 + (1-2\theta)c = 0.$

For $\underline{\theta} < \overline{\theta}$ to hold, we must have **(iv)** $3k > 2mh - 3c$, which implies $\underline{\theta} < \overline{\theta} < \frac{1}{3}$. Conditions (ii) and (iv) lead to $\frac{3c}{2m} < h < \frac{3(k+c)}{2m}$. Therefore, h must be in this mid-range in order for the window $\underline{\theta} < \overline{\theta}$ to exist for firm L to obtain higher profit than firm H. The width of the window $l(\theta) = \overline{\theta} - \underline{\theta}$ can be quite large. For example, at $t = \dfrac{mh^2}{3(2mh-3c)}$, $k = mh - \frac{3c}{2}$, and $mh/3<c<2mh/5$, we have $l(\theta) = \frac{2}{7}$.

Similar to the basic model, the condition for $D_{HP} < D_{LP}$ under this case is $\theta > \underline{\theta}$ and $\theta < \frac{1}{2}$. We have $\overline{\theta} = \dfrac{2hm-3c}{4hm+3k-6c} < \frac{1}{2}$. Hence, the window for firm L to obtain market share advantage can also exist and this window is larger than that for profit advantage.

The above derivations show that the main implications from Proposition 1-2 still hold.

<u>Proof of Remark 2</u> From the above derivations, we have that $\overline{\theta} = \dfrac{2hm-3c}{4hm+3k-6c}$ is not a function of t and $sign(\frac{\partial \underline{\theta}}{\partial t}) = \dfrac{sign\frac{\partial f}{\partial t}|_{\underline{\theta}}}{sign\frac{\partial f}{\partial \theta}|_{\underline{\theta}}}$. Because $\frac{\partial f}{\partial \theta}|_{\underline{\theta}} > 0$ and $sign(\frac{\partial f}{\partial t}|_{\underline{\theta}}) = sign(1-3\underline{\theta}) > 0$ as $\underline{\theta} < \overline{\theta} < \frac{1}{3}$, we have

$sign(\frac{\partial \theta}{\partial t})>0$. Therefore, the window of θ in which firm L has both profit and market share advantage becomes smaller as t increases, given that the restrictions on t, i.e. conditions (i) and (iii) are satisfied (i.e., $t > \max[\frac{h}{3}, \frac{mh^2}{3(2mh-3c)}]$). Q.E.D.

<u>Proof of Remark 3</u> From the above derivations, if $\underline{\theta} < \theta < \overline{\theta}$, we have that $p_{HP}>p_{LP}$. In addition, $p_{HS}<p_{LS}$ and $D_{HS}<D_{LS}$ as $D_{HP}<D_{LP}$ under $\underline{\theta} < \theta < \overline{\theta}$. Q.E.D.