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Trading up: A Strategic Analysis of Reference Group Effects

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Technical Appendices
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Appendix A

Proof of Lemma 1

First, consider the decision of leaders. In order to make their purchase decisions, the leaders must form expectations about the second-period demand. If all the followers are expected to buy, namely $y^e = \beta$, then the number of leaders who will buy the product is given by:

$$x_1(y^e = \beta) = 1 - (p_1 + g(\beta)) \tag{A1}$$

We know that such an expectation will be correct only if $h(x_1(y^e = \beta)) > c$ so that it is profitable for the firm to sell to the followers in the second period. In other words, we need $x_1(y^e = \beta) \geq h^{-1}(c)$. Then from (A1) we have:

$$1 - (p_1 + g(\beta)) \geq h^{-1}(c) \tag{A2}$$

which reduces to the condition:

$$p_1 \leq (1 - h^{-1}(c)) - g(\beta) = \hat{p}_1 \tag{A3}$$

Thus $\hat{p}_1$ forms the lower bound for the price that the firm might charge the leaders.
In the other polar case where none of the followers are expected to buy the product, we have $y^e = 0$. In this case, the first-period demand will be (compare with equation A1):

$$x_1(y^e = 0) = 1 - p_1 \tag{A4}$$

This implies that:

$$p_1 \geq \big(1 - h^{-1}(c)\big) = \tilde{p}_1 \tag{A5}$$

Hence it is rational for leaders to expect none of the followers to buy the product, only if the price is above $\tilde{p}_1$. Further it follows from (A3) that $\tilde{p}_1 > \hat{p}_1$.
In order to complete the proof we now need to consider the case when $p_1 \in (\tilde{p}_1, \hat{p}_1)$. When $p_1 \in (\hat{p}_1, \tilde{p}_1)$, only some of the followers purchase the product. This can only happen when the followers are indifferent between buying and not buying. Furthermore, $p_2 = c$. To see this, consider the case where $p_2 > c$ then the firm can strictly increase its profits by charging $p_2 - \epsilon$ and selling to all the followers. Let $z(p_1)$ followers buy the product. In a rational expectations equilibrium, we must have:

$$x_1 = 1 - (p_1 + g(z)) \tag{A6}$$

where:

$$h\left(1-p_1-g(z)\right)-c=0 \tag{A7}$$

i.e.,

$$z(p_1)=g^{-1}\left(1-p_1-h^{-1}(c)\right) \tag{A8}$$

Note that $z(p_1)$ is continuous and is decreasing in p_1 since $g(\cdot)$ is (weakly) increasing in its argument. Thus, the second period demand function is:

$$y(p_1)=\begin{cases}\beta & \text{if } p_1<\hat{p}_1\\ z(p_1) & \text{if } p_1\in(\hat{p}_1,\tilde{p}_1)\\ 0 & \text{otherwise.}\end{cases} \tag{A9}$$

Since $z(\hat{p}_1)=1$ and $z(\tilde{p}_1)=0$, $y(p_1)$ is continuous. The demand in the first period is given by:

$$x_1(p_1)=\begin{cases}1-p_1-g(\beta) & \text{if } p_1<\hat{p}_1\\ 1-p_1-g(z(p_1)) & \text{if } p_1\in(\hat{p}_1,\tilde{p}_1)\\ 1-p_1 & \text{otherwise.}\end{cases} \tag{A10}$$

□

Proof of Proposition 1a

We establish the proposition via a series of claims.

Claim 1 *If $y=0$ then:*

$$p_1=\begin{cases}1-h^{-1}(c), & \textit{if } h(\frac{1-c}{2})\geq c\\ \frac{1+c}{2}, & \textit{otherwise.}\end{cases} \tag{A11}$$

Also:

$$\Pi_1(y=0)=\begin{cases}\Pi_{12}\equiv h^{-1}(c)\Big[1-h^{-1}(c)-c\Big], & \textit{if } h(\frac{1-c}{2})\geq c\\ \Pi_{11}\equiv\frac{(1-c)^2}{2} & \textit{otherwise.}\end{cases} \tag{A12}$$

Proof: In this case, the firm solves the following optimization problem:

$$\max_{p_1}\Pi(y=0) \quad = \quad (1-p_1)(p_1-c) \tag{A13}$$

s.t.

$$p_1\geq\tilde{p}_1 \tag{A14}$$

If the constraint does not bind, then the first order conditions imply that:

$$p_1=\frac{1+c}{2} \tag{A15}$$

However, this is only valid as long as $p_1 \geq \tilde{p}_1$, which implies that:

$$\frac{1+c}{2} \geq 1 - h^{-1}(c) \tag{A16}$$

In other words:

$$h^{-1}(c) \geq \frac{1-c}{2} \tag{A17}$$

Thus, the unconstrained solution is valid only if $h(\frac{1-c}{2}) \leq c$. If the constraint binds, then $p_1 = \tilde{p}_1 = 1 - h^{-1}(c)$. The profit function follows immediately by substitution. □

Claim 2 *In any optimal solution $p_1 \notin [\hat{p}_1, \tilde{p}_1)$. Furthermore, if $y = \beta$ then firm's optimal profits are given by:*

$$\max_{p_1} \Pi(y = \beta) \quad \equiv \quad \Pi_{13} = \max_{p_1} (1 - p_1 - g(\beta))(p_1 - c) + \beta \left[h(1 - p_1 - g(\beta)) - c \right] \tag{A18}$$

Proof If $p_1 \in [\hat{p}_1, \tilde{p}_1)$ then the firm sells to some followers. However, $p_2 = c$. Thus, the firm's second period profits are zero and the first period price is $p_1 \in [\hat{p}_1, \tilde{p}_1)$. Consider a deviation by the firm to a price $\tilde{p}_1$. In this case, the firm sells to only the leaders and since $\tilde{p}_1 > p_1$, the profits are strictly greater.

If $y = \beta$ then the firm's problem is to choose p_1 such that:

$$\max_{p_1} \Pi(y = \beta) \quad \equiv \quad \Pi_{13} = (1 - p_1 - g(\beta))(p_1 - c) + \beta \left[h(1 - p_1 - g(\beta)) - c \right] \tag{A19}$$

s.t.

$$p_1 \leq \tilde{p}_1 \tag{A20}$$

Since in any optimal solution, the constraint does not bind as established earlier, we only need to examine the unconstrained case. □

Claim 3 *If c is small, then there exists a λ_l^* such that $y = \beta$ for $\lambda_l < \lambda_l^*$ and $y = 0$ otherwise.*

Proof: First, note that for small c, at $\lambda_l = 0$, $y = \beta$ is clearly optimal. Also, note that Π_{11} and Π_{12} are independent of λ_l. Also, note that the Envelope theorem implies:

$$\frac{\partial \Pi_{13}}{\partial \lambda_l} = -\tilde{g}(\beta)(p_1 - c) - \beta h' \tilde{g}(\beta) = -\tilde{g}(\beta) \left[p_1 - c + \beta h' \right] \tag{A21}$$

The first order conditions imply that:

$$\frac{\partial \Pi_{13}}{\partial p_1} = 1 - p_1 - g(\beta) - p_1 + c - \beta h'(1 - p_1 - g(\beta)) = 0 \tag{A22}$$

The second order condition is:

$$-2 + \beta h'' < 0 \tag{A23}$$

Note that since $\beta h'' < 1$, the second order condition for maximization is satisfied.

Using the first order condition we have:

$$1 - p_1 - g(\beta) = p_1 - c + \beta h' \tag{A24}$$

Substituting in (A21) we have:

$$\frac{\partial \Pi_{13}}{\partial \lambda_l} = -\tilde{g}(\beta)\left(1 - p_1 - g(\beta)\right) < 0 \tag{A25}$$

where the last inequality follows since $(1 - p_1 - g(\beta)) = x_1 > 0$.

Thus, if c is small, as λ_l increases, firm finds it less profitable to serve the followers and if λ_l is sufficiently high it does not sell to the followers. The result then follows from the intermediate value theorem. □

Claim 4 *If $y = \beta$ then p_1^* and p_2^* are decreasing in λ_l*

Proof:

Using the implicit function theorem and the concavity of the profit function, it follows that:

$$\begin{aligned} \operatorname{sgn}\left(\frac{\partial p_1}{\partial \lambda_l}\right) &= \operatorname{sgn}\left(\frac{\partial^2 \Pi_{13}}{\partial \lambda_l \partial p_1}\right) && \text{(A26)} \\ &= -\tilde{g}(\beta) + \beta h'' \tilde{g}(\beta) = (\beta h'' - 1)\tilde{g}(\beta) < 0 && \text{(A27)} \end{aligned}$$

Also, the second period price is given by:

$$p_2 = h\left(x_1\right) = h\left(1 - p_1 - g(\beta)\right) \tag{A28}$$

Thus:

$$\begin{aligned} \frac{\partial p_2}{\partial \lambda_l} &= -h' \frac{\partial p_1}{\partial \lambda_l} - h' \tilde{g}(\beta) && \text{(A29)} \\ &= -h'\left(\tilde{g}(\beta) + \frac{\partial p_1}{\partial \lambda_l}\right) && \text{(A30)} \\ &= -h'\left(\tilde{g}(\beta) + \frac{(\beta h'' - 1)\tilde{g}(\beta)}{2 - \beta h''}\right) && \text{(A31)} \\ &= -\tilde{h}\tilde{g}(\beta)\left(\frac{2 - \beta h'' + \beta h'' - 1}{2 - \beta h''}\right) && \text{(A32)} \\ &= -\tilde{h}\tilde{g}(\beta)\left(\frac{1}{2 - \beta h''}\right) < 0 && \text{(A33)} \end{aligned}$$

where the last inequality follows since $\beta h'' < 1$.

□

Claim 5 *If* $y = 0$ *then* p_1^* *and* p_2^* *are independent of* λ_l*.*

Proof: Immediate by inspection. □

Proposition 1a then follows from Claim 3, Claim 4 and Claim 5. □

Proof of Proposition 1b

We again prove this proposition via a series of claims.

Claim 6 $h^{-1}(\cdot)$ *is decreasing in* λ_f*.*

Proof: Note that for any constant y:

$$h^{-1}(h(y)) = y \tag{A34}$$

Thus:

$$\frac{\partial y}{\partial \lambda_f} = \frac{\partial h^{-1}}{\partial h}\frac{\partial h}{\partial \lambda_f} + \frac{\partial h^{-1}}{\partial \lambda_f} = 0 \tag{A35}$$

We know that $h^{-1}(\cdot)$ is increasing in its arguments since $h(\cdot)$ is increasing. Also, $h(\cdot)$ is increasing in λ_f. Therefore, we must have $\partial h^{-1}/\partial \lambda_f < 0$. □

Claim 7 *There exists a* λ_f^* *such that* $y = 0$ *for* $\lambda_f < \lambda_f^*$ *and* $y = \beta$ *otherwise.*

Proof: If $\lambda_f = 0$ then it clearly follows that the firm will prefer to sell to only the leaders since the followers have no value for the product. For large enough λ_f the firm will find it profitable to sell it to the followers. Also using Envelope Theorem

$$\frac{\partial \Pi_{13}}{\partial \lambda_f} = \beta \tilde{h}' > 0 \tag{A36}$$

Also, note that Π_{11} is independent of λ_f. Using the Envelope theorem, it follows that:

$$\begin{aligned} \frac{\partial \Pi_{12}}{\partial \lambda_f} &= \frac{\partial h^{-1}}{\partial \lambda_f}\left(1 - h^{-1} - c\right) + h^{-1}\left(-\frac{\partial h^{-1}}{\partial \lambda_f}\right) && \text{(A37)} \\ &= \frac{\partial h^{-1}}{\partial \lambda_f}\left(1 - c - 2h^{-1}\right) < 0 && \text{(A38)} \end{aligned}$$

where the inequality follows since $\partial h^{-1}/\partial \lambda_f < 0$ by Claim 6 and for Π_{12} to apply we must have:

$$h\left(\frac{1-c}{2}\right) > c \tag{A39}$$

which implies that $1 - c > 2h^{-1}$. Thus, as λ_f increases it becomes more attractive for the firm to sell to the followers. The result then follows from the intermediate value theorem. □

Claim 8 *If* $y = 0$ *then* p_1 *weakly increases in* λ_f*.*

Proof: When the constraint does not bind it follows by inspection of (A11). To see this in the case when the constraint binds, note that by previous claim $h^{-1}(\cdot)$ is decreasing in λ_f. Thus, $p_1 = 1 - h^{-1}(c)$ is increasing. □

Claim 9 *If* $y = \beta$ *then* p_1 *is decreasing in* λ_f *and* p_2 *is increasing in* λ_f*.*

Proof: Using the implicit function theorem and the concavity of the profit function, it follows that:
Also,

$$\begin{aligned} \operatorname{sgn}\left(\frac{\partial p_1}{\partial \lambda_f}\right) &= \operatorname{sgn}\left(\frac{\partial^2 \Pi_{13}}{\partial \lambda_f \partial p_1}\right) && \text{(A40)} \\ &= -\beta \tilde{h}' < 0 && \text{(A41)} \end{aligned}$$

The second period price is given by:

$$p_2 = h\left(x_1\right) = h\left(1 - p_1 - g(\beta)\right) \tag{A42}$$

Similarly,

$$\frac{\partial p_2}{\partial \lambda_f} = \tilde{h} - h' \frac{\partial p_1}{\partial \lambda_f} > 0 \tag{A43}$$

□

Proposition 1b then follows using Claims 7, 8 and 9. □

Proof of Proposition 2

We will prove the proposition via a series of claims.

Claim 10 *Define* c_2 *such that:*

$$c_2 = \left\{ c : h\left(\frac{1-c}{2}\right) - c = 0 \right\} \tag{A44}$$

Then c_2 *exists and is unique. Furthermore, if* $\exists$ *a* c_r *such that* $h\left(\frac{1-c_r}{2}\right) - c_r < 0$ *then* $h\left(\frac{1-c}{2}\right) - c < 0 \; \forall c > c_r$*. Similarly, if* $\exists$ *a* c_r *such that* $h\left(\frac{1-c_r}{2}\right) - c_r > 0$ *then* $h\left(\frac{1-c}{2}\right) - c > 0 \; \forall c < c_r$*. Thus,* $h\left(\frac{1-c_2}{2}\right) - c > 0$ *if* $c \in [0, c_2)$*.*

Proof: Note that $h\left(\frac{1-c}{2}\right) - c$ is positive for $c = 0$ and negative for $c = 1$, thus establishing existence. Also note that $h\left(\frac{1-c}{2}\right) - c$ is monotonically decreasing in c, which establishes uniqueness of c_2 and the last part of the claim. □

Claim 11 $1 - 2h^{-1} - c > 0 \forall c \in [0, c_2)$.

Proof: Note that by definition:

$$h\left(\frac{1-c_2}{2}\right) - c_2 = 0 \tag{A45}$$

Using (10) it follows that:

$$h\left(\frac{1-c}{2}\right) - c > 0 \ \forall c \in [0, c_2) \tag{A46}$$

which implies that:

$$\frac{1-c}{2} > h^{-1}(c) \ \forall c \in [0, c_2) \tag{A47}$$

Thus:

$$1 - 2h^{-1} - c > 0 \ \forall c \in [0, c_2) \tag{A48}$$

Claim 12 $\Pi_{11} = (1-c)^2/4$ *is decreasing in* c.

Proof: Immediately follows by differentiation. □

Claim 13 Π_{13} *is decreasing in* c.

Proof: Differentiating we have:

$$\frac{\partial \Pi_{13}}{\partial c} = -1 - \beta + p_1 + g(\beta) \tag{A49}$$

Note that $p_1 < \hat{p}_1$ implies that $p_1 + g(\beta) < 1 - h^{-1}(c)$. This implies that:

$$\frac{\partial \Pi_{13}}{\partial c} < -1 - \beta + 1 - h^{-1}(c) = -\beta - h^{-1}(c) < 0 \tag{A50}$$

□

Claim 14 $\Pi_{11} - \Pi_{13}$ *is increasing in* c.

Proof: Differentiating, we have:

$$\begin{aligned}
\frac{\partial}{\partial c}\left(\Pi_{11} - \Pi_{13}\right) &= -\left(\frac{1-c}{2}\right) - \left(-1 - \beta + p_1 + g(\beta)\right) && \text{(A51)}\\
&> -\left(\frac{1-c}{2}\right) - \left(-\beta - h^{-1}(c)\right) && \text{(A52)}\\
&= -\left(\frac{1-c}{2}\right) + \beta + h^{-1}(c) && \text{(A53)}\\
&= \frac{(2\beta - 1) + c}{2} + h^{-1}(c) > 0 && \text{(A54)}
\end{aligned}$$

where the first inequality follows using (A50) and $\beta \geq 1/2$. □

Claim 15 *$\Pi_{12} - \Pi_{13}$ is increasing in c for $c \in [0, c_2)$.*

Proof: Differentiating, we have:

$$\begin{aligned}
\frac{\partial}{\partial c}(\Pi_{12} - \Pi_{13}) &= \left[\left(h^{-1}\right)'(1 - 2h^{-1} - c) - h^{-1}\right] - [-1 - \beta + p_1 + g(\beta)] && \text{(A55)}\\
&= 1 + \beta + \left[\left(h^{-1}\right)'(1 - 2h^{-1} - c) - h^{-1} - p_1 - g(\beta)\right] && \text{(A56)}\\
&= 1 + \beta + \left[\left(h^{-1}\right)'\left(1 - 2h^{-1} - c\right) - h^{-1} - p_1 - g(\beta)\right] && \text{(A57)}\\
&= \left(h^{-1}\right)'\left[1 - 2h^{-1} - c\right] - h^{-1} + (1 + \beta - p_1 - g(\beta)) && \text{(A58)}\\
&= \left(h^{-1}\right)'\left[1 - 2h^{-1} - c\right] - h^{-1} - (-1 - \beta + p_1 + g(\beta)) && \text{(A59)}\\
&> \left(h^{-1}\right)'\left[1 - 2h^{-1} - c\right] - h^{-1} - (-\beta - h^{-1}) && \text{(A60)}\\
&= \left(h^{-1}\right)'\left[1 - 2h^{-1} - c\right] + \beta > 0 && \text{(A61)}
\end{aligned}$$

where the first inequality follows using (A50). The second inequality follows using Claim 11. □

Claim 16 *If Π_{13} intersects with Π_{12} at c^* in $[0, c_2)$ then the intersection is unique. Furthermore, in this case Π_{13} does not intersect with Π_{11}. Also, $y = \beta$ for $c < c^*$ and $y = 0$ otherwise.*

Proof: The uniqueness of intersection follows because, as proved earlier $\Pi_{12} - \Pi_{13}$ is increasing in c. The second part follows since $\Pi_{11} > \Pi_{12} \forall c$. Thus, if $\exists$ a c such that $\Pi_{13} < \Pi_{12}$ then this implies that $\Pi_{13} < \Pi_{11}$. The last part follows from uniqueness and by noting that Π_{13} must intersect Π_{12} from above since $\Pi_{12} - \Pi_{13}$ is increasing in c. □

Claim 17 *If Π_{13} and Π_{12} intersect at c^* then c^* is decreasing in λ_l and increasing in λ_f.*

Proof: We have:

$$\frac{\partial c^*}{\partial \lambda_f} = -\text{sgn}\left(\frac{\partial \Pi_{12}}{\partial \lambda_f} - \frac{\partial \Pi_{13}}{\partial \lambda_f}\right) \tag{A62}$$

Note that Π_{13} is increasing in λ_f from (A36). Also:

$$\begin{aligned}
\frac{\partial \Pi_{12}}{\partial \lambda_f} &= \frac{\partial h^{-1}}{\partial \lambda_f}\left[1 - h^{-1} - c\right] - \frac{\partial h^{-1}}{\partial \lambda_f} h^{-1} && \text{(A63)}\\
&= \frac{\partial h^{-1}}{\partial \lambda_f}\left[1 - 2h^{-1} - c\right] < 0 && \text{(A64)}
\end{aligned}$$

where the inequality follows since $\partial h^{-1}/\partial \lambda_f < 0$ and $(1 - 2h^{-1} - c) > 0$ for $c \in [0, c_2)$. Using (A62) it follows that c^* is increasing in λ_f. Also, note that:

$$\frac{\partial c^*}{\partial \lambda_l} = -\text{sgn}\left(\frac{\partial \Pi_{12}}{\partial \lambda_l} - \frac{\partial \Pi_{13}}{\partial \lambda_l}\right) = \text{sgn}\left(\frac{\partial \Pi_{13}}{\partial \lambda_l}\right) < 0 \tag{A65}$$

where the last inequality follows from (A25). □

Claim 18 *If* Π_{13} *intersects with* Π_{11} *at* c^* *then the intersection is unique and* $y = 0$ *for* $c > c^*$ *and* $y = \beta$ *otherwise.*

Proof: The uniqueness of intersection follows because, as proved earlier $\Pi_{11} - \Pi_{13}$ is increasing in c. The second part follows from uniqueness and the fact that Π_{13} must intersect with Π_{11} from above. □

Claim 19 *If* Π_{13} *and* Π_{11} *intersect at* c^* *then* c^* *is decreasing in* λ_l *and increasing in* λ_f*.*

Proof: Since Π_{11} is independent of λ_l and λ_f, the result follows from (A25) and (A36). □
The proposition then follows from Claims 16-19. □

Proof of Proposition 3

We will prove the proposition via a series of claims.

Claim 20 Π_{12} *is increasing in* c *at* $c = 0$ *and decreasing at* $c = c_2$*.*

Proof: First note that $h' > 0$ implies that $(h^{-1})' > 0$. We have:

$$\begin{aligned}
\frac{\partial \Pi_{12}}{\partial c} &= \left(h^{-1}\right)'\left[1 - h^{-1} - c\right] + h^{-1}(c)\left[-\left(h^{-1}\right)' - 1\right] && \text{(A66)}\\
&= \left(h^{-1}\right)' - h^{-1}\left(h^{-1}\right)' - c\left(h^{-1}\right)' - h^{-1}\left(h^{-1}\right)' - h^{-1} && \text{(A67)}\\
&= \left(h^{-1}\right)'(1-c) - 2h^{-1}\left(h^{-1}\right)' - h^{-1} && \text{(A68)}\\
&= \left(h^{-1}\right)'\left(1 - 2h^{-1} - c\right) - h^{-1} && \text{(A69)}
\end{aligned}$$

At $c = 0$ note that $h^{-1}(0) = 0$. Therefore, $\partial \Pi_{12}/\partial c = 1 > 0$ at $c = 0$. At $c > c_2$, $(1 - 2h^{-1} - c) < 0$. Thus, $\partial \Pi_{12}/\partial c < 0$ for $c \geq c_2$. □

Claim 21 $\Pi_{12} = h^{-1}(c)\left[1 - h^{-1}(c) - c\right]$ *is concave in* c *for* $c \in [0, c_2)$*. Therefore* $c_1 < c_2$ *exists and is unique.*

Proof: First note that $h'' \geq 0$ implies that $(h^{-1})'' = -1/h'' \leq 0$. We have:

$$\begin{aligned}
\frac{\partial^2 \Pi_{12}}{\partial c^2} &= \left(h^{-1}\right)''(1 - 2h^{-1} - c) + \left(h^{-1}\right)'\left(-2\left(h^{-1}\right)' - 1\right) - \left(h^{-1}\right)' && \text{(A70)}\\
&= \left(h^{-1}\right)''(1 - 2h^{-1} - c) - 2\left[\left(h^{-1}\right)'\right]^2 - 2\left(h^{-1}\right)' && \text{(A71)}\\
&= \left(h^{-1}\right)''(1 - 2h^{-1} - c) - 2\left(h^{-1}\right)'\left(1 + \left(h^{-1}\right)'\right) < 0 && \text{(A72)}
\end{aligned}$$

where the inequality follows since $(h^{-1})' > 0$, $(h^{-1})'' \leq 0$ and $(1 - 2h^{-1} - c) > 0$ since $c \in [0, c_2)$. The existence of c_1 follows from the previous claim and the intermediate value theorem. Uniqueness follows from concavity and the inequality $c_1 < c_2$ follows since $c_1 \in [0, c_2)$. □

We have therefore established that if $y = 0$ then the profit function is increasing in $c \in (0, c_1)$. To complete the proof, we only need establish that $c^* < c_1$. Note that if $g(\beta)$ is sufficiently large then $\hat{p}_1 < 0$. Since by assumption $p_1 \geq 0$, this implies that $y = 0$. This completes the proof. □

Proof of Lemma 2

The relevant rational expectations equation is:

$$\Omega(x_1) = x_1 - (1 - p_1 - g(Q - x_1)) \tag{A73}$$

We have: $\Omega(0) < 0$ and $\Omega(1) > 0$. Thus, there exists a $x \in (0, 1)$ such that $\Omega(x) = 0$. Also:

$$\Omega'(x) = 1 - g' > 0 \tag{A74}$$

which establishes uniqueness. □

Proof of Proposition 4

If $\beta > g^{-1}(1)$ then the firm does not sell to the followers when there is only one product. Furthermore, the condition $h\left(\frac{1-c}{2}\right) > c$ implies that the firm's first period price is $1 - h^{-1}(c)$ and the profits with a single product are given by:

$$\Pi(1) = h^{-1}(c)(1 - h^{-1}(c) - c) < \left(\frac{1-c}{2}\right)^2 \tag{A75}$$

Now consider the situation where the firm introduces k variants such that :

$$h\left(\frac{1-c}{2k}\right) < c \tag{A76}$$

Since $h(0) = 0$, clearly this holds for large enough k. Now consider the firm's problem of setting prices under the assumption that no follower's buy

$$\Pi(k) = \max_{p_1(k)} k\left(\frac{1 - p_1(k)}{k}\right)(p_1(k) - c) - k\phi \tag{A77}$$

subject to the condition that no follower finds it optimal to buy i.e.,

$$h\left(\frac{1-p_1(k)}{k}\right) < c \tag{A78}$$

The solution for the unconstrained problem is:

$$p_1(k) = \frac{1+c}{2} \tag{A79}$$

If the constraint does not bind, the first period demand for any variant is:

$$x_1(k) = \frac{1-p_1(k)}{k} = \frac{1-c}{2k} \tag{A80}$$

But by the assumption,

$$h\left(\frac{1-c}{2k}\right) < c \tag{A81}$$

it follows that the constraint does not bind and the firm's profits are:

$$\Pi(k) \geq \left(\frac{1-c}{2}\right)^2 - k\phi > \Pi(1) = h^{-1}(c)(1-h^{-1}(c)-c) \tag{A82}$$

where the inequality holds for small ϕ. □.

We will first establish Proposition 6 and then prove Proposition 5.

Proof of Proposition 6

We first establish the result for the case of increasing costs. If $c \in (c^*, c_2)$ then $y = 0$ and the relevant profit function is Π_{12}. By Claim 21, we know that $c_1 < c_2$. Also, the firm prefers to increase its cost when $c \in (c^*, c_1)$. Therefore, in this situation the relevant profit function is Π_{12}.

We note however that $\Pi_{12} \leq (1-c)^2/4$ since $(1-c)^2/4$ is the solution to the unconstrained solution. If $c \in (c^*, c_2)$ then we know that the constraint binds and $p_1 = \tilde{p}_1$ and $x_1^* = 1-\tilde{p}_1 = h^{-1}(c)$. Consider the case when the firm announces a $Q = (1-c)/2$ and $p_1 = (1+c)/2$. Note that:

$$x_1 = h^{-1}(c) < \frac{1-c}{2} = Q \tag{A83}$$

where the last inequality follows since $h\left(\frac{1-c}{2}\right) - c > 0$ if $c < c_2$. Under the proposed limited edition scenario, first period's demand is given by the solution of the rational expectations equation which is:

$$\Omega(x_1) = x_1 - \left(1 - \frac{1+c}{2}\right) - g\left(\frac{1-c}{2} - x_1\right) = 0 \tag{A84}$$

In other words:

$$x_1 - \left(\frac{1-c}{2}\right) - g\left(\frac{1-c}{2} - x_1\right) = 0 \tag{A85}$$

The unique rational expectations equilibrium is $x_1 = (1-c)/2$. Thus, the firm's profits under the limited edition strategy is at least $(1-c)^2/4 > \Pi_{12}$. This establishes the first part of the proposition.

Now, we consider the alternative of establishing multiple product variants. We will establish this part of the proposition using a series of claims:

Claim 22 *If $y_i = 0\ \forall i$ for the variant case then using a limited edition is more profitable.*

Proof: First consider the case when the firm does not sell to the followers under the multiple variant case. In this situation, the firm sells a total unit which is given by:

$$X_1(k) = 1 - p_1(k) \tag{A86}$$

The firm can at least make the same profits with limited edition by setting the price as $p_1(k)$ and limiting sales to $X_1(k)$. For $\phi > 0$, the limited edition would be strictly more profitable. □

Claim 23 *If the firm introduces k variants and finds it profitable to do so, then the first period sales of each product must be the same. Furthermore, if the second period sales are positive then it equals β/k.*

Proof: Suppose not and $\exists$ an $i \neq j$ such that $x_i > x_j$ then the utility for product i is given by:

$$U_i = v - p_i - g(y_i) > U_j = v - p_j - g(y_j) \tag{A87}$$

but this implies that $x_j = 0$. But then the firm can do better by dropping j and saving ϕ. So this cannot be an equilibrium and therefore $x_i = x_j\ \forall i, j$. It follows that the firm will charge the same price for all products in the second period and the demand will be β/k. □

Claim 24 *If $y(k) > 0$ under the k variant case, then the firm can do better by using a limited edition strategy.*

Proof: From the previous claim, it follows that the second period sales is β/k. The firms' problem is therefore:

$$\Pi(k) = \max_{p_1(k)}\ k\left(\frac{1 - p_1(k) - g(\frac{\beta}{k})}{k}\right)(p_1(k) - c) + \beta\left(h\left(\frac{1 - g(\frac{\beta}{k}) - p_1(k)}{k}\right) - c\right) - k\phi \tag{A88}$$

Consider an alternate in which firm offers only a single product, charges a price $p_1^*(k)$ and limits the total quantity to:

$$Q_1 = \left(\frac{1 - g(\frac{\beta}{k}) - p_1^*(k)}{k}\right) + \frac{\beta}{k} \tag{A89}$$

The profits under this limited-edition strategy is given by:

$$\tilde{\Pi} = k\left(\frac{1 - p_1^*(k) - g(\frac{\beta}{k})}{k}\right)(p_1^*(k) - c) + \left(\frac{\beta}{k}\right)\left[h\left(1 - g\left(\frac{\beta}{k}\right) - p_1^*(k)\right) - c\right] \tag{A90}$$

Therefore:

$$\tilde{\Pi} - \Pi^*(k) = \left(\frac{\beta}{k}\right)\left[h\left(1 - g\left(\frac{\beta}{k}\right) - p_1^*(k)\right) - c\right] - \beta\left(h\left(\frac{1 - g(\frac{\beta}{k}) - p_1(k)}{k}\right) - c\right) + k\phi \tag{A91}$$

Note that for a weakly convex function:

$$h\left(\frac{z}{k}\right) \leq \frac{1}{k} \cdot h\left(z\right) + \frac{k-1}{k} \cdot h(0) = \frac{1}{k} \cdot h\left(z\right) \tag{A92}$$

Using (A92) in (A91) it follows that:

$$\tilde{\Pi} - \Pi^*(k) > 0 \tag{A93}$$

which completes the proof. □

Thus, using Claims 22 and 24 the result follows. □

Proof of Proposition 5

As shown earlier if $g(\beta)$ is large enough, without limited edition $y = 0$. As shown in the proof of the proposition 6, limited edition can achieve a profit of at least $(1-c)^2/4$. In this case, the firm only sells to the leaders at a price $(1+c)/2$ and sets $Q = (1-c)/2$. Consider a different strategy in which the firm sets $Q_0 = (1-c)/2 + \epsilon$, where $\epsilon > 0$ and charges a first period price $\tilde{p} = (1+c)/2 - g(\epsilon)$. The first period demand is then given by the solution of:

$$x_1 - \left(\frac{1-c}{2} - g(\epsilon)\right) - g\left(\frac{1-c}{2} + \epsilon - x_1\right) = 0 \tag{A94}$$

The unique equilibrium is $x_1 = (1-c)/2$. In this alternate strategy the firm sells the same units as before although at a lower price and sells ϵ units to the followers. The change in profits from this alternate strategy is:

$$\Delta\Pi = -g(\epsilon)\left(\frac{1-c}{2}\right) + \epsilon\left[h\left(\frac{1-c}{2}\right) - c\right] \tag{A95}$$

where the first term is the change in first period profits and the second term represents the change in second period profits. Since by assumption $h\left(\frac{1-c}{2}\right) > \frac{1+c}{2}$, we have:

$$\Delta\Pi > \left(\frac{1-c}{2}\right)(\epsilon - g(\epsilon)) \simeq \left(\frac{1-c}{2}\right)\epsilon\,(1 - g'(0)) > 0 \tag{A96}$$

Thus, the firm can strictly benefit by deviating and selling $y \in (0, \beta)$. Note also that for small ϵ, $Q_0 > h^{-1}(c)$ since by assumption $h\left(\frac{1-c}{2}\right) > \frac{1+c}{2}$. Thus, Q_0 is more than what the firm would have sold absent a limited-edition strategy. Furthermore, $p_2^* = h(\frac{1-c}{2})$ and $p_1^* \geq \frac{1+c}{2} > p_2^*$. Finally note that since $\epsilon < \beta$ and some followers buy, it follows that there is a “buying frenzy”. $\square$

Appendix B

In this appendix, we consider the case when λ_f and λ_l are correlated. In particular, we consider the case when $\lambda_f = \lambda_l \equiv \lambda$. We have the following result:

Proposition B1 *The first period price weakly decreases as the reference group effects become stronger i.e., λ increases. If λ is sufficiently high then the firm charges a first period price which is below c. Furthermore, if $g(x) = \lambda x$ and $h(y) = \lambda y$ then the second period price increases as the reference group effect increases.*

Proof: The result closely parallels the proof for Proposition 1. First, consider the case when $y = 0$. In this case, the first period price is independent of λ. If $y = \beta$ then the sign of the firm's first period price is given by:

$$\frac{\partial p_1^*}{\partial \lambda} = -\frac{\frac{\partial^2 \Pi_{13}}{\partial p_1 \partial \lambda}}{\frac{\partial^2 \Pi_{13}}{\partial p_1^2}} \tag{B1}$$

$$= \frac{-\tilde{g}(1-\beta h'') - \beta \tilde{h}'}{2 - \beta h''} < 0 \tag{B2}$$

where the inequality follows since $\beta h'' < 1$. The second part of the proposition follows immediately from this.

Now consider p_2^*. For the linear case, we can solve for prices explicitly. The first period pricing problem is:

$$\max_{p_1} (1 - p_1 - \lambda\beta)(p_1 - c) + \beta(\lambda(1 - p_1 - \lambda\beta) - c) \tag{B3}$$

The first order condition is:

$$-2p_1 + c + 1 - 2\lambda\beta = 0 \tag{B4}$$

which implies that:

$$p_1^* = \frac{c + 1 - 2\lambda\beta}{2} \tag{B5}$$

Substituting, we have:

$$p_2^* = \frac{\lambda(1-c)}{2} \tag{B6}$$

which is clearly increasing in λ. □

Proposition B2 *Assume,* $g(y) = \lambda y$ *and* $h(x) = \lambda x$ *then for* $c < c^*(\lambda)$ *a "class-to-mass" strategy is optimal where* c^* *is weakly increasing in* λ*. In other words, as the reference group effects become stronger the firm is less likely to sell to followers.*

Proof: The proof closely parallels the proof in Appendix A for Proposition 2. Note that Claims 10-16 are still valid. First consider:

$$\Pi_{13} = (1 - p_1^* - \lambda\beta)(p_1^* - c) + \beta(p_2^* - c) \tag{B7}$$

Using the derivations for the linear case in the previous proposition and substituting, we get:

$$\Pi_{13} = \frac{-2c + 1 + c^2 - 4\beta c}{4} \tag{B8}$$

which implies that Π_{13} is independent of λ. Also, we have:

$$\Pi_{12} = \left(\frac{c}{\lambda}\right)\left(1 - \frac{c}{\lambda} - c\right) \tag{B9}$$

Therefore:

$$\frac{\partial \Pi_{12}}{\partial \lambda} = \frac{-c}{\lambda^2}\left(1 - \frac{c}{\lambda} - c\right) + \frac{c^2}{\lambda^3} \tag{B10}$$

which becomes:

$$\frac{\partial \Pi_{12}}{\partial \lambda} = \frac{c(-\lambda + c\lambda + 2c)}{\lambda^3} \tag{B11}$$

which is negative for:

$$c > \frac{\lambda}{2 + \lambda} = c_2 \tag{B12}$$

where c_2 is defined in Appendix A as:

$$h\left(\frac{1 - c_2}{2}\right) - c_2 = 0 \tag{B13}$$

which reduces to that in (B12). Then using the arguments in Claims 17 it follows that if c^* is the result of intersection of Π_{13} and Π_{12} then c^* must be decreasing in λ. If c^* is the result of intersection of Π_{11} and Π_{13} then they are both independent of λ and therefore c^* is invariant to λ. Thus, in general c^* is weakly increasing in reference group effects i.e., λ □