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#1: [x1 :ε Real [0, 1], x2 :ε Real [0, 1], x3 :ε Real [0, 1], p1 :ε Real [0, ∞), p2 :ε Real [0, ∞), p3 :ε Real [0, ∞)] User
#2: F(p) := α·ē-β·p User
#3: [ x1 1 - x1 ] · [ F(p2) + a 1 - F(p2) - a / F(p2) 1 - F(p2) ] = [ x2 1 - x2 ] User
#4: [ x2 1 - x2 ] · [ F(p3) + a 1 - F(p3) - a / F(p3) 1 - F(p3) ] = [ x3 1 - x3 ] User
#5: [ x3 1 - x3 ] · [ F(p1) + a 1 - F(p1) - a / F(p1) 1 - F(p1) ] = [ x1 1 - x1 ] User
#6: [ α·ē-β·p2 + a·x1 = x2 - α·ē-β·p2 - a·x1 + 1 = 1 - x2 ] Simp(#3)
#7: [ α·ē-β·p3 + a·x2 = x3 - α·ē-β·p3 - a·x2 + 1 = 1 - x3 ] Simp(#4)
#8: [ α·ē-β·p1 + a·x3 = x1 - α·ē-β·p1 - a·x3 + 1 = 1 - x1 ] Simp(#5)
#9: SOLVE([α·ē-β·p2 + a·x1 = x2, α·ē-β·p3 + a·x2 = x3, α·ē-β·p1 + a·x3 = x1], [x1, x2, x3]) User
Simp(#9)
#10: [ x1 = (α·ē-β·(p1+p2+p3) · (a·ēβ·p1 · (ēβ·p2 + a·ēβ·p3) + ēβ·(p2+p3))) / (1 - a3) x2 = (α·ē-β·(p1+p2+p3) · (ēβ·p1 · (a·ē2·β·p2 + ēβ·p3) + a·ēβ·(p2+p3))) / (1 - a3) x3 = (α·ē-β·(p1+p2+p3) · (ēβ·p1 · (ēβ·p2 + a·ēβ·p3) + a·ē2·β·(p2+p3))) / (1 - a3) ]
User
#11: [ x1st := (α·ē-β·(p1+p2+p3) · (a·ēβ·p1 · (ēβ·p2 + a·ēβ·p3) + ēβ·(p2+p3))) / (1 - a3) x2st := (α·ē-β·(p1+p2+p3) · (ēβ·p1 · (a·ē2·β·p2 + ēβ·p3) + a·ēβ·(p2+p3))) / (1 - a3) x3st := (α·ē-β·(p1+p2+p3) · (ēβ·p1 · (ēβ·p2 + a·ēβ·p3) + a·ē2·β·(p2+p3))) / (1 - a3) ]
#12: p1·x1st + p2·x2st + p3·x3st User
#13: (α·ē-β·(p1+p2+p3) · (ēβ·p1 · (ēβ·p2 · (a·p2 + a·p1 + p3) + ēβ·p3 · (a·p1 + a·p3 + p2)) + ēβ·(p2+p3) · (a·p3 + a·p2 + p1)) / (1 - a3) Simp(#12)
#14: (α·ē-β·(p1+p2+p3) · (ēβ·p1 · (ēβ·p2 · (a·p2 + a·p1 + p2) + ēβ·p2 · (a·p1 + a·p2 + p2)) + ēβ·(p2+p2) · (a·p2 + a·p2 + p1)) / (1 - a3) Sub(#13)

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#15: 
$$\frac{\alpha \cdot \tilde{e}^{-\beta \cdot (p1 + p2)} \cdot (\tilde{e}^{\beta \cdot p1})^2 \cdot (a \cdot (p1 + p2) + a \cdot (p1 + p2) + 2 \cdot p2) + \tilde{e}^{\beta \cdot p2} \cdot (a \cdot p2 + a \cdot p2 + p1)}{1 - a^3}$$
 Simp(#14)

#16: rev := 
$$\frac{\alpha \cdot \tilde{e}^{-\beta \cdot (p1 + p2)} \cdot (\tilde{e}^{\beta \cdot p1})^2 \cdot (a \cdot (p1 + p2) + a \cdot (p1 + p2) + 2 \cdot p2) + \tilde{e}^{\beta \cdot p2} \cdot (a \cdot p2 + a \cdot p2 + p1)}{1 - a^3}$$
 User

#17: 
$$\left[ \frac{d}{d p1} rev, \frac{d}{d p2} rev \right]$$
 User

#18: 
$$\left[ \frac{\alpha \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a \cdot \beta \cdot p2 + a \cdot \beta \cdot p2 + \beta \cdot p1 - 1)}{a^3 - 1} + \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a + 1)}{1 - a^3}, \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a + 1)}{1 - a^3} + \frac{\alpha \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a \cdot (\beta \cdot (p1 + p2) - 1) + a \cdot (\beta \cdot (p1 + p2) - 1) + 2 \cdot (\beta \cdot p2 - 1))}{a^3 - 1} \right]$$

#19: 
$$\left[ \frac{\alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot \left( a \cdot \beta \cdot \frac{1}{\beta} + a \cdot \beta \cdot \frac{1}{\beta} + \beta \cdot \frac{1}{\beta} - 1 \right)}{a^3 - 1} + \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a + 1)}{1 - a^3}, \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a + 1)}{1 - a^3} + \frac{\alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot \left( a \cdot \left( \beta \cdot \left( \frac{1}{\beta} + \frac{1}{\beta} \right) - 1 \right) + a \cdot \left( \beta \cdot \left( \frac{1}{\beta} + \frac{1}{\beta} \right) - 1 \right) + 2 \cdot \left( \beta \cdot \frac{1}{\beta} - 1 \right) \right)}{a^3 - 1} \right]$$

#20: [0, 0] Simp(#19)

#21: 
$$\left[ \left( \frac{d}{d p1} \right)^2 rev, \frac{d}{d p2} \frac{d}{d p1} rev \right]$$
 User

#22: 
$$\left[ \frac{\alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a \cdot \beta \cdot p2 + a \cdot \beta \cdot p2 + \beta \cdot p1 - 2)}{1 - a^3}, \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a + 1)}{a^3 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a + 1)}{a^3 - 1} \right]$$
 Simp(#21)

#22: 
$$\left[ \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a + 1)}{a^3 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a + 1)}{a^3 - 1}, \frac{\alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a \cdot (\beta \cdot (p1 + p2) - 2) + a \cdot (\beta \cdot (p1 + p2) - 2) + 2 \cdot (\beta \cdot p2 - 2))}{1 - a^3} \right]$$

#23: 
$$\left[ \frac{\alpha \cdot \beta \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( a \cdot \beta \cdot \frac{1}{\beta} + a \cdot \beta \cdot \frac{1}{\beta} + \beta \cdot \frac{1}{\beta} - 2 \right)}{1 - a^3} \quad \frac{a \cdot \alpha \cdot \beta \cdot e^{-\beta \cdot (1/\beta)} \cdot (a + 1)}{a^3 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot e^{-\beta \cdot (1/\beta)} \cdot (a + 1)}{a^3 - 1} \right]$$

$$\frac{a \cdot \alpha \cdot \beta \cdot e^{-\beta \cdot (1/\beta)} \cdot (a + 1)}{a^3 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot e^{-\beta \cdot (1/\beta)} \cdot (a + 1)}{a^3 - 1} \quad \frac{\alpha \cdot \beta \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( a \cdot \beta \cdot \left( \frac{1}{\beta} + \frac{1}{\beta} \right) - 2 \right) + a \cdot \beta \cdot \left( \frac{1}{\beta} + \frac{1}{\beta} \right) - 2 + 2 \cdot \left( \beta \cdot \frac{1}{\beta} - 2 \right)}{1 - a^3}$$

Sub(#22)

#24: 
$$\left[ \frac{\alpha \cdot \beta \cdot e^{-1} \cdot (a^2 + a - 1)}{1 - a^3} \quad \frac{2 \cdot a \cdot \alpha \cdot \beta \cdot e^{-1} \cdot (a + 1)}{a^3 - 1} \right]$$

$$\frac{2 \cdot a \cdot \alpha \cdot \beta \cdot e^{-1} \cdot (a + 1)}{a^3 - 1} \quad \frac{2 \cdot \alpha \cdot \beta \cdot e^{-1}}{a^3 - 1}$$

Simp(#23)

User

#25: "We have just obtained the Hessian matrix of the profit fuction at the point p1 = p2 = p\* = 1/β. The Hessian is negative definite at a = 0, and remains so for small positive values of a. We now calculate the smallest value of a for which this will no longer be true"

#26: DET 
$$\left[ \frac{\alpha \cdot \beta \cdot e^{-1} \cdot (a^2 + a - 1)}{1 - a^3} \quad \frac{2 \cdot a \cdot \alpha \cdot \beta \cdot e^{-1} \cdot (a + 1)}{a^3 - 1} \right]$$

$$\frac{2 \cdot a \cdot \alpha \cdot \beta \cdot e^{-1} \cdot (a + 1)}{a^3 - 1} \quad \frac{2 \cdot \alpha \cdot \beta \cdot e^{-1}}{a^3 - 1}$$

User

#27: 
$$-\frac{2 \cdot \alpha \cdot \beta \cdot e^{-2} \cdot (2 \cdot a^2 + 2 \cdot a - 1)}{(a - 1)^2 \cdot (a^2 + a + 1)}$$

Simp(#26)

#28: 
$$\left[ a = \frac{\sqrt{3}}{2} - \frac{1}{2}, a = -\frac{\sqrt{3}}{2} - \frac{1}{2}, a = \infty, a = -\infty \right]$$

Solve(#27,a)

#29: [a = 0.366025, a = -1.36602, a = ∞, a = -∞]

0.0s Approx(#28)

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User
#1: [x1 :ε Real [0, 1], x2 :ε Real [0, 1], x3 :ε Real [0, 1], x4 :ε Real [0, 1], p1 :ε Real [0, ∞), p2 :ε Real [0, ∞), p3 :ε Real [0, ∞), p4 :ε Real [0, ∞)]
#2: F(p) := α·ē-β·p User
#3: [ x1 1 - x1 ] · [ F(p2) + a 1 - F(p2) - a / F(p2) 1 - F(p2) ] = [ x2 1 - x2 ] User
#4: [ x2 1 - x2 ] · [ F(p3) + a 1 - F(p3) - a / F(p3) 1 - F(p3) ] = [ x3 1 - x3 ] User
#5: [ x3 1 - x3 ] · [ F(p4) + a 1 - F(p4) - a / F(p4) 1 - F(p4) ] = [ x4 1 - x4 ] User
#6: [ x4 1 - x4 ] · [ F(p1) + a 1 - F(p1) - a / F(p1) 1 - F(p1) ] = [ x1 1 - x1 ] User
#7: [ α·ē-β·p2 + a·x1 = x2 - α·ē-β·p2 - a·x1 + 1 = 1 - x2 ] Simp(#3)
#8: [ α·ē-β·p3 + a·x2 = x3 - α·ē-β·p3 - a·x2 + 1 = 1 - x3 ] Simp(#4)
#9: [ α·ē-β·p4 + a·x3 = x4 - α·ē-β·p4 - a·x3 + 1 = 1 - x4 ] Simp(#5)
#10: [ α·ē-β·p1 + a·x4 = x1 - α·ē-β·p1 - a·x4 + 1 = 1 - x1 ] Simp(#6)
#11: SOLVE([α·ē-β·p2 + a·x1 = x2, α·ē-β·p3 + a·x2 = x3, α·ē-β·p4 + a·x3 = x4, α·ē-β·p1 + a·x4 = x1], [x1, x2, x3, x4]) User
Simp(#11)
#12: [ x1 = (α·ē-β·(p1+p2+p3+p4) · (a·ēβ·p1 · (ēβ·p2 · (ēβ·p3 + a·ēβ·p4) + a·ē2·β·(p3+p4)) + ēβ·(p2+p3+p4))) / (1 - a4) x2 =
      (α·ē-β·(p1+p2+p3+p4) · (ēβ·p1 · (a·ēβ·p2 · (ēβ·p3 + a·ēβ·p4) + ēβ·(p3+p4)) + a·ēβ·(p2+p3+p4))) / (1 - a4) x3 =
      (α·ē-β·(p1+p2+p3+p4) · (ēβ·p1 · (ēβ·p2 · (a·ēβ·p3 + ēβ·p4) + a·ēβ·(p3+p4)) + a·ē2·β·(p2+p3+p4))) / (1 - a4) x4 =
      (α·ē-β·(p1+p2+p3+p4) · (ēβ·p1 · (ēβ·p2 · (ēβ·p3 + a·ēβ·p4) + a·ē2·β·(p3+p4)) + a3·ēβ·(p2+p3+p4))) / (1 - a4) ]

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User

$$\#13: \left[ \begin{aligned} x1st &:= \frac{\alpha \cdot \dot{e}^{-\beta \cdot (p1 + p2 + p3 + p4)} \cdot (a \cdot \dot{e}^{\beta \cdot p1} \cdot (\dot{e}^{\beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p3} + a \cdot \dot{e}^{\beta \cdot p4}) + a \cdot \dot{e}^{2 \cdot \beta \cdot (p3 + p4)}) + \dot{e}^{\beta \cdot (p2 + p3 + p4)})}{1 - a^4}, x2st := \\ x3st &:= \frac{\alpha \cdot \dot{e}^{-\beta \cdot (p1 + p2 + p3 + p4)} \cdot (\dot{e}^{\beta \cdot p1} \cdot (a \cdot \dot{e}^{2 \cdot \beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p3} + a \cdot \dot{e}^{\beta \cdot p4}) + \dot{e}^{\beta \cdot (p3 + p4)}) + a \cdot \dot{e}^{\beta \cdot (p2 + p3 + p4)})}{1 - a^4}, x4st := \\ &\frac{\alpha \cdot \dot{e}^{-\beta \cdot (p1 + p2 + p3 + p4)} \cdot (\dot{e}^{\beta \cdot p1} \cdot (\dot{e}^{\beta \cdot p2} \cdot (a \cdot \dot{e}^{3 \cdot \beta \cdot p3} + \dot{e}^{\beta \cdot p4}) + a \cdot \dot{e}^{\beta \cdot (p3 + p4)}) + a \cdot \dot{e}^{2 \cdot \beta \cdot (p2 + p3 + p4)})}{1 - a^4}, \\ &\frac{\alpha \cdot \dot{e}^{-\beta \cdot (p1 + p2 + p3 + p4)} \cdot (\dot{e}^{\beta \cdot p1} \cdot (\dot{e}^{\beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p3} + a \cdot \dot{e}^{\beta \cdot p4}) + a \cdot \dot{e}^{2 \cdot \beta \cdot (p3 + p4)}) + a \cdot \dot{e}^{3 \cdot \beta \cdot (p2 + p3 + p4)})}{1 - a^4} \end{aligned} \right]$$

#14: p1·x1st + p2·x2st + p3·x3st + p4·x4st

User

Simp(#14)

$$\#15: \frac{\alpha \cdot \dot{e}^{-\beta \cdot p1 - \beta \cdot p2 - \beta \cdot p3 - \beta \cdot p4} \cdot (\dot{e}^{\beta \cdot p1} \cdot (\dot{e}^{\beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p3} \cdot (a \cdot p1 + a \cdot p2 + a \cdot p3 + p4) + \dot{e}^{\beta \cdot p4} \cdot (a \cdot p1 + a \cdot p2 + p3 + a \cdot p4)) + \dot{e}^{\beta \cdot p3 + \beta \cdot p4} \cdot (a \cdot p1 + p2 + a \cdot (p3 + a \cdot p4))) + a \cdot p4)) + \dot{e}^{\beta \cdot p2 + \beta \cdot p3 + \beta \cdot p4} \cdot (p1 + a \cdot (p2 + a \cdot (p3 + a \cdot p4)))}{1 - a^4}$$

Sub(#15)

$$\#16: \frac{\alpha \cdot \dot{e}^{-\beta \cdot p1 - \beta \cdot p2 - \beta \cdot p2 - \beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p1} \cdot (\dot{e}^{\beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p2} \cdot (a \cdot p1 + a \cdot p2 + a \cdot p2 + p2) + \dot{e}^{\beta \cdot p2} \cdot (a \cdot p1 + a \cdot p2 + p2 + a \cdot p2)) + \dot{e}^{\beta \cdot p2 + \beta \cdot p2} \cdot (a \cdot p1 + p2 + a \cdot (p2 + a \cdot p2))) + a \cdot p2)) + \dot{e}^{\beta \cdot p2 + \beta \cdot p2 + \beta \cdot p2} \cdot (p1 + a \cdot (p2 + a \cdot (p2 + a \cdot p2)))}{1 - a^4}$$

$$\#17: \frac{\alpha \cdot \dot{e}^{-\beta \cdot p1 - \beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p1} \cdot (a \cdot p1 \cdot (a^2 + a + 1) + p2 \cdot (2 \cdot a^3 + 2 \cdot a^2 + 2 \cdot a + 3)) + \dot{e}^{\beta \cdot p2} \cdot (p1 + a \cdot p2 \cdot (a^2 + a + 1)))}{1 - a^4}$$

Simp(#16)

$$\#18: rev := \frac{\alpha \cdot \dot{e}^{-\beta \cdot p1 - \beta \cdot p2} \cdot (\dot{e}^{\beta \cdot p1} \cdot (a \cdot p1 \cdot (a^2 + a + 1) + p2 \cdot (2 \cdot a^3 + 2 \cdot a^2 + 2 \cdot a + 3)) + \dot{e}^{\beta \cdot p2} \cdot (p1 + a \cdot p2 \cdot (a^2 + a + 1)))}{1 - a^4}$$

User

$$\#19: \left[ \frac{d}{d p1} rev, \frac{d}{d p2} rev \right]$$

User

Simp(#19)

$$\#20: \left[ \frac{\alpha \cdot \tilde{e}^{-\beta \cdot p1} \cdot (\beta \cdot p1 + a \cdot \beta \cdot p2 \cdot (a^2 + a + 1) - 1)}{a^4 - 1} + \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a^2 + a + 1)}{1 - a^4}, \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a^2 + a + 1)}{1 - a^4} + \frac{\alpha \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a \cdot \beta \cdot p1 \cdot (a^2 + a + 1) + (2 \cdot a^3 + 2 \cdot a^2 + 2 \cdot a + 3) \cdot (\beta \cdot p2 - 1))}{a^4 - 1} \right]$$

Sub(#20)

$$\#21: \left[ \frac{\alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot \left( \beta \cdot \frac{1}{\beta} + a \cdot \beta \cdot \frac{1}{\beta} \cdot (a^2 + a + 1) - 1 \right)}{a^4 - 1} + \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a^2 + a + 1)}{1 - a^4}, \frac{a \cdot \alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a^2 + a + 1)}{1 - a^4} + \frac{\alpha \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot \left( a \cdot \beta \cdot \frac{1}{\beta} \cdot (a^2 + a + 1) + (2 \cdot a^3 + 2 \cdot a^2 + 2 \cdot a + 3) \cdot \left( \beta \cdot \frac{1}{\beta} - 1 \right) \right)}{a^4 - 1} \right]$$

#22: [0, 0]

Simp(#21)

#23: 
$$\left[ \begin{array}{c} \left( \frac{d}{d \cdot p1} \right)^2 \text{ rev } \frac{d}{d \cdot p2} \frac{d}{d \cdot p1} \text{ rev} \\ \frac{d}{d \cdot p1} \frac{d}{d \cdot p2} \text{ rev } \left( \frac{d}{d \cdot p2} \right)^2 \text{ rev} \end{array} \right]$$

User

#24: 
$$\left[ \frac{\alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p1} \cdot (\beta \cdot p1 + a \cdot \beta \cdot p2 \cdot (a^2 + a + 1) - 2)}{1 - a^4}, \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a^2 + a + 1)}{a^4 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a^2 + a + 1)}{a^4 - 1} \right]$$

$$\frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p1} \cdot (a^2 + a + 1)}{a^4 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a^2 + a + 1)}{a^4 - 1} + \frac{\alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot p2} \cdot (a \cdot \beta \cdot p1 \cdot (a^2 + a + 1) + (2 \cdot a^3 + 2 \cdot a^2 + 2 \cdot a + 3) \cdot (\beta \cdot p2 - 2))}{1 - a^4}$$

Simp(#23)

Sub(#24)

$$\#25: \left[ \frac{\alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot \left( \beta \cdot \frac{1}{\beta} + a \cdot \beta \cdot \frac{1}{\beta} \cdot (a^2 + a + 1) - 2 \right)}{1 - a^4}, \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a^2 + a + 1)}{a^4 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a^2 + a + 1)}{a^4 - 1} \right]$$

$$\frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a^2 + a + 1)}{a^4 - 1} + \frac{a \cdot \alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot (a^2 + a + 1)}{a^4 - 1} + \frac{\alpha \cdot \beta \cdot \tilde{e}^{-\beta \cdot (1/\beta)} \cdot \left( a \cdot \beta \cdot \frac{1}{\beta} \cdot (a^2 + a + 1) + (2 \cdot a^3 + 2 \cdot a^2 + 2 \cdot a + 3) \cdot \left( \beta \cdot \frac{1}{\beta} - 2 \right) \right)}{1 - a^4}$$

#26: 
$$\begin{bmatrix} \frac{\alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^3 + a^2 + a - 1)}{1 - a^4} & \frac{2 \cdot a \cdot \alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^2 + a + 1)}{a^4 - 1} \\ \frac{2 \cdot a \cdot \alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^2 + a + 1)}{a^4 - 1} & \frac{\alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^3 + a^2 + a + 3)}{a^4 - 1} \end{bmatrix}$$

Simp(#25)

User

#27: "We have just obtained the Hessian matrix of the profit fuction at the point p1 = p2 = p\* = 1/β. The Hessian is negative definite at a = 0, and remains so for small positive values of a. We now calculate the smallest value of a for which this will no longer be true"

#28: DET 
$$\begin{bmatrix} \frac{\alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^3 + a^2 + a - 1)}{1 - a^4} & \frac{2 \cdot a \cdot \alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^2 + a + 1)}{a^4 - 1} \\ \frac{2 \cdot a \cdot \alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^2 + a + 1)}{a^4 - 1} & \frac{\alpha \cdot \beta \cdot \hat{e}^{-1} \cdot (a^3 + a^2 + a + 3)}{a^4 - 1} \end{bmatrix}$$

User

#29: 
$$-\frac{\alpha^2 \cdot \beta^2 \cdot \hat{e}^{-2} \cdot (5 \cdot a^3 + 5 \cdot a^2 + 5 \cdot a - 3)}{(a - 1)^2 \cdot (a^3 + a^2 + a + 1)}$$

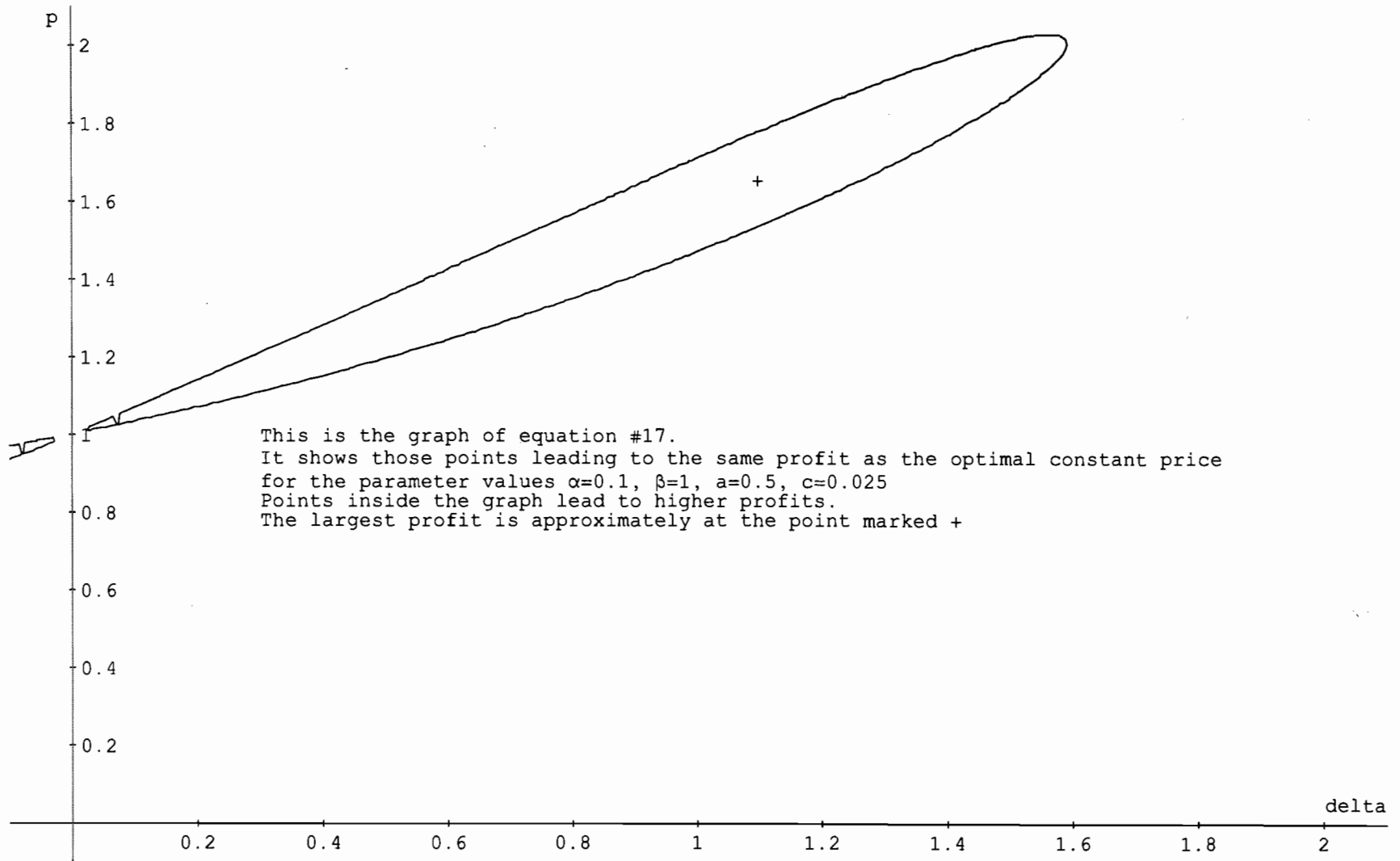
Simp(#28)

Solve(#29,a)

#30: 
$$\left[ a = -\left(\frac{2 \cdot \sqrt{11}}{15} - \frac{58}{135}\right)^{1/3} + \left(\frac{2 \cdot \sqrt{11}}{15} + \frac{58}{135}\right)^{1/3} - \frac{1}{3}, a = \infty, a = -\infty, a = \left(\frac{\sqrt{11}}{60} - \frac{29}{540}\right)^{1/3} - \left(\frac{\sqrt{11}}{60} + \frac{29}{540}\right)^{1/3} - \frac{1}{3} + i \cdot \left(\left(\frac{\sqrt{33}}{20} - \frac{29 \cdot \sqrt{3}}{180}\right)^{1/3} + \left(\frac{29 \cdot \sqrt{3}}{180} + \frac{\sqrt{33}}{20}\right)^{1/3}\right), a = \left(\frac{\sqrt{11}}{60} - \frac{29}{540}\right)^{1/3} - \left(\frac{\sqrt{11}}{60} + \frac{29}{540}\right)^{1/3} - \frac{1}{3} - i \cdot \left(\left(\frac{\sqrt{33}}{20} - \frac{29 \cdot \sqrt{3}}{180}\right)^{1/3} + \left(\frac{29 \cdot \sqrt{3}}{180} + \frac{\sqrt{33}}{20}\right)^{1/3}\right) \right]$$

#31: [a = 0.389365, a = ∞, a = -∞, a = -0.694682 + 1.02877·i, a = -0.694682 - 1.02877·i]

0.1s Approx(#30)



#1:  $[x1 : c \text{ Real } [0, 1], x2 : \epsilon \text{ Real } [0, 1]]$  User

#2:  $[F(p) := \alpha \cdot e^{-\beta \cdot p}, K(\delta) := c \cdot \delta^2]$  User

#3:  $[x2 \ 1 - x2] \cdot \begin{bmatrix} F(p) + a - K(\delta) & 1 - F(p) - a + K(\delta) \\ F(p) & 1 - F(p) \end{bmatrix} = [x1 \ 1 - x1]$  User

#4:  $[x1 \ 1 - x1] \cdot \begin{bmatrix} F(p - \delta) + a & 1 - F(p - \delta) - a \\ F(p - \delta) & 1 - F(p - \delta) \end{bmatrix} = [x2 \ 1 - x2]$  User

#5:  $[\alpha \cdot e^{-\beta \cdot p} + x2 \cdot (a - c \cdot \delta^2) = x1 - \alpha \cdot e^{-\beta \cdot p} - a \cdot x2 + c \cdot \delta^2 \cdot x2 + 1 = 1 - x1]$  Simp(#3)

#6:  $[\alpha \cdot e^{-\beta \cdot (\delta - p)} + a \cdot x1 = x2 - \alpha \cdot e^{-\beta \cdot (\delta - p)} - a \cdot x1 + 1 = 1 - x2]$  Simp(#4)

#7: SOLVE( $[\alpha \cdot e^{-\beta \cdot p} + x2 \cdot (a - c \cdot \delta^2) = x1, \alpha \cdot e^{-\beta \cdot (\delta - p)} + a \cdot x1 = x2], [x1, x2]$ ) User

#8:  $\left[ x1 = -\frac{\alpha \cdot e^{-\beta \cdot p} \cdot (\epsilon^{\beta \cdot \delta} \cdot (a - c \cdot \delta^2) + 1)}{a^2 - a \cdot c \cdot \delta^2 - 1}, x2 = -\frac{\alpha \cdot e^{-\beta \cdot p} \cdot (\epsilon^{\beta \cdot \delta} + a)}{a^2 - a \cdot c \cdot \delta^2 - 1} \right]$  Simp(#7)

#9:  $\left[ x1st := -\frac{\alpha \cdot e^{-\beta \cdot p} \cdot (\epsilon^{\beta \cdot \delta} \cdot (a - c \cdot \delta^2) + 1)}{a^2 - a \cdot c \cdot \delta^2 - 1}, x2st := -\frac{\alpha \cdot e^{-\beta \cdot p} \cdot (\epsilon^{\beta \cdot \delta} + a)}{a^2 - a \cdot c \cdot \delta^2 - 1} \right]$  User

#10: profit := p·x1st + (p - δ)·x2st User

#11:  $-\frac{\alpha \cdot e^{-\beta \cdot p} \cdot (\epsilon^{\beta \cdot \delta} \cdot (a \cdot p - c \cdot \delta^2 \cdot p - \delta + p) + a \cdot (p - \delta) + p)}{a^2 - a \cdot c \cdot \delta^2 - 1}$  0.2s Simp(#10)

#12: "We set  $p=1/\beta$  and  $\delta=0$  to obtain the profit using the optimal constant price." User

#13:  $-\frac{\alpha \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( \epsilon^{\beta \cdot 0} \cdot \left( a \cdot \frac{1}{\beta} - c \cdot 0 \cdot \frac{1}{\beta} - 0 + \frac{1}{\beta} \right) + a \cdot \left( \frac{1}{\beta} - 0 \right) + \frac{1}{\beta} \right)}{a^2 - a \cdot c \cdot 0^2 - 1}$  Sub(#11)

#14:  $\frac{2 \cdot \alpha \cdot e^{-1}}{\beta \cdot (1 - a)}$  0.1s Simp(#13)

#15:  $-\frac{\alpha \cdot e^{-\beta \cdot p} \cdot (\epsilon^{\beta \cdot \delta} \cdot (a \cdot p - c \cdot \delta^2 \cdot p - \delta + p) + a \cdot (p - \delta) + p)}{a^2 - a \cdot c \cdot \delta^2 - 1} = \frac{2 \cdot \alpha \cdot e^{-1}}{\beta \cdot (1 - a)}$  User

User

#16: "We display the graph of this equation for  $\alpha=0.1, \beta=1, a=0.5, c=0.025$ . The graph shows that we can increase the profit by choosing  $p$  larger than  $1/\beta$  and  $\delta$  positive, i.e., by promoting. We can also numerically search the interior of the graph to find the optimal values of  $p$  and  $\delta$ ."

#17:  $-\frac{0.1 \cdot e^{-1 \cdot p} \cdot (\epsilon^{1 \cdot \delta} \cdot (0.5 \cdot p - 0.025 \cdot \delta^2 \cdot p - \delta + p) + 0.5 \cdot (p - \delta) + p)}{0.5^2 - 0.5 \cdot 0.025 \cdot \delta^2 - 1} = \frac{2 \cdot 0.1 \cdot e^{-1}}{1 \cdot (1 - 0.5)}$  Sub(#15)

User

#18: "To see what other values of the parameters lead to similar behavior of the profit function, we first verify that the first order conditions for optimality always hold if  $p=1/\beta$  and  $\delta=0$ , and then calculate the determinant of the Hessian matrix at that point."

#19:  $\left[ \frac{d}{dp} \text{profit}, \frac{d}{d\delta} \text{profit} \right]$

User

#20: 
$$\left[ \frac{\alpha \cdot e^{-\beta \cdot p} \cdot (\beta \cdot \delta \cdot (a \cdot (\beta \cdot p - 1) - \beta \cdot (c \cdot \delta^2 \cdot p + \delta - p) + c \cdot \delta^2 - 1) - a \cdot (\beta \cdot (\delta - p) + 1) + \beta \cdot p - 1)}{a^2 - a \cdot c \cdot \delta^2 - 1}, \frac{a \cdot \alpha \cdot e^{-\beta \cdot p} \cdot (a^2 + a \cdot c \cdot \delta \cdot (\delta - 2 \cdot p) - 2 \cdot c \cdot \delta \cdot p - 1)}{(a^2 - a \cdot c \cdot \delta^2 - 1)^2} \right]$$

#21: 
$$\left[ \frac{\alpha \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( e^{\beta \cdot 0} \cdot \left( a \cdot \left( \beta \cdot \frac{1}{\beta} - 1 \right) - \beta \cdot \left( c \cdot 0 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) + c \cdot 0^2 - 1 \right) - a \cdot \left( \beta \cdot \left( 0 - \frac{1}{\beta} \right) + 1 \right) + \beta \cdot \frac{1}{\beta} - 1 \right)}{a^2 - a \cdot c \cdot 0^2 - 1}, \frac{a \cdot \alpha \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( a^2 + a \cdot c \cdot 0 \cdot \left( 0 - 2 \cdot \frac{1}{\beta} \right) - 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} - 1 \right)}{(a^2 - a \cdot c \cdot 0^2 - 1)^2} \right]$$

0.1s Simp(#21)

#22: [0, 0]

#23: 
$$\left[ \left( \frac{d}{dp} \right)^2 \text{profit} \quad \frac{d}{d\delta} \frac{d}{dp} \text{profit} \right]$$

User

2.6s Simp(#23)

#24:

$$\frac{\alpha \cdot \beta \cdot \delta^{-\beta \cdot p} \cdot (\delta^{\beta \cdot \delta} \cdot (a \cdot (\beta \cdot p - 2) - \beta \cdot (c \cdot \delta^2 \cdot p + \delta - p) + 2 \cdot (c \cdot \delta^2 - 1)) - a \cdot (\beta \cdot (\delta - p) + 2) - \beta \cdot p}{a^2 - a \cdot c \cdot \delta^2 - 1} + \frac{\alpha \cdot \delta^{-\beta \cdot p} \cdot (\delta^{\beta \cdot \delta} \cdot (a \cdot \beta \cdot (\beta \cdot p - 1) + a^2 \cdot \beta \cdot (2 \cdot (c \cdot \delta^2 - 1) - \beta \cdot (2 \cdot c \cdot \delta^2 \cdot p + \delta - p)) + a \cdot (\beta \cdot (c \cdot \delta^2 \cdot p + c \cdot \delta \cdot (\delta - p) - p) - \beta \cdot (c \cdot \delta^2 - 2 \cdot c \cdot \delta \cdot p - 1) - 2 \cdot c \cdot \delta) + \beta^2 \cdot (c \cdot \delta^2 \cdot p + \delta - p) + \beta \cdot p - 2)}{(a^2 - a \cdot c \cdot \delta^2 - 1)^2}$$

) +  $\beta \cdot p - 2$

$$\frac{\beta \cdot (2 - c \cdot \delta \cdot (\delta - 2 \cdot p)) - 2 \cdot c \cdot \delta - a \cdot (a \cdot \beta + a \cdot c \cdot \delta \cdot (\beta \cdot (\delta - 2 \cdot p) + 2) - \beta \cdot (2 \cdot c \cdot \delta \cdot p + 1) + 2 \cdot c \cdot \delta)}{(a^2 - a \cdot c \cdot \delta^2 - 1)^2}$$

$$\frac{2 \cdot a \cdot \alpha \cdot c \cdot \delta^{-\beta \cdot p} \cdot (a \cdot (3 \cdot \delta - p) + a^2 \cdot (c \cdot \delta^2 \cdot (\delta - 3 \cdot p) - p) - a \cdot (3 \cdot c \cdot \delta^2 \cdot p + 3 \cdot \delta - p) + p)}{(a^2 - a \cdot c \cdot \delta^2 - 1)^3} + \frac{\alpha \cdot \delta^{-\beta \cdot p} \cdot (\delta^{\beta \cdot \delta} \cdot (a \cdot \beta \cdot (\beta \cdot p - 1) + a^2 \cdot \beta \cdot (2 \cdot (c \cdot \delta^2 - 1) - \beta \cdot (2 \cdot c \cdot \delta^2 \cdot p + \delta - p)) + a \cdot (\beta \cdot (c \cdot \delta^2 \cdot p + c \cdot \delta \cdot (\delta - p) - p) - \beta \cdot (c \cdot \delta^2 - 2 \cdot c \cdot \delta \cdot p - 1) - 2 \cdot c \cdot \delta) + \beta^2 \cdot (c \cdot \delta^2 \cdot p + \delta - p) + \beta \cdot p - 2)}{(a^2 - a \cdot c \cdot \delta^2 - 1)^2}$$

$$\frac{\alpha \cdot \delta^{-\beta \cdot p} \cdot (\delta^{\beta \cdot \delta} \cdot (a \cdot \beta \cdot p - a \cdot \beta \cdot (\beta \cdot (3 \cdot c \cdot \delta^2 \cdot p + \delta - p) + 2) + a \cdot (\beta \cdot (3 \cdot c \cdot \delta^2 \cdot p + 2 \cdot c \cdot \delta \cdot (\delta - p) - 2 \cdot p) + 4 \cdot \beta \cdot c \cdot \delta \cdot p + 2 \cdot c \cdot (p - 3 \cdot \delta)) - a \cdot (\beta \cdot (c \cdot \delta^2 \cdot p + c \cdot \delta \cdot (\delta - p) - 4 \cdot c \cdot \delta^2 \cdot p + \delta - p) + \beta \cdot (2 - c \cdot \delta \cdot (\delta - 2 \cdot p)) - 2 \cdot c \cdot \delta)}{(a^2 - a \cdot c \cdot \delta^2 - 1)^2} + \frac{a \cdot \alpha \cdot \delta^{-\beta \cdot p} \cdot (a \cdot \beta + a \cdot c \cdot \delta \cdot (\beta \cdot (\delta - 2 \cdot p) + 2) - \beta \cdot (2 \cdot c \cdot \delta \cdot p + 1) + 2 \cdot c \cdot \delta)}{(a^2 - a \cdot c \cdot \delta^2 - 1)^2}$$

$$\frac{p \cdot 2 \cdot (\delta - p) - 2 \cdot \beta \cdot (c \cdot \delta^2 \cdot (\delta - 2 \cdot p) + 2 \cdot c \cdot \delta \cdot p + 2) + 2 \cdot c \cdot (c \cdot \delta^2 \cdot (\delta - 3 \cdot p) - p) - a \cdot (\beta \cdot (2 \cdot c \cdot \delta^2 \cdot p + 2 \cdot c \cdot \delta \cdot (\delta - p) - p) + 4 \cdot \beta \cdot c \cdot \delta \cdot p \cdot (c \cdot \delta^2 + 1) - 2 \cdot c \cdot (3 \cdot c \cdot \delta^2 \cdot p + 3 \cdot \delta - p))}{(a^2 - a \cdot c \cdot \delta^2 - 1)^3}$$

$$\frac{\beta^2 \cdot (c \cdot \delta^2 \cdot p + \delta - p) - 2 \cdot (\beta \cdot (2 \cdot c \cdot \delta \cdot p + 1) + c \cdot p)}{}$$

Sub(#24)

#25:

$$\frac{\alpha \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( a \cdot \beta \cdot \left( \beta \cdot \frac{1}{\beta} - 2 \right) - \beta \cdot \left( c \cdot 0^2 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) + 2 \cdot (c \cdot 0^2 - 1) \right)}{a^2 - a \cdot c \cdot 0^2 - 1} - \frac{\alpha \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( a^2 \cdot \beta \cdot \left( 2 \cdot (c \cdot 0^2 - 1) - \beta \cdot \left( 2 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) \right) + a \cdot \left( \beta \cdot \left( c \cdot 0^2 \cdot \frac{1}{\beta} + c \cdot 0^2 \cdot \left( 0 - \frac{1}{\beta} \right) - \frac{1}{\beta} \right) - \beta \cdot \left( c \cdot 0^2 \cdot 4 - 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} - 1 \right) - 2 \cdot c \cdot 0 \right) + \beta^2}{(a^2 - a \cdot c \cdot 0^2 - 1)^2}$$

$$a \cdot \left( \beta \cdot \left( 0 - \frac{1}{\beta} \right) + 2 \right) + \beta \cdot \frac{1}{\beta} - 2$$

$$\left( c \cdot 0^2 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) + \beta \cdot \left( 2 - c \cdot 0 \cdot \left( 0 - 2 \cdot \frac{1}{\beta} \right) \right) - 2 \cdot c \cdot 0 - a \cdot \left( a^2 \cdot \beta + a \cdot c \cdot 0 \cdot \left( \beta \cdot \left( 0 - 2 \cdot \frac{1}{\beta} \right) + 2 \right) - \beta \cdot \left( 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} + 1 \right) + 2 \cdot c \cdot 0 \right)$$

$$\frac{2 \cdot a \cdot \alpha \cdot c \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( a^3 \cdot \left( 3 \cdot 0 - \frac{1}{\beta} \right) + a^2 \cdot \left( c \cdot 0^2 \cdot \left( 0 - 3 \cdot \frac{1}{\beta} \right) - \frac{1}{\beta} \right) - a \cdot \left( 3 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} + 3 \cdot 0 - \frac{1}{\beta} \right) + \frac{1}{\beta} \right)}{(a^2 - a \cdot c \cdot 0^2 - 1)^3}$$

$$\frac{\alpha \cdot e^{\beta \cdot (0 - 1/\beta)} \cdot \left( a^3 \cdot \beta \cdot \left( \beta \cdot \frac{1}{\beta} - 1 \right) + a^2 \cdot \beta \cdot \left( 2 \cdot (c \cdot 0^2 - 1) - \beta \cdot \left( 2 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) \right) + a \cdot \left( \beta \cdot \left( c \cdot 0^2 \cdot \frac{1}{\beta} + c \cdot 0^2 \cdot \left( 0 - \frac{1}{\beta} \right) - \frac{1}{\beta} \right) - \beta \cdot \left( c \cdot 0^2 \cdot 4 - 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} - 1 \right) - 1 \right)}{(a^2 - a \cdot c \cdot 0^2 - 1)^2}$$

$$\frac{\alpha \cdot e^{\beta \cdot (0 - 1/\beta)} \cdot \left( a^5 \cdot \beta^2 \cdot \frac{1}{\beta} - a^4 \cdot \beta \cdot \left( \beta \cdot \left( 3 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) + 2 \right) + a^3 \cdot \left( \beta \cdot \left( 3 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} + 2 \cdot c \cdot 0^2 \cdot \left( 0 - \frac{1}{\beta} \right) - 2 \cdot \frac{1}{\beta} \right) + 4 \cdot \beta \cdot c \cdot 0 \cdot \frac{1}{\beta} + 2 \cdot c \cdot \left( \frac{1}{\beta} - 3 \cdot 0 \right) \right) - a^2 \cdot \left( \beta \cdot \left( c \cdot 0^2 \cdot \frac{1}{\beta} + c \cdot 0^2 \cdot \left( 0 - \frac{1}{\beta} \right) - \frac{1}{\beta} \right) - \beta \cdot \left( c \cdot 0^2 \cdot 4 - 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} - 1 \right) - 1 \right)}{(a^2 - a \cdot c \cdot 0^2 - 1)^2}$$

$$\frac{\left( -2 \cdot c \cdot 0 \right) + \beta^2 \cdot \left( c \cdot 0^2 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) + \beta \cdot \left( 2 - c \cdot 0 \cdot \left( 0 - 2 \cdot \frac{1}{\beta} \right) \right) - 2 \cdot c \cdot 0}{a \cdot \alpha \cdot e^{-\beta \cdot (1/\beta)} \cdot \left( a^2 \cdot \beta + a \cdot c \cdot 0 \cdot \left( \beta \cdot \left( 0 - 2 \cdot \frac{1}{\beta} \right) + 2 \right) - \beta \cdot \left( 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} + 1 \right) + 2 \cdot c \cdot 0 \right)} - \frac{\left( 0 \cdot \left( 0 - \frac{1}{\beta} \right) - 4 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} - 2 \cdot \left( 0 - \frac{1}{\beta} \right) \right) - 2 \cdot \beta \cdot \left( c \cdot 0^2 \cdot \left( 0 - 2 \cdot \frac{1}{\beta} \right) + 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} + 2 \right) + 2 \cdot c \cdot \left( c \cdot 0^2 \cdot \left( 0 - 3 \cdot \frac{1}{\beta} \right) - \frac{1}{\beta} \right) - a \cdot \left( \beta \cdot \left( 2 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} + 2 \cdot c \cdot 0 \cdot \left( 0 - \frac{1}{\beta} \right) - \frac{1}{\beta} \right) + 4 \cdot \beta \cdot c \cdot 0 \right)}{\left( a^2 - a \cdot c \cdot 0^2 - 1 \right)^2}$$

$$\frac{\left( \frac{1}{\beta} \cdot (c \cdot 0^2 + 1) - 2 \cdot c \cdot \left( 3 \cdot c \cdot 0^2 \cdot \frac{1}{\beta} + 3 \cdot 0 - \frac{1}{\beta} \right) \right) - \beta^2 \cdot \left( c \cdot 0^2 \cdot \frac{1}{\beta} + 0 - \frac{1}{\beta} \right) - 2 \cdot \left( \beta \cdot \left( 2 \cdot c \cdot 0 \cdot \frac{1}{\beta} + 1 \right) + c \cdot \frac{1}{\beta} \right)}{\left( a^2 - a \cdot c \cdot 0^2 - 1 \right)^3}$$

#26:  $\left[ \begin{array}{cc} \frac{2 \cdot \alpha \cdot \beta \cdot e^{-1}}{a - 1} & \frac{\alpha \cdot \beta \cdot e^{-1}}{1 - a} \\ \frac{\alpha \cdot \beta \cdot e^{-1}}{1 - a} & \frac{\alpha \cdot e^{-1} \cdot (a \cdot \beta^2 + 2 \cdot a \cdot (c - \beta)^2 + \beta^2 + 2 \cdot c)}{\beta \cdot (a + 1) \cdot (a - 1)^2} \end{array} \right]$  0.6s Simp(#25)

#27: DET  $\left[ \begin{array}{cc} \frac{2 \cdot \alpha \cdot \beta \cdot e^{-1}}{a - 1} & \frac{\alpha \cdot \beta \cdot e^{-1}}{1 - a} \\ \frac{\alpha \cdot \beta \cdot e^{-1}}{1 - a} & \frac{\alpha \cdot e^{-1} \cdot (a \cdot \beta^2 + 2 \cdot a \cdot (c - \beta)^2 + \beta^2 + 2 \cdot c)}{\beta \cdot (a + 1) \cdot (a - 1)^2} \end{array} \right]$  User

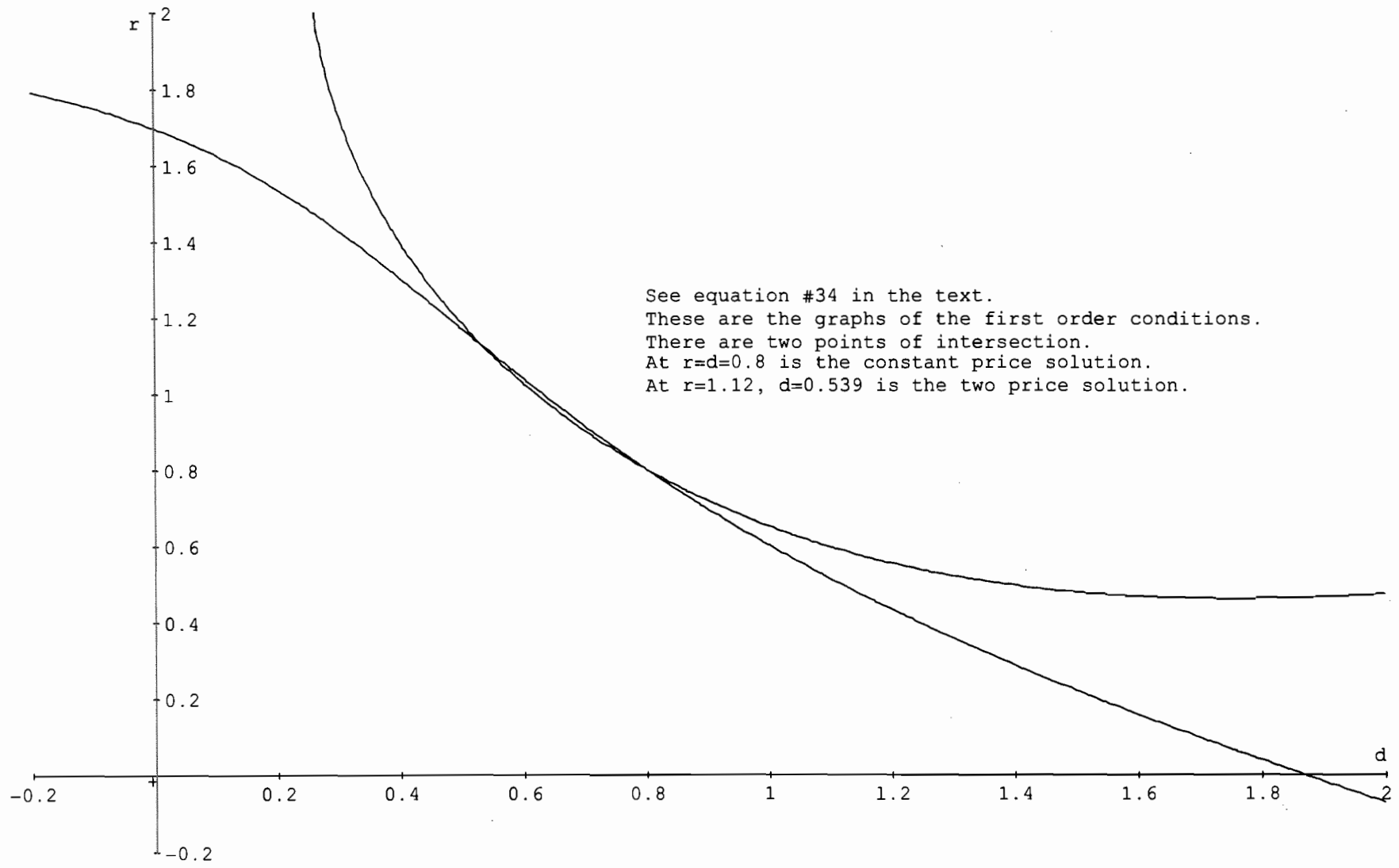
#28:  $\frac{\alpha^2 \cdot e^{-2} \cdot (3 \cdot a^2 \cdot \beta^2 + 4 \cdot a \cdot (c - \beta)^2 + \beta^2 + 4 \cdot c)}{(a + 1) \cdot (1 - a)^3}$  0.0s Simp(#27)

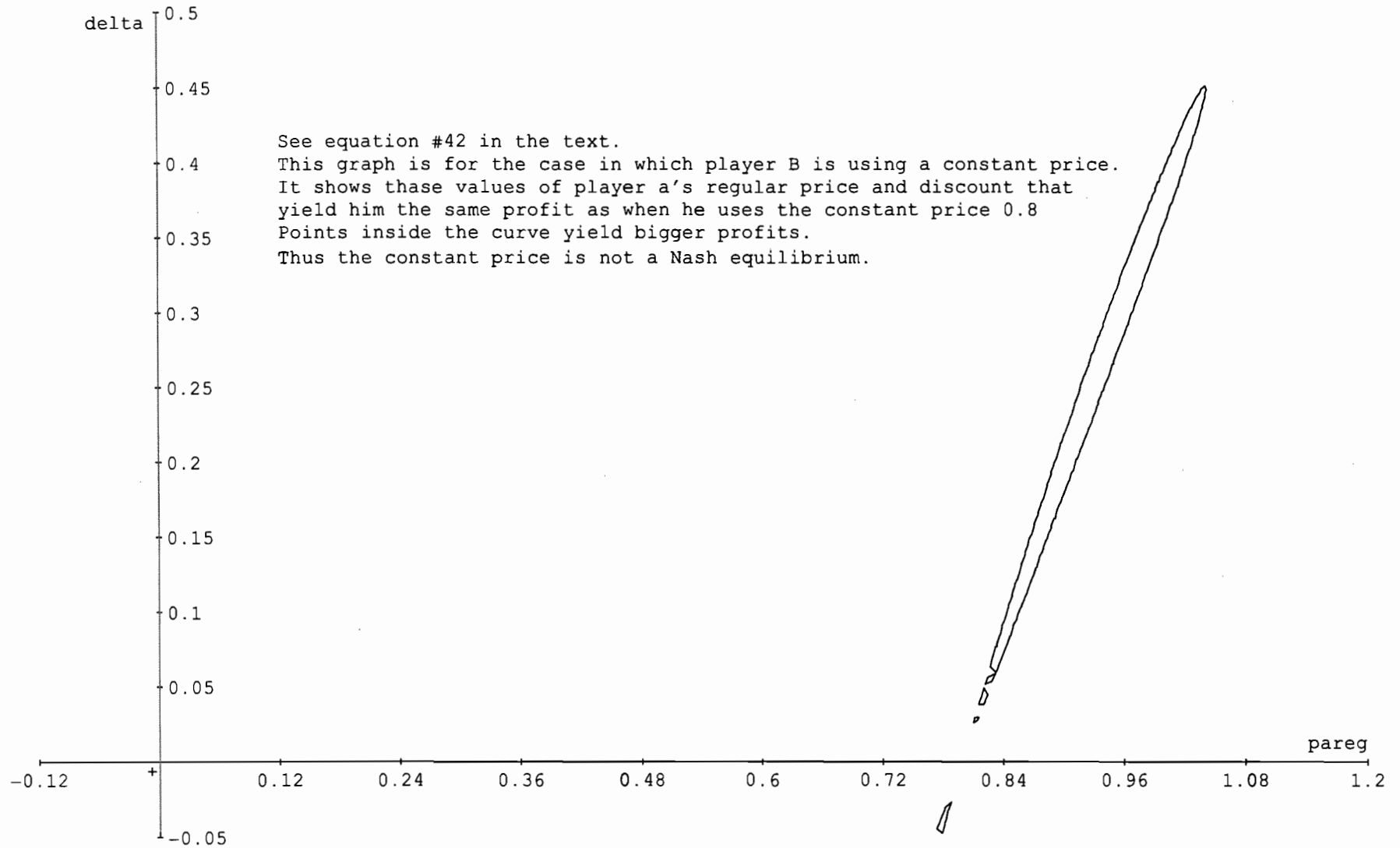
#29: "The sign of this determinant is the same as the sign of" User

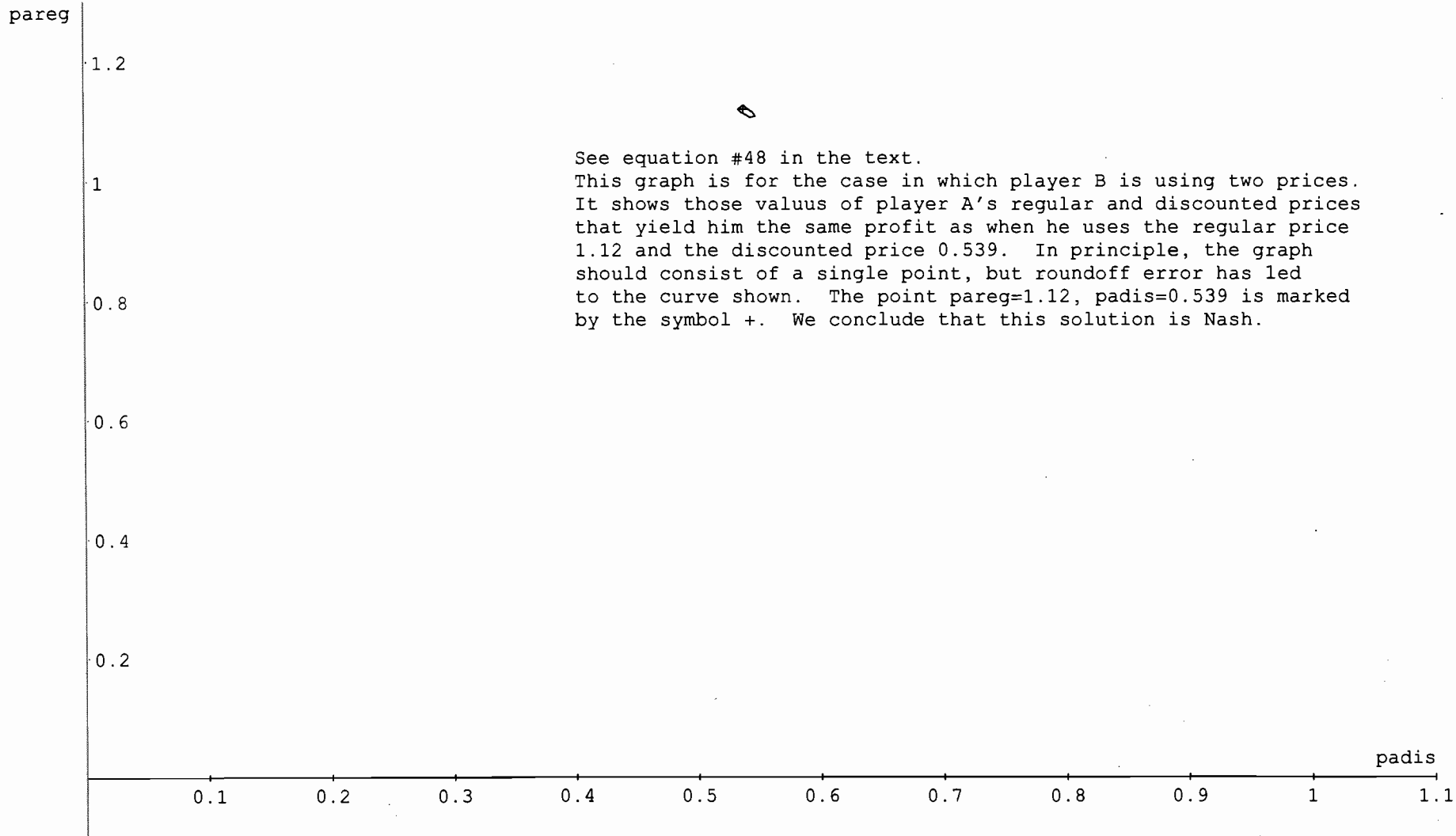
#30:  $3 \cdot a^2 \cdot \beta^2 + 4 \cdot a \cdot (c - \beta)^2 + \beta^2 + 4 \cdot c$  User

#31: "Whenever c is less than the following expression, the profit function will exhibit the behavior described above." User

#32:  $\left[ c = - \frac{\beta^2 \cdot (3 \cdot a^2 - 4 \cdot a + 1)}{4 \cdot (a + 1)} \right]$  0.0s Solve(#30,c)







See equation #48 in the text.  
This graph is for the case in which player B is using two prices. It shows those values of player A's regular and discounted prices that yield him the same profit as when he uses the regular price 1.12 and the discounted price 0.539. In principle, the graph should consist of a single point, but roundoff error has led to the curve shown. The point  $pareg=1.12$ ,  $padis=0.539$  is marked by the symbol +. We conclude that this solution is Nash.

```
#1: [FA(α, β, p) :=, FB(α, β, q) :=, H(δ) :=] User
#2: [aa :c Real [0, ∞), ab :c Real [0, ∞), ba :c Real [0, ∞), bb :c Real [0, ∞)] User
#3: [pareg :c Real [0, ∞), padis :c Real [0, ∞), pbreg :c Real [0, ∞), pbdis :c Real [0, ∞)] User
#4: [xal :c Real [0, ∞), xa2 :c Real [0, ∞), xbl :c Real [0, ∞), xb2 :c Real [0, ∞)] User
#5: VariableOrder := [x, y, z, xal, xa2, xbl, xb2, padis, pareg, pbdis, pbreg] User
```

```
User
#6: [ xal xbl 1 - xal - xbl ] = [ xal xbl 1 - xal - xbl ] · 
$$\begin{bmatrix} FA(\alpha, \beta, padis) + aa & FB(\alpha, \beta, pbreg) + bb \\ FA(\alpha, \beta, padis) + ba & FB(\alpha, \beta, pbreg) + ab - H(pbreg - pbdis) & 1 - FA(\alpha, \\ FA(\alpha, \beta, padis) & FB(\alpha, \beta, pbreg) \\ 1 - FA(\alpha, \beta, padis) - FB(\alpha, \beta, pbreg) - aa - bb \\ \beta, padis) - FB(\alpha, \beta, pbreg) - ba - ab + H(pbreg - pbdis) \\ 1 - FA(\alpha, \beta, padis) - FB(\alpha, \beta, pbreg) \\ \beta, pareg) - FB(\alpha, \beta, pbdis) - aa - bb + H(pareg - padis) \\ 1 - FA(\alpha, \beta, pareg) - FB(\alpha, \beta, pbdis) - ba - ab \\ 1 - FA(\alpha, \beta, pareg) - FB(\alpha, \beta, pbdis) \end{bmatrix} \cdot \begin{bmatrix} FA(\alpha, \beta, pareg) + aa - H(pareg - padis) & FB(\alpha, \beta, pbdis) + bb & 1 - FA(\alpha, \\ FA(\alpha, \beta, pareg) + ba & FB(\alpha, \beta, pbdis) + ab \\ FA(\alpha, \beta, pareg) & FB(\alpha, \beta, pbdis) \end{bmatrix}$$

```

Expd(#6)

```
#7: [ xal = - FA(α, β, padis)·H(pareg - padis) + aa·FA(α, β, padis) + FA(α, β, pareg) + ba·FB(α, β, pbreg) - aa·xal·H(pareg - padis) - ba·xbl·H(pareg - padis) - ba·xbl·H(pbreg - pbdis) + xal·(aa2 + ba·bb) + xbl·(aa·ba + ab·ba) xbl = bb·FA(α, β, padis) + FB(α, β, pbdis) + ab·FB(α, β, pbreg) - ab·xbl·H(pbreg - pbdis) + xal·(aa·bb + ab·bb) + xbl·(ab2 + ba·bb) -xal - xbl + 1 = FA(α, β, padis)·H(pareg - padis) - (aa + bb)·FA(α, β, padis) - FA(α, β, pareg) - FB(α, β, pbdis) - (ab + ba)·FB(α, β, pbreg) + aa·xal·H(pareg - padis) + ba·xbl·H(pareg - padis) + xbl·(ab + ba)·H(pbreg - pbdis) - xal·(aa2 + aa·bb + bb·(ab + ba)) - xbl·(aa·ba + ab2 + ab·ba + ba·bb) + 1 ]
```

User

```
#8: SOLVE([ xal = - FA(α, β, padis)·H(pareg - padis) + aa·FA(α, β, padis) + FA(α, β, pareg) + ba·FB(α, β, pbreg) - aa·xal·H(pareg - padis) - ba·xbl·H(pareg - padis) - ba·xbl·H(pbreg - pbdis) + xal·(aa2 + ba·bb) + xbl·(aa·ba + ab·ba), xbl = bb·FA(α, β, padis) + FB(α, β, pbdis) + ab·FB(α, β, pbreg) - ab·xbl·H(pbreg - pbdis) + xal·(aa·bb + ab·bb) + xbl·(ab2 + ba·bb) ], [xal, xbl])
```

0.7s Simp(#8)

#9: xa1 = -

$$\frac{\text{FA}(\alpha, \beta, \text{padis}) \cdot (\text{H}(\text{pareg} - \text{padis}) \cdot (\text{ab} \cdot \text{H}(\text{pbreg} - \text{pbdis}) - \text{ab}^2 + 1) + (\text{ba} \cdot \text{bb} - \text{aa} \cdot \text{ab}) \cdot \text{H}(\text{pbreg} - \text{pbdis}) + \text{aa} \cdot (\text{ab}^2 - 1) - \text{ab} \cdot \text{ba} \cdot \text{bb}) - \text{FA}(\alpha, \beta, \text{pareg}) \cdot (\text{ab} \cdot \text{H}(\text{pbreg} - \text{pbdis}) - \text{ab}^2 - \text{ba} \cdot \text{bb} + 1) + \text{ba} \cdot (\text{FB}(\alpha, \beta, \text{pbdis}) \cdot (\text{H}(\text{pareg} - \text{padis}) + \text{H}(\text{pbreg} - \text{pbdis}) - \text{aa} - \text{ab}) + \text{FB}(\alpha, \beta, \text{pbreg}) \cdot (\text{ab} \cdot \text{H}(\text{pareg} - \text{padis}) - \text{aa} \cdot \text{ab} + \text{ba} \cdot \text{bb} - 1))}{\text{H}(\text{pareg} - \text{padis}) \cdot (\text{aa} \cdot \text{ab} \cdot \text{H}(\text{pbreg} - \text{pbdis}) + \text{aa} \cdot (1 - \text{ab}^2) + \text{ab} \cdot \text{ba} \cdot \text{bb}) - (\text{aa}^2 \cdot \text{ab} - \text{aa} \cdot \text{ba} \cdot \text{bb} - \text{ab}) \cdot (\text{H}(\text{pbreg} - \text{pbdis}) + \text{aa} \cdot (\text{ab}^2 - 1) - 2 \cdot \text{aa} \cdot \text{ab} \cdot \text{ba} \cdot \text{bb} - \text{ab}^2 + (\text{ba} \cdot \text{bb} - 1)^2)} \quad \text{xb1} = -$$

$$\frac{\text{bb} \cdot \text{FA}(\alpha, \beta, \text{padis}) \cdot (\text{ab} \cdot \text{H}(\text{pareg} - \text{padis}) - \text{aa} \cdot \text{ab} + \text{ba} \cdot \text{bb} - 1) - \text{bb} \cdot (\text{aa} + \text{ab}) \cdot \text{FA}(\alpha, \beta, \text{pareg}) - \text{FB}(\alpha, \beta, \text{pbdis}) \cdot (\text{aa} \cdot \text{H}(\text{pareg} - \text{padis}) - \text{aa}^2 - \text{ba} \cdot \text{bb} + 1) - \text{FB}(\alpha, \beta, \text{pbreg}) \cdot (\text{aa} \cdot \text{ab} \cdot \text{H}(\text{pareg} - \text{padis}) - \text{aa} \cdot \text{ab} + \text{ba} \cdot \text{bb} + \text{ab})}{\text{H}(\text{pareg} - \text{padis}) \cdot (\text{aa} \cdot \text{ab} \cdot \text{H}(\text{pbreg} - \text{pbdis}) + \text{aa} \cdot (1 - \text{ab}^2) + \text{ab} \cdot \text{ba} \cdot \text{bb}) - (\text{aa}^2 \cdot \text{ab} - \text{aa} \cdot \text{ba} \cdot \text{bb} - \text{ab}) \cdot (\text{H}(\text{pbreg} - \text{pbdis}) + \text{aa} \cdot (\text{ab}^2 - 1) - 2 \cdot \text{aa} \cdot \text{ab} \cdot \text{ba} \cdot \text{bb} - \text{ab}^2 + (\text{ba} \cdot \text{bb} - 1)^2)}$$

User

#10: [ xa2 xb2 1 - xa2 - xb2 ] = [ xa2 xb2 1 - xa2 - xb2 ] ·

$\text{FA}(\alpha, \beta, \text{pareg}) + \text{aa} - \text{H}(\text{pareg} - \text{padis})$	$\text{FB}(\alpha, \beta, \text{pbdis}) + \text{bb} - 1 - \text{FA}(\alpha, \beta, \text{pareg}) + \text{ba}$	$\text{FB}(\alpha, \beta, \text{pbdis}) + \text{ab}$
$\beta, \text{pareg}) - \text{FB}(\alpha, \beta, \text{pbdis}) - \text{aa} - \text{bb} + \text{H}(\text{pareg} - \text{padis})$	$\text{FA}(\alpha, \beta, \text{pareg}) + \text{ba}$	$\text{FB}(\alpha, \beta, \text{pbdis})$
$1 - \text{FA}(\alpha, \beta, \text{pareg}) - \text{FB}(\alpha, \beta, \text{pbdis}) - \text{ba} - \text{ab}$	$\text{FA}(\alpha, \beta, \text{pareg})$	$\text{FB}(\alpha, \beta, \text{pbdis})$
$1 - \text{FA}(\alpha, \beta, \text{pareg}) - \text{FB}(\alpha, \beta, \text{pbdis})$	$\text{FA}(\alpha, \beta, \text{pareg}) + \text{aa}$	$\text{FB}(\alpha, \beta, \text{pbreg}) + \text{bb}$
$1 - \text{FA}(\alpha, \beta, \text{padis}) - \text{FB}(\alpha, \beta, \text{pbreg}) - \text{aa} - \text{bb}$	$\text{FA}(\alpha, \beta, \text{padis}) + \text{ba}$	$\text{FB}(\alpha, \beta, \text{pbreg}) + \text{ab} - \text{H}(\text{pbreg} - \text{pbdis})$
$\beta, \text{padis}) - \text{FB}(\alpha, \beta, \text{pbreg}) - \text{ba} - \text{ab} + \text{H}(\text{pbreg} - \text{pbdis})$	$\text{FA}(\alpha, \beta, \text{padis})$	$\text{FB}(\alpha, \beta, \text{pbreg})$
$1 - \text{FA}(\alpha, \beta, \text{padis}) - \text{FB}(\alpha, \beta, \text{pbreg})$		

Expd(#10)

#11: xa2 =  $\text{FA}(\alpha, \beta, \text{padis}) + \text{aa} \cdot \text{FA}(\alpha, \beta, \text{pareg}) + \text{ba} \cdot \text{FB}(\alpha, \beta, \text{pbdis}) - \text{aa} \cdot \text{xa2} \cdot \text{H}(\text{pareg} - \text{padis}) + \text{xa2} \cdot (\text{aa}^2 + \text{ba} \cdot \text{bb}) + \text{xb2} \cdot (\text{aa} \cdot \text{ba} + \text{ab} \cdot \text{ba})$      $\text{xb2} = \text{bb} \cdot \text{FA}(\alpha, \beta, \text{pareg}) - \text{FB}(\alpha, \beta, \text{pbdis}) \cdot \text{H}(\text{pbreg} - \text{pbdis}) + \text{ab} \cdot \text{FB}(\alpha, \beta, \text{pbdis}) + \text{FB}(\alpha, \beta, \text{pbreg}) - \text{bb} \cdot \text{xa2} \cdot \text{H}(\text{pareg} - \text{padis}) - \text{bb} \cdot \text{xa2} \cdot \text{H}(\text{pbreg} - \text{pbdis}) - \text{ab} \cdot \text{xb2} \cdot \text{H}(\text{pbreg} - \text{pbdis}) + \text{xa2} \cdot (\text{aa} \cdot \text{bb} + \text{ab} \cdot \text{bb}) + \text{xb2} \cdot (\text{ab}^2 + \text{ba} \cdot \text{bb})$

$- \text{xa2} - \text{xb2} + 1 = - \text{FA}(\alpha, \beta, \text{padis}) - (\text{aa} + \text{bb}) \cdot \text{FA}(\alpha, \beta, \text{pareg}) + \text{FB}(\alpha, \beta, \text{pbdis}) \cdot \text{H}(\text{pbreg} - \text{pbdis}) - (\text{ab} + \text{ba}) \cdot \text{FB}(\alpha, \beta, \text{pbdis}) - \text{FB}(\alpha, \beta, \text{pbreg}) + \text{xa2} \cdot (\text{aa} + \text{bb}) \cdot \text{H}(\text{pareg} - \text{padis}) + \text{bb} \cdot \text{xa2} \cdot \text{H}(\text{pbreg} - \text{pbdis}) + \text{ab} \cdot \text{xb2} \cdot \text{H}(\text{pbreg} - \text{pbdis}) - \text{xa2} \cdot (\text{aa}^2 + \text{aa} \cdot \text{bb} + \text{bb} \cdot (\text{ab} + \text{ba})) - \text{xb2} \cdot (\text{aa} \cdot \text{ba} + \text{ab}^2 + \text{ab} \cdot \text{ba} + \text{ba} \cdot \text{bb}) + 1$

#12: "For some unknown reason the software command SOLVE would not solve these equations. To get around this difficulty required a number of steps" User

#13: xa2 =  $\frac{\text{FA}(\alpha, \beta, \text{padis}) + \text{aa} \cdot \text{FA}(\alpha, \beta, \text{pareg}) + \text{ba} \cdot (\text{FB}(\alpha, \beta, \text{pbdis}) + \text{xb2} \cdot (\text{aa} + \text{ab}))}{\text{aa} \cdot \text{H}(\text{pareg} - \text{padis}) - \text{aa}^2 - \text{ba} \cdot \text{bb} + 1}$  Solve(#11', xa2)

#14: xa2 =  $\frac{\text{bb} \cdot \text{FA}(\alpha, \beta, \text{pareg}) + \text{FB}(\alpha, \beta, \text{pbdis}) \cdot (\text{ab} - \text{H}(\text{pbreg} - \text{pbdis})) + \text{FB}(\alpha, \beta, \text{pbreg}) - \text{xb2} \cdot (\text{ab} \cdot \text{H}(\text{pbreg} - \text{pbdis}) - \text{ab}^2 - \text{ba} \cdot \text{bb} + 1)}{\text{bb} \cdot (\text{H}(\text{pareg} - \text{padis}) + \text{H}(\text{pbreg} - \text{pbdis}) - \text{aa} - \text{ab})}$  Solve(#11', xa2)



User

xalst := -

#21:

xb2st := -

$$\begin{aligned}
& \frac{FA(\alpha, \beta, padis) \cdot (H(pareg - padis) \cdot (ab \cdot H(pbreg - pbdis) - ab^2 + 1) + (ba \cdot bb - aa \cdot ab) \cdot H(pbreg - pbdis) + aa \cdot (ab^2 - 1) - ab \cdot ba \cdot bb) - FA(\alpha, \beta, pareg) \cdot (ab \cdot H(pbreg - pbdis) - ab^2 - ba \cdot bb + 1) + ba \cdot (FB(\alpha, \beta, pbdis) \cdot (H(pareg - padis) + H(pbreg - pbdis) - aa - ab) + FB(\alpha, \beta, pbreg) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1))}{H(pareg - padis) \cdot (aa \cdot ab \cdot H(pbreg - pbdis) + aa \cdot (1 - ab^2) + ab \cdot ba \cdot bb) - (aa^2 \cdot ab - aa \cdot ba \cdot bb - ab) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1) + ba \cdot (FB(\alpha, \beta, pbdis) \cdot (H(pareg - padis) + H(pbreg - pbdis) - aa - ab) + FB(\alpha, \beta, pbreg) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1))} \\
xblst := & - \frac{bb \cdot FA(\alpha, \beta, padis) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1) - bb \cdot (aa + ab) \cdot FA(\alpha, \beta, pareg) - FB(\alpha, \beta, pbdis) \cdot (aa \cdot H(pareg - padis) - aa^2 \cdot ab + aa \cdot ba \cdot bb + ab)}{H(pareg - padis) \cdot (aa \cdot ab \cdot H(pbreg - pbdis) + aa \cdot (1 - ab^2) + ab \cdot ba \cdot bb) - (aa^2 \cdot ab - aa \cdot ba \cdot bb - ab) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1) + ba \cdot (FB(\alpha, \beta, pbdis) \cdot (H(pareg - padis) + H(pbreg - pbdis) - aa - ab) + FB(\alpha, \beta, pbreg) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1))} \\
xa2st := & \frac{FA(\alpha, \beta, padis) \cdot (ab \cdot H(pbreg - pbdis) - ab^2 - ba \cdot bb + 1) + FA(\alpha, \beta, pareg) \cdot (aa \cdot ab \cdot H(pbreg - pbdis) + aa \cdot (1 - ab^2) + ab \cdot ba \cdot bb) - (aa^2 \cdot ab - aa \cdot ba \cdot bb - ab) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1) + ba \cdot (FB(\alpha, \beta, pbdis) \cdot (H(pareg - padis) + H(pbreg - pbdis) - aa - ab) + FB(\alpha, \beta, pbreg) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1))}{H(pareg - padis) \cdot (aa \cdot ab \cdot H(pbreg - pbdis) + aa \cdot (1 - ab^2) + ab \cdot ba \cdot bb) - (aa^2 \cdot ab - aa \cdot ba \cdot bb - ab) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1) + ba \cdot (FB(\alpha, \beta, pbdis) \cdot (H(pareg - padis) + H(pbreg - pbdis) - aa - ab) + FB(\alpha, \beta, pbreg) \cdot (ab \cdot H(pareg - padis) - aa \cdot ab + ba \cdot bb - 1))} \\
& \frac{H(pbreg - pbdis) + aa^2 \cdot (ab^2 - 1) - 2 \cdot aa \cdot ab \cdot ba \cdot bb - ab^2 + (ba \cdot bb - 1)^2}{H(pbreg - pbdis) + aa^2 \cdot (ab^2 - 1) - 2 \cdot aa \cdot ab \cdot ba \cdot bb - ab^2 + (ba \cdot bb - 1)^2} \\
& - \frac{padis - aa^2 - ba \cdot bb + 1 - FB(\alpha, \beta, pbreg) \cdot (aa \cdot ab \cdot H(pareg - padis) - aa^2 \cdot ab + aa \cdot ba \cdot bb + ab)}{H(pbreg - pbdis) + aa^2 \cdot (ab^2 - 1) - 2 \cdot aa \cdot ab \cdot ba \cdot bb - ab^2 + (ba \cdot bb - 1)^2} \\
& ) - \frac{ba \cdot (FB(\alpha, \beta, pbdis) \cdot (aa \cdot H(pbreg - pbdis) - aa \cdot ab + ba \cdot bb - 1) - (aa + ab) \cdot FB(\alpha, \beta, pbreg))}{(pbreg - pbdis) + aa^2 \cdot (ab^2 - 1) - 2 \cdot aa \cdot ab \cdot ba \cdot bb - ab^2 + (ba \cdot bb - 1)^2} \\
& \frac{(pbreg - pbdis) - aa \cdot ab + ba \cdot bb + (1 - aa^2) \cdot H(pbreg - pbdis) + aa^2 \cdot ab - aa \cdot ba \cdot bb - ab - FB(\alpha, \beta, pbreg) \cdot (aa \cdot H(pareg - padis) - aa^2 - ba \cdot bb + 1)}{H(pbreg - pbdis) + aa^2 \cdot (ab^2 - 1) - 2 \cdot aa \cdot ab \cdot ba \cdot bb - ab^2 + (ba \cdot bb - 1)^2}
\end{aligned}$$

#22: pareg·xalst + padis·xa2st

User

```

User
#23: "This is player A's profit with general functions. Even after we substitute specific functions and specific parameter values alpha=0.1, beta=1, aa=ab=0.5, ba=bb=-0.1, and c=0.025,
the resulting function of the four prices is analytically intractable."
#24: [FA(alpha, beta, p) := alpha*epsilon^(-beta*p), FB(alpha, beta, q) := alpha*epsilon^(-beta*q), H(delta) := c*delta^2]
User
Simp(#22)
#25: (alpha*epsilon^(-beta*(padis + pareg + pbdis + pbreg)) * (epsilon^beta * padis * (ba*epsilon^beta * pareg + pbdis) * (epsilon^beta * pbdis * (aa*(ab*pareg + padis) + ab*(padis - c*pareg*(padis - pareg)^2) - pareg*(ba*bb - 1)) + epsilon^beta * pbreg * (aa*(a
b*padis - c*padis*(pbdis - pbreg)^2 + pareg) + ab*pareg - ba*bb*padis - c*pareg*(padis^2 - 2*padis*pareg + pareg^2 + (pbdis - pbreg)^2 + padis)) - epsilon^beta * (pbdis + pbreg) * (aa*padis
aa^2 * (ab^2 - ab*c*(pbdis - pbreg)^2 - 1) - aa*(ab^2 * c*(padis - pareg)^2 + ab*(2*ba*bb - c*(padis - pareg)
(ab^2 - ab*c*(pbdis - pbreg)^2 - 1) + ab^2 * pareg - ab*(ba*bb*padis + c*pareg*(pbdis - pbreg)^2) + pareg*(ba*bb - 1))) - epsilon^beta * (pareg + pbdis + pbreg) * (aa*pareg*(ab^2 - ab*c*(pbdis
(pbdis - pbreg)^2) - c*(ba*bb*(pbdis - pbreg)^2 + (padis - pareg)^2)) - ab^2 + ab*c*(ba*bb*(padis - pareg)^2 + (pbdis - pbreg)^2) + (ba*bb - 1)^2
- pbreg)^2 - 1) + ab^2 * (padis - c*pareg*(padis - pareg)^2) + ab*(c*(pbdis - pbreg)^2 * (c*pareg*(padis - pareg)^2 - padis) - ba*bb*pareg) + ba*bb*(c*pareg*(pbdis - pbreg)^2 + padi
s) + c*pareg*(padis - pareg)^2 - padis))
Sub(#25)
#26: (0.1*epsilon^(-1*(padis + pareg + pbdis + pbreg)) * (epsilon^1 * padis * ((-0.1)*epsilon^1 * pareg * (epsilon^1 * pbdis * (0.5*(0.5*pareg + padis) + 0.5*(padis - 0.025*pareg*(padis - pareg)^2) - pareg*((-0.1)*(-0.1) -
1)) + epsilon^1 * pbreg * (0.5*(0.5*padis - 0.025*padis*(pbdis - pbreg)^2 + pareg) + 0.5*pareg - (-0.1)*(-0.1)*padis - 0.025*pareg*(padis^2 - 2*padis*pareg + pareg^2 + (pbdis - pbreg)^2)
0.5*(0.5^2 - 0.5*0.025*(pbdis - pbreg)^2 - 1) - 0.5*(0.5^2 * 0.025*(padis - pa
+ padis)) - epsilon^1 * (pbdis + pbreg) * (0.5*padis*(0.5^2 - 0.5*0.025*(pbdis - pbreg)^2 - 1) + 0.5^2 * pareg - 0.5*((-0.1)*(-0.1)*padis + 0.025*pareg*(pbdis - pbreg)^2) + pareg*((-0.1)*(-
reg)^2 + 0.5*(2*(-0.1)*(-0.1) - 0.025^2 * (padis - pareg)^2 * (pbdis - pbreg)^2) - 0.025*((-0.1)*(-0.1)*(pbdis - pbreg)^2 + (padis - pareg)^2)) - 0.5^2 + 0.5*0.025*((-0.1)*(-0.1)*(pad
0.1) - 1))) - epsilon^1 * (pareg + pbdis + pbreg) * (0.5*pareg*(0.5^2 - 0.5*0.025*(pbdis - pbreg)^2 - 1) + 0.5 * (padis - 0.025*pareg*(padis - pareg)^2) + 0.5*(0.025*(pbdis - pbreg)^2 * (0.0
is - pareg)^2 + (pbdis - pbreg)^2) + ((-0.1)*(-0.1) - 1)^2
25*pareg*(padis - pareg)^2 - padis) - (-0.1)*(-0.1)*pareg) + (-0.1)*(-0.1)*(0.025*pareg*(pbdis - pbreg)^2 + padis) + 0.025*pareg*(padis - pareg)^2 - padis))

```

0.7s Fctr(#26)

$$\begin{aligned}
& \frac{-\text{pdis} - \text{pareg} - \text{pbdis} - \text{pbreg}}{\text{e}} \cdot \left( \frac{\text{pdis}}{\text{e}} \cdot \left( \frac{\text{pareg}}{\text{e}} \cdot \left( 4 \cdot \frac{\text{pbdis}}{\text{e}} \cdot (5 \cdot \text{pdis} \cdot \text{pareg} - 10 \cdot \text{pdis} \cdot (\text{pareg} + 40) + \text{pareg} \cdot (5 \cdot \text{pareg} - 496)) + 4 \cdot \frac{\text{pbreg}}{\text{e}} \cdot (10 \cdot \text{pdis} \cdot \text{pareg} - \text{pdis} \cdot (20 \cdot \text{pareg} \right. \right. \right. \\
& \left. \left. \left. + 5 \cdot \text{pbdis}^2 + 10 \cdot \text{pbdis} \cdot \text{pbreg} - 5 \cdot \text{pbreg}^2 + 496) + 10 \cdot \text{pareg} \cdot (\text{pareg}^2 + \text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 - 40) \right) \right) + 20 \cdot \frac{\text{pbdis} + \text{pbreg}}{\text{e}} \cdot (\text{pdis} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + \right. \\
& \left. \left. \left. 304) + 2 \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296) \right) \right) - \frac{\text{pareg} + \text{pbdis} + \text{pbreg}}{\text{e}} \cdot (5 \cdot \text{pdis}^2 \cdot \text{pareg} \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) - 10 \cdot \text{pdis} \cdot (\text{pareg}^2 \cdot (\text{pbdis}^2 - \right. \\
& \left. \left. \left. \text{eg}^2 + 304) + 5 \cdot \text{pareg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 16 \cdot (95 \cdot \text{pbdis}^2 - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376) \right. \right. \\
& \left. \left. \left. + 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) + 4 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296) \right) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) - 32 \cdot (3 \cdot \text{pbdis}^2 - 6 \cdot \text{pbdis} \cdot \text{pbreg} + 3 \cdot \right. \right. \\
& \left. \left. \left. \text{breg}^2 + 190) \right) \right) \right)
\end{aligned}$$

User

#28: profita :=

$$\begin{aligned}
& \frac{-\text{pdis} - \text{pareg} - \text{pbdis} - \text{pbreg}}{\text{e}} \cdot \left( \frac{\text{pdis}}{\text{e}} \cdot \left( \frac{\text{pareg}}{\text{e}} \cdot \left( 4 \cdot \frac{\text{pbdis}}{\text{e}} \cdot (5 \cdot \text{pdis} \cdot \text{pareg} - 10 \cdot \text{pdis} \cdot (\text{pareg} + 40) + \text{pareg} \cdot (5 \cdot \text{pareg} - 496)) + 4 \cdot \frac{\text{pbreg}}{\text{e}} \cdot (10 \cdot \text{pdis} \cdot \text{pareg} - \text{pdis} \cdot (20 \cdot \text{pareg} \right. \right. \right. \\
& \left. \left. \left. - 5 \cdot \text{pbdis}^2 + 10 \cdot \text{pbdis} \cdot \text{pbreg} - 5 \cdot \text{pbreg}^2 + 496) + 10 \cdot \text{pareg} \cdot (\text{pareg}^2 + \text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 - 40) \right) \right) + 20 \cdot \frac{\text{pbdis} + \text{pbreg}}{\text{e}} \cdot (\text{pdis} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + \right. \\
& \left. \left. \left. + 304) + 2 \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296) \right) \right) - \frac{\text{pareg} + \text{pbdis} + \text{pbreg}}{\text{e}} \cdot (5 \cdot \text{pdis}^2 \cdot \text{pareg} \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) - 10 \cdot \text{pdis} \cdot (\text{pareg}^2 \cdot (\text{pbdis}^2 - \right. \\
& \left. \left. \left. \text{breg}^2 + 304) + 5 \cdot \text{pareg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 16 \cdot (95 \cdot \text{pbdis}^2 - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376) \right. \right. \\
& \left. \left. \left. - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) + 4 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296) \right) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) - 32 \cdot (3 \cdot \text{pbdis}^2 - 6 \cdot \text{pbdis} \cdot \text{pbreg} + 3 \cdot \right. \\
& \left. \left. \left. \text{pbreg}^2 + 190) \right) \right) \right)
\end{aligned}$$

$$\#29: \left[ \frac{d}{d \text{ pareg}} \text{ profita}, \frac{d}{d \text{ pdis}} \text{ profita} \right]$$

User



$$\begin{aligned}
& \frac{\text{pbreg} + 5 \cdot \text{pbreg}^2 + 304 + 25 \cdot \text{pbdis}^4 - 100 \cdot \text{pbdis}^3 \cdot \text{pbreg} + 30 \cdot \text{pbdis}^2 \cdot (5 \cdot \text{pbreg}^2 + 18) - 20 \cdot \text{pbdis} \cdot \text{pbreg} \cdot (5 \cdot \text{pbreg}^2 + 54) + 25 \cdot \text{pbreg}^4 + 540 \cdot \text{pbreg}^2 - 62784 - 5 \cdot \text{padis} \cdot \text{pareg} \cdot (4 \cdot \text{pa} \\
& \hspace{15em} (5 \cdot \text{padis}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) \\
& \frac{\text{reg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 5 \cdot \text{pbdis}^4 - 20 \cdot \text{pbdis}^3 \cdot \text{pbreg} + 2 \cdot \text{pbdis}^2 \cdot (15 \cdot \text{pbreg}^2 + 512) - 4 \cdot \text{pbdis} \cdot \text{pbreg} \cdot (5 \cdot \text{pbreg}^2 + 512) + 5 \cdot \text{pbreg}^4 + 1024 \cdot \text{pbreg}^2 + 3865 \\
& - 10 \cdot \text{padis} \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 5 \cdot \text{pareg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 16 \cdot (95 \cdot \text{pbdis}^2 - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376) \\
& 6) + 5 \cdot \text{pareg}^4 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) - \text{pareg}^2 \cdot (25 \cdot \text{pbdis}^4 - 100 \cdot \text{pbdis}^3 \cdot \text{pbreg} + 10 \cdot \text{pbdis}^2 \cdot (15 \cdot \text{pbreg}^2 - 404) + 20 \cdot \text{pbdis} \cdot \text{pbreg} \cdot (404 - 5 \cdot \text{pbreg}^2) + 25 \cdot \text{pbreg} \\
& )^2 \\
& - 4040 \cdot \text{pbreg}^2 - 318848) + 16 \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 - 40) \cdot (95 \cdot \text{pbdis}^2 - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376)} + \\
& \frac{4 \cdot \text{e}^{-\text{pbreg}} \cdot (25 \cdot \text{padis}^4 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) - 100 \cdot \text{padis}^3 \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 10 \cdot \text{padis}^2 \cdot (15 \cdot \text{pareg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbd} \\
& \hspace{15em} (5 \cdot \text{padi} \\
& \text{is} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) - 16 \cdot (155 \cdot \text{pbdis}^2 - 310 \cdot \text{pbdis} \cdot \text{pbreg} + 155 \cdot \text{pbreg}^2 + 9624) - 20 \cdot \text{padis} \cdot \text{pareg} \cdot (5 \cdot \text{pareg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 8 \cdot (65 \cdot \text{pbdis}^2 \\
& \text{s} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) - 10 \cdot \text{padis} \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 5 \cdot \text{pareg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 16 \cdot ( \\
& 130 \cdot \text{pbdis} \cdot \text{pbreg} + 65 \cdot \text{pbreg}^2 + 3152) + 25 \cdot \text{pareg}^4 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 640 \cdot \text{pareg}^2 \cdot (55 \cdot \text{pbdis}^2 - 110 \cdot \text{pbdis} \cdot \text{pbreg} + 55 \cdot \text{pbreg}^2 + 3194) - 7936 \cdot (95 \cdot \text{pbd} \\
& 95 \cdot \text{pbdis}^2 - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376) \\
& \text{s} - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376) \\
& \frac{\text{e}^{-\text{padis}} \cdot (25 \cdot \text{padis}^4 \cdot \text{pareg} \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) - 100 \cdot \text{padis}^3 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) \cdot (\text{par} \\
& \text{eg} \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) + 2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296) + 10 \cdot \text{padis}^2 \cdot (15 \cdot \text{pareg}^3 \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pb} \\
& \text{dis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 16 \cdot \text{pareg} \cdot (95 \cdot \text{pbdis}^4 - 380 \cdot \text{pbdis}^3 \cdot \text{pbreg} + 2 \cdot \text{pbdis}^2 \cdot (285 \cdot \text{pbreg}^2 + 5588) - 4 \cdot \text{pbdis} \cdot \text{pbreg} \cdot (95 \cdot \text{pbreg}^2 + 5588) + 95 \cdot \text{pbreg}^4 + 11176 \cdot \text{pbreg}^2 + 328480) \\
& - 20 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296) \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) - 20 \cdot \text{padis} \cdot (5 \cdot \text{pareg}^4 \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{p} \\
& \text{bdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 2 \cdot \text{pareg}^2 \cdot (385 \cdot \text{pbdis}^4 - 1540 \cdot \text{pbdis}^3 \cdot \text{pbreg} + 14 \cdot \text{pbdis}^2 \cdot (165 \cdot \text{pbreg}^2 + 3172) - 28 \cdot \text{pbdis} \cdot \text{pbreg} \cdot (55 \cdot \text{pbreg}^2 + 3172) + 385 \cdot \text{pbreg}^4 + 44408 \cdot \text{pbreg}^2 + 12 \\
& (5 \cdot \text{padis}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) - 10 \cdot \text{padis} \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 5 \cdot \text{pareg}^2 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2
\end{aligned}$$

$$\frac{78080 + 40 \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296) + 32 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} + 304) + 16 \cdot (95 \cdot \text{pbdis}^2 - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376)}{2}$$

#31: "Because the coefficients are symmetric, we seek symmetric solutions pareg=pbreg=p, and padis=pbdis=d, of these first order conditions"

User

Sub(#30)

#32: -

$$\begin{aligned}
& \frac{-d \cdot (25 \cdot d^4 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 100 \cdot d^3 \cdot r \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 10 \cdot d^2 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (15 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 32 \cdot (15 \cdot d^2 - 30 \cdot d \cdot r + 15 \cdot r^2 + 1012)) - 20 \cdot d \cdot r \cdot (5 \cdot r^2 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 4 \cdot (255 \cdot d^4 - 1020 \cdot d^3 \cdot r + 18 \cdot d^2 \cdot (85 \cdot r^2 + 1628 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304)) - 12 \cdot d \cdot r \cdot (85 \cdot r^2 + 4884) + 255 \cdot r^4 + 29304 \cdot r^2 + 840320) + 25 \cdot r^4 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 80 \cdot r^2 \cdot (315 \cdot d^4 - 1260 \cdot d^3 \cdot r + 2 \cdot d^2 \cdot (945 \cdot r^2 + 18476) - 4 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))^2}{\cdot d \cdot r \cdot (315 \cdot r^2 + 18476) + 315 \cdot r^4 + 36952 \cdot r^2 + 1083200} - 512 \cdot (3 \cdot d^2 - 6 \cdot d \cdot r + 3 \cdot r^2 + 190) \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376)} \\
& \frac{20 \cdot \tilde{e} \cdot (5 \cdot d^3 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304)^2 - 20 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) \cdot (4 \cdot r + 5 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60)) - d \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) \cdot (25 \cdot r^2 \cdot (3 \cdot d^2 - 6 \cdot d \cdot r + 3 \cdot r^2 + 176) - 10 \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376)) + 2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296) \cdot (5 \cdot r^3 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))^2}{\cdot r + 5 \cdot r^2 + 304) + 16 \cdot r \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376) - 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))} + \\
& \frac{40 \cdot \tilde{e} \cdot (5 \cdot d^4 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 20 \cdot d^3 \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 2 \cdot d^2 \cdot (15 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 25 \cdot d^4 - 100 \cdot d^3 \cdot r + 30 \cdot d^2 \cdot (5 \cdot r^2 + 18) - 20 \cdot d \cdot r \cdot (5 \cdot r^2 + 54) + 25 \cdot r^4 + 540 \cdot r^2 - 62784) - 5 \cdot d \cdot r \cdot (4 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot d^4 - 20 \cdot d^3 \cdot r + 2 \cdot d^2 \cdot (15 \cdot r^2 + 512) - 4 \cdot d \cdot r \cdot (5 \cdot r^2 + 512) + 5 \cdot r^4 + 1024 \cdot r^2 + 38656) + 5 \cdot r^4 \cdot (5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))^2}{(5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))} \\
& \frac{4 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - r^2 \cdot (25 \cdot d^4 - 100 \cdot d^3 \cdot r + 10 \cdot d^2 \cdot (15 \cdot r^2 - 404) + 20 \cdot d \cdot r \cdot (404 - 5 \cdot r^2) + 25 \cdot r^4 - 4040 \cdot r^2 - 318848) + 16 \cdot (d^2 - 2 \cdot d \cdot r + r^2 - 40) \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))}{95 \cdot r^2 + 5376)} +
\end{aligned}$$

$$\frac{4 \cdot e^{-r} \cdot (25 \cdot d^4 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 100 \cdot d^3 \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 10 \cdot d^2 \cdot (15 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 16 \cdot (155 \cdot d^2 - 310 \cdot d \cdot r + 155 \cdot r^2 + 9624)) - 20 \cdot d \cdot r \cdot (5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 8 \cdot (65 \cdot d^2 - 130 \cdot d \cdot r + 65 \cdot r^2 + 3152)) + 25 \cdot r^4 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 640 \cdot r^2 \cdot (55 \cdot d^2 - 110 \cdot d \cdot r + 55 \cdot r^2 + 3194) - 7936 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))}{(5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))} \cdot (190 \cdot d \cdot r + 95 \cdot r^2 + 5376)$$

$$\frac{-d \cdot e^{-d} \cdot (25 \cdot d^4 \cdot r \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 100 \cdot d^3 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) \cdot (r \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) + 2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296)) + 10 \cdot d^2 \cdot (15 \cdot r^3 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot r \cdot (95 \cdot d^4 - 380 \cdot d^3 \cdot r + 2 \cdot d^2 \cdot (285 \cdot r^2 + 5588) - 4 \cdot d \cdot r \cdot (95 \cdot r^2 + 5588) + 95 \cdot r^4 + 11176 \cdot r^2 + 328480) - 20 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304)) - 20 \cdot d \cdot (5 \cdot r^4 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 2 \cdot r^2 \cdot (385 \cdot d^4 - 1540 \cdot d^3 \cdot r + 14 \cdot d^2 \cdot (165 \cdot r^2 + 3172) - 28 \cdot d \cdot r \cdot (55 \cdot r^2 + 3172) + 385 \cdot r^4 + 44408 \cdot r^2 + 1278080) + 40 \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296)^2 + 32 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296) \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376)) + 25 \cdot r^5 \cdot (d^2 - 2 \cdot d \cdot r + r^2 + 60) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))}{(5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))} \cdot (10 \cdot d^2 \cdot (75 \cdot r^2 + 1484) - 20 \cdot d \cdot r \cdot (25 \cdot r^2 + 1484) + 125 \cdot r^4 + 14840 \cdot r^2 + 440448) - 512 \cdot r \cdot (3 \cdot d^2 - 6 \cdot d \cdot r + 3 \cdot r^2 + 190) \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376) + 640 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296) \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376)$$

$$\frac{20 \cdot e^{-r} \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) \cdot (5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 20 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296) - 5 \cdot r^2 \cdot (25 \cdot d^2 - 50 \cdot d \cdot r + 25 \cdot r^2 + 1488) - 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))}{(5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))} \cdot (95 \cdot r^2 + 5376)$$

$$\frac{4 \cdot e^{-d} \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 - 496) \cdot (5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 20 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296) - 5 \cdot r^2 \cdot (25 \cdot d^2 - 50 \cdot d \cdot r + 25 \cdot r^2 + 1488) - 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))}{(5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))} \cdot (95 \cdot r^2 + 5376)$$

$$\frac{95 \cdot r^2 + 5376}{1} +$$

$$\frac{1600 \cdot e^{-r} \cdot (5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 20 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 296) - 5 \cdot r^2 \cdot (25 \cdot d^2 - 50 \cdot d \cdot r + 25 \cdot r^2 + 1488) - 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))}{(5 \cdot d^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) - 10 \cdot d \cdot r \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 5 \cdot r^2 \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 304) + 16 \cdot (95 \cdot d^2 - 190 \cdot d \cdot r + 95 \cdot r^2 + 5376))}$$

6.1s Simp(#32)

#33:

$$\frac{-d^6 \cdot (25 \cdot d^6 - 150 \cdot d^5 \cdot r + 5 \cdot d^4 \cdot (75 \cdot r^2 + 68) - 20 \cdot d^3 \cdot r \cdot (25 \cdot r^2 + 398) + d^2 \cdot (375 \cdot r^4 + 21840 \cdot r^2 - 108544) - 2 \cdot d \cdot r \cdot (75 \cdot r^4 + 10580 \cdot r^2 + 88576) + 25 \cdot r^6 + 6940 \cdot r^4 + 285696 \cdot r^2 - 1720)}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

320)

$$\frac{4 \cdot e^{-r} \cdot (125 \cdot d^5 \cdot (125 \cdot d^5 - 25 \cdot d^4 \cdot (10 \cdot r + 21) - 50 \cdot d^3 \cdot (5 \cdot r^2 - 27 \cdot r - 344) + 20 \cdot d^2 \cdot (50 \cdot r^3 - 45 \cdot r^2 - 20 \cdot r - 2432) - 5 \cdot d \cdot (175 \cdot r^4 + 30 \cdot r^3 + 10160 \cdot r^2 - 10496 \cdot r - 116736) + 250 \cdot r^5 + 225 \cdot r^4)}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

$$+ 34000 \cdot r^3 - 3840 \cdot r^2 + 1136640 \cdot r - 946176)$$

$$\frac{-d^6 \cdot (25 \cdot d^6 \cdot r - 50 \cdot d^5 \cdot (3 \cdot r^2 + 20) + 5 \cdot d^4 \cdot (75 \cdot r^3 + 1388 \cdot r - 220) - 20 \cdot d^3 \cdot (25 \cdot r^4 + 888 \cdot r^2 + 110 \cdot r + 6800) + d^2 \cdot r \cdot (375 \cdot r^4 + 21640 \cdot r^2 + 13200 \cdot r + 319936) - 2 \cdot d \cdot (75 \cdot r^6 + 6380 \cdot r^4)}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

$$+ 7700 \cdot r^3 + 115936 \cdot r^2 + 197120 \cdot r + 2273280) + 25 \cdot r^7 + 2940 \cdot r^5 + 5500 \cdot r^4 + 47936 \cdot r^3 + 394240 \cdot r^2 - 2334720 \cdot r + 3784704)$$

4)

$$\frac{20 \cdot e^{-r} \cdot (25 \cdot d^4 \cdot (25 \cdot d^4 + 50 \cdot d^3 \cdot r - 300 \cdot d^2 \cdot r^2 + 70 \cdot d \cdot r \cdot (5 \cdot r^2 + 128) - 125 \cdot r^4 - 1792 \cdot (5 \cdot r^2 + 48))}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

User  
#34: -

$$\frac{-d \cdot (25 \cdot d^6 - 150 \cdot d^5 \cdot r + 5 \cdot d^4 \cdot (75 \cdot r^2 + 68) - 20 \cdot d^3 \cdot r \cdot (25 \cdot r^2 + 398) + d^2 \cdot (375 \cdot r^4 + 21840 \cdot r^2 - 108544) - 2 \cdot d \cdot r \cdot (75 \cdot r^4 + 10580 \cdot r^2 + 88576) + 25 \cdot r^6 + 6940 \cdot r^4 + 285696 \cdot r^2 - 1720}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

320)

$$\frac{4 \cdot \hat{e}^{-r} \cdot (125 \cdot d^5 - 25 \cdot d^4 \cdot (10 \cdot r + 21) - 50 \cdot d^3 \cdot (5 \cdot r^2 - 27 \cdot r - 344) + 20 \cdot d^2 \cdot (50 \cdot r^3 - 45 \cdot r^2 - 20 \cdot r - 2432) - 5 \cdot d \cdot (175 \cdot r^4 + 30 \cdot r^3 + 10160 \cdot r^2 - 10496 \cdot r - 116736) + 250 \cdot r^5 + 225 \cdot r^4}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

$$+ \frac{34000 \cdot r^3 - 3840 \cdot r^2 + 1136640 \cdot r - 946176}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2} = 0,$$

$$\frac{-d \cdot (25 \cdot d^6 \cdot r - 50 \cdot d^5 \cdot (3 \cdot r^2 + 20) + 5 \cdot d^4 \cdot (75 \cdot r^3 + 1388 \cdot r - 220) - 20 \cdot d^3 \cdot (25 \cdot r^4 + 888 \cdot r^2 + 110 \cdot r + 6800) + d^2 \cdot r \cdot (375 \cdot r^4 + 21640 \cdot r^2 + 13200 \cdot r + 319936) - 2 \cdot d \cdot (75 \cdot r^6 + 6380 \cdot r^4)}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

$$+ \frac{7700 \cdot r^3 + 115936 \cdot r^2 + 197120 \cdot r + 2273280 + 25 \cdot r^7 + 2940 \cdot r^5 + 5500 \cdot r^4 + 47936 \cdot r^3 + 394240 \cdot r^2 - 2334720 \cdot r + 3784704}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2}$$

4)

$$\frac{20 \cdot \hat{e}^{-r} \cdot (25 \cdot d^4 + 50 \cdot d^3 \cdot r - 300 \cdot d^2 \cdot r^2 + 70 \cdot d \cdot r \cdot (5 \cdot r^2 + 128) - 125 \cdot r^4 - 1792 \cdot (5 \cdot r^2 + 48))}{(5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 224) \cdot (5 \cdot d^2 - 10 \cdot d \cdot r + 5 \cdot r^2 + 384)^2} = 0$$

User

#35: "When we solve these equations numerically we find two solutions: a constant price solution at r=d=0.8 and a two price solution at r=1.12, d=0.539. We first investigate the constant price solution d=r analytically, and verify that it occurs at d=r=4/5=0.8"

Sub(#34)

#36:

$$\frac{-x \cdot (25x^6 - 150x^5 \cdot x + 5x^4 \cdot (75x^2 + 68) - 20x^3 \cdot x \cdot (25x^2 + 398) + x^2 \cdot (375x^4 + 21840x^2 - 108544) - 2x \cdot x \cdot (75x^4 + 10580x^2 + 88576) + 25x^6 + 6940x^4 + 285696x^2 - 1720)}{(5x^2 - 10x \cdot x + 5x^2 + 224) \cdot (5x^2 - 10x \cdot x + 5x^2 + 384)^2}$$

320)

$$\frac{4 \cdot \frac{-x \cdot (125x^5 - 25x^4 \cdot (10x + 21) - 50x^3 \cdot (5x^2 - 27x - 344) + 20x^2 \cdot (50x^3 - 45x^2 - 20x - 2432) - 5x \cdot (175x^4 + 30x^3 + 10160x^2 - 10496x - 116736) + 250x^5 + 225x^4)}{(5x^2 - 10x \cdot x + 5x^2 + 224) \cdot (5x^2 - 10x \cdot x + 5x^2 + 384)^2}}{+ 34000x^3 - 3840x^2 + 1136640x - 946176} = 0,$$

$$\frac{-x \cdot (25x^6 \cdot x - 50x^5 \cdot (3x^2 + 20) + 5x^4 \cdot (75x^3 + 1388x - 220) - 20x^3 \cdot (25x^4 + 888x^2 + 110x + 6800) + x^2 \cdot (375x^4 + 21640x^2 + 13200x + 319936) - 2x \cdot (75x^6 + 6380x^4) + 7700x^3 + 115936x^2 + 197120x + 2273280) + 25x^7 + 2940x^5 + 5500x^4 + 47936x^3 + 394240x^2 - 2334720x + 3784704}{(5x^2 - 10x \cdot x + 5x^2 + 224) \cdot (5x^2 - 10x \cdot x + 5x^2 + 384)^2}$$

4)

$$\frac{20 \cdot \frac{-x \cdot (25x^4 + 50x^3 \cdot x - 300x^2 \cdot x^2 + 70x \cdot x \cdot (5x^2 + 128) - 125x^4 - 1792 \cdot (5x^2 + 48))}{(5x^2 - 10x \cdot x + 5x^2 + 224) \cdot (5x^2 - 10x \cdot x + 5x^2 + 384)^2}}{= 0}$$

#37:  $\left[ \frac{-x \cdot (4 - 5x)}{24} = 0, \frac{-x \cdot (4 - 5x)}{24} = 0 \right]$

0.2s Simp(#36)

Sub(#27)

$$\#38: \frac{e^{-0.8} - 0.8 - 0.8 - 0.8 - 0.8 \cdot (e^{0.8} \cdot (e^{0.8} \cdot (4 \cdot e^{0.8} \cdot (5 \cdot 0.8^2 \cdot 0.8 - 10 \cdot 0.8 \cdot (0.8^2 + 40) + 0.8 \cdot (5 \cdot 0.8^2 - 496)) + 4 \cdot e^{0.8} \cdot (10 \cdot 0.8^2 \cdot 0.8 - 0.8 \cdot (20 \cdot 0.8^2 - 5 \cdot 0.8^2 + 10 \cdot 0.8 \cdot 0.8 - 5 \cdot 0.8^2 + 496)) + 10 \cdot 0.8 \cdot (0.8^2 + 0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 - 40)) + 20 \cdot e^{0.8 + 0.8} \cdot (0.8 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 304) + 2 \cdot 0.8 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 296)) - e^{0.8 + 0.8 + 0.8} \cdot (5 \cdot 0.8^2 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 304) - 10 \cdot 0.8 \cdot 0.8 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 304) + 5 \cdot 0.8^2 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 304) + 16 \cdot (95 \cdot 0.8^2 \cdot (5 \cdot 0.8^2 \cdot 0.8 \cdot (0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 + 60) - 10 \cdot 0.8 \cdot (0.8^2 \cdot (0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 + 60) + 4 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 296)) + 0.8 \cdot (5 \cdot 0.8^2 \cdot (0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 - 190 \cdot 0.8 \cdot 0.8 + 95 \cdot 0.8^2 + 5376) + 60) - 32 \cdot (3 \cdot 0.8^2 - 6 \cdot 0.8 \cdot 0.8 + 3 \cdot 0.8^2 + 190))}}{4 \cdot e^{-4/5}}$$

#39:  $\frac{4 \cdot e^{-4/5}}{15}$  0.1s Simp(#38)

User

$$\#40: \frac{e^{-\text{padis} - \text{pareg} - \text{pbdis} - \text{pbreg}} \cdot (e^{\text{padis}} \cdot (e^{\text{pareg}} \cdot (4 \cdot e^{\text{pbdis}} \cdot (5 \cdot \text{padis}^2 \cdot \text{pareg} - 10 \cdot \text{padis} \cdot (\text{pareg}^2 + 40) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 - 496)) + 4 \cdot e^{\text{pbreg}} \cdot (10 \cdot \text{padis}^2 \cdot \text{pareg} - \text{padis} \cdot (20 \cdot \text{pareg}^2 + 496) + 10 \cdot \text{pbdis} \cdot \text{pbreg} - 5 \cdot \text{pbreg}^2 + 496) + 10 \cdot \text{pareg} \cdot (\text{pareg}^2 + \text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 - 40)) + 20 \cdot e^{\text{pbdis} + \text{pbreg}} \cdot (\text{padis} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 2 \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296)) - e^{\text{pareg} + \text{pbdis} + \text{pbreg}} \cdot (5 \cdot \text{padis} \cdot \text{pareg} \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) - 10 \cdot \text{padis} \cdot (\text{pareg} \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) + 5 \cdot \text{pareg} \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 304) + 16 \cdot (95 \cdot \text{pbdis}^2 - 190 \cdot \text{pbdis} \cdot \text{pbreg} + 95 \cdot \text{pbreg}^2 + 5376) + 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) + 4 \cdot (5 \cdot \text{pbdis}^2 - 10 \cdot \text{pbdis} \cdot \text{pbreg} + 5 \cdot \text{pbreg}^2 + 296)) + \text{pareg} \cdot (5 \cdot \text{pareg} \cdot (\text{pbdis}^2 - 2 \cdot \text{pbdis} \cdot \text{pbreg} + \text{pbreg}^2 + 60) - 32 \cdot (3 \cdot \text{pbdis}^2 - 6 \cdot \text{pbdis} \cdot \text{pbreg} + 3 \cdot \text{pbreg}^2 + 190))}}{4 \cdot e^{-4/5}}$$

15

User

#41: "This is the equation satisfied by those prices that lead to the same value of A's profit as does the constant price solution found above. We next investigate what happens when player B uses these constant prices but player A deviates from them."

Sub(#40)

$$\frac{-\delta \cdot (\text{pareg} - \delta) - \text{pareg} - 0.8 - 0.8 \cdot \text{pareg} - \delta \cdot \text{pareg} \cdot (4 \cdot \delta^{0.8} \cdot (5 \cdot (\text{pareg} - \delta) \cdot \text{pareg} - 10 \cdot (\text{pareg} - \delta) \cdot (\text{pareg}^2 + 40) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 - 496)) + 4 \cdot \delta^{0.8} \cdot (10 \cdot (\text{pareg} - \delta) \cdot \text{pareg}^2 - (20 \cdot \text{pareg}^2 - 5 \cdot 0.8^2 + 10 \cdot 0.8 \cdot 0.8 - 5 \cdot 0.8^2 + 496) + 10 \cdot \text{pareg} \cdot (\text{pareg}^2 + 0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 - 40))) + 20 \cdot \delta^{0.8 + 0.8} \cdot ((\text{pareg} - \delta) \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 304) + 2 \cdot \text{pareg} \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 296))) - \delta^{0.8 + 0.8} \cdot (5 \cdot (\text{pareg} - \delta) \cdot \text{pareg} \cdot (0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 + 60) - 10 \cdot (\text{pareg} - \delta) \cdot (\text{pareg} \cdot (0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 + 304) + 2 \cdot \text{pareg} \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 296)))}{5 \cdot \text{pareg}^2 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 304) + 16 \cdot (95 \cdot 0.8^2 - 190 \cdot 0.8 \cdot 0.8 + 95 \cdot 0.8^2 + 5376) \cdot 0.8 + 0.8^2 + 60) + 4 \cdot (5 \cdot 0.8^2 - 10 \cdot 0.8 \cdot 0.8 + 5 \cdot 0.8^2 + 296) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 \cdot (0.8^2 - 2 \cdot 0.8 \cdot 0.8 + 0.8^2 + 60) - 32 \cdot (3 \cdot 0.8^2 - 6 \cdot 0.8 \cdot 0.8 + 3 \cdot 0.8^2 + 190))} = \frac{-4/5}{15}$$

User

#43: "When we graph this expression we see that there are points, with pareg larger than 0.8 and δ positive, which increase player A's profit. Thus the constant price solution found above is not a Nash equilibrium."

#44: "Finally. we investigate the two price solution r=1.12, d=0.539"

User

Sub(#27)

$$\frac{-\text{padis} - \text{pareg} - 0.539 - 1.12 \cdot \text{padis} \cdot \text{pareg} \cdot (4 \cdot \delta^{0.539} \cdot (5 \cdot \text{padis} \cdot \text{pareg} - 10 \cdot \text{padis} \cdot (\text{pareg}^2 + 40) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 - 496)) + 4 \cdot \delta^{1.12} \cdot (10 \cdot \text{padis} \cdot \text{pareg} - \text{padis} \cdot (20 \cdot \text{pareg}^2 - 5 \cdot \text{pareg}^2 + 496) + 10 \cdot \text{pareg} \cdot (\text{pareg}^2 + 0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 - 40))) + 20 \cdot \delta^{0.539 + 1.12} \cdot (\text{padis} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) + 2 \cdot \text{pareg} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 296))) - \delta^{0.539 + 1.12} \cdot (5 \cdot \text{padis} \cdot \text{pareg} \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) - 10 \cdot \text{padis} \cdot (\text{pareg} \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 304) + 2 \cdot \text{pareg} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 296)))}{+ 5 \cdot \text{pareg}^2 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) + 16 \cdot (95 \cdot 0.539^2 - 190 \cdot 0.539 \cdot 1.12 + 95 \cdot 1.12^2 + 5376) \cdot 1.12^2 + 60) + 4 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 296) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) - 32 \cdot (3 \cdot 0.539^2 - 6 \cdot 0.539 \cdot 1.12 + 3 \cdot 1.12^2 + 190))}$$

Sub(#44)

$$\frac{-0.539 - 1.12 - 0.539 - 1.12 \cdot e^{0.539} \cdot (e^{1.12} \cdot (4 \cdot e^{0.539} \cdot (5 \cdot 0.539^2 \cdot 1.12 - 10 \cdot 0.539 \cdot (1.12^2 + 40) + 1.12 \cdot (5 \cdot 1.12^2 - 496)) + 4 \cdot e^{1.12} \cdot (10 \cdot 0.539^2 \cdot 1.12 - 0.539 \cdot (20 \cdot 1.12^2 - 5 \cdot 0.539^2 + 10 \cdot 0.539 \cdot 1.12 - 5 \cdot 1.12^2 + 496) + 10 \cdot 1.12 \cdot (1.12^2 + 0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 - 40))) + 20 \cdot e^{0.539 + 1.12} \cdot (0.539 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) + 2 \cdot 1.12 \cdot (5 \cdot 0.539^2 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) - 10 \cdot 0.539 \cdot 1.12 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) + 5 \cdot 1.12^2 \cdot 0.539 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 296)) - e^{1.12 + 0.539 + 1.12} \cdot (5 \cdot 0.539^2 \cdot 1.12 \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) - 10 \cdot 0.539 \cdot (1.12^2 \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) + 2 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) + 16 \cdot (95 \cdot 0.539^2 - 190 \cdot 0.539 \cdot 1.12 + 95 \cdot 1.12^2 + 5376) - 4 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 296)) + 1.12 \cdot (5 \cdot 1.12^2 \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) - 32 \cdot (3 \cdot 0.539^2 - 6 \cdot 0.539 \cdot 1.12 + 3 \cdot 1.12^2 + 190)))}{}$$

#47: 0.119984 0.1s Approx(#46)

User

$$\frac{-\text{padis} - \text{pareg} - 0.539 - 1.12 \cdot \text{padis} \cdot \text{pareg} \cdot e^{0.539} \cdot (5 \cdot \text{padis} \cdot \text{pareg} - 10 \cdot \text{padis} \cdot (\text{pareg}^2 + 40) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 - 496)) + 4 \cdot e^{1.12} \cdot (10 \cdot \text{padis} \cdot \text{pareg} - \text{padis} \cdot (20 \cdot \text{pareg}^2 - 5 \cdot 0.539^2 + 10 \cdot 0.539 \cdot 1.12 - 5 \cdot 1.12^2 + 496) + 10 \cdot \text{pareg} \cdot (\text{pareg}^2 + 0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 - 40))) + 20 \cdot e^{0.539 + 1.12} \cdot (\text{padis} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) + 2 \cdot 5 \cdot \text{padis} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) - 10 \cdot \text{padis} \cdot \text{pareg} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) - \text{pareg} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 296)) - e^{\text{pareg} + 0.539 + 1.12} \cdot (5 \cdot \text{padis} \cdot \text{pareg} \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) - 10 \cdot \text{padis} \cdot (\text{pareg}^2 \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) + 5 \cdot \text{pareg} \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 304) + 16 \cdot (95 \cdot 0.539^2 - 190 \cdot 0.539 \cdot 1.12 + 95 \cdot 1.12^2 + 5376) - 1.12^2 + 60) + 4 \cdot (5 \cdot 0.539^2 - 10 \cdot 0.539 \cdot 1.12 + 5 \cdot 1.12^2 + 296)) + \text{pareg} \cdot (5 \cdot \text{pareg}^2 \cdot (0.539^2 - 2 \cdot 0.539 \cdot 1.12 + 1.12^2 + 60) - 32 \cdot (3 \cdot 0.539^2 - 6 \cdot 0.539 \cdot 1.12 + 3 \cdot 1.12^2 + 190)))}{}$$

0.119984

#49: "This equation fixes pbreg=1.12 and pbdis=0.539. When we graph it to see if player A can do better, we find that he can not. Since the situation is symmetric, the same would also hold for player B. Thus this solution is a Nash equilibrium."