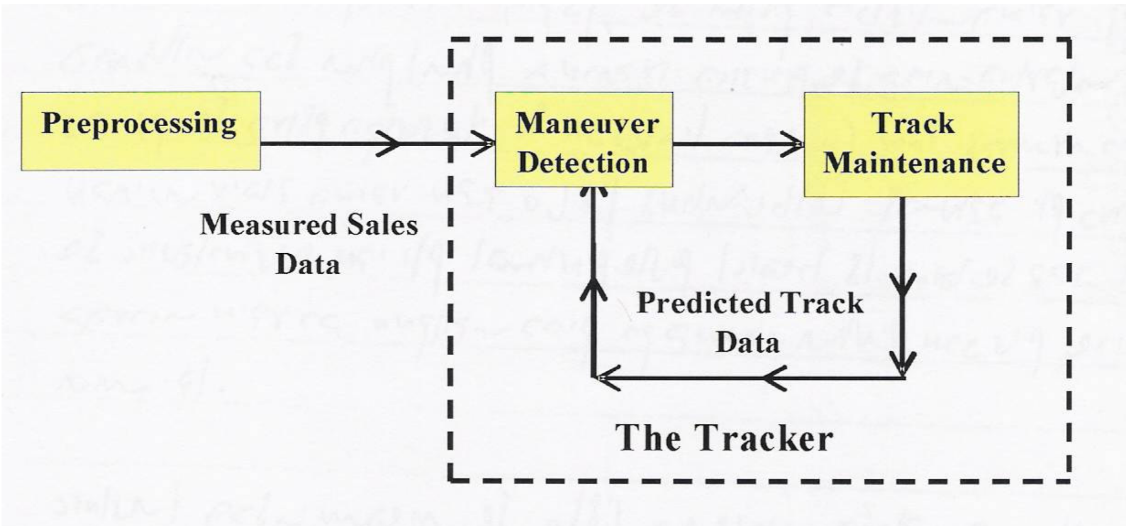
Appendix B: The Sales Movement Tracker

In this Appendix, we describe the operational mechanism of the sales movement tracker that was applied in our empirical validation. First, we draw a scheme of the basic components of the tracker. Next, we offer a step-by-step outline of the sales tracking algorithm.

Figure B1:

A box diagram of the tracker design



Prior to the activation of the tracking routine itself, and in order to assemble the actual rough sales data to carry out an applicable track at any given time t, a preprocessing procedure was conducted. In this procedure, preliminary statistical measures of the sales rate mean and the sales rate volatility are calculated. Then, the tracker is fed with the preprocessing output data serially, one at a time. The incoming measurements are recursively handled by the tracker in a processing loop, as shown in Figure B1.

The Maneuver Detection phase is designed to identify radical changes in the current trend. This phase is only set in motion if the current track is not in an initialization or reinitialization stage (i.e., more than a predetermined number of measurements so that an adequate and stable state vector is conducted). In this phase, the absolute difference between the current measured sales rate mean and the average sales rate prediction of the maintained track is tested vis-à-vis the current maneuver threshold. If the observation-to-track distance is greater than the current maneuver threshold, a track maneuver is declared. Otherwise, the track is asserted as non-maneuver. The value of the current maneuver threshold is proportional to the sum of the predicted track covariance and the variance of the observed sales rate within the current sampling period.

In the Track Maintenance phase, the sales-rate mean measurement is incorporated in the updated track estimator according to the Kalman-filter weights. The smaller the diagonal elements of the predicted track's covariance matrix, the more accurate the track is considered and the lower the weight of the current measurement in the current track estimation, and vice-versa. If no maneuver was indicated in the Maneuver Detection phase, the track's diagonal covariance elements are reduced. Conversely, a track maneuver declaration leads to an increase of the diagonal components of the track's covariance matrix to enlarge the weights of the following observation. An extreme occurrence, in which the number of continuous maneuver detections extends beyond a predetermined threshold, implies that the current track fails to represent the actual existing trend of sales. In such a case, the track is reinitialized according to the last relevant measurements. To complete the Track Maintenance phase, the updated track state vector and covariance data are promoted to the next scan. Those predictions are transmitted in return to the Maneuver Detection phase.

Tracker performance depends on the specific adjustment of system parameters such as the maximal range for the track time history, the non-maneuver gate factor, the number of accepted continuous maneuver detections before reinitializing the track or the sampling period of the tracker, which is the averaging window length for calculating the sales rate means and variances in the preprocessing procedure. The particular optimal choices of those system parameters may hold different values for different kinds of products and different markets due to their unique structure and characteristics. Furthermore, the operational objectives of the tracker design have a significant influence on the numerical choice of the system parameters. For instance, if interest is in relatively short sales movement forecasting, the designer will utilize small maximal range for the track time history, lower the non-maneuver gate factor, reduce the maximal accepted number of continuous maneuver detections and shorten the length of the tracker's sampling period. This drift will make the tracker more sensitive to the changes in the current selling trend. Although able to conduct early maneuver detections and identify short trends of sales, such a tracker may possibly maintain an unstable track which can easily lose the line of the primary sales movement. Alternatively, increasing those parameters will enable the tracker to manage longer sales movement tracking while treating the short movements as being just consuming fluctuations. This situation is viewed as a trade-off, because it slows and tones down the mechanism of identifying changing trends, where in outermost cases the tracker can even miss existing selling trends. To conclude, the tuning of the tracker's system parameters is a question of engineering optimization, which should take into account operational considerations as well as characteristic attributes of the specific product and the particular market.

The values of the system parameters for the four data sets of the e-mail software tool and the three data sets of the movie rental records we used are presented in Tables B1 and B2 respectively. For each case study, we chose two sets of parameters. One set is suitable for applying short sales movement tracking while the other can fit for learning longer patterns of sales, which we define as intermediate sales movement tracking.

Table B1: The tracker's parameters in the short (S) and intermediate (I) tracking modes – The email software tool datasets

	Argentina (S)	Argentina (I)	India (S)	India (I)	Poland (S)	Poland (I)	Sweden (S)	Sweden (I)
τ The sampling period of the tracker (given in days)	7	7	7	7	7	7	7	7
T_H The maximum length of track history (given in days)	400	400	600	600	200	200	200	200
TM The track maturation parameter (given in sampling periods)	4	8	4	8	4	4	4	4
g The non-maneuver gate factor	0.01	5	1	5	0.01	0.5	0.05	1
S The number of sampling periods for maneuver detections counting	10	5	10	5	10	10	5	5
A The accepted number of maneuver detections in the near past for track reinitialization decision	8	1	0	1	8	8	1	1

Table B2: The tracker's parameters in the short (S) and intermediate (I) tracking modes- The movie rental record datasets

	Case A (S)	Case A (I)	Case B (S)	Case B (I)	Case C (S)	Case C (I)
τ The sampling period of the tracker (given in days)	7	7	7	7	7	7
T_H The maximum length of track history (given in days)	50	800	50	500	50	500
TM The track maturation parameter (given in sampling periods)	4	8	4	4	4	8
g The non-maneuver gate factor	0.1	3	0.1	2	0.1	3
S The number of sampling periods for maneuver detections counting	10	10	10	10	10	10
A The accepted number of maneuver detections in the near past for track reinitialization decision	0	0	0	0	0	0

The stages of the preprocessing protocol and the sales tracking algorithm are given below in a step-by-step outline.

B.1 The Preprocessing Protocol

1. Given T, the index of the present time, the track initialization time T_0 is calculated according to the maximum length for track history T_H and the averaging window τ (which is also the sampling period of the tracker) as follows:

$$T_0 = T_1 + (T - T_1) \bmod \tau \tag{B1}$$

where

$$T_1 = \max(T - T_H, \tau) \tag{B2}$$

and *mod* denotes the modulus operator.

2. The time series of daily sales observations received between the track initialization time T_0 and the present time T is partitioned to $(T - T_0)/\tau$, sequentially ordered subseries, where each subseries consists of τ daily sales observations. The statistical measures of the sales rate mean and the sales rate volatility of a certain subseries $z(t-\tau+1), z(t-\tau+2), \ldots, z(t)$ are given by:

$$y(t) = \frac{1}{\tau}\sum_{j=0}^{\tau-1} z(t-j) \tag{B3}$$

and

$$R(t) = \frac{1}{\tau-1}\sum_{j=0}^{\tau-1}\left(z(t-j) - y(t)\right)^2 \tag{B4}$$

As a final point, the output of the preprocessing protocol consists of the two time series: $y(T_0), y(T_0+\tau), y(T_0+2\tau)\ldots, y(T)$ and $R(T_0), R(T_0+\tau), R(T_0+2\tau)\ldots, y(T)$ of the sales rate means and the sales rate volatilities, respectively. These two series are the inputs of the sales tracking algorithm, where the averaging window length τ becomes the sampling period of the tracker.

B.2 The Sales Tracking Algorithm

Objective. The tracking algorithm adaptively incorporates the observed sales rate means and volatilities to estimate and smooth the true track state vector $\vec{x}(t) = \begin{pmatrix} x(t) \\ \dot{x}(t) \end{pmatrix}$, where $x(t)$ and $\dot{x}(t)$ are the true values of the sales rate mean and its temporal derivative at a given time t.

Notations.

$y(t)$ is the observed sales rate mean at a given time t.

$R(t)$ is the observed sales rate volatility at a given time t.

$\vec{x}_s(t) = \begin{pmatrix} x_s(t) \\ \dot{x}_s(t) \end{pmatrix}$ is the smoothed track state vector at a given time t.

$\vec{x}_p(t) = \begin{pmatrix} x_p(t) \\ \dot{x}_p(t) \end{pmatrix}$ is the prediction of the track state vector at a given time t.

$P_s(t) = \mathrm{cov}(\vec{x}_s(t) - \vec{x}(t))$ is the smoothed covariance matrix of the smoothed track at a given time t.

$P_p(t) = \mathrm{cov}(\vec{x}_p(t) - \vec{x}(t))$ is the prediction of the tracks' covariance matrix at a given time t.

$H = (1 \ \ 0)$ is the transformation matrix from track to measurement spaces.

$\Phi = \begin{pmatrix} 1 & \tau \\ 0 & 1 \end{pmatrix}$ is the transition matrix of the track dynamics, where τ is the sampling period of the tracker.

Initial status.

- The track quality $TQ = 0$.
- The track is not in Reinitialization status.
- The current index of time is $t = T_0$, where T_0 is the track initialization time.

The process of the algorithm.

Step I Collect the current observed sales rate mean and volatility $y(t)$ and $R(t)$.

Step II Maneuver detection

1. If the track quality $TQ < TM$, where TM is the track maturation parameter, skip to step III(a). (The track is still premature and hence a non-maneuver tracking has been conducted.)
2. If the track is in Reinitialization status, skip to step III(d) (Reinitialized reinitialized track maintenance).
3. Check whether the current sales rate mean observation is within a non-maneuver gate around the predicted track. Namely, if $\left|y(t) - x_p(t)\right| < g\sqrt{HP_p(t)H' + R(t)}$, where g is the parameter of the non-maneuvering gate factor and H' is the transposed form of the track to measurement space transformation matrix H and go to step III(a) (Non-maneuvering track maintenance). Otherwise, a track maneuver is detected, thus go to step III(b) (Maneuvering track maintenance).

Step III Track maintenance

Step III(a) Non-maneuvering track maintenance

1. If the track quality is $TQ > 1$, incorporate the current sales observations by updating the track state vector and covariance matrix via the Kalman-filtering procedure. Namely,

$$K(t) = P_p(t)H'\left(HP_p(t)H' + R(t)\right)^{-1} \quad \text{(B5)}$$

$$\vec{x}(t) = \left(I - K(t)H\right)\vec{x}_p(t) + K(t)y(t) \quad \text{(B6)}$$

$$P_s(t) = \left(I - K(t)H\right)P_p(t) \quad \text{(B7)}$$

where $K(t)$ is the Kalman gain, I is the identity matrix, and H' is the transposed form of the track to measurement space transformation matrix H. If the track quality $TQ = 1$,

$$\vec{x}_s(t) = \begin{pmatrix} y(t) \\ \dfrac{y(t) - y(t-\tau)}{\tau} \end{pmatrix} \quad \text{(B8)}$$

$$P_s(t) = \begin{pmatrix} R(t) & 0 \\ 0 & \dfrac{R(t) + R(t-\tau)}{\tau^2} \end{pmatrix} \quad \text{(B9)}$$

If the track quality $TQ = 0$, $x_s(t) = y(t)$.

2. Increase the track quality index to the value $TQ = 1$.

3. If the track quality $TQ > 0$, predict the track state vector and covariance matrix in the next sampling period via the relations:

$$\vec{x}_p(t+\tau) = \Phi\,\vec{x}_s(t) \quad \text{(B10)}$$

$$P_p(t+\tau) = \Phi\,P_s(t)\Phi' \quad \text{(B11)}$$

where Φ' is the transposed form of the dynamics transition matrix Φ.

4. Promote the current time index to $t+\tau$, and then go back to step I.

Step III(b) Maneuvering track maintenance

1. Incorporate the current sales observations by updating the track state vector using the Kalman-filtering procedure and increasing the track covariance. Namely,

$$K(t) = P_p(t)H'\left(HP_p(t)H' + R(t)\right)^{-1} \tag{B12}$$

$$\vec{x}_s(t) = \left(I - K(t)H\right)\vec{x}_p(t) + K(t)y(t) \tag{B13}$$

$$P_s(t) = P_p(t) \tag{B14}$$

where $K(t)$ is the Kalman gain, I is the identity matrix, and H' is the transposed form of the track to measurement space transformation matrix H.

2. Predict the track state vector and covariance matrix for the next sampling period via the relations:

$$\vec{x}_p(t+\tau) = \Phi\,\vec{x}_s(t) \tag{B15}$$

$$P_p(t+\tau) = \Phi P_s(t)\Phi' \tag{B16}$$

where Φ' is the transposed form of the dynamics transition matrix Φ.

3. If the number of maneuver detections within the last S sampling periods (S is a system parameter that determines the number of sampling periods for maneuver detections counting) does <u>not</u> exceed a threshold value A, promote the current time index to $t+\tau$ and go back to step I. Otherwise continue to step III(c) to reinitialize a new track.

Step III(c) New track reinitialization

1. Initialize an additional hidden track and then apply a non-maneuvering track maintenance (on the hidden track) based on the last sales rate mean and volatility measurements $y(t_0), y(t_0 + \tau), y(t_0 + 2\tau)\ldots, y(t)$ and $R(t_0), R(t_0 + \tau), R(t_0 + 2\tau)\ldots, R(t)$, where t is the current time index and t_0 is the time of the first maneuver event detected in the last S sampling periods. The process is handled by repeated executions of step III(a), while collecting the observed sales rate mean and volatility pair $(y(t'), R(t'))$ $(t_0 \leq t' \leq t)$ one by one, serially.
2. If the hidden track quality is larger than TM, the track maturation parameter, replace the current system track with the hidden track. Otherwise, keep both the two tracks (the current system track and the hidden track) and turn the track to Reinitialization status.
3. Promote the current time index to $t+\tau$, and go back to step I.

Step III(d) Reinitialized track maintenance

1. Apply the III(a)1, III(a)2 and III(a)3 of the non-maneuvering track maintenance on the hidden track.
2. If the hidden track quality is larger than TM, the track maturation parameter, replace the current track with the hidden track. Otherwise, apply steps III(b)1 and III(b)2 of the maneuvering track maintenance on the current system track.
3. Promote the current time index to $t+\tau$, and go back to step I.

Halting criteria: When the current time index $t > T$, where T is the index of the present time.

B.3 Future Sales Forecasting

The sales rates' track state vector prediction for a future time index t (where $t > T$) is given by:

$$x_p(t) = x_s(T) + (t - T)\dot{x}_s(t) \tag{B17}$$

$$\dot{x}_p(t) = \dot{x}_s(T) \tag{B18}$$

That is, the future sales movement is assumed to evolve in line with the current selling trend.