Technical Appendix to: Price Competition in Markets with Consumer Variety Seeking

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Technical Appendix to: Price Competition in Markets with Consumer Variety Seeking

This is the technical appendix to the paper "Price Competition in Markets with Consumer Variety Seeking". It is intended for review purpose only, not for publication. The exposition in this appendix follows Klemperer (1987a, 1987b) very closely at various places.

1 Symmetric Pure-Strategy Equilibrium under the Undifferentiated Products Model: Solution and Uniqueness

In our manuscript (page 9), we note that the only information about the distribution of variety-seeking in the market that matters is the density of consumers who do not seek variety, $\gamma(0)$. These are the marginal consumers who are sensitive to a small deviation in one firm's price from its competitor's price. However, the rest of the distribution is important in determining whether the prices satisfying the first-order conditions are global best responses for the firms. We now discuss the symmetric pure-strategy equilibrium in more detail. First, we discuss the existence of a symmetric pure-strategy equilibrium in the second period under various cases (assuming symmetric market shares). Second, we discuss its uniqueness. Third, we show that the unique symmetric pure-strategy equilibrium derived in the symmetric market shares case is also a pure-strategy equilibrium in the asymmetric market shares case, provided $\gamma(0) = 0$.

1.1 Solution

To illustrate the symmetric pure-strategy equilibrium solution using a concrete example, we assume a linear demand function and a linear total cost function, i.e., $p = \alpha - \beta q$, $c = Cq$. We assume symmetric market shares for the two firms, i.e., $\sigma_{A1} = \sigma_{B1} = \frac{1}{2}$. The first two cases that we consider are the two extreme cases of $\gamma(0) = 0$ and $\gamma(0) \to \infty$. The third case that we consider is the case of $\gamma(0)$ being positive and finite.

1. When $\gamma(0) = 0$, all consumers have a positive staying cost. In this case, the only possible symmetric pure-strategy equilibrium is for each firm to choose the monopoly price for an

otherwise identical market without variety-seeking, i.e., $p = p_{A2} = p_{B2} = (\alpha + C)/2$ (assuming that a monopolist's profit function would be quasi-concave if there were no staying costs). At any lower common price, each firm has an incentive to slightly increase its price, which more fully exploits the variety-seeking customers without losing any to the other firm. One can check (by comparing equilibrium profits and deviation profits) that the first order conditions do, in fact, define an equilibrium – neither firm has an incentive to make a deviation – if $s \geq (\alpha - C)(\frac{\sqrt{5}-1}{4})$. In fact, this condition is sufficient for the joint profit maximizing outcome to be a non-cooperative equilibrium. For s large enough (as in the given condition), each firm's sales are $q_m/2$, i.e., half the monopoly output, and each firm's profits are $\pi_m/2$, i.e., half the monopoly profit, where q_m and π_m stand for the monopoly output and the monopoly profit, respectively, in an otherwise identical market without variety-seeking. When $s \leq (\alpha - C)(\frac{\sqrt{5}-1}{4})$, however, firms have incentives to deviate and only mixed-strategy equilibria exist. We do not derive these mixed-strategy equilibria.[1]

2. When $\gamma(0) = \infty$, there is an atom of consumers without staying costs. In this case, the only possible symmetric pure-strategy equilibrium is for each firm to price at marginal cost, i.e., $p = p_{A2} = p_{B2} = C$. At any greater common price, each firm has an incentive to slightly lower its price and capture the entire atom of consumers without staying costs. Clearly this is not an equilibrium in general – it is not, for example, an equilibrium with constant marginal costs if any consumers have positive staying costs – so mixed-strategy equilibria arise in this case because of a discontinuity in firms' profit functions.

3. When $0 < \gamma(0) < \infty$, there is a positive density (but no atom) of consumers with zero staying costs. Consider staying costs uniformly distributed on the interval $[0, k]$, i.e., $\gamma(s) = 1/k$, for $s \leq k$, and $\gamma(s) = 0$ for $s > k$. Then the second order conditions are globally satisfied and a unique equilibrium in pure strategies exists and is given by

[1]The illustrations of obtaining mixed-strategy equilibria solutions can be found in Shilony (1977) and Narasimhan (1988). The main message of our paper is conveyed through the symmetric pure-strategy equilibrium.

$p = p_{A2} = p_{B2} = \{k + (\frac{\alpha+C}{2}) - \sqrt{k^2 + (\frac{\alpha-C}{2})^2}\}$ for $k \in [0, \infty)$. Interestingly, as k increases, the market price and industry profits increase monotonically from the competitive equilibrium, $p = C$ at $k = 0$, to the fully collusive outcome, $p = (\alpha + C)/2$, as $k \to \infty$.

1.2 Uniqueness

We can prove that no asymmetric pure-strategy equilibria exist in the symmetric market shares case. To prove that this is true for any demand (under the linear costs assumption), let us assume that $p_{A2} < p_{B2}$ in equilibrium and denote $\Gamma(p_{B2} - p_{A2}) = \Delta$. In that case, $q_{A2} = \sigma_{B1}h(p_{A2}) + \sigma_{A1}h(p_{B2})\Delta + \sigma_{A1}(h(p_{A2}) - h(p_{B2}))x\Delta$ for firm A, and $q_{B2} = \sigma_{A1}h(p_{B2}) - \sigma_{A1}h(p_{B2})\Delta$ for firm B, where

$$x = \frac{\int\limits_{r=p_{A2}}^{p_{B2}} \Gamma(r - p_{A2})[-dh(r)]}{\int\limits_{r=p_{A2}}^{p_{B2}} \Gamma(p_{B2} - p_{A2})[-dh(r)]} \in (0, 1)$$

For firm A not to charge p_{B2}, we require $(p_{A2} - C)q_{A2} \geq (p_{B2} - C)\sigma_{B1}h(p_{B2})$, which implies that (by plugging in for q_{A2} on the left hand side from equation (9) in the manuscript)

$$(p_{A2} - C)h(p_{A2}) + \frac{\sigma_{A1}}{\sigma_{B1}}\{(p_{A2} - C)\Delta[xh(p_{A2}) + (1 - x)h(p_{B2})]\} \geq (p_{B2} - C)h(p_{B2})$$

For firm B not to charge p_{A2}, we require $(p_{B2} - C)q_{B2} \geq (p_{A2} - C)\sigma_{A1}h(p_{A2})$, which implies that

$$(p_{B2} - C)h(p_{B2}) \geq (p_{A2} - C)h(p_{A2}) + (p_{B2} - C)\Delta h(p_{B2})$$

Together these above 2 inequalities require that

$$\frac{\sigma_{A1}}{\sigma_{B1}}(p_{A2} - C)[xh(p_{A2}) + (1 - x)h(p_{B2})] \geq (p_{B2} - C)h(p_{B2})$$

which implies that (since $h(p_{A2}) > [xh(p_{A2}) + (1 - x)h(p_{B2})]$) for $x \in (0, 1)$)

$$\frac{\sigma_{A1}}{\sigma_{B1}}(p_{A2} - C)h(p_{A2}) > (p_{B2} - C)h(p_{B2})$$

Since we know that $(p_{A2} - C)h(p_{A2}) < (p_{B2} - C)h(p_{B2})$, the above inequality implies that $\sigma_{B1} < \sigma_{A1}$. Therefore, the only way that unequal prices can be an equilibrium is for firms to have unequal market shares. The reason that the firm with the higher market share in the first period (firm A) has a lower price in the second period is because the higher market share firm stands more to lose than its competing firm from the fact that consumers have staying costs and, therefore, seek variety in their second period choices. The higher market share firm has, therefore, to price low to combat such switching behavior of its consumers (who are larger in number than its competitors' first period consumers).

1.3 Asymmetric Market Shares Case

In the previous sub-section, we see that the higher market share firm chooses a lower price in an asymmetric equilibrium. Here, we show that when $\gamma(0) = 0$, the unique symmetric pure-strategy equilibrium derived under the symmetric market share case is also a pure-strategy equilibrium (which may not be unique in this case) under the asymmetric market share case.

Under a symmetric pure-strategy equilibrium, since $p_{A2} = p_{B2} = p$, the first order condition given in equation (12) of the manuscript reduces to the following when $\gamma(0) = 0$.

$$
\begin{aligned}
0 &= \sigma_{B1} h(p_{A2}) + \sigma_{A1}\Gamma(p_{B2} - p_{A2}) h(p_{B2}) + \sigma_{A1} \int_{r=p_{A2}}^{p_{B2}} \Gamma(r - p_{A2})[-dh(r)] \\
&\quad + \left[p_{A2} - \frac{\partial c_A}{\partial q_{A2}}\right] \left[\sigma_{B1} h'(p_{A2}) - \sigma_{A1}\gamma(p_{B2} - p_{A2}) h(p_{B2}) + \sigma_{A1} \int_{r=p_{A2}}^{p_{B2}} -\gamma(r - p_{A2})\{-dh(r)\}\right] \\
&= \sigma_{B1} h(p_{A2}) + \left[p_{A2} - \frac{\partial c_A}{\partial q_{A2}}\right] \sigma_{B1} h'(p_{A2})
\end{aligned}
$$

which implies that

$$
0 = h(p_{A2}) + \left[p_{A2} - \frac{\partial c_A}{\partial q_{A2}}\right] h'(p_{A2})
$$

which is the first order condition of a monopolist. Therefore, the unique symmetric pure-strategy equilibrium derived under the symmetric market share case is also a pure-strategy equilibrium under the asymmetric market share case, when $\gamma(0) = 0$.

2 Pure-Strategy Equilibrium under the Differentiated Products Model: Discussion

The fact that consumers do not know before their first-period purchase whether their tastes for the underlying product characteristics will change, or whether they will leave the market, guarantees that in the first period all consumers to the left of $\sigma_{A1}L$ buy from firm A, while all those to the right buy from firm B, for some $\sigma_{A1} \in [0, 1]$. Further, equations (25)-(27) in the manuscript define the unique symmetric pure-strategy equilibrium of the differentiated products model under the assumption of $\mu + \nu \neq 0$, provided that $|\sigma_{A1} - \sigma_{B1}| < s/L$. In the symmetric market shares case, $\sigma_{A1} = \sigma_{B1}$, this condition is automatically satisfied. The second-order conditions always hold locally. Each firm's profit function is concave in its price in the region in which equation (22) of the manuscript holds. However, this is not sufficient to show that equation (25) in the manuscript always defines a Nash equilibrium. A firm may want to deviate from its strategy in the "candidate"equilibrium defined by the first-order conditions, by choosing a strategy outside the range in which equation (22) of the manuscript holds. However, if ν is large enough, the first-order conditions define an equilibrium both for the second period and for the full game. They also define an equilibrium if μ is large enough and s is small. In the second-period sub-game, with $\sigma_{A1} = \sigma_{B1}$ and $s = L/2$, for example, equations (25)-(27) of the manuscript describe an equilibrium for (1) $\mu > 0.38$ if $\nu = 0$, (2)$\nu > 0.43$ if $\mu = 0$, and (3) always if $1 - \mu - \nu = 0$. Any of these sets of conditions ensures that firms choose strategies in the range in which equation (22) of the manuscript holds. For small μ and ν there exists no symmetric pure-strategy equilibrium.

Two special cases of interest are (1) the case in which all consumers were in the market in the first period and have mirror-image preferences for the underlying attributes in the second period, i.e., $\mu + \nu = 0$, and (2) the case in which all consumers who bought in the previous period have tastes that are independent of their previous period's tastes, i.e., $\mu + \nu = 1$. We cover these two cases next.

2.1 Collusive Outcome When All Consumers are Attribute-Based Variety-seekers: $\nu + \mu = 0$

In this case, the analysis provided in the manuscript (and the above discussion) does not suffice because of the constraint imposed by consumers' reservation price r. Provided $|(p_{A2} + \sigma_{B1} L) - (p_{B2} + \sigma_{A1} L)| < s$ and $p_{A2} < r - \sigma_{B1} L$, we have $\frac{\partial \pi_{A2}}{\partial p_{A2}} = \sigma_{B1} L > 0$. However, for prices such that $p_{A2} > r - \sigma_{B1} L + s$, $p_{B2} > r - \sigma_{A1} L + s$, in the second period no consumer will buy from the firm he purchased from in the first period, so each firm acts as a monopolist on the other firm's customers who bought in the first period. Thus only the collusive prices (i.e., the firms' joint profit maximizing prices) satisfy the first-order conditions for a symmetric equilibrium. For $r \geq c + L$, full market coverage obtains. For $r < c + L$, only partial market coverage obtains. Sufficient conditions for the first-order conditions to specify global best responses for the firms, that is, to define an equilibrium, are given by $r \leq c + s + 3\frac{L}{2} + 2\sqrt{sL}$ for $s \leq L$, and $r \leq c + 2s + 5\frac{L}{2}$ for $s \geq L$. By breaking the consumers into two distinct groups so that neither firm can retain its previous consumers except by a large price cut, staying costs cause the fully collusive outcome to arise as a non-cooperative equilibrium.

2.2 Competitive Outcome When No Attribute-Based Variety-Seekers Exist: $\nu + \mu = 1$

In this case, each "old"consumer's preferences for the underlying product attributes are independent of his previous preferences, and so independent of his first-period choice. Equations (28)-(30) of the manuscript show that in a symmetric equilibrium, the prices and profits reduce to those in a market without variety-seeking (even if $\mu > 0$, i.e., some consumers have staying costs)! In other words, $p_{A2} = c + L$ and $\pi_{A2} = \frac{L^2}{2}$.

The reason is that a fraction σ_{A1} of consumers are uniformly distributed on $(0, L)$ but they have shifted s away from A, while the complementary fraction σ_{B1} are uniformly distributed on $(0, L)$ but have shifted s away from B. In the case, in a symmetric equilibrium, the identity of the marginal consumers is altered by the staying costs but their density, 1, is not, so that the market is as competitive as if there were no staying costs. Said differently, although consumers incur staying costs for their previously tried product, they could develop new preferences for its attributes in the second period!

Finally, if we compare the equilibrium prices and profits of the collusive and competitive outcomes (discussed above) with those of the general case (given in equations (28)-(30) of the manuscript), we find that the equilibrium solutions of the general case lie between the collusive and competitive solutions.

2.3 Naive Consumer Expectations

Here we assume that consumers do not take expected second period prices into account while making their first period choices. In this case first-period market shares are determined exactly as if there were no staying costs, as shown below.

$$\sigma_{A1}(p_{A1}, p_{B1}) = \frac{L + p_{B1} - p_{A1}}{2L} \tag{1}$$

$$\sigma_{B1}(p_{A1}, p_{B1}) = \frac{L + p_{A1} - p_{B1}}{2L} \tag{2}$$

Plugging these into firm A's discounted profit function in the first period (see page 15 of the manuscript), it follows that

$$\pi_A(p_{A1}, p_{B1}) = (p_{A1} - c)\frac{L + p_{B1} - p_{A1}}{2} + \frac{\lambda}{2(\nu + \mu)}\left\{L + \frac{\left[\frac{p_{A1} - p_{B1}}{L}\right][(1 - \nu - \mu)L + \mu s]}{3}\right\}^2$$

The first-order condition then becomes

$$\frac{\partial \pi_A}{\partial p_{A1}} = \frac{1}{2}(L + p_{B1} - 2p_{A1} + c) - \frac{\lambda}{\nu + \mu}\left\{L + \frac{\left[\frac{p_{A1} - p_{B1}}{L}\right][(1 - \nu - \mu)L + \mu s]}{3}\right\}\frac{1}{3L}[(1 - \nu - \mu)L + \mu s]$$

In a symmetric equilibrium

$$p_{A1} = p_{B1} = c + L + \frac{2\lambda}{3(\nu + \mu)}[(1 - \nu - \mu)L + \mu s] \tag{3}$$

and

$$p_{A2} = p_{B2} = c + \frac{L}{\nu + \mu} \tag{4}$$

Total discounted profits of firm A are

$$\pi_A = \pi_B = \frac{L^2}{2}\left\{1 + \frac{\lambda}{\nu + \mu} + \frac{2\lambda}{3(\nu + \mu)}\left[(1 - \nu - \mu) + \frac{\mu s}{L}\right]\right\} \tag{5}$$

By contrast, if there is no variety-seeking in the market, the two periods are not connected, so that $p_{A1} = p_{B1} = c + L$, and total discounted profits per firm are $\pi_A = \pi_B = \frac{L^2}{2}(1 + \lambda)$.

It is clear that first-period prices are always higher than in a market without variety-seeking. The higher is the degree of variety-seeking in the market, the more collusive the pricing in the first period, i.e., $\frac{\partial p_{A1}}{\partial s} > 0$.

2.4 Rational Consumer Expectations

Here we assume that consumers take expected second period prices into account while making their first period choices. Under this case, we can compute the fraction $\sigma_{A1}(p_{A1}, p_{B1})$ by locating the marginal consumer, for whom the difference in their expected second-period surpluses from buying from the different firms in the first period equals the difference in his first-period surpluses from buying from different firms.

Consider a first-period consumer located at z. That consumer's first-period surpluses from buying from firms A and B are $(r - p_{A1} - z)$ and $(r - p_{B1} - L + z)$ respectively.

In the second period, this same consumer could be one of the three types of consumers we assumed.

With probability μ the consumer's tastes in the second period are uniformly distributed on $(0, L)$, and he will buy in the second-period equilibrium. Conditional on buying from A in period one and on being in this group of consumers, the consumer will again buy from firm A in the second period if his second-period location is at $x \leq \frac{[L+p_{B2}(\sigma_{A1})-p_{A2}(\sigma_{B1})-s]}{2}$. The consumer's expected second-period surplus is then given by

$$\frac{\lambda}{L}\left[\begin{array}{c} \int_{x=0}^{\frac{L+p_{B2}(\sigma_{A1})-p_{A2}(\sigma_{B1})-s}{2}} \{r - s - p_{A2}(\sigma_{B1}) - x\}\, dx \\ + \int_{x=\frac{L+p_{B2}(\sigma_{A1})-p_{A2}(\sigma_{B1})-s}{2}}^{L} \{r - p_{B2}(\sigma_{A1}) - L + x\}\, dx \end{array}\right]$$

where $p_{A2}(\sigma_{B1})$ and $p_{B2}(\sigma_{A1})$ are firms' second-period prices as functions of their competitors' first-period market shares.

With probability $(1 - \nu - \mu)$, the consumer changes his taste and is now located at $(L - z)$ in the second period. If he buys B in the second-period, his second period surplus conditional on buying from A in the first period is $\lambda[r - p_{B2}(\sigma_{A1}) - z]$. In this case, consumers whose tastes perfectly change, in a desire for variety, will not buy from the firm from which they previously bought.

Lastly, With probability ν the consumer is not in the market, and so gets zero surplus in the second period.

Writing the corresponding equations for second-period surplus conditional on the consumer's buying from firm B in period one and performing the integrations, we find that the gain in expected total surplus (i.e., first-period surplus plus expected second-period surplus) resulting from buying from firm A rather than from firm B in the first period is

$$[p_{B1}-p_{A1}+L-2z]+\lambda\left[\mu\frac{s}{L}\left\{p_{A2}(\sigma_{B1})-p_{B2}(\sigma_{A1})\right\}+(1-\nu-\mu)\{p_{A2}(\sigma_{B1})-p_{B2}(\sigma_{A1})+L-2z\}\right]$$

By equation (25) of the manuscript,

$$p_{A2}(\sigma_{B1})-p_{B2}(\sigma_{A1})=\frac{2\,(1-2\sigma_{A1})}{3(\nu+\mu)}[(1-\nu-\mu)L+\mu s]$$

Finally, the marginal consumer has $z=\sigma_{A1}L$ and is indifferent between buying from A and buying from B, so that

$$\begin{aligned}0=\lambda&\left[\left\{\frac{2(1-2\sigma_{A1})}{3(\nu+\mu)}[(1-\nu-\mu)L+\mu s]\right\}\left\{[1-\nu-\mu]+\mu\frac{s}{L}\right\}+(1-\nu-\mu)(L-2\sigma_{A1}L)\right]\\&+[(p_{B1}-p_{A1})+(L-2\sigma_{A1}L)]\end{aligned}$$

It follows, therefore, that

$$\sigma_{A1}(p_{A1},p_{B1})=\frac{L+\frac{1}{y}(p_{B1}-p_{A1})}{2L}$$

where

$$y=1+\lambda\left\{(1-\nu-\mu)+\frac{2}{3(\nu+\mu)}\left[(1-\nu-\mu)+\frac{\mu s}{L}\right]^2\right\}$$

Note that $y\geq 1$. Further, $y=1$ if either $\nu=1$ (all the first-period consumers leave the market and are replaced by new consumers in the second period) or $\lambda=0$ (consumers ignore the second period in making first-period decisions - the case of naive expectations). When $y>1$, first-period market shares are less responsive to price changes than when consumers are naive or when there are no staying costs in the second period.

Now, therefore,

$$\pi_A(p_{A1},p_{B1}) = (p_{A1}-c)\left[\frac{L+\frac{1}{y}(p_{B1}-p_{A1})}{2}\right]+\frac{\lambda}{2(\nu+\mu)}\left\{L+\frac{\left(\frac{p_{A1}-p_{B1}}{ty}\right)[(1-\nu-\mu)L+\mu s]}{3}\right\}^2$$

so that

$$\frac{\partial\pi_A}{\partial p_{A1}} = \frac{1}{2}\left\{L+\frac{1}{y}(p_{B1}-2p_1^A+c)\right\}-\frac{\lambda}{\nu+\mu}\left\{L+\frac{\left(\frac{p_{A1}-p_{B1}}{ty}\right)[(1-\nu-\mu)L+\mu s]}{3}\right\}\frac{1}{3Ly}[(1-\nu-\mu)L+\mu s]$$

In symmetric equilibrium

$$p_{A1} = p_{B1} = c+Ly+\frac{2\lambda}{3(\nu+\mu)}\{(1-\nu-\mu)L+\mu s\} \tag{6}$$

and

$$p_{A2} = p_{B2} = c+\frac{L}{\nu+\mu} \tag{7}$$

Total discounted profits are

$$\pi_A = \pi_B = \frac{L^2}{2}\left\{y+\frac{\lambda}{\nu+\mu}+\frac{2\lambda}{3(\nu+\mu)}[(1-\nu-\mu)+\frac{\mu s}{L}]\right\} \tag{8}$$

Comparison of equation (3) with equation (6) shows that the first period is more collusive when consumers have rational expectations than when they have naive expectations (since $y \geq 1$). But the symmetric second-period equilibrium is unaffected, and a comparison of equations (6) and (8) shows that when $\upsilon+\mu=1$, so $y>1$, first-period prices are even higher than second-period prices!