Technical Appendix for "Service Cancellation and Competitive Refund Policy"

I will show that any refund $R_j \in [H, V_1 + H]$, $j = A, B$, will lead to the same equilibrium payoff for firm j as that when $R_j = H$, and any refund $R_j < H$ is payoff equivalent to $R_j = 0$. To this end, I will analyze the scenarios when $R_A \in [H, V_1 + H]$ and $R_B \in [H, V_1 + H]$ and when $R_A \in [H, V_1 + H]$ and $R_B < H$, corresponding to Section 3.1.2 and 3.1.3, respectively. The proof closely follows those for Proposition 1 and 2, respectively. Note first that allowing R_j to be greater than H does not change the equilibrium in the spot period nor the firms' optimal reservation limit N_j in the advance period. I will then focus on investigating the equilibrium advance prices and the firms' equilibrium expected profits. Moreover, note that the consumers' gross expected valuation for a firm's service (i.e., willingness to pay) in the advance period is given by $V_1/2 + (R_j - H)/2 = (V_1 + R_j - H)/2$ when the firm offers $R_j \in [H, V_1 + H]$, and by $V_1/2$ when $R_j < H$.

Both Firms Offer Partial Refund: $R_A \in [H, V_1 + H]$ **and** $R_B \in [H, V_1 + H]$

In this scenario the firms' optimal reservation limit is given by N_j^{**} as that in Lemma 2. Consider first $(1 - \alpha)M/2 < N \leq (2 - \alpha)M/4$. In this case, we have $N_A^{**} + N_B^{**} \leq \alpha M$ and thus the firms are local monopolies in the advance period. It follows that in equilibrium $P_{1A}^{**} = (V_1 + R_A - H)/2$ and $P_{1B}^{**} = (V_1 + R_B - H)/2$, while $\Pi_A^{**} = \Pi_B^{**} = N_j^{**}(P_{1j}^{**} - R_j/2) + (1 - \alpha)MV_2/2 = [2N - (1 - \alpha)M)](V_1 - H)/2 + (1 - \alpha)MV_2/2$.

Consider then $(2 - \alpha)M/4 < N < M/2$. In this case, the equilibrium is in mixed strategy. When $(V_1 + R_A - H)/2 - P_{1A} > (V_1 + R_B - H)/2 - P_{1B}$ or equivalently $R_A/2 - P_{1A} > R_B/2 - P_{1B}$, firm A will sell N_A^{**} and firm B will sell $\alpha M - N_A^{**} = M - 2N$ in the advance period. Conversely, when $(V_1 + R_A - H)/2 - P_{1A} < (V_1 + R_B - H)/2 - P_{1B}$ or equivalently $R_A/2 - P_{1A} < R_B/2 - P_{1B}$, firm A will sell $\alpha M - N_B^{**} = M - 2N$ and firm B will sell N_B^{**} in the advance period. As a result, the lowest advance price a firm would like to charge is given by, respectively:

$$(\underline{P}_A - R_A/2)[2N - (1 - \alpha)M] = (M - 2N)(V_1 - H)/2 \Longrightarrow \underline{P}_A^{**} = \frac{R_A}{2} + \frac{(M - 2N)(V_1 - H)/2}{2N - (1 - \alpha)M},$$

and

$$(\underline{P}_B - R_B/2)[2N - (1 - \alpha)M] = (M - 2N)(V_1 - H)/2 \Longrightarrow \underline{P}_B^{**} = \frac{R_B}{2} + \frac{(M - 2N)(V_1 - H)/2}{2N - (1 - \alpha)M}.$$

Note that $R_A/2 - \underline{P}_A^{**} = R_B/2 - \underline{P}_B^{**}$. Therefore, the equilibrium support for the firms' advance prices is $P_{1A}^{**} \in [\underline{P}_A^{**}, (V_1 + R_A - H)/2]$ and $P_{1B}^{**} \in [\underline{P}_B^{**}, (V_1 + R_B - H)/2]$, respectively. Nevertheless, the firms' equilibrium expected payoffs are $\Pi_A^{**} = \Pi_B^{**} = (N - N_j^{**})(V_1 - H)/2 + (1-\alpha)MV_2/2$. It is then evident that the firms' equilibrium expected profits are the same as that in (7).

Asymmetric Refund Policies: $R_A \in [H, V_1 + H]$ **and** $R_B < H$

In this scenario the firms' optimal reservation limit is given by N_A^o and N_B^o, respectively, as in Section 3.1.3. Consider first $(1-\alpha)M/2 < N \leq (3-\alpha)M/6$. In this case, we have $N_A^o + N_B^o \leq \alpha M$ and thus the firms remain local monopolies in the advance period. It is then straightforward that in equilibrium $P_{1A}^o = (V_1 + R_A - H)/2$ and $P_{1B}^o = V_1/2$, and the firms' equilibrium expected payoffs are the same as those in Section 3.1.3.

Consider then the mixed-strategy equilibrium when $(3-\alpha)M/6 < N < M/2$. In this case, when $(V_1 + R_A - H)/2 - P_{1A} > V_1/2 - P_{1B}$ or equivalently $(R_A - H)/2 > P_{1A} - P_{1B}$, firm A will sell N_A^o and firm B will sell $\alpha M - N_A^o = M - 2N$ in the advance period. Conversely, when $(V_1 + R_A - H)/2 - P_{1A} < V_1/2 - P_{1B}$ or equivalently $(R_A - H)/2 < P_{1A} - P_{1B}$, firm A will sell $\alpha M - N_B^o = (1+\alpha)M/2 - N$ and firm B will sell N_B^o in the advance period. As a result, the lowest advance price a firm would like to charge is given by, respectively:

$$(\underline{P}_A - R_A/2)[2N - (1-\alpha)M] = [(1+\alpha)M/2 - N](V_1 - H)/2 \Longrightarrow \underline{P}_A^o = \frac{R_A}{2} + \frac{[(1+\alpha)M/2 - N](V_1 - H)/2}{2N - (1-\alpha)M},$$

and

$$\underline{P}_B[N - (1-\alpha)M/2] = (M - 2N)V_1/2 \Longrightarrow \underline{P}_B^o = \frac{(M-2N)V_1/2}{N - (1-\alpha)M/2}.$$

Note that $(R_A - H)/2 < \underline{P}_A^o - \underline{P}_B^o$. Therefore, the equilibrium support for the firms' advance prices is $P_{1A}^{**} \in [\underline{P}_A^o, (V_1 + R_A - H)/2]$ and $P_{1B}^{**} \in [\underline{P}_A^o - (R_A - H)/2, V_1/2)$, respectively. Moreover, we can also see that the firms' equilibrium expected advance-period payoffs are given by $\Pi_{1A}^o = [(1+\alpha)M/2 - N](V_1 - H)/2$ and $\Pi_{1B}^o = [\underline{P}_A^o - (R_A - H)/2][N - (1-\alpha)M/2] = [(1+\alpha)M - 2N]V_1/8 + [6N - (3-\alpha)M]H/8$. It follows that the firms' equilibrium expected profits are the same as those in (8) and (9), respectively. This completes the proof.