

Technical Appendix

B. Lease Brokering in the U.S. Automobile Industry

In our analysis, we represent lease brokering as though the manufacturer sets a *wholesale* lease rate of \hat{w} to which the dealer adds his own leasing margin. Although this is not exactly what occurs in the automotive industry, we will show that what does occur is equivalent to our representation. In practice, automotive dealers own all of the vehicles that are on their lots in the sense that they have agreed to pay the manufacturer the wholesale selling price, w , for each unit. For lease brokering, the manufacturer sets several parameters: Let ρ , RV , and CB be the per-period interest rate, residual value, and cash back incentive offered by the manufacturer on brokered leases.⁹ To broker a lease, the dealer negotiates an implicit selling price, which we denote by P , with the consumer, and the consumer's single payment for a one period lease is:

$$LP = P - CB - RV(1 + \rho)^{-1} \quad (\text{T.1})$$

Although the consumer's lease payment goes to the manufacturer, the manufacturer pays the dealer an amount equal to the implicit selling price, P , that the dealer negotiated with the consumer¹⁰. For the dealer, this arrangement is equivalent to one in which he receives a margin on the lease equal to:

$$LM = P - w \quad (\text{T.2})$$

Subtracting (T.2) from (T.1), we can see that the manufacturer's net revenue from the lease is:

$$\hat{w} = w - CB - RV(1 + \rho)^{-1} \quad (\text{T.3})$$

In the language of our model, if the dealer brokers \hat{q} leases, then the lease payment at which the market clears is $r(\hat{q}) = a - \hat{q}$. Equating this to (T.1), we can see that in order to broker \hat{q} leases, the dealer would need to offer an implicit selling price of:

$$P = a - \hat{q} + CB + RV(1 + \rho)^{-1}$$

⁹Note that the interest rate ρ offered by the manufacturer is not necessarily equal to the actual cost of capital. This distinction is inconsequential for a single period lease, but may matter in leases that involve multiple payments.

¹⁰Net of any amount of the original wholesale price, w , still owed to the manufacturer by the dealer.

Since the dealer earns a leasing margin of $LM = P - w$ on each of the \hat{q} leases that he brokers, it is easy to see from first order conditions that he would optimally set $\hat{q} = \left(a - w + CB + RV(1 + \rho)^{-1} \right) / 2 = (a - \hat{w}) / 2$, which is consistent with what we obtain in (5).

C. Derivation of Second Period Dealer Quantities

1. Dealer Specific Wholesale Pricing

Differentiating (24) w.r.t. to q_{i2} , we have that

$$q_{i2} = \frac{a - 2q_{il} - q_{is} - w_{2i} - \sum_{j=1..n, j \neq i} (q_{jl} + q_{js} + q_{2j}) \theta}{2} \quad \forall i = 1..n \quad (\text{T.4})$$

Summing eq. T.4 over all values of $i = 1..n$, we have that

$$q_{i2} + \sum_{j \neq i} (q_{2j}) = \frac{na - 2 \left(q_{il} + \sum_{j \neq i} (q_{jl}) \right) - \left(q_{is} + \sum_{j \neq i} (q_{js}) \right) - \sum_i (w_j) - \sum_{j \neq i} (n-1) (q_{jl} + q_{js} + q_{2j}) \theta}{2} \quad (\text{T.5})$$

It is easy to see that eq. T.4 and eq. T.5 are simultaneous equations in q_{2i} and $\sum_i (q_{2j})$. Solving for q_{2i} , we have that

$$q_{i2}^* (\mathbf{q}_l, \mathbf{q}_s, \mathbf{w}_2) = \left[\frac{a(2 - \theta) - 2(q_{is} + w_{i2}) - \theta \left(\sum_{j \neq i} (q_{js} - w_{j2}) - (2 - n)w_{i2} + q_{is}(n(1 - \theta) - 2 + \theta) \right)}{(2 - \theta)(2 + (n - 1)\theta)} - q_{il} \right]^+ \quad (\text{T.6})$$

2. Uniform Wholesale Pricing

For this, we have to substitute $w_{2i} = w_2$ into T.6

$$q_{i2}^* (\mathbf{q}_l, \mathbf{q}_s, \mathbf{w}_2) = \left[\frac{a(2 - \theta) - 2(q_{is} + w_2) - \theta \left(\sum_{j \neq i} (q_{js} - w_2) - (2 - n)w_2 + q_{is}(n(1 - \theta) - 2 + \theta) \right)}{(2 - \theta)(2 + (n - 1)\theta)} - q_{il} \right]^+$$

Substituting above equation into the manufacturer profit function and differentiating it w.r.t. to w_2 gives us the optimal second period wholesale price.

Decision	Dealer-Specific Pricing	Uniform Pricing		
	Dealer: Pure Leasing	Dealer: Pure Leasing	Dealer: Hybrid Policy	Dealer: Pure Selling
q_{i2}	$\frac{a(1+n(6-\theta)-2\theta+3n^2\theta)}{2(2+(n-1)\theta)(2+\alpha(n,\theta))}$	$\frac{a(1+6n+(n-1)(2+3n)\theta)}{(2+(n-1)\theta)(2+\beta(n,\theta))}$	$\frac{a(4+16n(3n-2)+\Omega(n,\theta))}{(2+(n-1)\theta)(16n(10n-3)+\Lambda(n,\theta))}$	$\frac{-a(-2(5+36n)+\xi(n,\theta))}{(24+184n+\eta(n,\theta))(2+(n-1)\theta)}$
w_2	$\frac{a(1+n(6-\theta)-2\theta+3n^2\theta)}{2(2+\alpha(n,\theta))}$	$\frac{a(1+6n+(n-1)(2+3n)\theta)}{2(2+\beta(n,\theta))}$	$\frac{a((n-1)\theta+2)(4+16n(3n-2)+\Omega(n,\theta))}{(2+(n-1)\theta)(16n(10n-3)+\Lambda(n,\theta))}$	$\frac{-a(-2(5+36n)+\xi(n,\theta))}{24+184n+\eta(n,\theta)}$
p_{i2}	$\frac{a(5+\delta(n,\theta))}{2(2+(n-1)\theta)(2+\alpha(n,\theta))}$	$\frac{a(5+26n+(n-1)(7+19n)\theta+\beta(n,\theta))}{(2+(n-1)\theta)(2+\beta(n,\theta))}$	$\frac{a(4n(-7+26n)+\chi(n,\theta))}{16n(10n-3)+\Lambda(n,\theta)}$	$\frac{-a(-6(5+36n)+\xi(n,\theta)(3+(n-1)\theta))}{(24+184n+\eta(n,\theta))(2+(n-1)\theta)}$
q_{il}	$\frac{a(1+2n(2-\theta)+2n^2\theta)}{(2+(n-1)\theta)(2+\alpha(n,\theta))}$	$\frac{a(1+2n(2-\theta)+2n^2\theta)}{(2+(n-1)\theta)(2+\beta(n,\theta))}$	$\frac{a(4(16n^2+10n-3)+\Theta(n,\theta))}{(2+(n-1)\theta)(16n(10n-3)+\Lambda(n,\theta))}$	-
q_{is}	0	-	$\frac{a(4-24n+\Gamma(n,\theta))}{16n(10n-3)+\Lambda(n,\theta)}$	$\frac{-a(-2-20n+\zeta(n,\theta))}{(24+184n+\eta(n,\theta))}$
w_1	$\frac{a(7+3(n-1)\theta)^2}{2(2+(n-1)\theta)(12+5(n-1)\theta)}$	$\frac{a((1+6n)^2+\kappa(n,\theta))}{2n(2+(n-1)\theta)(2+\beta(n,\theta))}$	$\frac{a(4-24n+Y(n,\theta))}{16n(10n-3)+\Lambda(n,\theta)}$	$\frac{a(12+32n(8+39n)+\varphi(n,\theta))}{2n(2-\theta)(2+(n-1)\theta)^2(24+184n+\eta(n,\theta))}$
p_{il}	$\frac{a(11+\gamma(n,\theta))}{2(2+(n-1)\theta)(2+\alpha(n,\theta))}$	$\frac{a(3+16n+(n-1)(5+14n)\theta+\beta(n,\theta))}{(2+(n-1)\theta)(2+\beta(n,\theta))}$	$\frac{a(4+8n(32n-11)+\Psi(n,\theta))}{(2+(n-1)\theta)(16n(10n-3)+\Lambda(n,\theta))}$	-
p_{is}	-	-	$\frac{a(4+16n(29n-9)+\psi(n,\theta))}{(2+(n-1)\theta)(16n(10n-3)+\Lambda(n,\theta))}$	$\frac{-a(-14(5+36n)+\mu(n,\theta))}{(24+184n+\eta(n,\theta))(2+(n-1)\theta)}$

Table 1: Optimal Decisions under Competition between Dealers when Manufacturer Sells

where:

$$\begin{aligned}
\beta(n, \theta) &= (n-1)^2 (2+3n) \theta^2 \\
\alpha(n, \theta) &= n(10-3\theta) - 2\theta + 5n^2\theta \\
\delta(n, \theta) &= 26n + (n-1)(7+19n)\theta + (n-1)^2(2+3n)\theta^2 \\
\gamma(n, \theta) &= 58n + (n-1)(17+47n)\theta + 3(n-1)^2(2+3n)\theta^2 \\
\Omega(n, \theta) &= 4(-1+4n(4+n(-11+6n)))\theta + (-3+2n(-3+2n)(5+9n(-3+2n)))\theta^2 \\
&\quad + (n-1)(-4+n(3+2n(29+2n(-19+6n))))\theta^3 + (n-1)^2(-1+n(3+n(6+n(-10+3n))))\theta^4 \\
\Psi(n, \theta) &= (-4+8n(33+2n(-65+38n)))\theta + (-11+2n(-133+n(769+3n(-299+96n))))\theta^2 \\
&\quad + (n-1)\left(-20+n\left(-104+n\left(897-966n+272n^2\right)\right)\right)\theta^3 \\
&\quad + (n-1)^2(-11+n(-7+(-2+n)n(-129+64n)))\theta^4 + (n-2)(n-1)^3(1+n(-1+3n(-5+2n)))\theta^5 \\
\Theta(n, \theta) &= 4(5+4n(-4+n(-11+8n)))\theta + (-7+2n(-5+n(165+n(-193+48n))))\theta^2 \\
&\quad + (n-1)\left(2+n\left(-56+n\left(301-230n+32n^2\right)\right)\right)\theta^3 \\
&\quad + (n-1)^2(1+n(-25+n(118+n(-65+4n))))\theta^4 - n(n-1)^3(3+n(-17+7n))\theta^5 \\
\Gamma(n, \theta) &= -4(1+2(-4+n)n)\theta + (-1+2n(2+n(-23+19n)))\theta^2 \\
&\quad (n-1)(-1+n(14+n(-59+32n)))\theta^3 + n(n-1)^2(3+n(-17+7n))\theta^4 \\
\Lambda(n, \theta) &= 2\left(2+62n-280n^2+160n^3\right)\theta + 2\left(-5-51n+344n^2-394n^3+120n^4\right)\theta^2 \\
&\quad + 2\left(4+12n-172n^2+312n^3-196n^4+40n^5\right)\theta^3 + 2\left(-1+n+28n^2-78n^3+78n^4-33n^5+5n^6\right)\theta^4 \\
\Upsilon(n, \theta) &= 2(-1+n(57+2n(-125+72n)))\theta + (-8+n(-77+2n(293+2n(-173+54n))))\theta^2 \\
&\quad + (n-1)(-8+n(-13+n(252+n(-269+72n))))\theta^3 + (n-1)^2(-2+(-2+n)n(-1+3n(-7+3n)))\theta^4 \\
\chi(n, \theta) &= 2(1+n(34+n(-163+90n)))\theta + (-5+n(-51+n(361+2n(-200+57n))))\theta^2 \\
&\quad + (n-1)(-4+n(-13+n(148+n(137+31n))))\theta^3 + (n-1)^2\left(-1+n^2(23+n(-17+3n))\right)\theta^4 \\
\psi(n, \theta) &= 4n(107+4n(-114+67n))\theta + (-23+2n(-217+n(1327+2n(-775+246n))))\theta^2 \\
&\quad + (n-1)(-33+n(-181+2n(777+4n(-205+56n))))\theta^3 \\
&\quad + (n-1)^2(-17+n(-20+n(n-2)(101n-226)))\theta^4 + (n-1)^3(-3+n(3+n(n-2)(9n-26)))\theta^5 \\
\kappa(n, \theta) &= (n-1)(1+2n(7+18n))\theta + (n-1)^2n(4+9n)\theta^2 \\
\xi(n, \theta) &= (29+12n(9-8n))\theta - 3(n-1)(-10+n(-25+14n))\theta^2 \\
&\quad - (n-1)^2(-13+6(-4+n)n)\theta^3 + (n-1)^3(2+3n)\theta^4 \\
\eta(n, \theta) &= (-68+4n(-76+67n))\theta + 2(n-1)(-34+n(-113+64n))\theta^2 \\
&\quad + 4(n-1)^2(-7+n(-19+5n))\theta^3 - 2(n-1)^3(2+5n)\theta^4 \\
\varsigma(n, \theta) &= (3+2(13-9n)n)\theta - (n-1)(-1+4(-3+n)n)\theta^2 + 2(n-1)^2n\theta^3 \\
\varphi(n, \theta) &= \left(-52+8n\left(-125-448n+390n^2\right)\right)\theta + \left(91+4n\left(409+n\left(981-2092n+774n^2\right)\right)\right)\theta^2 \\
&\quad + 2(n-1)(41+n(773+2n(851+3n(-465+127n))))\theta^3 \\
&\quad + (n-1)^2(40+n(864+n(1819+12n(31n-175))))\theta^4 + (n-1)^3(10+n(286+n(597+4n(9n-106))))\theta^5 \\
&\quad - (n-1)^4(-1+n(-52+3n(-37+12n)))\theta^6 + (n-1)^5n(4+9n)\theta^7 \\
\mu(n, \theta) &= \left(233+942n-888n^2\right)\theta - 3(n-1)(-99+n(-283+194n))\theta^2 - (n-1)^2(-181+3n(-131+56n))\theta^3 \\
&\quad - (n-1)^3(-53+3n(-31+6n))\theta^4 + 3(n-1)^4(2+3n)\theta^5 \\
\beta(n, \theta) &= (n-1)^2(2+3n)\theta^2 \\
\alpha(n, \theta) &= n(10-3\theta) - 2\theta + 5n^2\theta
\end{aligned}$$