<u>Online Technical Companion to Accompany "*Trade-ins in Durable Goods Markets: Theory and Evidence*"</u>

This appendix is divided into six main sections which are ordered in a sequence corresponding to their appearance in the paper itself. We start with a section (TO1) that contains the expressions for a baseline model that has no adverse selection and no trade-ins - in the interest of space, this model is not discussed in the paper. Next, we provide expressions for variables corresponding to the model with adverse selection but no trade-ins (section TO2), which corresponds to section 4.2 of the text. Section TO3 produces the omitted expressions in the section 4.3 of the text. We also provide the details of the numerical simulations performed to check the robustness of the key results with respect to different values of α (section TO4) and asymmetrical peaches and lemons (TO5) - these two sections correspond to section 4.3.4 of the text. Finally, we provide the list of the make-models of automobiles used in our analysis (section TO6).

TO1: NO ADVERSE SELECTION AND NO TRADE-INS

In this section, we analyze the benchmark case where there is no adverse selection problem in the used good market, and the producer does not offer trade-ins. The realization of used good quality is not known *ex ante*. However, after it has deteriorated stochastically into either a peach (high quality used good), or a lemon (low quality used good), its quality is known to *both* the seller and buyers in the used goods market.

Each consumer buys a good which provides the highest utility to her and buys it every period. In this frictionless market, no buyer holds a purchased good more than one period. Each buyer always updates to her optimal vintage every period (which includes the 'no buy' option). Essentially, there are three 'products' in the market with three different qualities – new goods, peaches, and lemons. This in turn leads to four different types of consumer behavior, leading to four consumer segments based upon their valuation for quality (see Figure T1). It is important to emphasize that these four types of behavior enumerated above cover the gamut of possible behaviors. We do not impose the existence of segments on the equilibrium solution – if any of these behaviors is not observed in equilibrium, our solution will reflect this fact. The four segments are:

Segment 1: This segment consists of the highest valuation types who buy new goods every period.
Segment 2: This segment buys peaches every period.
Segment 3: This segment buys lemons every period.
Segment 4: This segment consists of the lowest valuation types who do not buy any product in any period (these buyers can be said to buy the 'outside option').

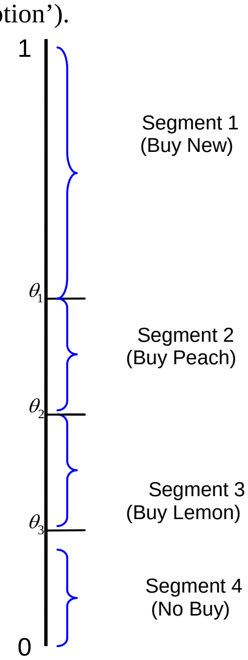

Figure T1: Consumer Segments with No Adverse Selection

Now, the discounted utility, V, of a segment 1 consumer is:

$$V^n(\theta) = v\theta - p_n + \delta[E(p_u) + V^n(\theta)]$$

where δ is the discount factor and $E(p_u)$ is the expected price of the used good. Since a new good might deteriorate into a peach or a lemon with equal probability, we have:

$$E(p_u) = (1/2)(p_e + p_l)$$

where p_e is the price of the peach and p_l is the price of the lemon. This gives:

$$V^n(\theta) = \frac{v\theta - p_n + (1/2)\delta(p_e + p_l)}{(1-\delta)} \quad \text{(A1)}$$

Similarly, the discounted utilities for consumers in segment 2 (peach buyers) and segment 3 (lemon buyers) are respectively:

Peach buyers: $V^e(\theta) = \dfrac{(1+s)\theta - p_e}{(1-\delta)}$ (A2)

Lemon buyers: $V^l = \dfrac{(1-s)\theta - p_l}{(1-\delta)}$ (A3)

The non-buyers get zero utility.

We use the value functions above to derive expressions for segment boundaries, prices and quality. The following lemma is useful in what follows.

Lemma TO1: *When the quality of the used good is uncertain ex-ante but is observable ex-post, no consumer holds any good beyond one period.*

Proof: We prove one case by contradiction; all other cases can be ruled out similarly. Suppose that a consumer in segment 1 finds it optimal to hold high quality realizations but sell low quality realizations, instead of buying new every period. The Bellman equation for a consumer exhibiting this behavior can be written as.

$$V^{nee}(\theta) = v\theta - p_n + \underbrace{(1/2)\delta[\theta(1+s) + \delta V^{nee}(\theta)]}_{High\ realization} + \underbrace{(1/2)\delta[p_l + V^{nee}(\theta)]}_{Low\ realization}.$$

This gives:

$$V^{nee}(\theta) = \frac{v\theta - p_n + (1/2)\delta p_l + (1/2)\delta(1+s)\theta}{1 - (1/2)\delta - (1/2)\delta^2}.$$

This behavior is optimal for a new good buyer compared to buying a new good every period if:

$V^{nee}(\theta) > V^n(\theta)$.

This implies:

$(1+s)\theta - p_e > v\theta - p_n + (1/2)\delta(p_e + p_l)$.

But this implies that this buyer cannot belong to segment 1 (since she would have a higher utility in segment 2). This is a contradiction. Also note that if a consumer buys a used good (which implies she gets a positive utility), she would not hold on to it in the next period since it would give her zero utility.

Q.E.D.

TO2: MARKETS WITH ADVERSE SELECTION BUT WITHOUT TRADE-INS

This section of the Technical Appendix corresponds to Section 4.2 of the text.

For $\alpha = (1/2)$, the equation (10) in the text can be written as:

$$w_s = \frac{(1-\theta_1)(1) + (1/3)(\theta_1 - \theta_2)(1-s)}{(1-\theta_1)(1) + (1/3)(\theta_1 - \theta_2)} \quad . \quad \text{(A4)}$$

Inserting relevant values from equations (5)-(9) of the main paper, we can solve for the used good quality. This is given by[i]:

$$w = \frac{2(1+s)(2+\delta) + \sqrt{y}\sqrt{A} - y(2 + (4+3s)\delta + 6v)}{2(1-2y)(2+\delta)}, \quad \text{(A5)}$$

[i] The other root of the quadratic equation is discarded since it gives negative used good quality within the parameter range.

where,

$$A = s^2[24(2+\delta) + y(-32 + (-32+\delta)\delta] + 12s[-4 + (-2+3y)\delta](-1+v) + 36y(-1+v)^2$$

Given the stationary equilibrium, the producer profit per-period can be expressed as $\pi = yp$.

Hence:

$$\pi = \frac{y\left\{\sqrt{y}(1+2\delta)\sqrt{A} + 6(2+\delta)(\delta + s\delta + v) - y(s(-4+\delta(5+8\delta)) + 6(1+\delta)(1+2\delta+3v))\right\}}{6+2\delta} \qquad \text{(A6)}$$

The above is a highly non-linear function, but a closed form optimal solution can be obtained using a first order condition with respect to y[ii].

TO3: MARKETS WITH ADVERSE SELECTION AND TRADE-INS

This section corresponds to Section 4.3 of the paper. Using equations (14), (15) and (16), and solving (17) and (18) simultaneously yields expressions for new and used good prices, as well as used good quality.

$$p_T = \frac{-3(-1+y)\delta(1+s-v)(\delta+s\delta+2v) + p(-3(1+s)\delta + 2(-1+\delta)v + (2+\delta)w)}{-(1+s)(-1+\delta)\delta - 2(1+2\delta)v + (1+\delta)(2+\delta)w}$$
$$+\frac{\delta w(1+s)(-2-\delta^2 + y(-2+\delta)(1+2\delta)) + 3(-\delta + y(2+3\delta))v - (-1+2y)(1+\delta)(2+\delta)w)}{-(1+s)(-1+\delta)\delta - 2(1+2\delta)v + (1+\delta)(2+\delta)w} \qquad \text{(A7)}$$

$$p_u = (1-2y)w. \qquad \text{(A8)}$$

The average used good quality supplied in the secondary market is given by:

$$w_s = \frac{(1-\theta_1)(1) + (1/3)(\theta_1 - \theta_2)(1-s)}{(1-\theta_1)(1) + (1/3)(\theta_1-\theta_2)}. \qquad \text{(A9)}$$

In equilibrium:

$$w_s = w. \qquad \text{(A10)}$$

Substituting equations (14)-(16) and (A7)-(A9) into (A10), we get:

$$w = 1 + s + \frac{2sy(1+2\delta)(1+s-v)}{s\left\{-\delta(2+\delta) + y(-2+(-2+\delta)\delta)\right\} - 2(\delta+v) + y(\delta + 2\delta^2 + 4v + 5\delta v) + (2+\delta)p} \qquad \text{(A11)}$$

Finally, the per-period profit of the producer when using trade-ins (equation 9) is given by:

$$\pi = (1-\theta_1)p_T + (1/3)(\theta_1-\theta_2)p_T + (1/3)(\theta_1-\theta_2)p. \qquad \text{(A12)}$$

The above expression can be optimized for p_T and p. The key results are given in Table 3 of the paper. To facilitate exposition, the reported results are for $\delta \to 1$[iii].

Proposition TO1: *In stationary equilibrium, when a producer accepts trade-ins, he will charge a new good price of p_T to consumers who trade-in and a new good price of $p > p_T$ to consumers who do not trade-in.*

Proof: Solving for the first order conditions from equation A12 leads to these prices. It is easy to verify that the concavity of the profit function is also satisfied. The analytic expressions for p_T and p are listed in Table 3. The difference between the new good price without and with trade-ins is given by:

$$p - p_T = \frac{2s(1+s-v)(2s^2 - 4(-1+v)(9+v) + s(37+5v))}{(4+5s-4v)(s^2+8s(3+v) - 8(-1+v)(3+v))}.$$

It is a straightforward algebraic exercise to show that $p - p_T > 0$ as long as assumptions A1 and A2 (see page 5 in the paper) hold. Q.E.D.

Corollary 1: *In the absence of quality uncertainty, the producer charges a single price to all new good buyers, regardless of whether they trade-in, i.e., $p = p_T$.*

[ii] The expressions are available upon request from the authors.

[iii] This is without loss of generality since none of the implications of our model are affected by the discount rate.

Proof: Putting s = 0 in the above expression for p - p_T gives us p - p_T = 0. Q.E.D.

Proposition TO2: *Profits are higher with trade-ins in a market with a positive level of quality uncertainty.*
Proof: This proposition can be shown without explicitly comparing the profits under trade-ins and no trade-ins regimes. If trade-ins do not work, then the new good price (p_T) for consumers trading in should equal the new good price (p) for consumers not trading in a used good. If these prices are always unequal, then we should conclude that trade-ins always work in our set-up. From Table 3, we have:

$$p_T = \frac{s^4 - 36s(-1+\nu)(1+\nu)(3+\nu) + 16(-1+\nu)^2(1+\nu) + s^3(21+5\nu) + 16s^2(5+\nu(4+\nu))}{(4+5s-4\nu)(s^2+8s(3+\nu)-8(-1+\nu)(3+\nu)} \quad \text{(A13)}$$

and,

$$p = \frac{s^3 - 4(-1+\nu)(1+\nu)(3+\nu) + s^2(19+3\nu) + 2s(15+\nu^2)}{s^2+8s(3+\nu)-8(-1+\nu)(3+\nu)} \quad . \quad \text{(A14)}$$

If 'no trade-ins' were to be an optimal policy, then $p_T = p$.
Equating (A13) and (A14) immediately implies that one of the following must hold:

$s = 0$, (A15a)

$s = -1+\nu$, (A15b)

$s = (1/4)(-37-5\nu-\sqrt{1081+626+57\nu^2}$, (A15c)

$s = (1/4)(-37-5\nu+\sqrt{1081+626+57\nu^2}$. (A15d)

Equation A15a implies quality certainty, and trade-ins have no role to play for s=0. Equation A15b violates our assumption that a used good deteriorates relative to a new good, i.e., $\nu > (1+s)$. Equation A15c implies a negative s, violating assumption A1, i.e., $0 < s < 1$. Finally, A15d violates assumption A2, i.e., $s < (4/5)(\nu - 1)$. Q.E.D.

Proposition TO3: *As the quality uncertainty increases, the (expected) quality of used goods decreases*.
Proof: From Table 3, we have:

$$w = 1 + \frac{s^2}{4+5s-4\nu} .$$

Differentiating with respect to s gives:

$$-\frac{5s^2}{(4+5s-4\nu)^2} + \frac{2s}{4+5s-4\nu} .$$

This is negative as long as Assumption 2 holds. Q.E.D.

Proposition TO4: *The steeper the deterioration rate, the higher the (expected) quality of used goods.*
Proof: Differentiating the expected quality expression from Table 3 gives:

$$\frac{4s^2}{(4+5s-4\nu)^2} .$$

This is positive. Q.E.D

TO4: DETAILS OF CALCULATIONS OF NUMERICAL SOLUTIONS FOR DIFFERENT LEVELS OF *α*

This section corresponds to Section 4.3.4 of the paper.
The results described in the paper were for the specific value of α= ½. This value was used for tractability and ease of exposition. Observe that for a general α, the deterioration is given as $d \triangleq \frac{v}{\alpha(1+s)+(1-\alpha)(1-s)}$, but for α= ½ this value becomes equal to v, which greatly simplifies our analysis. To check the robustness of our results, we ran a series of simulations for a wide range of values of α. Specifically; we obtain numerical solutions for a series of values of α going from 0.2 to 0.8 in increments of 0.1. We find that all four propositions derived above are robust to these different values. In other words, for the entire range of α that we simulate, we find that for a given α, i) the trade-in incentive increases as quality uncertainty (s) increases ii) the volume of trade decreases as quality uncertainty increases, iii) the trade-in incentive decreases as deterioration (d) increases, and iv) the volume of trade increases as deterioration increases. Numerical solutions for different values of α are derived as follows.

1. **Formulation of Value Functions**

Segment 1r (Compulsive Buyers):

$$V^b(\theta) = \frac{v\theta - p_T + \delta p_u}{1-\delta}$$

Segment 2r (Strategic Holders)

$$V^h(\theta) = \frac{v\theta - ((1-\alpha)p_T + \alpha p) + (1-\alpha)\delta p_u + (\alpha)(1+s)\theta}{1-(1-\alpha)\delta - \alpha\delta^2}$$

Segment 3r (Cheapskates)

$$V^c(\theta) = \frac{w\theta - p_u}{1-\delta}$$

Segment 4r (Non-buyers)

Do not buy and get zero utility.

2. **Calculating Segment Sizes**

The segment sizes are calculated through the identification of marginal consumers:

$$\theta_1 = \frac{p + (1+\delta)(-p_T + \delta p_u)}{\delta(1+s-v)}$$

$$\theta_2 = \frac{(1-\alpha)p_T + \alpha p - (1+\delta)p_u}{(1+s)\alpha\delta + v - (1+\alpha\delta)w}$$

$$\theta_3 = p_u / w$$

3. **Applying Stationarity**

Since we are considering a stationary equilibrium, the total amount of new goods purchased each period must remain constant. Application of Lemma TA2 yields:

$$\underbrace{(1-\theta_1)}_{Segment\,1r} + (1/(1+\alpha))\underbrace{(\theta_1-\theta_2)}_{Segment\,2r} = y$$

4. **Applying Market Clearing**

Markets must clear at the end of each period, i.e., the number of consumers who supply the used goods each period must equal the number of consumers who purchase the used goods during the period. Applying TA2 yields:

$$(1-\theta_1)+(\frac{(1-\alpha)}{(1+\alpha)})(\theta_1-\theta_2)=\underbrace{(\theta_2-\theta_3)}_{Segment\,3r}$$

5. **Applying Rational Expectations**

The willingness to pay for the new good as well as for the used good depends upon consumers' expectation about the quality of the used good. Under the assumption of rational expectations, this expectation turns out to be exactly what is realized. In other words, the (average) used quality supplied equals the (average) quality expected:

$$\underbrace{w_s}_{Used\,Quality\,Supplied} = \frac{(1-\theta_1)(\alpha(1+s)+(1-\alpha)(1-s))+(\frac{1-\alpha}{1+\alpha})(\theta_1-\theta_2)(1-s)}{(1-\theta_1)+(\frac{1-\alpha}{1+\alpha})(\theta_1-\theta_2)} = \underbrace{w}_{Used\,Quality\,Expected}$$

6. **Profit Maximization**

The final step involves the optimization of the profits subject to the above conditions. The manufacturer maximizes per-period profits by picking p and p_T :

$$\pi=(1-\theta_1)p_T+(\frac{1-\alpha}{1+\alpha})(\theta_1-\theta_2)p_T+(\frac{\alpha}{1+\alpha})(\theta_1-\theta_2)p$$

7. **Solutions for Different Values of *α***

Steps 1-6 can be used to construct a solution assuming a particular value of α that lies between 0 and 1. Our aim is to check the robustness of our results reported in the main body of the paper at α = ½. Notice that, for any given α**,** the quality uncertainty is given by s, the spread of the used good quality. On the other hand, the deterioration is given by the ratio of new good quality to the expected quality of the used good purchased. In other words, the expected deterioration is given by:

$$d \triangleq \frac{v}{\alpha(1+s)+(1-\alpha)(1-s)}.$$

Notice that in the paper, deterioration is measured by v, since for α =½, d=v. However, this is not true for $\alpha \neq$ ½. In our numerical solutions, obtained at a particular α from steps 1-6, we substitute the following for new product quality:

$$v = d\,(\alpha(1+s)+(1-\alpha)(1-s))$$

This helps us obtain the analytical solution in terms of d and s, which means we can keep the value of deterioration constant when we obtain the numerical solution for different values of s. Notice that if we fix v at a particular value and change s, then we would be confounding the effect of quality uncertainty with the change in deterioration. Figures T2-T5 below provide a sample of the numerical solutions- notice that these figures represent one example from numerical simulations carried out across a wide range of parameters values.

Figure T2 graphically depicts the change in trade-in incentive with the change in s for different levels of α, keeping deterioration (d) constant at 3.5. This figure is consistent with **Proposition 1**.

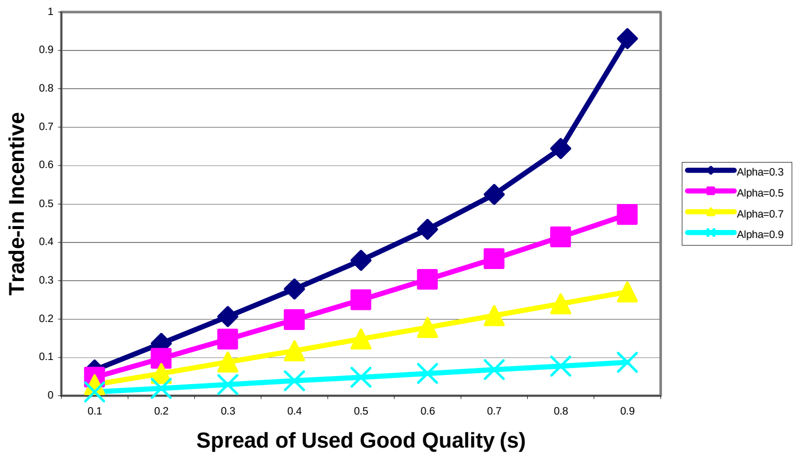

Figure T2 (Deterioration=3.5)

Figure T3 graphically depicts the change in VOT with the change in s for different levels of α, keeping deterioration (d) constant at 4.0. This figure is consistent with **Proposition 2**.

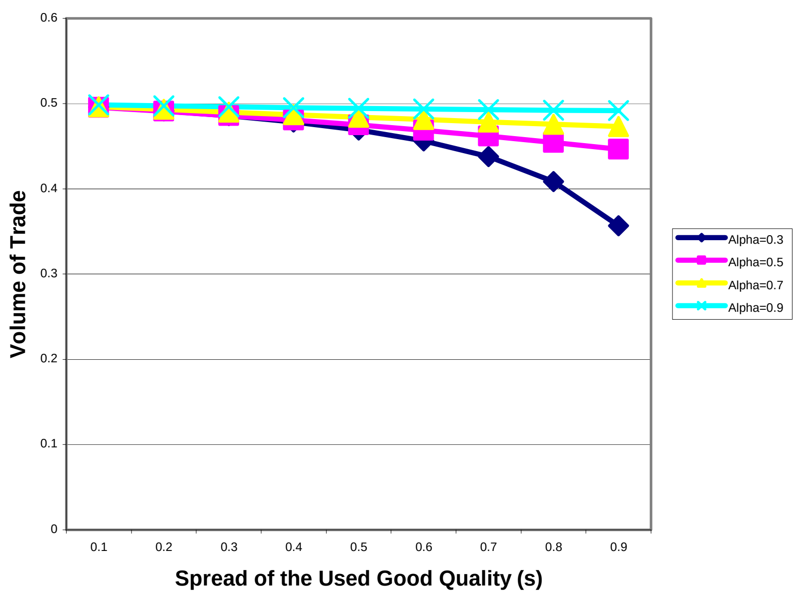

Figure T3 (Deterioration=4.0)

Figure T4 graphically depicts the change in trade-in incentive with change in d for different levels of α, keeping s constant at 0.7. Figure T4 is consistent with **Proposition 3**.

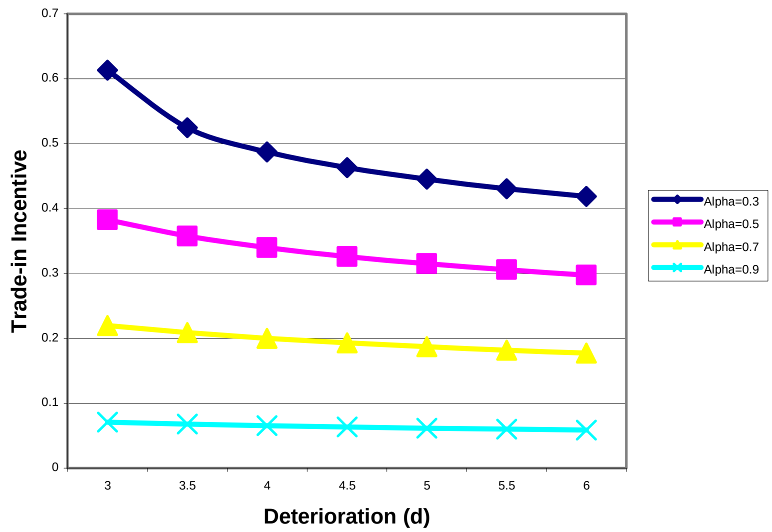

Figure T4 (*s*=0.7)

Figure T5 graphically depicts the change in VOT with change in *d* for different levels of *α*, keeping *s* constant at 0.7. This figure is consistent with **Proposition 4**.

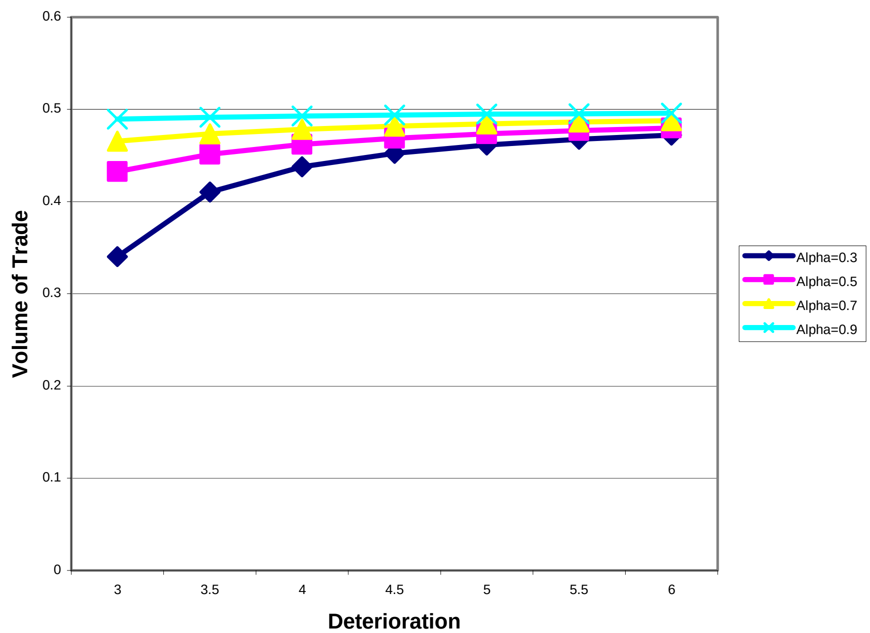

Figure T5 (s=0.7)

TO5: DETAILS OF CALCULATIONS OF NUMERICAL SOLUTIONS FOR ASYMMETRICAL PEACHES AND LEMONS

This section corresponds to Section 4.3.4 of the paper.

In the analysis in the paper, we have assumed that the quality of a peach is (1+*s*) while the quality of a lemon is (1-*s*). To check the robustness of our results for asymmetrical peaches and lemons, i.e., a peach with quality (1+s_1) and a lemon with quality (1-s_2), we ran a series of numerical simulations with s_1 taking values from 0.1 to 0.8 and s_2 taking values from 0.1 to 0.8 in steps of 0.1.

To understand the results, it helps to express the asymmetric case in terms of our existing model. First, note that in the current model, i.e., the symmetric uncertainty case, the severity of adverse selection can be represented as:

$$\Delta_{adv} = \text{(Quality of peach)-(Expected Used Good Quality)}$$

A large Δ_{adv} implies that the quality of a peach is high compared to the expected used good quality. Hence, any consumer who realizes a peach will be loath to sell, since he will receive the used good price for it, which, because it reflects expected used good quality, is much lower than the price he would have got if there were no asymmetry of information. Adverse selection is therefore a big problem for large Δ_{adv} .

For symmetrical peaches and lemons, we have:

$$\begin{aligned}\Delta_{adv} &= (1+s) - \left[\alpha(1+s) + (1-\alpha)(1-s)\right] \\ &= 2(1-\alpha)s\end{aligned}$$

In the case of asymmetric peaches and lemons, this can be written as:

$$\begin{aligned}\Delta_{adv} &= (1+s_1) - \left[\alpha(1+s_1) + (1-\alpha)(1-s_2)\right] \\ &= (1-\alpha)(1+s_1) - (1-\alpha)(1-s_2) \\ &= (1-\alpha)(s_1+s_2)\end{aligned}$$

Hence, note that the effects of (s_1 + s_2) are akin to the effects of *s* when peaches and lemons are symmetric. In other words, for a given *α*, (s_1 + s_2) represents the spread of the used quality and hence the quality uncertainty. We find that <u>all the four propositions</u> are robust to the asymmetric used good quality case. Numerical solutions when peaches take the value (1+s_1) and lemons take the value (1-s_2) are derived as follows.

1. **Formulation of Value Functions**

<u>Segment 1r (Compulsive Buyers)</u>:

$$V^b(\theta) = \frac{v\theta - p_T + \delta p_u}{1-\delta}$$

<u>Segment 2r (Strategic Holders)</u>

$$V^h(\theta) = \frac{v\theta - ((1-\alpha)p_T + \alpha p) + (1-\alpha)\delta p_u + (\alpha)(1+s_1)\theta}{1-(1-\alpha)\delta - \alpha\delta^2}$$

Notice that in the above expression the quality of the peach is $(1+s_1)$.

Segment 3r (Cheapskates)

$$V^c(\theta) = \frac{w\theta - p_u}{1-\delta}$$

Segment 4r (Non-buyers)

Do not buy and get zero utility.

2. **Calculating Segment Sizes**

The segment sizes are calculated through the identification of marginal consumers:

$$\theta_1 = \frac{p + (1+\delta)(-p_T + \delta p_u)}{\delta(1+s-v)}$$

$$\theta_2 = \frac{(1-\alpha)p_T + \alpha p - (1+\delta)p_u}{(1+s_1)\alpha\delta + v - (1+\alpha\delta)w}$$

$$\theta_3 = p_u / w$$

3. **Applying Stationarity**

Since we are considering a stationary equilibrium, the total amount of new goods purchased each period must remain constant. The application of Lemma TA2 yields:

$$\underbrace{(1-\theta_1)}_{Segment\,1r} + (1/(1+\alpha))\underbrace{(\theta_1 - \theta_2)}_{Segment\,2r} = y$$

4. **Applying Market Clearing**

Markets must clear at the end of each period, i.e., the number of consumers who supply the used goods each period must equal the number of consumers who purchase the used goods during the period. Applying TA2 yields:

$$(1-\theta_1) + (\frac{(1-\alpha)}{(1+\alpha)})(\theta_1 - \theta_2) = \underbrace{(\theta_2 - \theta_3)}_{Segment\,3r}$$

5. **Applying Rational Expectations**

The willingness to pay for the new good as well as for the used good depends upon consumers' expectation about the quality of the used good. Under the assumption of rational expectations, this expectation turns out to be exactly what is realized. In other words, the (average) used quality supplied equals the (average) quality expected:

$$\underbrace{w_s}_{Used\,Quality\,Supplied} = \frac{(1-\theta_1)(\alpha(1+s_1) + (1-\alpha)(1-s_2)) + (\frac{1-\alpha}{1+\alpha})(\theta_1 - \theta_2)(1-s_2)}{(1-\theta_1) + (\frac{1-\alpha}{1+\alpha})(\theta_1 - \theta_2)} = \underbrace{w}_{Used\,Quality\,Expected}$$

Notice that in the above expression, the compulsive buyer segment supplies a quality of $(\alpha(1+s_1) + (1-\alpha)(1-s_2))$ while the strategic holder segment supplies a quality of $(1-s_2)$.

6. **Profit Maximization:**

The final step involves the optimization of the profits subject to the above conditions. The manufacturer maximizes per-period profits by picking p and p_T :

$$\pi = (1-\theta_1)p_T + (\frac{1-\alpha}{1+\alpha})(\theta_1-\theta_2)p_T + (\frac{\alpha}{1+\alpha})(\theta_1-\theta_2)p$$

7. **Solutions for a Range of s_1 and s_2 :**

The main aim of the analysis in this section is to examine how the trade-in incentive and VOT respond when peaches and lemons are asymmetrical. As before, we examine the impact of changes in s_1 and s_2 keeping the expected deterioration constant. Notice that the expected deterioration is given by:

$$d \triangleq \frac{v}{\alpha(1+s_1)+(1-\alpha)(1-s_2)}$$

We fix a specific numerical value of α and obtain a series of numerical solutions for different values of s_1 and s_2 in increments of 0.1. Tables T1-T4 provide a sample of the numerical solutions.

Tables T1 and T2 contain the results of numerical solutions for the trade-in incentive for different values of s_1 and s_2, with α =0.5 and the deterioration level fixed at 3.0 and 4.0 respectively. In each of the tables, as we go down a column, we are increasing the value of s_1 keeping s_2 constant. As we go down each column the trade-in incentive goes up since (s_1 + s_2) goes up. This is consistent with **Proposition 1** in the paper. In each row, as we go from left to right, the trade-in incentive goes up since (s_1 + s_2) goes up. This again is consistent with **Proposition 1** in the paper.

Furthermore, notice that each element of **Table T2** (which has higher d) is smaller than the corresponding element of **Table T1**; this is consistent with **Proposition 3** in the paper that states that the trade-in incentive goes down as deterioration increases.

	s_2=0.1	s_2=0.2	s_2=.03	s_2=0.4	s_2=0.5	s_2=0.6	s_2=0.7	s_2=0.8
s_1=0.1	0.0504389	0.0760731	0.10209	0.128614	0.155829	0.184031	0.213716	0.245798
s_1=0.2	0.0759628	0.101857	0.128171	0.15504	0.18266	0.21134	0.241594	0.27435
s_1=0.3	0.10167	0.127826	0.15444	0.181652	0.209671	0.23882	0.269625	0.303018
s_1=0.4	0.127549	0.153968	0.18088	0.208434	0.236848	0.266454	0.29779	0.331787
s_1=0.5	0.153586	0.180269	0.207479	0.235373	0.264175	0.294227	0.326074	0.360644
s_1=0.6	0.179774	0.20672	0.234227	0.262455	0.291639	0.322126	0.354465	0.389578
s_1=0.7	0.206101	0.23331	0.261111	0.289671	0.319229	0.350138	0.382952	0.418579
s_1=0.8	0.23256	0.26003	0.288123	0.31701	0.346934	0.378255	0.411524	0.44764

Table T1 (d=3.0; α =0.5)

	s_2=0.1	s_2=0.2	s_2=.03	s_2=0.4	s_2=0.5	s_2=0.6	s_2=0.7	s_2=0.8
s_1=0.1	0.0466737	0.0702337	0.0939884	0.117987	0.142299	0.167021	0.192298	0.218359
s_1=0.2	0.0701749	0.0938681	0.117768	0.141926	0.16641	0.191319	0.216795	0.243056
s_1=0.3	0.093771	0.117594	0.141635	0.165946	0.190595	0.21568	0.24134	0.267785
s_1=0.4	0.117453	0.141403	0.16558	0.190037	0.214844	0.240095	0.265927	0.29254
s_1=0.5	0.141212	0.165285	0.189595	0.214193	0.239149	0.264557	0.290549	0.317317
s_1=0.6	0.165042	0.189235	0.213672	0.238405	0.263504	0.28906	0.315202	0.342112
s_1=0.7	0.188936	0.213245	0.237805	0.262669	0.287904	0.3136	0.339881	0.366924
s_1=0.8	0.212889	0.237311	0.26199	0.286979	0.312343	0.338171	0.364584	0.391749

Table T2 (d=4.0; α =0.5)

Tables T3 and T4 contain the results of numerical solutions for VOT for different values of s_1 and s_2, with α =0.5 and the deterioration level fixed at 3.0 and 4.0 respectively. In each of the tables, as we go down a column, we are increasing the value of s_1 keeping s_2 constant. As we go down each column VOT goes down, since ($s_1 + s_2$) goes up. This is consistent with **Proposition 2** in the paper. In each row, as we go from left to right, VOT goes down since ($s_1 + s_2$) goes up. This again is consistent with **Proposition 2** in the paper.

Furthermore, notice that each element of **Table T4** (which has higher *d*) is larger than the corresponding element of **Table T3**; this is consistent with **Proposition 4** in the paper that states that VOT goes up as the deterioration increases.

	s_2=0.1	s_2=0.2	s_2=.03	s_2=0.4	s_2=0.5	s_2=0.6	s_2=0.7	s_2=0.8
s_1=0.1	0.493421	0.489286	0.484375	0.478448	0.471154	0.461957	0.45	0.433824
s_1=0.2	0.490385	0.486111	0.481061	0.475	0.467593	0.458333	0.446429	0.430556
s_1=0.3	0.4875	0.483108	0.477941	0.471774	0.464286	0.455	0.443182	0.427632
s_1=0.4	0.484756	0.480263	0.475	0.46875	0.461207	0.451923	0.440217	0.425
s_1=0.5	0.482143	0.477564	0.472222	0.465909	0.458333	0.449074	0.4375	0.422619
s_1=0.6	0.479651	0.475	0.469595	0.463235	0.455645	0.446429	0.435	0.420455
s_1=0.7	0.477273	0.472561	0.467105	0.460714	0.453125	0.443966	0.432692	0.418478
s_1=0.8	0.475	0.470238	0.464744	0.458333	0.450758	0.441667	0.430556	0.416667

Table T3 (*d*=3.0*;* α =0.5)

	s_2=0.1	s_2=0.2	s_2=.03	s_2=0.4	s_2=0.5	s_2=0.6	s_2=0.7	s_2=0.8
s_1=0.1	0.49569	0.493056	0.49	0.486413	0.482143	0.476974	0.470588	0.4625
s_1=0.2	0.49375	0.491071	0.487981	0.484375	0.480114	0.475	0.46875	0.460937
s_1=0.3	0.491935	0.489224	0.486111	0.4825	0.478261	0.473214	0.467105	0.459559
s_1=0.4	0.490234	0.4875	0.484375	0.480769	0.476563	0.471591	0.465625	0.458333
s_1=0.5	0.488636	0.485887	0.482759	0.479167	0.475	0.470109	0.464286	0.457237
s_1=0.6	0.487132	0.484375	0.48125	0.477679	0.473558	0.46875	0.463068	0.45625
s_1=0.7	0.485714	0.482955	0.479839	0.476293	0.472222	0.4675	0.461957	0.455357
s_1=0.8	0.484375	0.481618	0.478516	0.475	0.470982	0.466346	0.460938	0.454545

Table T4 (*d*=4.0*;* α =0.5)

TO6: VEHICLES IN THE DATA

Our data contain details on new and used car transactions covering over 100 make-models of cars. To keep the analysis tractable, we focus on only the 20 most popular make-models, which cover the vast majority of the transactions in our data.

Buick Lesabre
Chevrolet Blazer
Ford F-150
Ford Mustang
Ford Ranger
Saturn SL
Honda Accord
Honda Civic
Jeep Cherokee
Pontiac Grand AM
Pontiac Grand Prix
Toyota Corolla
Ford Explorer
Chevrolet Silverado
Chevrolet Cavalier
Chevrolet S-10
Nissan Sentra
Toyota Camry
Dodge Caravan
Ford Taurus

Table T5: Vehicle Make-Models in Data