Online Appendices for "Optimal Referral Bonuses with Asymmetric Information: Firm-Offered and Interpersonal Incentives" by Laura J. Kornish and Qiuping Li

Appendix 1: Cases for Recommendation Risk Aversion

With recommendation risk aversion to close friends, there are three possible formulations of the firm's profit maximization problem. The first two—sell to both groups and sell to close friends only—are the same objective functions and constraints as in the risk neutral development, with the exception that now $\hat{m}_c = p - B/\beta_c + \sigma^2/(2R)$. A third possibility must be considered, that of selling only to distant friends, which will happen if $\sigma^2/(2R)$ is so high that customers don't want to recommend to any close friends.

$$\max_{p,B} \alpha(p-B)\left[q(1-\hat{m}_d)\right]$$

Subject to

$2p - 1 \le \hat{m}_d \le 2p$	(Signals to distant friends must be credible.)
$0 \le \hat{m}_d \le 1$	(Logical restriction based on $m_i \in [0,1]$.)
$\hat{m}_c \ge 1$	(No buy recommendations to close friends.)
$p \ge 0$, $B \ge 0$	(Price and bonus are non-negative.)

With $\hat{m}_d = p - B/\beta_d$, $\hat{m}_c = p - B/\beta_c + \sigma^2/(2R)$

The three possibilities (sell to both groups, close only, distant only) lead to five cases. For each of the three possibilities, we must consider the conditions under which the objective function grows without bound (as it did in the risk neutral cases) for simultaneous increases vs. decreases in *p* and *B*. Growing without bound for increases means that upper bound constraints will be binding; growing without bound for decreases means that lower bound constraints will be binding. In the explanation that follows, we call the former "at a high price" and the latter "at a low price." We rule out one of the six combinations of (both, close, distant) x (high price, low price): the problem of selling to close friends only "at a low price" is not feasible.

The conditions for each case presented below are of three types. The first type of condition characterizes the behavior of the objective function, and therefore implies which constraints are binding. The second type of condition ensures that the constraints in the problem formulation hold. The third type of condition ensures that this case applies because it yields the highest profit. We did not need to compare the optimal profit in every case to every other case; a comparison between two cases was unnecessary if constraints of the first type made the cases mutually exclusive. (For example, Cases 1 and 2 don't have to be compared because Case 1 has the condition $q < \frac{\beta_d(\beta_c - 1)}{(\beta_c - \beta_d)}$ and Case 2 has the opposite condition.) The following optimal profit expressions are used in the five cases for the third type of condition.

$$\pi_{Case1} = \frac{\alpha}{4}\left(1 - \frac{\sigma^2}{2R}(1-q)\right)^2$$

$$\pi_{Case2} = \frac{\alpha\left(\beta_d(1-q)\left(1-\beta_c\frac{\sigma^2}{2R}\right) + \beta_c\left(1+q-\frac{\sigma^2}{2R}(1-q)\right)\right)^2}{4(1+\beta_d)\beta_c(\beta_d(1-q)+\beta_c(1+q))}$$

$$\pi_{Case3} = \alpha(1-q)\frac{\left(1-\beta_c\frac{\sigma^2}{2R}\right)^2}{2(1+\beta_c)}$$

$$\pi_{Case4} = \frac{\alpha q}{4}$$

$$\pi_{Case5} = \frac{\alpha q}{2(1+\beta_d)}$$

Case 1: The "sell to both groups at a low price" case.

The constraint $B \geq 0$ is binding.

$$B^* = 0, \; p^* = \frac{1-(1-q)\frac{\sigma^2}{2R}}{2}$$

Conditions

$q < \frac{\beta_d(\beta_c - 1)}{(\beta_c - \beta_d)}$ (Objective function increasing with simultaneous decreases in *p* and *B*.)

$\frac{\sigma^2}{2R} \leq \frac{1}{1+q}$ (This condition ensures $\hat{m}_c \leq 1$.)

$\pi_{Case1} \geq \pi_{Case4}$, $\pi_{Case1} \geq \pi_{Case5}$

Case 2: The "sell to both groups at a high price" case.

The constraint $\hat{m}_d \geq 2p-1$ is binding.

$$B^* = \frac{\beta_d\left(\beta_d(1-q)\left(1+\beta_c\frac{\sigma^2}{2R}\right)+\beta_c\left((1+q)+(1-q)\frac{\sigma^2}{2R}\right)\right)}{2(1+\beta_d)(\beta_d(1-q)+\beta_c(1+q))},$$

$$p^* = 1-\frac{\left(\beta_d(1-q)\left(1+\beta_c\frac{\sigma^2}{2R}\right)+\beta_c\left((1+q)+(1-q)\frac{\sigma^2}{2R}\right)\right)}{2(1+\beta_d)(\beta_d(1-q)+\beta_c(1+q))}$$

Conditions

$q > \frac{\beta_d(\beta_c-1)}{(\beta_c-\beta_d)}$ (Objective function increasing with simultaneous increases in *p* and *B*.)

$\frac{\sigma^2}{2R} \le \frac{(\beta_d+\beta_c)(\beta_d(1-q)+\beta_c(1+q))}{\beta_c(1+\beta_d)(\beta_d(1-q)+\beta_c(1+3q))}$ (This condition ensures $\hat{m}_c \le 1$.)

$\pi_{Case2} \ge \pi_{Case3}$, $\pi_{Case2} \ge \pi_{Case5}$

Case 3: The “sell to close friends only at a high price” case.

The constraint $\hat{m}_c \ge 2p-1$ is binding.

$$B^* = \frac{\beta_c\left(1+(2+\beta_c)\frac{\sigma^2}{2R}\right)}{2(1+\beta_c)},\ p^* = 1+\frac{\sigma^2}{2R}-\frac{\left(1+(2+\beta_c)\frac{\sigma^2}{2R}\right)}{2(1+\beta_c)}$$

Conditions

$\beta_c < 1$ (Objective function increasing with simultaneous increases in *p* and *B*.)

$\pi_{Case3} \ge \pi_{Case2}$, $\pi_{Case3} \ge \pi_{Case5}$

Case 4: The “sell to distant friends only at a low price” case.

The constraint $B \ge 0$ is binding.

$$B^* = 0,\ p^* = \frac{1}{2}.$$

Conditions

$\beta_d > 1$ (Objective function increasing with simultaneous decreases in *p* and *B*.)

$\frac{\sigma^2}{2R} \ge \frac{1}{2}$ (This condition ensures $\hat{m}_c \ge 1$.)

$\pi_{Case4} \ge \pi_{Case1}$

Case 5: The "sell to distant friends only at a high price" case.

The constraint $\hat{m}_d \geq 2p - 1$ is binding.

$$B^* = \frac{\beta_d}{2(1+\beta_d)}, \quad p^* = 1 - \frac{1}{2(1+\beta_d)}$$

Conditions

$\beta_d < 1$ (Objective function increasing with simultaneous increases in *p* and *B*.)

$\frac{\sigma^2}{2R} \geq \frac{\beta_d + \beta_c}{2\beta_c(1+\beta_d)}$ (This condition ensures $\hat{m}_c \geq 1$.)

$\pi_{Case5} \geq \pi_{Case1}$, $\pi_{Case5} \geq \pi_{Case2}$, $\pi_{Case5} \geq \pi_{Case3}$

Appendix 2: Proof of Proposition 4

1) *A bonus should not be used (B* = 0) if* β_d *and* β_c *are each above a threshold.*

Cases 1 and 4 are the ones in which $B^* = 0$. The conditions in both of those cases can be rewritten as lower bounds on β_d and β_c.

Case 1 conditions:

a) $q < \frac{\beta_d(\beta_c - 1)}{(\beta_c - \beta_d)}$. This is the same as the boundary between Cases 1 and 2 in Figure 1. It can be rewritten as $\beta_d > \frac{\beta_c q}{\beta_c - 1 + q}$ and $\beta_c > \frac{\beta_d(1-q)}{\beta_d - q}$.

b) $\frac{\sigma^2}{2R} \leq \frac{1}{1+q}$. This is not a function of β_d or β_c.

c) $\pi_{Case1} \geq \pi_{Case4}$. This is not a function of β_d or β_c.

d) $\pi_{Case1} \geq \pi_{Case5}$. Rewriting, it becomes $\frac{\alpha}{4}\left(1 - \frac{\sigma^2}{2R}(1-q)\right)^2 \geq \frac{\alpha q}{2(1+\beta_d)}$, which is a lower bound on β_d.

Case 4 conditions:

a) $\beta_d > 1$. This is a lower bound on β_d.

b) $\frac{\sigma^2}{2R} \geq \frac{1}{2}$. This is not a function of β_d or β_c.

c) $\pi_{Case4} \geq \pi_{Case1}$. This is not a function of β_d or β_c.

2) *If a bonus is optimal (B* > 0), it is strictly increasing, at least locally, in* β_d *if distant friends are targeted and/or* β_c *if close friends are targeted.*

Case 2: close and distant

$$\frac{dB}{d\beta_d} = \frac{\beta_c^{\,2}\left(1+2q+q^2+\frac{\sigma^2}{2R}\left(1-q^2\right)\right)+2\beta_d\beta_c\left(1-q^2\right)\left(1+\beta_c\frac{\sigma^2}{2R}\right)+\beta_d^{\,2}\left(1-q\right)\left(1-q+\beta_c^{\,2}\frac{\sigma^2}{2R}\left(1+q\right)\right)}{2\left(1+\beta_d\right)^2\left(\beta_d\left(1-q\right)+\beta_c\left(1+q\right)\right)^2} > 0$$

$$\frac{dB}{d\beta_c} = \frac{\beta_d^{\,2}\left(1-q\right)^2\frac{\sigma^2}{2R}}{2\left(\beta_d\left(1-q\right)+\beta_c\left(1+q\right)\right)^2} > 0$$

Case 3: close only

$$\frac{dB}{d\beta_c} = \frac{1+\left(2+2\beta_c+\beta_c^{\,2}\right)\frac{\sigma^2}{2R}}{2\left(1+\beta_c\right)^2} > 0$$

Case 5: distant only

$$\frac{dB}{d\beta_d} = \frac{1}{2\left(1+\beta_d\right)^2} > 0$$

3) *If a bonus is optimal (B* > 0) and close friends are targeted, the bonus is strictly increasing, at least locally, in* $\sigma^2/\left(2R\right)$.

We define $X \equiv \frac{\sigma^2}{2R}$.

Case 2: close and distant

$$\frac{dB}{dX} = \frac{\beta_d\beta_c\left(1-q\right)}{2\beta_d\left(1-q\right)+2\beta_c\left(1+q\right)} > 0$$

Case 3: close only

$$\frac{dB}{dX} = \frac{\beta_c\left(2+\beta_c\right)}{2\left(1+\beta_c\right)} > 0$$

4) *If a bonus is optimal (B* > 0) and distant friends are targeted, price is strictly increasing, at least locally, in* β_d.

Case 2: close and distant

$$\frac{dp}{d\beta_d}=\frac{2\beta_d\beta_c(1-q)\left(1+q+\frac{\sigma^2}{2R}(1-q)\right)+\beta_d{}^2(1-q)^2\left(1+\beta_c\frac{\sigma^2}{2R}\right)+\beta_c\left(\beta_c(1+q)^2+(1-q)^2\frac{\sigma^2}{2R}\right)}{2(1+\beta_d)^2(\beta_d(1-q)+\beta_c(1+q))^2}>0$$

Case 5: distant only

$$\frac{dp}{d\beta_d}=\frac{1}{2(1+\beta_d)^2}>0$$

5) *If a bonus is optimal (B* > 0) and only close friends are targeted, price is strictly increasing, at least locally, in* β_c *and* $\sigma^2/(2R)$*; but if both close and distant friends are targeted, price is strictly decreasing, at least locally, in* β_c *and* $\sigma^2/(2R)$.

Case 3: close only

$$\frac{dp}{d\beta_c}=\frac{1+\frac{\sigma^2}{2R}}{2(1+\beta_c)^2}>0$$

$$\frac{dp}{dX}=\frac{\beta_c}{2(1+\beta_c)}>0$$

Case 2: close and distant

$$\frac{dp}{d\beta_c}=-\frac{\beta_d(1-q)^2\frac{\sigma^2}{2R}}{2(\beta_d(1-q)+\beta_c(1+q))^2}<0$$

$$\frac{dp}{dX}=-\frac{\beta_c(1-q)}{2\beta_d(1-q)+2\beta_c(1+q)}<0$$

Appendix 3: Heterogeneity in Risk Aversion

For comparison purposes, we present a case in which customers are recommendation risk averse with all friends; the risk tolerance parameter for distant friends is R_d and for close friends it is R_c, with $R_d > R_c$. In this case, there are two possible objective functions, depending on whether price and bonus are set to target both close and distant friends or just close friends. For brevity, we present only the "sell to both" formulation:

$$\max_{p,B} \alpha(p-B)\left[q(1-\hat{m}_d)+(1-q)(1-\hat{m}_c)\right]$$

$2p-1 \le \hat{m}_d \le 2p$, $2p-1 \le \hat{m}_c \le 2p$ (Signals to both groups must be credible.)

$0 \le \hat{m}_d \le 1$, $0 \le \hat{m}_c \le 1$ (Logical restriction based on $m_i \in [0,1]$.)

$p \ge 0$, $B \ge 0$

With $\hat{m}_d = \dfrac{\sigma^2}{2R_d} + p - B/\beta_d$ and $\hat{m}_c = \dfrac{\sigma^2}{2R_c} + p - B/\beta_c$

If $\dfrac{q}{\beta_d} + \dfrac{1-q}{\beta_c} > 1$ (i.e., the same condition as before for the objective function to be unbounded in simultaneous increases in *p* and *B*), then $\hat{m}_d \ge 2p-1$ is binding, and the optimal solution is

$$B^* = \frac{\beta_d\left(\beta_d\left(1-q+\beta_c(1+q)\right)\dfrac{\sigma^2}{2R_d} + \beta_d(1-q)\left(1+\beta_c\dfrac{\sigma^2}{2R_c}\right) + \beta_c(1+q)\left(1+2\dfrac{\sigma^2}{2R_d}\right) + \beta_c\dfrac{\sigma^2}{2R_c}(1-q)\right)}{2(1+\beta_d)\left(\beta_d(1-q)+\beta_c(1+q)\right)}$$

$$p^* = 1 + \frac{\sigma^2}{2R_d} - \frac{\left(\beta_d\left(1-q+\beta_c(1+q)\right)\dfrac{\sigma^2}{2R_d} + \beta_d(1-q)\left(1+\beta_c\dfrac{\sigma^2}{2R_c}\right) + \beta_c(1+q)\left(1+2\dfrac{\sigma^2}{2R_d}\right) + \beta_c\dfrac{\sigma^2}{2R_c}(1-q)\right)}{2(1+\beta_d)\left(\beta_d(1-q)+\beta_c(1+q)\right)}$$

The additional condition to ensure that $\hat{m}_c \le 1$ is $\dfrac{\sigma^2}{2R_c} + p^* - B^*/\beta_c \le 1$.

We omit the details, but we have shown that (defining $X_d \equiv \dfrac{\sigma^2}{2R_d}$ and $X_c \equiv \dfrac{\sigma^2}{2R_c}$):

$$\frac{dB^*}{d\beta_d} > 0,\ \frac{dB^*}{d\beta_c} > 0,\ \frac{dB^*}{dX_d} > 0,\ \frac{dB^*}{dX_c} > 0$$

$$\frac{dp^*}{d\beta_d} > 0,\ \frac{dp^*}{d\beta_c} < 0,\ \frac{dp^*}{dX_d} > 0,\ \frac{dp^*}{dX_c} < 0$$

Appendix 4: Friends Risk Averse Instead of Customers

In this appendix, we discuss a variation on the analysis in Section 5 in which it is the friends instead of the customers that are risk averse. The closest comparison to Section 5 would be to just consider close friends risk averse. However, if the risk aversion resides with the friends and not the customers, we felt it was more reasonable to assume risk aversion was independent from the closeness of the relationship.

Such a model follows. The $\hat{m}_i$ (i.e., the *m* cutoff for the close friends who get a recommendation) is $\hat{m}_i = \frac{\sigma^2}{2R} + p - B/\beta_i$ (which is the same as it was in Section 5 for $\hat{m}_c$), and now the risk adjusting term $\frac{\sigma^2}{2R}$ comes from the customer's concern for the friends' risk aversion. The credibility constraint also changes. If friends are risk averse, a recommendation to buy will only be credible to them if $\frac{\hat{m}_i + 1}{2} - \frac{\sigma^2}{2R} \geq p$.

The objective function and the other constraints are the same as the firm's profit maximization problem presented in (1). Looking at the part of the parameter space in which $\frac{q}{\beta_d} + \frac{1-q}{\beta_c} > 1$ (i.e., the objective function is unbounded in simultaneous increases in *p* and *B*, which includes all of $\beta_d < \beta_c < 1$), then upper bounds on *p* and *B* are binding. As in Section 5, the credibility constraint for the close friends is a weaker constraint than for the distant friends, so it is the binding upper bound: $p = 1 - B/\beta_d - \frac{\sigma^2}{2R}$.

The optimal price and bonus in this case are

$$B^* = \frac{\beta_d\left(1-\sigma^2/(2R)\right)}{2(1+\beta_d)} \text{ and } p^* = 1 - \frac{\left(1-\sigma^2/(2R)\right)}{2(1+\beta_d)} - \frac{\sigma^2}{2R}$$

Similar to the other variations we have studied, over the range that this solution applies, they are increasing in β_d . In the analysis in Section 5 and the heterogeneous risk averse case presented in Appendix 3, β_i and $\frac{\sigma^2}{2R}$ work in the same direction. However, in this variation, they work in opposite directions: higher $\frac{\sigma^2}{2R}$ lowers the optimal price and bonus. The customers' concern for the friends' outcomes has the same effect of increasing the discriminatory power of the signal. However, the friends' risk aversion puts downward pressure on the price, and also reduces the impact of the bonuses.

Appendix 5: Variation on Customer Utility to Include "Pain Avoided" for a "Not Recommended" Signal

In this appendix, we discuss a variation on the sender utility in Section 3 in which the customers gain positive utility from having their signal followed, even if the signal results in no purchase. In the analysis in Section 3, the sender has utility of 0 for the not recommended or no signal decision, because the receiver does not buy based on that. (In the notation of Section 3, an accepted "buy" recommendation yields the recommender $B + E_x\left[j(x-p) \mid m\right]$—and we assume $E_x\left[j(x-p) \mid m\right]$ is increasing in *m*—and "no signal" or "not recommended" yields 0.)

Here we consider how the model and results change with "pain avoided" utility: the customer gets utility from not recommending too, related to how much disutility the customer helps the friend avoid, e.g., $E_x\left[h(p-x) \mid m\right]$, with ($E_x\left[h(p-x) \mid m\right]$ decreasing in *m*). Such an extension maintains the structure of the signaling equilibrium in Proposition 1.

In the linear utility case, the specific solutions change, but the structural properties of the solution remain the same. The $\hat{m}$ cutoff, i.e., the value of *m* above which recommendations are made, would come from $B + E_x\left[j(x-p) \mid m\right] = E_x\left[h(p-x) \mid m\right]$ instead of $B + E_x\left[j(x-p) \mid m\right] = 0$. For the linear utility case, instead of $\hat{m} = p - B/\beta$, we would have $\hat{m} = p - B/(2\beta)$. The conditions for the friend to accept the recommendation would stay at $2p - 1 \leq \hat{m} \leq 2p$, resulting in a binding constraint of $p = 1 - B/(2\beta_d)$ when an upper bound on *p* and *B* is binding and *q* is large enough to warrant selling to both segments. The optimal price and bonus are therefore $B^* = \dfrac{\beta_d}{(1+2\beta_d)}$ $p^* = 1 - \dfrac{1}{2(1+2\beta_d)}$. This price and bonus are higher compared to the results for Case 2 of Proposition 2 in Section 4, but they are both still increasing in β_d.

We conclude that when the customer gains utility from "pain avoided" for his friend, he is even *more* discriminating in his recommendations, allowing the firm to charge a higher price and making it optimal for them to increase the bonus to leverage that price as well.

Appendix 6: Multiple Periods

In this appendix, we examine the robustness of our main results to the time horizon. In our single period model, we did not allow for the possibility that someone who receives a recommendation can subsequently make a recommendation himself. That possibility affects initial recommendation behavior because a friend's utility from a recommendation would be more than the expected value of the product minus the price ($m - p$); it would also include anticipated bonuses and anticipated interpersonal utility from the recommendations that *he* will make. The multiple period model presented here does not lend itself to a complete analytical solution, so we illustrate the robustness of our results with a numerical analysis.

Model setup

To set up the multi-period model, we describe the recommendation behavior and the firm's profit maximization problem. Our extension accounts for the credibility and signaling issues we addressed in the single period as well as market saturation over time. To keep the multi-period model set-up and interpretation reasonable, we treat the risk neutral case and consider only one "level" of relationship, represented by β (instead of close and distant). As before, we normalize the size of the group each person knows to 1.

Recommendation behavior

Taking into account future bonuses and interpersonal utility, in period *k* (the subscript indicates the period throughout this section), a customer will recommend to friends who have m values (mean values of the product) such that $B + E_x\left[j(u_R) \mid m\right] \geq 0$, as before. But now $E_x\left[j(u_R) \mid m\right] = \beta(m - p + f_k)$, with f_k as the anticipated (*f*uture) utility the friend will get when he subsequently makes recommendations. We use the same definitions of *B*, *p*, *m*, and β as previously, and α_k and $\hat{m}_k$ represent the same ideas as before (the interconnectedness of the network and the cutoff value above which a recommendation is given, respectively), but now with a subscript to indicate the period. Using that notation, we derive f_k recursively as

$$f_k = \alpha_{k+1}\int_{\hat{m}_{k+1}}^{1} B + \beta(m - p + f_{k+1})dm = \alpha_{k+1}\left(\left(B - \beta p + f_{k+1}\right)\left(1 - \hat{m}_{k+1}\right) + \beta\frac{1 - \hat{m}_{k+1}{}^2}{2}\right)$$

and $\hat{m}_k$ the minimum level of *m* such that $B + \beta(m - p + f_k) \geq 0$, that is, $\hat{m}_k = p - \dfrac{B}{\beta} - f_k$.

The logic of the signaling model is similar to the single period model. A credible signal is one that the friend will follow: the $\hat{m}_k$ cutoff has to be high enough such that people who get a recommendation have an expected value (now including both the value of using the product and any future value) higher than the price. Adjusting the credibility constraints derived in Proposition 1 to account for this future value yields $2(p - f_k) - 1 \le \hat{m}_k \le 2(p - f_k)$ for all *k*. In addition, as before, we need $0 \le \hat{m}_k \le 1$.

Firm's profit maximization problem

The profit maximization objective function is

$$\max_{p,B}(p-B)\sum_{k=1}^{\infty}\prod_{j=1}^{k}\alpha_j\left(1-\hat{m}_j\right).$$

As before, $(p-B)$ is the marginal value to the firm of an accepted recommendation (still assuming zero marginal cost). The term $\alpha_j\left(1-\hat{m}_j\right)$ represents the (expected) quantity of accepted recommendations in period *j*.

The product term reflects the fact that the quantity of accepted recommendations in a later period builds on accepted recommendations in all earlier periods. The sum is over the periods. We capture market saturation with a decreasing α_k sequence. That assumption also gives a decreasing f_k sequence (i.e., the future prospects from successful recommendations decrease as the market saturates). That decreasing sequence means that the maximum possible price is more constrained by considerations farther down in the recommendation tree compared to earlier periods. Like our single period analysis, the binding constraint for $\beta < 1$ is $p = 1 - B/\beta$, which means that the firm will not set the price to make potential customers rely on future recommendation rewards. The logic of trying to expropriate those future rewards unravels: a higher price would mean that future recommendations were not credible, so those anticipated rewards would not materialize.

To evaluate this numerically, we use a forecast horizon of 10 periods. In the examples we looked at, going beyond 10 periods did not add measurably to the optimal profit or affect the values of the decision variables.

Numerical Results

We find strong numerical evidence that our main result is robust to a multi-period horizon: as long as the firm can't more efficiently motivate recommendations with price cuts, the more concern customers have for friends, the higher the price and bonus should be. For $\beta > 1$ (the cutoff is exactly at 1 due to the homogeneity of β), the optimal bonus once again drops down to 0, and the price drops too. These findings, both the increasing pattern and the drop, are consistent with the single period analysis.

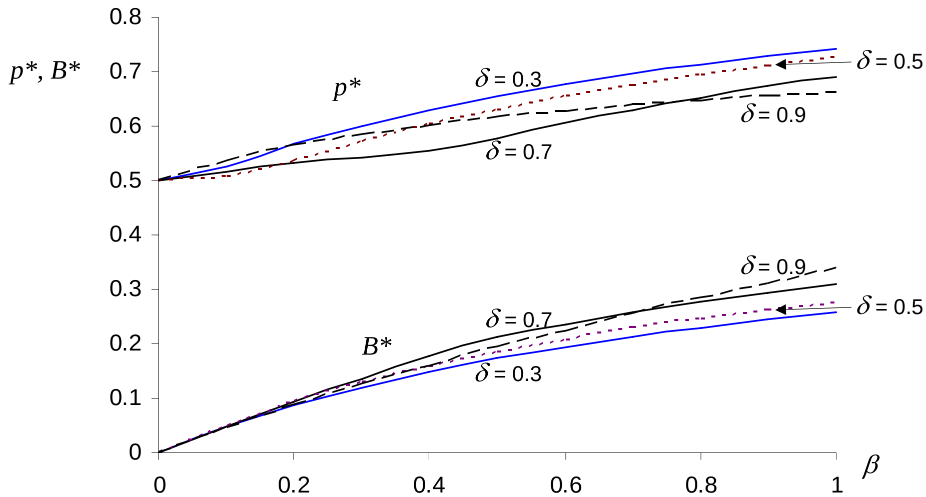

Figure A-1: Price and bonus are increasing for $0 < \beta < 1$. The figure shows analysis for $\alpha_k = \delta^k$.

In a multi-period analysis, we can also look at the effect of the "salience" of the future. Here we capture that salience with δ, setting $\alpha_k = \delta^k$, $\delta < 1$. The figure shows that, in many cases, the more salient the future (the higher the δ), the lower the price and the higher the bonus for high enough β. That is, the more important the future, the more the firm wants to invest in increasing quantity. Increases in quantity in the early periods pay rewards to the firm in future periods, and the more potential that future has, the more attractive quantity-increasing strategies (i.e., lower price and higher bonus) are. We have found these patterns presented here through numerical examples to apply over the range of the parameter space.

Notation Summary—Additional Notation Introduced in Online Appendix

R_d	Risk tolerance parameter for distant friends.
R_c	Risk tolerance parameter for close friends.
f_k	Anticipated future utility a friend will get when he subsequently makes recommendations, in period *k* of multi-period model.
α_k	Analogous to α, but in period *k* of multi-period model.
δ	A parameter used to represent the weight on future referrals in multi-period numerical example, $\alpha_k = \delta^k$
$\hat{m}_k$	Analogous to $\hat{m}$, but in period *k* of multi-period model.