Technical Appendix to "Information Provision in a Vertically Differentiated Competitive Marketplace"

PROOF OF PROPOSITION 5

The monopolist has 4 options of quality-information combination.

If the monopolist **offers low quality product** (q_1) **without providing preference-revealing information**, the optimal price is

(A19) $p_{(1,NI)} = V + q_1/2$ with profit $\pi_{(1,NI)} = V + q_1/2$,

If the monopolist **offers high quality product** (q_2) **without providing preference-revealing information**, the optimal price is

(A20) $p_{(2,NI)} = V + q_2/2 - \delta$ with profit $\pi_{(2,NI)} = V + q_2/2 - \delta$,

as long as $0 < \delta < V + q_2/2$ (note this is satisfied when $0 < \delta < \Delta/2$). Otherwise, the monopolist cannot make any sale by offering q_2 without preference information provision.

If the monopolist **offers low quality product** (q_1) **and provides preference information**, the optimal price is

(A21) $p_{(1,I_\theta)} = V + q_1/2$ with profit $\pi_{(1,I_\theta)} = (V + q_1)^2/4q_1 - c_{I\theta}$

when $0 < V < q_1$. Otherwise, the optimal price is

(A22) $p_{(1,I_\theta)} = V$ with profit $\pi_{(1,I_\theta)} = V - c_{I\theta}$

If the monopolist **offers high quality product** (q_2) **and provides preference information**, the optimal price is

(A23) $p_{(2,I_\theta)} = V + q_2 + \delta/2$ with profit $\pi_{(2,I_\theta)} = (V + q_2 - \delta)^2/4q_2 - c_{I\theta}$

when $\max\{0, \delta - q_2\} < V < \delta + q_2$ where the monopolist covers only part of the market. Otherwise, the optimal price becomes

(A24) $p_{(2,I_\theta)} = V$ with profit $\pi_{(2,I_\theta)} = V - \delta - c_{I\theta}$

Comparing the above profit levels leads to the monopolist's optimal quality-information combinations under different ranges of $\delta, c_{I\theta}$ as stated in Proposition 5. This completes the proof.

PROOF OF LEMMA 2

If neither store provides preference information, similarly to this case of Lemma 1, we derive that the equilibrium prices and profits are as in the first row of equation (A3), which is reported in the upper left entry of Table 3. All consumers buy from store 2.

Now, consider if **store 2 alone provides preference information**. Consumers always going to store 1 first and buying there can not be an equilibrium, because it would result in the same prices as in the above case and lead to consumers all preferring to buy from store 2. Suppose consumers go to store 2 to learn their θ first. The marginal consumer who is indifferent between buying from store 2 once being there and incurring an additional t and buying from store 1 is found to have θ such that

(A25) $$\theta_2 q_2 - p_2 = \theta_2 q_1 - p_1 - t \rightarrow \theta_2 = (p_2 - p_1 - t)/\Delta$$

If $\theta_2 < 0$, consumers always go to store 1 first and buy there; if $\theta_2 > 1$, consumers always go to store 2 and buy there; if $\theta_2 \in (0,1)$,consumers go to store 2 to acquire preference information first, they buy from store 2 if discovered their $\theta > \theta_2$ and buy from store 1 otherwise. In this case, a consumer decides on which store to go first by comparing the expected utility of going to store 1 and buying there ($\mu_1 = q_1/2 - p_1$) with the expected utility of going to store 2 first, learning her θ and then buying from store 2 or going to store 1 (spending an additional t) and buying there, a strategy with the expected utility

(A26) $$\mu_2 = (1-\theta_2)(\frac{1+\theta_2}{2} q_2 - p_2) + \theta_2(\frac{\theta_2}{2} q_1 - p_1 - t)$$

If consumers go to store 2 first and $\theta_2 \le 1$, then the market shares of store 1 and 2 are θ_2 and $1-\theta_2$ correspondingly, and profits are

(A27) $\pi_1 = p_1\theta_2,\ \pi_2 = (p_2 - \delta)(1-\theta_2) - c_{I\theta}$ with θ_2 defined in the above equation (A25).

Simultaneously solving for the first order conditions of the profit functions in (A27) leads to

(A28) $$p_1 = (\Delta + \delta - t)/3,\ \ p_2 = (2\Delta + 2\delta + t)/3$$

Substituting (A28) back to (A27) results in the upper-right entry of Table 3. To check that this is the equilibrium prices, we need to verify that a) $\theta_2 \in (0,1)$ with prices in the equation (A28); b) $\mu_2 > \mu_1$; c) Store 1 does not have unilateral incentive of deviating to a sufficient price reduction so that $\mu_2 < \mu_1$ and consumers go to store 1 immediately. The sufficient and necessary condition for a)-c) to hold is that t/Δ is smaller than the smaller root (for x) of $(2-\partial)^2 + x^2 + 2x(2-\partial) - 9\sqrt{2x}$ (so that this expression would be positive). This condition is necessary for c) to hold. If it is not satisfied, the equilibrium will be in mixed strategies, where store 1 would sometimes deviate to a lower price.

Consider the case of **store 1 alone providing preference information**. Going through similar process as the above case, we can derive the marginal consumer who is indifferent between buying from store 1 once being there and incurring an additional t and buying from store 2. The following prices follow from the profit maximization practice similar to the previous case

(A29) $$p_1 = (\Delta + \delta + t)/3, \quad p_2 = (2\Delta + 2\delta - t)/3$$

Substituting back to the profit expression results in the bottom left entry of Table 3. For the condition under which this is the equilibrium, we derive that t/Δ must be less than the smaller root (for x) of $(1+\partial)^2 + x^2 + 2x(1+\partial) - 9\sqrt{2x}$ (so that this expression would be positive).

It also cannot be an equilibrium that consumers always go to store 2 first when store 1 is the one providing preference-revealing information. It can be argued by contradiction. If it were so, the prices would have to be as in the case of no store providing preference-revealing information. But given those prices, the expected utility of going to store 1 first (defined through a similar procedure as the above μ_2 in A26) will be

(A30) $$\mu_1 = q_1/2 + (\Delta - 2t)^2/8\Delta$$

This is higher than the expected utility of going to store 2 and buying there, $q_2/2$. So consumers should optimally go to store 1 first, which is a contradiction.

It remains to consider the case **when both stores provide preference information**. Now, consumers going to either store first will be able to learn their preference θ and make a non-trivial decision where to buy. We then consider two sub-cases separately: 1) consumers first go to store 1, and 2) consumers first go to store 2. The first sub-case leads to exactly same solution as when store 1 is the only one providing preference information, but it turns out that given the prices derived there, consumers should prefer to first go to store 2. Hence, this sub-case leads to no equilibrium solutions in pure strategy. The second sub-case is likewise very similar to the earlier case of only store 2 providing preference information, and it is an equilibrium as long as

(A31) $$\frac{t}{\Delta} \le \frac{3\sqrt{17 + 60\partial + 68\partial^2} - 13\partial - 11}{16}$$

The right hand side of the above equation is positive as far as $\partial < 1/2$. The resulted profits are in the bottom right entry of Table 3. This completes the proof of Lemma 2.

PROOF OF PROPOSITION 6

Comparing the top right with bottom right entries of Table 3, we have

(A32) $$\pi_1((q_1, NI);(q_2, I_\theta)) - \pi_1((q_1, I_\theta);(q_2, I_\theta)) > 0$$

as long as $c_{I\theta} < (\Delta + \delta - t)^2/9\Delta$. This implies that store 1 would not want to provide preference information given that store 2 is doing so. Next compare the top right with top left entries of Table 3, we have

(A33) $$\pi_2((q_1, NI);(q_2, I_\theta)) - \pi_2((q_1, NI);(q_2, NI)) > 0$$

if $c_{I\theta} < c_{I\theta}*$. This implies that store 2 prefers to provide preference information if store 1 does not do so. (A32), (A33) together proves the first part of proposition 6.

If $c_{I\theta} > c_{I\theta}*$, then the above (A33) becomes negative implying that given neither store provides preference information, store 2 does not have unilateral incentive to deviate and provide preference information. Further compare the bottom left and top left entries of Table 3, we have

(A34) $$\pi_1((q_1, I_\theta);(q_2, NI)) - \pi_1((q_1, NI);(q_2, NI)) > 0$$

if $c_{I\theta} < c_{I\theta}**$. This implies that store 1 wants to provide preference information if nobody does. However, compare bottom left and bottom right entries of Table 3, we know that

(A35) $$\pi_2((q_1, I_\theta);(q_2, I_\theta)) - \pi_2((q_1, I_\theta);(q_2, NI)) > 0$$

if $c_{I\theta} < 4(2-\partial)t/9$. This implies that if store 1 provides preference information, then store 2 wants to do so too. Further it can be easily verified that $c_{I\theta}* < c_{I\theta}** < 4(2-\partial)t/9$, which proves the second part of proposition 6.

Finally, if $c_{I\theta} > c_{I\theta}**$, similarly to the above, one derives that neither store would provide preference-revealing information. This completes the proof of Proposition 6.

STATEMENT AND PROOF OF THE MODIFICATION OF LEMMA 1 FOR LOWER *V*

Statement: When $V < \dfrac{q_1(\Delta+\delta)}{(q_1+2q_2)}$, Table 2'(a) states the profits for the differentiated-quality firms depending on their decisions of providing preference information. When $\dfrac{q_1(\Delta+\delta)}{(q_1+2q_2)} \le V \le \dfrac{\Delta+\delta}{3}$ and $0 < \delta < \dfrac{\Delta}{2}$, Table 2'(b) states the profits for the differentiated–quality firms depending on their decisions of providing preference information.

Proof: If **neither firm provides preference information**, the profits for differentiated-quality firms are the same as when $V > \dfrac{\Delta+\delta}{3}$. This is because when neither firm provides preference information, every consumer acts the same, market is either fully covered or zero covered.

Now consider **one of the firms (or both) providing preference information**. Then the profits gross of information cost are the same regardless of who provides this type of information (since, information cost is sunk), but consumers are differentiated. Given *V* is small, there could be some consumers who do not purchase any product due to low θ such that $V + \theta_0 q_1 - p_1 = 0$, i.e., $\theta_0 = \dfrac{p_1 - V}{q_1}$. As far as $\theta_0 \in [0,1]$, it represents the part of market uncovered. The consumer who is indifferent between buying from firm 1 and 2 has $\theta = \theta^*$ such that $V + \theta^* q_1 - p_1 = V + \theta^* q_2 - p_2$, i.e., $\theta^* = \frac{p_2 - p_1}{\Delta}$. Thus $\theta^* - \theta_0$ represent the demand of the low-quality firm, while the demand of high-quality firm is $1 - \theta^*$.

The profit (gross of information provision cost) of the low-quality firm is $\pi_1(p_1, p_2) = (\theta^*(p_1, p_2) - \theta_0) p_1$, and the profit (gross of information provision cost) of the high-quality firm is $\pi_2(p_1, p_2) = (1 - \theta^*(p_1, p_2))(p_2 - \delta)$. The first order conditions yield a unique solution $\left\{ p_1 = \frac{q_1^2 - q_1 q_2 + 2Vq_1 - 2Vq_2 - \delta q_1}{q_1 - 4q_2},\ p_2 = \frac{Vq_1 - Vq_2 + 2q_1 q_2 - 2q_2^2 - 2q_2\delta}{q_1 - 4q_2} \right\}$, with the profits reported in Table 2'(a).

For high-quality firm to have positive demand, we need $1 - \theta^* > 0$, which yields $0 < \delta < \frac{(V + 2q_2)\Delta}{2q_2 - q_1}$, otherwise high-quality firm is not in the market (note $\frac{(V + 2q_2)\Delta}{2q_2 - q_1} > \frac{\Delta}{2}$). And it is straightforward to check $\theta^* - \theta_0 > 0$ such that the low-quality firm would have positive demand. Further $\theta_0 > 0$ implies $V < \frac{q_1(\Delta + \delta)}{(q_1 + 2q_2)}$ which is the condition for incomplete market coverage. When $\delta > \frac{(V + 2q_2)\Delta}{2q_2 - q_1}$, we obtain the boundary solution with the high-quality firm pricing at the marginal cost and obtaining zero demand and profits. The solution results are reported in Table 2'(a) for case of $\delta > \frac{(V + 2q_2)\Delta}{2q_2 - q_1}$.

Note that $\frac{q_1(\Delta + \delta)}{(q_1 + 2q_2)} - \frac{\Delta + \delta}{3} < 0$, thus the equilibrium is different when $\frac{q_1(\Delta + \delta)}{(q_1 + 2q_2)} \le V \le \frac{\Delta + \delta}{3}$. In this range, price of low quality product is bounded to the valuation of all consumers in the market, i.e., $p_1 = V$. And the high-quality firm maximizes profit by setting $p_2 = \frac{\Delta + V + \delta}{2}$. The demand for high-quality firm is positive as long as $\delta < V + \Delta$. The profits are reported in Table 2'(b), and the proof is complete.

STATEMENT AND PROOF OF THE MODIFICATION OF PROPOSITION 2 FOR LOWER *V*

To state the modification of proposition 2 for lower V, Condition (1) needs to be adjusted to:

(1') $0 \le \delta < \min\{\frac{(\Delta + \delta)q_1}{(q_1 + 2q_2)}, 2\Delta\},\ 0 < c_{I\theta} < \min\{\frac{q_2((\Delta + \delta)q_1 + 2\Delta V)^2}{\Delta q_1 (4q_2 - q_1)^2}, \frac{(2(\Delta - \delta)q_2 + \delta q_1 + \Delta V)^2}{\Delta (4q_2 - q_1)^2}\}$

Proposition 2': Assume Condition (1') and $V < \frac{q_1(\Delta+\delta)}{(q_1+2q_2)}$. Then: When $0<\delta<\Delta/2$:

1) If $0 \le c_{I\theta} \le \frac{2(V\Delta+\delta q_1+2q_2(\Delta-\delta))^2+\Delta(q_1-4q_2)^2(2\delta-\Delta)}{2\Delta(q_1-4q_2)^2}$, both quality-information combinations are possible in equilibrium: either the high- or low-quality firm may provide preference-revealing information.
2) Otherwise, the unique quality-information equilibrium combination is when the low-quality firm provides preference information, but the high-quality firm does not.

Alternatively when $\delta > \Delta/2$:

1) If $0 \le c_{I\theta} \le \frac{2q_2(q_1(\Delta+\delta)+2V\Delta)^2+\Delta q_1(\Delta-2\delta)(q_1-4q_2)^2}{2\Delta q_1(q_1-4q_2)^2}$, both quality-information combinations are possible in equilibrium: either the high- or low- quality firm may provide preference-revealing information.
2) Otherwise, the unique quality-information equilibrium outcome is when the high-quality firm provides preference-revealing information, but the low-quality firm does not.

Proof: As in the proof for proposition 2 in the main text, under condition (1'), firms differentiated in quality and furthermore, preference information is provided in equilibrium by one of the firms. Let us now consider whether the high-quality or the low-quality firm would provide this type of information given condition (1'). First consider the case of $\delta<\Delta/2$, where the profits in the pricing subgame are given by the first matrix in Table 2'(a). We have already noted that if one of the firms provides preference information, the other can free-ride on it and does not want to provide preference information. Comparing the top left with top right boxes of the matrix, we derive that if neither firm provides preference-revealing information, the high-quality firm wants to do so if and only if $c_{I\theta} \le \frac{2(V\Delta+\delta q_1+2q_2(\Delta-\delta))^2+\Delta(q_1-4q_2)^2(2\delta-\Delta)}{2\Delta(q_1-4q_2)^2}$. Comparing the top left with the bottom left boxes, we derive that the low-quality firm wants to provide preference information if and only if $c_{I\theta} \le \frac{q_2((\Delta+\delta)q_1+2\Delta V)^2}{\Delta q_1(4q_2-q_1)^2}$. Hence if both inequalities are satisfied, either firm providing preference information can be an equilibrium outcome, while if the first one is not satisfied and the second one is, the equilibrium outcome must be that the low-quality firm providing preference-revealing information. Through similar procedure, one can verify the other case ($\delta>\Delta/2$), and the proof is complete.

SEQUENTIAL (q-I-p) GAME STRUCTURE AND THE EXOGENOUS QUALITY CASE

Let us first consider the sequential game structure (q-I-p) with simultaneous quality choices first, followed by simultaneous information provision decision, and then, as before, simultaneous price choices. It turns out that the results in this case are exactly the same as in the simultaneous quality and information provision decision model in the main text. To see this, note first that Lemma 1 and its proof will hold without modifications since it is about the price subgames that have not changed at all.

To see the intuition for why the sequential game structure does not change any results, consider the following. The firms' quality decision at the first stage of q-I-p game structure results in 4 subgames corresponding to the possible quality choice combination, namely, (q_1, q_1), (q_1, q_2), (q_2, q_1), and (q_2, q_2). In the two subgames where the firms' quality levels are the same, regardless of the information provision decision, they will be engaged in an undifferentiated-Bertrand competition, resulting in zero profits gross of information cost. Therefore, if firms are not differentiated in quality, neither firm would want to provide information in the second stage, and undifferentiated Bertrand price competition results in equilibrium price at the marginal cost level which yields zero profit. Since in the pure (in q) strategy equilibrium, the difference between the sequential and simultaneous quality and information provision is that in one case, quality is known when deciding on information provision, and in the other case, quality is expected correctly, the only difference in the outcome must be coming from a firm's incentive to affect the other firm's information provision decision through own quality choice. However, since if a firm deviates and chooses the quality that it expects the other firm would choose in the equilibrium, it is guaranteed zero or negative profit. Therefore, there is no incentive to deviate from the expected equilibrium behavior due to the observability of the quality choice before the information provision decision.

Formally, the proofs of Propositions 1, 2, 3, 4, 5, and 6 as well as Corollaries 1-3 hold without any changes in the case of sequential game structure above.

By the same logic, the exogenously different quality levels of the firms will not change any results about the connection between the firm's quality (chosen or exogenously existing) and the information provision that correspond to the case where pure strategy equilibria involved only differentiated-in-quality firms. Obviously, the mixed strategy in quality result will not be applicable as well as any parts of the propositions about the quality choice itself.

MARGINAL COST OF INFORMATION PROVISION (SECTION 5.3)

The solution in this case is very similar to the case of fixed costs, except that marginal costs of information provision mean that 1) the prices are affected by the cost if information is provided and 2) the firm with higher market share ends up spending more on the information. We start with the case $\delta < \Delta/2$. The equilibrium profits in the case of information provision are as follows:

If firm 1 alone provides information,

(A36) $\pi_1 = (\Delta + \delta - c_{I\theta})^2 / 9\Delta,\ \pi_2 = (2\Delta - \delta + c_{I\theta})^2 / 9\Delta;$ if $0 < c_{I\theta} < \Delta + \delta$

If firm 2 alone provides information,

(A37) $\pi_1 = (\Delta + \delta + c_{I\theta})^2 / 9\Delta,\ \pi_2 = (2\Delta - \delta - c_{I\theta})^2 / 9\Delta;$ if $0 < c_{I\theta} < 2\Delta - \delta$

If both firm 1 and 2 provide information,

(A38) $\pi_1 = (\Delta + \delta)^2 / 9\Delta,\ \pi_2 = (2\Delta - \delta)^2 / 9\Delta$; if $0 < c_{I\theta} < 2\Delta - \delta$

Comparing the above to the profits in the case of no information provision, we obtain that the high quality firm always has lower incentive than the low quality firm to provide information and the incentive to the high quality firm is negative when $\delta < (3\sqrt{3} - 5)\Delta / 2$.

Therefore, we obtain that Proposition 1 holds without changes. Going through the similar procedure of derivation as in Proposition 2 for the case $\delta < \Delta/2$, we have both quality-information combinations possible in equilibrium if

(A39) $0 < c_{I\theta} \leq (2(2\Delta - \delta) - 3\sqrt{2\Delta(\Delta - 2\delta)}) / 2$

Otherwise ($(2(2\Delta - \delta) - 3\sqrt{2\Delta(\Delta - 2\delta)}) / 2 < c_{I\theta} < \Delta + \delta$), the unique quality-information equilibrium combination is when the low-quality firm provides preference-revealing information, but the high-quality firm does not. Compared to Proposition2, this results in a larger parameter range where only the low quality firm can provide information in the equilibrium.

The case $\delta < \Delta/2$ is similar and results in the following change to the range of single equilibrium where only high-quality firm provides information:

(A40) $(2(\Delta + \delta) - 3\sqrt{2\Delta(2\delta - \Delta)}) / 2 < c_{I\theta} < 2\Delta - \delta$

which is again enlarged when compared to range in the case of fixed costs. Therefore we obtain the qualitative results of Proposition 2 and the upper bound of Condition (1) is expanded to $0 < c_{I\theta} < \min\{\Delta + \delta, 2\Delta - \delta\}$.

PARTIAL RESOLUTION OF UNCERTAINTY (SECTION 5.3)

We illustrate the solution on the case $\delta < \Delta/2$ (the case $\delta > \Delta/2$ is similar with results reversed). We first calculate the profits if information is provided as to result in a given *a*. It turns out that for $a < (\Delta - 2\delta)/6\Delta$, we have the corner solution with $\pi_1 = 0$ and $\pi_2 = (½ - a)\Delta - \delta$. For $a > (\Delta - 2\delta)/6\Delta$, we obtain: $\pi_1 = (\Delta - 2\delta - 6a\Delta)^2/36\Delta$ and $\pi_2 = (\Delta - 2\delta + 6a\Delta)^2/36\Delta$.

Taking derivative of these profits with respect to a, we find that for both firms increase profits from an increase in information provided when $a > (\Delta - 2\delta)/6\Delta$, and the high quality firm is strictly worse off when from a small amount of additional information provision when $a < (\Delta - 2\delta)/6\Delta$. One obtains that the high quality firm only wants to provide information (and in that case, reveal all information) if a is already above $(\Delta^2 - 10\delta\Delta - 2\delta^2)/18\Delta^2$, whereas the low quality firm always prefers full information provision to any level of information provision.

ASYMMETRIC DISTRIBUTION OF θ (SECTION 5.5)

Given the P.D.F. of θ being $f(x) = a + (2-a)\cdot x$ for $0 < a < 2$, the expected θ in the case of no information provision is $(4-a)/6$. In the case information is provided, the marginal θ for a consumer decision to buy from the low- or high-quality firm is still $\theta^* = (p_2 - p_1)/\Delta$, just as in the uniform distribution case, but the market share of the low-quality firm is now $\theta^*(a + (1-a)\theta^*)$. The first order conditions result in a quadratic equation on p_1 and p_2. The root corresponding to the maximum profits is identified by $0 < \theta < 1$. Given the number of parameters, the solution is too long to report here. However, substituting $\delta = \delta/\Delta$, we have that the resulting profits are linear in Δ, after which it is easy to graphically check that only low quality firm is willing to provide information when δ is small enough, both are willing to provide information when d is close to Δ/2 and only the high quality firm is willing to provide information when d is close to 2Δ. Hence, Proposition 2 is qualitatively unchanged.

MODEL EXTENSION WITH PRODUCT RETURNS (SECTION 5.6)

We provide in the following proofs for results of Proposition 7 when $\delta < \Delta/2$. The verification for the case of $\delta > \Delta/2$ follows the similar logic and procedure.

If neither firm provides information through product return, equilibrium prices and profits for the two firms are the same as those in Prop 2.

If firm 1 alone provides information through product return, the marginal consumer who is indifferent between buying from firm 1 or firm 2 still has $\theta^* = (p_2 - p_1)/\Delta$. The profits of firm 1 and 2 are, respectively

(A41) $\pi_1 = p_1 \min\{1, \theta^*\} - c_R \left(1 - \min\{1, \theta^*\}\right),\ \pi_2 = (p_2 - \delta)\left(1 - \min\{1, \theta^*\}\right)$

Equilibrium prices can be solved as

(A42) $p_1 = (\Delta + \delta - 2c_R)/3,\ p_2 = (2\Delta + 2\delta - c_R)/3$

with equilibrium profits become

(A43) $\pi_1 = [c_R^2 + (\Delta+\delta)^2 - c_R(7\Delta - 2\delta)]/9\Delta, \pi_2 = (2\Delta - \delta - c_R)^2/9\Delta;$

if $c_R < (7\Delta - 2\delta - 3\sqrt{\Delta(5\Delta - 4\delta)})/2$.

Similarly, i**f firm 2 alone provides information** through product return, we have

(A44) $p_1 = (\Delta+\delta-c_R)/3,\ p_2 = (2\Delta+2\delta-2c_R)/3$

(A45) $\pi_1 = (\Delta+\delta-c_R)^2/9\Delta,\ \pi_2 = [c_R^2 + (2\Delta-\delta)^2 - c_R(5\Delta+2\delta)]/9\Delta;$

if $c_R < (5\Delta + 2\delta - 3\sqrt{\Delta(\Delta+4\delta)})/2$.

If both firm 1 and 2 provide information through product return, we have

(A46) $p_1 = (\Delta+\delta-3c_R)/3,\ p_2 = (2\Delta+2\delta-3c_R)/3$

(A47) $\pi_1 = (\Delta+\delta)^2/9\Delta - c_R,\ \pi_2 = (2\Delta-\delta)^2/9\Delta - c_R;$ if $c_R < (\Delta+\delta)^2/9\Delta$.

Going through the similar procedure of payoff comparison as in Proposition 2 for the case $\delta < \Delta/2$, we found that if neither firm provides information through product return, the high quality firm wants to do so if and only if

(A48) $c_R \le [(5-3\sqrt{3})\Delta - 2\delta]/2$

While low-quality firm wants to provide information through product return when the high-quality firm does not if and only if

(A49) $c_R \le (7\Delta - 2\delta - 3\sqrt{\Delta(5\Delta-4\delta)})/2$

Further it is straightforward to see the high-quality firm has no incentive to deviate and provide information as well when the low-quality firm chooses to do so. Note that $\frac{(5-3\sqrt{3})\Delta - 2\delta}{2} < \frac{7\Delta - 2\delta - 3\sqrt{\Delta(5\Delta-4\delta)}}{2}$ when $\delta < \Delta/2$, which leads to the range of unique equilibrium result shown in the first part of Proposition 7. Finally the full market coverage condition is checked by ensuring the lowest valuation consumer (θ=0) must be willing to purchase q_1 rather than buying nothing. Substituting the equilibrium price into $V - q_1 > 0$ yields that $V > (\Delta+\delta-C_R)/3$ is the condition for market being fully covered. This completes the proof.

Table 2'(a): Payoffs for Differentiated-Product Firms, zero shopping costs ($V < \frac{q_1(\Delta+\delta)}{(q_1+2q_2)}$)

When $0 \le \delta < \frac{\Delta}{2}$

1\2	(q_2, NI)	(q_2, I_θ)
(q_1, NI)	$(0, \frac{\Delta}{2}-\delta)$	$(\frac{q_2((\Delta+\delta)q_1+2\Delta V)^2}{(4q_2-q_1)^2\Delta q_1}, \frac{(\Delta V+\delta q_1+2(\Delta-\delta)q_2)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta})$
(q_1, I_θ)	$(\frac{q_2((\Delta+\delta)q_1+2\Delta V)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta}, \frac{(\Delta V+\delta q_1+2(\Delta-\delta)q_2)^2}{(4q_2-q_1)^2\Delta q_1})$	$(\frac{q_2((\Delta+\delta)q_1+2\Delta V)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta}, \frac{(\Delta V+\delta q_1+2(\Delta-\delta)q_2)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta})$

When $\frac{\Delta}{2} < \delta < \frac{(V+2q_2)\Delta}{(2q_2-q_1)}$

1\2	(q_2, NI)	(q_2, I_θ)
(q_1, NI)	$(\delta-\frac{\Delta}{2}, 0)$	$(\frac{q_2((\Delta+\delta)q_1+2\Delta V)^2}{(4q_2-q_1)^2\Delta q_1}, \frac{(\Delta V+\delta q_1+2(\Delta-\delta)q_2)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta})$
(q_1, I_θ)	$(\frac{q_2((\Delta+\delta)q_1+2\Delta V)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta}, \frac{(\Delta V+\delta q_1+2(\Delta-\delta)q_2)^2}{(4q_2-q_1)^2\Delta q_1}$	$(\frac{q_2((\Delta+\delta)q_1+2\Delta V)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta}, \frac{(\Delta V+\delta q_1+2(\Delta-\delta)q_2)^2}{(4q_2-q_1)^2\Delta q_1}-c_{I\theta})$

When $\delta > \frac{(V+2q_2)\Delta}{(2q_2-q_1)}$

1\2	(q_2, NI)	(q_2, I_θ)
(q_1, NI)	$(\delta-\frac{\Delta}{2}, 0)$	$(\frac{(\delta q_1+\Delta V)^2}{4\Delta q_1 q_2}, -c_{I\theta})$
(q_1, I_θ)	$(\frac{(\delta q_1+\Delta V)^2}{4\Delta q_1 q_2}-c_{I\theta}, 0)$	$(\frac{(\delta q_1+\Delta V)^2}{4\Delta q_1 q_2}-c_{I\theta}, -c_{I\theta})$

Table 2'(b): Payoffs for Differentiated-Product Firms, zero shopping costs ($\frac{q_1(\Delta+\delta)}{(q_1+2q_2)} \le V \le \frac{\Delta+\delta}{3}$)

Firm 1\2	(q_2, NI)	(q_2, I_θ)
(q_1, NI)	$(0, \frac{\Delta}{2}-\delta)$	$(\frac{V(\Delta+\delta-V)}{2\Delta}, \frac{(\Delta+V-\delta)^2}{4\Delta}-c_{I\theta})$
(q_1, I_θ)	$(\frac{V(\Delta+\delta-V)}{2\Delta}-c_{I\theta}, \frac{(\Delta+V-\delta)^2}{4\Delta})$	$(\frac{V(\Delta+\delta-V)}{2\Delta}-c_{I\theta}, \frac{(\Delta+V-\delta)^2}{4\Delta}-c_{I\theta})$