Online Appendix
Advertising Effectiveness, Digital Video Recorders and Product Market Competition

Naive Consumer model

Conditions for Existence and Uniqueness of Equilibrium

We need to ensure that the equilibrium of the Naive Consumer Model presented in table 1 of the paper satisfies three conditions - (i) All prices above $p = (v - t)$ are strictly dominated in equilibrium. (ii) No firm has an incentive to deviate to the corner $p = (v - t)$ and (iii) The equilibrium specified above is unique. The objective of the following analysis is to find restrictions on the parameters β, θ and t such that conditions (i-iii) hold. The parameter restrictions are shown in table 1 of the paper. The structure of the proof closely follows Soberman (2004).

Domination of prices above $v - t$

First, we show that for $t < \frac{v-c}{2}$ all prices greater than $v - t$ are strictly dominated and hence are never observed in equilibrium. When firm i charges $p_i > v - t$, with $i = A, B$, the appropriate profit function is such that $x_i < 1$ in the demand function for firm i (equation 1 in the paper). In a symmetric equilibrium when both firms charge $p_i = p = v - t$, dropping the subscript i from prices, we have:

$$\frac{\partial \Pi_i}{\partial p} = -\frac{\phi_i(-v + 2t + c)(-2 + \phi_{-i} - \alpha(1 - \beta)(-2 + \phi_{-i} + S\beta\phi_{-i}))}{2t} \tag{N1}$$

where the subscript $-i$ refers to the rival of firm i. It is straightforward to verify that $\frac{\partial \Pi_i}{\partial p} < 0$ for all $t < \left(\frac{v-c}{2}\right)$. That is, profit monotonically decreases with price for all prices strictly greater than $v - t$. Please also note that $\frac{\partial \Pi_i}{\partial p}$ has a supremum at $p = v - t$. In order to see this, we differentiate $\frac{\partial \Pi_i}{\partial p}$ with respect to p

$$\frac{\partial^2 \Pi_i}{\partial p^2} = \frac{\phi_i(-2+\phi_{-i}-\alpha(1-\beta)(-2+\phi_{-i}+S\beta\phi_{-i}))}{t} < 0 \text{ for } \phi \in [0,1] \tag{N2}$$

Equations (N1) and (N2) together imply (i) profit is strictly decreasing for all prices greater than $(v-t)$ and (ii) Change in profit is at its maximum value at $p = v-t$ beyond which it strictly goes down. Therefore, all prices strictly greater than $v-t$ are never observed in equilibrium given $t < \frac{v-c}{2}$.

For $t < \frac{v-c}{2}$, maximizing the profit function assuming $x_i = 1$ yields the equilibrium prices p_n^*, advertising reach ϕ_n^* and profit Π_n^* as stated in table 1. We also checked for the second order conditions for a maximizer and they are satisfied at the candidate equilibrium.

As mentioned in the paper we restrict θ such that $0 < \phi_n^* \leq 1$.The following lower bound on θ ensures the same:

$$\theta \geq \theta_{Ln} = \frac{t(1-\alpha(1-\beta)(1-S\beta))^2}{4(1+\alpha(1-\beta)(1-S\beta))} \tag{N3}$$

To obtain a tighter restriction we determine the supremum of the lower bound θ_{Ln} with respect to α. We define $\underline{\theta_n} = \underset{\alpha}{max}(\theta_{Ln})$.

$$\underline{\theta_n} = \max\left[\frac{t}{4}, \frac{t(1+S(1-\beta))^2\beta}{4-4S(1-\beta))}\right] \tag{N4}$$

Non Deviation

The pure strategy equilibrium must be stable to defections by either firm to $p = v-t$. The profit earned by charging $v-t$ to partially informed consumers is

$$\Pi(\phi^*, v-t) = \left(\alpha\left[\begin{array}{c} S(\phi^*(1-\phi^*)\beta + \phi^{*2}\beta(1-\beta)) \\ +(1-S)(\phi^*(1-\phi^*)\beta) \end{array}\right] + (1-\alpha)\phi^*(1-\phi^*)\right)(v-t-c) - \theta\phi^{*2} \tag{N5}$$

$\Pi^*(\phi^*, p^*) > \Pi(\phi^*, v-t)$ is necessary for a pure strategy equilibrium in prices, where

$\Pi^*(\phi^*, p^*)$ is the equilibrium profit reported in table 1 in the paper. Substituting ϕ^* in $\Pi(\phi^*, v-t)$ and computing $\Xi_D = \Pi^*(\phi^*, p^*) - \Pi(\phi^*, v-t)$ yields

$$\Xi_D = \frac{2t(1-\alpha(1-\beta))^3(4\theta t(1-\alpha(1-\beta)) - (c+t-v)(t(1-\alpha(1-\beta))(1-\alpha(1-\beta)(1+S\beta)) - 2\sqrt{\theta t((1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta))})))}{\left(t(1-\alpha(1-\beta))(1-\alpha(1-\beta)(1+S\beta)) + 2\sqrt{\theta t((1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta))}\right)^2} \tag{N6}$$

The denominator of Ξ_D is strictly positive. However, one can show that the numerator is also strictly positive for

$$t \in \left(\frac{v-c}{5}, \frac{v-c}{2}\right) \tag{N7}$$

Thus, a deviation is never profitable for $t \in \left(\frac{v-c}{5}, \frac{v-c}{2}\right)$.

Uniqueness

In order to ensure uniqueness of the symmetric pure strategy equilibrium, we need to exclude the possibility of a corner equilibrium where $p_i = p_{-i} = v - t$. All other prices are not viable because the profit function is concave in prices in the open interval $(0, v-t)$. If there exists a corner equilibrium at $p = v - t$, the following conditions should hold around the corner:

$$\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^-}} > 0, \frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^+}} < 0 \tag{N8}$$

Our strategy is to come up with a contradiction to conditions (N8), guaranteeing the equilibrium is unique in the interval $[0, v-t]$. From equation (N1) above, we know that $\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^+}} < 0$ for $p_i > v - t$. Consider the case for $p_i \leq v - t$,

$$\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^-}} = \frac{\phi_i(2t(1-\alpha(1-\beta)) + (c-v)(1-\alpha(1-\beta)(1+S\beta))\phi_{-i})}{2t} \tag{N9}$$

Even though $t < \frac{v-c}{2}$, for small enough ϕ_{-i} it can be that $\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^-}} > 0$. In other words, for low values of ϕ_{-i} it is possible that $p_i = p_{-i} = v - t$ is a symmetric equilbrium.

Focusing on the case when $p_i = v - t$, the profit function for each firm can be derived by substituting $x_i = 1$ in equation 1 in the paper. Thus, the profit function for firm i, with $i = A$, B, as a function of ϕ_i is given as follows:

$$\Pi_i = \frac{\phi_i(-2\theta\phi_i + (-v + t + c)(-2 + \phi_{-i} - \alpha(1-\beta)(-2 + \phi_{-i} + S\beta\phi_{-i})))}{2} \tag{N10}$$

Maximizing Π_i given in equation (N10) with respect to ϕ_i and assuming $\phi_i = \phi_{-i}$ in equilbrium we obtain ϕ^*_{nd}:

$$\phi^*_{nd} = \frac{2(v - t - c)(1 - \alpha(1-\beta))}{4\theta + (v - t - c)(1 - \alpha(1-\beta)(1 + S\beta))} \tag{N11}$$

Substituting (N11) into (N9) we find that $\frac{\partial \Pi_i}{\partial p_i}|_{p_i \to (v-t)^-} < 0$ for $t < \frac{v-c}{2}$ and $\theta < \theta_{Hn} = \dfrac{(v-t-c)^2(1-\alpha(1-\beta)(1+S\beta))}{4t}$. In other words, for $\theta < \theta_{Hn}$ there cannot exist a symmetric equilibrium such that both firm charge $p = v - t$.

To obtain a tighter restriction, we determine the minimum of θ_{Hn} with respect to α as $\overline{\theta_n} = \underset{\alpha}{min}(\theta_{Hn}) = \dfrac{(v-t-c)^2(1-S(1-\beta))\beta}{4t}$. In summary, when $\theta < \bar{\theta}_n$, the candidate interior equilibrium is unique for all α. Next, we summarize all conditions sufficient for the existence of equilibrium. Towards that end we also ensure that the equilibrium exists for non-empty intervals of θ, t and β.

Note that the equilibrium exists as long as $\theta \in \left[\underline{\theta_n}, \bar{\theta}_n\right]$ which has a positive measure as long as:

$$t < \overline{t_n} = \min\left[\frac{(1 - S(1-\beta))(v-c)}{2}, \frac{(v-c)\sqrt{\beta(1-S(1-\beta))}}{1 + \sqrt{\beta(1-S(1-\beta))}}\right] \tag{N12}$$

Equation (N7) above shows that the minimum value of t is $\underline{t_n} = \left(\frac{v-c}{5}\right)$. Therefore, as our final step, we need to ensure that the support $[\underline{t_n}, \overline{t_n}]$ has a positive measure which is guaranteed for $\beta \geq \frac{2}{5}$.

Proof of Propositions 1, 2

Proof of Proposition 1

Differentiation of p_n^* with respect to α yields

$$\frac{\partial p_n^*}{\partial \alpha} = \frac{\theta^2 t^2 (1-\alpha(1-\beta))^3 (1-\beta)(1+S\beta)}{(\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta)))^{\frac{3}{2}}} > 0 \tag{N13}$$

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Proof of Proposition 2

Differentiating of ϕ_n^* with respect to α yields

$$\begin{aligned}\frac{\partial \phi_n^*}{\partial \alpha} &= \frac{2t^2(1-\alpha(1-\beta))^2(-1+\beta)}{\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta))}} \\ &* \frac{\left(-S\beta\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta))} + \theta(1-\alpha(1-\beta))(1-S\beta-\alpha(1-\beta)(1+S\beta))\right)}{\left[(t(1-\alpha(1-\beta))(1-\alpha(1-\beta)(1+S\beta)) + 2\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta))}\right]^2}\end{aligned} \tag{N14}$$

The denominator of the above expression is strictly greater than 0. Define

$$\Delta = \left(-S\beta\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta))} + \theta(1-\alpha(1-\beta))(1-S\beta-\alpha(1-\beta)(1+S\beta))\right) \tag{N15}$$

One can see that the $sgn(\frac{\partial \phi_n^*}{\partial \alpha})$ is determined by the $sgn(\Delta)$ where $\frac{\partial \phi_n^*}{\partial \alpha} > 0$ when $\Delta < 0$ and $\frac{\partial \phi_n^*}{\partial \alpha} < 0$ when $\Delta > 0$[1].

Note $\Delta < 0$ for all values of $\alpha \in [0,1]$ if the following conditions on θ and S hold:

$$\theta < \theta_n^* \text{ and } S > 0, \text{ where } \theta_n^* = \frac{t(S\beta)^2}{(1-S\beta)^2} \tag{N16}$$

These conditions establishes Proposition 2(i).

Furthermore, $\Delta > 0$ for all values of $\alpha \in [0,1]$ if the following conditions on θ and S hold:

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[1] Please note that $\frac{\partial \Pi_n^*}{\partial \alpha} > 0$ when $\Delta < 0$ and $\frac{\partial \Pi_n^*}{\partial \alpha} < 0$ when $\Delta > 0$, i.e. the results on profit are identical to the ones on ad reach as stated in proposition 2.

$$\theta > \overline{\theta_n^*} \text{ and } S < S_n^*, \text{ where } \overline{\theta_n^*} = \frac{S^2 t(1 - S(1-\beta))\beta}{(1 - S(2-\beta))^2} \text{ and } S_n^* = \frac{1}{2-\beta} \tag{N17}$$

These conditions establish Proposition 2(ii).

Furthermore, $\Delta < 0$ for $\alpha > \alpha_n^*$ and $\Delta > 0$ for $\alpha < \alpha_n^*$ if conditions (i) or (ii) are fulfilled, where

$$(i)\ \left(\theta_n^* < \theta < \overline{\theta_n^*}\right) \cap S > 0 \tag{N18}$$

$$(ii)\ (\theta > \theta_n^*) \cap (S > S_n^*) \tag{N19}$$

$$\alpha_n^* = \frac{1}{2}\left(\frac{-(S\beta)^{\frac{3}{2}}\sqrt{t(4\theta + St\beta)} - (S\beta)^2 t + 2\theta(1 - S\beta)}{\theta(1-\beta)(1+S\beta)}\right) \tag{N20}$$

These conditions establish Proposition 2(iii).■

Comparative Statics with respect to β

Lemma 1 *In the unique symmetric equilibrium, if the base viewing rate, β, is sufficiently high, ad reach and profit decrease with the viewing rate, β, if S is high.*

Differentiating equilibrium advertising reach with respect to β gives:

$$\frac{\partial \phi_n^*}{\partial \beta} = \frac{2t^2\alpha(1-\alpha(1-\beta))^2(\theta(1-\alpha(1-\beta))((1+S)(1-\alpha) + (S(-2+\alpha)+\alpha)\beta) - S(-1+\alpha(1-\beta)^2+2\beta)\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta)(1+S\beta))}}{\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta^2))}\left(t(1-\alpha(1-\beta))(1-\alpha(1-\beta^2)) + 2\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta^2))}\right)^2} \tag{N21}$$

A necessary condition for the direction of $\frac{\partial \phi_n^*}{\partial \beta}$ is complex to derive. However, one can see that

$$\frac{\partial \phi_n^*}{\partial \beta}|_{\beta \to 1} = -\frac{2t^2(\theta(S-1) + S\sqrt{\theta t})\alpha}{\sqrt{\theta t}(t + 2\sqrt{\theta t})^2} < 0 \text{ for } S > S_n^{**} \tag{N22}$$

$$S_n^{**} = \begin{cases} \frac{\theta - \sqrt{\theta t}}{\theta - t} & if\ \theta > t \\ \frac{\sqrt{\theta t} - \theta}{t - \theta} & if\ \theta < t \end{cases} \tag{N23}$$

Given ϕ_n^* is a continuous function of both β and S in the interval [0,1], it follows that there must exist a non-empty interval close to $\beta = 1$,such that $\frac{\partial \phi^{n*}}{\partial \beta} < 0$ for S sufficiently high.[2] ■

Selective Consumer Model

Conditions for Existence and Uniqueness of Equilibrium

The derivation for existence of this equilibrium follows the same logic as for the Naive Consumer Model. First, we ensure that all prices strictly greater than $p = v - t$ are never observed in equilibrium. Second, the equilibrium must be deviation proof. Finally, we require uniqueness of the equilibrium.

Domination of prices above $v - t$

Note for prices $p \in (v - t, \infty)$ the appropriate profit functions for the firms must reflect that the market for partially informed consumers is not completely covered, i.e . $x_A < 1$ and $x_B < 1$. We maximize the profit functions, (equation 5 in the paper), with respect to prices and ad reach. Note that $p_i = p$ and $\phi_i = \phi$, for $i = A, B$, in any symmetric equilibrium. When both firms charge $p_i = p = v - t$, dropping the subscript i from prices, we have:

$$\frac{\partial \Pi_i}{\partial p} = \frac{\phi(6(c+2t-v)(1-\alpha(1-\beta)) - 6(c+t-v)\alpha\epsilon - (3(c+2t-v)(1-\alpha(1-\beta^2)) - (6c+7t-6v)\alpha\epsilon^2)\phi)}{6t} \tag{S1}$$

The sign of $\frac{\partial \Pi_i}{\partial p}$ depends on the numerator. The numerator is strictly negative for:

$$t < t_{\max} = \frac{3(v-c)(-2+\phi+\alpha(2-2\beta+2\epsilon-\phi+\beta^2\phi-2\epsilon^2\phi))}{6(\phi-2)+\alpha(12-12\beta+6\epsilon-6\phi+6\beta^2\phi-7\epsilon^2\phi)} \tag{S2}$$

The above threshold is a function of α, ϵ and ϕ. We simplify the above threshold in two steps. First, we note that $t_{\max}$ is a continuous and strictly decreasing function of α.

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[2] One can compute comparative statics for low values of β. A detailed comparative statics is available from the authors.

Therefore, we obtain $\min\limits_{\alpha}(t_{\max}) = t_{\max\,|\alpha\to 1}$.

$$t_{\max\,|\alpha\to 1} = \frac{3(v-c)(-2\beta+\beta^2\phi+2\epsilon(1-\epsilon\phi))}{-12\beta+6\beta^2\phi+\epsilon(6-7\epsilon\phi)} \tag{S3}$$

Second, $t_{\max\,|\alpha\to 1}$ is a continuous and strictly decreasing function of ϕ and ϵ as long as $\epsilon \leq \left(\frac{\sqrt{37}-5}{2}\right)\beta$. Therefore, we can substitute $\phi = 1$ and $\epsilon = \left(\frac{\sqrt{37}-5}{2}\right)\beta$, to get a strict infimum for $t_{\max}$. In summary we require that $t < \overline{t_s}$, where

$$\overline{t_s} = \inf(t_{\max}) = \frac{6(v-c)(\sqrt{37}-7+5\beta(\sqrt{37}-6))}{6(\sqrt{37}-9)+5(7\sqrt{37}-41)\beta} \tag{S4}$$

Note that the above cutoff is less than $\left(\frac{v-c}{2}\right)$. Therefore, as long as $t < \overline{t_s}$ holds, all prices $p > (v-t)$ are strictly dominated and therefore are never observed in equilibrium.

Next, we establish that $\frac{\partial \Pi_i}{\partial p}$ in equation (S1) has a supremum at $p = v-t$. We differentiate $\frac{\partial \Pi_i}{\partial p}$ with respect to p

$$\frac{\partial^2 \Pi_i}{\partial p^2} = \frac{\phi(4t((2v+c-3p)\alpha\epsilon - t(1-\alpha(1-\beta-\epsilon))) + (3t^2(1-\alpha(1-\beta^2)) - 4(2p+t-2v)(4p+t-2v-2c)\alpha\epsilon^2)\phi)}{2t^3} \tag{S5}$$

Therefore, $\frac{\partial^2 \Pi_i}{\partial p^2} < 0$ for all ϕ if $\phi < \phi^h$ where

$$\phi^h = \frac{4t((3p-2v-c)\alpha\epsilon + t(1-\alpha(1-\beta-\epsilon)))}{3t^2(1-\alpha(1-\beta^2)) - 4(2p+t-2v)(4p+t-2v-2c)\alpha\epsilon^2} \tag{S6}$$

In the following we show that $\phi^h > 1$ for all $p > v-t$. One can show that ϕ^h increases with price for all prices greater than $(v-t)$. In order to see that, choose a price $\widehat{p} = v-t+\eta$ for $\eta > 0$. Substituting $\widehat{p} = v-t+\eta$ in equation (S6) above and differentiating ϕ^h with respect to η we can show that $\frac{\partial \phi^h}{\partial \eta} > 0$ for $t < \frac{2}{5}(v-c)$ which is automatically satisfied for $t < \overline{t_s}$. In other words, ϕ^h increases with p for all $p > v-t$.

Furthermore at $p_i = v-t$,

$$\left|\phi^h\right|_{p=v-t} = \left(\frac{4t(1-\alpha(1-\beta)) + 4(v-c-2t)\alpha\epsilon}{3t(1-\alpha(1-\beta^2)) + 4(2v-2c-3t)\alpha\epsilon^2}\right) \tag{S7}$$

Note that $\left|\phi^h\right|_{p=v-t} > 1$ for $t < \left(\frac{v-c}{2}\right)$. Since, in the discussion following (S6) above, we also established that ϕ^h increases with p and increases further above 1 for all $p > (v-t)$, it must imply that $\frac{\partial^2 \Pi_i}{\partial p^2} < 0$ for all $p > v - t$ and $\phi \in [0,1]$. Therefore, $\frac{\partial \Pi_i}{\partial p}$ attains a supremum at $p = v - t$.

To summarize, for $t < \overline{t_s}$ and $\epsilon \leq \min\left[(1-\beta), \left(\frac{\sqrt{37}-5}{2}\right)\beta\right]$ prices greater than $(v-t)$ are not observed in equilibrium and we look for an interior equilibrium in the interval $(0, v-t]$.

Maximizing the profit function assuming that $x_i = 1$ yields the equilibrium prices p_s^*, advertising reach ϕ_s^* and profit Π_s^* as stated in table 1. We also checked for the second order conditions for a maximizer and they are satisfied at the candidate equilibrium. As mentioned in the paper we restrict θ such that $0 < \phi_s^* \leq 1$.The following condition lower bound on θ ensures the same:

$$\theta > \theta_{Ls} = \frac{t(3 + \alpha(-3(1-\beta)^2 + \epsilon^2))^2}{36(1 - \alpha(1-\beta^2))} \tag{S8}$$

We simplify the above threshold by finding the supremum of θ_{Ls} as a function of α. We obtain the following threshold $\underline{\theta_s}$.

$$\underline{\theta_s} = \frac{t(3(2-\beta)\beta + \epsilon^2)^2}{36\beta^2} \tag{S9}$$

Non Deviation

The pure strategy equilibrium must be stable to defections by either firm to $v - t$. The profits earned by charging $v - t$ to partially informed consumers are

$$\Pi(\phi^*, v-t) = \left[\alpha \left\{ \int_0^1 (\phi^*(1-\phi^*)(\beta + \epsilon - 2\epsilon x))\, dx + \int_0^1 \phi^{*2}\left((\beta + \epsilon - 2\epsilon x)(1 - \beta + \epsilon - 2\epsilon x)\right) dx \right\} (1-\alpha)\phi^*(1-\phi^*))(v - t - c)\right] - \theta\phi^{*2} \tag{S10}$$

$\Pi(\phi^*, p^*) > \Pi(\phi^*, v-t)$ is necessary for stability in pure strategy in prices, where $\Pi(\phi^*, p^*)$ is the equilibrium profit reported in table 1. Substituting ϕ^* in $\Pi(\phi^*, v-t)$ and computing $\Xi_D = \Pi(\phi^*, p^*) - \Pi(\phi^*, v-t)$ yields the following:

$$\Xi_D = \frac{6t(1-\alpha(1-\beta))^3(-12\theta t(1-\alpha(1-\beta)) + (c+t-v)(-6\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta^2))} + t(1-\alpha(1-\beta))(3+\alpha(-3+3\beta^2-\epsilon^2)))))}{\left(6\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta^2))} + t(1-\alpha(1-\beta))(3+\alpha(-3+3\beta^2-\epsilon^2))\right)^2} \tag{S11}$$

Note that $sgn(\Xi_D)$ depends on

$$sgn[(-12\theta t(1-\alpha(1-\beta)) + (c+t-v)(-6\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta^2))} + t(1-\alpha(1-\beta))(3+\alpha(-3+3\beta^2-\epsilon^2)))))] \tag{S12}$$

The sign of the above polynomial is strictly less than 0 for:

$$t > t_{\text{min}} = \frac{3(v-c)(1-\alpha(1-\beta^2))}{15-\alpha(15-15\beta^2+4\epsilon^2)} \tag{S13}$$

Again we simplify the above threshold on t by finding the supremum of t_{min}which will give us a sufficient condition for non-deviation. Please note that t_{min} is a continuous and increasing function of α. Therefore, we substitute $\alpha = 1$ to get the supremum threshold $\underline{t_s}$ leading to the following condition

$$t > \underline{t_s} = \frac{3(v-c)\beta^2}{15\beta^2 - 4\epsilon^2} \tag{S14}$$

Uniqueness

In order to ensure uniqueness of the symmetric pure strategy equilibrium, we need to consider the possibility of a corner symmetric equilibrium wherein $p_i = p_{-i} = v - t$. All other prices are not viable or firm $i = A, B$ because the profit function is globally concave in prices interior to the open interval $(0, v-t)$. For a corner symmetric equilibrium with $p_i = p_{-i} = v - t$ we need the following to hold:

$$\frac{\partial \Pi_i}{\partial p_i|}_{p_i \to (v-t)^-} > 0, \frac{\partial \Pi_i}{\partial p_i|}_{p_i \to (v-t)^+} < 0 \tag{S15}$$

Our strategy is to come up with a contradiction to condition (S15), guaranteeing the equilibrium is unique in the interval $[0, v-t]$. From above we know that$\frac{\partial \Pi_i}{\partial p_i|}_{p_i \to (v-t)^+} < 0$ for

$p_i > v - t$. Now, consider the case for $p_i \leq v - t$,

$$\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^-}} = \frac{\phi_i(3(c-v)(1-\alpha(1-\beta^2))\phi_{-i} + t(6+\alpha(-6+6\beta+\epsilon^2\phi_{-i})))}{6t} \tag{S16}$$

Even though $t < \frac{6(v-c)(\sqrt{37}-7+5\beta(\sqrt{37}-6))}{6(\sqrt{37}-9)+5(7\sqrt{37}-41)\beta}$, for small enough ϕ_{-i}, it is possible that $\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^-}} > 0$ and thus, that $p_i = v - t$ is a symmetric equilbrium for both firms. Focusing on the case when $p_i = v - t$, we derive the appropriate profit functions by setting $x_A = x_B = 1$ in equation 5 in the paper. Maximizing the profit function with respect to ϕ_i, $i = A, B$, and assuming $\phi_i = \phi_{-i}$ in equilbrium, we get ϕ^*_{sd}

$$\phi^*_{sd} = \frac{6(v-t-c)(1-\alpha(1-\beta))}{12\theta + (v-t-c)(3-\alpha(-3+3\beta^2-\epsilon^2))} \tag{S17}$$

Substituting (S17) into (S16) we get

$$\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^-}} = \frac{18(v-t-c)(1-\alpha(1-\beta))^2(4\theta t - (v-t-c)^2(1-\alpha(1-\beta^2)))}{t(12\theta + (v-t-c)(3-\alpha(-3+3\beta^2-\epsilon^2)))^2} \tag{S18}$$

Now note that$\frac{\partial \Pi_i}{\partial p_i|_{p_i \to (v-t)^-}} < 0$ for

$$\theta < \theta_{Hs} = \frac{(v-t-c)^2(1-\alpha(1-\beta^2))}{4t} \tag{S19}$$

In other words for $\theta < \theta_{Hs}$ a contradiction to (S15) is established. Therefore, if $\theta < \theta_{Hs}$ there does not exist a symmetric corner equilibrium such that $p^* = v - t$. For parsimony we again simplify the restriction by finding the minimum of θ_{Hs} in α.Since, θ_{Hs} is a decreasing function of α, the minimum of θ_{Hs} is found for $\alpha = 1$. Hence, $\theta_{Hs}|_{\alpha=1} = \overline{\theta_s}$, where,

$$\overline{\theta_s} = \frac{(v-t-c)^2}{4t}\beta^2 \tag{S20}$$

Therefore the condition $\theta < \overline{\theta_s}$ guarantees uniqueness of the equilibrium.

To summarize, condition on θ for which the equilibrium is defined is

$$\frac{t(3(2-\beta)\beta+\epsilon^2)^2}{36\beta^2} = \underline{\theta}^s < \theta < \overline{\theta}^s = \frac{(v-t-c)^2}{4t}\beta^2 \tag{S21}$$

For the interval $\left[\underline{\theta_s}, \overline{\theta_s}\right]$ to have a a positive measure we need

$$t < \beta\left(\frac{v-c}{2}\right) \tag{S22}$$

However, there is another upper bound on t which is given by (S4).

Therefore, the condition on t for which the candidate equilibrium holds is:

$$\frac{3(v-c)\beta^2}{15\beta^2-4\epsilon^2} = \underline{t_s} < t < \overline{t_s} = \min\left[\frac{6(v-c)(\sqrt{37}-7+5\beta(\sqrt{37}-6))}{6(\sqrt{37}-9)+5(7\sqrt{37}-41)\beta}, \beta\left(\frac{v-c}{2}\right)\right] \tag{S23}$$

Finally, we ought to ensure $\overline{t_s} > \underline{t_s}$. Note that $\frac{6(v-c)(\sqrt{37}-7+5\beta(\sqrt{37}-6))}{6(\sqrt{37}-9)+5(7\sqrt{37}-41)\beta} > \underline{t_s}$ for all permissable parameter values. However, to ensure that $\beta\left(\frac{v-c}{2}\right) > \underline{t_s}$ we require $\beta > \underline{\beta_s}$, where $\underline{\beta_s} = \frac{1}{15}(3+\sqrt{9+60\epsilon^2})$.[3] Therefore, in our proposed equilibrium

$$\beta \in \left[\frac{1}{15}(3+\sqrt{9+60\epsilon^2}), (1-\epsilon)\right] \tag{S24}$$

Finally, please note that we previously restricted ϵ such that $\epsilon \leq \left(\frac{\sqrt{37}-5}{2}\right)\beta$. For parsimony, we substitute the lower bound on β as given in equation (S24) in $\epsilon \leq \left(\frac{\sqrt{37}-5}{2}\right)\beta$, and obtain the following simpler threshold on ϵ:

$$\epsilon \leq \overline{\epsilon_s} = \frac{12}{45+\sqrt{37}} \tag{S25}$$

Inequalities (S21, S23-S25) together provide the restrictions on the parameters θ, t, β and ϵ as outlined in Table 1.

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[3] For $\beta\left(\frac{v-c}{2}\right) > \underline{t_s}$,we also require that $\epsilon < \frac{6}{11}$ which is automatically fulfilled since $\epsilon \in [0, \frac{1}{2}]$.

Proof of Propositions 3 and 4

Proof of Proposition 3

The proposition is established by differentiation of p_s^*, ϕ_s^* and Π_s^* with respect to ϵ.

$$\frac{\partial p_s^*}{\partial \epsilon} = 0 \tag{S26}$$

$$\frac{\partial \phi_s^*}{\partial \epsilon} = \frac{12t^2\alpha(1-\alpha(1-\beta))\epsilon}{(t(3-\alpha(3(1-\beta^2)+\epsilon^2))+6\sqrt{\theta t(1-\alpha(1-\beta^2))})^2} \geq 0 \tag{S27}$$

$$\frac{\partial \Pi_s^*}{\partial \epsilon} = \frac{144\theta t^3\alpha(1-\alpha(1-\beta))^2\epsilon}{(t(3-\alpha(3(1-\beta^2)+\epsilon^2))+6\sqrt{\theta t(1-\alpha(1-\beta^2))})^3} \geq 0 \tag{S28}$$

■

Lemma 2 *Price elasticity of demand for fully informed consumers increases monotonically with* ϵ.

The demand from fully informed consumers is given in equation 4 in the paper. We evaluate the elasticity around the interior equilibrium to determine how it changes with respect to the ad selectivity parameter ϵ.The price elasticity of demand around the interior equilibrium is given by:

$$\xi = \left|\frac{\partial F_i}{\partial p}\frac{p}{F_i}\right|_{p=p^*,\phi=\phi^*} = \frac{3c(1-\alpha(1-\beta))(1-\alpha(1-\beta^2))+6\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta^2))}}{t(1-\alpha(1-\beta))(3-\alpha(3-3\beta^2+\epsilon^2))} \tag{S29}$$

Differentiating ξ in equation (S29) with respect to ϵ yields:

$$\frac{d\xi}{d\epsilon} = \frac{6\alpha\left(c(1-\alpha(1-\beta))(1-\alpha(1-\beta^2)) + 2\sqrt{\theta t(1-\alpha(1-\beta))^2(1-\alpha(1-\beta^2))}\right)\epsilon}{t(1-\alpha(1-\beta))(3-\alpha(3-3\beta^2+\epsilon^2))^2} > 0 \tag{S30}$$

In other words, price elasticity of demand monotonically increases with ϵ.■

Proof of Proposition 4

In order to show the result we first determine how equilibrium ad reach and profit changes with α. Differentiating ϕ_s^* and Π_s^* with respect to α we find that $\frac{\partial\phi_s^*}{\partial\alpha} > 0$ and $\frac{\partial\Pi_s^*}{\partial\alpha} > 0$ for all (i) $\theta < \theta_s^*$ or for (ii) ($\alpha \geq \min[0, \alpha_s^*]$ and $\theta \geq \theta_s^*$)where,

$$\begin{aligned}\alpha_s^* = {} & \frac{18\theta(1-\beta)^3 - t(3(1-\beta)\beta+\epsilon^2)^2}{18\theta(1-\beta)^3(1+\beta)} \\ & - \frac{\sqrt{t}(3(1-\beta)\beta+\epsilon^2)\sqrt{36\theta t(1-\beta)^2\beta + t(3(1-\beta)\beta+\epsilon^2)^2}}{18\theta(1-\beta)^3(1+\beta)}\end{aligned} \tag{S31}$$

and

$$\theta_s^* = \frac{t(3(1-\beta)\beta+\epsilon^2)^2}{9(1-\beta)^4} \tag{S32}$$

Next, differentiating α_s^* with respect to ϵ yields

$$\begin{aligned}\frac{\partial\alpha_s^*}{\partial\epsilon} = {} & -\frac{4\theta\sqrt{t}\beta\epsilon}{(1-\beta)^2\sqrt{36\theta(1-\beta)^2\beta + t(3(1-\beta)\beta+\epsilon^2)^2}} \\ & - \frac{2t\epsilon(3(1-\beta)\beta+\epsilon^2)\left(\sqrt{t}(3(1-\beta)\beta+\epsilon^2) + \sqrt{36\theta(1-\beta)^2\beta + t(3(1-\beta)\beta+\epsilon^2)^2}\right)}{9\theta(1-\beta)^3(1+\beta)\sqrt{36\theta(1-\beta)^2\beta + t(3(1-\beta)\beta+\epsilon^2)^2}} < 0\end{aligned} \tag{S33}$$

In other words, the threshold above which ad reach and profit strictly increase with DVR penetration α_s^* decreases with ϵ.■