Online Technical Appendix To Accompany "*An Empirical Investigation of Private Label Supply by National Label Producers*"

In the technical appendix we describe in detail the various supply games that we estimate, which corresponds to Sections 2.3 and 4.3.2 in the text. We also explain the approach used to selected the most appropriate game form (Section 4.3.2 in the text), and display the elasticity matrix derived from our demand estimates (Section 4.3.1 in the text). In order to make the Technical Appendix as self-contained as possible, we occasionally reproduce some of the material from the text, including the relevant references.

SUPPLY GAMES

At the retail market, the products sold include both national brands and two kinds of private labels (skim milk and whole milk) for each retailer. We denote the set of products at retailer *r* as F_r; thus, the full set of products in the retail market as $F_R = \cup F_r$ with measure N_R. Denote the set of national brands and private labels at retail level as F_R^n and F_R^b respectively[1]. Given the two types of products (skim and whole milk), denote F_R^{n1} and F_R^{n2} as the sets of these two types of NL products at the retail level, and F_R^{b1} and F_R^{b2} as the corresponding sets of PL products at the retail level. Note that superscript 1 denotes skim while 2 denotes whole milk. By definition, $F_R = F_R^n \cup F_R^b$, $F_R^n = F_R^{n1} \cup F_R^{n2}$, and $F_R^b = F_R^{b1} \cup F_R^{b2}$. Finally, we use q_j and p_j to represent the quantity and price of brand *j* in the retail market.

In the wholesale market, we denote the set of NL products that manufacturer *m* sells as F_m^n with measure N_m^n, and the full set of NL products as $F_M^n = \cup F_m^n$ with measure N_M^n. Let F_M^b denote the set of PL products supplied in the wholesale market with measure N_M^b. Since private labels are homogeneous products in the wholesale market, each manufacturer either supplies F_M^b or does not[2]. Let q_j^n and w_j^n represent the quantity and wholesale price for national brands, x_{mi}^b represent the quantity of private label *i* supplied by manufacturer *m*, and w_i^b the wholesale price for private label *i*.

TA.1 Specifying The Retailer's Problem

The retailers maximize their category profit,

[1] NL and PL products carry the *n* and *b* superscripts respectively.
[2] We constrain each manufacturer to either supply both whole and skim private label milk, or neither. This reflects our institutional context – no producer in this market focuses on one kind of milk exclusively.

$$\underset{p_j, j \in F_r}{Max}\, \Pi_r = \sum_{j \in F_r} (p_j - w_j) q_j(p)$$

where q_j is the quantity and w_j is the wholesale price for product j ; p is a vector of retail prices. The first order conditions are:

$$q_j + \sum_{k \in F_r} (p_k - w_k) \frac{\partial q_k}{\partial p_j} = 0 \qquad \forall \ j \in F_r \tag{1}$$

Written in matrix form, the price-cost margins for all retailers (R) are

$$PCM^R = p - w = -\left(T_R \cdot \Delta_1\right)^{-1} q \tag{2}$$

where p, w, and q are vectors for retail price, wholesale price, and quantity respectively. Δ_1 is an $N_R \times N_R$ matrix of marketing response to retail price, with

$$\Delta_1(i, j) = \frac{\partial q_j}{\partial p_i} \quad \forall i, j \in F_R$$

We show how to calculate Δ_1 in section TA3. T_R is an $N_R \times N_R$ matrix indicating the retailer's pricing strategy; in the category maximization case, $T_R(i, j) = 1 \ \ \forall i, j \in F_r$. Finally, $T_R \cdot \Delta_1$ is the element by element product of the matrices T_R and Δ_1.

TA.2 Specifying The Manufacturers' Problem

The model we specify has upstream manufacturers picking quantities for the private label and wholesale prices for the national brand. Now, manufacturers can use various profit maximization strategies with respect to private labels and national brands.

In particular, we consider three cases: i) *Product management*, wherein the manufacturer maximizes profits separately for each 'product' in his portfolio. ii) *Account management*, wherein the manufacturer maximizes profits for NL products (viewed as one account) separately from PL products (viewed as a different account). iii) *Category management*, wherein the manufacturer maximizes joint profits across all products.

We next discuss the model for scenario (i) in detail. The other two scenarios are straightforward extensions of scenario (i).

In scenario (i), manufacturers maximize joint profits by choosing prices for national brands and quantities for the private label they supply:

Manufacturers Who Supply Private Labels: These manufacturers maximize profits by choosing prices for national brands and quantities for the private label they supply:

$$\underset{w_j^n, x_{mi}^b}{Max}\Pi_m = \sum_{j\in F_m^n} w_j^n q_j^n + \sum_{i\in F_M^b} w_i^b x_{mi}^b - \sum_{j\in F_m^n} c_j^n q_j^n - \sum_{i\in F_M^b} c_i^b x_{mi}^b$$

where w_j^n is the wholesale price for national brand *j*, w_i^b is the wholesale price for PL *i*, c_j^n is the marginal cost for national brand *j*, and c_i^b is the marginal cost for PL *i*.   .

Note that w_i^b is endogenously determined by imposing the condition that markets clear, i.e.,

$$\sum_{m\in M} x_{mi}^b = \sum_{j\in F_R^{bi}} q_j\left(p\left(w^n, w^b\right)\right) \quad \text{for } i=1,2$$

Therefore, the PL wholesale prices can be considered as a function of the total quantity of private labels in the wholesale market, and the NL wholesale prices, i.e.,

$$w^b\left(\sum_{m\in M} x_{m1}^b, \sum_{m\in M} x_{m2}^b, w^n\right).$$

Hence, the first order conditions are:

$$\frac{\partial \Pi_m}{\partial w_k^n} = q_k^n + \sum_{j\in F_m^n} w_j^n \frac{\partial q_j^n}{\partial w_k^n} + \sum_{i\in F_M^b} \frac{\partial w_i^b}{\partial w_k^n} x_{mi}^b - \sum_{j\in F_m^n} c_j^n \frac{\partial q_j^n}{\partial w_k^n} = 0 \quad \forall\ k \in F_m^n \tag{3a}$$

$$\frac{\partial \Pi_m}{\partial x_{mk}^b} = \sum_{j\in F_m^n} w_j^n \frac{\partial q_j^n}{\partial x_{mk}^b} + w_k^b + \sum_{i\in F_M^b} \frac{\partial w_i^b}{\partial x_{mk}^b} x_{mi}^b - \sum_{j\in F_m^n} c_j^n \frac{\partial q_j^n}{\partial x_{mk}^b} - c_k^b = 0 \quad \forall\ k \in F_M^b \tag{3b}$$

In the above,

$$\frac{\partial q_j^n}{\partial w_k^n} = \sum_{l\in F_R} \frac{\partial q_j^n}{\partial p_l} \frac{\partial p_l}{\partial w_k^n}$$

$$\frac{\partial q_j^n}{\partial x_{mk}^b} = \sum_{l\in F_R} \frac{\partial q_j^n}{\partial p_l} \sum_{i\in F_M^b} \frac{\partial p_l}{\partial w_i^b} \frac{\partial w_i^b}{\partial x_{mk}^b}$$

Manufacturers Who Don't Supply Private Labels: These manufacturers maximize profits by choosing prices for their NL products, i.e.,

$$\underset{w_j^n}{Max}\Pi_m = \sum_{j\in F_m^n} w_j^n q_j^n - \sum_{j\in F_m^n} c_j^n q_j^n$$

The first order conditions are obtained from equation 3a by setting terms involving w^b to zero:

$$\frac{\partial \Pi_m}{\partial w_k^n} = q_k^n + \sum_{j\in F_m^n} w_j^n \frac{\partial q_j^n}{\partial w_k^n} - \sum_{j\in F_m^n} c_j^n \frac{\partial q_j^n}{\partial w_k^n} = 0 \qquad \forall\ k \in F_m^n. \tag{4}$$

With expressions 3a, 3b, and 4 in place, we can now derive the manufacturers' price cost margins for both national brands and private labels, in matrix form.

<u>*Matrix Expression for Manufacturer Price Cost Margins*</u>: Consider national brands first. It is straightforward to combine equation 3a and 4 as:

$$\sum_{j\in F_m^n}\left(w_j^n - c_j^n\right)\frac{\partial q_j^n}{\partial w_k^n} = -\left(q_k^n + \phi(m)\sum_{i\in F_M^b}\frac{\partial w_i^b}{\partial w_k^n}x_{mi}^b\right)$$

where $\phi(m)$ is an indicator function to reflect whether manufacturer *m* supplies private labels or not. Define an $N_M^n \times 1$ matrix G_1 as:

$$G_1(k) = q_k^n + \phi(m)\sum_{i\in F_M^b}\frac{\partial w_i^b}{\partial w_k^n}x_{mi}^b \qquad \forall\ k \in F_M^n$$

Then we can write manufacturers' margins on national brands as:

$$PCM_M^n = -\left(T_M^n \square \Delta_2^n\right)^{-1} G_1 \tag{5}$$

where PCM_M^n is an $N_M^n \times 1$ vector of manufacturers' margins on their national brands, with $PCM_M^n(j) = \left(w_j^n - c_j^n\right)\ \forall\ j \in F_M^n$, Δ_2^n is an $N_M^n \times N_M^n$ matrix with $\Delta_2^n(k, j) = \frac{\partial q_j^n}{\partial w_k^n}$ $\forall\ k, j \in F_M^n$, and T_M^n is an $N_M^n \times N_M^n$ matrix indicating manufacturers' pricing strategies for their national brand. Note that for the category management case, $T_M^n(i, j) = 1\ \ \forall i, j \in F_m^n$. Finally, $T_M^n \square \Delta_2^n$ is the element by element product of the matrices T_M^n and Δ_2^n.

Next, we turn to manufacturers' margins from private labels, if they supply those. Equation 3b can be written as:

$$\left(w_k^b - c_k^b\right) = -\sum_{i\in F_m^b}\frac{\partial w_i^b}{\partial x_{mk}^b}x_{mi}^b - \sum_{j\in F_m^n}\left(w_j^n - c_j^n\right)\frac{\partial q_j^n}{\partial x_{mk}^b}.$$

Define an $N_M^b \times 1$ matrix G_{2m} as $G_{2m}(k) = \sum_{i\in F_M^b}\frac{\partial w_i^b}{\partial x_{mk}^b}x_{mi}^b$ for $k \in F_M^b$. Therefore manufacturer *m*'s margin from private labels can be written as:

$$PCM_m^b = -G_{2m} - \Delta_{3m} PCM_m^n \tag{6}$$

where PCM_m^b is an $N_M^b \times 1$ matrix with $PCM_m^b(k) = \left(w_k^b - c_k^b\right)$ for $k \in F_M^b$. Δ_{3m} is an $N_M^b \times N_m^n$ matrix with $\Delta_{3m}(k, j) = \dfrac{\partial q_j^n}{\partial x_{mk}^b}$ for $m \in M$, $k \in F_M^b$ and $j \in F_m^n$.

Note that in order to calculate the manufacturer's margin using equation 5 and 6, we need to know Δ_2^n, Δ_{3m}, G_1, and G_{2m}. Furthermore, to calculate G_1, we need to know an $N_M^n \times N_M^b$ matrix $\Delta_{w1}(k,i) = \dfrac{\partial w_i^b}{\partial w_k^n}$ for $k \in F_M^n$ and $i \in F_M^b$. To calculate G_{2m}, we need to know an $N_M^b \times N_M^b$ matrix $\Delta_{w2m}(k,i) = \dfrac{\partial w_i^b}{\partial x_{mk}^b}$ for $k \in F_M^b$ and $i \in F_M^b$.

First we show how to calculate Δ_2^n and Δ_{w1}. Notice that $\Delta_2^n = \Delta_R^{n'}\Delta_{1n}$, where Δ_R^n is an $N_R \times N_M^n$ matrix with $\Delta_R^n(i, j) = \dfrac{dp_i}{dw_j^n}$ for $i \in F_R$ and $j \in F_M^n$, and Δ_{1n} is a subset of Δ_1 with $\Delta_{1n}(i, j) = \dfrac{\partial q_j}{\partial p_i}$ for $j \in F_M^n$ and $i \in F_R$.

We fully differentiate equation 1 with respect to a wholesale price w_f^n for national brand *f*. Observing that retail price p_k and private label's wholesale price w_i^b are implicit functions of w_f^n, we get

$$\sum_{k\in F_R}\left[\frac{\partial q_j}{\partial p_k} + \sum_{i\in F_R}\left[T_R(i, j)\frac{\partial^2 q_i}{\partial p_j \partial p_k}(p_i - w_i)\right] + T_r(k, j)\frac{\partial q_k}{\partial p_j}\right]\frac{dp_k}{dw_f^n} - \sum_{i\in F_M^b}\frac{\partial w_i^b}{\partial w_f^n}\sum_{r\in R}T_R(ri, j)\frac{\partial q_{ri}}{\partial p_j} - T_R(f, j)\frac{\partial q_f}{\partial p_j} = 0 \tag{7}$$

There are N_R such equations with $N_R + N_M^b$ unknowns ($\dfrac{dp_k}{dw_f^n}$, *k=1…N*$_R$, $\dfrac{\partial w_i^b}{\partial w_f^n}$, *i=1…* N_M^b).

For the private label $i \in F_M^b$, we have the market clearing constraint,

$$\sum_{m\in M} x_{mi}^b - \sum_{j\in F_R^{bi}} q_j\left(p\left(w^n, w^b\right)\right) = 0 \tag{8}$$

Fully differentiating equation 8 w.r.t. w_f^n, we have

$$\sum_{j\in F_R^{bi}}\sum_k \frac{\partial q_j}{\partial p_k}\frac{dp_k}{dw_f^n} = \sum_k\sum_{j\in F_R^{bi}}\frac{\partial q_j}{\partial p_k}\frac{dp_k}{dw_f^n} = 0 \tag{9}$$

This gives us N_M^b equations.

Combine (7) and (9), define an $N_R \times N_R$ matrix S_1,

$$S_1(j,k) = \frac{\partial q_j}{\partial p_k} + \sum_{i\in F_R}\left[T_R(i, j)\frac{\partial^2 q_i}{\partial p_j \partial p_k}(p_i - w_i)\right] + T_R(k, j)\frac{\partial q_k}{\partial p_j}$$

$N_R \times N_M^n$ matrix S_2,

$$S_2(j,i) = -\sum_{r\in R} T_R(ri,j)\frac{\partial q_{ri}}{\partial p_j}$$

$N_M^n \times N_R$ matrix S_3,

$$S_3(i,k) = \sum_{j\in F_R^{bi}} \frac{\partial q_j}{\partial p_k}$$

$\left(N_R + N_M^n\right)\times\left(N_R + N_M^n\right)$ matrix S,

$$S = \begin{pmatrix} S_1 & S_2 \\ S_3 & 0 \end{pmatrix}$$

$N_R \times 1$ matrix H_{1f},

$$H_{1f}\left(j,1\right) = T_R\left(f,j\right)\frac{\partial q_f}{\partial p_j}$$

and an $\left(N_R + N_M^n\right)\times 1$ matrix H_f

$$H_f = \begin{bmatrix} H_{1f} \\ 0 \end{bmatrix}$$

where the expression for $\frac{\partial q_i}{\partial p_k}$ and $\frac{\partial^2 q_i}{\partial p_j \partial p_k}$ is given later. Then equations 7 and 9 can be written in matrix form as:

$$S\begin{bmatrix} \frac{dp}{dw_f^n} & \frac{dw^b}{dw_f^n} \end{bmatrix}' - H_f = 0$$

Therefore

$$\begin{bmatrix} \frac{dp}{dw_f^n} & \frac{dw^s}{dw_f^n} \end{bmatrix}' = S^{-1}H_f$$

Now we show how to calculate Δ_{3m} and Δ_{w2m}. We fully differentiate equation 1 with respect to a wholesale price x_{mi}^b for national brand *f*. Observing that retail price p_k and private label's wholesale price w_i^b are implicit functions of x_{mi}^b, we get

$$\sum_{k\in F_R}\left[\frac{\partial q_j}{\partial p_k} + \sum_{i\in F_R}\left[T_R(i,j)\frac{\partial^2 q_i}{\partial p_j \partial p_k}(p_i - w_i)\right] + T_R(k,j)\frac{\partial q_k}{\partial p_j}\right]\frac{dp_k}{dx_{mi}^b} - \sum_{i\in F_M^b}\frac{\partial w_i^b}{\partial x_{mi}^b}\sum_{r\in R} T_R\left(ri,j\right)\frac{\partial q_{ri}}{\partial p_j} = 0 \quad (10)$$

There are N_R such equations with $N_R + N_M^b$ unknowns ($\frac{dp_k}{dx_{mi}^b}$, $k=1\ldots N_R$, $\frac{\partial w_i^b}{\partial x_{mi}^b}$, $i=1\ldots N_M^b$).

For the private label $l \in F_M^b$, we have the market clearing constraint,

$$\sum_{m\in M} x_{ml}^b - \sum_{j\in F_R^{bi}} q_j^n\left(p\left(w^n, w^b\right)\right) = 0 \tag{11}$$

Fully differentiating equation 11 w.r.t. x_{mi}^b, we have

$$\sum_{j\in F_R^{bi}} \sum_{k\in F_R} \frac{\partial q_j}{\partial p_k}\frac{\partial p_k}{\partial x_{mi}^b} - 1 = \sum_{k\in F_R} \sum_{j\in F_R^{bi}} \frac{\partial q_j}{\partial p_k}\frac{\partial p_k}{\partial x_{mi}^b} - 1 = 0 \qquad \forall l = i$$

$$\sum_{j\in F_R^{bl}} \sum_{k\in F_R} \frac{\partial q_j}{\partial p_k}\frac{\partial p_k}{\partial x_{mi}^b} - 0 = \sum_{k\in F_R} \sum_{j\in F_R^{bl}} \frac{\partial q_j}{\partial p_k}\frac{\partial p_k}{\partial x_{mi}^b} = 0 \qquad \forall l \neq i \tag{12}$$

This gives us N_M^b equations.

Combining (10) and (12), we define an $N_R \times N_R$ matrix S_1,

$$S_1(j,k) = \frac{\partial q_j}{\partial p_k} + \sum_{i\in F_R}\left[T_R(i,j)\frac{\partial^2 q_i}{\partial p_j \partial p_k}(p_i - w_i)\right] + T_R(k,j)\frac{\partial q_k}{\partial p_j}$$

an $N_M^n \times N_R$ matrix S_2,

$$S_2(j,i) = -\sum_{r\in R} T_R\left(ri, j\right)\frac{\partial q_{ri}}{\partial p_j}$$

An $N_M^n \times N_R$ matrix S_3,

$$S_3(l,k) = \sum_{j\in F_R^{bl}} \frac{\partial q_j}{\partial p_k}$$

an $\left(N_R + N_M^n\right)\times\left(N_R + N_M^n\right)$ matrix S,

$$S = \begin{pmatrix} S_1 & S_2 \\ S_3 & 0 \end{pmatrix}$$

An $N_M^n \times 1$ matrix H_1,

$$H_1(j,1) = 1 \text{ for } j = mi$$

$$H_1(j,1) = 0 \text{ otherwise.}$$

and an $\left(N_R + N_M^n\right)\times 1$ matrix H_{mi},

$$H_{mi} = \begin{bmatrix} 0 \\ H_1 \end{bmatrix}$$

where the expressions for $\frac{\partial q_i}{\partial p_k}$ and $\frac{\partial^2 q_i}{\partial p_j \partial p_k}$ are given later. Equations 10 and 12 can be written in matrix form as:

$$S\begin{bmatrix} \frac{dp}{dx_{mi}^b} & \frac{dw^b}{dx_{mi}^b} \end{bmatrix}' - H_{mi} = 0$$

Therefore,

$$\left[\begin{array}{cc}\frac{dp}{dx_{mi}^{b}} & \frac{dw^{b}}{dx_{mi}^{b}}\end{array}\right]' = S^{-1}H_{mi}$$

and

$$\Delta_{3m}\left(k, j\right) = \frac{\partial q_j^n}{\partial x_{mk}^b} = \sum_{k\in F_R}\frac{\partial q_j^n}{\partial p_k}\frac{dp_k}{dx_{mk}^b}$$

TA.3: Market Response Function

The market response functions are given as:

$$\frac{\partial q_j}{\partial p_i} = \frac{\partial \overline{Q}_j}{\partial p_i}\overline{M}$$

where $\overline{Q}_j$ is the average quantity per person and $\overline{M}$ is the measure of the market. We have

$$\overline{Q}_j = \int \frac{y_h}{p_j}\frac{\mu\alpha_3}{\alpha_h}\frac{e^v}{\sum e^v}\ln\left(1+\sum e^v\right)\phi\left(\Lambda\right)d\Lambda$$

$$= \mu\alpha_3\int\frac{y_h}{\alpha_h}\frac{1}{p_j}\cdot s_{1j}\ln C\cdot\phi\left(\Lambda\right)d\Lambda$$

$$= \frac{\mu\alpha_3}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\frac{1}{p_j}\cdot s_{1j}\ln C$$

where h indicates the individual (household), $s_{1j} = \frac{e^{v_j}}{\sum e^{v_k}}$, $C = 1+\sum e^{v_k}$, and

$s_{2j} = \frac{e^{v_j}}{1+\sum e^{v_k}}$.

We can then derive the following expression for $\frac{\partial\overline{Q_j}}{\partial p_i}$:

If $i = j$:

$$\frac{\partial\overline{Q_j}}{\partial p_j} = \frac{\mu\alpha_3}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\left\{-\frac{1}{p_j^2}s_{1j}\ln C+\frac{1}{p_j}\left(-\alpha_h\right)s_{1j}\left(1-s_{1j}\right)\frac{1}{p_j}\ln C+\frac{1}{p_j}s_{1j}\cdot\frac{1}{C}\cdot e^{v_j}\cdot\left(-\alpha_h\right)\frac{1}{p_j}\right\}$$

$$= -\frac{\mu\alpha_3}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\left(\frac{1}{p_j^2}\right)s_{1j}\left\{\ln C+\alpha_h\left(1-s_{1j}\right)\ln C+s_{2j}\alpha_h\right\}$$

If $i \neq j$:

$$\frac{\partial\overline{Q}_j}{\partial p_i} = \frac{\mu\alpha_3}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\left\{\frac{1}{p_j}\alpha_h s_{1j}s_{1i}\frac{1}{p_i}\ln C+\frac{1}{p_j}s_{1j}s_{2i}\left(-\alpha_h\right)\frac{1}{p_i}\right\}$$

$$= \frac{\mu\alpha_3}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\frac{1}{p_i p_j}\alpha_h s_{1j}\left(s_{1i}\ln C - s_{2i}\right)$$

$$= \frac{\mu\alpha_3}{N}\sum_{h=1}^{N}\frac{y_h}{p_i p_j}s_{1j}\left(s_{1i}\ln C - s_{2i}\right)$$

Similarly, we can derive the following expression for $\frac{\partial^2 \overline{Q}_i}{\partial p_j \partial p_k}$:

If $i = j = k$:

$$\frac{\partial^2 \overline{Q}_i}{\partial p_i \partial p_i} = -\frac{\mu\alpha_3}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\left\{-\frac{2}{p_i^3}s_{1i}\left[\ln C + \alpha_h\left(1-s_{1i}\right)\ln C + s_{2i}\alpha_h\right]\right.$$
$$+\frac{1}{p_i^2}\left(-\alpha_h\right)s_{1i}\left(1-s_{1i}\right)\frac{1}{p_i}\left[\ln C + \alpha_h\left(1-s_{1i}\right)\ln C + s_{2i}\alpha_h\right]$$
$$\left.+\frac{s_{1i}}{p_i^2}\left[s_{2i}\left(-\alpha_h\right)\frac{1}{p_i} + \alpha_h\alpha_h s_{1i}\left(1-s_{1i}\right)\frac{1}{p_i}\ln C + \alpha_h\left(1-s_{1i}\right)s_{2i}\left(-\alpha_h\right)\frac{1}{p_i} + \left(-\alpha_h\right)s_{2i}\left(1-s_{2i}\right)\frac{1}{p_i}\alpha_h\right]\right\}$$

If $i = j \;\&\; j \neq k$:

$$\frac{\partial^2 \overline{Q}_i}{\partial p_i \partial p_k} = -\frac{\alpha_3\mu}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\frac{1}{p_i^2 p_k}\left\{\begin{array}{l}\alpha_h s_{1i}s_{1k}\left[\ln C + \alpha_h\left(1-s_{1i}\right)\ln C + \alpha_h s_{2i}\right] \\ +s_{1i}\left[-\alpha_h s_{2k} - \alpha_h^2 s_{1i}s_{1k}\ln C - \alpha_h^2\left(1-s_{1i}\right)s_{2k} + \alpha_h^2 s_{2i}s_{2k}\right]\end{array}\right\}$$

If $i \neq j \;\&\; j \neq k \;\&\; i \neq k$

$$\frac{\partial^2 \overline{Q}_i}{\partial p_j \partial p_k} = \frac{\alpha_3\mu}{N}\sum_{h=1}^{N}y_h\frac{1}{p_i p_j p_k}\left\{2\alpha_h s_{1i}s_{1j}s_{1k}\ln C - \alpha_h s_{1i}s_{1j}s_{2k} - \alpha_h s_{1i}s_{2j}s_{1k} - \alpha_h s_{1i}s_{2j}s_{2k}\right\}$$

If $i \neq j \;\&\; i = k$

$$\frac{\partial^2 \overline{Q}_i}{\partial p_j \partial p_i} = -\frac{\alpha_3\mu}{N}\sum_{h=1}^{N}\frac{y_h}{\alpha_h}\frac{1}{p_i^2 p_j}\left\{\begin{array}{l}\alpha_h s_{1i}s_{1j}\left[\ln C + \alpha_h\left(1-s_{1i}\right)\ln C + \alpha_h s_{2i}\right] \\ +s_{1i}\left[-\alpha_h s_{2j} - \alpha_h^2 s_{1i}s_{1j}\ln C - \alpha_h^2\left(1-s_{1i}\right)s_{2j} + \alpha_h^2 s_{2i}s_{2j}\right]\end{array}\right\}$$

If $i \neq j \;\&\; j = k$

$$\frac{\partial^2 \overline{Q}_i}{\partial p_j \partial p_j} = -\frac{\alpha_3\mu}{N}\sum_{h=1}^{N}y_h\frac{1}{p_i p_j^2}\left\{\begin{array}{l}-s_{1i}\left[s_{1j}\ln C - s_{2j}\right] + \alpha_h s_{1i}s_{1j}\left[s_{1j}\ln C - s_{2j}\right] \\ +s_{1i}\left[-\alpha_h s_{1j}\left(1-s_{1j}\right)\ln C - \alpha_h s_{1i}s_{2j} + \alpha_h s_{2j}\left(1-s_{2i}\right)\right]\end{array}\right\}$$

PICKING APPROPRIATE GAME FORM

Recall that we had specified and estimated a number of possible competitive interactions and retailer behaviors. It is important at this stage to decide the most appropriate 'game form'. Prior literature has employed various ways of picking across games, including i) a conduct parameter approach, with estimated values of the conduct parameter suggesting the appropriate game (Bresnahan 1989; Kadiyali et al. 2000); ii) statistical fit tests, such as the Vuong (1989) or Smith (1992) tests (e.g., Sudhir 2001); iii) data patterns that pin down the nature of the game form (Porter 1983); and iv) a comparison of the margins/costs predicted by various models with those obtained externally (Nevo 2001). The key issue is whether there is sufficient information in the data to pin down *both* the nature of competitive interaction (and hence the price-cost margins) and the marginal cost. As Reiss and Wolak (2007) point out, this is almost never the case. With regard to the first alternative, various authors have suggested fundamental problems with the interpretation of the conduct parameters, and with the data requirements for identification in differentiated product markets (Corts 1999; Nevo 2001). Statistical testing is also problematic – among other issues, these tests are not transitive, which means that we can have a situation where model A dominates model B, model B dominates model C, but model C dominates model A. Finding data patterns to identify the nature of the game is extremely hard in practice. This leaves the last alternative, which is in many ways the most attractive, but one that requires valid external information.

Fortunately, we do have access to external information on average margins at the retailer and manufacturer level for a number of milk producers in the same geographical region as our data. We therefore follows Nevo (2001) closely, comparing the predicted margins from each of the possible game forms we estimate to these external numbers, and picking the game that gives us numbers closest to the external value.

Because of the complexity of the vertical and horizontal interactions that we model, the comparison of estimated margins to actual margins is fairly involved. To recap, the focus here is on comparing our *proposed model*, where PL products are homogenous upstream (Cournot-Nash) and differentiated downstream (Bertrand-Nash), with the *alternative model* where PL products are differentiated both upstream and downstream (Bertrand-Nash). In addition, we wish to distinguish between three possible forms of manufacturer and retailer behavior. Before we proceed, it is useful to set some nomenclature. We define a 'product' upstream as consisting of the identity of the manufacturer (*M1*, *M2*), the 'brand' (NL, PL), and the type (skim, whole). Downstream the definition is similar, except that the identity of the retailer is also relevant for PL products. Examples of upstream products include: NL skim milk from *M2*, PL whole milk from

M1, etc. Equivalent downstream products, at say *R2*, would be NL skim milk from *M1* and *M2*, and PL whole milk.

With this in place, we outline the three possible forms of manufacturer and retailer behavior. i) *Product management*, wherein the manufacturer (retailer) maximizes profits separately for each 'product' in his portfolio. ii) *Account management*, wherein the manufacturer (retailer) maximizes profits for NL products (viewed as one account) separately from PL products (viewed as a different account). As an example, upstream, *M1* would maximize profits for its NL skim and whole milk jointly, but separate from its PL skim and whole milk. Downstream, retailer *R2* would maximize profits jointly for NL products such as *M1* skim and whole, and *M2* skim and whole, but separate from its own PL skim and whole milk products. iii) *Category management*, wherein the manufacturer (retailer) maximizes joint profits across all products.

This gives us a total of 18 games to pick from – nine for the proposed model and identically nine for the alternative. For each game we estimate three margins (total, retailer, and manufacturer) giving us a total of 54 margins (18 x 3). Table TA1 displays all the estimated margins, along with confidence intervals. Cells where the true value of the margin falls within the estimated confidence interval are bolded, to highlight that this mode of conduct is indistinguishable statistically from the 'true' behavior. Note that there is only one game for which all three margins are bolded – this is the proposed model with both manufacturers and retailers practicing account management. To reiterate, the best fitting game is one where i) PL products compete Cournot-Nash upstream and Bertrand-Nash downstream; ii) NL products compete Bertrand-Nash both upstream and downstream; iii) both manufacturers and retailers practice account management, i.e., they separately maximize profits for NL and PL products. This is the game we use in all subsequent analyses.

Table TA1: Supply Model Selection

		Manufacturer_Category	Manufacturer_Account	Manufacturer_Product
Proposed Model				
Retailer_Category	Total	**62.7 (62.1,63.5)**	62.4 (61.7,63.2)	61.9 (61.2,62.8)
	Retailer	**40.5 (39.9,41.1)**	**40.5 (39.9,41.1)**	**40.5 (39.9,41.1)**
	Manufacturer	22.3 (22.1,22.4)	21.9 (21.7,22.1)	21.5 (21.2,21.8)
Retailer_Acccount	Total	**63.1 (62.5,63.9)**	***62.8 (62.2,63.7)***	62.3 (61.6,63.2)
	Retailer	**40.1 (39.6,40.8)**	***40.1 (39.6,40.8)***	**40.1 (39.6,40.8)**
	Manufacturer	23.0 (22.8,23.1)	***22.7 (22.6,22.9)***	22.2 (22.0,22.4)
Retailer_Product	Total	**63.2 (62.6,64.0)**	**63.0 (62.3,63.8)**	62.2 (61.4,63.0)
	Retailer	39.9 (39.3,40.5)	39.9 (39.3,40.5)	39.9 (39.3,40.5)
	Manufacturer	23.3 (23.2,23.5)	**23.1 (22.9,23.3)**	22.3 (22.1,22.5)
Alternative Model				
Retailer_Category	Total	**63.1 (62.3,63.7)**	**62.6 (61.7,63.4)**	62.1 (61.4,63.0)
	Retailer	**40.5 (39.9,41.1)**	**40.5 (39.9,41.1)**	**40.5 (39.9,41.1)**
	Manufacturer	22.6 (22.4,22.6)	22.1 (21.9,22.3)	21.7 (21.4,22.0)
Retailer_Acccount	Total	**63.5 (62.8,64.3)**	**63.2 (62.6,64.0)**	**62.4 (61.7,63.3)**
	Retailer	**40.1 (39.6,40.8)**	**40.1 (39.6,40.8)**	**40.1 (39.6,40.8)**
	Manufacturer	23.3 (23.2,23.5)	23.1 (23.0,23.2)	22.3 (22.1,22.5)
Retailer_Product	Total	**63.7 (63.0,64.5)**	**63.2 (62.5,64.0)**	**62.9 (62.2,63.7)**
	Retailer	39.9 (39.3,40.5)	39.9 (39.3,40.5)	39.9 (39.3,40.5)
	Manufacturer	23.8 (23.6,23.9)	23.3 (23.2,23.5)	23.0 (22.8,23.1)

i) Proposed Model: PL products homogeneous upstream and differentiated downstream; NL differentiated upstream and downstream.
Alternative Model: PL products differentiated both upstream and downstream; NL differentiated upstream and downstream.
ii) Reported numbers are median margins, with the 95% confidence interval in parenthesis.
iii) Numbers are bolded when the confidence interval covers the observed value from external data. The best fitting game has values italicized and bolded.
iv) Observed average margins (external data): Total 63.3%, Retailer 40.6%; Manufacturer 22.7%.

Table TA2: Price Elasticity Matrix

		R1			R2			R3			R4			R5		
		M1	**M2**	**P**	**M1**	**M2**	**P**	**M1**	**M2**	**P**	**M1**	**M2**	**P**	**M1**	**M2**	**P**
R1	**M1**	-2.0878	0.0160	0.0333	0.0273	0.0085	0.0262	0.0018	0.0004	0.001	0.0174	0.0057	0.0228	0.0014	0.0003	0.0007
	M2	0.0363	-2.1981	0.0475	0.0163	0.0091	0.0343	0.0009	0.0002	0.0006	0.0131	0.0093	0.0389	0.0008	0.0002	0.0004
	P	0.0251	0.0158	-2.2460	0.0132	0.0090	0.0551	0.0005	0.0001	0.0005	0.0135	0.0099	0.0623	0.0004	0.0001	0.0003
R2	**M1**	0.0545	0.0143	0.0350	-2.182	0.0085	0.0359	0.0018	0.0005	0.0024	0.0169	0.0058	0.0308	0.0016	0.0004	0.002
	M2	0.0341	0.0160	0.0476	0.0170	-2.289	0.0478	0.0011	0.0004	0.0029	0.0140	0.0089	0.0480	0.0011	0.0003	0.0025
	P	0.0155	0.0090	0.0434	0.0107	0.0071	-2.4823	0.0006	0.0003	0.0102	0.0110	0.0071	0.0582	0.0007	0.0003	0.0085
R3	**M1**	0.0062	0.0014	0.0022	0.0031	0.0009	0.0035	-3.1775	0.0238	0.0432	0.0015	0.0003	0.0007	0.0828	0.0320	0.0461
	M2	0.0030	0.0007	0.0011	0.0017	0.0007	0.0038	0.0483	-3.2206	0.0474	0.0007	0.0002	0.0003	0.0830	0.0334	0.0508
	P	0.0013	0.0003	0.0008	0.0016	0.0010	0.0229	0.0167	0.0090	-3.0021	0.0003	0.0001	0.0002	0.0270	0.0113	0.0598
R4	**M1**	0.0462	0.0154	0.0477	0.0226	0.0093	0.0492	0.0011	0.0003	0.0006	-2.1753	0.008	0.0511	0.0009	0.0002	0.0004
	M2	0.0265	0.0192	0.0616	0.0135	0.0104	0.0555	0.0004	0.0001	0.0002	0.0141	-2.2557	0.0647	0.0003	0.0001	0.0002
	P	0.0193	0.0145	0.0699	0.0131	0.0102	0.0827	0.0002	0.0000	0.0001	0.0162	0.0117	-2.2766	0.0001	0.0000	0.0001
R5	**M1**	0.0032	0.0008	0.0012	0.0018	0.0006	0.0026	0.0540	0.0267	0.0456	0.0007	0.0002	0.0003	-3.2598	0.0360	0.0489
	M2	0.0016	0.0004	0.0007	0.0011	0.0005	0.0029	0.0529	0.0272	0.0485	0.0004	0.0001	0.0002	0.0912	-3.2828	0.0523
	P	0.0009	0.0002	0.0007	0.0014	0.0009	0.0202	0.0188	0.0102	0.0630	0.0002	0.0000	0.0002	0.0305	0.0129	-3.0458

Note:

1. The matrix is for skim milk only. The complete matrix for skim and whole milk is available from the authors.
2. An element in the *i*th row and *j*th column represents the percentage change in *i*'s quantity as a result of a 1% change in *j*'s price.

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