Online Supplement

To accompany the following manuscript,

The Impact of Customer Community Participation on Customer Behaviors: An Empirical Investigation

CONTENTS

I The Participation Model

a) *Probit Model*

The "Probit Model for Participation" is specified by equations (1) thru (3) in the manuscript (*Section 4.1*). These equations are reproduced below[1],

$$P_h^* \sim \text{Normal}\left[\rho_h,\ 1\right] \tag{1}$$

$$\begin{aligned} P_h &= 1 \quad \text{if} \quad P_h^* > 0 \\ P_h &= 0 \quad \text{if} \quad P_h^* \le 0 \end{aligned} \tag{2}$$

$$\rho_h = \rho_0 + \rho_1 German_h + \rho_2 Age_h + \rho_3 Gender_h + \rho_4 Memlength_h + \rho_5 PosFB_h + \rho_6 NegFB_h + \rho_7 Invite_h \tag{3}$$

where P_h^* is a latent participation variable for customer *h*, and P_h is a dummy variable which equals 1 if the customer participated in the community in the time period T_2, else it equals 0. This leads to a "Probit" model of choice.

In these set of equations (1 thru 3) we need to set our priors over the parameters ρ_0 thru ρ_7. We specify a prior distribution of Normal(0,100) over all these parameters.

b) *Linear Probability Model*

The Linear Probability Model is specified by equations (A3.1) and (A3.2) in *Appendix 3* of the manuscript. These are reproduced below,

$$P_h \sim \text{Normal}\left[Propen_h,\ \xi^2\right] \tag{A3.1}$$

[1] We maintain the same numbering for the equations as used in the manuscript.

$$Propen_h = \eta_0 + \eta_1 German_h + \eta_2 Age_h + \eta_3 Gender_h + \eta_4 Memlength_h + \eta_5 PosFB_h + \eta_6 NegFB_h + \eta_7 Invite_h \quad \text{(A3.2)}$$

P_h is a dummy variable which equals 1 if the customer participated in the community during time period T_2, else it equals 0. Further, we interpret $Propen_h$ as the propensity to participate in a community for customer *h*.

II The Outcome Model

The "Outcome Model" is a multivariate tobit type I model and is specified by equations 4a thru 6 in the manuscript (*Section 4.2*). These equations are reproduced below,

Define a latent outcome vector $\mathbf{Outcome^*_{ht}} = \begin{vmatrix} Bids^*_{ht} \\ Listings^*_{ht} \\ Amnt^*_{ht} \\ Revenue^*_{ht} \end{vmatrix}$ such that,

$$\begin{aligned} Bids_{ht} &= 0 \quad if\ Bids^*_{ht} \le 0 \\ Bids_{ht} &= Bids^*_{ht} \quad if\ Bids^*_{ht} > 0 \end{aligned} \tag{4a}$$

$$\begin{aligned} Listings_{ht} &= 0 \quad if\ Listings^*_{ht} \le 0 \\ Listings_{ht} &= Listings^*_{ht} \quad if\ Listings^*_{ht} > 0 \end{aligned} \tag{4b}$$

$$\begin{aligned} Amnt_{ht} &= 0 \quad if\ Amnt^*_{ht} \le 0 \\ Amnt_{ht} &= Amnt^*_{ht} \quad if\ Amnt^*_{ht} > 0 \end{aligned} \tag{4c}$$

$$\begin{aligned} Revenue_{ht} &= 0 \quad if\ Revenue^*_{ht} \le 0 \\ Revenue_{ht} &= Revenue^*_{ht} \quad if\ Revenue^*_{ht} > 0 \end{aligned} \tag{4d}$$

Further,

$$\mathbf{Outcome^*_{ht}} \sim MVNormal\left(\boldsymbol{\beta}_{\mathbf{ht}}, \boldsymbol{\Sigma}\right) \tag{5}$$

where

$$\begin{aligned} \boldsymbol{\beta}_{\mathbf{ht}} = \ & \boldsymbol{\beta}_{\mathbf{0}} + \boldsymbol{\beta}_{\mathbf{1}} Propen_h + \boldsymbol{\beta}_{\mathbf{2}} T_{2t} + \boldsymbol{\beta}_{\mathbf{3}} T_{3t} + \boldsymbol{\beta}_{\mathbf{4}} T_{4t} + \boldsymbol{\beta}_{\mathbf{5}} German_h + \boldsymbol{\beta}_6 Age_h + \\ & \boldsymbol{\beta}_7 Gender_h + \boldsymbol{\beta}_8 PosFB_h + \boldsymbol{\beta}_9 NegFB_h + \boldsymbol{\beta}_{10} Memlength_h \end{aligned} \tag{6a}$$

and $\boldsymbol{\Sigma}$ is a 4x4 variance-covariance matrix.

The priors used to estimate the various parameters in equations 4a thru 6a are provided in *Appendix 2* of the manuscript.

III The set of Full Conditional distributions

The two models (the "Participation" and the "Outcome" model) are estimated together by setting up an MCMC sampler involving a data augmentation process. In each iteration, the sampler samples thru the following set of full conditional distributions[2].

The Participation Model:

a) *Probit Model of Participation*

1a.1 Draw ρ_0

$\Pi[\rho_0 / ....]$ $\propto$ **Probit likelihood x Normal prior**

$$\propto \left[\prod_{h=1}^{N}\left\{\Phi\left(P_h^* = 0\right)\right\}^{1-P_h}\left\{1-\Phi\left(P_h^* = 0\right)\right\}^{P_h}\right]\frac{1}{\sqrt{2\pi * 100}}\, e^{-\frac{1}{2*100}(\rho_0 - 0)^2}$$

$$\log\ \Pi[\rho_0 / ....] \quad \propto \quad \sum_{h=1}^{N}(1-P_h)\log\left\{\Phi\left(P_h^* = 0\right)\right\} + \sum_{h=1}^{N} P_h \log\left\{1-\Phi\left(P_h^* = 0\right)\right\} - \frac{1}{2*100}(\rho_0 - 0)^2$$

where,

$P_h = 1$ if $P_h^* > 0$ else it is $= 0$

$P_h^* \sim \text{Normal}[\rho_h,\ 1]$

$$\rho_h = \rho_0 + \rho_1 German_h + \rho_2 Age_h + \rho_3 Gender_h + \rho_4 Memlength_h + \rho_5 PosFB_h + \rho_6 NegFBr_h + \rho_7 Invite_h$$

1a.2 Draw ρ_1

$\Pi[\rho_1 / ....]$ $\propto$ **Probit likelihood x Normal prior**

[2] Many of the conditional distributions in the sampler are from non-conjugate distributions. We employ a random walk Metropolis Hastings algorithm to obtain draws from these distributions.

$$\propto \left[\prod_{h=1}^{N} \left\{ \Phi\left(P_h^* = 0\right) \right\}^{1-P_h} \left\{ 1 - \Phi\left(P_h^* = 0\right) \right\}^{P_h} \right] \frac{1}{\sqrt{2\pi * 100}} e^{-\frac{1}{2*100}(\rho_1 - 0)^2}$$

$$\log \Pi[\rho_1 / ....] \quad \propto \quad \sum_{h=1}^{N} (1 - P_h) \log\left\{ \Phi\left(P_h^* = 0\right) \right\} + \sum_{h=1}^{N} P_h \log\left\{ 1 - \Phi\left(P_h^* = 0\right) \right\} - \frac{1}{2*100} (\rho_1 - 0)^2$$

where,

$P_h = 1$ if $P_h^* > 0$ else it is $= 0$

$P_h^* \sim \text{Normal}[\rho_h, \ 1]$

$$\rho_h = \rho_0 + \rho_1 German_h + \rho_2 Age_h + \rho_3 Gender_h + \rho_4 Memlength_h + \rho_5 PosFB_h + \rho_6 NegFBr_h + \rho_7 Invite_h$$

1a.3
1a.4
1a.5
1a.6
1a.7
1a.8

Draw $\rho_2, \ \rho_3, \ \rho_4, \ \rho_5, \ \rho_6,$ and ρ_7 respectively, similarly as per step 1a.2

b) Linear Probability Model

1b.1 Draw η_0

$\Pi[\eta_0 / ....]$ $\propto$ **Normal likelihood x Normal prior**

$$\propto \left[\prod_{h=1}^{N} \frac{1}{\sqrt{2\pi\xi^2}} e^{-\frac{1}{2\xi^2}(P_h - Propen_h)^2} \right] \frac{1}{\sqrt{2\pi * 100}} e^{-\frac{1}{2*100}(\eta_0 - 0)^2}$$

$\propto$ Normal (Mean, Variance)

where,

$$\text{Mean} = \frac{\left\{ \frac{\sum_{h=1}^{N} (P_h - zz_h)}{\xi^2} + \frac{0}{100} \right\}}{\left\{ \frac{N}{\xi^2} + \frac{1}{100} \right\}} \quad \text{and} \quad \text{Variance} = \frac{N}{\xi^2} + \frac{1}{100}$$

In the above expressions $zz_h = Propen_h - \eta_0$, P_h is a dummy variable for participation and

$$Propen_h = \eta_0 + \eta_1 German_h + \eta_2 Age_h + \eta_3 Gender_h + \eta_4 Memlength_h + \eta_5 PosFB_h + \eta_6 NegFB_h + \eta_7 Invite_h$$

1b.2 Draw η_1

$\Pi[\eta_1 / ....]$ ∞ **Normal likelihood x Normal prior**

$$\infty \left[\prod_{h=1}^{N} \frac{1}{\sqrt{2\pi\xi^2}} e^{-\frac{1}{2\xi^2}(P_h - Propen_h)^2} \right] \frac{1}{\sqrt{2\pi * 100}} e^{-\frac{1}{2*100}(\eta_1 - 0)^2}$$

∞ Normal (Mean, Variance)

where,

$$\text{Mean} = \frac{\left\{ \frac{\sum_{h=1}^{N} (P_h - zz_h) German_h}{\xi^2} + \frac{0}{100} \right\}}{\left\{ \frac{\sum_{h=1}^{N} (German_h)^2}{\xi^2} + \frac{1}{100} \right\}} \quad \text{and} \quad \text{Variance} = \left\{ \frac{\sum_{h=1}^{N} (German_h)^2}{\xi^2} + \frac{1}{100} \right\}$$

In the above expressions $zz_h = Propen_h - \eta_1 German_h$, P_h is a dummy variable for participation and

$$Propen_h = \eta_0 + \eta_1 German_h + \eta_2 Age_h + \eta_3 Gender_h + \eta_4 Memlength_h + \eta_5 PosFB_h + \eta_6 NegFB_h + \eta_7 Invite_h$$

1b.3
1b.4
1b.5
1b.6
1b.7
1b.8

Draw $\eta_2,\ \eta_3,\ \eta_4,\ \eta_5,\ \eta_6,$ and η_7 respectively, similarly as per step 1b.2

1.9 Calculate $Propen_h$

$$Propen_h = \eta_0 + \eta_1 German_h + \eta_2 Age_h + \eta_3 Gender_h + \eta_4 Memlength_h + \eta_5 PosFB_h + \eta_6 NegFB_h + \eta_7 Invite_h$$

1.10 Data augmentation step

(--------Data augmentation details are provided at the end of this document--------)

The Outcome Model:

1.11 Draw $\boldsymbol{\beta_0}$

$\Pi[\boldsymbol{\beta_0} / ....]$ $\propto$ **MVNormal likelihood x MVNormal prior**

$$\propto \left[\prod_{h=1}^{N}\prod_{t=1}^{4} \exp\left\{-\tfrac{1}{2}\left(\mathbf{Outcome^*_{ht}} - \boldsymbol{\beta_{ht}}\right)^T (\Sigma)^{-1}\left(\mathbf{Outcome^*_{ht}} - \boldsymbol{\beta_{ht}}\right)\right\}\right] \exp\left\{-\tfrac{1}{2}(\boldsymbol{\beta_0} - \mathbf{0})^T [\mathbf{100I}]^{-1}(\boldsymbol{\beta_0} - \mathbf{0})\right\}$$

$\propto$ Normal (**Mean**, **Variance**)

where,

$$\textbf{Mean} = \frac{\mathbf{0}*[\mathbf{100I}]^{-1}+\left\{\sum_{h=1}^{N}\sum_{t=1}^{4}\left(\mathbf{Outcome}^{*}_{\mathbf{ht}}-\mathbf{ZZ}_{\mathbf{ht}}\right)\right\}\sum^{-1}}{[\mathbf{100I}]^{-1}+Nall\sum^{-1}} \quad \text{and} \quad [\textbf{Variance}]^{-1} = [\mathbf{100I}]^{-1} + Nall\sum^{-1}$$

In the above expressions $\mathbf{ZZ}_{\mathbf{ht}} = \boldsymbol{\beta}_{\mathbf{ht}} - \boldsymbol{\beta}_{\mathbf{0}}$, $Nall = \sum_{h=1}^{N}\sum_{t=1}^{4} 1.0$ and ,

$$\boldsymbol{\beta}_{\mathbf{ht}} = \boldsymbol{\beta}_{\mathbf{0}} + \boldsymbol{\beta}_{\mathbf{1}} Propen_h + \boldsymbol{\beta}_{\mathbf{2}} T_2 + \boldsymbol{\beta}_{\mathbf{3}} T_3 + \boldsymbol{\beta}_{\mathbf{4}} T_4 + \boldsymbol{\beta}_{\mathbf{5}} German_h + \boldsymbol{\beta}_{\mathbf{6}} Age_h + \boldsymbol{\beta}_{\mathbf{7}} Gender_h$$
$$+ \boldsymbol{\beta}_{\mathbf{8}} PosFB_h + \boldsymbol{\beta}_{\mathbf{9}} NegFB_h + \boldsymbol{\beta}_{\mathbf{10}} Memlength_h$$

1.12 Data augmentation step

(--------Data augmentation details are provided at the end of this document--------)

1.13 Draw $\boldsymbol{\beta}_1$

$\Pi[\boldsymbol{\beta}_1 / \ldots\ldots\ldots..] \propto$ **MVNormal likelihood x MVNormal prior**

$$\propto \left[\prod_{h=1}^{N}\prod_{t=1}^{4} \exp\left\{-\tfrac{1}{2}\left(\mathbf{Outcome}^{*}_{\mathbf{ht}} - \boldsymbol{\beta}_{\mathbf{ht}}\right)^{T}\left(\Sigma\right)^{-1}\left(\mathbf{Outcome}^{*}_{\mathbf{ht}} - \boldsymbol{\beta}_{\mathbf{ht}}\right)\right\}\right] \exp\left\{-\tfrac{1}{2}\left(\boldsymbol{\beta}_{\mathbf{1}} - \mathbf{0}\right)^{T}[\mathbf{100I}]^{-1}\left(\boldsymbol{\beta}_{\mathbf{1}} - \mathbf{0}\right)\right\}$$

$\propto$ Normal (**Mean**, **Variance**)

where,

$$\textbf{Mean} = \frac{\mathbf{0} * [\mathbf{100I}]^{-1} + \left\{ \sum_{h=1}^{N} \sum_{t=1}^{4} \left(\mathbf{Outcome}^{*}_{\mathbf{ht}} - \mathbf{ZZ}_{\mathbf{ht}} \right) Propen_h \right\} \sum^{-1}}{[\mathbf{100I}]^{-1} + \left\{ \sum_{h=1}^{N} \sum_{t=1}^{4} (Propen_h)^2 \right\} \sum^{-1}} \quad \text{and}$$

$$[\mathbf{Variance}]^{-1} = [\mathbf{100I}]^{-1} + \left\{ \sum_{h=1}^{N} \sum_{t=1}^{4} (Propen_h)^2 \right\} \sum^{-1}$$

In the above expressions $\mathbf{ZZ}_{\mathbf{ht}} = \beta_{\mathbf{ht}} - \beta_{\mathbf{1}} Propen_h$ and ,

$$\begin{aligned} \beta_{\mathbf{ht}} = {} & \beta_{\mathbf{0}} + \beta_{\mathbf{1}} Propen_h + \beta_{\mathbf{2}} T_2 + \beta_{\mathbf{3}} T_3 + \beta_{\mathbf{4}} T_4 + \beta_{\mathbf{5}} German_h + \beta_{\mathbf{6}} Age_h + \beta_{\mathbf{7}} Gender_h \\ & + \beta_{\mathbf{8}} PosFB_h + \beta_{\mathbf{9}} NegFB_h + \beta_{\mathbf{10}} Memlength_h \end{aligned}$$

1.14 Data augmentation step

(--------Data augmentation details are provided at the end of this document--------)

1.15
1.16
⋮
1.29
1.30
1.31

Draw β_2 thru β_{10} similarly as per step 1.13. Every drawing is followed by a data augmentation step as per step 1.14

1.32 Draw Σ

$\Pi[\sum / \ldots\ldots\ldots\ldots] \propto$ **MVNormal likelihood x InvWishart prior**

$$[\Sigma]^{-1} \propto \text{Wishart}\left(\left[\left\{\sum_{h=1}^{N}\sum_{t=1}^{4}\left(\mathbf{Outcome}^{*}_{\mathbf{ht}} - \boldsymbol{\beta}_{\mathbf{ht}}\right)\left(\mathbf{Outcome}^{*}_{\mathbf{ht}} - \boldsymbol{\beta}_{\mathbf{ht}}\right)^{T}\right\} + \mathbf{10I}\right]^{-1},\ N+10\right)$$

1.33 Data augmentation step

(--------Data augmentation details are provided at the end of this document--------)

→ Go back to Step 1a.1 and keep repeating the MCMC chain till convergence

IV Description of the Data Augmentation Step

The Outcome model is as specified in equations 4a thru 6a in the manuscript.

We observe $\mathbf{Outcome_{ht}} = \begin{vmatrix} Bids_{ht} \\ Listings_{ht} \\ Amnt_{ht} \\ Revenue_{ht} \end{vmatrix}$.

$\mathbf{Outcome_{ht}}$ is a censored version of the latent vector $\mathbf{Outcome^*_{ht}} = \begin{vmatrix} Bids^*_{ht} \\ Listings^*_{ht} \\ Amnt^*_{ht} \\ Revenue^*_{ht} \end{vmatrix}$ as described in the manuscript.

The data augmentation aims to augment the censored data so as to recreate the latent $\mathbf{Outcome^*_{ht}}$ vector. Subsequently the estimation of the parameters becomes a simple process.

The augmentation step is performed after each updation of a parameter in the MCMC chain. The augmentation step uses the fact that the conditional distribution of any dimension in a multivariate normal is also distributed normal.

The following steps constitute the data augmentation procedure,

Step 0: Initialize $\mathbf{Outcome^*_{ht}}$

(Performed only once at the very beginning of the estimation process)

Set $\mathbf{Outcome^*_{ht}} = \mathbf{Outcome_{ht}}$ for all *h*, *t*

That is, set $Bids^*_{ht} = Bids_{ht}$, $Listings^*_{ht} = Listings_{ht}$, $Amnt^*_{ht} = Amnt_{ht}$, and $Revenue^*_{ht} = Revenue_{ht}$

The following four Steps (which update the $\mathbf{Outcome}^{*}_{\mathbf{ht}}$ vector) are the augmentation steps that are performed regularly during the MCMC chain for all h, t

Step 1: Update $Bids^{*}_{ht}$

if $Bids^{*}_{ht} > 0$ then let $Bids^{*}_{ht} = Bids^{*}_{ht}$

else if $Bids^{*}_{ht} \leq 0$ draw a value between $-\infty$ and 0 from the following univariate normal distribution and set $Bids^{*}_{ht}$ = *the drawn value*

$$f(Bids^{*}_{ht} / ..) \quad = \quad \text{Normal}\left(mean, \quad variance\right)$$

where

$$mean \;=\; \beta^{(1)}_{ht} + \begin{bmatrix}\Sigma_{12} & \Sigma_{13} & \Sigma_{14}\end{bmatrix}\begin{bmatrix}\Sigma_{22} & \Sigma_{23} & \Sigma_{24}\\ \Sigma_{32} & \Sigma_{33} & \Sigma_{34}\\ \Sigma_{42} & \Sigma_{43} & \Sigma_{44}\end{bmatrix}^{-1}\begin{bmatrix}Listings^{*}_{ht} - \beta^{(2)}_{ht}\\ Amnt^{*}_{ht} - \beta^{(3)}_{ht}\\ Revenue^{*}_{ht} - \beta^{(4)}_{ht}\end{bmatrix}$$

$$variance \;=\; \Sigma_{11} - \begin{bmatrix}\Sigma_{12} & \Sigma_{13} & \Sigma_{14}\end{bmatrix}\begin{bmatrix}\Sigma_{22} & \Sigma_{23} & \Sigma_{24}\\ \Sigma_{32} & \Sigma_{33} & \Sigma_{34}\\ \Sigma_{42} & \Sigma_{43} & \Sigma_{44}\end{bmatrix}^{-1}\begin{bmatrix}\Sigma_{21}\\ \Sigma_{31}\\ \Sigma_{41}\end{bmatrix}$$

Please note that $\beta^{(1)}_{ht}, \beta^{(2)}_{ht}, \beta^{(3)}_{ht}, \beta^{(4)}_{ht}$ are the four elements of the $\boldsymbol{\beta}_{\mathbf{ht}}$ vector.

Similarly $\Sigma_{11}, \Sigma_{11}, \ldots, \Sigma_{43}, \Sigma_{44}$ are the individual elements of the 4x4 variance covariance matrix Σ

Step 2: Update $Listings^{*}_{ht}$

if $Listings^{*}_{ht} > 0$ then let $Listings^{*}_{ht} = Listings^{*}_{ht}$

else if $Listings^{*}_{ht} \leq 0$ draw a value between $-\infty$ and 0 from the following univariate normal distribution and set $Listings^{*}_{ht}$ = *the drawn value*

$$f(Listings^{*}_{ht} / ..) \quad = \quad \text{Normal}\left(mean, \quad variance\right)$$

where

$$mean \;=\; \beta_{ht}^{(2)} + \begin{bmatrix}\Sigma_{21} & \Sigma_{23} & \Sigma_{24}\end{bmatrix}\begin{bmatrix}\Sigma_{11} & \Sigma_{13} & \Sigma_{14}\\ \Sigma_{31} & \Sigma_{33} & \Sigma_{34}\\ \Sigma_{41} & \Sigma_{43} & \Sigma_{44}\end{bmatrix}^{-1}\begin{bmatrix}Bids_{ht}^{*} - \beta_{ht}^{(1)}\\ Amnt_{ht}^{*} - \beta_{ht}^{(3)}\\ Revenue_{ht}^{*} - \beta_{ht}^{(4)}\end{bmatrix}$$

$$variance \;=\; \Sigma_{22} - \begin{bmatrix}\Sigma_{21} & \Sigma_{23} & \Sigma_{24}\end{bmatrix}\begin{bmatrix}\Sigma_{11} & \Sigma_{13} & \Sigma_{14}\\ \Sigma_{31} & \Sigma_{33} & \Sigma_{34}\\ \Sigma_{41} & \Sigma_{43} & \Sigma_{44}\end{bmatrix}^{-1}\begin{bmatrix}\Sigma_{12}\\ \Sigma_{32}\\ \Sigma_{42}\end{bmatrix}$$

Step 3: Update $Amnt_{ht}^{*}$

if $Amnt_{ht}^{*} > 0$ then let $Amnt_{ht}^{*} = Amnt_{ht}^{*}$
else if $Amnt_{ht}^{*} \leq 0$ draw a value between $-\infty$ and 0 from the following univariate normal distribution and set $Amnt_{ht}^{*} = \textit{the drawn value}$

$$f(Amnt_{ht}^{*}/..) \;=\; \text{Normal}\left(mean,\;\; variance\right)$$

where

$$mean \;=\; \beta_{ht}^{(3)} + \begin{bmatrix}\Sigma_{31} & \Sigma_{32} & \Sigma_{34}\end{bmatrix}\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & \Sigma_{14}\\ \Sigma_{21} & \Sigma_{22} & \Sigma_{24}\\ \Sigma_{41} & \Sigma_{42} & \Sigma_{44}\end{bmatrix}^{-1}\begin{bmatrix}Bids_{ht}^{*} - \beta_{ht}^{(1)}\\ Listings_{ht}^{*} - \beta_{ht}^{(2)}\\ Revenue_{ht}^{*} - \beta_{ht}^{(4)}\end{bmatrix}$$

$$variance \;=\; \Sigma_{33} - \begin{bmatrix}\Sigma_{31} & \Sigma_{32} & \Sigma_{34}\end{bmatrix}\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & \Sigma_{14}\\ \Sigma_{21} & \Sigma_{22} & \Sigma_{24}\\ \Sigma_{41} & \Sigma_{42} & \Sigma_{44}\end{bmatrix}^{-1}\begin{bmatrix}\Sigma_{13}\\ \Sigma_{23}\\ \Sigma_{43}\end{bmatrix}$$

Step 4: Update $Revenue_{ht}^{*}$

if $Revenue_{ht}^{*} > 0$ then let $Revenue_{ht}^{*} = Revenue_{ht}^{*}$
else if $Revenue_{ht}^{*} \leq 0$ draw a value between $-\infty$ and 0 from the following univariate normal distribution and set $Revenue_{ht}^{*} = \textit{the drawn value}$

$$f(Revenue^*_{ht} / ..) \;=\; \text{Normal}\left(mean,\;\; \text{var}iance\right)$$

where

$$mean \;=\; \beta^{(4)}_{ht} + \begin{bmatrix} \Sigma_{41} & \Sigma_{42} & \Sigma_{43} \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}^{-1} \begin{bmatrix} Bids^*_{ht} - \beta^{(1)}_{ht} \\ Listings^*_{ht} - \beta^{(2)}_{ht} \\ Amnt^*_{ht} - \beta^{(3)}_{ht} \end{bmatrix}$$

$$variance \;=\; \Sigma_{44} - \begin{bmatrix} \Sigma_{41} & \Sigma_{42} & \Sigma_{43} \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}^{-1} \begin{bmatrix} \Sigma_{14} \\ \Sigma_{24} \\ \Sigma_{34} \end{bmatrix}$$

Once again please note that $\beta^{(1)}_{ht}, \beta^{(2)}_{ht}, \beta^{(3)}_{ht}, \beta^{(4)}_{ht}$ are the four elements of the $\boldsymbol{\beta}_{\mathbf{ht}}$ vector. Similarly $\Sigma_{11}, \Sigma_{11}, \ldots, \Sigma_{43}, \Sigma_{44}$ are the individual elements of the 4x4 variance covariance matrix Σ

***