Internet Channel Entry:

A Strategic Analysis of Mixed Channel Structures

Technical Supplement A:
Application of Balasubramanian's model (1998) to our game setting

This appendix provides the results and proof mentioned in "Consumer Heterogeneity and Demand Derivation" section in 2.2. Demand section. We applied Balasubramanian's model setting (the constant disutility of Internet shopping: μ) to our channel structures and rules of the game.

Numerical Example When μ = (1/3)V

	VI	D	VIM	PIM	HIM	DM
W*	*0*	*0.5V*	*0*	*0.66667 V*	*0.66667 V*	*0.66667 V*
P_N*	*0*	*0*	*0.66667 V*	*0.66667 V*	*0.66667 V*	*0.66667 V*
P_S*	*0.5V*	*0.75V*	*0.83333 V*	*0.83333 V*	*0.83333 V*	*0.83333 V*
Ave. Retail Price	*0.5V*	*0.75V*	*0.69445 V*	*0.69445 V*	*0.69445 V*	*0.69445 V*
Q_N	*0*	*0*	$0.83333V^2$	$0.83333V^2$	$0.83333V^2$	$083333V^2$
Q_S	$0.5V^2$	$0.25V^2$	$0.16667V^2$	$0.16667V^2$	$0.16667V^2$	$0.16667V^2$
Tot. Q	$0.5V^2$	$0.25V^2$	V^2	V^2	V^2	V^2
N Profit	*0*	*0*	0	*0*	*0*	*0*
S Profit	*0*	$0.0625V^3$	0	$0.02778V^3$	$0.02778V^3$	$0.02778V^3$
Tot. Retail Profit	*0*	$0.0625V^3$	*0*	$0.02778V^3$	$0.02778V^3$	$0.02778V^3$
M Profit w/ N	*0*	*0*	$0.55556V^3$	$0.55556V^3$	$0.55556V^3$	$0.55556V^3$
M Profit w/ S	$0.25V^3$	$0.125V^3$	$0.13889V^3$	$0.11111V^3$	$0.11111V^3$	$0.11111V^3$
Tot. M Profit	$0.25V^3$	$0.125V^3$	$0.69445V^3$	$0.66667V^3$	$0.66667V^3$	$0.66667V^3$
Channel Profit	$0.25V^3$	$0.1875V^3$	$0.69445V^3$	$0.69445V^3$	$0.69445V^3$	$0.69445V^3$
Consumer Surplus	$0.125V^3$	$0.03125V^3$	$0.01389V^3$	$0.01389V^3$	$0.01389V^3$	$0.01389V^3$

Numerical Example When μ = (1/2)V

	VI	D	VIM	PIM	HIM	DM
W*	*0*	*0.5V*	*0*	*0.5V*	*0.5V*	*0.5V*
P_N*	*0*	*0*	*0.5V*	*0.5V*	*0.5V*	*0.5V*
P_S*	*0.5V*	*0.75V*	*0.75V*	*0.75V*	*0.75V*	*0.75V*
Ave. Retail Price	*0.5V*	*0.75V*	*0.5625 V*	*0.5625V*	*0.5625V*	*0.5625V*
Q_N	*0*	*0*	$0.75V^2$	$0.75V^2$	$0.75V^2$	$0.75V^2$
Q_S	$0.5V^2$	$0.25V^2$	$0.25V^2$	$0.25V^2$	$0.25V^2$	$0.25V^2$

Tot. Q	$0.5V^2$	$0.25V^2$	V^2	V^2	V^2	V^2
N Profit	0	0	0	0	0	0
S Profit	0	$0.0625V^3$	0	$0.0625V^3$	$0.0625V^3$	$0.0625V^3$
Tot. Retail Profit	0	$0.0625V^3$	0	$0.0625V^3$	$0.0625V^3$	$0.0625V^3$
M Profit w/ N	0	0	$0.375V^3$	$0.375V^3$	$0.375V^3$	$0.375V^3$
M Profit w/ S	$0.25V^3$	$0.125V^3$	$0.1875V^3$	$0.125V^3$	$0.125V^3$	$0.125V^3$
Tot. M Profit	$0.25V^3$	$0.125V^3$	$0.5625V^3$	$0.5V^3$	$0.5V^3$	$0.5V^3$
Channel Profit	$0.25V^3$	$0.1875V^3$	$0.5625V^3$	$0.5625V^3$	$0.5625V^3$	$0.5625V^3$
Consumer Surplus	$0.125V^3$	$0.03125V^3$	$0.03125V^3$	$0.03125V^3$	$0.03125V^3$	$0.03125V^3$

This appendix provides the proofs of the mathematical solutions for the equilibrium prices shown in the tables above. We do not show how to obtain other results such as consumer surplus, since they can be easily obtained from the equilibrium prices. All other details are available from us upon request.

Vertically Integrated Channel (VI)

The demand for the physical store, which is vertically integrated to the manufacture, is described as:

$$Q_S = V(V - P_S) = V^2 - VP_S.$$

Manufacturer maximizes its profit through the vertically integrated physical store.

$$\Pi_M = (V^2 - VP_S)P_S$$

We can obtain the FOC by taking the first derivative w.r.t. the retailer's retail price (Ps) from the manufacturer's profit function.

$$FOC_M^{P_S} = -2VP_S + V^2$$

By solving $FOC_M=0$, we obtain the equilibrium price, $P_S^* = \frac{1}{2}V$.

The Second Order Condition (SOC) is satisfied as follows:

$$SOC_M^{P_S P_S} = -2V<0.$$

The demand at the equilibrium price is $Q_S^* = \frac{1}{2}V^2$.

Decentralized Channel (D)

Since we assume that the manufacturer is the Stackelberg leader, we need to solve the independent physical retailer's profit maximization problem first to obtain the sub-game perfect equilibrium. The demand for the independent physical retailer is $Q_S = V^2 - VP_S$, and it maximizes its profit conditional on the manufacture's wholesale price.

$$\pi_S = (V^2 - VP_S)(P_S - W)$$

Then, we can get the following FOC by taking the first derivative w.r.t. the retailer's retail price (P_S).

$$FOC_S^{P_S} = V^2 - VP_S - (P_S - W)V$$

By solving *FOC=0*, we are able to obtain the retailer's equilibrium price conditional on the manufacturer's wholesale price (W), $P_S^* = \frac{1}{2}V + \frac{1}{2}W$. The SOC is satisfied as follows:

$$SOC_S^{P_S P_S} = -2V<0.$$

Solving backward, we then solve the manufacturer's profit maximization problem by incorporating the optimal retail price, $P_S^* = \frac{1}{2}V + \frac{1}{2}W$.

$$\Pi_M = W(V^2 - \frac{1}{2}V(V+W))$$

The manufacturer's FOC w.r.t. W is

$$FOC_M^W = V^2 - \frac{1}{2}V(V+W) - \frac{1}{2}VW.$$

By solving *FOC_M=0*, we obtain the manufacturer's equilibrium wholesale price.

$$W^* = \frac{1}{2}V$$

The SOC is satisfied as follows:

$$SOC_M^{WW} = -V<0.$$

With the equilibrium wholesale price, we can obtain the other solutions.

$$P_S^* = \frac{3}{4}V, Q_S^* = \frac{1}{4}V^2$$

Vertically Integrated Mixed Channel (VIM)

Since the disutility of shopping from an Internet store (δ_{Ni}) is fixed to μ across all consumers in Balasubramanian's model, consumer i's utility can be described as follows:

Consumer *i*'s utility of purchasing from the Internet store: $U_{Ni} = V - P_N - \mu$ (1)

Consumer *i*'s utility of purchasing from the physical store: $U_{Si} = V - P_S - \delta_{S\,i}$ (2)

When $P_N > V - \mu$, $Q_N = 0$ and $Q_S = V(V - P_S)$. This problem is identical to VI.
When $P_N \leq V - \mu$, we can obtain the location of the indifferent consumer between the two channels by equating (1) and (2): $P_N - P_S + \mu$.
The demand for the Internet store and that for the physical store are described as follows:

$$Q_N = V(V - P_N + P_S - \mu)$$
$$Q_S = V(P_N - P_S + \mu)$$
.

With both the Internet and the physical stores, the manufacturer maximizes the joint profits from both channels.

$$\Pi_M = P_N Q_N + P_S Q_S$$

Since $P_N \leq V - \mu$, the aggregate demand is inelastic and, thus, the manufacturer sets P_N^* to be $V - \mu$ as long as $P_N^* \leq P_S^*$ (i.e., the manufacturer has no incentive to switch a consumer from the physical store to the Internet store by offering $P_N < V - \mu$). This leads to:

$$\Pi_M = VP_S(V - \mu) + V(V - P_S)P_S$$

We can calculate the following FOC w.r.t. P_S.

$$FOC_M^{P_S} = V(V - \mu) - VP_S + V(V - P_S)$$

By equating the FOC to 0, we can get the equilibrium prices.

$$P_S{}^* = V - \frac{1}{2}\mu .$$

Since $P_S{}^* = V - \frac{1}{2}\mu > V - \mu$, it satisfies the condition, $P_N^* \leq P_S^*$.

It can be shown that the above pricing strategy ($P_N = V - \mu$) leads to a greater manufacturer profit than $P_N > V - \mu$.

The SOC is satisfied as follows:

$$SOC_M^{P_S P_S} = -2V<0.$$

Partially Integrated Mixed Channel (PIM)

The demand for the Internet store (Q_N) and that for the physical store (Q_S) are described as follows:

$$Q_N = V(V - P_N + P_S - \mu)$$
$$Q_S = V(P_N - P_S + \mu).$$

Since the independent physical store is the Stackelberg follower, we need to solve the independent retailer's profit maximization problem first.

$$\pi_S = V(P_N - P_S + \mu)(P_S - W)$$

Then the FOC of the physical retailer w.r.t. Ps is

$$FOC_S^{P_S} = -V(P_S - W) + V(P_N - P_S + \mu).$$

Solving $FOC_S=0$, we can calculate the retailer's equilibrium price.

$$P_S{}^* = \frac{1}{2}(W + P_N + \mu)$$

The SOC is satisfied as shown below:

$$SOC_S^{P_S P_S} = -2V<0.$$

Solving backward, we now solve the manufacturer's profit maximization problem incorporating the retailer's optimal pricing decision.

$$\Pi_M = P_N V(V - P_N + P_S - \mu) + WV(P_N - P_S + \mu)$$

When $P_N \le V - \mu$, the aggregate demand is inelastic and, therefore, the manufacturer sets P_N^* to be $V - \mu$.

After substituting $P_S^* = \frac{1}{2}(W + P_N + \mu)$ and $P_N^* = V - \mu$ into the manufacturer's profit function above, we can obtain the manufacturer's profit function as follows:

$$\Pi_M = \frac{1}{2}V(V-\mu)(W+V) + \frac{1}{2}WV(V-W).$$

The FOC w.r.t. the wholesale price (W) for the independent physical retailer is as follows:

$FOC_M^W = \frac{1}{2}V(V-\mu) + \frac{1}{2}V(V-W) - \frac{1}{2}VW = 0$, which leads to

$W* = V - \frac{\mu}{2}.$

The SOC is satisfied as shown below:

$SOC_M^{WW} = -V<0.$

However, the feasible range of prices must be $V - \mu \geq P_N \geq W$ to prevent an arbitrage problem. Furthermore, it can be shown that $FOC_M > 0$ when this constraint is satisfied. Therefore, the optimal wholesale price maximizing the manufacturer's profit is as follows:

$W^* = V - \mu .$

It can be easily shown that the above solution is more profitable for the manufacturer than $P_N > V - \mu$, which reduces PIM into D (i.e., no sales in the Internet store).

Horizontally Integrated Mixed Channel (HIM)

$$Q_N = V(V - P_N + P_S - \mu)$$
$$Q_S = V(P_N - P_S + \mu)$$

Under this channel structure, the independent retailer coordinates both outlets and maximizes the joint profit conditional on the manufacturer's wholesale price.

$$\pi_S = V(V - P_N + P_S - \mu)(P_N - W) + V(P_N - P_S + \mu)(P_S - W)$$

Since the demand is inelastic when $P_N \leq V - \mu$, the retailer sets P_N^* to be $V - \mu$.

After substituting $P_N^* = V - \mu$ into the retailer's profit function above, we can obtain the following profit function:

$$\pi_S = VP_S(V - \mu - W) + V(V - P_S)(P_S - W)$$

The FOC w.r.t. Ps is:

$$FOC_S^{P_S} = V(V-\mu-W)-V(P_S-W)+V(V-P_S).$$

We then solve the physical retail price that makes FOC 0.

$$P_S{}^* = V-\frac{1}{2}\mu$$

The SOC is satisfied as shown below:

$$SOC_S^{P_S P_S} = -2V<0.$$

Solving backward, we solve the manufacturer's profit maximization problem incorporating the retailer's optimal pricing decision.

$$\Pi_M = W(V(V-P_N+P_S-\mu)+V(P_N-P_S+\mu))$$

After we substitute $P_N^* = V-\mu$ and $P_S{}^* = V-\frac{1}{2}\mu$ to the manufacturer's profit function, we get

$$\Pi_M = W(V(V-\frac{1}{2}\mu)+\frac{1}{2}V\mu) = WV^2.$$

Therefore, the manufacturer sets the optimal wholesale price as high as it is feasible, which leads to:

$$W^* = V-\mu.$$

Decentralized Mixed Channel (DM)

$$Q_N = V(V-P_N+P_S-\mu)$$
$$Q_S = V(P_N-P_S+\mu)$$

Since there are two independent retailers in this structure, the first step is to solve a Nash game between the retailers who set retail prices conditional on wholesale prices. The profit functions of each retailer are

$$\pi_S = V(P_N-P_S+\mu)(P_S-W), \text{ and}$$
$$\pi_N = V(V-P_N+P_S-\mu)(P_N-W).$$

If $0 \le W < \frac{1}{3}V-\frac{2}{3}\mu$, the FOCs of each retailer are

$$FOC_S^{P_S} = -V(P_S-W)+V(P_N-P_S+\mu), \text{ and}$$

$$FOC_N^{P_N} = -V(P_N - W) + V(V - P_N + P_S - \mu).$$

We can solve the two retail prices that simultaneously make both FOC's 0.

$$P_S^* = W + \frac{1}{3}(V + \mu), \text{ and } \text{----------------------------------} \quad (3)$$

$$P_N^* = W + \frac{1}{3}(2V - \mu) \quad \text{--------------------------------------} \quad (4)$$

If $W \geq \frac{1}{3}V - \frac{2}{3}\mu$, the Internet retailer does not increase the price above $V - \mu$ as W increases. Therefore, $P_N^* = V - \mu$.

The FOC of the PS is $FOC_S = -V(P_S - W) + V(P_N - P_S + \mu)$

We can solve the retail price that make the FOC 0, after substituting P_N with $V - \mu$.

$$P_S^* = \frac{1}{2}(w + V) \quad \text{----------------------------------} \quad (5)$$

The SOC's are satisfied as shown below:

$$SOC_S^{P_S P_S} = -2V<0 \text{ and } SOC_N^{P_N P_N} = -2V<0.$$

Solving backward, we then solve the manufacturer's profit maximization problem incorporating the solutions of the retail level game:

$$\Pi_M = W(V(P_N - P_S + \mu) + V(V - P_N + P_S - \mu) = WV^2$$

Therefore, the manufacturer sets the optimal wholesale price as high as it is feasible, which leads to:

$W^* = V - \mu$. It satisfies the condition, $W \geq \frac{1}{3}V - \frac{2}{3}\mu$.

Technical Supplement B: Proof of Mathematical Solutions in Table 1 a

This appendix provides the proofs of the mathematical solutions for the equilibrium prices shown in Table 1 a. In this appendix, we do not show how to obtain other results such as consumer surplus, since they can be easily obtained from the equilibrium prices. All other details are available from the authors upon request.

Vertically Integrated Channel (VI)

The demand for the physical store, which is vertically integrated to the manufacture, is described as:

$$Q_S = V^2 - VP_S .$$

Manufacturer maximizes its profit through the vertically integrated physical store.

$$\Pi_M = (V^2 - VP_S)P_S$$

We can obtain the FOC by taking the first derivative w.r.t. the retailer's retail price (Ps) from the manufacturer's profit function.

$$FOC_M^{P_S} = -2VP_S + V^2$$

By solving the above $FOC_M^{P_S} =0$, we obtain the equilibrium price, $P_S^* = \frac{1}{2}V$. The demand at the equilibrium price is $Q_S^* = \frac{1}{2}V^2$.

The Second Order Condition (SOC) is satisfied as follows:

$$SOC_M^{P_S P_S} = -2V<0.$$

Decentralized Channel (D)

Since we assume that the manufacturer is the Stackelberg leader, we need to solve the independent physical retailer's profit maximization problem first to obtain the sub-game perfect equilibrium. The demand for the independent physical retailer is $Q_S = V^2 - VP_S$, and it maximizes its profit conditional on the manufacture's wholesale price.

$$\pi_S = (V^2 - VP_S)(P_S - W)$$

Then, we can get the following FOC by taking the first derivative w.r.t. the retailer's retail price (P_S).

$$FOC_S^{P_S} = V^2 - VP_S - (P_S - W)V$$

By solving $FOC_S^{P_S}$ *=0,* we are able to obtain the retailer's equilibrium price conditional on the manufacturer's wholesale price (W), $P_S^* = \frac{1}{2}V + \frac{1}{2}W$. Now, we check SOC, and it is satisfied as shown below:

$$SOC_S^{P_S P_S} = -2V<0.$$

Solving backward, we then solve the manufacturer's profit maximization problem by incorporating the optimal retail price, $P_S^* = \frac{1}{2}V + \frac{1}{2}W$.

$$\Pi_M = W(V^2 - \frac{1}{2}V(V+W))$$

The manufacturer's FOC w.r.t. W is

$$FOC_M^W = V^2 - \frac{1}{2}V(V+W) - \frac{1}{2}VW\ .$$

By solving FOC_M*=0,* we obtain the manufacturer's equilibrium wholesale price.

$$W^* = \frac{1}{2}V$$

The SOC is satisfied as shown below:

$$SOC_M^{WW} = -V<0.$$

With the equilibrium wholesale price, we can obtain the other solutions.

$$P_S^* = \frac{3}{4}V, Q_S^* = \frac{1}{4}V^2$$

Vertically Integrated Mixed Channel (VIM)

With both the Internet and the physical stores, the manufacturer maximizes the combined profits from both channels. As shown in Figure 3(b), we have to consider two cases: $P_S \geq P_N$ and $P_S < P_N$ to solve the problem. To save space, we only show the condition where the equilibrium exists.

When $P_S \geq P_N$, the demand for the Internet store and that for the physical store are described as follows:

$$Q_N = P_S(V-P_N)+\frac{1}{2}(V-P_S)(P_S-2P_N+V)$$
$$Q_S = P_N(V-P_S)+\frac{1}{2}(V-P_S)^2.$$

The manufacturer maximizes the joint profits from both outlets.

$$\Pi_M = P_N(P_S(V-P_N)+\frac{1}{2}(V-P_S)(P_S-2P_N+V))+P_S(P_N(V-P_S)+\frac{1}{2}(V-P_S)^2)$$

We can calculate the following FOC's w.r.t. each retail price.

$$FOC_M^{P_N} = -VP_N+P_S(V-P_N)+\frac{1}{2}(V-P_S)(P_S-2P_N+V))+P_S(V-P_S)$$
$$FOC_M^{P_S} = 2P_N(V-P_S)+(-P_N+P_S-V)P_S+\frac{1}{2}(V-P_S)^2$$

By equating the two FOC's to 0 and simultaneously solving them, we can get the equilibrium prices.

$$P_S^* = P_N^* = \frac{\sqrt{3}}{3}V$$

The SOC's are satisfied at the equilibrium prices:

$$SOC_M^{P_N P_N} = -2V < 0$$
$$SOC_M^{P_S P_S} = -2V < 0$$
$$SOC_M^{P_s P_N} = V(2-\sqrt{3})$$
$$SOC_M^{P_N P_N} \times SOC_M^{P_S P_S} - (SOC_M^{P_S P_N})^2 = V^2(4\sqrt{3}-3) > 0\ .$$

Partially Integrated Mixed Channel (PIM)

When $P_S \geq P_N$, the demand for the Internet store (Q_N) and that for the physical store (Q_S) are described as follows:

$$Q_N = P_S(V-P_N)+\frac{1}{2}(V-P_S)(P_S-2P_N+V)$$
$$Q_S = P_N(V-P_S)+\frac{1}{2}(V-P_S)^2.$$

Since the independent physical store is the Stackelberg follower, we need to solve the independent retailer's profit maximization problem first.

$$\pi_S = (P_S - W)(P_N(V - P_S) + \frac{1}{2}(V - P_S)^2)$$

Then the FOC of the physical retailer w.r.t. Ps is

$$FOC_S^{P_S} = P_N(V - P_S) + (-P_N + P_S - V)(P_S - W) + \frac{1}{2}(V - P_S)^2 .$$

Solving $FOC_S=0$, we can calculate the retailer's equilibrium price.

$$P_S^* = \frac{2}{3}P_N + \frac{1}{3}W + \frac{2}{3}V - \frac{1}{3}\sqrt{4P_N^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$

The SOC is satisfied at the equilibrium prices:

$$SOC_S^{P_S P_S} = -1.38496V<0.$$

Solving backward, we now solve the manufacturer's profit maximization problem incorporating the retailer's optimal pricing decision.

$$\Pi_M = P_N(P_S(V - P_N) + \frac{1}{2}(V - P_S)(P_S - 2P_N + V)) + W(P_N(V - P_S) + \frac{1}{2}(V - P_S)^2)$$

After substituting P_S^* into the manufacturer's profit function above, we can obtain the manufacturer's profit function as follows:

$$\begin{aligned}\Pi_M &= W(P_N(\frac{1}{3}V - \frac{2}{3}P_N - \frac{1}{3}W + \frac{1}{3}\sqrt{4P_N^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})\\
&+ \frac{1}{2}(\frac{1}{3}V - \frac{2}{3}P_N - \frac{1}{3}W + \frac{1}{3}\sqrt{4P_N^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})^2)\\
&+ P_N((\frac{2}{3}P_N + \frac{1}{3}W + \frac{2}{3}V - \frac{1}{3}\sqrt{4P_N^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})(V - P_N)\\
&+ \frac{1}{2}(\frac{1}{3}V - \frac{2}{3}P_N - \frac{1}{3}W + \frac{1}{3}\sqrt{4P_N^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})\\
&(-\frac{4}{3}P_N + \frac{1}{3}W + \frac{5}{3}V - \frac{1}{3}\sqrt{4P_N^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}))\end{aligned}$$

The FOC's w.r.t each of the wholesale price for the independent physical retailer and the retail price for the Internet channel (P_N) are as follows:

$$FOC_M^{P_N} = \frac{1}{9}(4WV\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$-12P_N{}^2\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$-16P_N V\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$-6WP_N\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$+8V^2\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2} + 5WP_N V + 3W^2 + 24P_N{}^3 + 6WP_N{}^2$$
$$-6P_N W^2 + 5WV^2 - 7VW^2 + P_N V^2 + 2P_N V^2 - V^3$$
$$-3W^2\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})/(\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})$$

$$FOC_M^{W} = -\frac{1}{9}(4WV\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$+3P_N{}^2\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$-4P_N V\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$+6WP_N\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2}$$
$$-V^2\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2} + 14WP_N V + 3W^2 - 6P_N{}^3 + 12WP_N{}^2$$
$$-9P_N W^2 + 5WV^2 - 7VW^2 - 5P_N V^2 - 7P_N{}^2 V - V^3$$
$$-3W^2\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})/(\sqrt{4Pn^2 - 2P_N W + 2P_N V + W^2 - 2VW + V^2})$$

By equating the two FOC's to 0 and simultaneously solving them, we can solve the equilibrium price for the Internet retail price and the wholesale price for the independent physical retailer.

$P_N^*=0.54759V$ and $W^*=0.53834V$

The SOC's are satisfied at the equilibrium prices:

$$SOC_M^{P_N P_N} = -1.99288V < 0$$
$$SOC_M^{WW} = -0.88982V < 0$$
$$SOC_M^{P_N W} = 0.08964V$$
$$SOC_M^{P_N P_N} \times SOC_M^{WW} - (SOC_M^{P_N W})^2 = 1.7653V > 0$$

Horizontally Integrated Mixed Channel (HIM)

When $P_S \geq P_N$,

$$Q_N = P_S(V - P_N) + \frac{1}{2}(V - P_S)(P_S - 2P_N + V)$$
$$Q_S = P_N(V - P_S) + \frac{1}{2}(V - P_S)^2$$

Under this channel structure, the independent retailer coordinates both outlets and maximizes the joint profit conditional on the manufacturer's wholesale price.

$$\pi_S = (P_N - W)(P_S(V - P_N) + \frac{1}{2}(V - P_S)(P_S - 2P_N + V)) + (P_S - W)(P_N(V - P_S) + \frac{1}{2}(V - P_S)^2)$$

The FOC's w.r.t. each of the retail prices are

$$FOC_S^{P_N} = P_S(V - P_N) + \frac{1}{2}(V - P_S)(P_S - 2P_N + V) - (P_N - W)V + (P_S - W)(V - P_S),$$

and

$$FOC_S^{P_S} = (P_N - W)(V - P_S) + P_N(V - P_S) + \frac{1}{2}(V - P_S)^2 + (P_S - W)(-P_N - V + P_S) .$$

We then solve the two retail prices that simultaneously make both FOC's 0, we find

$$P_N^* = P_S^* = \frac{1}{3}W + \frac{1}{3}\sqrt{W^2 + 3V^2} .$$

The SOC's are satisfied at the equilibrium prices:

$$SOC_S^{P_N P_N} = -2V < 0$$
$$SOC_S^{P_S P_S} = -2V < 0$$
$$SOC_S^{P_S P_N} = 0.18265V$$
$$SOC_S^{P_N P_N} \times SOC_S^{P_S P_S} - (SOC_S^{P_N P_S})^2 = 3.96664V > 0$$

Solving backward, we solve the manufacturer's profit maximization problem incorporating the retailer's optimal pricing decision.

$$\Pi_M = W((P_S(V - P_N) + \frac{1}{2}(V - P_S)(P_S - 2P_N + V)) + (P_N(V - P_S) + \frac{1}{2}(V - P_S)^2))$$

After we substitute $P_N^* = P_S^* = \frac{1}{3}W + \frac{1}{3}\sqrt{W^2 + 3V^2}$ to the manufacturer's profit function, we get

$$\Pi_M = W(V + \frac{1}{3}W + \frac{1}{3}\sqrt{W^2 + 3V^2})(V - \frac{1}{3}W - \frac{1}{3}\sqrt{W^2 + 3V^2}) .$$

Then, the FOC w.r.t. W is

$$FOC_M^W =$$

$$\left(V+\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{\mathrm{W}^2+3\,V^2}\right)\left(V-\frac{1}{3}\mathrm{W}-\frac{1}{3}\sqrt{\mathrm{W}^2+3\,V^2}\right)$$
$$+\mathrm{W}\left(\frac{1}{3}+\frac{\frac{1}{3}\mathrm{W}}{\sqrt{\mathrm{W}^2+3\,V^2}}\right)\left(V-\frac{1}{3}\mathrm{W}-\frac{1}{3}\sqrt{\mathrm{W}^2+3\,V^2}\right)$$
$$+\mathrm{W}\left(V+\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{\mathrm{W}^2+3\,V^2}\right)\left(-\frac{1}{3}-\frac{1}{3}\frac{\mathrm{W}}{\sqrt{\mathrm{W}^2+3\,V^2}}\right)$$

The equilibrium wholesale price that makes the FOC 0 is

$$W^* = \frac{\sqrt{2}}{2}\sqrt{-3V^2+\sqrt{13}V^2}\ .$$

The SOC is satisfied at the equilibrium prices.

$$SOC_M^{WW} = -1.72310V < 0$$

Decentralized Mixed Channel (DM)

When $P_S \geq P_N$,

$$Q_N = P_S(V-P_N)+\frac{1}{2}(V-P_S)(P_S-2P_N+V)$$
$$Q_S = P_N(V-P_S)+\frac{1}{2}(V-P_S)^2$$

Since there are two independent retailers in this structure, the first step is to solve a Nash game between the retailers who set retail prices conditional on wholesale prices. The profit functions of each retailer are

$$\pi_N = (P_N-W)(P_S(V-P_N)+\frac{1}{2}(V-P_S)(P_S-2P_N+V)),\text{ and}$$
$$\pi_S = (P_S-W)(P_N(V-P_S)+\frac{1}{2}(V-P_S)^2).$$

The FOCs of each retailer are

$$FOC_N^{P_N} = -V(P_N-W)+\frac{1}{2}(V-P_S)(P_S-2P_N+V)+(V-P_N)P_S,\text{ and}$$

$$FOC_S^{P_S} = P_N(V - P_S) + (-P_N + P_S - V)(P_S - W) + \frac{1}{2}(V - P_S)^2.$$

We can solve the two retail prices that simultaneously make both FOC's 0.

$$P_N^* = P_S^* = -V + \sqrt{2V^2 + 2WV}$$

The SOC's are satisfied at the equilibrium prices.

$$SOC_S^{P_S P_S} = -1.77726V < 0$$
$$SOC_N^{P_N P_N} = -2V < 0$$

Solving backward, we then solve the manufacturer's profit maximization problem incorporating the solutions of the retail level game.

$$\Pi_M = W((P_S(V - P_N) + \frac{1}{2}(V - P_S)(P_S - 2P_N + V)) + (P_N(V - P_S) + \frac{1}{2}(V - P_S)^2))$$

After we incorporate the retailers' pricing strategies, $P_N^* = P_S^* = -V + \sqrt{2V^2 + 2wV}$, into the manufacturer's profit function, we get

$$\Pi_M = W\sqrt{2V^2 + 2WV}(2V - \sqrt{2V^2 + 2VW}).$$

Then, the FOC w.r.t. W is

$$FOC_M^W = \sqrt{2\,V^2 + 2\,\mathrm{W}\,V}\,(2\,V - \sqrt{2\,V^2 + 2\,\mathrm{W}\,V}) + \frac{\mathrm{W}\,(2\,V - \sqrt{2\,V^2 + 2\,\mathrm{W}\,V})\,V}{\sqrt{2\,V^2 + 2\,\mathrm{W}\,V}} - \mathrm{W}\,V$$

The equilibrium wholesale price that makes the FOC 0 is

$$W^* = 0.52191V.$$

The SOC is satisfied at the equilibrium prices.

$$SOC_M^{WW} = -1.66439V < 0.$$

Technical Supplement C: Proof of Mathematical Solutions in Table 1 b

This appendix provides the proofs of the mathematical solutions for the equilibrium prices shown in Table 1 b. The demands are adjusted by multiplying ½ to make consistent comparisons of the market outcomes between the models described in Figure 2 (a) and (b). In this appendix, we do not show how to obtain other results, such as channel profits and consumer surplus, since they can be easily obtained from the equilibrium prices. All other details are available from the authors upon request.

Vertically Integrated Channel with Two Physical Stores (VI2: Local Monopoly)

The optimal prices for each of the two physical stores under local monopolies, which are vertically integrated to the manufacture, are the same as those of the physical store under VI structure in Table 1 a, when they have enough demands to cover.

$$P_{PS1}^{*} = P_{PS2}^{*} = \frac{1}{2}V.$$

$$Q_{PS1} = Q_{PS2} = \frac{1}{2}V^2.$$

Vertically Integrated Channel with Two Physical Stores (VI2: Competition)

When each of the two physical stores is located –a and a, respectively, and they are close enough to have competition, the demands for each store can be described as:

$$Q_{PS1} = (\frac{P_{PS2} - P_{PS1}}{2} - (-a - (V - P_{PS1})))\frac{V}{2} \text{ and}$$

$$Q_{PS2} = (a + V - P_{PS2} - \frac{(P_{PS2} - P_{PS1})}{2})\frac{V}{2}$$

The manufacturer's objective function is:

$$\Pi_M = P_{PS1}Q_{PS1} + P_{PS2}Q_{PS2}$$

Then, we can get the following FOCs by taking the first derivative w.r.t. each retail price.

$$FOC_M^{PS1} = -\frac{3}{4}VP_{PS1} + \frac{1}{2}(\frac{1}{2}P_{PS2} - \frac{3}{2}P_{PS1} + a + V)V + \frac{1}{4}VP_{PS2}$$

$$FOC_M^{PS2} = \frac{1}{4}VP_{PS1} - \frac{3}{4}VP_{PS2} + \frac{1}{2}(a + V - \frac{3}{2}P_{PS2} + \frac{1}{2}P_{PS1})V$$

By solving the two *FOCs=0* simultaneously, we are able to obtain the optimal retail prices conditional on their locations (a), $P_{PS1}{}^{*} = P_{PS2}{}^{*} = \frac{1}{2}a + \frac{1}{2}V$.

The SOC's are satisfied as follows:

$$SOC_M^{PS1PS1} = -\frac{3}{2}V < 0$$
$$SOC_M^{PS2PS2} = -\frac{3}{2}V < 0$$
$$SOC_M^{PS1PS2} = \frac{1}{2}V$$
$$SOC_M^{PS1PS1} \times SOC_M^{PS2PS2} - SOC_M^{PS1PS2} = 2V^2 > 0.$$

Decentralized Channel with Two Physical Stores (D2: Local Monopoly)

The wholesale price of the manufacturer and the retail prices for each of the two independent physical stores under local monopolies are the same as the wholesale price and retail price of D structure in Table 1 a, when the stores have enough demands to cover.

$$W^* = \frac{1}{2}V,\ P_{PS1}{}^{*} = P_{PS2}{}^{*} = \frac{3}{4}V.$$
$$Q_{PS1} = Q_{PS2} = \frac{1}{4}V^2.$$

Decentralized Channel with Two Physical Stores (D2: Competition)

When each of the two physical stores are located –a and a, respectively, and they are close enough to have competition, the demands for each store can be described as:

$$Q_{PS1} = (\frac{P_{PS2} - P_{PS1}}{2} - (-a - (V - P_{PS1})))\frac{V}{2} \text{ and}$$

$$Q_{PS2} = (a + V - P_{PS2} - \frac{(P_{PS2} - P_{PS1})}{2})\frac{V}{2}$$

The physical stores' objective functions are:

$$\pi_{PS1} = (P_{PS1} - W)Q_{PS1}$$
$$\pi_{PS2} = (P_{PS2} - W)Q_{PS2}$$

Then, we can get the following FOCs by taking the first derivative of each objective function w.r.t. the corresponding retail price.

$$FOC_{PS1}^{PS1} = -\frac{3}{4}V(P_{PS1} - W) + \frac{1}{2}(\frac{1}{2}P_{PS2} - \frac{3}{2}P_{PS1} + a + V)V$$

$$FOC_{PS2}^{PS2} = -\frac{3}{4}V(P_{PS2} - W) + \frac{1}{2}(\frac{1}{2}P_{PS1} - \frac{3}{2}P_{PS2} + a + V)V$$

By solving the two *FOCs=0* simultaneously, we are able to obtain the optimal retail prices conditional on their locations (a) and the manufacturer's wholesale price (*W*).

$$P_{PS1}{}^{*} = P_{PS2}{}^{*} = \frac{3}{5}W + \frac{2}{5}(a+V)\ .$$

The SOC's are satisfied:

$$SOC_{PS1}^{PS1PS1} = SOC_{PS2}^{PS2PS2} = -\frac{3}{2}V < 0\ .$$

Solving backward, we then solve the manufacturer's profit maximization problem incorporating the retailers' pricing strategies shown above.

$$\Pi_M = \frac{3}{5}VW(-W + a + V)$$

Then, the FOC w.r.t. W is

$$FOC_M^W = \frac{3}{5}V(-2W + a + V).$$

The equilibrium wholesale price that makes the FOC 0 is

$$W^* = \frac{1}{2}(a+V)\ .$$

The SOC is satisfied:

$$SOC_M^{WW} = -\frac{6}{5}V < 0\ .$$

Technical Supplement D: 5.1. Impact of Asymmetric Heterogeneity

Market Outcomes When 0<δ_N< V*

*We set V=1 without loss of generality

	W*	Pn*	Ps*	Qn*	Qs*	Total Q	Retail Profit for N	Retail Profit for PS	Total Retail Profit	M Profit Thru N	M Profit Thru PS	Total M Profit	Total Channel Profit	Consumer Surplus
VI (0<δs<V/2)	0	0	0.50000	0	1.00000	1.00000	0	0	0	0	0.50000	0.50000	0.50000	0.25000
VI (0<δs<V)	0	0	0.50000	0	0.50000	0.50000	0	0	0	0	0.25000	0.25000	0.25000	0.12500
VI (0<δs<2V)	0	0	0.50000	0	0.25000	0.25000	0	0	0	0	0.12500	0.12500	0.12500	0.06250
DC (0<δs<V/2)	0.50000	0	0.75000	0	0.50000	0.50000	0	0.12500	0.12500	0	0.25000	0.25000	0.37500	0.06250
DC (0<δs<V)	0.50000	0	0.75000	0	0.25000	0.25000	0	0.06250	0.06250	0	0.12500	0.12500	0.18750	0.03125
DC (0<δs<2V)	0.50000	0	0.75000	0	0.12500	0.12500	0	0.03125	0.03125	0	0.06250	0.06250	0.09375	0.01563
VIM (0<δs<V/2)	0	0.66667	0.58333	0.16667	0.72222	0.88889	0	0	0	0.11111	0.42130	0.53241	0.53241	0.15201
VIM (0<δs<V)	0	0.57735	0.57735	0.33333	0.33333	0.66666	0	0	0	0.19245	0.19245	0.38490	0.38490	0.14088
VIM (0<δs<2V)	0	0.53829	0.57182	0.41588	0.16108	0.57696	0	0	0	0.22386	0.09211	0.31597	0.31597	0.11769
PIM (0<δs<V/2)	0.54490	0.59587	0.75455	0.34388	0.35276	0.69664	0	0.07209	0.07209	0.20491	0.19222	0.39713	0.46922	0.09996
PIM (0<δs<V)	0.53834	0.54759	0.74952	0.42103	0.16855	0.58958	0	0.03559	0.03559	0.23055	0.09074	0.32129	0.35688	0.10843
PIM (0<δs<2V)	0.52570	0.52570	0.74155	0.45760	0.08463	0.54223	0	0.01827	0.01827	0.24056	0.04449	0.28505	0.30332	0.11492
HIM (0<δs<V/2)	0.56610	0.80955	0.79717	0.14946	0.36939	0.51885	0.03639	0.08535	0.12174	0.08461	0.20911	0.29372	0.41546	0.04719
HIM (0<δs<V)	0.55025	0.78922	0.78922	0.18858	0.18858	0.37716	0.04506	0.04506	0.09013	0.10377	0.10377	0.20753	0.29766	0.03975
HIM (0<δs<2V)	0.53271	0.77396	0.78093	0.21404	0.09677	0.31081	0.05164	0.02402	0.07566	0.11402	0.05155	0.16557	0.24123	0.03307
DM (0<δs<V/2)	0.53635	0.73759	0.75263	0.20122	0.42611	0.62733	0.04049	0.09216	0.13265	0.10792	0.22854	0.33647	0.46912	0.07042
DM (0<δs<V)	0.52191	0.74465	0.74465	0.22275	0.22275	0.44550	0.04962	0.04962	0.09923	0.11626	0.11626	0.23251	0.33174	0.05688
DM (0<δs<2V)	0.51387	0.74906	0.74120	0.23421	0.11366	0.34787	0.05508	0.02584	0.08092	0.12035	0.05841	0.17876	0.25968	0.04165

Market Outcomes When 0<δs< V*

*We set V=1 without loss of generality

	W*	Pn*	Ps*	Qn*	Qs*	Total Q	Retail Profit for N	Retail Profit for PS	Total Retail Profit	M Profit Thru N	M Profit Thru PS	Total M Profit	Total Channel Profit	Consumer Surplus
VI (0<δ_N<V/2)	0	0	0.50000	0	0.50000	0.50000	0	0	0	0	0.25000	0.25000	0.25000	0.12500
VI (0<δ_N<V)	0	0	0.50000	0	0.50000	0.50000	0	0	0	0	0.25000	0.25000	0.25000	0.12500
VI (0<δ_N<2V)	0	0	0.50000	0	0.50000	0.50000	0	0	0	0	0.25000	0.25000	0.25000	0.12500
D (0<δ_N<V/2)	0.50000	0	0.75000	0	0.25000	0.25000	0	0.06250	0.06250	0	0.12500	0.12500	0.18750	0.03125
D (0<δ_N<V)	0.50000	0	0.75000	0	0.25000	0.25000	0	0.06250	0.06250	0	0.12500	0.12500	0.18750	0.03125
D (0<δ_N<2V)	0.50000	0	0.75000	0	0.25000	0.25000	0	0.06250	0.06250	0	0.12500	0.12500	0.18750	0.03125
VIM (0<δ_N<V/2)	0	0.58333	0.66667	0.72222	0.16667	0.88889	0	0	0	0.42130	0.11111	0.53241	0.53241	0.15201
VIM (0<δ_N<V)	0	0.57735	0.57735	0.33333	0.33333	0.66666	0	0	0	0.19245	0.19245	0.38490	0.38490	0.14088
VIM (0<δ_N<2V)	0	0.57182	0.53829	0.16108	0.41588	0.57696	0	0	0	0.22386	0.09211	0.31597	0.31597	0.11769
PIM (0<δ_N<V/2)	0.56886	0.56886	0.73085	0.78984	0.10951	0.89935	0	0.01774	0.01774	0.44931	0.06230	0.51160	0.52934	0.16745
PIM (0<δ_N<V)	0.53834	0.54759	0.74952	0.42103	0.16855	0.58958	0	0.03559	0.03559	0.23055	0.09074	0.32129	0.35688	0.10843
PIM (0<δ_N<2V)	0.51420	0.54681	0.74829	0.21076	0.21051	0.42127	0	0.04928	0.04928	0.11525	0.10824	0.22349	0.27277	0.07025
HIM (0<δ_N<V/2)	0.56610	0.79717	0.80955	0.36939	0.14946	0.51885	0.08535	0.03639	0.12174	0.20911	0.08461	0.29372	0.41546	0.04719
HIM (0<δ_N<V)	0.55025	0.78922	0.78922	0.18858	0.18858	0.37716	0.04506	0.04506	0.09013	0.10377	0.10377	0.20753	0.29766	0.03975
HIM (0<δ_N<2V)	0.53271	0.78093	0.77396	0.09677	0.21404	0.31081	0.02402	0.05164	0.07566	0.05155	0.11402	0.16557	0.24123	0.03307
DM (0<δ_N<V/2)	0.53635	0.75263	0.73759	0.42611	0.20122	0.62733	0.09216	0.04049	0.13265	0.22854	0.10792	0.33647	0.46912	0.07042
DM (0<δ_N<V)	0.52191	0.74465	0.74465	0.22275	0.22275	0.44550	0.04962	0.04962	0.09923	0.11626	0.11626	0.23251	0.33174	0.05688
DM (0<δ_N<2V)	0.51387	0.74120	0.74906	0.11366	0.23421	0.34787	0.02584	0.05508	0.08092	0.05841	0.12035	0.17876	0.25968	0.04165

Technical Supplement E: 5.2. Absence of Channel Price Leadership

Market Outcomes for Various Channel Structures under Vertical Nash

	VI	D	VIM	PIM	HIM	DM
W*	*0*	*0.33333V*	*0*	*0.44536V*	*0.35355V*	*0.40303V*
P_N*	*0*	*0*	*0.57735V*	*0.54464V*	*0.7071V*	*0.6751 V*
P_S*	*0.5V*	*0.66666V*	*0.57735V*	*0.69536V*	*0.7071V*	*0.67513V*
Ave. Retail Price	*0.5V*	*0.66666V*	*0.57735V*	*0.59615V*	*0.7071V*	*0.67513V*
Q_N	*0*	*0*	$0.33333V^2$	$0.40896V^2$	$0.25V^2$	$0.2721V^2$
Q_S	$0.5V^2$	$0.33333V^2$	$0.33333V^2$	$0.21232V^2$	$0.25V^2$	$0.2721V^2$
Tot. *Q*	$0.5V^2$	$0.33333V^2$	$0.66666V^2$	$0.62128V^2$	$0.37716V^2$	$0.5442V^2$
N Profit	*0*	*0*	*0*	*0*	$0.08839V^3$	$0.07404V^3$
S Profit	*0*	$0.11111V^3$	*0*	$0.05308V^3$	$0.08839V^3$	$0.07404V^3$
Tot. Retail Profit	*0*	$0.11111V^3$	*0*	$0.05308V^3$	$0.17678V^3$	$0.14808V^3$
M Profit w/ N	*0*	*0*	$0.19245V^3$	$0.22274V^3$	$0.08839V^3$	$0.10966V^3$
M Profit w/ S	$0.25V^3$	$0.11111V^3$	$0.19245V^3$	$0.09456V^3$	$0.08839V^3$	$0.10966V^3$
Tot. M Profit	$0.25V^3$	$0.11111V^3$	$0.38490V^3$	$0.3173V^3$	$0.17678V^3$	$0.21932V^3$
Channel Profit	$0.25V^3$	$0.22222V^3$	$0.38490V^3$	$0.37038V^3$	$0.35356V^3$	$0.3647V^3$
Consumer Surplus	$0.125V^3$	$0.05555V^3$	$0.14088V^3$	$0.11361V^3$	$0.06577V^3$	$0.07836V^3$

This appendix provides the proofs of the mathematical solutions for the equilibrium prices. In this appendix, we do not show how to obtain other results such as consumer surplus, since they can be easily obtained from the equilibrium prices. All other details are available from the authors upon request.

Vertically Integrated Channel (VI)

Same as the results for VI in Table 1 a.

Decentralized Channel (D)

The manufacturer is not a Stackelberg leader anymore, we need to solve the profit maximization problems of both independent physical retailer and the manufacturer simultaneously.

The demand is $Q_S = V^2 - VP_S$, and the Ps is composed of the manufacturer's wholesale price (W) and the retailer's margin (r).

$$\pi_S = (V^2 - V(W + r))r$$

The manufacturer's profit function is:

$$\Pi_M = (V^2 - V(W + r))W$$

Then, we can get the following FOCs by taking the first derivative of each of the profit functions above w.r.t. the corresponding margin.

$$FOC_S^r = V^2 - V(W + r) - rV$$
$$FOC_M^W = V^2 - V(W + r) - WV$$

By solving the two *FOCs=0*, we are able to obtain the retailer's equilibrium price and the manufacturer's wholesale price. $W^* = r^* = \frac{1}{3}V$

The SOC's are satisfied as follows:

$$SOC_M^{WW} = SOC_S^{rr} = -2V < 0 .$$

Vertically Integrated Mixed Channel (VIM)

Same as the results for VIM in Table 1 a.

Partially Integrated Mixed Channel (PIM)

When $P_S \geq P_N$, the demand for the Internet store (Q_N) and that for the physical store (Q_S) are described as follows:

$$Q_N = (W + r)(V - P_N) + \frac{1}{2}(V - W - r)(W + r - 2P_N + V)$$
$$Q_S = P_N(V - W - r) + \frac{1}{2}(V - W - r)^2 .$$

Since this is a vertical nash game, we need to solve the profit maximization problems of both manufacturer and independent retailer simultaneously.

$$\pi_S = rQ_S$$
$$\Pi_M = WQ_S + P_N Q_N$$

Then the physical retailer' FOC w.r.t. r is

$$FOC_S^r = P_N(V - W - r) + (-P_N + W + r - V)r + \frac{1}{2}(V - W - r)^2 .$$

The SOC of the store is satisfied at the equilibrium prices:

$$SOC_S^{rr} = -1.44856V < 0.$$

The manufacturer's FOCs w.r.t. W and P_N are

$$FOC_M^{P_N} = W(V-W-r)+(W+r)(V-P_N)+\frac{1}{2}(V-W-r)(W+r-2P_N+V)-P_N V$$
$$FOC_M^{W} = 2P_N(V-W-r)+\frac{1}{2}(V-W-r)^2+W(-P_N-V+W+r)$$

Solving the three $FOCs=0$ simultaneously, we can obtain the equilibrium wholesale and retail prices.

$$W^* = 0.44536V, r^* = 0.25000V, P_N^* = 0.54464V$$

The SOC's of the manufacturer is also satisfied at the equilibrium prices:

$$SOC_M^{P_N P_N} = -2V < 0$$
$$SOC_M^{WW} = -1.79784V < 0$$
$$SOC_M^{P_N W} = 0.16392V$$
$$SOC_M^{P_N P_N} \times SOC_M^{WW} - (SOC_M^{P_N W})^2 = 3.56881V^2 > 0.$$

Horizontally Integrated Mixed Channel (HIM)

When $P_S \geq P_N$ where Ps is composed of the manufacturer's wholesale price (W) and the retailer's margin through the physical store (r) and P_N is composed of W and the retailer's margin through the Internet Channel (n),

$$Q_N = (W+r)(V-(W+n))+\frac{1}{2}(V-W-r)(-W+r-2n+V)$$
$$Q_S = (W+r)(V-(W+r))+\frac{1}{2}(V-W-r)^2$$

Under this channel structure, the independent retailer coordinates both outlets and maximizes the joint profit conditional on the manufacturer's wholesale price.

$$\pi_S = nQ_N + rQ_S$$

The retailer's FOC's w.r.t. each of the retail margins are

$$FOC_S^n = r(V-W-r)+(W+r)(V-W-n)+\frac{1}{2}(V-W-r)(-W+r-2n+V)-nV$$

, and

$$FOC_S^r = r(W+n)(V-W-n)+\frac{1}{2}(V-W-r)^2+r(-n-V+r)+n(V-W-r).$$

The SOC's are satisfied at the equilibrium prices:

$$SOC_S^{rr} = SOC_S^{nn} = -2V < 0$$
$$SOC_S^{rn} = 0.23225V$$
$$SOC_S^{rr} \times SOC_S^{nn} - (SOC_S^{rn})^2 = 3.94606V^2 > 0.$$

The manufacturer also maximizes its own profit conditional on the retailer's margins (n and r).

$$\Pi_M = W(Q_N + Q_S)$$

The FOC w.r.t. W is

$$FOC_M^W = (W+r)(V-W-n) + \frac{1}{2}(V-w-r)(-w+r-2n+V)$$
$$+ (W+n)(V-W-r) + \frac{1}{2}(V-W-r)^2 + W(-2W-n-r)$$

Solving the three *$FOC_S=0$* simultaneously, we can obtain the equilibrium wholesale price and retail margins.

$$W^* = n^* = r^* = 0.35355V\ .$$

The SOC is satisfied at the equilibrium prices:

$$SOC_M^{WW} = -3.53550V < 0.$$

Decentralized Mixed Channel (DM)

When $P_S \geq P_N$,

$$Q_N = (W+r)(V-W-n) + \frac{1}{2}(V-W-r)(r-W-2n+V)$$
$$Q_S = (W+n)(V-W-r) + \frac{1}{2}(V-W-r)^2$$

Since there are two independent retailers in this structure, we need to solve a Nash game among the two physical retailers and the manufacturer. The profit functions of each player are

$$\pi_N = nQ_N,\ \pi_S = rQ_S, \text{ and } \Pi_M = W(Q_S + Q_N).$$

The FOCs of each player are

$$FOC_N^n = (r+W)(V-W-n)+\frac{1}{2}(V-W-r)(r-W-2n+V)-nV$$

$$FOC_S^r = (W+n)(V-W-r)+\frac{1}{2}(V-W-r)^2 + r(-n-V+r), and$$

$$FOC_M^W = (W+n)(V-W-r)+\frac{1}{2}(V-W-r)^2 + (W+r)(V-W-n)$$

$$+\frac{1}{2}(V-W-r)(r-W-2n+V)+W(-2W-r-n).$$

Solving the three *FOCs=0* simultaneously, we can obtain the equilibrium wholesale and retail margins.

$$W^* = 0.40303V, n^* = r^* = 0.27210V .$$

The SOC's are satisfied at the equilibrium prices:

$$SOC_S^{rr} = -1.72790V < 0$$

$$SOC_N^{nn} = -2V < 0$$

$$SOC_M^{WW} = -3.50658V < 0.$$

Technical Supplement F: 5.3. Cases with Two Physical Stores

Market Outcomes with Two Physical Stores and an Internet Channel

	VIM2 (Local Monopoly at (-1/2)V and (1/2)V)	VIM2 (Competition at (-1/10)V and (1/10)V)	PIM2 (Local Monopoly at (-1/2)V and (1/2)V)	PIM2 (Competition at (-1/10)V and (1/10)V)
W*	*0*	*0*	*0.5423V*	*0.57713V*
P_{PS1}*	*0.58333V*	*0.62498V*	*0.75306V*	*0.76563V*
P_{PS2}*	*0.58333V*	*0.62498V*	*0.75306V*	*0.76563V*
P_N*	*0.66667V*	*0.59702V*	*0.59587V*	*0.57713V*
Q_{PS1}	$0.36111V^2$	$0.19321V^2$	$0.17763V^2$	$0.11944V^2$
Q_{PS2}	$0.36111V^2$	$0.19321V^2$	$0.17763V^2$	$0.11944V^2$
Q_N	$0.16667V^2$	$0.30016V^2$	$0.34315V^2$	$0.37697V^2$
Tot. *Q*	$0.88889V^2$	$0.68658V^2$	$0.69842V^2$	$0.61585V^2$
PS 1 Profit	*0*	*0*	$0.03744V^3$	$0.02251V^3$
PS 2 Profit	*0*	*0*	$0.03744V^3$	$0.02251V^3$
Tot. Retail Profit	*0*	*0*	$0.07488V^3$	$0.04503V^3$
M Profit w/ PS 1	$0.21065V^3$	$0.12075V^3$	$0.09633V^3$	$0.06893V^3$
M Profit w/ PS 2	$0.21065V^3$	$0.12075V^3$	$0.09633V^3$	$0.06893V^3$
M Profit w/ N	$0.11111V^3$	$0.17920V^3$	$0.20447V^3$	$0.21756V^3$
Tot. M Profit	$0.53241V^3$	$0.42070V^3$	$0.39713V^3$	$0.35542V^3$
Channel Profit	$0.53241V^3$	$0.42070V^3$	$0.47201V^3$	$0.40045V^3$
Consumer Surplus	$0.15201V^3$	$0.10107V^3$	$0.10023V^3$	$0.09605V^3$

The demands are adjusted by multiplying ½ to make consistent comparisons of the market outcomes between the models described in Figure 2 (a) and (b).

VIM2 (Local Monopolies)

When $P_{PS1} = P_{PS2} \le P_N$, the adjusted demand functions for the Internet channel, physical store 1, and physical store 2 are

$$Q_N = \frac{1}{2}(2(V-P_N)V-(V-P_{PS1})^2-(V-P_{PS2})^2)$$
$$Q_{PS1} = \frac{1}{2}(2P_N(V-P_{PS1})+(V-P_{PS1})^2)$$
, respectively.
$$Q_{PS2} = \frac{1}{2}(2P_N(V-P_{PS2})+(V-P_{PS2})^2)$$

The manufacturer maximizes the joint profit across these three outlets.

$$\Pi_M = Q_N P_N + Q_{PS1} P_{PS1} + Q_{PS2} P_{PS2}$$

The manufacturer's FOCs w.r.t. each price are

$$FOC_M^{P_N} = \frac{1}{2}(2(V - P_N)V - (V - P_{PS1})^2 - (V - P_{PS2})^2) - P_N V$$
$$+ P_{PS1}(V - P_{PS1}) + (V - P_{PS2})P_{PS2}$$

$$FOC_M^{PS1} = P_N (V - P_{PS1}) + \frac{1}{2}(2P_N (V - P_{PS1}) + (V - P_{PS1})^2)$$
$$+ \frac{1}{2} P_{PS1}(-2P_N - 2V - 2P_{PS1})$$
$$FOC_M^{PS2} = P_N (V - P_{PS2}) + \frac{1}{2}(2P_N (V - P_{PS2}) + (V - P_{PS2})^2)$$
$$+ \frac{1}{2} P_{PS2}(-2P_N - 2V - 2P_{PS2})$$

The equilibrium prices making all the FOCs 0 are

$$P_N^* = 0.66667V, P_{PS1}^* = P_{PS2}^* = 0.58333V$$

The SOC's are satisfied at the equilibrium prices:

$$|H| = \begin{vmatrix} SOC_M^{P_N P_N} & SOC_M^{P_N P_{PS!}} & SOC_M^{P_N P_{PS2}} \\ SOC_M^{P_{PS1} P_N} & SOC_M^{P_{PS1} P_{PS1}} & SOC_M^{P_{PS1} P_{PS2}} \\ SOC_M^{P_{PS2} P_N} & SOC_M^{P_{PS2} P_{PS1}} & SOC_M^{P_{PS2} P_{PS2}} \end{vmatrix} = \begin{vmatrix} -2V & 0.25001V & 0.25001V \\ 0.25001V & -2.25002V & 0 \\ 0.25001V & 0 & -2.25002V \end{vmatrix}$$

$$|H_1| = -2V < 0$$
$$|H_2| = 4.43754V^2 > 0$$
$$|H_3| = -9.98454V^3 < 0.$$

VIM2 (Competition)

When each of the two physical stores is located –a and a, respectively, and they are close enough, competition between the two physical stores occurs.
When $P_{PS1} = P_{PS2} \geq P_N$, the adjusted demand functions for the Internet channel, physical store 1, and physical store 2 are:

$$Q_N = \frac{1}{2}\{2V(V-P_N) - ((P_N - P_{PS2}) + (V - P_{PS2}))(V - P_N)$$
$$-((P_N - P_{PS1}) + (V - P_{PS1}))(V - P_N) + (V - a - \frac{P_{PS1} + P_{PS2}}{2})^2\}$$
$$Q_{PS1} = \frac{1}{2}\{(\frac{P_{PS2} - P_{PS1}}{2} - (-a - (V - P_{PS1})))P_N + (V -$$
$$P_N)((P_N - P_{PS1}) + (V - P_{PS1})) - \frac{1}{2}(V - a - \frac{(P_{PS1} + P_{PS2})}{2})^2\}$$
$$Q_{PS2} = \frac{1}{2}\{(a + V - P_{PS2} - \frac{P_{PS2} - P_{PS1}}{2})P_N + (V -$$
$$P_N)((P_N - P_{PS2}) + (V - P_{PS2})) - \frac{1}{2}(V - a - \frac{(P_{PS1} + P_{PS2})}{2})^2\}$$

When *a=(1/10)V*, the demand functions for the Internet channel, physical store 1, and physical store 2 are:

$$Q_N = V(V - P_N) - \frac{1}{2}(V - P_{PS1})^2 - \frac{1}{2}(V - P_{PS2})^2 + \frac{1}{2}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$
$$Q_{PS1} = \frac{1}{2}(\frac{1}{2}P_{PS2} - \frac{3}{2}P_{PS1} + \frac{11}{10}V)P_N + \frac{1}{2}(V - P_{PS1})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$
$$Q_{PS2} = \frac{1}{2}(\frac{11}{10}V - \frac{3}{2}P_{PS2} + \frac{1}{2}P_{PS1})P_N + \frac{1}{2}(V - P_{PS2})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$

, respectively

The manufacturer maximizes the joint profit across these three outlets.

$$\Pi_M = Q_N P_N + Q_{PS1} P_{PS1} + Q_{PS2} P_{PS2}$$

The FOCs w.r.t. each price are

$$FOC_M^{P_N} = P_{PS1}(\frac{11}{20}V - \frac{3}{4}P_{PS1} + \frac{1}{4}P_{PS2}) + P_{PS2}(\frac{11}{20}V - \frac{3}{4}P_{PS2} + \frac{1}{4}P_{PS1})$$
$$+V(V-P_N) - \frac{1}{2}(V-P_{PS1})^2 - \frac{1}{2}(V-P_{PS2})^2 + \frac{1}{2}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2 - P_N V$$
$$FOC_M^{PS1} = \frac{1}{2}(\frac{1}{2}P_{PS2} - \frac{3}{2}P_{PS1} + \frac{11}{10}V)P_N + \frac{1}{2}(V-P_{PS1})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$
$$+P_{PS1}(-\frac{3}{4}P_N - \frac{31}{40}V + \frac{7}{8}P_{PS1} - \frac{1}{8}P_{PS2}) + P_{PS2}(\frac{1}{4}P_N + \frac{9}{40}V - \frac{1}{8}P_{PS1} - \frac{1}{8}P_{PS2})$$
$$+P_N(\frac{11}{20}V - \frac{3}{4}P_{PS1} + \frac{1}{4}P_{PS2})$$
$$FOC_M^{PS2} = \frac{1}{2}(\frac{1}{2}P_{PS1} - \frac{3}{2}P_{PS2} + \frac{11}{10}V)P_N + \frac{1}{2}(V-P_{PS2})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$
$$+P_{PS2}(-\frac{3}{4}P_N - \frac{31}{40}V + \frac{7}{8}P_{PS2} - \frac{1}{8}P_{PS1}) + P_{PS1}(\frac{1}{4}P_N + \frac{9}{40}V - \frac{1}{8}P_{PS1} - \frac{1}{8}P_{PS2})$$
$$+P_N(\frac{11}{20}V - \frac{3}{4}P_{PS2} + \frac{1}{4}P_{PS1})$$

The equilibrium prices making all the FOCs 0 are

$$P_N^* = 0.59702V, P_{PS1}^* = P_{PS2}^* = 0.62498V$$

The SOC's are satisfied at the equilibrium prices:

$$|H| = \begin{vmatrix} SOC_M^{P_N P_N} & SOC_M^{P_N P_{PS!}} & SOC_M^{P_N P_{PS2}} \\ SOC_M^{P_{PS1} P_N} & SOC_M^{P_{PS1} P_{PS1}} & SOC_M^{P_{PS1} P_{PS2}} \\ SOC_M^{P_{PS2} P_N} & SOC_M^{P_{PS2} P_{PS1}} & SOC_M^{P_{PS2} P_{PS2}} \end{vmatrix} = \begin{vmatrix} -2V & 0.16253V & 0.16253V \\ 0.16253V & -1.48709V & 0.42903V \\ 0.16253V & 0.42903V & -1.48709V \end{vmatrix}$$

$$|H_1| = -2V < 0$$
$$|H_2| = 2.94776V^2 > 0$$
$$|H_3| = -3.95351V^3 < 0.$$

PIM2 (Local Monopolies)

When $P_{PS1} = P_{PS2} \geq P_N$, the adjusted demand functions for the Internet channel, physical store 1, and physical store 2 are

$$Q_N = V(V - P_N) - \frac{1}{2}(V - P_{PS1})^2 - \frac{1}{2}(V - P_{PS2})^2$$
$$Q_{PS1} = (V - P_{PS1})P_N + \frac{1}{2}(V - P_{PS1})^2$$
$$Q_{PS2} = (V - P_{PS2})P_N + \frac{1}{2}(V - P_{PS2})^2$$

, respectively.

Since the independent physical stores are the Stackelberg followers, we need to solve the independent retailers' profit maximization problems first.

$$\pi_{PS1} = (P_{PS1} - W)Q_{PS1}$$
$$\pi_{PS2} = (P_{PS2} - W)Q_{PS2}$$

Then the FOCs of the physical retailers w.r.t. P_{PS1} and P_{PS2} are

$$FOC_{PS1}^{PS1} = P_N(V - P_{PS1}) + (-P_N + P_{PS1} - V)(P_{PS1} - W) + \frac{1}{2}(V - P_{PS1})^2$$
$$FOC_{PS2}^{PS2} = P_N(V - P_{PS2}) + (-P_N + P_{PS1} - V)(P_{PS1} - W) + \frac{1}{2}(V - P_{PS1})^2$$

, respectively.

Solving *FOCs=0*, we can obtain the retailers' equilibrium prices.

$$P_{PS1}^* = P_{PS2}^* = \frac{2}{3}P_N + \frac{1}{3}W + \frac{2}{3}V - \frac{1}{3}\sqrt{4P_N^2 - 2P_NW + 2P_NV + W^2 - 2VW + V^2}$$

The SOC's are satisfied at the equilibrium prices:

$$SOC_{PS1}^{PS1PS1} = -1.47486V < 0$$
$$SOC_{PS2}^{PS2PS2} = -1.47486V < 0.$$

Solving backward, we now solve the manufacturer's profit maximization problem incorporating the retailers' optimal pricing decisions.

$$\Pi_M = P_N Q_N + W(Q_{PS1} + Q_{PS2})$$

After substituting P_{PS1}^* and P_{PS2}^* into the manufacturer's profit function above, the FOCs w.r.t. each of the wholesale price (W) and the retail price for the Internet channel (P_N) are as follows:

$$FOC_M^W =$$

$$\left(2\left(-\frac{1}{3}+\frac{\frac{1}{6}\left(-2\,V-2\,Pn+2\,\mathrm{W}\right)}{\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}}\right)Pn+2\left(\frac{1}{3}V-\frac{2}{3}Pn-\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}\right)\left(-\frac{1}{3}+\frac{\frac{1}{6}\left(-2\,V-2\,Pn+2\,\mathrm{W}\right)}{\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}}\right)\right)\mathrm{W}$$
$$+2\left(\frac{1}{3}V-\frac{2}{3}Pn-\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}\right)Pn$$
$$+\left(\frac{1}{3}V-\frac{2}{3}Pn-\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}\right)^2-2\left(\frac{1}{3}V-\frac{2}{3}Pn-\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}\right)\left(-\frac{1}{3}+\frac{\frac{1}{6}\left(-2\,V-2\,Pn+2\,\mathrm{W}\right)}{\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}}\right)Pn$$

$FOC_M^N =$

$$\left(2\left(-\frac{2}{3}+\frac{\frac{1}{6}\left(2\,V+8\,Pn-2\,\mathrm{W}\right)}{\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}}\right)Pn+\frac{2}{3}V-\frac{4}{3}Pn-\frac{2}{3}\mathrm{W}+\frac{2}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}+2\left(\frac{1}{3}V-\frac{2}{3}Pn-\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}\right)\left(-\frac{2}{3}+\frac{\frac{1}{6}\left(2\,V+8\,Pn-2\,\mathrm{W}\right)}{\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}}\right)\right)\mathrm{W}$$
$$+\left(-V-2\left(\frac{1}{3}V-\frac{2}{3}Pn-\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}\right)\left(-\frac{2}{3}+\frac{\frac{1}{6}\left(2\,V+8\,Pn-2\,\mathrm{W}\right)}{\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}}\right)\right)Pn+V\left(V-Pn\right)$$
$$-\left(\frac{1}{3}V-\frac{2}{3}Pn-\frac{1}{3}\mathrm{W}+\frac{1}{3}\sqrt{V^2+2\,Pn\,V-2\,\mathrm{W}\,V+4\,Pn^2-2\,\mathrm{W}\,Pn+\mathrm{W}^2}\right)^2$$

By equating the two FOC’s to 0 and simultaneously solving them, we can solve the equilibrium price for the Internet and the wholesale price.

P_N^{*}*=0.59587V* and W^{*}*=0.54230V*

The SOC's are satisfied at the equilibrium prices:

$$SOC_M^{P_N P_N} = -1.98857V < 0$$
$$SOC_M^{WW} = -1.89288V < 0$$
$$SOC_M^{P_N W} = 0.16615V > 0$$
$$SOC_M^{P_N P_N} \times SOC_M^{WW} - (SOC_M^{P_N W})^2 = 3.73652V^2 > 0.$$

PIM2 (Competition)

When $P_{PS1} = P_{PS2} \geq P_N$, the adjusted demand functions for the Internet channel, physical store 1, and physical store 2 are

$$Q_N = V(V - P_N) - \frac{1}{2}(V - P_{PS1})^2 - \frac{1}{2}(V - P_{PS2})^2 + \frac{1}{2}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$
$$Q_{PS1} = \frac{1}{2}(\frac{1}{2}P_{PS2} - \frac{3}{2}P_{PS1} + \frac{11}{10}V)P_N + \frac{1}{2}(V - P_{PS1})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$
$$Q_{PS2} = \frac{1}{2}(\frac{1}{2}P_{PS1} - \frac{3}{2}P_{PS2} + \frac{11}{10}V)P_N + \frac{1}{2}(V - P_{PS2})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$

, respectively.

Since the independent physical stores are the Stackelberg followers, we need to solve the independent retailers' profit maximization problems first.

$$\pi_{PS1} = (P_{PS1} - W)Q_{PS1}$$
$$\pi_{PS2} = (P_{PS2} - W)Q_{PS2}$$

Then the FOCs of the physical retailers w.r.t. P_{PS1} and P_{PS2} are

$$FOC_{PS1}^{PS1} = \frac{1}{2}(\frac{1}{2}P_{PS2} - \frac{3}{2}P_{PS1} + \frac{11}{10}V)P_N + \frac{1}{2}(V - P_{PS1})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS1} - \frac{1}{2}P_{PS2})^2$$
$$+ (P_{PS1} - W)(-\frac{3}{4}P_N - \frac{31}{40}V + \frac{7}{8}P_{PS1} - \frac{1}{8}P_{PS2})$$

$$FOC_{PS2}^{PS2} = \frac{1}{2}(\frac{1}{2}P_{PS1} - \frac{3}{2}P_{PS2} + \frac{11}{10}V)P_N + \frac{1}{2}(V - P_{PS2})^2 - \frac{1}{4}(\frac{9}{10}V - \frac{1}{2}P_{PS2} - \frac{1}{2}P_{PS1})^2$$
$$+ (P_{PS2} - W)(-\frac{3}{4}P_N - \frac{31}{40}V + \frac{7}{8}P_{PS2} - \frac{1}{8}P_{PS1})$$

, respectively.

Solving *$FOC_S=0$*, we can obtain the retailers' equilibrium prices.

$$P_{PS1}^{*}=P_{PS2}^{*}=\frac{5}{8}P_N+\frac{3}{8}W+\frac{53}{80}V-\frac{1}{80}\sqrt{2500P_N^{\,2}-1800P_NW+1780P_NV+900W^2-1780VW+905V^2}$$

The SOC's are satisfied at the equilibrium prices:

$$SOC_{PS1}^{PS1PS1}=-1.10231V<0$$
$$SOC_{PS2}^{PS2PS2}=-1.10231V<0.$$

Solving backward, we now solve the manufacturer's profit maximization problem incorporating the retailer's optimal pricing decision.

$$\Pi_M=P_NQ_N+W(Q_{PS1}+Q_{PS2})$$

P_{PS1}^{*} and P_{PS2}^{*} are substituted into the manufacturer's profit function above, and $P_N=W$ is imposed to deal with an arbitrage problem ($P_N^{*}<W^{*}$). The manufacturer's FOCs w.r.t. the wholesale price is as follows:

$$\begin{aligned}
FOC_M^W =\;& \left(-\mathrm{W}+\frac{7}{16}V-\frac{1}{80}\sqrt{1600\,\mathrm{W}^2+905\,V^2}\right)\mathrm{W}+\mathrm{W}\left(\left(-1-\frac{20\,\mathrm{W}}{\sqrt{1600\,\mathrm{W}^2+905\,V^2}}\right)\mathrm{W}-\mathrm{W}\right.\\
&+\frac{7}{16}V-\frac{1}{80}\sqrt{1600\,\mathrm{W}^2+905\,V^2}\\
&+2\left(\frac{27}{80}V-\mathrm{W}-\frac{1}{80}\sqrt{1600\,\mathrm{W}^2+905\,V^2}\right)\left(-1-\frac{20\,\mathrm{W}}{\sqrt{1600\,\mathrm{W}^2+905\,V^2}}\right)\\
&\left.-\left(\frac{19}{80}V-\mathrm{W}-\frac{1}{80}\sqrt{1600\,\mathrm{W}^2+905\,V^2}\right)\left(-1-\frac{20\,\mathrm{W}}{\sqrt{1600\,\mathrm{W}^2+905\,V^2}}\right)\right)+\left(-V\right.\\
&-2\left(\frac{27}{80}V-\mathrm{W}-\frac{1}{80}\sqrt{1600\,\mathrm{W}^2+905\,V^2}\right)\left(-1-\frac{20\,\mathrm{W}}{\sqrt{1600\,\mathrm{W}^2+905\,V^2}}\right)\\
&\left.+\left(\frac{19}{80}V-\mathrm{W}-\frac{1}{80}\sqrt{1600\,\mathrm{W}^2+905\,V^2}\right)\left(-1-\frac{20\,\mathrm{W}}{\sqrt{1600\,\mathrm{W}^2+905\,V^2}}\right)\right)\mathrm{W}\\
&+V\,(V-\mathrm{W})
\end{aligned}$$

The equilibrium wholesale price ($W^{*}=P_N^{*}$) making the FOC 0 is *$0.57713V$*.

The SOC is satisfied at the equilibrium prices:

$$SOC_M^{WW}=-2.82652V<0.$$