Technical Appendix for "Bricks & Clicks": The Impact of Product Returns on the Strategies of Multi-Channel Retailers

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This Technical Appendix provides formal details of the monopoly case with not fully covered segments, the duopoly setup when k_H is not high enough to ensure that high shopping trip cost consumers purchase online, the extra analysis described in Section 5 of the paper, and a summary of comparative statics for the basic model duopoly cases.

1 Monopoly Analysis with Non-Fully Covered Markets

In this section we extend our monopoly analysis to a case in which the market is not covered. In particular, we examine a case in which both the low and high shopping trip cost segments a partially covered. That is, there are consumers in each segment who make a purchase but not all of the consumers make purchases. This ensures that we obtain an interior solution both with respect to prices and shopping assistance levels. The model setup is the same as before. There is a monopolist that own both outlets (at the two ends of the Hotelling line). We show that the monopolist is better off in the Bricks & Clicks case, since profits are higher and store assistance levels are lower than in the Bricks Only case. First, we determine the demand that the monopolist faces in the two segments by calculating the locations of the consumers who are indifferent between making a purchase and not. The total demand in the Bricks Only case is [1]

$$\mu\frac{2(v-p-(1-\lambda_S)c)}{t}+(1-\mu)\frac{2(v-p-(1-\lambda_S)c-k_H)}{t},$$

whereas in the Bricks and Clicks case it is

$$\mu\frac{2(v-p-(1-\lambda_S)c)}{t}+(1-\mu)\frac{2(v-p-c)}{t}.$$

[1] We assume $th>(c+r)^2$ to ensure an interior solution.

Solving for the optimal price and service assistance levels yield

$$\hat{\lambda}_{BO} = \frac{(c+r)(v-c-r-(1-\mu)k_H)}{ht-(c+r)^2} \; \hat{\lambda}_{BC} = \frac{\mu(c+r)(v-c-r)}{ht-\mu^2(c+r)^2-4\mu(1-\mu)cr}.$$

The profits are

$$\hat{\pi}_{BO} = \frac{h(v-c-r-(1-\mu)k_H)^2}{2(ht-(c+r)^2)} \; \hat{\pi}_{BC} = \frac{(ht-4\mu(1-\mu)cr)(v-c-r)^2}{2t(ht-\mu^2(c+r)^2-4\mu(1-\mu)cr)}$$

To make the comparison tractable, we set $c = r = 1$ and $\mu = 1/2$. Then $\hat{\lambda}_{BO} > \hat{\lambda}_{BC}$ if $ht(v-2-k_H)+k_H \geq 0$ which follows from $v \geq 2 + k_H\left(\frac{3}{2} - \frac{4}{ht}\right)$. One can check that this follows from $v \geq \hat{p}_{BO} + c(1-\hat{\lambda}_{BO} + k_H$. This assumption is needed to ensure that a positive portion of the high shopping trip cost consumers makes a purchase. Similarly, $\hat{\pi}_{BC} > \hat{\pi}_{BO}$ follows from the assumption that $v \geq 2 + k_H/2$, which ensures that some portion of high shopping trip customers in the Brick and Clicks case makes a purchase.

2 Characterizing Equilibria for All k_H Values

Throughout the paper we assumed that k_H is high enough such that high shopping trip customers always shop online. In most of the cases, the assumption $k_H > c$ ensures this. In what follows we relax this assumption and generalize the findings that we reported in Table 3 of the paper. Recall that type k_H consumers prefer to shop online from retailer i if and only if the cost of making the trip to the retailer's store is greater than the expected reduction in returns cost they would achieve due to store assistance (i.e., $c\lambda_{S_i} \leq k_H$). This implies that the choice of store assistance levels not only affects how intensely the two retailers compete for the k_L type consumers, who always shop in stores, but also determines where k_H type consumers will shop. In the following proposition, we summarize the results for k_H values that are not necessarily high.

Proposition 9 *There exists a $\overline{k}(c,r,h,t,\mu) < c$, and $\underline{k}(c,r,h,t,\mu) = \underline{k} \leq \overline{k}$ such that*

- *If $0 < k_H \leq \underline{k}$, there is a unique symmetric sub-game perfect equilibrium where every consumer shops in physical stores and store assistance levels and prices are equal to the Bricks-Only case, that is, $\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BO}$ and $\hat{p}_{S_1} = \hat{p}_{S_2} \equiv \hat{p}_{BO}$.*
- *If $\underline{k} < k_H \leq \overline{k}$, there is no pure-strategy symmetric sub-game perfect equilibrium.*
- *if $\overline{k} \leq k_H$, there is a unique symmetric sub-game perfect equilibrium where k_L type consumers shop in physical stores while k_H type consumers shop on the Internet and we obtain the results presented in the right hand column of Table 3*

PROOF: Let us examine the case when $k_H \le c$ as we have already completed the analysis for $k_H > c$. In this case k_L type consumers shop in the physical stores and k_H type consumers preferring retailer i shop online if and only if $c\lambda_i \ge k_H$. There are four possible cases. If both $c\lambda_1 \ge k_H$ and $c\lambda_2 \ge k_H$, (denote the region by SS) then k_H consumers of both retailers shop in the physical stores. Hence the pricing equilibrium is the same as in the Bricks-Only case. Plugging these prices back to the profit function of retailer 2, we get $\pi_2^{SS}(\lambda_1, \lambda_2)$, which is identical to that of the Bricks-Only case. Similarly, if both $c\lambda_1 < k_H$ and $c\lambda_2 < k_H$ (region NN) then k_H consumers of both retailers shop on the Internet leading to the pricing equilibrium of the Bricks & Clicks case with $c < k_H$, that we have characterized in the beginning of the proof. Plugging these prices back into the profit function we obtain $\pi_2^{NN}(\lambda_1, \lambda_2)$, the profit function of the Bricks & Clicks case. If $c\lambda_1 \ge k_H$ and $c\lambda_2 < k_H$ (region SN) then k_H type consumers of retailer 1 shop in the physical store, whereas k_H type consumers of retailer 2 shop on the Internet. Calculating the indifferent consumer in segments k_L and k_H and plugging them into the profit functions, we get the equilibrium prices

$$p_1 = t + r + \frac{\lambda_1(c-2r) - \lambda_2\mu(c+r)}{3}, \quad p_2 = t + r + \frac{\lambda_2\mu(c-2r) - \lambda_1(c+r)}{3}.$$

Plugging these back into the profit function we obtain $\pi_2^{SN}(\lambda_1, \lambda_2)$. Similarly we can calculate $\pi_2^{NS}(\lambda_1, \lambda_2)$ in the NS region. Note that $\pi_2^{SS}(\lambda_1, \lambda_2) < \pi_2^{SN}(\lambda_1, \lambda_2)$ and $\pi_2^{NS}(\lambda_1, \lambda_2) < \pi_2^{NN}(\lambda_1, \lambda_2)$.

We now determine the symmetric equilibria in the SA level choice stage. Since the profit functions in the SS and NN regions are the same as in the previously discussed cases, the best response functions are identical to those identified in (A4) and (A6), respectively. Hence, whenever the previously determined equilibria fall in the correct region, that is, when $\hat{\lambda}_{BO}$ falls in SS and $\hat{\lambda}_{BC}$ falls in NN, they constitute local equilibria in this case and there are no other symmetric local equilibria. We still have to check whether these equilibria are global. Since profit functions are symmetric, we only check if firm 2 has an incentive to deviate. In the case of $\hat{\lambda}_{BO}$ one can check that

$$\hat{\pi}_{BO} \ge \max_{\lambda_2} \pi_2^{SN}(\hat{\lambda}_{BO}, \lambda_2),$$

that is, $\hat{\lambda}_{BO}$ always constitutes a global equilibrium if $\hat{\lambda}_{BO} \ge k_H/c$. In the case of $\hat{\lambda}_{BC}$, it is a global equilibrium if and only if $k_H > \overline{k}$, where

$$\overline{k}(c, r, h, t, \mu) = \min\{c, \frac{ct(c+r)(36ht - 4\mu^2(c^2+r^2) + (10\mu - 18)\mu cr + 6\sqrt{D})}{6(9ht - (c+r)^2)(\mu^2 c - \mu cr + 2ht)}\}, \text{ and} \quad (1)$$

$$D = (1-\mu)\left(36t^2h^2(1+\mu) - 4ht(1+\mu)\mu^2(c^2+r^2) + (28\mu^2 - 8\mu - 36)\mu crht + 9(1-\mu)\mu^2c^2r^2\right).$$

Note that $\overline{k} > c\hat{\lambda}_{BO}$, hence we get the equilibrium results stated in Table 3 with

$$\underline{k}(c, r, h) = c\hat{\lambda}_{BO} = \frac{c(c+r)}{3h}. \tag{2}$$

□

What Proposition 9 tells us, is that the SA level retailers set when there is only a physical store outlet ($\hat{\lambda}_{BO}$) is sufficient to entice k_H type consumers to visit stores when the shopping trip cost they have to incur is small. In this case, even though the online outlet exists and can feature the product, in effect, all consumers purchase at the physical stores.

3 Formal Analysis of Model Extensions Described in Section 5 of the Paper

3.1 Bricks & Clicks Retailers Selling Multiple Products

In what follows, we generalize the basic model presented in Section 3 of the paper so that retailers offer two separate product categories. We seek to answer the following questions. Do firms sell more product categories on the Internet or in the physical store? If a certain type of product has a higher baseline probability of return, under what conditions will it be sold on both channels vis-a-vis other conditions where it is sold through only one of the channels?

Let there be two distinct products that differ in terms of their baseline probability of fit: one product, which we will call the "safe" product, has a lower likelihood of being returned without physical inspection than the other product, which we will call the "risky" product. Retailers choose the SA level to provide and price to charge for each product. It is straightforward that retailers can price each product differently, but SA levels can also differ– for example, a retailer could devote more salespeople to certain products, set aside more demo units and special equipment to try certain goods, or allocate more prime store space for some of the categories it carries. On a given shopping occasion, some consumers are interested in buying the safe product while the remaining consumers are interested in buying the risky product (and each consumer buys one unit). There are thus four consumer types: high vs. low shopping trip costs that seek to buy the safe vs. risky product (and, independently, each consumer has a preference for each retailer that is uniformly distributed). Table 6 provides additional notation for this extended setup.

Table 6: Additional Notation: Multiple Categories Setup

	Notation:
Variable:	
Index for each product type	β denotes safe product α denotes risky product
Likelihood of a product return without physical inspection	$1-\lambda_\beta$ and $1-\lambda_\alpha$
Store assistance level chosen by retailer i for each product	$\lambda_{S_i\alpha}$ and $\lambda_{S_i\beta}$
Cost of providing SA levels $\lambda_{S_i\alpha}$ and $\lambda_{S_i\beta}$	$\frac{1}{2}h(\lambda_{S_i\alpha})^2$ and $\frac{1}{2}h(\lambda_{S_i\beta})^2$
Price chosen by retailer i for each product	$p_{i\alpha}$ and $p_{i\beta}$
Proportion of consumers that wish to buy risky product	ω

Given that we assume (without loss of generality) $\lambda_\alpha < \lambda_\beta$, the baseline consumer expected return costs are $c_\alpha \equiv (1-\lambda_\alpha)m > c_\beta \equiv (1-\lambda_\beta)m$. All other assumptions are identical to those of the basic model, and whether a consumer is looking for product α or β is independent of that consumer's shopping trip cost.

Consistent with our previous notation and findings, the equilibrium SA level as a function of c is $\hat{\lambda}_{BO}(c) = \frac{c+r}{3h}$ in the Bricks-Only case and $\hat{\lambda}_{BC}(c) = \frac{2\mu t(c+r)}{6ht-3rc\mu(1-\mu)}$ in the Bricks & Clicks case. Note that by setting prices and SA levels, firms affect consumers' shopping behavior. In particular, whether the k$_H$ type consumers purchase online or in the store. The case where no consumer purchases a product over the Internet is equivalent to not offering that product for sale to begin with through the online channel. The following proposition characterizes the multiple-product equilibria.

Proposition 10 *There exist k_1, k_2, k_3, k_4, $0 < k_1 < k_2 \le k_4$ and $k_1 < k_3 < k_4$, such that the unique symmetric sub-game perfect equilibria are*

- *If $k_H > k_4$: all k_L type consumers shop in physical stores while all k_H type consumers shop on the Internet. SA levels and prices are: $\hat{\lambda}_{S_{1\alpha}} = \hat{\lambda}_{S_{2\alpha}} \equiv \hat{\lambda}_{BC}(c_\alpha)$, $\hat{\lambda}_{S_{1\beta}} = \hat{\lambda}_{S_{2\beta}} \equiv \hat{\lambda}_{BC}(c_\beta)$ and $\hat{p}_{1\alpha} = \hat{p}_{2\alpha} \equiv \hat{p}_{BC}(c_\alpha)$, $\hat{p}_{1\beta} = \hat{p}_{2\beta} \equiv \hat{p}_{BC}(c_\beta)$.*
- *If $k_3 \ge k_H > k_2$: product α consumers shop in physical stores while product β consumers shop on the Internet if and only if their type is k_H. SA levels and prices are: $\hat{\lambda}_{S_{1\alpha}} = \hat{\lambda}_{S_{2\alpha}} \equiv \hat{\lambda}_{BO}(c_\alpha)$, $\hat{\lambda}_{S_{1\beta}} = \hat{\lambda}_{S_{2\beta}} \equiv \hat{\lambda}_{BC}(c_\beta)$ and $\hat{p}_{1\alpha} = \hat{p}_{2\alpha} \equiv \hat{p}_{BO}(c_\alpha)$, $\hat{p}_{1\beta} = \hat{p}_{2\beta} \equiv \hat{p}_{BC}(c_\beta)$.*
- *If $k_1 \ge k_H$: every consumer shops in physical stores. SA levels and prices are equal to the Bricks-Only case: $\hat{\lambda}_{S_{1\alpha}} = \hat{\lambda}_{S_{2\alpha}} \equiv \hat{\lambda}_{BO}(c_\alpha)$, $\hat{\lambda}_{S_{1\beta}} = \hat{\lambda}_{S_{2\beta}} \equiv \hat{\lambda}_{BO}(c_\beta)$ and $\hat{p}_{1\alpha} = \hat{p}_{2\alpha} \equiv \hat{p}_{BO}(c_\alpha)$, $\hat{p}_{1\beta} = \hat{p}_{2\beta} \equiv \hat{p}_{BO}(c_\beta)$.*

Otherwise, there is no pure strategy sub-game perfect equilibrium.

Proof: Since the demand for the two products is comprised of disjunct sets of consumers, we can determine the equilibria separately for the two products. Using the results of Proposition 2, we get the following. If $k_H > \overline{k}(c_\alpha) = \overline{k}(c_\alpha, r, h, t, \mu)$ and $k_H > \overline{k}(c_\beta)$ then we get the first type of equilibria for both products described in Table 3 of the paper, where all k_L type consumers shop in physical stores while k_H type consumers shop on the Internet and $\hat{\lambda}_{S_{1\alpha}} = \hat{\lambda}_{S_{2\alpha}} \equiv \hat{\lambda}_{BC}(c_\alpha)$, $\hat{\lambda}_{S_{1\beta}} = \hat{\lambda}_{S_{2\beta}} \equiv \hat{\lambda}_{BC}(c_\beta)$ and $\hat{p}_{1\alpha} = \hat{p}_{2\alpha} \equiv \hat{p}_{BC}(c_\alpha)$, $\hat{p}_{1\beta} = \hat{p}_{2\beta} \equiv \hat{p}_{BC}(c_\beta)$. If $\underline{k}(c_\alpha) \geq k_H > \overline{k}(c_\beta)$, then for product β we get the same type of equilibrium as before, but product α is only sold in the physical stores. That is, every consumer of product α shops in physical stores and consumers of product β shop on the Internet if and only if their type is k_H. Store assistance levels are $\hat{\lambda}_{S_{1\alpha}} = \hat{\lambda}_{S_{2\alpha}} \equiv \hat{\lambda}_{BO}(c_\alpha)$, $\hat{\lambda}_{S_{1\beta}} = \hat{\lambda}_{S_{2\beta}} \equiv \hat{\lambda}_{BC}(c_\beta)$ and $\hat{p}_{1\alpha} = \hat{p}_{2\alpha} \equiv \hat{p}_{BO}(c_\alpha)$, $\hat{p}_{1\beta} = \hat{p}_{2\beta} \equiv \hat{p}_{BC}(c_\beta)$. If $\underline{k}(c_\beta) \geq k_H$, then note that $\underline{k}(c_\alpha) \geq k_H$ also holds, since $\underline{k}(c)$ is increasing in c. In this case we get an equilibrium where every consumer shops in physical stores and store assistance levels and prices are equal to the Bricks-Only case, that is, $\hat{\lambda}_{S_{1\alpha}} = \hat{\lambda}_{S_{2\alpha}} \equiv \hat{\lambda}_{BO}(c_\alpha)$, $\hat{\lambda}_{S_{1\beta}} = \hat{\lambda}_{S_{2\beta}} \equiv \hat{\lambda}_{BO}(c_\beta)$ and $\hat{p}_{1\alpha} = \hat{p}_{2\alpha} \equiv \hat{p}_{BO}(c_\alpha)$, $\hat{p}_{1\beta} = \hat{p}_{2\beta} \equiv \hat{p}_{BO}(c_\beta)$. In any other case there is no pure-strategy symmetric equilibrium. In summary, we get the results stated in Proposition 10 with

$$
\begin{aligned}
k_1 &= \underline{k}(c_\beta) = \frac{c_\beta(c_\beta + r)}{3h},\ k_2 = \overline{k}(c_\beta), \\
k_3 &= \underline{k}(c_\alpha) = \frac{c_\alpha(c_\alpha + r)}{3h},\ k_4 = \max\left(\overline{k}(c_\beta), \overline{k}(c_\alpha)\right). \quad \square
\end{aligned}
$$

Figure 3.1 depicts the different regions of the equilibrium in terms of where each product is sold. When the segment that values the convenience of online shopping has to incur a very high shopping trip cost, $k_H > k_4$, retailers set prices and SA levels to induce purchases of both products through the Internet (to the k_H types) and the store (to the k_L types). When at the other extreme the shopping trip cost is very low, $k_1 \geq k_H$, retailers end up serving all consumers in physical stores only. In the mid region, when the shopping trip cost satisfies $k_2 < k_H \leq k_3$, we get the intriguing result whereby the retailers set an SA level for the riskier product (α) such that all consumers shop for it only in the store, whereas the safer product (β) is purchased in both channels. Based on our findings in Proposition 2 of the paper, the intuition for this outcome is as follows. Because of the difference in base return probabilities for the two products, the cutoff value of k_H that makes high shopping trip cost consumers indifferent between shopping online and offline, i.e., $U_{S_i}(k_H) = U_{N_i}(k_H)$, is different for the two products. Moreover, given that retailers want to manage the supply side cost associated with returns, they tend to set higher SA levels for products that have a greater likelihood of being returned; this further entices consumers to buy such products in the store. Combining these pieces together, we get that when the difference in return probabilities between the

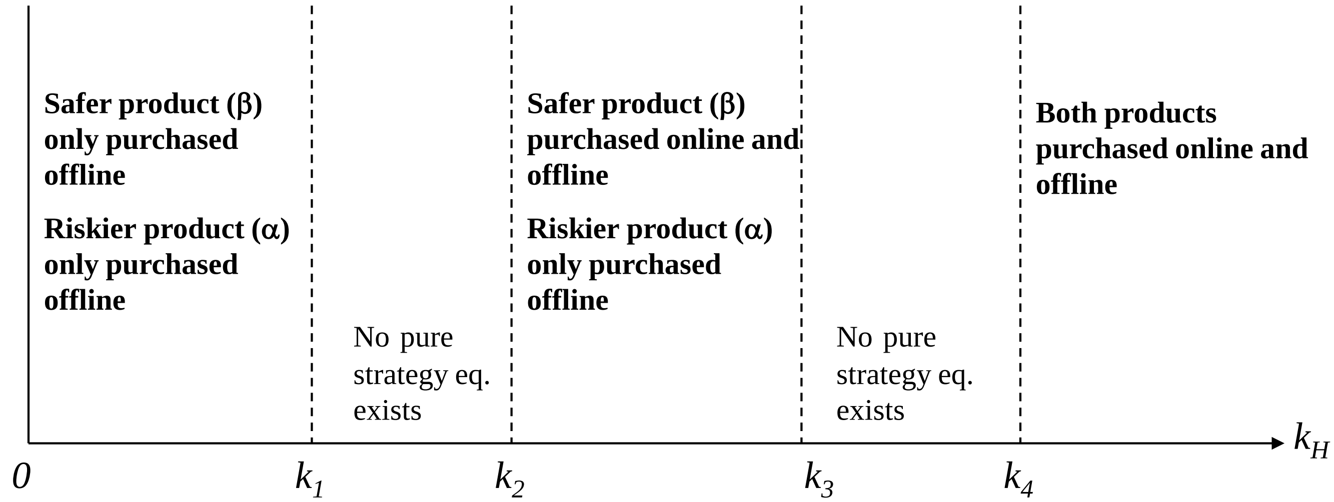

Figure 5: Consumer Product Purchase per Channel in Equilibrium

two products is sufficiently high, retailers will end up selling only the safer of the two on both channels and the riskier one only in stores.

The following corollary identifies a sufficient condition under which the equilibrium with limited online product assortment exists.

Corollary 5 *If* $c_\beta < \frac{c_\alpha(c_\alpha+r)}{3h}$ *then* $k_2 < k_3$.

Proof : From the proof of Proposition *10* we have: $k_2 < k_3$ iff $\overline{k}(c_\beta) < \underline{k}(c_\alpha) = \frac{c_\alpha(c_\alpha+r)}{3h}$. From the definition of $\overline{k}$ in the proof of Proposition 9, a sufficient condition for $k_2 < k_3$ is thus $c_\beta < \frac{c_\alpha(c_\alpha+r)}{3h}$.

Hence, if the expected consumer return cost of the safe product (c_β) is sufficiently lower than that of the risky product (c_α), we will always be able to find a region of shopping trip costs (k_H) such that firms will sell one product only in stores while selling the other through both channels. For retailers that carry a variety of different product lines, such as general merchandise retailers, there will likely be some categories for which this condition is met. We remind the reader that c_α and c_β are functions of λ_α and λ_β, respectively, hence the condition in Corollary 5 is more likely met the greater the difference between the two products in the baseline probability of match without physical inspection.

We would like to highlight several implications of the equilibrium in Proposition 10. First, it suggests that retailers should not automatically assume that their entire set of products will sell across the two channels. If indeed the end result is that some product categories won't sell online, retailers need to be ready to handle the volume of shoppers in stores (and stock accordingly) and may have an incentive to save on any fixed distribution costs associated with allowing online sales. Of course, this does not preclude allowing consumers to view all the products online; but firms may seek to limit the variety that can be purchased online. Second, and somewhat counter to the common wisdom that the Internet will surely lead to sales of esoteric less mainstream products across the board (Anderson 2006),[2] in categories where physical inspection helps reduce the likelihood of returns the assortment on the Internet may actually be limited to mainstream products. It is also important to understand this finding in the context of the options retailers have. We have shown that there can exist conditions under which retailers will prefer to offer high enough SA levels to induce all shopping to take place in the physical store for risky products, rather than offer lower SA levels to induce a portion of consumers to shop online.

3.2 Price Differences across Outlets

In the main model, we assumed that each retailer sets a single price, i.e., consumers were charged the same price in both channels. This is consistent with the practice in most multi-channel settings. It is instructive, however, to examine what happens when retailers are able to charge a different price in each channel. We examine two settings. In the first, there is no bound on the price difference across outlets. In the second, prices can diverge but the difference is bounded. In terms of notation, p_{S_i} and p_{N_i} will denote the prices retailer i sets in each channel and, when relevant, δ will denote the upper bound on the price difference across channels $|p_{N_i} - p_{S_i}| < \delta$.

Proposition 11 *If there is no limit on the price difference across channels, then in equilibrium*

$$\hat{p}_{N_1} = \hat{p}_{N_2} \equiv \hat{p}_{BC^N_\infty} = t + r \text{ and } \hat{p}_{S_1} = \hat{p}_{S_2} \equiv \hat{p}_{BC^S_\infty} = t + r(1 - \hat{\lambda}_{BC_\infty}), \tag{3}$$

$$\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BC_\infty} = \frac{\mu(c+r)}{3h} \text{ and } \hat{\pi}_{BC_1} = \hat{\pi}_{BC_2} \equiv \hat{\pi}_{BC_\infty} = \frac{1}{2}(t - h\hat{\lambda}^2_{BC_\infty}).$$

Proof: Given the proposed consumer behavior, firm 1's profits in the two distribution outlets are the following:

$$\pi_{S_1} = \mu\left(\frac{p_{S_2} - p_{S_1} + (\lambda_1 - \lambda_2)c + t}{2t}\right)(p_{S_1} - r(1-\lambda_1)) - \frac{1}{2}h\lambda_1^2, \quad \pi_{N_1} = (1-\mu)\left(\frac{p_{N_2} - p_{N_1} + t}{2t}\right)(p_{N_1} - r).$$

[2]Reference: Anderson, C. 2006. *The Long Tail: Why the Future of Business is Selling Less of More.* Hyperion, New York, NY.

Differentiating with respect to both store and Internet prices that are set simultaneously, and solving as before, we obtain the equilibrium prices and SA levels in (3). In this case, we also need to check under what conditions the equilibrium strategies are consistent with consumer behavior. First, note that $k_L = 0$ type consumers do not want to switch to the Internet because prices are higher there. $k_H > 0$ type consumers may consider the stores, however, because lower store prices may compensate them for their higher shopping trip costs. Substituting prices in the utility of a consumer located at x, the condition for k_H type consumers to not deviate to stores is:

$$v - (r + t) - tx - c \geq v - (r + t - \hat{\lambda}_{BC_\infty} r) - tx - (1 - \hat{\lambda}_{BC_\infty})c - k_H.$$

After simplification, this condition reduces to $k_H > \frac{\mu(c+r)^2}{3h}$, which we assume holds. Thus we have proved the proposition. $\square$

Proposition 11 shows that if prices online are not bound in any way by prices offline, then B&C firms will always set a lower SA level and earn higher profits relative to the BO case. We explain the intuition by again analyzing the direct and strategic effects that accompany a change in the SA level. Under complete freedom in pricing, firms maximize the profits in each channel separately. Therefore, in considering a marginal increase in the SA level, firm i needs to take into account the effect on downstream store price competition ($\hat{p}_{S_j}$) and separately on online price competition ($\hat{p}_{N_j}$). This decoupling across channels results in a large negative strategic effect on $\hat{p}_{S_j}$ (since all store customers are affected by the marginal change in SA level) and no effect on $\hat{p}_{N_j}$ (as online customers do not benefit from the change). By contrast, when prices were equal across channels the fact that online customers were not benefiting from the change in SA level had a tempering effect on the ensuing single price competition (since the rival did not have to worry about losing online customers due to a marginal increase in $\hat{\lambda}_{BC_i}$). Consequently, if we compare the strategic effects of different vs. same pricing across channels we have $\left|\frac{\partial \pi_{BC_i}}{\partial p_{S_j}} \frac{\partial \hat{p}_{S_j}}{\partial \lambda_{S_i}}\right| > \left|\frac{\partial \pi_{BC_i}}{\partial p_{BC_j}} \frac{\partial \hat{p}_{BC_j}}{\partial \lambda_{BC_i}}\right|$ (we show this formally in the Appendix). Because of the greater negative strategic effect, firms have less of an incentive to increase the SA level in the different pricing case than in the same pricing case. Yet in both cases the direct effect of marginally increasing the SA level is the same and smaller than in the BO case— which is why in the B&C separate pricing scenario the end result is a lower SA level. Higher profitability immediately follows from this.

Examining the equilibrium solution in Proposition 11 reveals that the price differential across channels is $\hat{p}_{BC^N_\infty} - \hat{p}_{BC^S_\infty} = r\hat{\lambda}_{BC_\infty} = \frac{r\mu(c+r)}{3h}$. If this differential is large, for example if handling returns from the firm's perspective is costly (large r) and the fraction of low-shopping trip cost consumers is substantial (large μ),[3] then a firm may find it difficult to sustain the price discrepancy across channels. In particular, there is serious potential for consumer backlash or dissatisfaction at such a practice, which would undermine the profitability of separate pricing (for instance, a percent of consumers could shun the firm if they discover such a practice or generate negative buzz about it using various social online tools).[4] Indeed, according to Kurt Salmon Associates, a consultancy firm specializing in retailing, "It's not clear to the customer why a price should be different within the same retail organization, so there is pressure on retailers to normalize pricing within their own [multi-channel] operation" (Wagner 2007). Given this strong pressure for prices to not be too dissimilar across channels, we seek to examine what happens if prices can diverge but only by a limited amount δ that satisfies $\frac{r\mu(c+r)}{3h} > \delta$. (Based on the preceding discussion, δ reflects the difference in prices that customers would bear without it bothering them).

Proposition 12 *If prices across channels can differ at most by an amount δ, then in equilibrium*

$$\hat{p}_{N_1} = \hat{p}_{N_2} \equiv \hat{p}_{BC_\delta^N} = t + r\left(1 - \mu\left(\hat{\lambda}_{BC_\delta} - \frac{\delta}{r}\right)\right) \text{ and } \hat{p}_{S_1} = \hat{p}_{S_2} \equiv \hat{p}_{BC_\delta^S} = \hat{p}_{BC_\delta^N} - \delta, \quad (4)$$

$$\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BC_\delta} = \frac{2\mu t(c+r) - 3c\delta\mu(1-\mu)}{6ht - 3cr\mu(1-\mu)} \quad \text{and} \quad \hat{\pi}_1 = \hat{\pi}_2 \equiv \hat{\pi}_{BC_\delta} = \frac{1}{2}(t - h\hat{\lambda}^2_{BC_\delta}).$$

PROOF: In order to solve the case in which there is an upper bound on the price difference ($\frac{r\mu(c+r)}{3h} > \delta$), one can check that the best response functions are such that firms always want to set a higher price online, attaining the maximal possible price difference (i.e., the constraint on prices is binding). Then, setting $p_{S_i} = p_{N_i} - \delta$, we follow the same steps as in the proof of Proposition 2 and obtain the results. □

Thus, if only limited discrepancy in prices is allowed, firms will take advantage of the full amount possible. However, even though prices differ, the main findings obtained in the equal pricing case of Section 3.3 qualitatively hold. In particular, as the next corollary shows, SA levels can be higher and profits lower than in the Bricks-Only case.

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[3] For example, if $r = 1$, $c = 1$, $\mu = 0.75$, $t = 0.5$, $h = 1$ (values for which all solutions are valid), then prices online would be 33% higher than offline.

[4] Take the following example reported in the media. J&R Electronics Inc. once maintained different prices on its web site and in its Manhattan store. However, J&R found that customers regularly browsed the site and then expected to see the same price in the store, which wasn't always the case. According to the firm's executive vice president of e-commerce "Having the different pricing wasn't a good experience for them". The firm now charges the same prices across channels (Wagner 2007).

Corollary 6 *If prices across channels can differ at most by an amount δ, then $\hat{\lambda}_{BO} < \hat{\lambda}_{BC_\delta}$ and $\hat{\pi}_{BO} > \hat{\pi}_{BC_\delta} \Leftrightarrow t < \underline{t} := \frac{\mu cr}{2h} - \frac{3\mu c\delta}{2(c+r)}$.*

Proof: Straightforward calculation comparing the expressions in Table 3 and (4) yields that $\hat{\lambda}_{BC_\delta} > \hat{\lambda}_{BO}$ and $\hat{\pi}_{BC_\delta} < \hat{\pi}_{BO}$ hold if and only if $t < \frac{\mu cr}{2h} - \frac{3\mu c\delta}{2(c+r)}$. □

Thus, retailers may be worse off in the B&C case relative to the BO case, despite charging a higher price online than offline. This occurs, once again, when the differentiation between retailers is low enough. The intuition is similar to that provided in connection with the findings of Proposition 2. Because the price differential is bounded, the price online is connected to the price offline and the strategic effect in the B&C case is smaller relative to the one in the BO case, which can induce B&C firms to select a higher SA level. We note that the condition for the SA level to be higher and profits lower is more stringent when prices can differ (formally: $\bar{t} = \frac{\mu cr}{2h} > \underline{t} = \frac{\mu cr}{2h} - \frac{3\mu c\delta}{2(c+r)} > 0$).

3.3 Shipping Costs, Variable Costs

As we discussed above, there may be some differences between offline and online prices. One justification retailers have to price differently across outlets is the existence of shipping and handling expenses. All non-digital products bought online have to be processed and shipped to the customer's designated location. The related costs can either be borne by the retailer or passed (at least in part) onto the customer. In this section we would like to examine how the existence of shipping costs affects the analysis in the Section 3.3. As previously, p_{S_i} and p_{N_i} denote the prices retailer i sets in each channel and δ denotes the upper bound on the price difference across channels $|p_{N_i} - p_{S_i}| < \delta$. The new component is c_s denoting the shipping costs per unit that the retailer incurs. We allow the maximal difference to cover the shipping costs, that is, $c_s \leq \delta$. The following proposition summarizes the equilibrium.

Proposition 13 *If $c_s + \frac{\mu r(c+r)}{3h} < \delta$, then in equilibrium*

$$\hat{p}_{N_1} = \hat{p}_{N_2} \equiv \hat{p}_{BC^N_\infty} = t + c_s + r \text{ and } \hat{p}_{S_1} = \hat{p}_{S_2} \equiv \hat{p}_{BC^S_\infty} = t + r(1 - \hat{\lambda}_{BC_\infty}), \tag{5}$$
$$\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BC_\infty} = \frac{\mu(c+r)}{3h} \text{ and } \hat{\pi}_1 = \hat{\pi}_2 \equiv \hat{\pi}_{BC_\infty} = \frac{1}{2}(t - h\hat{\lambda}^2_{BC_\infty}).$$

otherwise

$$\hat{p}_{N_1} = \hat{p}_{N_2} \equiv \hat{p}_{BC^N_\delta} = t + c_s + r\left(1 - \mu\left(\hat{\lambda}_{BC_\delta} - \frac{\delta - c_s}{r}\right)\right) \text{ and } \hat{p}_{S_1} = \hat{p}_{S_2} \equiv \hat{p}_{BC^S_\delta} = \hat{p}_{BC^N_\delta} - \delta,$$
$$\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BC_\delta} = \frac{2\mu t(c+r) - 3c(\delta - c_s)\mu(1-\mu)}{6ht - 3cr\mu(1-\mu)} \text{ and } \hat{\pi}_1 = \hat{\pi}_2 \equiv \hat{\pi}_{BC_\delta} = \frac{1}{2}(t - h\hat{\lambda}^2_{BC_\delta}). \tag{6}$$

Proof : The proof follows the same steps as those of Propositions 4 and 5. First, we solve the case in which there is no restriction on the price differences. Given the proposed consumer behavior, firm 1's profits in the two distribution outlets are the following:

$$\pi_1^S = \mu \left(\frac{p_{S_2} - p_{S_1} + (\lambda_1 - \lambda_2)c + t}{2t} \right) (p_{S_1} - r(1 - \lambda_1)) - \frac{1}{2} h \lambda_1^2,$$

$$\pi_1^N = (1 - \mu) \left(\frac{p_{N_2} - p_{N_1} + t}{2t} \right) (p_{N_1} - r - c_s).$$

Differentiating with respect to both store and Internet prices that are set simultaneously, and solving as before, we obtain the equilibrium prices and SA levels. As before, we also make sure that the equilibrium strategies are consistent with consumer behavior.

In order to solve the second part, where the upper bound on the price difference is binding, one can check that the best response functions are such that firms always want to set a higher price online, attaining the maximal possible price difference. Then, setting $p_{S_i} = p_{N_i} - \delta$, we follow the same steps as in the proof of Proposition 2 and obtain the results. □

The equilibrium outcome reveals findings similar to those of Section 3.3. First, if firms have extensive leeway in the prices they can charge across channels, they transfer the entire shipping costs (c_s) *and* the entire expected cost of handling returns (r) onto consumers through much higher prices on the Internet than in the stores. If only limited discrepancy in prices is allowed ($c_s \leq \delta < c_s + \frac{\mu r(c+r)}{3h}$), firms set the price difference to the maximum possible amount. That is, they transfer shipping costs onto online customers and also try to transfer part of the expected return costs. In the special (yet plausible) case of $\delta = c_s$, that is, charging customers more up to the actual shipping and handling costs incurred is acceptable— firms price exactly c_s dollars higher online than offline. Importantly, the equilibrium solution in this case— in terms of store prices, SA levels and firm profits— is otherwise equivalent to the Bricks & Clicks case with equal prices across channels and no shipping costs (per Table 3).

We now examine the effects of variable costs on the Brick & Clicks model. If variable costs are identical online and offline then the results essentially do not change. Let MC denote the variable costs. Then the store assistance levels and profits remain the same as in Section 3.3, whereas prices increase by MC. Let us now assume that marginal costs are different online and offline and let ΔMC denote the difference (positive offline is higher). Then we obtain that

$$\hat{\lambda}_{BC}^{\Delta MC} = \frac{2\mu t(c+r) - 3\mu(1-\mu)c\Delta MC}{6ht - 3cr\mu(1-\mu)}, \ \ \hat{p}_{BC}^{\Delta MC} = MC - (1-\mu)\Delta MC + t + r(1 - \mu\hat{\lambda}_{BC}^{\Delta MC})$$

$$\hat{\pi}_{BC}^{\Delta MC} = \frac{1}{2}(t - h(\hat{\lambda}_{BC}^{\Delta MC})^2).$$

It is clear that a higher marginal cost online leads to lower store assistance levels, since offline consumers are less attractive to the firms. Consequently the marginal cost difference results in a profit increase when cost are lower online.

3.4 Returns Policy: Imposing a Restocking Fee

In this section we examine the possibility of the retailer imposing a restocking fee on returned products. Although customers only incur this cost if they return the product, at the time of purchase they (and the retailers) only know the probability of mismatch. Therefore, the players only take into account the expected restocking expenses when making their decisions. Note that the existence of a restocking fee is similar to the case that we examined in Section 3.2 of this Technical Appendix, where retailers could charge different prices across outlets to account for shipping costs. In this case, if the retailer introduces a restocking fee it is a higher expected burden for online customers because the probability of fit is lower, therefore the expected amount spent on incurring the restocking fee is higher for purchases made online. Formally, we use a similar model as in Section 3.2. We allow the retailer to charge a restocking fee of r_f, which is bounded by δ_f. Thus, similar to the case of shipping costs, we assume that the retailers cannot charge arbitrarily higher online (for the product + its expected returns charge); we call the difference in the amount paid for a returned vs. non-returned product the restocking fee. Furthermore, we assume again that $k_H > c$. Otherwise, the model is the same as in Section 3.3. The following proposition gives the results when the maximal restocking fee is not too high.[5]

Proposition 14 *If δ_f is not too high, then in equilibrium the retailers will charge the highest possible restocking fee ($r_{f_1} = r_{f_2} = \delta_f$) and*

$$\begin{aligned}\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BCr} &= \frac{2\mu t(c+r)}{6ht - 3(r-\delta_f)(c+\delta_f)\mu(1-\mu)},\\ \hat{p}_1 = \hat{p}_2 \equiv \hat{p}_{BCr} &= t + (r-\delta_f)(1-\mu\hat{\lambda}_{BCr}),\\ \hat{\pi}_1 = \hat{\pi}_2 \equiv \hat{\pi}_{BCr} &= \frac{1}{2}(t - h\hat{\lambda}_{BCr}^2).\end{aligned} \tag{7}$$

[5] If, for example, $t = h = r = c = 1$ and $\mu = 1/2$, then δ_f can be as high as about 20% of the regular Bricks & Clicks equilibrium price. Such a percentage seems in line with restocking fees in practice.

PROOF: We have already seen in the proof of Proposition 12 that when the upper bound on the price difference is binding, the best response functions are such that firms always want to set a higher price online, attaining the maximal possible price difference. In our case this translates to $r_{f_1} = r_{f_2} = \delta_f$. Then, we follow the same steps as in the proof of Proposition 2 and obtain the results. $\square$

It is clear from the equilibrium expressions in the proposition that the existence of such restocking fees does not effect our overall results and the outcome is very similar to the basic Bricks & Clicks case.

3.5 Shopping Assistance Online

We assumed throughout the paper that retailers can only offer store assistance in their physical stores. However, for certain product categories, such as digital goods, it might be more convenient to offer assistance on the Internet and it may involve lower costs to offer the same level of assistance as in physical stores. In this section, we examine the effects of cost differences between offering SA offline and online. Let h_S and h_N denote the scaling parameters of the cost functions in the store and online, respectively. That is, the cost of offering λ_{S_i} assistance offline for firm i is $\frac{1}{2}h_S(\lambda_{S_i})^2$, whereas the cost of offering λ_{N_i} assistance online is $\frac{1}{2}h_N(\lambda_{N_i})^2$. Otherwise, the model is identical to the basic Bricks & Clicks setup with two symmetric firms and we assume that $k_H > c$. The following proposition summarizes the results.

Proposition 15 *If $h_N < h_S$, then firms only offer store assistance online and*

$$\begin{aligned}
\hat{\lambda}_{S_1} &= \hat{\lambda}_{S_2} \equiv 0\,, \;\; \hat{\lambda}_{N_1} = \hat{\lambda}_{N_2} \equiv \hat{\lambda}_{BO} = \frac{c+r}{3h}, \\
\hat{p}_1 &= \hat{p}_2 \equiv \hat{p}_{BO} = t + r(1 - \hat{\lambda}_{BO}), \\
\hat{\pi}_1 &= \hat{\pi}_2 \equiv \hat{\pi}_{BO} = \frac{1}{2}(t - h\hat{\lambda}^2_{BO}).
\end{aligned}$$

If h_N is significantly higher than h_S, then

$$\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{S_{BC}} = \frac{2\mu t(c+r)h_N - (1-\mu)(c+r)cr}{6th_Sh_N - 3rc\mu(1-\mu)(h_S+h_N)},$$

$$\hat{\lambda}_{N_1} = \hat{\lambda}_{N_2} \equiv \hat{\lambda}_{N_{BC}} = \frac{2(1-\mu)t(c+r)h_S - \mu(1-\mu)(c+r)cr}{6th_Sh_N - 3rc\mu(1-\mu)(h_S+h_N)},$$

$$\hat{p}_1 = \hat{p}_2 \equiv \hat{p}_{BC} = t + r(1 - \mu\hat{\lambda}_{S_{BC}} - (1-\mu)\hat{\lambda}_{N_{BC}}),$$

$$\hat{\pi}_1 = \hat{\pi}_2 \equiv \hat{\pi}_{BC} = \frac{1}{2}(t - h_S\hat{\lambda}^2_{S_{BC}} - h_N\hat{\lambda}^2_{N_{BC}}).$$

PROOF: First, let us deal with the case $h_N < h_S$. In this case, a firm's best response is to only offer assistance on the Internet, since it can offer higher assistance online for the same cost and even with the same level of assistance online it would be able to offer more utility to consumers than in the stores (as consumers save the shopping trip cost online). Given these best responses, the game will be equivalent to the Bricks-Only scenario, but selling everything online instead of offline.

If $h_N > h_S$, then it is not trivial to determine the best responses, since if the difference is low, it might be beneficial to offer all the store assistance online to please the high shopping trip cost customers and attract some of the low shopping trip cost customers. However, if h_N is high enough, the best response is to offer a higher assistance level in the store, leading to a situation similar to that of the Bricks & Clicks case (in the sense that k_L type consumers shop in stores and k_H type consumers shop online; yet both consumer types benefit from the SA provided in the channel they shop at). To derive the equilibrium we proceed as in the proof of Proposition 2. We derive the best response functions for both the online and offline assistance levels. Then we solve for the equilibrium and obtain the above results if the conditions for an internal solution hold. One can check that firms do not have an incentive to offer higher assistance levels online if h_N is high enough, in particular if both of the following inequalities hold:

$$h_N > h_S \frac{1-\mu}{\mu} + \frac{cr(1-\mu)^2}{2\mu t},$$

$$h_N > h_S \frac{\hat{\lambda}_{SBC}^2}{\hat{\lambda}_{SBC}^2 - \hat{\lambda}_{NBC}^2}.$$

□

Note that when h_N is much higher, the equilibrium is very similar to that in the Bricks & Clicks case. As $h_N \to \infty$, we have $\hat{\lambda}_{S_{BC}} \to \hat{\lambda}_{BC}$ (with $h = h_S$) and $\hat{\lambda}_{N_{BC}} \to 0$, that is, in the limit the two equilibria are the same.

3.6 Store Assistance Affects Consumer Utility–

In this section, we analyze a variant of the base model, in which store assistance not only decreases product returns, but increases consumer valuation. In particular, a level of λ_S store assistance increases consumer valuation of the product from v to $v(1+\lambda_S)$. Following the same steps we get the same results as in the basic model. If k_H is high enough,

$$\hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BC} = \frac{2\mu t(v+c+r)}{6ht - 3r(v+c)\mu(1-\mu)},$$

$$\begin{aligned} \hat{p}_1 = \hat{p}_2 \equiv \hat{p}_{BC} &= t + r(1 - \mu\hat{\lambda}_{BC}), \\ \hat{\pi}_1 = \hat{\pi}_2 \equiv \hat{\pi}_{BC} &= \frac{1}{2}(t - h\hat{\lambda}_{BC}^2). \end{aligned} \tag{8}$$

Comparing these results with the Bricks Only case, we get the same results as in Proposition 2 for $t < \frac{\mu(v+c)r}{2h}$.

3.7 Three Consumer Segments with Different Shopping Trip Costs

In this section, we relax the assumption that there are only two segments of consumers with respect to shopping trip costs. Here, we examine a model in which there are three segments: a μ_L proportion of consumers that have a low shopping trip cost, k_L, a μ_M proportion of consumers that have a medium shopping trip cost, k_M, whereas the remaining consumers ($\mu_H = 1 - \mu_L - \mu_M$) have a high shopping trip cost, $k_H > c$. We assume that $0 = k_L < k_M < k_H$ and that shopping trip costs are independent from retailer preferences. Otherwise, the model is identical to the basic Bricks & Clicks model.

Proposition 16 *There exists a $\bar{k}(c, r, h, t, \mu_L, \mu_M) < c$, such that if $\bar{k} \leq k_M$, there is a unique symmetric sub-game perfect equilibrium where k_L type consumers shop in physical stores while k_M and k_H type consumers shop on the Internet. In equilibrium*

$$\begin{aligned} \hat{\lambda}_{S_1} = \hat{\lambda}_{S_2} \equiv \hat{\lambda}_{BC3} &= \frac{2\mu_L t(c+r)}{6ht - 3rc\mu_L(1-\mu_L)}, \\ \hat{p}_1 = \hat{p}_2 \equiv \hat{p}_{BC3} &= t + r(1 - \mu_L\hat{\lambda}_{BC3}), \\ \hat{\pi}_1 = \hat{\pi}_2 \equiv \hat{\pi}_{BC3} &= \frac{1}{2}(t - h\hat{\lambda}_{BC3}^2). \end{aligned} \tag{9}$$

Proof: If $k_M > c$, then both k_M and k_H types always shop on the Internet. Therefore, we can merge these two segments and apply Table 3 with $\mu = \mu_L$. If, however, $k_M < c$ then we have to follow through the steps of the proof of Proposition 2. Since k_H type consumers always shop on the Internet in this setup, one can see that the proof of Proposition 2 can be trivially modified to obtain the above results. □

3.8 Different Probabilities of Product Match Across Consumers

In this section, we show that if consumers have different probabilities of returning the product they buy, then the situation is equivalent to the basic model as long as retailers are not allowed to price discriminate. It is plausible to assume this because it would be very difficult for the firms to offer different prices based on the ex-ante probability of the product matching a consumer's taste/needs; in most cases, such heterogeneity in product match is further unobservable.[6] We assume that consumers know their probability of fit which is drawn from a random distribution with mean λ and is independent from other preferences. Furthermore, we assume that $k_H > c$. Then, the analysis of this extended model is equivalent to the basic Bricks & Clicks model. Since a consumer has the same expected probability of fit for both retailers, it does not affect the comparison between the two retailers. The probability may influence whether the person shops online or offline, but if k_H is high enough, then k_H type consumers always shop online. Since firms cannot price discriminate based on the probability of product match (and they cannot even identify individual consumers' probabilities) only the expected probability of fit matters, which is λ. In this way, the model is equivalent to the basic Bricks & Clicks setup.

3.9 Consumers who Enjoy Shopping Online

In this section, we examine what happens if there is a consumer segment that likes shopping online. That is, these consumers not only have a high cost of visiting the physical store, but gain extra utility from shopping online. Specifically, we assume that a ν proportion of k_H type consumers gain e extra utility from shopping online.[7] This modification creates a third segment that prefers shopping online even more than regular k_H type consumers do. Although we do not solve this model fully, we note that if $k_H + e > c$, then the solution is equivalent to that in Section 3.7 of this Technical Appendix with $\mu_L = \mu$, $\mu_M = (1-\nu)(1-\mu)$, $k'_M = k_H$, and $k'_H = k_H + e$, therefore, we can apply Proposition 16.

3.10 Limited Consumer Access to the Internet

Finally, we examine the effect of limited Internet access on our results. We assume that (independently from other preferences and costs) only a $0 < \kappa < 1$ proportion of consumers has access to the Internet and the remaining consumers can only purchase offline. Otherwise, the model is identical to the basic Bricks & Clicks setup. Since consumers with no Internet access cannot buy online, and we assume that they have high enough valuations for the products, they will all buy a product in one of the physical stores. It is easy to check that

[6] In some cases, perhaps online, sellers could use previous purchase and return data to estimate these probabilities per individual. Even then, it is difficult to imagine that consumer advocate groups would allow price discrimination based on this form of heterogeneity.

[7] We could also assume that there are some consumers who have no cost of shopping in stores, but gain extra utility from shopping online.

this model will be equivalent to the basic Brick & Clicks setup with $\mu' = \mu + (1-\kappa)(1-\mu)$. That is, including a segment of customers with no Internet access is equivalent to having more low shopping trip cost consumers.

3.11 Consumer Risk Aversion

Throughout the paper we assume that both firms and consumers are risk neutral. However, consumers might exhibit risk averse or even risk seeking behaviors when considering the possibility of returns. Furthermore, the extent of risk aversion may vary across channels as different type of consumers shop in different outlets. Without assuming specific risk averse or risk seeking behavior, we show that our main results hold independently of consumer risk aversion.

Let y denote the payoff of a particular consumer. We will decompose the payoff to a certain and a risky component. If consumers shop in the physical stores of retailer 1 or 2, they receive payoffs of

$$\underline{y}_{S_1} = v - p_1 - tx - k_j - m, \;\; \underline{y}_{S_2} = v - p_1 - t(1-x) - k_j - m \tag{10}$$

for certain, even if there is a return. If they shop online then the certain payoffs are

$$\underline{y}_{N_1} = v - p_1 - tx - m, \;\; \underline{y}_{S_2} = v - p_1 - t(1-x) - m. \tag{11}$$

Let F_{S_i} and F_{N_i} denote the random variables that indicate the occurrence of a fit offline and online (taking the value of 1 if there is a fit, 0 otherwise). Then, consumers receive the following payoffs offline and online:

$$y_{S_i} = \underline{y}_{S_i} + F_{S_i} m, \quad y_{N_i} = \underline{y}_{N_i} + F_{N_i} m. \tag{12}$$

To model consumers' risk aversion we assume that they use $\underline{y}$ as a reference point, since they receive that amount of payoff for certain. Let the increasing $U(y)$ function denote their utility over the uncertain portion of the payoff (with $U(0) = 0$). That is, a consumer's total utility is $\underline{y}_{S_i} + U(F_{S_i} m)$, or $\underline{y}_{N_i} + U(F_{N_i} m)$ in the store and on the Internet, respectively. If $U(y)$ is a linear function of y, that is, consumers are risk neutral then expected utility is simply $\underline{y} + m\ \mathbf{Pr}(F = 1)$, where $\mathbf{Pr}(F_{S_i} = 1) = 1 - (1 - \lambda_{S_i})(1 - \lambda)$ and $\mathbf{Pr}(F_{N_i} = 1) = \lambda$, as in our main model, yielding the formulas in (1) and (2).

However, if consumers are risk averse (or risk seeking), $U()$ may be concave (or convex). Furthermore, consumers with a preference for online shopping may have a different utility function. Let $U_H()$ denote the utility function of high shopping trip customers and let $U_L()$

denote that of the low shopping trip customers. The more concave a consumer's utility function the higher s/he will perceive the expected costs of a return. However, since we are in a duopoly setting, this effect is the same for both firms. What matters for firms at the stage when deciding about store assistance levels is the sensitivity of consumers to a change in return probability. Since risk averse consumers value less the extra benefit of not having to return, they will be less sensitive to investments in store assistance levels. With these, consumer utilities for a type $j \in \{L, H\}$ consumer become

$$U_{S_1} = \underline{y}_{S_1} + (1 - (1 - \lambda_{S_1})(1 - \lambda))U_j(m) = v - p_1 - tx - k'_j - (1 - \lambda_{S_i})c_j, \tag{13}$$

$$U_{S_2} = \underline{y}_{S_2} + (1 - (1 - \lambda_{S_2})(1 - \lambda))U_j(m) = v - p_1 - t(1 - x) - k'_j - (1 - \lambda_{S_i})c_j, \tag{14}$$

$$U_{N_1} = \underline{y}_{N_1} + \lambda U_j(m) = v - p_1 - tx + l_j - c_j, \ \ and \tag{15}$$

$$U_{N_2} = \underline{y}_{N_2} + \lambda U_j(m) = v - p_1 - t(1 - x) + l_j - c_j, \tag{16}$$

where $c_j = (1 - \lambda)U_j(m)$, $k'_j = k_j + m - U_j(m)$ and $l_j = m - U_j(m)$. It is easy to see that this model is very similar to the one specified in (1) and (2). Determining the equilibria can be achieved exactly as in the basic model and the solutions show that as long as k_H is high enough, we get the same results as in that model, but with $c = c_L$. If $k_H > c_H$, high shopping trip cost consumers always shop online and low shopping trip consumers shop in the store. That is, unless high shopping trip cost consumers are very risk seeking they do not go to the physical store and in equilibrium consumers self select to the appropriate outlets. In these equilibria the outcome only depends on the offline consumers' risk aversion since firms can only compete on store assistance offline. In general, the more risk averse these consumers are, the lower the store assistance levels. This is due to the above mentioned reduction in sensitivity to store assistance investments. Otherwise, the results of Proposition 2 hold as before. When the differentiation between the two retailers is low, store assistance levels can go up and profits can go down when opening online arms. The level of differentiation below which this happens ($\bar{t} = \frac{\mu c_L r}{2h}$) decreases as offline consumers are more risk averse.

3.12 Consumers Exiting the Market after a Return

Our main model describes the effects of product mismatch and subsequent product return. Our modeling captures the idea that consumers may not buy the product that best fits them initially, and that better store assistance enhances the chances of product match. We further assume that the store carries the model/design/variant that fits the consumer, and that after incurring a return the consumer will get the item that fits. In this extension, we take another route and assume that consumers who buy a product that does not fit return it, get their money back and exit the market immediately. (If the store does indeed carry a product variant that fits the consumer, as we assume, such behavior is unlikely; though for completeness we study it here).

In order to accommodate this assumption, we have to slightly modify the utility functions of consumers to reflect that they only pay the price and enjoy the benefits of the product if it fits upon initial purchase. Therefore, consumers get the following utility in the store:

$$U_{S_1}(x, k_j) = (v - p_1 - tx)\left(1 - (1 - \lambda_{S_1})(1 - \lambda)\right) - (1 - \lambda_{S_1})(1 - \lambda)m - k_j, \tag{17}$$

$$U_{S_2}(x, k_j) = (v - p_2 - t(1 - x))\left(1 - (1 - \lambda_{S_2})(1 - \lambda)\right) - (1 - \lambda_{S_2})(1 - \lambda)m - k_j. \tag{18}$$

When the consumer buys on the Internet, the expected utility from either retailer is

$$U_{N_1}(x, k_j) = (v - p_1)(1 - \lambda) - tx - (1 - \lambda)m, \quad U_{N_2}(x, k_j) = (v - p_2 - t(1 - x))(1 - \lambda) - (1 - \lambda)m. \tag{19}$$

Since firms have to refund the price in case of a return, their profit functions also change.

$$\pi_i = \left(p_i\left(1 - (1 - \lambda_{S_i})(1 - \lambda)\right) - (1 - \lambda_{S_i})(1 - \lambda)g\right)D_{S_i}(\vec{p}, \vec{\lambda}) + (p_i\lambda - (1 - \lambda)g)D_{N_i}(\vec{p}, \vec{\lambda}) - 1/2h(\lambda_{S_i})^2. \tag{20}$$

In order to solve for the equilibria, we follow the same steps as in the basic model. We calculate the equilibrium prices given fixed store assistance levels and plug that into the profit function. Because the profit functions become high degree polynomials of the decision variables (in particular store assistance levels) there is no closed form solution for the equilibria. Given the high level of complexity, we determined the equilibria numerically for given parameter levels. Our main goal is to see whether similar results can hold as in our basic model. In order to achieve this, we fix the parameters m, g, h, μ, λ and plot store assistance levels and profits as functions of t. Figure 3.12 shows the results. It depicts the symmetric equilibrium store assistance levels and profits for both the Bricks Only and Bricks and Clicks cases. The figure shows that indeed we get similar results as in our main model. When t is high stores invest more in assistance when they have no online arms (BO case) and make less profits, whereas when t is low, store assistance levels are higher in the presence of online outlets (B&C case) and profits are lower than when there is no Internet channel.

4 Comparative Statics for the Basic Model Duopoly Cases

In the paper we discussed the comparative statics with respect to μ and h. We now discuss other comparative static findings. First note that firms increase SA levels the greater the

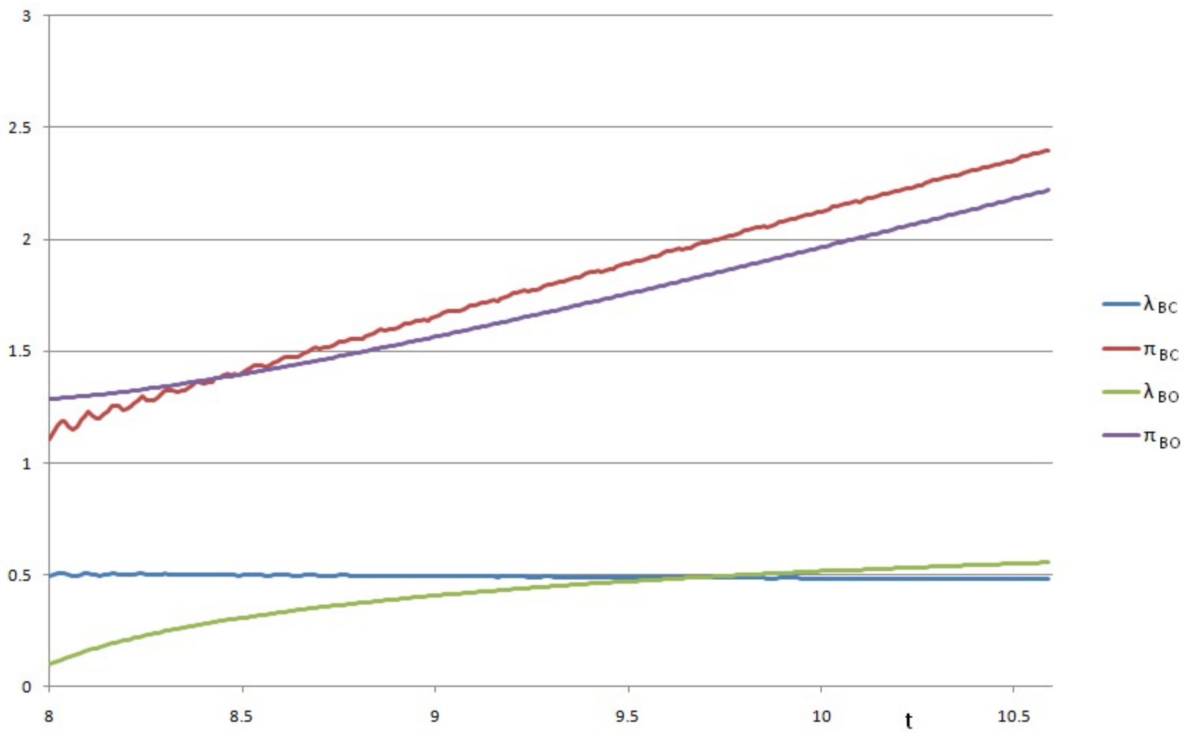

Figure 6: λ_{BO}, λ_{BC}, π_{BO} and π_{BC} as functions of t where $\lambda = 0.8$, $c = 5.3$, $r = 10$, $h = 9.6$, $\mu = 0.54$.

Table 7: Summary of Comparative Statics

	Bricks-Only			Bricks & Clicks		
	$\hat{\lambda}_{BO}$	$\hat{p}_{BO}$	$\hat{\pi}_B$	$\hat{\lambda}_{BC}$	$\hat{p}_{BC}$	$\hat{\pi}_{BC}$
Parameter: μ	0	0	0	$+\ iff$ $\mu < \underline{\mu}$	$+\ iff$ $\mu > 2\underline{\mu}^2$	$+iff$ $\mu > \underline{\mu}$
c	$+$	$-$	$-$	$+$	$-$	$-$
r	$+$	$+\ iff$ $\frac{1}{2}(3h-c) > r$	$-$	$+$	$+\ iff$ $\frac{1}{2}(3h-c) > r$	$-$
t	0	$+$	$+$	$-$	$+$	$+$
h	$-$	$+$	$+$	$-$	$+$	$+$

Signs reflect the derivative of each equilibrium quantity with respect to the parameters in the first column. Note again that λ, the baseline probability of fit, is negatively (and linearly) related to c and r.

baseline expected cost of returns to the consumer or to the firm (c, r). The impact of the latter is intuitive: when r increases firms wish to directly reduce expected supply-side costs by lowering the probability of returns taking place. Increasing SA levels when c increases stems from the need to mitigate the decrease in willingness to pay of consumers (demand-side costs).[8] Note that prices are a function of the expected returns costs, through the terms $r(1-\hat{\lambda}_{BO})$ and $r(1-\mu\hat{\lambda}_{BO})$. This is because from the firm's perspective, the possibility that each product sold may be returned acts like a "variable cost" of doing business, which is at least partly passed on to the consumer. Whenever firms increase SA levels, and thus reduce the probability of returns, this has a negative effect on prices (much like the effect of a drop in variable cost in the standard hotelling model). This leads firms to lower prices as c increases (through the dependence of $\hat{\lambda}$ on c as explained above). Note though that the impact of an increase in r on prices has both a direct positive effect (because of the desire to transfer return costs onto consumers) and an indirect negative effect (through $\hat{\lambda}$). Initially as r increases firms will prefer to pass on some of the expense of handling returns onto consumers through higher prices. But as the expected cost of handling a return (r) goes up further, firms will be induced to manage the high returns costs by increasing SA levels. Since the indirect effect resulting from increasing SA level dominates, firms will now *decrease* prices as r rises; leading to an inverted-U pattern for prices as a function of r.

[8] Recall that $r \equiv (1-\lambda)g$ and $c \equiv (1-\lambda)m$. Hence, the equilibrium price and SA level are a function of λ and as the ex-ante probability of a return goes up retailers will offer more in store assistance, i.e., $\frac{\partial\hat{\lambda}_{BO}}{\partial(1-\lambda)} > 0$.