Technical Appendix: Uninformative Advertising as an Invitation to Search

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1 Proof of Lemma 2

To prove Lemma 2 in the Appendix, we first demonstrate the following results which provide a range of prices and beliefs under which search is the best response for the consumer.

Lemma 1 *[TA1] Assume that* $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$.

1. *Consider the case where the firm engages in attribute-focused advertising,* $A_1 = (a_j,\, p)$ *and the consumer's belief is* $\mu^j = (0, \mu_M^j, \mu_H^j)$*. Consumer chooses to search iff*

$$\begin{aligned} \underline{p}^j(\mu_H^j) &\leq p \leq \overline{p}^j(\mu_H^j) \\ \underline{\mu} &\leq \mu_H^j \leq \overline{\mu} \end{aligned}$$

where $\underline{p}^j(\mu_H^j) = \frac{\mu_H^j(1-\gamma_H)\overline{V} + (1-\mu_H^j)(1-\gamma_M)\frac{\overline{V}}{2} + c}{\mu_H^j(1-\gamma_H) + (1-\mu_H^j)(1-\gamma_M)}$, $\overline{p}^j(\mu_H^j) = \frac{\mu_H^j\gamma_H\overline{V} + (1-\mu_H^j)\gamma_M\frac{\overline{V}}{2} - c}{\mu_H^j\gamma_H + (1-\mu_H^j)\gamma_M}$, $\underline{\mu} = \frac{1}{2} - \frac{\sqrt{\frac{\overline{V}^2}{4}(\gamma_H-\gamma_M)^2 - 2\overline{V}c(\gamma_H-\gamma_M)}}{(\gamma_H-\gamma_M)\overline{V}} \geq 0$, $\overline{\mu} = \frac{1}{2} + \frac{\sqrt{\frac{\overline{V}^2}{4}(\gamma_H-\gamma_M)^2 - 2\overline{V}c(\gamma_H-\gamma_M)}}{(\gamma_H-\gamma_M)\overline{V}} \leq 1$.

2. *Consider the case when the firm engages in uninformative advertising,* $A_1 = (a_0,\, p)$ *and the consumer's belief is* $\mu^0 = (\mu_L^0, \mu_M^0, \mu_H^0)$*. Consumer chooses to search iff*

$$\begin{aligned} &\underline{p}^0(\mu^0) \leq p \leq \overline{p}^0(\mu^0) \\ &\underline{\mu}(\mu_L^0) \leq \mu_H^0 \leq \overline{\mu}(\mu_L^0) \\ \textit{and} \quad &\mu_L^0 \leq \frac{1}{2}\left(1 + \sqrt{1 - \frac{4c}{\overline{V}(\gamma_H - \gamma_L)}}\right), \end{aligned}$$

where $\underline{p}^0(\mu^0) = \frac{\mu_H^0(1-\gamma_H)\overline{V} + \mu_M^0(1-\gamma_M)\frac{\overline{V}}{2} + c}{\mu_H^0(1-\gamma_H) + \mu_M^0(1-\gamma_M) + \mu_L^0(1-\gamma_L)}$, $\overline{p}^0(\mu^0) = \frac{\mu_H^0\gamma_H\overline{V} + \mu_M^0\gamma_M\frac{\overline{V}}{2} - c}{\mu_H^0\gamma_H + \mu_M^0\gamma_M + \mu_L^0\gamma_L}$, $\underline{\mu}(\mu_L^0) = \frac{1}{2}(1 + \frac{(\gamma_H-\gamma_L)\mu_L^0}{(\gamma_H-\gamma_M)} - \frac{\Psi(\mu_L^0)}{(\gamma_H-\gamma_M)\overline{V}})$, $\overline{\mu}(\mu_L^0) = \frac{1}{2}\left(1 + \frac{(\gamma_H-\gamma_L)\mu_L^0}{(\gamma_H-\gamma_M)} + \frac{\Psi(\mu_L^0)}{(\gamma_H-\gamma_M)\overline{V}}\right)$, *and* $\Psi(\mu_L^0) = (\overline{V}^2(\gamma_H - \gamma_M + \mu_L^0(\gamma_H - \gamma_L))^2 + 4\overline{V}(\gamma_H - \gamma_M)(\overline{V}(\gamma_M - \gamma_L)(1 - \mu_L^0)\mu_L^0 - 2c))^{\frac{1}{2}}$.

Proof.

1. Consider the case the firm sends an attribute advertising, $A_1 = (a_j,\, p)$.

   First, suppose that $E(V|\mu_H^j) - p \geq 0 \Leftrightarrow \overline{V}\mu_H^j + \frac{\overline{V}}{2}(1 - \mu_H^j) \geq p$. Consumer will search if $c \leq \Pr(\underline{s}|\mu_H^j)[p - E(V|\mu_H^j, \underline{s})]$ or

$$p \geq \frac{\mu_H^j(1-\gamma_H)\overline{V} + (1-\mu_H^j)(1-\gamma_M)\frac{\overline{V}}{2} + c}{\mu_H^j(1-\gamma_H) + (1-\mu_H^j)(1-\gamma_M)} \equiv \underline{p}^j(\mu_H^j) \tag{1}$$

If the price is sufficiently low such that $p < \min(\underline{p}^j(\mu_H^j), E(V|\mu_H^j))$, the consumer will not search but still purchase without search since $p \leq E(V|\mu_H^j)$. Hence, consumer will search if

$$\underline{p}^j(\mu_H^j) < p \leq E(V|\mu_H^j). \tag{2}$$

Note that at $p = \underline{p}^j(\mu_H^j)$, the consumer's best response is to mix between "purchasing the product without search" and "search". Hence, the function $p = \underline{p}^j(\mu_H^j)$ specifies the indifference curve for the consumer between "purchasing the product without search" and "search" when $p \leq E(V|\mu_H^j)$.

By equating the lower and the upper bound on p, we can show that the consumer will search as long as $\underline{\mu} \leq \mu_H^j \leq \overline{\mu}$, where $\underline{\mu} = \frac{1}{2} - \frac{\sqrt{\frac{\overline{V}^2}{4}(\gamma_H-\gamma_M)^2-2\overline{V}c(\gamma_H-\gamma_M)}}{(\gamma_H-\gamma_M)\overline{V}}$ and $\overline{\mu} = \frac{1}{2} + \frac{\sqrt{\frac{\overline{V}^2}{4}(\gamma_H-\gamma_M)^2-2\overline{V}c(\gamma_H-\gamma_M)}}{(\gamma_H-\gamma_M)\overline{V}}$. Note that the above set of beliefs is not empty if $c \leq \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$ (since $c \leq \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$ implies that $\sqrt{\frac{\overline{V}^2}{4}(\gamma_H-\gamma_M)^2 - 2\overline{V}c(\gamma_H-\gamma_M)} \geq 0$). Also note that $[\underline{\mu}, \overline{\mu}] \subseteq [0,1]$ since $\frac{\sqrt{\frac{\overline{V}^2}{4}(\gamma_H-\gamma_M)^2-2\overline{V}c(\gamma_H-\gamma_M)}}{(\gamma_H-\gamma_M)\overline{V}} \leq \frac{1}{2}$. Second, suppose that $E(V|\mu_H^j) - p < 0$ or $\overline{V}\mu_H^j + \frac{\overline{V}}{2}(1-\mu_H^j) < p$. Consumer will search if $c \leq \Pr(\overline{s}|\mu_H^j)[E(V|\mu_H^j, \overline{s}) - p]$ or

$$p \leq \frac{\mu_H^j\gamma_H\overline{V} + (1-\mu_H^j)\gamma_M\frac{\overline{V}}{2} - c}{\mu_H^j\gamma_H + (1-\mu_H^j)\gamma_M} \equiv \overline{p}^j(\mu_H^j) \tag{3}$$

If the price is sufficiently high such that $p > \max(\overline{p}^j(\mu_H^j), E(V|\mu_H^j))$, the consumer will not search and will not purchase a product since $p > E(V|\mu_H^j)$. Hence, consumer will search if

$$E(V|\mu_H^j) < p < \overline{p}^j(\mu_H^j). \tag{4}$$

Also note that at $p = \overline{p}^j(\mu_H^j)$, the consumer's best response is to mix between "not purchasing the product" and "search". The function $p = \overline{p}^j(\mu_H^j)$ specifies the indifference curve for the consumer between "not purchasing the product" and "search" when $p > E(V|\mu_H)$. By equating the lower and the upper bound on p, we can show that the consumer will search as long as $\underline{\mu} \leq \mu_H^j \leq \overline{\mu}$: which yields the same bounds on beliefs and prices as above. Hence, combining the equations (2) and (4) suggests that when $c \leq \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$, if $\underline{p}^j(\mu_H^j) \leq p \leq \overline{p}^j(\mu_H^j)$ and $\mu_H^j \in [\underline{\mu}, \overline{\mu}]$, search is the consumer's best response.

At $[\underline{\mu}, \underline{p}^j]$, and at $[\overline{\mu}, \overline{p}^j]$, the consumer's best response is to either mix between "search" and "not buying the product" or mix between "search" and "buying the product."

2. Consider the case the firm sends a uninformative advertising, $A_1 = (a_0, \, p)$.

First, suppose that $E(V|\mu^0) - p \geq 0 \Leftrightarrow \overline{V}\mu_H^0 + \frac{\overline{V}}{2}\mu_M^0 \geq p$. Consumer will search if $c \leq \Pr(\underline{s}|\mu^0)[p - E(V|\mu^0, \underline{s})]$ or

$$p \geq \frac{\mu_H^0(1-\gamma_H)\overline{V} + \mu_M^0(1-\gamma_M)\frac{\overline{V}}{2} + c}{\mu_H^0(1-\gamma_H) + \mu_M^0(1-\gamma_M) + \mu_L^0(1-\gamma_L)} \equiv \underline{p}^0(\mu^0) \tag{5}$$

If the price is sufficiently low such that $p < \min(\underline{p}^0(\mu^0), E(V|\mu^0))$, the consumer will not search but still purchase without search since $p \leq E(V|\mu^0)$. Hence, consumer will search if

$$\underline{p}^0(\mu^0) \; < p \leq E(V|\mu^0). \tag{6}$$

Note that at $p = \underline{p}^0(\mu^0)$, the consumer's best response is to mix between "purchasing the product without search" and "search". Hence, the function $p = \underline{p}^0(\mu^0)$ specifies the indifference curve for the consumer between "purchasing the product without search" and "search" when $p \leq E(V|\mu^0)$. By equating the lower and the upper bound on p for a given μ_L, we can show that for a given μ_L, the consumer will search as long as $\underline{\mu}(\mu_L^0) \leq \mu_H^0(\mu_L) \leq \overline{\mu}(\mu_L^0)$, where $\underline{\mu}(\mu_L^0) = \frac{1}{2}\left(1 + \frac{(\gamma_H-\gamma_L)\mu_L^0}{(\gamma_H-\gamma_M)} - \frac{\Psi(\mu_L^0)}{(\gamma_H-\gamma_M)\overline{V}}\right) \leq \overline{\mu}(\mu_L^0) = \frac{1}{2}\left(1 + \frac{(\gamma_H-\gamma_L)\mu_L^0}{(\gamma_H-\gamma_M)} + \frac{\Psi(\mu_L^0)}{(\gamma_H-\gamma_M)\overline{V}}\right)$, and $\Psi(\mu_L^0) = \sqrt{\overline{V}^2(\gamma_H - \gamma_M + \mu_L^0(\gamma_H-\gamma_L))^2 + 4\overline{V}(\gamma_H-\gamma_M)(\overline{V}(\gamma_M-\gamma_L)(1-\mu_L^0)\mu_L^0 - 2c)}$. Note that $\Psi(\mu_L^0) \geq 0 \Leftrightarrow c \leq \frac{\overline{V}\left(\gamma_H-\gamma_M+2(\gamma_H-3\gamma_L+2\gamma_M)\mu_L^0\right)}{8} + \frac{\overline{V}\left((\gamma_H+\gamma_L-2\gamma_M)^2(\mu_L^0)^2\right)}{8(\gamma_H-\gamma_M)}$. Here, we can see that $c \leq \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$ implies $c \leq \frac{\overline{V}\left(\gamma_H-\gamma_M+2(\gamma_H-3\gamma_L+2\gamma_M)\mu_L^0\right)}{8} + \frac{\overline{V}\left((\gamma_H+\gamma_L-2\gamma_M)^2(\mu_L^0)^2\right)}{8(\gamma_H-\gamma_M)}$ for any $\mu_L^0 \in [0,1]$, because $(\gamma_H - 3\gamma_L + 2\gamma_M)\mu_L^0 \geq 0$ and $\frac{(\gamma_H+\gamma_L-2\gamma_M)^2(\mu_L^0)^2}{\gamma_H-\gamma_M} \geq 0$. Hence, if $c \leq \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$, the above set of beliefs, $\left[\underline{\mu}(\mu_L^0), \overline{\mu}(\mu_L^0)\right]$, is not empty for a given $\mu_L^0 \in [0,1]$.

We also have to consider another constraint on the set of beliefs: as the level of μ_L^0 increases, the feasible range for $\mu_H^0(\mu_L)$ gets smaller since $\mu_H^0(\mu_L) + \mu_L^0 \leq 1$. Hence, at the large value of μ_L^0 (for example, $\mu_L^0 = 1$), the corresponding $\mu_H^0(\mu_L) \in \left[\underline{\mu}(\mu_L^0), \overline{\mu}(\mu_L^0)\right]$, which ensures the search is not simply feasible. Given $\mu_L^0 \in [0,1]$, for the corresponding $\mu_H^0(\mu_L)$ to exist, it must be that $\underline{\mu}(\mu_L^0) + \mu_L^0 \leq 1$.

$$\underline{\mu}(\mu_L^0) + \mu_L^0 \leq 1 \Leftrightarrow \mu_L^0 \leq \frac{1}{2}\left(1 + \sqrt{1 - \frac{4c}{\overline{V}(\gamma_H-\gamma_L)}}\right) \tag{7}$$

Note that the above set of beliefs is not empty since $c \leq \frac{\overline{V}(\gamma_H-\gamma_M)}{8} \leq \frac{\overline{V}(\gamma_H-\gamma_L)}{4}$.

Second, suppose that $E(V|\mu^0) - p < 0$ or $\overline{V}\mu_H^0 + \frac{\overline{V}}{2}\mu_M^0 < p$. Consumer will search if $c \leq \Pr(\overline{s}|\mu^0)[E(V|\mu^0, \overline{s}) - p]$ or

$$p \leq \frac{\mu_H^0 \gamma_H \overline{V} + \mu_M^0 \gamma_M \frac{\overline{V}}{2} - c}{\mu_H^0 \gamma_H + \mu_M^0 \gamma_M + \mu_L^0 \gamma_L} \equiv \overline{p}^0(\mu^0) \tag{8}$$

If the price is sufficiently high such that $p > \max(\overline{p}^0(\mu^0), E(V|\mu^0))$, the consumer will not search and will not purchase a product since $p > E(V|\mu^0)$. Hence, consumer will search if

$$E(V|\mu^0) < p < \overline{p}^0(\mu^0). \tag{9}$$

Also note that at $p = \overline{p}^0(\mu^0)$, the consumer's best response is to mix between "not purchasing the product" and "search". The function $p = \overline{p}^0(\mu^0)$ specifies the indifference curve for the consumer between "not purchasing the product" and "search" when $p > E(V|\mu^0)$.

By equating the lower and the upper bound on p, we can show that the consumer will search as long as $\underline{\mu}(\mu_L^0) \leq \mu_H^0 \leq \overline{\mu}(\mu_L^0)$: which yields the same bounds on beliefs and prices as above. Hence, combining the equations (6) and (9) suggests that when $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$, if $\underline{p}^0(\mu_L^0) \leq p \leq \overline{p}(\mu_L^0)$ and $\underline{\mu}(\mu_L^0) \leq \mu_H^0 \leq \overline{\mu}(\mu_L^0)$, search is the consumer's best response.

Finally, at $[\underline{\mu}(\mu_L^0), \underline{p}^0(\mu_L^0)]$, and at $[\overline{\mu}(\mu_L^0), \overline{p}(\mu_L^0)]$, the consumer's best response is to either mix between "search" and "not buying the product" or mix between "search" and "buying the product."

■

Using this result, we can easily prove Lemma 2 (in the Appendix of main body of the paper).

1. From Lemma TA1 above, it directly follows that there exists a belief ($\underline{\mu} \leq \mu_H^j \leq \overline{\mu}$) under which search is the best response for the consumer as long as $\underline{p}^j \leq p \leq \overline{p}^j$, where $\underline{p}^j = \underline{p}^j(\underline{\mu}) = \frac{3}{4}\overline{V} - \frac{\sqrt{\frac{\overline{V}^2}{4}(\gamma_H - \gamma_M)^2 - 2\overline{V}c(\gamma_H - \gamma_M)}}{2(\gamma_H - \gamma_M)} \geq \frac{\overline{V}}{2}$ and $\overline{p}^j = \overline{p}^j(\overline{\mu}) = \frac{3}{4}\overline{V} + \frac{\sqrt{\frac{\overline{V}^2}{4}(\gamma_H - \gamma_M)^2 - 2\overline{V}c(\gamma_H - \gamma_M)}}{2(\gamma_H - \gamma_M)} \leq \overline{V}$. Moreover, when $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$, the region is non-empty. The rest directly follows from the Proposition above.

2. Similarly, it directly follows from Lemma TA1 that there exists a belief ($\underline{\mu}(\mu_L^0) \leq \mu_H^0 \leq \overline{\mu}(\mu_L^0)$) under which search is the best response for the consumer as long as $\underline{p}^0 \leq p \leq \overline{p}^0$ and $\mu_L^0 \leq \frac{1}{2}(1 + \sqrt{1 - \frac{4c}{\overline{V}(\gamma_H - \gamma_L)}})$, where $\underline{p}^0 \equiv \min_{0 \leq \mu_L^0 \leq \widehat{\mu}_L} \underline{p}^0(\mu_L^0) \leq \underline{p}^0(0) = \underline{p}^j$. and $\overline{p}^0 \equiv \max_{0 \leq \mu_L^0 \leq \widehat{\mu}_L} \overline{p}^0(\mu_L^0) \geq \overline{p}^0(0) = \overline{p}^j$. Moreover, when $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8} \leq \frac{\overline{V}(\gamma_H - \gamma_L)}{4}$, the region is non-empty. The rest directly follows from the Proposition above.

Q.E.D.

2 HL Equilibrium without Search

Proposition *A semi-separating HL equilibrium without consumer search does not exist.*

Proof. We show this result by contradiction. Suppose that there exists an HL equilibrium without consumer search: $(a_H = a_L = a_0, p_H = p_L \equiv p^{ns}_{HL}, a_M = a_j, p_M = \frac{\overline{V}}{2})$. Note that in equilibrium, given a prior beliefs, the belief following (a_0, p_{HL}) must be $\mu^0_H = \frac{1}{2}$, $\mu^0_L = \frac{1}{2}$. Hence, applying Lemma 1, we know that $p^{ns}_{HL} \leq \underline{p}^0 = \frac{\frac{1}{2}(1-\gamma_H)\overline{V}+c}{\frac{1}{2}(1-\gamma_H)+\frac{1}{2}(1-\gamma_L)}$. Note that $\underline{p}^0 < \frac{\overline{V}}{2}$ as long as $c < \frac{\overline{V}(\gamma_H-\gamma_L)}{4}$. Hence, this implies that if $c < \frac{\overline{V}(\gamma_H-\gamma_L)}{4}$, $p^{ns}_{HL} < \frac{\overline{V}}{2}$ and H-type prefers to deviate to M's strategy, which would destroy the proposed equilibrium. Finally, note that $\frac{\overline{V}(\gamma_H-\gamma_L)}{4} > \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$. Hence, for $c < \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$, this equilibrium similarly does not exist. Note that even under the general prior (μ^0_H) this equilibrium does not survive *D1*. We adopt this less general proof for brevity. ■

3 Proof of Proposition 5 (Other equilibria: HML and HM)

We prove Proposition 5 of the main body of the paper showing the following two propositions. First proposition deals with HML equilibrium while the second proves the case of HM.

3.1 HML Equilibrium

Proposition HML *Consider HML equilibrium, where all types engage in the same type of advertising and post the same price ($a_\theta = a^*_{HML}$ and $p_\theta = p^*_{HML}$, where $\theta \in \{L, M, H\}$).*

1. *A full pooling equilibrium (HML) without consumer search does not exist.*

2. *A full pooling equilibrium HML where the consumer chooses to search after observing (a_0, p^*_{HML}), exists if γ_M is high enough and the price is in the intermediate range. Here, $\Pi^*(H) > \Pi^*(M) > \Pi^*(L)$. Moreover, this equilibrium survives D1.*

Proof. *1. Proof of part 1:*

Suppose that there exists an HML equilibrium, where all types pool on image advertising and price (a_0, p^{ns}_{HML}), and the consumer does not search. In equilibrium, the beliefs following (a_0, p^{ns}_{HML}) must be equal to the a priori beliefs, which are $\mu^0_H = \frac{\rho}{2}$, $\mu^0_M = 1-\rho$, $\mu^0_L = \frac{\rho}{2}$. According to Lemma 2 in the Appendix, the maximum price that the firms can charge in equilibrium such that the consumer does not search (given these beliefs) is $\underline{p}^0_{HML} = \underline{p}^0(\rho) = \frac{(1-\gamma_H)\rho\overline{V}+(1-\gamma_M)(1-\rho)\overline{V}+2c}{(2-\gamma_H-\gamma_L)\rho+2(1-\gamma_M)(1-\rho)} < \frac{\overline{V}}{2}$. This in turn implies that H and M types would prefer to deviate to $(a_j, p^{dev} = \frac{\overline{V}}{2})$, which credibly signals that the firm is not L type and hence results in purchase. This destroys this potential equilibrium. Note that we can show that even under the most general priors, $\mu^0_H = \alpha$, $\mu^0_M = \beta$, $\mu^0_L = 1-\alpha-\beta$, this equilibrium does not survive D1. We adopt this less general proof mainly for brevity. ■

2. Proof part 2:

Before we turn to the equilibrium conditions, we first examine the restrictions on out-of-equilibrium beliefs that are imposed by D1:

Lemma 2 *[TA2] The out-of-equilibrium beliefs that are consistent with D1 have the following properties:*

1. *When the consumer observes the deviation $A_1 = \{a_j, p^{dev}\}$, where $\max(\gamma_M p^*_{HML}, \underline{p}^j) < p^{dev} < \min(p^*_{HML}, \overline{p}^j)$ or $\gamma_M p^*_{HML} < p^{dev} < \min(\gamma_H p^*_{HML}, \underline{p}^j)$, the consumer forms the off-equilibrium belief that $\mu_M(A_1) = 1$.*

2. *When the consumer observes the deviation $A_1 = \{a_0, p^{dev}\}$,*

   (a) if $\max(\gamma_L p^*_{HML}, \underline{p}^0) < p^{dev} < \min(p^*_{HML}, \overline{p}^0)$ or $\gamma_L p^*_{HML} < p^{dev} < \min(\gamma_M p^*_{HML}, \underline{p}^0)$, the consumer forms the off-equilibrium belief that $\mu_L(A_1) = 1$,

   (b) if $\gamma_M p^*_{HML} < p^{dev} < \min(\gamma_H / p^*_{HML}, \underline{p}^0)$, the consumer forms the off-equilibrium belief that $\mu_H(A_1) = 0$.

Proof. We consider the HML equilibrium with search: all firms choose $(a_0,\ p^*_{HML})$ and the consumer searches in equilibrium. We first derive the restrictions that D1 imposes on the out-of-equilibrium beliefs following the deviation A_1. The consumer's mixed best response has 2 possible forms: (1) $\alpha_2 = \{0, \alpha_{22}, 1-\alpha_{22}\}$ (mixing between no purchase and search), and (2) $\alpha_2 = \{\alpha_{21}, 0, 1-\alpha_{21}\}$ (mixing between purchase and search). In the first case, $\Pi(A_1, \alpha_2, \theta) = (1-\alpha_{22})\gamma_\theta p^{dev}$; in the second case, $\Pi(A_1, \alpha_2, \theta) = (\alpha_{21} + (1-\alpha_{21})\gamma_\theta)\, p^{dev}$. Also, of course, $\Pi^*(\theta) = \gamma_\theta p^*_{HML}$ for $\theta = \{L, M, H\}$.

Let us first define the sets for $\theta = \{L, M, H\}$

$$D^0(\theta, A_1) = X_\theta^0 \cup Y_\theta^0 = \tag{10}$$
$$\left\{(0, \alpha_{22}, 1-\alpha_{22}) \mid \alpha_{22} = \frac{p^{dev} - p^*_{HML}}{p^{dev}}\right\} \cup \left\{(\alpha_{21}, 0, 1-\alpha_{21}) \,|\alpha_{21} = \frac{\gamma_\theta\left(p^*_{HML} - p^{dev}\right)}{(1-\gamma_\theta)p^{dev}}\right\},$$
$$D(\theta, A_1) = X_\theta \cup Y_\theta =$$
$$\left\{(0, \alpha_{22}, 1-\alpha_{22}) \mid \alpha_{22} < \frac{p^{dev} - p^*_{HML}}{p^{dev}}\right\} \cup \left\{(\alpha_{21}, 0, 1-\alpha_{21}) \,|\alpha_{21} > \frac{\gamma_\theta\left(p^*_{HML} - p^{dev}\right)}{(1-\gamma_\theta)p^{dev}}\right\}.$$

One can easily note that $X_L^0 = X_M^0 = X_H^0$, and $X_L = X_M = X_H$. Hence,

$$D^0(\theta, A_1) \cup D(\theta, A_1) = \widehat{X_\theta} \cup \widehat{Y_\theta} = \left(X_H^0 \cup X_H\right) \cup \left(Y_\theta^0 \cup Y_\theta\right) = \tag{11}$$
$$\left\{(0, \alpha_{22}, 1-\alpha_{22}) \mid \alpha_{22} \leq \frac{p^{dev} - p_{HML}}{p^{dev}}\right\} \cup \left\{(\alpha_{21}, 0, 1-\alpha_{21}) \,|\alpha_{21} \geq \frac{\gamma_\theta\left(p_{HML} - p^{dev}\right)}{(1-\gamma_\theta)p^{dev}}\right\}$$

Note that $0 \leq \alpha_{2k} \leq 1$ (where $k \in \{1, 2, 3\}$), and hence some of the sets may be empty depending on the value of p^{dev} relative to p^*_{HML}.

1. Consider a deviation to a price such that the consumer chooses not to purchase at any off-equilibrium belief: $A_1 = \left(a_j, p^{dev}\right)$ where $p^{dev} > \overline{p}^j$ or $A_1 = \left(a_0, p^{dev}\right)$ where $p^{dev} > \overline{p}^0$. Here, none of the types are better off than in equilibrium, which implies that D1 does not apply.

2. Next, consider a deviation to a price such that the consumer chooses to purchase without search at any off-equilibrium belief: $A_1 = \left(a_j, p^{dev}\right)$ where $p^{dev} < \underline{p}^j$ or $A_1 = \left(a_0, p^{dev}\right)$ where $p^{dev} < \underline{p}^0$, i.e., $\alpha_{21} = 1$ and $X_i = \varnothing$ for all $i \in \{L, M, H\}$. Here, a type i is (weakly) better off in deviation than in equilibrium if $p^{dev} \geq \gamma_i p^*_{HML}$ (we can also see this from examining the Y sets). Hence, D1 rules out type j if $\gamma_i p^*_{HML} < p^{dev} < \gamma_j p^*_{HML}$. Therefore, if $A_1 = \left(a_j, p^{dev}\right)$, for all $p^{dev} \in (\gamma_M p^*_{HML}, \min(\gamma_H p^*_{HML}, \underline{p}^j))$, D1 imposes that $\mu_H = 0$. Similarly, if $A_1 = \left(a_0, p^{dev}\right)$, for all $p^{dev} \in (\gamma_L p^*_{HML}, \min(\gamma_M p^*_{HML}, \underline{p}^0))$, $\mu_L = 1$, and $\mu_H = 0$ for all $p^{dev} \in (\gamma_M p^*_{HML}, \min(\gamma_H p^*_{HML}, \underline{p}^0))$.

3. Consider the deviation $A_1 = \left(a_j, p^{dev}\right)$, where $\underline{p}^j \leq p^{dev} \leq \overline{p}^j$ (so that there exists a belief where search is the best response based on Lemma 2).

   (a) Let us first identify the conditions under which $D^0(H, A_1) \cup D(H, A_1) \subset D(M, A_1)$ holds. This condition is equivalent to $\left(\widehat{X_H} \cup \widehat{Y_H}\right) \subset (X_H \cup Y_M)$ using the notation defined above. This would, of course, imply that D1 imposes the out-of-equilibrium belief $\mu_M(A_1) = 1$.

i. Note that $X_H \subseteq \widehat{X_H}$. Hence, it must be the case that $X_H = \widehat{X_H}$. Otherwise, $\left(\widehat{X_H} \cup \widehat{Y_H}\right) \subset (X_H \cup Y_M)$ would not hold. This, in turn, implies that $X_M = \widehat{X_H}$. The condition $X_M = \widehat{X_H}$ can hold if (i) $X_M = \widehat{X_H} = \varnothing$, which implies that $\alpha_{22} < \frac{p^{dev} - p_{HML}}{p^{dev}} < 0 \Leftrightarrow p^{dev} < p^*_{HML}$, or (ii) $\frac{p^{dev} - p^*_{HML}}{p^{dev}} > 1 \Leftrightarrow p^*_{HML} < 0$. Hence, for $X_H = \widehat{X_H}$ to hold under a non-negative equilibrium price, it must be the case that $p^{dev} < p^*_{HML}$.

ii. Note that $\widehat{Y_H} \subseteq Y_M$ since $\gamma_M < \gamma_H$. For the condition $\left(\widehat{X_H} \cup \widehat{Y_H}\right) \subset (X_H \cup Y_M)$ to hold, the range of prices must be such that $\widehat{Y_H} \subset Y_M$ since we just determined that $X_H = \widehat{X_H}$. Hence, the additional conditions needed are that (i) $\frac{\gamma_M\left(p^*_{HML} - p^{dev}\right)}{(1-\gamma_M)p^{dev}} < 1$ (Y_M is non-empty) $\Leftrightarrow \gamma_M p^*_{HML} < p^{dev}$, and (ii) $\underline{p}^j \leq p^{dev} \leq \overline{p}^j$ (search is best response based on Lemma 2).

Hence, it must be the case that when $\max(\gamma_M p^*_{HML}, \underline{p}^j) < p^{dev} < \min(p^*_{HML}, \overline{p}^j)$, D1 imposes the belief that $\mu_M(A_1) = 1$.

(b) Next, let's look at the conditions under which $D^0(M, A_1) \cup D(M, A_1) \subset D(H, A_1)$ or $\left(\widehat{X_H} \cup \widehat{Y_M}\right) \subset (X_H \cup Y_H)$, which would impose the belief that $\mu_H(A_1) = 1$. As we noted before, $X_H \subseteq \widehat{X_H}$, and we can see that $Y_H \subset \widehat{Y_M}$ since $\gamma_M < \gamma_H$, which rules out the case $D^0(M, A_1) \cup D(M, A_1) \subset D(H, A_1)$.

4. Consider the deviation $A_1 = \left(a_0, p^{dev}\right)$ where $\underline{p}^0 \leq p^{dev} \leq \overline{p}^0$ (so that there exists a belief where search is the best response based on Lemma 2). Using the same techniques as before, we can show that for $p^{dev} < p^*_{HML}$, $\widehat{X_H} = \widehat{X_M} = \widehat{X_L} = \varnothing$, and $\frac{\gamma_L\left(p^*_{HML} - p^{dev}\right)}{(1-\gamma_L)p^{dev}} < \frac{\gamma_M\left(p^*_{HML} - p^{dev}\right)}{(1-\gamma_M)p^{dev}} < \frac{\gamma_H\left(p^*_{HML} - p^{dev}\right)}{(1-\gamma_H)p^{dev}}$. Moreover, we can show that $\frac{\gamma_L\left(p^*_{HML} - p^{dev}\right)}{(1-\gamma_L)p^{dev}} < 1$ if $p^{dev} > \gamma_L p^*_{HML}$. This implies that $\widehat{Y_H} \subset Y_L$ and $\widehat{Y_M} \subset Y_L$ if $max(\underline{p}^0, \gamma_L p^*_{HML}) < p^{dev} < \min(p^*_{HML}, \overline{p}^0)$; therefore, D1 implies that $\mu_L = 1$ in this region. As before, if $p^{dev} > p^*_{HML}$, $\widehat{X_H} \nsubseteq X_M$, $\widehat{X_M} \nsubseteq X_H$, etc., which implies that D1 does not apply.

One example of an off-equilibrium belief which is consistent with the properties described above is $\mu_L = 1$ for all $(a_0, p \neq p^*_{HML})$ and $\mu_M = 1$ for all (a_j, p). This is the off-equilibrium beliefs we assume from now on. ■

Next we show that if $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$, $\gamma_M p^*_{HML} \geq \frac{\overline{V}}{2}$, and $p^*_{HML} \in [\underline{p}^0_{HML}, \overline{p}^0_{HML}]$, there exists HML equilibrium with search. According to Lemma 2 in the Appendix, in order for the consumer to search in equilibrium, it must be the case that $c \leq \frac{\overline{V}(\gamma_H - \gamma_M)}{8}$ and $p^*_{HML} \in [\underline{p}^0_{HML}, \overline{p}^0_{HML}]$, where $\underline{p}^0_{HML} = \underline{p}^0(\rho) = \frac{(1-\gamma_H)\rho\overline{V}+(1-\gamma_M)(1-\rho)\overline{V}+2c}{(2-\gamma_H-\gamma_L)\rho+2(1-\gamma_M)(1-\rho)} < \frac{\overline{V}}{2}$ and $\overline{p}^0_{HML} = \overline{p}^0(\rho) = \frac{\rho\gamma_H\overline{V}+(1-\rho)\gamma_M\overline{V}-2c}{\rho(\gamma_H+\gamma_L)+2(1-\rho)\gamma_M} > \frac{\overline{V}}{2}$. Either H or M type can deviate on both advertising and price. If it deviates on price alone $(a_0, p' \neq p^*_{HML})$, the consumer believes that the firm is L type, which cannot be profitable. If it deviates on advertising, the consumer believes that the firm is M type, which yields a maximum profit of $\frac{\overline{V}}{2}$. Hence, for equilibrium to exist, $\Pi^*(a_0, p^*_{HML}|q = H) = \gamma_H p^*_{HML} > Max_p \Pi(a_j, p|q = H) = \frac{\overline{V}}{2}$ and $\Pi^*(a_0, p^*_{HML}|q = M) = \gamma_M p^*_{HML} > Max_p \Pi(a_j, p|q = M) = \frac{\overline{V}}{2}$. The L-type by definition cannot deviate on advertising. A deviation on price can yield a maximum profit of 0. Hence, $\Pi^*(a_0, p^*_{HML}|q = L) = \gamma_L p^*_{HML} > 0$, which is trivial. Since $\Pi^*(a_0, p^*_{HML}|q = H) > \Pi^*(a_0, p^*_{HML}|q = M)$, the non-deviation condition for H-type is not binding if the non-deviation condition for M-type holds. This proves part 2 of Proposition 3.1 (HML). ■

This completes the proof of Proposition HML. **Q.E.D.**

3.2 HM Equilibrium

Next, we consider HM equilibrium.

Proposition HM *Suppose that search cost is low enough such that* $c \leq \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$.

1. *A semi-separating HM equilibrium without consumer search does not survive D1.*

2. *A semi-separating HM equilibrium, where the consumer chooses to search after observing* (a_j, p^*_{HM})*, exists if γ_M is sufficiently high and the price is in the higher range. Here,* $\Pi^*(H) > \Pi^*(M) > \Pi^*(L)$. *Moreover, this equilibrium survives D1.*

Proof. *1. Proof of part 1:*

We present a proof by contradiction. Suppose that there exists an HM equilibrium where both types pool on (a_j, p^{ns}_{HM}), and the consumer buys without search.

1. First, we show that $p^{ns}_{HM} < \overline{p}^j$ if $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$, where $\overline{p}^j$ is the upper bound in price where consumers may choose to search. Note that in equilibrium, the prior probabilities on the two types $\{H, M_j\}$ are $(\rho, 1-\rho)$. Given this, and using Lemma 2, we can show that the highest price that the firm can charge so that the consumer chooses not to search in equilibrium is $\max(p^{ns}_{HM}) = \frac{(1-\gamma_H)\rho\overline{V}+(1-\gamma_M)(1-\rho)\frac{\overline{V}}{2}+c}{\rho(1-\gamma_H)+(1-\rho)(1-\gamma_M)} = \underline{p}^j(\rho)$. We can also show that $p^{ns}_{HM} \leq \underline{p}^j(\rho) < \overline{p}^j$ as long as $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$.

2. Next we derive the restrictions that D1 imposes on the out-of-equilibrium beliefs following the deviation $A_1 = (a_j, p^{dev})$ where $p^{dev} \in (p^{ns}_{HM}, \overline{p}^j]$. From 1 above, as long as $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$, we know that there exists a belief for which search is the best response since $p^{dev} \leq \overline{p}^j$. Therefore, the consumer's mixed best response has 2 possible forms: (1) $\alpha_2 = \{0, \alpha_{22}, 1-\alpha_{22}\}$ and (2) $\alpha_2 = \{\alpha_{21}, 0, 1-\alpha_{21}\}$. In the first case, $\Pi(A_1, \alpha_2, H) = (1-\alpha_{22})\gamma_H p^{dev}$ and $\Pi(A_1, \alpha_2, M) = (1-\alpha_{22})\gamma_M p^{dev}$; in the second case, $\Pi(A_1, \alpha_2, H) = (\alpha_{21} + (1-\alpha_{21})\gamma_H)\, p^{dev}$ and $\Pi(A_1, \alpha_2, M) = (\alpha_{21} + (1-\alpha_{21})\gamma_M)\, p^{dev}$. Also, of course, $\Pi^*(H) = p^{ns}_{HM}$ and $\Pi^*(M) = p^{ns}_{HM}$.

   This yields the sets,

$$D^0(M, A_1) \cup D(M, A_1) = \widehat{X_M} \cup \widehat{Y_M} = \tag{12}$$
$$\left\{(0, \alpha_{22}, 1-\alpha_{22}) \mid \alpha_{22} \leq \frac{\gamma_M p^{dev} - p^{ns}_{HM}}{\gamma_M p^{dev}}\right\} \cup \left\{(\alpha_{21}, 0, 1-\alpha_{21}) \,|\alpha_{21} \geq \frac{p^{ns}_{HM} - \gamma_M p^{dev}}{(1-\gamma_M)p^{dev}}\right\}$$

$$D(H, A_1) = X_H \cup Y_H = \tag{13}$$
$$\left\{(0, \alpha_{22}, 1-\alpha_{22}) \mid \alpha_{22} < \frac{\gamma_H p^{dev} - p^{ns}_{HM}}{\gamma_H p^{dev}}\right\} \cup \left\{(\alpha_{21}, 0, 1-\alpha_{21}) \,|\alpha_{21} > \frac{p^{ns}_{HM} - \gamma_H p^{dev}}{(1-\gamma_H)p^{dev}}\right\}$$

   We can show that since $\gamma_H > \gamma_M$, $\frac{\gamma_M p^{dev} - p^{ns}_{HM}}{\gamma_M p^{dev}} < \frac{\gamma_H p^{dev} - p^{ns}_{HM}}{\gamma_H p^{dev}}$, which implies that $\widehat{X_M} \subseteq X_H$. In addition, since $\frac{\gamma_M p^{dev} - p^{ns}_{HM}}{\gamma_M p^{dev}} < 1 \Leftrightarrow p^{ns}_{HM} > 0$, it must be the case that $\widehat{X_M} \subset X_H$. Finally, we can similarly show that $\frac{p^{ns}_{HM} - \gamma_M p^{dev}}{(1-\gamma_M)p^{dev}} > \frac{p^{ns}_{HM} - \gamma_H p^{dev}}{(1-\gamma_H)p^{dev}}$, which implies that $\widehat{Y_M} \subseteq Y_H$. Hence, we have shown that $\left(\widehat{X_M} \cup \widehat{Y_M}\right) \subset (X_H \cup Y_H)$, which implies that $D^0(M, A_1) \cup D(M, A_1) \subset D(H, A_1)$ for all $A_1 = (a_j, p^{dev})$ where $p^{dev} \in (p^{ns}_{HM}, \overline{p}^j]$.

This implies that D1 constrains the belief to be $\mu^j_H = 1$ following $A_1 = (a_j, p^{dev})$, where $p^{ns}_{HM} < p^{dev} \leq \overline{p}^j$, which implies that both H and M types prefer to deviate to A_1, which, in turns, destroys this equilibrium. ■

2. Proof of part 2.

Before we turn to the equilibrium conditions, we first examine the restrictions on the out-of-equilibrium beliefs that are imposed by D1:

Lemma 3 *[TA3] The out-of-equilibrium beliefs that are consistent with D1 have the following properties:*

1. *When the consumer observes the deviation $A_1 = (a_j, p^{dev} \neq p^*_{HM})$, where $\max(\gamma_M p^*_{HM}, \underline{p}^j) < p^{dev} < \min(p^*_{HM}, \overline{p}^j)$ or $\gamma_M p^*_{HM} < p^{dev} < \min(\gamma_H p^*_{HM}, \underline{p}^j)$, she forms the belief $\mu_M(A_1) = 1$.*

2. *When the consumer observes the deviation* $A_1 = (a_0, p^{dev} \neq 0)$*, where* $0 < p^{dev} \leq \overline{p}^0$*, she forms the belief* $\mu_L(A_1) = 1$*.*

Proof. We follow the same logic as we do in the Proof for Lemma (TA2). For example, $D^0(H, A_1) \cup D(H, A_1)$ and $D^0(M, A_1) \cup D(M, A_1)$ are the same as equations (10) and (11). The only difference is that L type in equilibrium earns 0 profit. Hence,

$$D^0(L, A_1) = X_L^0 \cup Y_L^0 = \{(0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} = 1\} \cup \varnothing,$$

$$D(L, A_1) = X_L \cup Y_L = \{(0, \alpha_{22}, 1 - \alpha_{22}) \mid \alpha_{22} < 1\} \cup \{(\alpha_{21}, 0, 1 - \alpha_{21}) \, |0 \leq \alpha_{21} \leq 1\}.$$

1. Consider a deviation to a price such that. the consumer chooses not to purchase at any off-equilibrium belief: $A_1 = \left(a_j, p^{dev}\right)$ where $p^{dev} > \overline{p}^j$ or $A_1 = \left(a_0, p^{dev}\right)$ where $p^{dev} > \overline{p}^0$. Here D1 does not apply.

2. Next, consider a deviation to a price such that. the consumer chooses to purchase without search at any off-equilibrium belief: $A_1 = \left(a_j, p^{dev}\right)$ where $p^{dev} < \underline{p}^j$ or $A_1 = \left(a_0, p^{dev}\right)$ where $p^{dev} < \underline{p}^0$; i.e., $\alpha_{21} = 1$. Therefore, if $A_1 = \left(a_j, p^{dev}\right)$ where $p^{dev} \in (\gamma_M p^*_{HM}, \min(\gamma_H p^*_{HM}, \underline{p}^j))$, D1 imposes that $\mu_H = 0$. Similarly, if $A_1 = \left(a_0, p^{dev}\right)$ where $p^{dev} \in (0, \min(\gamma_M p^*_{HM}, \underline{p}^0))$, $\mu_L = 1$, and for all $p^{dev} \in (\gamma_M p^*_{HM}, \min(\gamma_H p^*_{HM}, \underline{p}^0))$, $\mu_H = 0$.

3. If the deviation is $A_1 = (a_j, p^{dev} \neq p^*_{HM})$ and $\underline{p}^j \leq p^{dev} \leq \overline{p}^j$, the proof is identical to the Proof for Lemma (TA2): D1 imposes the belief that $\mu_M(A_1) = 1$ if $\max(\gamma_M p^*_{HM}, \underline{p}^j) < p^{dev} < \min(p^*_{HM}, \overline{p}^j)$. Furthermore, we can see that $\underline{p}^j \leq \gamma_M p^*_{HM}$ since $p^*_{HM} \in [\underline{p}_{HM}, \overline{p}_{HM}]$, where $\overline{p}_{HM} = \overline{p}^j(\rho) \leq \overline{p}^j$ from Lemma 2. Also, we can show that $p^*_{HM} \leq \overline{p}^j$. This is so because M must prefer its equilibrium strategy to an optimal deviation, in equilibrium $\gamma_M p^*_{HM} > Max_{A_1} \Pi(A_1 | q = M)$. In particular, a firm can deviate and charge a price $p \leq \underline{p}^j$ such that the consumer chooses to purchase without search. Hence, $Max_{A_1} \Pi(A_1 | q = M) \geq \underline{p}^j$ and, therefore, $\gamma_M p^*_{HM} \geq \underline{p}^j$. Therefore, D1 imposes the belief that $\mu_M(A_1) = 1$ if $\gamma_M p^*_{HM} < p^{dev} < p^*_{HM}$.

4. Consider a deviation strategy of $A_1 = (a_0, p^{dev} < \overline{p}^0)$, where $\underline{p}^0 \leq p^{dev} \leq \overline{p}^0$. First, note that the consumer's response $(0, \alpha_{22} = 1, 0) \notin D^0(H, A_1) \cup D(H, A_1)$ and $(0, \alpha_{22} = 1, 0) \notin D^0(M, A_1) \cup D(M, A_1)$. This is true since in equilibrium the H and M types make non-zero profit. Hence, $\widehat{X_H} \subset X_L$ and $\widehat{X_M} \subset X_L$. Moreover, $\widehat{Y_H} \subset Y_M$ and $\widehat{Y_M} \subset Y_L$ since

$\gamma_L < \gamma_M < \gamma_H$. This implies that $D^0(H, A_1) \cup D(H, A_1) \subset D(L, A_1)$ and $D^0(M, A_1) \cup D(M, A_1) \subset D(L, A_1)$. Note that if $p^{dev} > \overline{p}^0$, the consumer's best response is no purchase, i.e., $\alpha_{22} = 1$, which in turn would imply that $D(L, A_1)$ is an empty set or that D1 does not apply. Hence, if $A_1 = (a_0, p^{dev} < \overline{p}^0)$, D1 implies that $\mu_L(A_1) = 1$.

One example of an off-equilibrium belief which is consistent with the properties described above is $\mu_L = 1$ for all (a_0, p) and $\mu_M = 1$ for all $(a_j, p \neq p^*_{HM})$, which we assume from now on. ■

Next, we show that the HM equilibrium with search exists if $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ and $Max\left[\frac{\overline{V}}{2\gamma_M}, \underline{p}_{HM}\right] < p^*_{HM} \leq \overline{p}_{HM}$.

On the equilibrium path, the probability that the firm is type H is ρ, and the probability that it is type M is $1-\rho$. As we can see from Lemma 2, search will not occur at *any* price unless $\underline{\mu} \leq \rho \leq \overline{\mu}$ or $c \leq \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ (see the proof of Lemma 2 above). In addition, in order for the consumer to engage in search at the equilibrium price, the price must be such that $p^*_{HM} \in [\underline{p}_{HM}, \overline{p}_{HM}]$, where $\overline{p}_{HM} = \frac{\gamma_H\rho\overline{V} + \gamma_M(1-\rho)\overline{V} - c}{\gamma_H\rho + \gamma_M(1-\rho)} \equiv \overline{p}^j(\rho)$ and $\underline{p}_{HM} = \frac{(1-\gamma_H)\rho\overline{V} + (1-\gamma_M)(1-\rho)\overline{V} + c}{(1-\gamma_H)\rho + (1-\gamma_M)(1-\rho)} \equiv \underline{p}^j(\rho)$ (see Lemma 2). Hence, when $c < \frac{\overline{V}}{2}\rho(1-\rho)(\gamma_H - \gamma_M)$ and $p^*_{HM} \in [\underline{p}_{HM}, \overline{p}_{HM}]$, the consumer chooses to search along the equilibrium path.

In order for the equilibrium to hold, all types must prefer the equilibrium profits to the optimal deviation. Given the assumed out-of-equilibrium beliefs, this reduces to the following conditions: $\Pi^*(a_j, p_{HM}|q = H) = \gamma_H p^*_{HM} > Max_{A_1}\Pi(A_1|q = H) = \frac{\overline{V}}{2}$ and $\Pi^*(a_j, p_{HM}|q = M) = \gamma_M p^*_{HM} > Max_{A_1}\Pi(A_1|q = M) = \frac{\overline{V}}{2}$. This of course reduces to the non-deviation condition on M-type: $\gamma_M p^*_{HM} > \frac{\overline{V}}{2}$ or $p^*_{HM} > \frac{\overline{V}}{2\gamma_M}$. This along with the condition that $p^*_{HM} \in [\underline{p}_{HM}, \overline{p}_{HM}]$ results in the necessary condition for existence: $Max\left[\frac{\overline{V}}{2\gamma_M}, \underline{p}_{HM}\right] < p^*_{HM} \leq \overline{p}_{HM}$. ■

This completes the proof of Proposition HM. **Q.E.D.**

4 Proof of Proposition 7: Advertising Costs and Endogenous Advertising Spending

Proposition 7 *HL equilibrium with consumer search exists if max* $\left\{\frac{\gamma_H(1-\gamma_H)\overline{V} + c(\gamma_L+\gamma_H)}{\gamma_H+\gamma_L-\gamma_L^2-\gamma_H^2}, \frac{\overline{V}}{2\gamma_H}\right\} <$ *min* $\left\{\frac{\gamma_H^2\overline{V} - c(\gamma_L+\gamma_H)}{\gamma_H^2+\gamma_L^2}, \frac{\overline{V}}{2\gamma_M}\right\}$. *Here* $\Phi^*_H = \gamma_H p^*_{HL} > \Phi^*_M = \frac{\overline{V}}{2} > \Phi^*_L = \gamma_L p^*_{HL}$ *and* $\Pi^*(H) = \frac{\gamma_H^2(p^*_{HL})^2}{2} > \Pi^*(M) = \frac{\overline{V}^2}{2} > \Pi^*(L) = \frac{\gamma_L^2(p^*_{HL})^2}{2}$.

Proof. The equilibrium concept is Perfect Bayesian Nash Equilibrium. To solve for the equilibrium investment in advertising, we assume (and later check) that there exists an HL equilibrium, where the H and L types pool on (a_0, p^*_{HL}), the M-type separates with $(a_j, \frac{\overline{V}}{2})$, and the consumer searches after observing (a_0, p^*_{HL}). The firm's maximization problem (by type) becomes: $\underset{\Phi_H}{max}\ \Phi_H(\gamma_H p^*_{HL}) - \frac{\Phi_H^2}{2}$, $\underset{\Phi_M}{max}\ \Phi_M(\frac{\overline{V}}{2}) - \frac{\Phi_M^2}{2}$, and $\underset{\Phi_L}{max}\ \Phi_L(\gamma_L p^*_{HL}) - \frac{\Phi_L^2}{2}$, respectively. Solving for the F.O.C.s yields:

$$\Phi^*_H = \gamma_H p^*_{HL} > \Phi^*_M = \frac{\overline{V}}{2} > \Phi^*_L = \gamma_L p^*_{HL}. \tag{14}$$

Also, the consumer who observes (a$_0$,p^*_{HL}) forms the following posterior belief:

$$Prob(\theta_H|a_0, p^*_{HL}) = \mu^0_H = \frac{\Phi^*_H \frac{\varrho}{2}}{\Phi^*_H \frac{\varrho}{2} + \Phi^*_L \frac{\varrho}{2}} = \frac{\gamma_H}{\gamma_H + \gamma_L}, \tag{15}$$

$$Prob(\theta_L|a_0, p^*_{HL}) = \mu^0_L = \frac{\Phi^*_L \frac{\varrho}{2}}{\Phi^*_H \frac{\varrho}{2} + \Phi^*_L \frac{\varrho}{2}} = \frac{\gamma_L}{\gamma_H + \gamma_L}. \tag{16}$$

Although the consumer does not observe the actual level of reach, she can invert each firm type's problem, and hence uses the equilibrium reach levels, Φ^*_H and Φ^*_L in her inference.

Using Lemma 1 in the Appendix, we can now derive the price bounds that will result in consumer searching after observing (a_0,p^*_{HL}), $\frac{\gamma_H(1-\gamma_H)\overline{V}+c(\gamma_H+\gamma_L)}{\gamma_H(1-\gamma_H)+\gamma_L(1-\gamma_L)} \leq p^*_{HL} \leq \frac{\gamma_H^2\overline{V}-c(\gamma_H+\gamma_L)}{\gamma_H^2+\gamma_L^2}$. Note that this set is non-empty if $c \leq \frac{\gamma_H\gamma_L(\gamma_H-\gamma_L)\overline{V}}{(\gamma_L+\gamma_H)^2}$. The proof of non-deviation conditions for firms is the same as before in Proposition ■. **Q.E.D.**

5 Proof of Proposition 8: Consumer Heterogeneity and Asymmetric Attributes

Proposition 8 *HL equilibrium with consumer search exists if* $min\left\{\frac{(1-\gamma_H)(u_0+\overline{V})+(1-\gamma_L)u_0+2c}{2-\gamma_H-\gamma_L}, \frac{u_0}{\lambda\gamma_L}, \frac{u_0+\phi\overline{V}}{\lambda\gamma_H}\right\} < max\left\{\frac{\gamma_H(u_0+\overline{V})+\gamma_L u_0-2c}{\gamma_H+\gamma_L}, \frac{u_0+(1-\phi)\overline{V}}{\gamma_M}\right\}$. *Here,* $\Pi^*(H) = \lambda\gamma_H p^*_{HL} > \Pi^*(M_\alpha) = \lambda(u_0+\phi\overline{V}) > \Pi^*(M_\beta) = \lambda(u_0+(1-\phi)\overline{V}) > \Pi^*(L) = \lambda\gamma_L p^*_{HL}$.

Proof. We first turn to the consumer's problem. When the firm engages in uninformative advertising, $A_1 = (a_0,\ p)$, the consumer's belief is $\mu^0 = (\mu^0_L, \mu^0_{M_\alpha}, \mu^0_{M_\beta}, \mu^0_H)$. Applying Lemma 1, in order for the consumer to search after seeing (a_0, p^*_{HL}), it must be the case that $p_{HL} \in [\frac{(1-\gamma_H)(u_0+\overline{V})+(1-\gamma_L)u_0+2c}{2-\gamma_H-\gamma_L}, \frac{\gamma_H(u_0+\overline{V})+\gamma_L u_0-2c}{\gamma_H+\gamma_L}]$(since the probabilities that the firm is H-type and L-type on the equilibrium path are $\frac{1}{2}$ and $\frac{1}{2}$). This set is not empty as long as c $\leq \frac{\overline{V}(\gamma_H-\gamma_L)}{4}$ (which is the case when $c < \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$).

Next, we need to ensure that all types prefer their equilibrium strategy to the maximal deviation. To show existence, we impose the following out-of-equilibrium belief: $\mu_L = 1$ for all $(a_0, p \neq p^*_{HL})$, $\mu_H = 0$ for all $(a_\alpha, p \neq u_0 + \phi\overline{V})$ and $\mu_H = 0$ for all $(a_\beta, p \neq u_0 + (1-\phi)\overline{V})$. While we omit the proof here for brevity, we can show that this belief is consistent with D1. Given the assumed out-of-equilibrium beliefs, the non-deviation conditions for H-type and M-type are the following:

$$\Pi^*(a_0, p^*_{HL}|q = H) = \gamma_H p^*_{HL} > Max_{A_1}\Pi(A_1|q = H) = u_0 + \phi\overline{V} \tag{17}$$

$$\Pi^*(a_j, p_{M_\alpha}|q = M_\alpha) = u_0 + \phi\overline{V} > Max_{A_1}\Pi(A_1|q = M_\alpha) = \gamma_M p^*_{HL} \tag{18}$$

$$\Pi^*(a_j, p_{M_\beta}|q = M_\beta) = u_0 + (1-\phi)\overline{V} > Max_{A_1}\Pi(A_1|q = M_\beta) = \gamma_M p^*_{HL} \tag{19}$$

$$\Pi^*(a_0, p_L|q = L) = \lambda\gamma_H p^*_{HL} > Max_{A_1}\Pi(A_1|q = L) = u_0 \tag{20}$$

Note that since $\phi > \frac{1}{2}$, $u_0 + (1-\phi)\overline{V} > \gamma_M p^*_{HL}$ implies that $u_0 + \phi\overline{V} > \gamma_M p^*_{HL}$. The rest of the proof is the same as before in Proposition. ■ **Q.E.D.**

6 Proof of Proposition 9: n-Attribute case

Lemma 4 *[TA4] Assume that* $c \leq \frac{(h-m)\overline{V}(\gamma_H - \gamma_M)}{4n}$.

Consider the case when the firm engages in uninformative advertising, $A_1 = (a_0, p)$ *and the consumer's belief is* $\mu^0 = (\mu^0_L, \mu^0_M, \mu^0_H)$. *Consumer chooses to search iff*

$$\underline{p}^0(\mu^0) \leq p \leq \overline{p}^0(\mu^0)$$

$$\underline{\mu}(\mu^0_L) \leq \mu^0_H \leq \overline{\mu}(\mu^0_L)$$

$$\text{and} \qquad \mu^0_L \leq \frac{1}{2\cdot h}\left(h + \sqrt{h\cdot\left(h - \frac{4\cdot n\cdot c}{\overline{V}(\gamma_H - \gamma_L)}\right)}\right),$$

where $\underline{p}^0(\mu^0) = \frac{\mu^0_H(1-\gamma_H)\frac{h\overline{V}}{n} + \mu^0_M(1-\gamma_M)\frac{m\overline{V}}{n} + c}{\mu^0_H(1-\gamma_H)+\mu^0_M(1-\gamma_M)+\mu^0_L(1-\gamma_L)}$, $\overline{p}^0(\mu^0) = \frac{\mu^0_H\gamma_H\frac{h\overline{V}}{n} + \mu^0_M\gamma_M\frac{m\overline{V}}{n} - c}{\mu^0_H\gamma_H + \mu^0_M\gamma_M + \mu^0_L\gamma_L}$,
$\underline{\mu}(\mu^0_L) = \frac{1}{2}\left(1 + \frac{(h(\gamma_M-\gamma_L)+m(\gamma_H-2\gamma_M+\gamma_L))\mu^0_L}{(h-m)(\gamma_H-\gamma_M)} - \frac{\Upsilon(\mu^0_L)}{(h-m)(\gamma_H-\gamma_M)\overline{V}}\right)$,
$\overline{\mu}(\mu^0_L) = \frac{1}{2}\left(1 + \frac{(h(\gamma_M-\gamma_L)+m(\gamma_H-2\gamma_M+\gamma_L))\mu^0_L}{(h-m)(\gamma_H-\gamma_M)} + \frac{\Upsilon(\mu^0_L)}{(h-m)(\gamma_H-\gamma_M)\overline{V}}\right)$, *and* $\Upsilon(\mu^0_L) = (\overline{V}^2(\gamma_H - \gamma_M + \mu^0_L(\gamma_H - \gamma_L))^2 + 4\overline{V}(\gamma_H - \gamma_M)(\overline{V}(\gamma_M - \gamma_L)(1-\mu^0_L)\mu^0_L - 2c))^{\frac{1}{2}}$.

Proof. The proof is basically identical to the proof of Lemma 2 above. First, suppose that $E(V|\mu^0) - p \geq 0 \Leftrightarrow \frac{h\overline{V}}{n}\mu^j_H + \frac{m\overline{V}}{n}(1-\mu^j_H) \geq p$. Consumer will search if $c \leq \Pr(\underline{s}|\mu^0)[p - E(V|\mu^0, \underline{s})]$ or $p \geq \frac{\mu^0_H(1-\gamma_H)\frac{h\overline{V}}{n} + \mu^0_M(1-\gamma_M)\frac{m\overline{V}}{n} + c}{\mu^0_H(1-\gamma_H)+\mu^0_M(1-\gamma_M)+\mu^0_L(1-\gamma_L)} \equiv \underline{p}^0(\mu^0)$. If the price is sufficiently low such that $p < \min(\underline{p}^0(\mu^0), E(V|\mu^0))$, the consumer will not search but still purchase without search since $p \leq E(V|\mu^0)$. Hence, consumer will search if $\underline{p}^0(\mu^0) < p \leq E(V|\mu^0)$. By equating the lower and the upper bound on p for a given μ_L, we can show that for a given μ_L, the consumer will search as long as $\underline{\mu}(\mu_L^0) \leq \mu_H^0(\mu_L) \leq \overline{\mu}(\mu_L^0)$, where $\underline{\mu}(\mu_L^0) = \frac{1}{2}\left(1 + \frac{(h(\gamma_M-\gamma_L)+m(\gamma_H-2\gamma_M+\gamma_L))\mu_L^0}{(h-m)(\gamma_H-\gamma_M)} - \frac{\Upsilon(\mu_L^0)}{(h-m)(\gamma_H-\gamma_M)\overline{V}}\right) \leq \overline{\mu}(\mu_L^0) = \frac{1}{2}\left(1 + \frac{(h(\gamma_M-\gamma_L)+m(\gamma_H-2\gamma_M+\gamma_L))\mu_L^0}{(h-m)(\gamma_H-\gamma_M)} + \frac{\Upsilon(\mu_L^0)}{(h-m)(\gamma_H-\gamma_M)\overline{V}}\right)$, and $\Upsilon(\mu_L^0) = \left(\overline{V}^2\left((h-m)(\gamma_H-\gamma_M) + \mu_L^0\left(h(\gamma_M-\gamma_L) + m(\gamma_H-2\gamma_M+\gamma_L)\right)\right)^2 + 4(h-m)(\gamma_H-\gamma_M)\overline{V}\left(m\overline{V}(\gamma_M-\gamma_L)(1-\mu_L^0)\mu_L^0 - n\cdot c\right)\right)^{\frac{1}{2}}$. Note that $\Psi(\mu_L^0) \geq 0 \Leftrightarrow c \leq \frac{\overline{V}(\gamma_M-\gamma_L)(1-\mu_L^0)\mu_L^0}{n} + \frac{\overline{V}\left((h-m)(\gamma_H-\gamma_M)+\mu_L^0(h(\gamma_M-\gamma_L)+m(\gamma_H-2\gamma_M+\gamma_L))\right)^2}{4n(h-m)(\gamma_H-\gamma_M)}$. Here, we can see that $c \leq \frac{(h-m)\overline{V}(\gamma_H-\gamma_M)}{4n}$ implies $c \leq \frac{\overline{V}\left((h-m)(\gamma_H-\gamma_M)+\mu_L^0(h(\gamma_M-\gamma_L)+m(\gamma_H-2\gamma_M+\gamma_L))\right)^2}{4n(h-m)(\gamma_H-\gamma_M)}$ for any $\mu_L^0 \in [0,1]$. Hence, if $c \leq \frac{(h-m)\overline{V}(\gamma_H-\gamma_M)}{4n}$, the above set of beliefs, $\left[\underline{\mu}(\mu_L^0), \overline{\mu}(\mu_L^0)\right]$, is not empty for a given $\mu_L^0 \in [0,1]$.

We also have to consider another constraint on the set of beliefs: as the level of μ_L^0 increases, the feasible range for $\mu_H^0(\mu_L)$ gets smaller since $\mu_H^0(\mu_L) + \mu_L^0 \leq 1$. Hence, at the large value of μ_L^0 (for example, $\mu_L^0 = 1$), the corresponding $\mu_H^0(\mu_L) \in \left[\underline{\mu}(\mu_L^0), \overline{\mu}(\mu_L^0)\right]$, which ensures the search is not simply feasible. Given $\mu_L^0 \in [0,1]$, for the corresponding $\mu_H^0(\mu_L)$ to exist, it must be that $\underline{\mu}(\mu_L^0) + \mu_L^0 \leq 1$.

$$\underline{\mu}(\mu_L^0) + \mu_L^0 \leq 1 \Leftrightarrow \mu_L^0 \leq \frac{1}{2\cdot h}\left(h + \sqrt{h\cdot\left(h - \frac{4\cdot n\cdot c}{\overline{V}(\gamma_H-\gamma_L)}\right)}\right) \tag{21}$$

Note that the above set of beliefs is not empty since $c \leq \frac{(h-m)\overline{V}(\gamma_H-\gamma_M)}{4n} \leq \frac{h\overline{V}(\gamma_H-\gamma_L)}{4n}$.

Second, suppose that $E(V|\mu^0) - p < 0$ or $\frac{h\overline{V}}{n}\mu_H^j + \frac{m\overline{V}}{n}(1-\mu_H^j) < p$. Consumer will search if $c \leq \Pr(\overline{s}|\mu^0)[E(V|\mu^0,\overline{s}) - p]$ or$p \leq \frac{\mu_H^0\gamma_H\frac{h\overline{V}}{n} + \mu_M^0\gamma_M\frac{m\overline{V}}{n} - c}{\mu_H^0\gamma_H + \mu_M^0\gamma_M + \mu_L^0\gamma_L} \equiv \overline{p}^0(\mu^0)$. If the price is sufficiently high such that $p > \max(\overline{p}^0(\mu^0), E(V|\mu^0))$, the consumer will not search and will not purchase a product since $p > E(V|\mu^0)$. Hence, consumer will search if $E(V|\mu^0) < p < \overline{p}^0(\mu^0)$. By equating the lower and the upper bound on p, we can show that the consumer will search as long as $\underline{\mu}(\mu_L^0) \leq \mu_H^0 \leq \overline{\mu}(\mu_L^0)$: which yields the same bounds on beliefs and prices as above.

Hence, combining these two cases suggests that when $c \leq \frac{\overline{V}(\gamma_H-\gamma_M)}{8}$, if $\underline{p}^0(\mu_L^0) \leq p \leq \overline{p}(\mu_L^0)$ and $\underline{\mu}(\mu_L^0) \leq \mu_H^0 \leq \overline{\mu}(\mu_L^0)$, search is the consumer's best response. **Q.E.D.**

Using the result from Lemma 4 above, we now prove Proposition 9 of the main paper.

Proposition 9 *When $k < m$, HL equilibrium with consumer search exists if (1) $c \leq \frac{(h-m)\overline{V}(\gamma_H-\gamma_M)}{4n}$, and (2) max $\left\{\frac{h(1-\gamma_H)\overline{V}+2nc}{n(2-\gamma_H-\gamma_L)}, \frac{m\overline{V}}{n\gamma_H}, \frac{l\overline{V}}{n\gamma_L}\right\} <$ min $\left\{\frac{h\gamma_H\overline{V}-2nc}{n(\gamma_H+\gamma_L)}, \frac{m\overline{V}}{n\gamma_M}\right\}$. Here, $\Pi^*(H) = \gamma_H p_{HL}^* > \Pi^*(M) = \frac{m\overline{V}}{n} > \Pi^*(L) = \gamma_L p_{HL}^*$.*

Proof. Above Lemma [TA 4] shows that when $c \leq \frac{(h-m)\overline{V}(\gamma_H-\gamma_M)}{4n}$ the consumer decides to search if $\underline{p}^0 \leq p \leq \overline{p}^0$, where $\underline{p}^0(\mu_L) = \frac{\mu_H^0(1-\gamma_H)\frac{h\overline{V}}{n}+\mu_M^0(1-\gamma_M)\frac{m\overline{V}}{n}+c}{\mu_H^0(1-\gamma_H)+\mu_M^0(1-\gamma_M)+\mu_L^0(1-\gamma_L)}$, $\overline{p}^0(\mu_L) = \frac{\mu_H^0\gamma_H\frac{h\overline{V}}{n}+\mu_M^0\gamma_M\frac{m\overline{V}}{n}-c}{\mu_H^0\gamma_H+\mu_M^0\gamma_M+\mu_L^0\gamma_L}$. In addition, on the equilibrium path, the probabilities that the firm is H-type and L-type following (a_0, p^*_{HL}) are $\frac{1}{2}$ and $\frac{1}{2}$ (i.e., $\frac{\frac{\rho}{2}}{\frac{\rho}{2}+\frac{\rho}{2}}$ and $\frac{\frac{\rho}{2}}{\frac{\rho}{2}+\frac{\rho}{2}}$ for H and L-type, respectively). Hence, $\overline{p}^0(\frac{1}{2}) = \frac{\gamma_H\frac{h\overline{V}}{n}-2c}{\gamma_H+\gamma_L}$ and $\underline{p}^0(\frac{1}{2}) = \frac{(1-\gamma_H)\frac{h\overline{V}}{n}+2c}{2-\gamma_H-\gamma_L}$. Hence, in order for the consumer to search in equilibrium, the price must be in the appropriate range: $p^*_{HL} \in [\frac{(1-\gamma_H)\frac{h\overline{V}}{n}+2c}{2-\gamma_H-\gamma_L}, \frac{\gamma_H\frac{h\overline{V}}{n}-2c}{\gamma_H+\gamma_L}]$.
Next, we need to ensure that all types prefer their equilibrium strategy to an optimal deviation. To show existence, we impose the following out-of-equilibrium belief: $\mu_L = 1$ for all $(a_0, p \neq p^*_{HL})$ and $\mu_H = 0$ for all $(a_j, p \neq \frac{\overline{V}}{2})$. Given the assumed out-of-equilibrium beliefs, the non-deviation conditions for H-type and M-type reduce to the following:

$$\Pi^*(a_0, p^*_{HL}|q = H) = \gamma_H p^*_{HL} > Max_{A_1}\Pi(A_1|q = H) = \frac{m\overline{V}}{n} \tag{22}$$
$$\Pi^*(a_j, p_M|q = M) = \frac{m\overline{V}}{n} > Max_{A_1}\Pi(A_1|q = M) = \gamma_M p^*_{HL}$$

The L-type by definition cannot deviate on advertising. A deviation on price only yields a maximum profit of 0 under the off-equilibrium beliefs. Hence, $\Pi^*(a_0, p_{HL}|q = L) = \gamma_L p^*_{HL} > 0$, which is trivially satisfied. The rest of the proof is the same as in Proposition 3 in the main body of the paper. ■ **Q.E.D.**

7 Experimental Investigation

In this Section we provide an example of consumer behavior in a laboratory setting that is consistent with the predictions made by the HL equilibrium: (1) consumers may not always view non-attribute ads as being associated with high quality products, and (2) compared to attribute ads, non-attribute ads may increase consumers' likelihood to search for information about the product.

We recruited respondents from a university-maintained online panel to participate in an online study on advertising effectiveness. The respondents were given three hypothetical scenarios featuring products in three different categories: hotel, restaurant, and books. For example, in the hotel scenario, the subjects were told, "Suppose that you are planning your summer vacation in Vermont and come across an ad for the hotel called The Green Mountains Inn." The subjects were randomly assigned to either (1) the *non-attribute-focused ad* condition or (2) the *attribute-focused ad* condition. Both types of ads contained the product name as well as the same image. The only difference across the conditions was that the attribute-focused ad also made an attribute-based product claim. (See Figures 1-3). After seeing the ad, each subject was asked to answer two questions. First, we asked the subjects to estimate the quality of the product, where the choices were, a) High, b) Average, c) Low, or d) Could be high or could be low, can't tell from the ad alone. Second, we asked the subjects what choice they would make about the product, where the choices were, a) Purchase the product, b) Search for more information, and c) Decide not to purchase the product.

We present the results of the analysis in Tables 1- 3.* The analysis in Tables 1 and 2 addresses the first question: we demonstrate that consumers may not always view non-attribute-focused ads as being associated with high quality products. Table 1 contains the results of a probit model, where the dependent variable is whether consumers select that the product could be high or low quality and the independent variable is the ad condition (as well as a constant term). Here we can see that both in the Hotel and the Book scenario, consumers are more likely to select "could be high or low" in the non-attribute ad condition. In contrast, as we can see from Table 2, consumers are more likely to estimate that the product's quality is "average" in attribute ad condition. This is consistent with the prediction of the HL equilibrium that an non-attribute ad will create uncertainty on the product quality. It is also important to note that there was no statistically significant difference between conditions in the restaurant scenario.†

Table 1: Dependent Variable: $Y = 1$ if "*Quality could be high or low*" is selected, 0 otherwise

	Hotel	Restaurant	Book
	(1)	(2)	(3)
Non-attribute-focused ad	0.74** (0.26)	0.08 (0.28)	0.97* (0.27)
LR Chi2(1)	8.12	0.08	13.56
Prob > Chi2	0.004	0.774	0.000

**Significant at 1%

*Significant at 5%

N=100

*For our analysis we remove the subjects who took an unreasonably short time to complete the study. That is, we remove all subjects who took less than a minute (in total) to complete all the scenarios.

†Since the goal of the study was to provide an example, we did not investigate further whether this pattern of results is stable and is perhaps driven by different equilibria that arise in different categories.

Table 2: Dependent Variable: $Y = 1$ if *"Quality is average"* is selected, 0 otherwise

	Hotel	Restaurant	Book
	(1)	(2)	(3)
Non-attribute-focused ad	-1.02^{**} (0.29)	-0.04 (0.26)	-0.66^{*} (0.29)
LR Chi2(1)	13.22	0.03	5.50
Prob > Chi2	0.000	0.867	0.020

**Significant at 1%

*Significant at 5%

N=100

The analysis in Table 3 addresses the second question: whether compared to attribute ads, non-attribute ads may increase consumers' likelihood to search for information about the product. In particular, we investigate whether the uncertainty created by non-attribute ads in the hotel and book scenarios (as we saw in Tables 1) leads to additional search as compared to the attribute-based ads. Equations (1) and (3) in Table 3 present the results of a probit model, where the dependent variable is whether the respondent chose "Search" and the independent variable is the respondent's ad condition (as well as a constant). As we see from the Table, in the book scenario, indeed more search occurs in the non-attribute-based ad ad condition (the effect is not seen in the hotel scenario). We can go further and test whether this effect of the ad condition is indeed driven by the uncertainty created by the non-attribute ad. Equation (4) presents the result of a model where we also add whether the subject indicated that her estimate of the product was "High or Low – can't tell from the ad alone." As we can see, once we control for the estimate of quality, the effect of the ad condition is no longer significant. On the other hand, the effect of "Quality High or Low" is positive and significant at 1%. Hence, the additional search in the non-attribute-focused condition is being driven by the extra uncertainty generated about product quality.

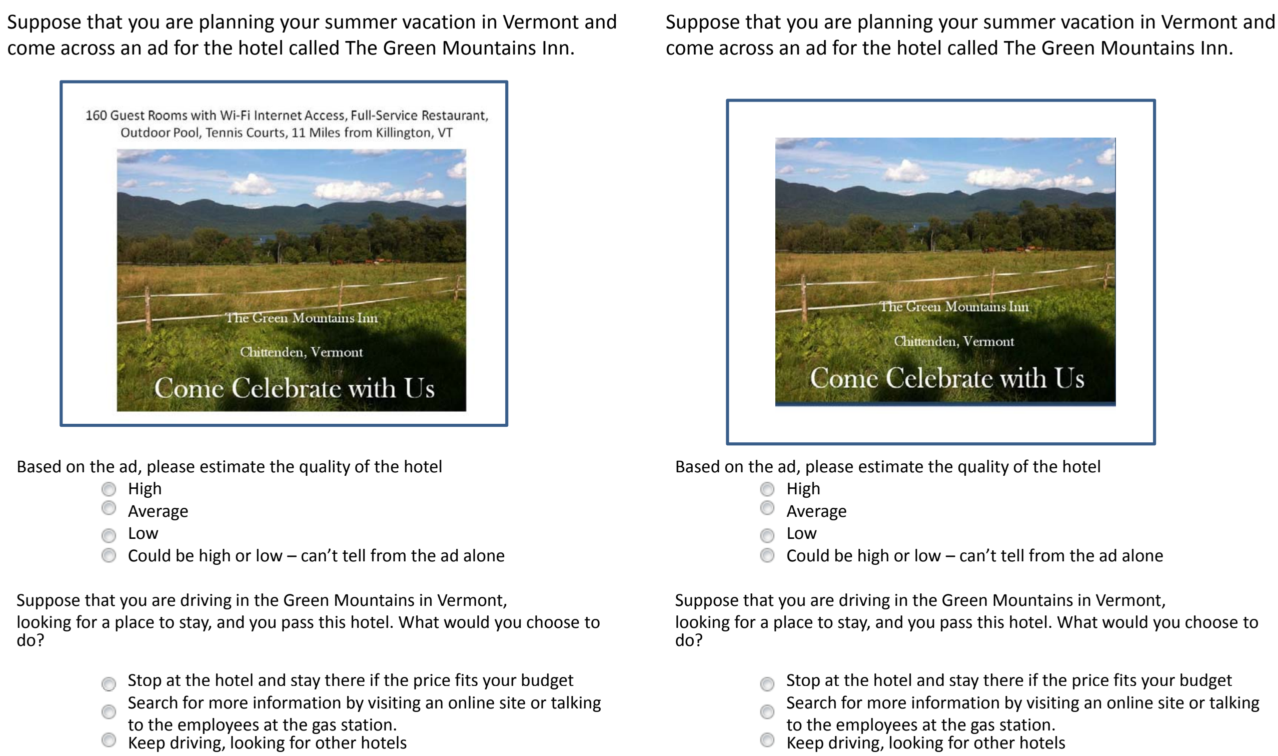
Table 3: Dependent Variable: $Y = 1$ if *"Search for more information"* is selected, 0 otherwise

	Hotel		Book	
	(1)	(2)	(3)	(4)
Non-attribute-focused ad	-0.39 (0.26)	-0.45 (0.27)	0.50^{*} (0.25)	0.23 (0.28)
Quality high or low		0.20 (0.27)		0.84^{**} (0.29)
LR Chi2(1)	1.36	2.93	3.89	12.53
Prob > Chi2	0.12	0.23	0.05	0.00

**Significant at 1%

*Significant at 5%

N=100



(a) Attribute Condition (b) Non-Attribute Condition

Figure 1: Hotel

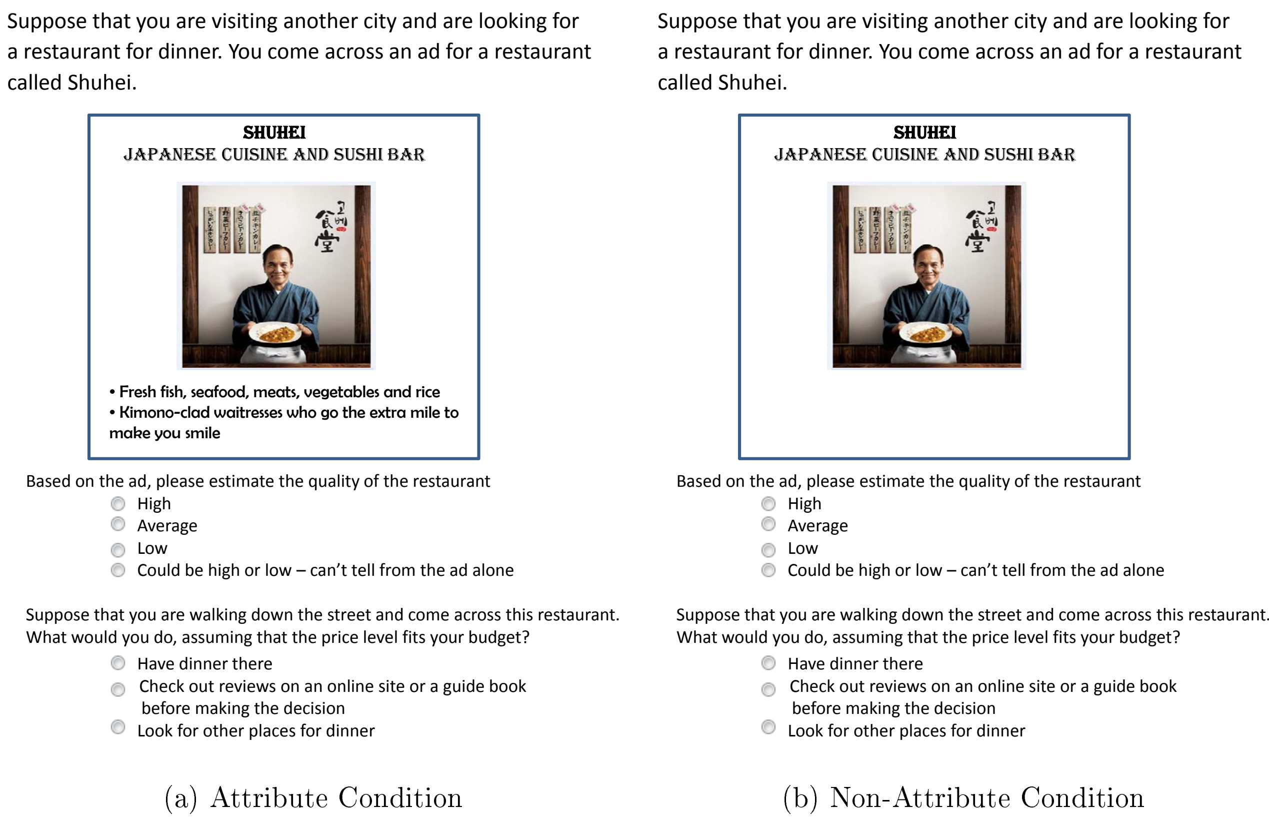
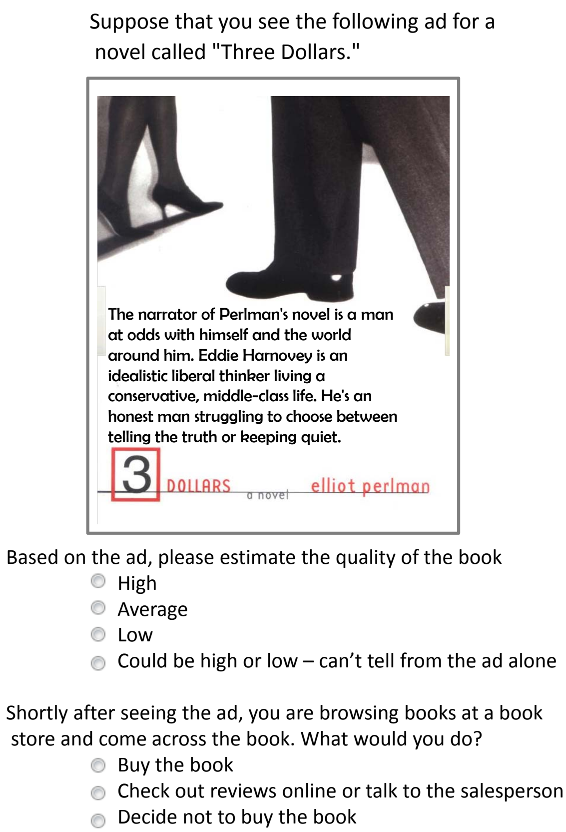
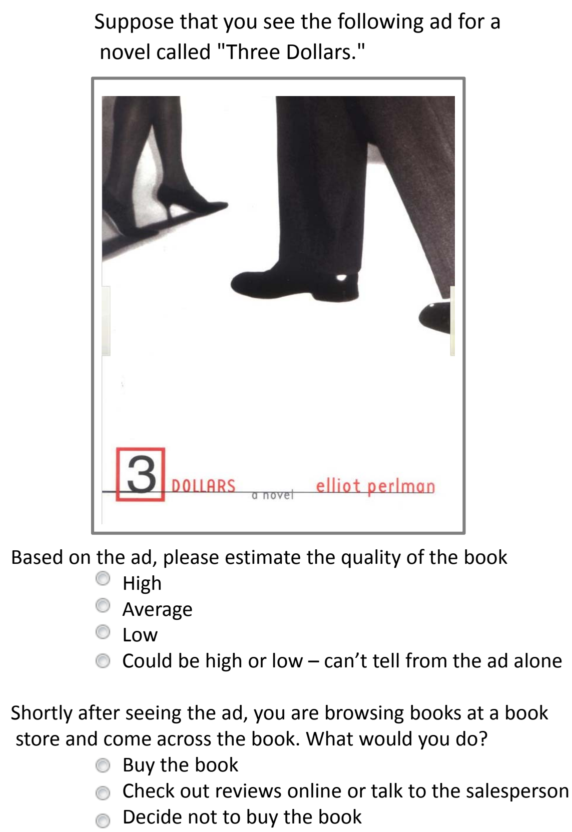


Figure 2: Restaurant



(a) Attribute Condition



(b) Non-Attribute Condition

Figure 3: Book