Online Supplements

Proof of Lemma 1: To compare equilibrium prices/rates, we need to solve the Nash equilibrium of the second stage game for Scenarios EE, EA, AE, and AA, respectively (see Economides and Salop, 1992). Following Eq. (2), in Scenario EE, the demand for each package is given by

$$\begin{aligned} D_{1a}^{EE} &= \frac{1}{4(1+\tau)} - \frac{p_1+p_a}{(1-\tau)(1+\tau)} + \frac{\tau(p_2+p_b)}{(1-\tau)(1+\tau)}, \\ D_{2b}^{EE} &= \frac{1}{4(1+\tau)} - \frac{p_2+p_b}{(1-\tau)(1+\tau)} + \frac{\tau(p_1+p_a)}{(1-\tau)(1+\tau)}, \\ D_{ij}^{EE} &= 0, \quad ij = 1b, 2a. \end{aligned}$$

In Scenario EA, the demand for each package is given by

$$\begin{aligned} D_{1a}^{EA} &= \frac{1}{4(1+2\tau)} - \frac{(1+\tau)(p_1+p_a)}{(1-\tau)(1+2\tau)} + \frac{\tau(2p_2+p_a+p_b)}{(1-\tau)(1+2\tau)}, \\ D_{2a}^{EA} &= \frac{1}{4(1+2\tau)} - \frac{(1+\tau)(p_2+p_a)}{(1-\tau)(1+2\tau)} + \frac{\tau(p_1+p_2+p_a+p_b)}{(1-\tau)(1+2\tau)}, \\ D_{2b}^{EA} &= \frac{1}{4(1+2\tau)} - \frac{(1+\tau)(p_2+p_b)}{(1-\tau)(1+2\tau)} + \frac{\tau(p_1+p_2+2p_a)}{(1-\tau)(1+2\tau)}, \\ D_{1b}^{EA} &= 0. \end{aligned}$$

In Scenario AE, the demand for each package is given by

$$\begin{aligned} D_{1a}^{AE} &= \frac{1}{4(1+2\tau)} - \frac{(1+\tau)(p_1+p_a)}{(1-\tau)(1+2\tau)} + \frac{\tau(p_1+p_2+2p_b)}{(1-\tau)(1+2\tau)}, \\ D_{1b}^{AE} &= \frac{1}{4(1+2\tau)} - \frac{(1+\tau)(p_1+p_b)}{(1-\tau)(1+2\tau)} + \frac{\tau(p_1+p_2+p_a+p_b)}{(1-\tau)(1+2\tau)}, \\ D_{2b}^{AE} &= \frac{1}{4(1+2\tau)} - \frac{(1+\tau)(p_2+p_b)}{(1-\tau)(1+2\tau)} + \frac{\tau(2p_1+p_a+p_b)}{(1-\tau)(1+2\tau)}, \\ D_{2a}^{AE} &= 0. \end{aligned}$$

In Scenario AA, the demand for each package is given by

$$\begin{aligned} D_{1a}^{AA} &= \frac{1}{4(1+3\tau)} - \frac{(1+2\tau)(p_1+p_a)}{(1-\tau)(1+3\tau)} + \frac{\tau(p_1+2p_2+p_a+2p_b)}{(1-\tau)(1+3\tau)}, \\ D_{1b}^{AA} &= \frac{1}{4(1+3\tau)} - \frac{(1+2\tau)(p_1+p_b)}{(1-\tau)(1+3\tau)} + \frac{\tau(p_1+2p_2+2p_a+p_b)}{(1-\tau)(1+3\tau)}, \\ D_{2a}^{AA} &= \frac{1}{4(1+3\tau)} - \frac{(1+2\tau)(p_2+p_a)}{(1-\tau)(1+3\tau)} + \frac{\tau(2p_1+p_2+p_a+2p_b)}{(1-\tau)(1+3\tau)}, \\ D_{2b}^{AA} &= \frac{1}{4(1+3\tau)} - \frac{(1+2\tau)(p_2+p_b)}{(1-\tau)(1+3\tau)} + \frac{\tau(2p_1+p_2+2p_a+p_b)}{(1-\tau)(1+3\tau)}. \end{aligned}$$

Following Eq. (3) without revenue sharing, we have

$$\Pi_i = \sum_{j=a,b} p_i D_{ij}, \ i = 1, 2,$$

$$\Pi_j = \sum_{i=1,2} p_j D_{ij},\ j = a, b.$$

To prove that there exists a unique equilibrium, following Cachon and Netessine (2004), we define the Hessian matrix as:

$$H \equiv \begin{bmatrix} \frac{\partial^2 \Pi_1}{\partial p_1^2} & \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_a} & \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_b} \\ \frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi_2}{\partial p_2^2} & \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_a} & \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_b} \\ \frac{\partial^2 \Pi_a}{\partial p_1 \partial p_a} & \frac{\partial^2 \Pi_a}{\partial p_2 \partial p_a} & \frac{\partial^2 \Pi_a}{\partial p_a^2} & \frac{\partial^2 \Pi_a}{\partial p_a \partial p_b} \\ \frac{\partial^2 \Pi_b}{\partial p_1 \partial p_b} & \frac{\partial^2 \Pi_b}{\partial p_2 \partial p_b} & \frac{\partial^2 \Pi_b}{\partial p_a \partial p_b} & \frac{\partial^2 \Pi_b}{\partial p_b^2} \end{bmatrix}.$$

Therefore, for Scenarios EE, EA, AE, and AA, we obtain their respective Hessian matrix as follows:

$$H^{EE} = \frac{1}{(1-\tau)(1+\tau)} \begin{bmatrix} -2 & \tau & -1 & \tau \\ \tau & -2 & \tau & -1 \\ -1 & \tau & -2 & \tau \\ \tau & -1 & \tau & -2 \end{bmatrix}.$$

$$H^{EA} = \frac{1}{(1-\tau)(1+2\tau)} \begin{bmatrix} -2(1+\tau) & 2\tau & -1 & \tau \\ 2\tau & -4 & -(1-2\tau) & -1 \\ -1 & -(1-2\tau) & -4 & 2\tau \\ \tau & -1 & 2\tau & -2(1+\tau) \end{bmatrix}.$$

$$H^{AE} = \frac{1}{(1-\tau)(1+2\tau)} \begin{bmatrix} -4 & 2\tau & -1 & -(1-2\tau) \\ 2\tau & -2(1+\tau) & \tau & -1 \\ -1 & \tau & -2(1+\tau) & 2\tau \\ -(1-2\tau) & -1 & 2\tau & -4 \end{bmatrix}.$$

$$H^{AA} = \frac{1}{(1-\tau)(1+3\tau)} \begin{bmatrix} -4(1+\tau) & 4\tau & -(1-\tau) & -(1-\tau) \\ 4\tau & -4(1+\tau) & -(1-\tau) & -(1-\tau) \\ -(1-\tau) & -(1-\tau) & -4(1+\tau) & 4\tau \\ -(1-\tau) & -(1-\tau) & 4\tau & -4(1+\tau) \end{bmatrix}.$$

First, from the Hessian matrixes, we can see each player's objective function is concave in its own decision variable. So the existence of Nash equilibrium holds. Moreover, according to Cachon and Netessine (2004) (Theorem 6), there is a unique Nash equilibrium, if $H + H^T$ is negative definite, which is true for all above scenarios given $\tau \in [0, 1)$. One can easily verify this conclusion from the above Hessian matrixes. For brevity, we omit the determinants.

We can then obtain the best response pricing function for each player given the other prices from their corresponding first order condition. Due to limited space, we hereby show only Scenario EE. Computation of Scenarios EA, AE, and AA follows the same procedure. In Scenario EE, the best response pricing functions for suppliers 1 and 2 and retailers a and b are given, respectively, by

$$\begin{aligned}
p_1 &= \frac{1}{8}\left(1-\tau+4\tau p_2-4p_a+4\tau p_b\right), \\
p_2 &= \frac{1}{8}\left(1-\tau+4\tau p_1+4\tau p_a-4p_b\right), \\
p_a &= \frac{1}{8}\left(1-\tau-4p_1+4\tau p_2+4\tau p_b\right), \\
p_b &= \frac{1}{8}\left(1-\tau+4\tau p_1-4p_2+4\tau p_a\right).
\end{aligned}$$

Combining the best response functions above, we can obtain equilibrium prices and then profits, as shown in Table 1. Due to symmetry, we provide only results for supplier 1 and retailer a. The equilibrium solutions of supplier 2 and retailer b in Scenarios EE and AA are the same as those of supplier 1 and retailer a. The equilibrium solutions of supplier 2 and retailer b in Scenario EA are the same as those of supplier 1 and retailer a, respectively, in Scenario AE. We can also show that all equilibria are located inside the feasible domain, since all demands under the equilibrium prices are nonnegative. Note that the demand under equilibrium can be computed by plugging the equilibrium prices into the above demand functions, which is skipped for brevity.

Table 1: Equilibrium solutions in Scenarios EE, EA, AE, and AA under.

	p_1^*	p_a^*	Π_1^*	Π_a^*
EE	$\frac{1-\tau}{12-8\tau}$	$\frac{1-\tau}{12-8\tau}$	$\frac{1-\tau}{16(3-2\tau)^2(1+\tau)}$	$\frac{1-\tau}{16(3-2\tau)^2(1+\tau)}$
EA	$\frac{3-\tau-2\tau^2}{4(9+5\tau-6\tau^2)}$	$\frac{3+\tau-4\tau^2}{36+20\tau-24\tau^2}$	$\frac{(1-\tau)(1+\tau)(3+2\tau)^2}{16(1+2\tau)(9+5\tau-6\tau^2)^2}$	$\frac{(1-\tau)(3+4\tau)^2}{8(1+2\tau)(9+5\tau-6\tau^2)^2}$
AE	$\frac{3+\tau-4\tau^2}{36+20\tau-24\tau^2}$	$\frac{3-\tau-2\tau^2}{4(9+5\tau-6\tau^2)}$	$\frac{(1-\tau)(3+4\tau)^2}{8(1+2\tau)(9+5\tau-6\tau^2)^2}$	$\frac{(1-\tau)(1+\tau)(3+2\tau)^2}{16(1+2\tau)(9+5\tau-6\tau^2)^2}$
AA	$\frac{1-\tau}{4(3-\tau)}$	$\frac{1-\tau}{4(3-\tau)}$	$\frac{(1-\tau)(1+\tau)}{8(3-\tau)^2(1+3\tau)}$	$\frac{(1-\tau)(1+\tau)}{8(3-\tau)^2(1+3\tau)}$

We next compare the prices. For supplier 1, we have

$$\begin{aligned}
p_1^{*EE}-p_1^{*EA} &= \frac{\tau\left(5-7\tau+2\tau^2\right)}{4\left(27-3\tau-28\tau^2+12\tau^3\right)} \geq 0, \\
p_1^{*EE}-p_1^{*AA} &= \frac{(1-\tau)\tau}{4\left(9-9\tau+2\tau^2\right)} \geq 0, \\
p_1^{*AE}-p_1^{*EA} &= \frac{(1-\tau)\tau}{2\left(9+5\tau-6\tau^2\right)} \geq 0,
\end{aligned}$$

$$\begin{aligned}
p_1^{*AE} - p_1^{*AA} &= \frac{\tau\left(2-\tau-\tau^2\right)}{2\left(27+6\tau-23\tau^2+6\tau^3\right)} \geq 0,\\
p_1^{*EE} - p_1^{*AE} &= -\frac{\tau\,(1-\tau)(1-2\tau)}{4\left(27-3\tau-28\tau^2+12\tau^3\right)},\\
p_1^{*EA} - p_1^{*AA} &= -\frac{\tau\,(1-\tau)(1-2\tau)}{2\left(27+6\tau-23\tau^2+6\tau^3\right)},
\end{aligned}$$

in which the inequalities can be verified easily given $\tau \in [0,1)$. For example, the numerator of the first inequality is positive as $5-7\tau+2\tau^2$ is decreasing in τ and its value is positive when τ approaches 1; the same argument leads to the positiveness of the denominator. And it is also easy to verify that when $\tau > 1/2$, the right hand sides of the last two equations are positive whereas they are negative when $\tau < 1/2$. Thus, we obtain the result regarding p_1^* as shown in Lemma 1. Similar reasoning leads to the result regarding p_a^* as shown in Lemma 1. □

Proof of Theorem 1: Continuing with Lemma 1, we consider the prices for packages. Based on Table 1, we have

$$\begin{aligned}
P_{1a}^{*EE} - P_{1a}^{*EA} &= \frac{(1-\tau)\tau}{27-3\tau-28\tau^2+12\tau^3} \geq 0,\\
P_{1a}^{*EA} - P_{1a}^{*AE} &= 0,\\
P_{1a}^{*EA} - P_{1a}^{*AA} &= \frac{\tau\left(1+2\tau-3\tau^2\right)}{2\left(27+6\tau-23\tau^2+6\tau^3\right)} \geq 0,
\end{aligned}$$

where the inequalities follow from that $\tau \in [0,1)$. Thus, the theorem is proved. □

Proof of Theorem 2: Recall the sequence of moves by the players in each channel. The supplier first suggests a channel structure (exclusive or not) to the retailer. Then if the supplier suggests an exclusive deal with the retailer, the retailer determines whether to accept it; otherwise, if the supplier decides to sell its product through both retailers, then the retailer has no choice but to accept the contract. Note that in this stage, if the retailer refuses to form an exclusive deal with the supplier, the supplier will just sell its product through both retailers. Finally, both the supplier and retailer set their prices. Both channels proceed simultaneously with the above sequence. We solve the game backwards.

To prove that forming an exclusive deal is a weakly dominant strategy for the retailers, we need to show that, a retailer is not worse off with an exclusive deal, regardless of whether the other supplier-retailer pair forms an exclusive deal. Similarly, if forming an exclusive deal is dominated for both suppliers, no supplier would choose an exclusive deal regardless of the other supplier-retailer pair's strategy. Due to the symmetry, we only show this result for retailer a and supplier 1. For retailer a, it is sufficient to prove that Scenario EE outperforms Scenario AE and Scenario EA outperforms Scenario AA, provided that supplier 1 offers an exclusive deal. Hence, based on the analysis of retailer a, for supplier 1, it suffices to show that its profit under EE is less than that under AE and its profit under EA is less than that under AA. Based on Table 1, for retailer a,

$$
\begin{aligned}
\Pi_a^{*EE} - \Pi_a^{*AE} &= \frac{(1-\tau)\left(\frac{1}{(3-2\tau)^2} - \frac{(1+\tau)^2(3+2\tau)^2}{(1+2\tau)(9+5\tau-6\tau^2)^2}\right)}{16(1+\tau)} \geq 0, \\
\Pi_a^{*EA} - \Pi_a^{*AA} &= \frac{1}{8}(1-\tau)\left(\frac{(3+4\tau)^2}{(1+2\tau)\left(9+5\tau-6\tau^2\right)^2} - \frac{1+\tau}{(3-\tau)^2(1+3\tau)}\right) \geq 0.
\end{aligned}
$$

The above inequalities become equalities only when $\tau = 0$. For supplier 1,

$$
\begin{aligned}
\Pi_1^{*EE} - \Pi_1^{*AE} &= \frac{1}{16}(1-\tau)\left(\frac{1}{(3-2\tau)^2(1+\tau)} - \frac{2(3+4\tau)^2}{(1+2\tau)\left(9+5\tau-6\tau^2\right)^2}\right) < 0, \\
\Pi_1^{*EA} - \Pi_1^{*AA} &= \frac{1}{16}(1-\tau)(1+\tau)\left(\frac{(3+2\tau)^2}{(1+2\tau)\left(9+5\tau-6\tau^2\right)^2} - \frac{2}{(3-\tau)^2(1+3\tau)}\right) < 0.
\end{aligned}
$$

Thus, no matter whether the other channel forms an exclusive deal, it is weakly dominant and, thus, optimal for retailer a to seek an exclusive deal with supplier 1. However, the reverse is true for supplier 1. Consequently, supplier 1 will not offer an exclusive deal contract to retailer a and, thus, forming exclusive deals without revenue sharing cannot be an equilibrium. □

Proof of Theorem 3: From Table 1, we obtain the total profit for the entire supply chain including both suppliers and both retailers as follows:

$$
\begin{aligned}
\Pi_{All}^{*EE} &= \frac{1-\tau}{4(3-2\tau)^2(1+\tau)}, \\
\Pi_{All}^{*EA} &= \Pi_{All}^{*AE} = \frac{27+42\tau-21\tau^2-44\tau^3-4\tau^4}{8(1+2\tau)\left(9+5\tau-6\tau^2\right)^2}, \\
\Pi_{All}^{*AA} &= \frac{(1-\tau)(1+\tau)}{2(3-\tau)^2(1+3\tau)}.
\end{aligned}
$$

We then visualize the proof in Figure 2. We define $\hat{\tau}_1$ as the intersection point between Scenarios AA and EA and $\hat{\tau}_2$ as the intersection point between Scenarios EE and EA. There, solving the single crossing points between AA and EA/AE by setting $\Pi_{All}^{*AA} = \Pi_{All}^{*EA}$ and between EA and EE by setting $\Pi_{All}^{*EE} = \Pi_{All}^{*EA}$ yields $\hat{\tau}_1 = 0.5633$ and $\hat{\tau}_2 = 0.6901$, respectively. We then observe that AA dominates Scenarios EA, AE, and EE when $0 \leq \tau < \hat{\tau}_1$; Scenarios EA and AE dominate EE and AA when $\hat{\tau}_1 \leq \tau < \hat{\tau}_2$; and Scenario EE dominates EA, AE, and AA, if $\hat{\tau}_2 \leq \tau < 1$. □

Proof of Lemma 2: With revenue sharing, the profit functions are given by Eq. (3). The demand functions are the same as those in the proof of Lemma 1. Because the Hessian matrixes are independent of the revenue sharing rate, they are identical to those in Lemma 1. Therefore, similar to the proof of Lemma 1, we obtain unique equilibrium solutions in Table 2.

Table 2: Equilibrium solutions in Scenarios EE, EA, AE, and AA with revenue sharing.

	p_1^*	p_a^*	Π_1^*	Π_a^*
EE	$\frac{1-\tau}{12-8\tau}-r$	$\frac{1-\tau}{12-8\tau}+r$	$\frac{1-\tau}{16(3-2\tau)^2(1+\tau)}$	$\frac{1-\tau}{16(3-2\tau)^2(1+\tau)}$
EA	$\frac{C_2-4rC_3}{4C_1}$	$\frac{C_4+4rC_5}{4C_1}$	$\frac{(1+\tau)(C_2+4rC_6)^2}{16(1+\tau-2\tau^2){C_1}^2}$	$\frac{{C_4}^2-2rC_7-8r^2C_8}{8(1+\tau-2\tau^2)C_1^2}$
AE	$\frac{C_4-4rC_9}{4C_1}$	$\frac{C_2+4rC_{10}}{4C_1}$	$\frac{(C_4-4rC_9)^2}{8(1+\tau-2\tau^2){C_1}^2}$	$\frac{(1+\tau)(C_2+4rC_{10})^2}{16(1+\tau-2\tau^2)C_1^2}$
AA	$\frac{1-\tau}{4(3-\tau)}$	$\frac{1-\tau}{4(3-\tau)}$	$\frac{(1-\tau)(1+\tau)}{8(3-\tau)^2(1+3\tau)}$	$\frac{(1-\tau)(1+\tau)}{8(3-\tau)^2(1+3\tau)}$

In Table 2, we have

$$
\begin{aligned}
C_1 &= 45+106\tau+33\tau^2-44\tau^3-12\tau^4>0,\\
C_2 &= 15+22\tau-13\tau^2-20\tau^3-4\tau^4\geq 0,\\
C_3 &= 33+71\tau+20\tau^2-24\tau^3-8\tau^4>0,\\
C_4 &= 15+32\tau-5\tau^2-34\tau^3-8\tau^4\geq 0,\\
C_5 &= 21+48\tau+13\tau^2-22\tau^3-8\tau^4>0,\\
C_6 &= 12+35\tau+13\tau^2-20\tau^3-4\tau^4>0,\\
C_7 &= 90+717\tau+1980\tau^2+1838\tau^3-1050\tau^4-2899\tau^5-1316\tau^6+288\tau^7+304\tau^8+48\tau^9\geq 0,\\
C_8 &= 603+3741\tau+8204\tau^2+6196\tau^3-2519\tau^4-5673\tau^5-1488\tau^6+920\tau^7+432\tau^8+48\tau^9>0,\\
C_9 &= 6+21\tau+17\tau^2-4\tau^3-4\tau^4>0,\\
C_{10} &= 3+12\tau+9\tau^2-8\tau^3-4\tau^4>0.
\end{aligned}
$$

Constraints are imposed for nonnegative marginal profits (product price plus shared revenue) and demands as follows:

$$
r\leq\hat{r}_0\equiv\min\left\{\frac{C_4}{4C_9},\frac{3-5\tau^2+2\tau^3}{4\left(3+6\tau-\tau^2-2\tau^3\right)}\right\}=\frac{3-5\tau^2+2\tau^3}{4\left(3+6\tau-\tau^2-2\tau^3\right)},
$$

where the first item guarantees nonnegative prices in Scenarios EA and AE and the second item guarantees nonnegative demand for package $2a$ in Scenario EA and package $1b$ for Scenario AE. Other marginal profits and demands are all nonnegative. From Table 2 and the above equations, it is quite obvious that, with revenue sharing, product prices decrease while service rates increase with the revenue sharing rate in Scenarios EE, EA, and AE. Consider the package price of P_{1a}^*.

$$
\begin{aligned}
\frac{dP_{1a}^{*EE}}{dr} &= 0,\\
\frac{dP_{1a}^{*EA}}{dr} &= -\frac{12+23\tau+7\tau^2-2\tau^3}{C_1}<0,
\end{aligned}
$$

$$\begin{aligned}\frac{dP_{1a}^{*AE}}{dr} &= -\frac{3+9\tau+8\tau^2+4\tau^3}{C_1}<0,\\ \frac{dP_{1a}^{*AA}}{dr} &= 0.\end{aligned}$$

Thus, package prices decrease with the revenue sharing rate in Scenarios EA and AE. □

Proof of Theorem 4: To show that forming exclusive deals is a subgame perfect equilibrium, we must demonstrate that no player will unilaterally deviate from Scenario EE in the first stage of the game. For supplier 1 and retailer a, Scenario EE must be no worse than Scenario AE; for supplier 2 and retailer b, Scenario EE must be no worse than Scenario EA. Due to the symmetry, $\Pi_1^{*AE}=\Pi_2^{*EA}$ and $\Pi_a^{*AE}=\Pi_b^{*EA}$. Hence, it is sufficient to prove that both supplier 1 and retailer a prefer Scenario EE to AE. Comparing the profits from Table 2,

$$\begin{aligned}\Delta\Pi_1^{EEAE}\equiv\Pi_1^{*EE}-\Pi_1^{*AE} &= \frac{1-\tau}{16(3-2\tau)^2(1+\tau)}-\frac{\left(C_4-4rC_9\right)^2}{8\left(1+\tau-2\tau^2\right)C_1{}^2},\\ \Delta\Pi_a^{EEAE}\equiv\Pi_a^{*EE}-\Pi_a^{*AE} &= \frac{1-\tau}{16(3-2\tau)^2(1+\tau)}-\frac{(1+\tau)\left(C_2+4rC_{10}\right)^2}{16\left(1+\tau-2\tau^2\right)C_1^2}.\end{aligned}$$

We need to identify the region of τ where $\Delta\Pi_1^{EEAE}\geq 0$ and $\Delta\Pi_a^{EEAE}\geq 0$ so that EE is an equilibrium. To this end, we take the second derivatives with respect to r as follows:

$$\begin{aligned}\frac{d^2\Delta\Pi_1^{EEAE}}{dr^2} &= -\frac{4(1+2\tau)\left(6+9\tau-\tau^2-2\tau^3\right)^2}{(1-\tau)C_1^2}<0,\\ \frac{d^2\Delta\Pi_a^{EEAE}}{dr^2} &= -\frac{2(1+\tau)(1+2\tau)\left(3+6\tau-3\tau^2-2\tau^3\right)^2}{(1-\tau)C_1^2}<0.\end{aligned}$$

Thus, both $\Delta\Pi_1^{EEAE}$ and $\Delta\Pi_a^{EEAE}$ are strictly concave in r. Solving $\Delta\Pi_1^{EEAE}=0$ and $\Delta\Pi_a^{EEAE}=0$ yields two roots for each, respectively. The equations for the roots are very lengthy, thus they are omitted here. The roots depend on a single parameter, τ. Moreover, we can show that the smaller root of $\Delta\Pi_1^{EEAE}=0$ is larger than that of $\Delta\Pi_a^{EEAE}=0$ whereas the larger root of $\Delta\Pi_1^{EEAE}=0$ is larger than that of $\Delta\Pi_a^{EEAE}=0$. As a result, we can identify the area defined by $\hat{r}_1(\tau)\leq r\leq\hat{r}_2(\tau)$, under which EE is the equilibrium, where $\hat{r}_1(\tau)$ is the smaller root of $\Delta\Pi_1^{EEAE}=0$ and $\hat{r}_2(\tau)$ is the minimum of the larger root of $\Delta\Pi_a^{EEAE}=0$ and $\hat{r}_0$ (defined in Lemma 2), as illustrated in Figure 3. Note that the complexity of $\hat{r}_1(\tau)$ and $\hat{r}_2(\tau)$ are mainly due to the asymmetry of Scenario AE and the game setting that players need to determine four prices simultaneously in a Nash game. From our previous discussion, we can easily infer that the retailer will prefer not to form an exclusive deal with the supplier as long as $r>\hat{r}_a(\tau)$, which results in non-exclusive deal (regardless whether the supplier offers exclusive contract) between the supplier and retailer. Similarly, if $r<\hat{r}_1(\tau)$, even if the retailer prefers to form an exclusive deal with the supplier, the supplier prefers to sell its product through both retailers. Hence, in both cases, forming an exclusive deal is not an equilibrium. Furthermore, the starting point of the overlapping area is given by $\tau = 0.34$ solved from $\hat{r}_1(\tau) = \hat{r}_2(\tau)$, and the ending point is at $\tau = 1$. $\square$

Proof of Theorem 5: It is sufficient to show that either a supplier or a retailer is more inclined to deviate from Scenario EE as ρ grows. Note the profit functions in Eq. (3) continue to hold for this new revenue sharing scheme. Due to the symmetry, $\Pi_1^{*AE} = \Pi_2^{*EA}$ and $\Pi_a^{*AE} = \Pi_b^{*EA}$. To show whether either a supplier or a retailer will deviate from Scenario EE, similar to the proof of Theorem 4, it is sufficient to prove that either supplier 1 or retailer a will unilaterally deviate from Scenario EE to Scenario AE. We compute players' profits and compare them as follows:

$$\begin{aligned}
\Delta\Pi_1^{EEAE} \equiv \Pi_1^{*EE} - \Pi_1^{*AE} &= \frac{1-\tau}{16(3-2\tau)^2(1+\tau)} - \frac{\left(C_4 - 4(1-\rho)rC_9\right)^2}{8\left(1+\tau-2\tau^2\right)C_1{}^2}, \\
\Delta\Pi_a^{EEAE} \equiv \Pi_a^{*EE} - \Pi_a^{*AE} &= \frac{1-\tau}{16(3-2\tau)^2(1+\tau)} - \frac{(1+\tau)\left(C_2 + 4(1-\rho)rC_{10}\right)^2}{16\left(1+\tau-2\tau^2\right)C_1^2}.
\end{aligned}$$

When $\rho = 0$, the functions above are the same as those in Theorem 4, such that for any given r satisfying $\hat{r}_1(\tau) \leq r \leq \hat{r}_2(\tau)$ for $\tau \in [0.34, 1)$,

$$\Delta\Pi_1^{EEAE} \geq 0 \text{ and } \Delta\Pi_a^{EEAE} \geq 0.$$

When $\rho = 1$, the above case is equivalent to that in Theorem 2, such that for any r, we have

$$\begin{aligned}
\Delta\Pi_1^{EEAE} &= \frac{1}{16}(1-\tau)\left(\frac{1}{(3-2\tau)^2(1+\tau)} - \frac{2(3+4\tau)^2}{(1+2\tau)\left(9+5\tau-6\tau^2\right)^2}\right) < 0, \\
\Delta\Pi_a^{EEAE} &= \frac{(1-\tau)\left(\frac{1}{(3-2\tau)^2} - \frac{(1+\tau)^2(3+2\tau)^2}{(1+2\tau)\left(9+5\tau-6\tau^2\right)^2}\right)}{16(1+\tau)} \geq 0.
\end{aligned}$$

Moreover, it is clear that $\Delta\Pi_1^{EEAE}$ is decreasing in ρ while $\Delta\Pi_a^{EEAE}$ is increasing in ρ since C_1, C_2, C_4, C_9, and C_{10} are all positive as demonstrated in the proof of Theorem 4 and $C_4 - 4(1-\rho)rC_9 > 0$ given $\hat{r}_1(\tau) \leq r \leq \hat{r}_2(\tau)$. Combining these results, we can easily infer that retailer a always prefers an exclusive deal given $\hat{r}_1(\tau) \leq r \leq \hat{r}_2(\tau)$ for $\tau \in [0.34, 1)$. However, supplier 1's preference of exclusive deal hinges upon the relative level (ρ) of revenue sharing under nonexclusivity. Furthermore, there exists a unique threshold value $\widehat{\rho}$ of ρ, such that supplier 1 no longer prefers EE to AE when $\widehat{\rho} < \rho \leq 1$, where $\widehat{\rho}$ is the value of ρ solved by $\Delta\Pi_1^{EEAE} = 0$. Overall, it is more likely for supplier 1 to deviate from EE (i.e., EE will be no longer an equilibrium) as ρ grows. $\square$

Proof of Theorem 6: Because the Hessian matrixes of the profit functions in Eq. (A-3) are independent of the revenue sharing rate and the fencing costs, they are identical to H^{AA} as in Lemma 1. Using the same techniques in the proof of Lemma 1, we obtain the unique equilibrium solutions for Scenarios EE, EA,

Table 3: Equilibrium solutions in Scenarios EE, EA, AE, and AA with fencing.

	p_1^*	p_a^*	Π_{All}^*
EE	$\frac{1-\tau}{4(3-\tau)} - \frac{k}{2}$	$\frac{1-\tau}{4(3-\tau)}$	$\frac{(1-\tau)^2(1+\tau)-2k^2(3-\tau)^2(1+3\tau)}{2(3-\tau)^2(1+2\tau-3\tau^2)}$
EA	$\frac{1-\tau}{4(3-\tau)} - \frac{k}{2}$	$\frac{2+2\tau-4\tau^2+k\left(3+8\tau-3\tau^2\right)}{8(3+5\tau-2\tau^2)}$	$\frac{8(1+\tau)\left(1+\tau-2\tau^2\right)^2-k^2(3-\tau)^2(1+3\tau)(5+\tau(18+17\tau))}{16(1-\tau)(1+3\tau)(3+(5-2\tau)\tau)^2}$
AE	$\frac{1-\tau}{4(3-\tau)}$	$\frac{2+2\tau-4\tau^2-k\left(3+8\tau-3\tau^2\right)}{8(3+5\tau-2\tau^2)}$	Π_{All}^{*EA}
AA	$\frac{1-\tau}{4(3-\tau)}$	$\frac{1-\tau}{4(3-\tau)}$	$\frac{(1-\tau)(1+\tau)}{2(3-\tau)^2(1+3\tau)}$

AE, and AA, as illustrated in Table 3. Comparing the profits of the entire supply chain in Scenarios EE, EA (same as AE), and AA, we have

$$\begin{aligned}
\Delta\Pi_{All}^{EAAA} \equiv \Pi_{All}^{*EA} - \Pi_{All}^{*AA} &= -\frac{k^2\left(5+18\tau+17\tau^2\right)}{16(1-\tau)(1+2\tau)^2} \leq 0, \\
\Delta\Pi_{All}^{EAEE} \equiv \Pi_{All}^{*EA} - \Pi_{All}^{*EE} &= \frac{k^2\left(11+46\tau+47\tau^2\right)}{16(1-\tau)(1+2\tau)^2} \geq 0.
\end{aligned}$$

This proves the first item of the theorem. We now consider the corner solution. In line with O'Brien and Shaffer (1993) and Ingene and Parry (2004) (Chapter 10), we adopt a price-out strategy where the switching cost k_i is set at a price such that demand for the undesirable package(s) in Scenarios EE, EA, and AE becomes zero. In Scenario EE, the values of k is set such that $D_{1b}^{EE} = D_{2a}^{EE} = 0$. In Scenario EA, $D_{1b}^{EA} = 0$, and in Scenario AE, $D_{2a}^{EA} = 0$. We replace the switching cost with the equivalence of product prices and service rates in the profit functions. For example, in Scenario EA, the price-out switching cost is set at

$$\bar{k}^{EA}(\tau) = \frac{1-\tau-4(1+\tau)p_1+8\tau p_2+8\tau p_a-4p_b-4\tau p_b}{4+8\tau}.$$

Placing this price-out switching cost into the corresponding demand functions, we resolve the first-order conditions and obtain the price-out switching cost as follows:

$$\bar{k}^{EA}(\tau) = \frac{3+4\tau-\tau^2-6\tau^3}{36+92\tau+16\tau^2-48\tau^3}.$$

The same price-out switching cost is applied to Scenario AE. For Scenario EE,

$$\bar{k}^{EE}(\tau) = \frac{1-\tau}{12+4\tau-8\tau^2}.$$

Based on the above price-out strategy, we then solve the equilibrium prices/rates and profits and obtain exactly the same solutions as those in Table 1. This demonstrates that the alternative model with price-out strategy is equivalent to our main model with exclusive channels as specified in Section 2. □

Proof of Lemma 3: In composite package (CP) competition, each package is determined to optimize its package profit (see Economides and Salop, 1992). The demand functions are given by Eq. (2) and are the same as those in the proof of Lemma 1 while combining p_i and p_j into P_{ij}. For example, in Scenario EE,

$$\begin{aligned} D_{1a}^{EE} &= \frac{1}{4(1+\tau)} - \frac{P_{1a}}{(1-\tau)(1+\tau)} + \frac{\tau P_{2b}}{(1-\tau)(1+\tau)}, \\ D_{2b}^{EE} &= \frac{1}{4(1+\tau)} - \frac{P_{2b}}{(1-\tau)(1+\tau)} + \frac{\tau P_{1a}}{(1-\tau)(1+\tau)}. \end{aligned}$$

The profit of package ij is thus given by

$$\Pi_{ij} = P_{ij} D_{ij},$$

where in Scenario EE, $ij = 1a, 2b$; in Scenario EA, $ij = 1a, 2a, 2b$; in Scenario AE, $ij = 1a, 1b, 2b$; and in Scenario AA, $ij = 1a, 1b, 2a, 2b$. Through reasoning similar to the proof of Lemma 1, we obtain the following Hessian matrixes.

$$H^{EE} = \begin{bmatrix} \frac{\partial^2 \Pi_{1a}}{\partial P_{1a}^2} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{2b}}{\partial P_{1a} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{2b}^2} \end{bmatrix} = \frac{1}{(1-\tau)(1+\tau)} \begin{bmatrix} -2 & \tau \\ \tau & -2 \end{bmatrix}.$$

$$H^{EA} = \begin{bmatrix} \frac{\partial^2 \Pi_{1a}}{\partial P_{1a}^2} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{2a}} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{2a}}{\partial P_{1a} \partial P_{2a}} & \frac{\partial^2 \Pi_{2a}}{\partial P_{2a}^2} & \frac{\partial^2 \Pi_{2a}}{\partial P_{2a} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{2b}}{\partial P_{1a} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{2a} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{2b}^2} \end{bmatrix} = \frac{1}{(1-\tau)(1+2\tau)} \begin{bmatrix} -2(1+\tau) & \tau & \tau \\ \tau & -2(1+\tau) & \tau \\ \tau & \tau & -2(1+\tau) \end{bmatrix}.$$

$$H^{AE} = \begin{bmatrix} \frac{\partial^2 \Pi_{1a}}{\partial P_{1a}^2} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{1b}} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{1b}}{\partial P_{1a} \partial P_{1b}} & \frac{\partial^2 \Pi_{1b}}{\partial P_{1b}^2} & \frac{\partial^2 \Pi_{1b}}{\partial P_{1b} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{2b}}{\partial P_{1a} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{1b} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{2b}^2} \end{bmatrix} = \frac{1}{(1-\tau)(1+2\tau)} \begin{bmatrix} -2(1+\tau) & \tau & \tau \\ \tau & -2(1+\tau) & \tau \\ \tau & \tau & -2(1+\tau) \end{bmatrix}.$$

$$\begin{aligned} H^{AA} &= \begin{bmatrix} \frac{\partial^2 \Pi_{1a}}{\partial P_{1a}^2} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{1b}} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{2a}} & \frac{\partial^2 \Pi_{1a}}{\partial P_{1a} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{1b}}{\partial P_{1a} \partial P_{1b}} & \frac{\partial^2 \Pi_{1b}}{\partial P_{1b}^2} & \frac{\partial^2 \Pi_{1b}}{\partial P_{1b} \partial P_{2a}} & \frac{\partial^2 \Pi_{1b}}{\partial P_{1b} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{2a}}{\partial P_{1a} \partial P_{2a}} & \frac{\partial^2 \Pi_{2a}}{\partial P_{1b} \partial P_{2a}} & \frac{\partial^2 \Pi_{2a}}{\partial P_{2a}^2} & \frac{\partial^2 \Pi_{2a}}{\partial P_{2a} \partial P_{2b}} \\ \frac{\partial^2 \Pi_{2b}}{\partial P_{1a} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{1b} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{2a} \partial P_{2b}} & \frac{\partial^2 \Pi_{2b}}{\partial P_{2b}^2} \end{bmatrix} \\ &= \frac{1}{(1-\tau)(1+3\tau)} \begin{bmatrix} -2(1+2\tau) & \tau & \tau & \tau \\ \tau & -2(1+2\tau) & \tau & \tau \\ \tau & \tau & -2(1+2\tau) & \tau \\ \tau & \tau & \tau & -2(1+2\tau) \end{bmatrix}. \end{aligned}$$

Similarly, according to Cachon and Netessine (2004), there is a unique Nash equilibrium in each scenario, because $H + H^T$ is negative definite for all above Hessian matrixes given $\tau \in [0, 1)$.

To obtain the equilibrium result, we then solve the first-order conditions with respect to package prices. For example, in Scenario EE, the packages' best response pricing functions are given by

$$\begin{aligned} P_{1a} &= \frac{1}{8}\left(1 - \tau + 4\tau P_{2b}\right), \\ P_{2b} &= \frac{1}{8}\left(1 - \tau + 4\tau P_{1a}\right). \end{aligned}$$

Due to symmetry, we show the unique equilibrium result for package $1a$ only, as illustrated in Table 4.

Table 4: Equilibrium solutions in Scenarios EE, EA, AE, and AA under CP competition.

	EE	EA	AE	AA
P^*_{1a}	$\frac{1-\tau}{4(2-\tau)}$	$\frac{1-\tau}{8}$	$\frac{1-\tau}{8}$	$\frac{1-\tau}{8+4\tau}$
Π^*_{1a}	$\frac{1-\tau}{16(2-\tau)^2(1+\tau)}$	$\frac{1-\tau^2}{64+128\tau}$	$\frac{1-\tau^2}{64+128\tau}$	$\frac{1+\tau-2\tau^2}{16(2+\tau)^2(1+3\tau)}$

Similar to that of Lemma 1, all equilibria are inside the feasible domain given $\tau \in [0, 1)$. Comparing the prices in Table 4, we can easily conclude $P^{*AA}_{1a} = P^{*EA}_{1a} = P^{*AE}_{1a} = P^{*EE}_{1a}$.

We define $\Delta P_{1a} \equiv P^{*CP}_{1a} - P^{*IO}_{1a}$ and denote the scenarios to the superscripts. We compare prices between IO and CP competition in all scenarios as follows:

$$\begin{aligned} \Delta P^{EE}_{1a} &= -\frac{1-\tau}{4\left(6 - 7\tau + 2\tau^2\right)} < 0, \\ \Delta P^{EA}_{1a} &= -\frac{3 + 4\tau - \tau^2 - 6\tau^3}{8\left(9 + 5\tau - 6\tau^2\right)} < 0, \\ \Delta P^{AE}_{1a} &= -\frac{3 + 4\tau - \tau^2 - 6\tau^3}{8\left(9 + 5\tau - 6\tau^2\right)} < 0, \\ \Delta P^{AA}_{1a} &= -\frac{1 + 2\tau - 3\tau^2}{4\left(6 + \tau - \tau^2\right)} < 0. \end{aligned}$$

Again, due to the symmetry, package prices for $1b$, $2a$, and $2b$ follow the same pattern. □

Proof of Lemma 4: From Table 4, we obtain the total profit for the entire supply chain including all packages as follows:

$$\Pi^{*EE}_{All} = \frac{1-\tau}{8(2-\tau)^2(1+\tau)},$$

$$\begin{aligned}\Pi_{All}^{*EA} &= \Pi_{All}^{*AE} = \frac{3(1-\tau^2)}{64+128\tau},\\ \Pi_{All}^{*AA} &= \frac{1+\tau-2\tau^2}{4(2+\tau)^2(1+3\tau)}.\end{aligned}$$

We define $\hat{\tau}_3$ as the intersection point between Scenarios AA and EA and $\hat{\tau}_4$ as the intersection point between Scenarios EE and EA. Solving the single crossing points between AA and EA/AE by setting $\Pi_{All}^{*AA} = \Pi_{All}^{*EA}$ and between EA and EE by setting $\Pi_{All}^{*EE} = \Pi_{All}^{*EA}$ yields $\hat{\tau}_3 = 0.3621$ and $\hat{\tau}_4 = 0.4468$, respectively. Similar to Figure 2, we observe that AA dominates Scenarios EA, AE, and EE when $0 \leq \tau < \hat{\tau}_3$; Scenarios EA and AE dominate EE and AA when $\hat{\tau}_3 \leq \tau < \hat{\tau}_4$; and Scenario EE dominates EA, AE, and AA, if $\hat{\tau}_4 \leq \tau < 1$. □

Proof of Theorem 7: Define $\Delta\Pi_{CPIO} = \Pi_{All}^{*CP} - \Pi_{All}^{*IO}$. We have

$$\begin{aligned}\Delta\Pi_{CPIO}^{EE} &= \frac{1-5\tau+6\tau^2-2\tau^3}{8(1+\tau)\left(6-7\tau+2\tau^2\right)^2},\\ \Delta\Pi_{CPIO}^{EA} &= \frac{27-66\tau-324\tau^2-98\tau^3+389\tau^4+180\tau^5-108\tau^6}{64(1+2\tau)\left(9+5\tau-6\tau^2\right)^2},\\ \Delta\Pi_{CPIO}^{AA} &= \frac{1-8\tau+7\tau^2}{4(3-\tau)^2(2+\tau)^2}.\end{aligned}$$

Since there is only one independent variable τ in all equilibrium profits, we can use a two-dimension graph to visually compare all profits for the entire supply chain in different scenarios. We observe that Π_{All}^{*CP} and Π_{All}^{*IO} have a single crossing point for each scenario during $\tau \in [0,1)$. Setting $\Delta\Pi_{CPIO} = 0$ for the above equations yields the single crossing points at $\tau = 0.2929, 0.2037, 0.1429$ for Scenarios EE, EA/AE, and AA, respectively. In other words, CP outperforms IO only if $\tau < 0.2929$ in Scenario EE, or if $\tau < 0.2037$ in Scenario EA/AE, or if $\tau < 0.1429$ in Scenario AA. As shown in Theorem 3 and Lemma 4, Scenario AA outperforms other scenarios for both IO and CP cases, as long as $\tau < 0.3621$. Therefore, the best of CP, either Scenario EE, EA/AE, or AA, outperforms the best of IO as long as $\tau < 0.1429$. When $\tau > 0.1429$, we can combine all scenarios in both IO and CP cases and then compare them. For a shortcut, we can also prove it visually, because we find the best of IO and the best of CP have a single crossing point, as uniquely illustrated in Figure 6. Therefore, the best of IO outperforms the best of CP if and only if $\tau \geq 0.1429$. □

Proof of Lemma 5: The computation process is similar to that of IO competition with revenue sharing, as shown in the proof of Lemma 2, except that we replace the original r with the new revenue sharing functions in Eq. (A-4). The new Hessian matrix is

$$H^{EE} = \frac{1}{2(1-\tau)^2(1+\tau)} \begin{bmatrix} -2 & \tau & -2(1-\tau) & \tau \\ \tau & -2 & \tau & -2(1-\tau) \\ -2(1-\tau) & \tau & -2 & \tau \\ \tau & -2(1-\tau) & \tau & -2 \end{bmatrix}.$$

It is easy to show that $H^{EE} + H^{EE^T}$ is negative definite. Therefore, there is a unique Nash equilibrium. Similarly, solving the first order conditions results in the overall channel profit in terms of η as follows:

$$\Pi_{1a}^{EE} = \Pi_1^{*EE} + \Pi_a^{*EE} = \frac{(1-\eta)(1-\tau)}{8(3-2\eta(1-\tau)-2\tau)^2(1+\tau)}.$$

Solving the first order condition yields the unique η in the feasible domain optimizing this exclusive channel profit as follows:

$$\eta^* = \frac{1-2\tau}{2-2\tau}.$$

Note the above η^* also optimizes the entire supply chain profit. Plugging the above η into the price and profit functions yields

$$\begin{aligned} p_1^{*EE} &= \frac{1}{16} + r_0 - \frac{r_0}{\tau}, \\ p_a^{*EE} &= \frac{1}{16} - r_0 + \frac{r_0}{\tau}, \\ \Pi_1^{*EE} &= \frac{1}{128+128\tau}, \\ \Pi_a^{*EE} &= \frac{1}{128+128\tau}. \end{aligned}$$

Immediately, we can obtain the equilibrium package price,

$$P_{1a}^{*EE} = \frac{1}{8},$$

and the optimal single channel profit,

$$\Pi_{1a}^{O} = \frac{1}{64+64\tau}.$$

The base revenue sharing rate r_0 does not affect the profit of any player. Due to the symmetry, we have $\Pi_{2b}^{O} = \Pi_{1a}^{O}$. □

Proof of Theorem 8: From the proof of Lemma 1,

$$\Pi_{1a}^{IO} = \Pi_1^{*EE} + \Pi_a^{*EE} = \frac{1-\tau}{8(3-2\tau)^2(1+\tau)}.$$

Comparing Π_{1a}^{IO} with Π_{1a}^{O} obtained in Lemma 5, we have

$$\Pi_{1a}^{O} - \Pi_{1a}^{IO} = \frac{(1-2\tau)^2}{64(3-2\tau)^2(1+\tau)} \geq 0.$$

Comparing Case CP with Case O results in

$$\Pi_{1a}^{O} - \Pi_{1a}^{CP} = \frac{1}{64 + 64\tau} - \frac{1-\tau}{16(2-\tau)^2(1+\tau)} = \frac{\tau^2}{64(2-\tau)^2(1+\tau)} \geq 0.$$

Thus, Case O always outperforms Cases IO and CP for the entire supply chain. □

Proof of Corollary 1: Recall that $P_{1a}^{*EE} = 1/8$ in Case O from the proof of Lemma 5 and

$$P_{1a}^{*AA} = \frac{1-\tau}{2(3-\tau)}$$

from the proof of either Lemma 1 or Lemma 2. Thus,

$$P_{1a}^{*AA} - P_{1a}^{*EE} = \frac{1-3\tau}{24-8\tau}.$$

Given that $\tau < 1$, thus $24 - 8\tau > 0$, we find that if $\tau < 1/3$, $P_{1a}^{*EE} < P_{1a}^{*AA}$; otherwise, the reverse is true.□