

**Online Technical Appendix**

**Accompanying**

**A Conjoint Model of Quantity Discounts**

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## Online Technical Appendix A: Model Estimation

In this Appendix, we describe model estimation. We estimate the surplus model in Equation (3) using a hierarchical Bayesian, multinomial logit approach. Consider a sample of  $I$  consumers, each choosing at most one product from a set of  $J$  products, where each product  $j=1, \dots, J$  is associated with  $q_j$  quantity. Let  $t$  indicate a choice task or occasion. If consumer  $i$  contributes  $T_i$  such observations, then the total number of observations in the data is given by  $T = \sum_{i=1}^I T_i$ . Let  $z_{ijt} = 1$  if the choice of product  $j$  is recorded for choice occasion  $t$ ; otherwise,  $z_{ijt} = 0$ . Let  $j = 0$  denote the index for the no-choice alternative. Thus,  $z_{i0t} = 1$  if the consumer chooses none of the products in choice task  $t$ . Let  $\psi_i$  be the joint vector of model parameters. Then the conditional likelihood,  $L_i | \psi_i$ , of observing the choices consumer  $i$  makes across the  $T_i$  choice occasions is given by

$$(A1) \quad L_i | (\psi_i) = \prod_{t=1}^{T_i} \prod_{j=0}^J \text{Pr}_{ijt}^{y_{ijt}},$$

where the  $\text{Pr}_{ijt}$  are the multinomial logit choice probabilities (see Equation 10).

To capture consumer heterogeneity, we assume that the individual-level regression parameters,  $\psi_i$ , are distributed multivariate normal with mean vector  $\bar{\psi}$  and covariance matrix  $\Sigma$ . Then, the unconditional likelihood,  $L$ , for a random sample of  $I$  consumers is given by

$$(A2) \quad L = \prod_{i=1}^I \int L_i | \psi_i f(\psi_i | \bar{\psi}, \Sigma) d\psi,$$

where  $f(\psi_i | \bar{\psi}, \Sigma)$  is the multivariate normal  $N(\bar{\psi}, \Sigma)$  density function.

The likelihood function in Equation (A2) is complicated because it involves multidimensional integrals, making classical inference using maximum likelihood methods difficult. We circumvent this complexity by adopting a Bayesian framework to make inferences about the parameters and using MCMC methods, which avoid the need for numerical integration. The MCMC methods yield random draws from the joint posterior distribution and inference is based on the distribution of the drawn samples.

For the Bayesian estimation, we use the following set of proper but noninformative priors for all the population-level parameters. Suppose  $\bar{\psi}$  is a  $p \times 1$  vector and  $\Sigma^{-1}$  is a  $p \times p$  matrix. Then the prior for  $\bar{\psi}$  is a multivariate normal with mean  $\eta_\psi = 0$  and covariance  $C_\psi = \text{diag}(10)$ . The prior for  $\Sigma^{-1}$  is a Wishart distribution,  $W(R, \rho)$  where  $\rho = p+1$  and  $R$  is a  $p \times p$  identity matrix.

## Online Technical Appendix B - Quantity Discount Schedule for a New Entrant in a Competitive Market

In this Appendix, we describe how we determine the quantity discount schedule that MovieMail should offer in the presence of competition. Suppose MovieMail is a new entrant in a market in which Netflix and Blockbuster already offer online movie rental services. Presently Netflix

offers four online movie rental plans whereas Blockbuster offers only three. Both Netflix and Blockbuster charge \$8.99, \$13.99, \$16.99 for the one, two, three DVDs out-at-a-time plans, respectively. For the four DVDs out at-a-time plan, Netflix charges \$23.99. At these prices, Netflix (Blockbuster) has a relative quantity-weighted share of 70% (30%).

Suppose MovieMail decides to offer four online movie rental plans. Suppose that the MovieMail service will be available to all consumers in the market and will enjoy full awareness immediately after launch. Suppose also that consumer switching costs are negligible and that Netflix and Blockbuster do not react to MovieMail pricing. Let  $p_1, p_2, p_3,$  and  $p_4$  be MovieMail prices (decision variables) for the one, two, three, and four DVDs out-at-a-time plans. In addition, let  $c_1, c_2, c_3,$  and  $c_4$  be the plan-specific marginal costs for the one, two, three, and four DVDs out-at-a-time plan, respectively. Please see Section 6 of the text for how we compute the marginal costs. Then, MovieMail chooses the discount pricing scheme  $\mathbf{p} = (p_1, p_2, p_3, p_4)$  to maximize the gross contribution:

$$(B1) \quad \sum_{i=1}^{250} \sum_{q=1}^4 (p_q - c_q) \text{Pr}_{iq}(\mathbf{p}),$$

subject to the incentive compatibility constraint that the per-DVD price is decreasing over successive quantities:

$$(B2) \quad \frac{p_q}{q} \leq \frac{p_{q-1}}{q-1}, \quad q = 2, \dots, 4,$$

where  $\text{Pr}_{iq}(\mathbf{p})$  is the choice probability of a MovieMail plan with  $q$  DVDs out-at-at-time by consumer  $i, i=1, \dots, 250$  conditional on the discount pricing scheme  $\mathbf{p}$  and the prices for the plans currently offered by Netflix and Blockbuster. We use the MCMC draws of the parameter to calculate these choice probabilities (see Equation 10). Note that, with MovieMail in the market, the choice set faced by consumers includes a total of eleven plan options: four new MovieMail plans, four Netflix plans, and three Blockbuster plans.

We maximize the constrained objective function in Equation (B1) using Proc NLP in SAS. The optimization results show that the optimal monthly price for the one, two, three, and four DVDs out-at-a-time plans are, respectively, \$8.22, \$12.69, \$16.40, and \$21.82.<sup>1</sup> These optimal prices are slightly lower than those offered by Netflix and Blockbuster which are, respectively, \$8.99, \$13.99, \$16.99, and \$23.99. At these prices, MovieMail expects to capture 15.25%, 8.37%, 8.20% and 4.75% market share for the one, two, three, and four DVDs out-at-a-time plans, respectively. The quantity-weighted shares of the online DVD rental market with MovieMail entry are as follows: MovieMail 33%, Netflix 49%, and Blockbuster 18%.

Note that these shares are derived under the assumption of full awareness, availability and no switching costs. Our optimization also assumes that Netflix and Blockbuster do not react to MovieMail entry by adjusting their prices and/or changing other attributes of their rental plans. While such an assumption is standard in conjoint simulators (Green and Srinivasan 1990), it does not lead to an equilibrium solution (e.g., Nash). A proper analysis should allow the incumbent firms to react to MovieMail entry as in Ferjani et al. (2009). However, such equilibrium analysis requires making assumptions about the nature of competition in the online DVD rental market.

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<sup>1</sup> We checked for the global optimality of this solution using a grid search. Both Proc NLP and grid search resulted in virtually identical quantity discount schemes.

### **Additional Reference**

Ferjani, M., K. Jedidi, S. Jagpal. 2009. A conjoint approach for consumer- and firm-level brand valuation. *J. Marketing Res.* **46**(6) 846–862.