Annex to "Intertemporal movie distribution: Versioning when customers can buy both versions"

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1 Simultaneous bargaining

The purpose of this Annex is to examine if our results of Section 3 are robust to a modification in the timing of the bargain between the producer and the channels. While in the main paper we studied the case where the producer negotiates sequentially, first with the exhibitor and then with the distributor, we now consider that the two negotiations are simultaneous. All the relevant notation, as well as the other assumptions of the model, are maintained here.

The timeline of the new model is represented in Panel I of Figure 1 below. The timing of the negotiations is as follows:

- At time t_{-1} the producer negotiates, separately and simultaneously, with the exhibitor and with the distributor. This means that two separate negotiators are sent to bargain with each channel. In the producer-exhibitor bargain, the two firms jointly decide the revenue share r_e, as well as the release time for the video version, d. If this negotiation breaks down, H is not shown, thus the exhibitor gets nothing, while the producer gets whatever is negotiated in the bargain with the distributor over the release of L. In the separate producer-distributor bargain, they jointly decide the revenue share r_d. If this negotiation breaks down, L is not shown, the distributor gets nothing, and the producer obtains the result of the bargain with the exhibitor over the release of H. Each bargain is solved using the two-person Nash solution.[1]

- After these contractual terms over r_e, d, and r_d are set, at time t_0 the exhibitor and the distributor independently set the retail prices p_H and p_L respectively.
- The exhibitor then releases H at time t_0 and the distributor offers L at t_1.

For ease of comparison, Panel II of Figure 1 reports the full timing sequence of the sequencing game analyzed in Section 3 (Figure 3 in the main text corresponds to the *upper* branch of Panel II of Figure 1 below, and shows what is eventually played along the equilibrium path). There are two differences between the two models. First, most obviously, with a simultaneous negotiation the result of the producer-distributor bargain cannot be fully anticipated. Second, the producer's outside options are different with the new timing. We now show that these changes are not material for our results of Section 3.

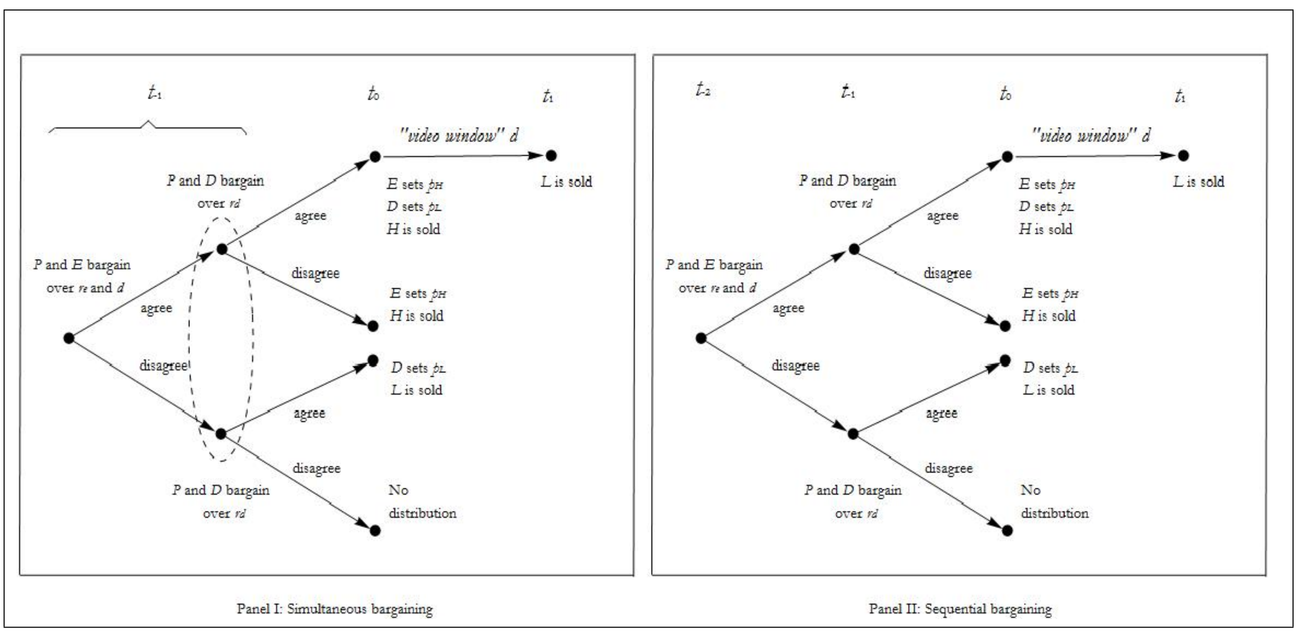

Figure 1: Timing in the channels' negotiation game

[1] The outcome therefore represents a Nash equilibrium in the Nash bargains (see, e.g., Horn and Wolinsky, RAND Journal of Economics, 1988).

1.1 Analysis

At time t_0, if agreements with the two channels are reached, prices p_H and p_L are determined in the same way as in Section 3. Pay-offs therefore are the same as those reported in eq. (17).

At time t_{-1}, the producer and the distributor bargain over the revenue share r_d. In case the negotiation breaks down, the distributor's outside option is zero, version H is shown at the earliest possible date, and the producer gets $\pi_H^p = r_e \frac{u_H}{4}$. Taking this into account, the firms solve the following problem:

$$\max_{r_d} \Omega_{LHB}^{p,d} = \left(\pi_{LHB}^p - r_e \frac{u_H}{4}\right)^\beta \left(\pi_{LHB}^d\right)^{1-\beta}.$$

This is exactly the same Nash problem as in Section 3, which therefore has the solution given by eq. (18).

Simultaneously, at time t_{-1}, the producer and the exhibitor bargain over the length of the video window and the revenue share r_e. In case the negotiation breaks down, the exhibitor's outside option is zero, version L is shown at the earliest possible date, and the producer earns $\pi_L^p = r_d \frac{u_L}{4}$. Taking into account these disagreement payoffs, the firms solve the following Nash bargain:

$$\max_{r_e, d} \Omega_{LHB}^{p,e} = \left(\pi_{LHB}^p - r_d \frac{u_L}{4}\right)^\alpha \left(\pi_{LHB}^e\right)^{1-\alpha}.$$

This is where the sequential and simultaneous solution differ. First, eq. (18) is *not* used into the expression for π_{LHB}^p, as the game is not sequential anymore. Second, in the producer's outside option we still have r_d which is determined in the other simultaneous bargain. Recall that in Section 3 we replaced it with β, which was the endogenous share that we computed in the ensuing subgame in case of a breakdown with the exhibitor.

These differences notwithstanding, the interior solution of this problem is still characterized by the following FOCs, which mirror (19) and (20):

$$\frac{\alpha}{1-\alpha} \frac{\pi_{LHB}^e}{\pi_{LHB}^p - r_d \frac{u_L}{4}} = -\frac{\partial \pi_{LHB}^e / \partial r_e}{\partial \left(\pi_{LHB}^p - r_d \frac{u_L}{4}\right) / \partial r_e}, \quad (1)$$

$$\frac{\alpha}{1-\alpha} \frac{\pi_{LHB}^e}{\pi_{LHB}^p - r_d \frac{u_L}{4}} = -\frac{\partial \pi_{LHB}^e / \partial d}{\partial \left(\pi_{LHB}^p - r_d \frac{u_L}{4}\right) / \partial d}. \quad (2)$$

1.2 Results

Solving (1) yields:

$$r_e = \alpha + \frac{(1-\alpha)r_d\{16k^2(2-s)^2+d^2[(5-s)^2+36k(3-4s+s^2)]-4dk[2(5-s)+9k(1-s)](2-s)-36d^3(1-s)\}}{4(k-d)[2k(2-s)-d(1+s)]^2}. \quad (3)$$

Solving (3) and (18) simultaneously, gives r_e^* and r_d^*, as a function of d. The substitution of these expressions in (2) results in one last equation in d, which can be solved as a function of the parameters of the model to get d^*. This solves the problem completely. We do not report the explicit value for d^*, as this involves a long expression, but this solution takes values in the appropriate interval [0, 1] when parameters k, α and β are in a relevant range. Figure 2, plots the equilibrium value of the video window d, which should be contrasted with the result of the sequential model shown in Figure 5 in the main text. The similarities are immediately evident.

We establish next the corner solutions. Take first the corner solution $d = 0$ (i.e., version L is not released). In this case, from (1) we obtain $r_e = \alpha + (1-\alpha)r_d/k$. Solving this equation and (18), gives r_e^0 and r_d^0. To define the range of validity of this solution, substitute them into (2), together with $d = 0$, to get

$$sign[\frac{\partial \Omega_{LH}^{p,e}}{\partial d}] < 0 \text{ iff } \alpha < \widetilde{\alpha}^0 = 1 - \frac{9k\beta(7-11s)(1-s)}{2(1-\beta)(1+s)[9k(1-s)-2(1+s)]}.$$

Hence the corner solution $d = 0$ can occur as long as $\widetilde{\alpha}^0$ takes plausible values (between 0 and 1). $\widetilde{\alpha}^0$ has the same interpretation as α^0 in the main text. It also behaves in the same way: for $s \leq 7/11$, $\widetilde{\alpha}^0$ is decreasing in β, and it takes its maximum value $\widetilde{\alpha}^0 = 1$ for $\beta = 0$. It is also increasing in k. Figure 3 presents a three-dimensional plot of $\widetilde{\alpha}^0$, and is the analog to Figure 4 in the main text.

Additionally, notice that for $1/2 \leq s \leq 7/11$ it is always $\alpha^0 > \widetilde{\alpha}^0 \geq 0$. This result implies that, for the relevant range of the parameters of the model, versioning appears *more frequently* when the negotiation is simultaneous than when it is sequential.

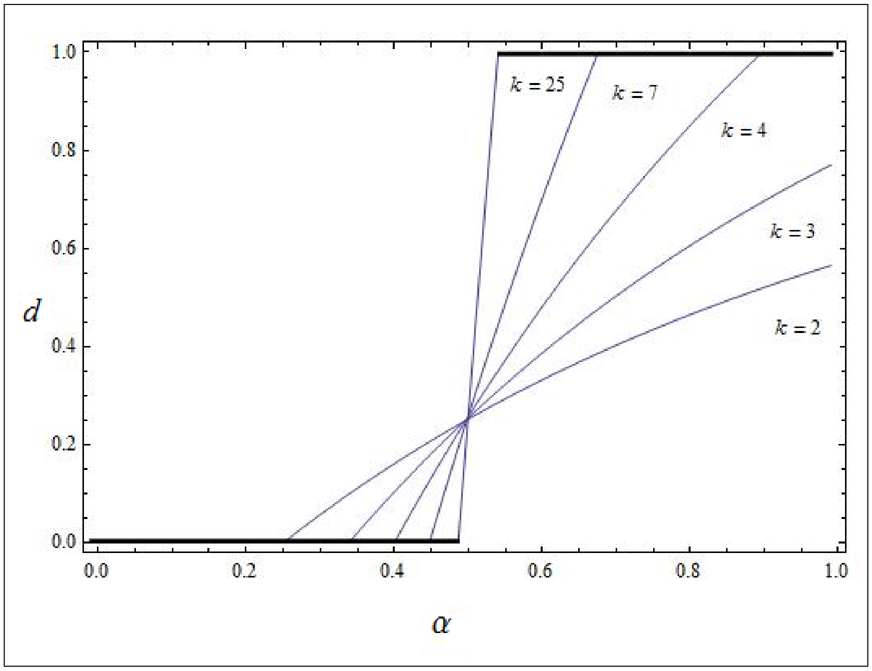

Figure 2: Negotiated video window with simultaneous bargain. The figure plots the optimal length of the video window d negotiated between a producer and an exhibitor. The window can fall into three areas according to the value taken by k: $d = 1$ corresponds to the simultaneous release of both versions ("day-and-date"), $0 < d < 1$ is the sequential release of both versions ("video window"), and $d = 0$ implies the release of the theatrical version only (Parameters values as in Figure 5: $u_L = 1$, $\beta = 0.5$ and $s = 0.55$).

Finally, consider the corner solution $d = 1$. The revenue share in (3) simplifies to

$$r_e = \alpha + \frac{(1-\alpha) r_d [11 - 26s - s^2 + 4k^2 \left(2 - 11s + 5s^2\right) - 4k \left(7 - 22s + 7s^2\right)]}{4(k-1)[2k(2-s) - 1 - s]^2}.$$

Solving this equation and (18), gives r_e^1 and r_d^1. To define the range of validity of this solution, substitute them into (2), together with $d = 1$, to get $sign[\frac{\partial \Omega_{LB}^{p,e}}{\partial d}] > 0$ iff $\alpha > \widetilde{\alpha}^1$. Hence the solution $d = 1$ can occur as long as $\widetilde{\alpha}^1$ takes admissible values (between 0 and 1). The expression for $\widetilde{\alpha}^1$ is complex and we do not report it here. However, it behaves in the same way as α^1: it is decreasing in β, and reaches its maximum value $\widetilde{\alpha}^1 = 1$ for $\beta = 0$. $\widetilde{\alpha}^1$ is also decreasing in k, it reaches $\widetilde{\alpha}^1 = 1$ for $k = 1$ and the limit value $\lim_{k\to\infty} \widetilde{\alpha}^1 = 1 - \frac{\beta(7-11s)}{2(1-\beta)(1+s)} = \lim_{k\to\infty} \widetilde{\alpha}^0 = \lim_{k\to\infty} \alpha^0 = \lim_{k\to\infty} \alpha^1$. Since $\widetilde{\alpha}^0$ is increasing in k, this implies that $\widetilde{\alpha}^1 > \widetilde{\alpha}^0$ for all plausible parameters range. Figure 3 reports the three-dimensional plot of $\widetilde{\alpha}^1$, again to be contrasted with Figure 4 in the main text.

Formally, Proposition 3 is still valid under simultaneous bargain, and exactly so if one simply replaces the thresholds $\widetilde{\alpha}^0$ and $\widetilde{\alpha}^1$, as just discussed. The similarities between Figure 2 and 3, and their counterparts Figure 5 and 4 in the text, are apparent.

We thus conclude that versioning is more likely to appear with simultaneous bargaining, i.e., for a wider range of parameter values. This is intuitive, as the exhibitor now does not enjoy a first-mover advantage and the distributor does not suffer from a second-mover disadvantage. However, the particular timing of the negotiations with the channels (sequential vs simultaneous) does not affect the nature of our results in any other noticeable way.

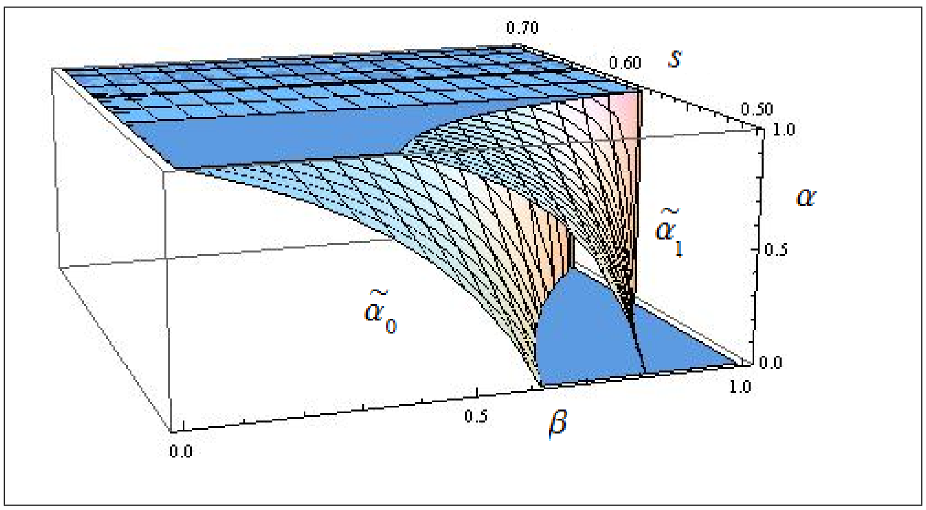

Figure 3: Versioning and sequencing with simultaneous bargaining. When $\alpha < \widetilde{\alpha}^0$, only H is released. When $\widetilde{\alpha}^0 < \alpha < \widetilde{\alpha}^1$, there is both versioning and sequencing. When $\alpha > \widetilde{\alpha}^1$, there is versioning and simultaneous release of both versions (Parameters values as in Figure 4: $u_L = 1$ and $k = 4$).