Appendix A

Optimal solution for the general problem

Since we seek only non-trivial solutions, we set $I = 1$ and will then check to see if the optimal detailing level α^* is > 0. We start by analyzing the sampling decision. There are two cases to analyze. First, when the number of samples is less than or equal to the number of identified patients without ability to pay, and second when the number of samples is greater than the number of identified patients without ability to pay.

(a) If $S \leq \phi\frac{1}{2}\alpha(1-\theta)$, then $S_e = 0$ by definition and we use the profit function in equation (3) to derive the first order condition with respect to S:

$$\frac{\partial\pi}{\partial S}|_{S\leq\phi\frac{1}{2}\alpha(1-\theta)} = -c$$

The first derivative is negative and hence the optimal level of samples is $S^* = 0$. Note that this satisfies the condition $S \leq \phi\frac{1}{2}\alpha(1-\theta)$ that we began with. Substituting this in the firm's profit function in equation (3) and taking the first order condition with respect to detailing α and solving we obtain the optimal level of detailing as:

$$\alpha_N^* = \frac{(1+\delta)\,\theta(p-c)}{4k} \quad (>0 \text{ for finite } k) \tag{6}$$

This results in optimal profit level, with no sampling, of:

$$\pi_N^* = \frac{(1+\delta)\,\theta^2(p-c)^2}{16k}$$

Since the fraction of identified patients cannot exceed $\frac{1}{2}$, we need $\alpha_N^* < 1$. This condition can be rewritten as $k > k_1^*$ where the exact form of k_1^* is given in the form of a Mathematica Notebook in Appendix B.

(b) If $S > \phi\frac{1}{2}\alpha(1-\theta)$, then the first order condition with respect to S is

$$\frac{\partial\pi}{\partial S}|_{S>\phi\frac{1}{2}\alpha(1-\theta)} = \delta\cdot\lambda\cdot\left(\frac{1-\alpha}{2-\alpha}\right)\theta(p-c) - c$$

Setting this derivative to be positive gives us the following condition:

$$(\delta \cdot \lambda \cdot \theta(p-c) - c)\, \alpha < \delta \cdot \lambda \cdot \theta(p-c) - 2c$$

We know that $\left(\frac{1-\alpha}{2-\alpha}\right) < 1$ for α in the range $[0,1]$ and for a positive derivative, we need $\delta \cdot \lambda \cdot \left(\frac{1-\alpha}{2-\alpha}\right) \cdot \theta(p-c) - c > 0$. Consequently, $\delta \cdot \lambda \cdot \theta(p-c) - c > 0$. This implies that we can rearrange the inequality as:

$$\alpha < \frac{\delta \cdot \lambda \cdot \theta(p-c) - 2c}{\delta \cdot \lambda \cdot \theta(p-c) - c}$$

Since $\alpha > 0$, we must also have:

$$\delta \cdot \lambda \cdot \theta(p-c) - 2c > 0$$

which on rearranging provides:

$$\theta > \frac{2c}{\delta \cdot \lambda \cdot (p-c)} \tag{7}$$

Since θ lies in the range $[0,1]$, we must also have:

$$p > \frac{2c}{\delta\lambda} + c \tag{8}$$

Since the firm's profit as a function of S is linear with a positive derivative for the given range of $S > \phi\frac{1}{2}\alpha(1-\theta)$, we can set S^* to its upper limit $\left(1 - \frac{1}{2}\alpha\right) + \phi\frac{1}{2}\alpha\,(1-\theta)$. Thus, the optimal level of samples is $S^* = \left(1 - \frac{1}{2}\alpha\right) + \phi\frac{1}{2}\alpha\,(1-\theta)$. Substituting this in the firm's profit function and taking the first order condition with respect to detailing α and solving we obtain the optimal level of detailing as:

$$\alpha_S^* = \frac{\theta\,(p-c)\,[1 + \delta\,(1-\lambda)] + c\,[1 - \phi + \theta\phi]}{4k} > 0 \tag{9}$$

and that of sampling as:

$$S^* = \left(1 - \frac{1}{2}\alpha_S^*\right) + \phi\frac{1}{2}\alpha_S^*\,(1-\theta) \tag{10}$$

. This leads to an optimal profit level of π_S^*. Exact expression is given in the form of a Mathematica Notebook in Appendix B. Once again we need $\alpha_S^* < 1$. This condition can be rewritten as $k > k_2^*$

where the exact form of k_2^* is given in the form of a Mathematica Notebook in Appendix B. We also need $S^* > 0$ which provides a condition $k > k_3^*$.

Further, the conditions for $S > 0$ as the optimal solution are:

$$\frac{\partial \pi}{\partial S}|_{S>\phi\frac{1}{2}\alpha_S^*(1-\theta)} > 0 \text{ and } \pi_S^* > \pi_N^*$$

$\pi_S^* > \pi_N^*$ provides another condition on k: $k > k_4^*$. We have shown earlier that for $\frac{\partial \pi}{\partial S}|_{S>\phi\frac{1}{2}\alpha_S^*(1-\theta)} > 0$, we need:

$$\frac{\delta \cdot \lambda \cdot \theta(p-c) - 2c}{\delta \cdot \lambda \cdot \theta(p-c) - c} > \alpha_S^*$$

This provides a further condition $k > k_5^*$. Putting these conditions together gives us a condition on k:

$$k > \max\{k_1^*, k_2^*, k_3^*, k_4^*, k_5^*\} = k_B^* \tag{11}$$

Exact expressions for k_1^* through k_5^* are given in Appendix B in the form of a Mathematica Notebook.

Proof of Proposition 1

Part 1): The required conditions are given by (11), (7) and (8).

Part 2): We use the expressions for α_S^* and S^* given in equations (9) and (10) and use the sampling optimality conditions from Part 1).

Proof of Proposition 2

Part 1) Obtained by examining the expression for $\frac{\partial \alpha_S^*}{\partial c}$ using the expression for α_S^* from equation (9) and using the sampling optimality conditions from (7), (8) and (11).

Part 2) Obtained by examining the expression for $\frac{\partial S^*}{\partial c}$ using the expression for S^* from equation (10) and using the sampling optimality conditions from (7), (8) and (11).

Proof of Proposition 3

The expressions for α_N^* and α_S^* for general values of θ, λ and δ are available from equations (6) and (9) respectively. We employ the expressions for α_N^* and α_S^* in equations (5) and (4) respectively. A comparison of the expressions at $c = 0$ and $\phi = 0$ shows that market coverage under sampling is lower when $0 < \lambda < \lambda^*$ and $\delta > \delta^*$. The exact expressions for λ^* and δ^* are given in a Mathematica Notebook in Appendix B.