

# Appendix

## A

### A.1 Proof of Result 1

The Lagrangian for the problem is as follows.

$$\mathcal{L}_1 = \frac{w[T_h - w + f(s^*) + g(a)]}{2} - a - L - \lambda_1 \left[ \frac{[T_h - w + f(s^*) + g(a)]^2}{4} - s^* + L \right], \quad (\text{A1})$$

where  $\lambda_1$  is the Lagrangian multiplier associated with the voluntary participation constraint.

When voluntary participation constraint does not bind:

We denote the optimal solution in this case by  $\{w_1^*, a_1^*, L_1^*\}$ . In this case,  $\lambda_1 = 0$ , and the first-order conditions for  $a$  and  $w$ , after applying the first-order condition for  $s$ , and simplifying, are :

$$\left(\frac{w}{2}\right)[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 = 0, \quad (\text{A2})$$

$$\left(\frac{1}{2}\right)[T_h - w + f(s^*) + g(a)] + \left(\frac{w}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] = 0. \quad (\text{A3})$$

Note that the non-negativity constraint on  $L$  is imposed by  $L\frac{\partial \mathcal{L}_1}{\partial L} = 0$ . Therefore,  $\frac{\partial \mathcal{L}_1}{\partial L} = -1 < 0 \Rightarrow L_1^* = 0$ .  $\{w_1^*, a_1^*\}$  are given by the above first-order conditions. Let the highest value of  $\underline{\pi}$  for which this result holds be  $\underline{\pi}^*$ .

When voluntary participation constraint binds:

We denote the optimal solution in this case by  $\{w_2^*, a_2^*, L_2^*\}$ . In this case,  $\lambda_1 > 0$ , and the first-order conditions for  $a$  and  $w$ , after simplification, are:  $\frac{w}{2}[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 + \frac{\lambda_1}{2}[T_h - w + f(s^*) + g(a)]g'(a) + \lambda_1 \frac{[T_h - w + f(s^*) + g(a)]f'(s^*)}{2} - 1 \frac{\partial s^*}{\partial a} = 0$ , and  $(\frac{1-\lambda_1}{2})[T_h - w + f(s^*) + g(a)] + (\frac{w}{2})[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] + \lambda_1 \frac{[T_h - w + f(s^*) + g(a)]f'(s^*)}{2} - 1 \frac{\partial s^*}{\partial w} = 0$ .

The retailer's first-order condition for  $s^*$ , after substituting for  $p^*$ , is  $\frac{[T_h - w + f(s^*) + g(a)]f'(s^*)}{2} - 1 = 0$ . Substituting this in the two equations above, the first-order conditions for  $a$  and  $w$  are simplified as follows.

$$\frac{w}{2}[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 + \frac{\lambda_1}{2}[T_h - w + f(s^*) + g(a)]g'(a) = 0, \quad (\text{A4})$$

$$\left(\frac{1 - \lambda_1}{2}\right)[T_h - w + f(s^*) + g(a)] + \left(\frac{w}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] = 0. \quad (\text{A5})$$

We show that  $L_2^* = 0$  by contradiction. Suppose  $L_2^* > 0$ . Then,  $\frac{\partial \mathcal{L}_1}{\partial L} = 1 - \lambda_1 = 0$ ; and  $\lambda_1 = 1$ . Substituting  $\lambda_1 = 1$  in the first-order condition for  $w$ , we get,

$$\left(\frac{w}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] = 0.$$

This suggests  $w = 0$ , which results in a negative profit for the firm with positive advertising and slotting allowance levels. Therefore,  $L_2^* = 0$ .

## A.2 Proof of Proposition 1

The Lagrangian,  $\mathcal{L}_2$  is defined as below.

$$\begin{aligned} \mathcal{L}_2 = & w \frac{[T_h - w + f(s^*) + g(a)]}{2} - a - L \\ & - \lambda_2 \frac{[T_h - w + f(s^*) + g(a)]^2}{4} - s^* + L \\ & - \mu_2 w \frac{[2T_l - T_h - w + f(s^*) + (2l - 1)g(a)]}{2} - a - L, \end{aligned} \quad (\text{A6})$$

where  $\lambda_2$  and  $\mu_2$  are Lagrangian multipliers.

The first-order conditions, after applying the first-order condition for  $s$ , and simplifying, are:

$$\begin{aligned} \frac{\partial \mathcal{L}_2}{\partial a} = & (1 - \mu_2) \left(\frac{w}{2}\right)[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 \\ & + \frac{\lambda_2}{2}[T_h - w + f(s^*) + g(a)]g'(a) + \mu_2(1 - l)wg'(a) = 0, \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \frac{\partial \mathcal{L}_2}{\partial w} = & \left(\frac{1 - \lambda_2}{2}\right)[T_h - w + f(s^*) + g(a)] + (1 - \mu_2)\left(\frac{w}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] \\ & - \frac{\mu_2}{2}[2T_l - T_h - w + f(s^*) + g(a)] + \mu_2(1 - l)g(a) = 0. \end{aligned} \quad (\text{A8})$$

As before, the non-negativity of  $L$  implies that  $L \frac{\partial \mathcal{L}_2}{\partial L} = 0$ .

We first show that if the voluntary participation constraint is not binding in the equilibrium, then the manufacturer does not offer a slotting allowance in the equilibrium. The proof is by contradiction. If the voluntary participation constraint is not binding, then  $\lambda_2 = 0$ . Suppose  $L_1^{**} > 0$ . Then  $\frac{\partial \mathcal{L}_2}{\partial L} = -1 + \mu_2 = 0$ , and  $\mu_2 = 1$ . Substituting  $\lambda_2 = 0$  and  $\mu_2 = 1$  in Equation(A8), we get  $T_h - T_l + (1 - l)g(a) = 0$ , giving a contradiction. Therefore,  $\mu_2 \neq 1$ , and  $L_1^{**} = 0$ .

$\{w_1^{**}, a_1^{**}\}$  are given by substituting  $\lambda_2 = 0$  in Equations (A7)-(A8). Note that by Kuhn-Tucker conditions,  $\frac{\partial \mathcal{L}_2}{\partial L} = -1 + \lambda_2 + \mu_2 \leq 0$ . Since both Lagrangian multipliers are non-negative,  $0 \leq \mu_2 \leq 1$  follows.

We now prove the direction of deviations in the separating equilibrium. Here we are considering those values of parameters for which the voluntary participation constraint is not binding in the separating equilibrium. With the current level of generality, we cannot establish whether  $\underline{\pi}^{**}$  is higher or lower than  $\underline{\pi}^*$ . Therefore, we compare  $\{w_1^{**}, a_1^{**}\}$  with both  $\{w_1^*, a_1^*\}$  and  $\{w_2^*, a_2^*\}$ .

We know that  $\Pi_l(w_1^{**}, a_1^{**}, L_1^{**} = 0) = 0$  and  $\Pi_l(w_1^*, a_1^*, L_1^* = 0) > 0$ . The function  $w \frac{[2T_l - T_h - w + f(s^*) + (2l-1)g(a)]}{2} - a - L$  is decreasing in  $w$  and  $a$  for any  $w > w_1^*$  and any  $a > a_1^*$ . Since  $L_1^{**} = L_1^* = 0$ , the difference between  $\Pi_l(w_1^{**}, a_1^{**}, L_1^{**} = 0) = 0$  and  $\Pi_l(w_1^*, a_1^*, L_1^* = 0) > 0$  must come from changes in  $w$  or  $a$ . This can happen in one of the three ways: (i)  $w$  is increased above  $w_1^*$  level, i.e.,  $w_1^{**} > w_1^*$ , (ii)  $a$  is increased above the  $a_1^*$  level, i.e.,  $a_1^{**} > a_1^*$  or (iii) both  $w_1^{**} > w_1^*$  and  $a_1^{**} > a_1^*$ . Note that if both  $w_1^{**} < w_1^*$  and  $a_1^{**} < a_1^*$ , then  $\Pi_l(w_1^{**}, a_1^{**}, L_1^{**} = 0) > \Pi_l(w_1^*, a_1^*, L_1^* = 0)$ , and the signaling constraint is not satisfied.

$a_1^{**}$  is defined by Equation (A7), which after substituting  $\lambda_2 = 0$ , can be written as,

$$\left(\frac{w}{2}\right)[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 + \frac{\mu_2}{(1 - \mu_2)}(1 - l)wg'(a) = 0.$$

$a_1^*$  is defined by,

$$\left(\frac{w}{2}\right)[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 = 0.$$

By using the implicit function rule on the first-order condition defining  $s^*(\cdot)$ , we get,

$$\frac{\partial s^*}{\partial a} = -\frac{f'(s^*)g'(a)}{[f'(s)]^2 + [T_h - w + f(\cdot) + g(\cdot)]f''(\cdot)} \quad \text{and} \quad f'(\cdot)\frac{\partial s^*}{\partial a} = -\frac{[f'(s^*)]^2 g'(a)}{[f'(s)]^2 + [T_h - w + f(\cdot) + g(\cdot)]f''(\cdot)}. \quad \text{By simple differentiation, we get,}$$

$$\frac{\partial [f'(s^*)\frac{\partial s^*}{\partial a}]}{\partial a} = -\frac{f'(\cdot)^2((f'(\cdot)^2 + (T_h - w + f(\cdot) + g(\cdot))f''(\cdot))^2 g''(\cdot) - (T_h - w + f(\cdot) + g(\cdot))g'(\cdot)^2(-3f''(\cdot)^2 + f'(\cdot)f'''(\cdot)))}{[f'(\cdot)^2 + (T_h - w + f(\cdot) + g(\cdot))f''(\cdot)]^3} < 0,$$

and,

$\frac{\partial[f'(s^*)\frac{\partial s^*}{\partial a}]}{\partial w} = \frac{(T_h - w + f(\cdot) + g(\cdot))f'(\cdot)g'(\cdot)(-3f''(\cdot)^2 + f'(\cdot)f'''(\cdot))}{[f'(\cdot)^2 + (T_h - w + f(\cdot) + g(\cdot))f''(\cdot)]^3} > 0$  when  $f'''(\cdot) \leq 0$ .<sup>1</sup> Therefore, the above two equations are not satisfied if  $w_1^{**} > w_1^*$  and  $a_1^{**} \leq a_1^*$ . That is,  $w_1^{**} > w_1^* \Rightarrow a_1^{**} > a_1^*$ .  $w_1^*$  is defined by

$$\left(\frac{1}{2}\right)[T_h - w_1^* + f(s^*) + g(a_1^*)] + \left(\frac{w_1^*}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] = 0.$$

$w_1^{**}$  is defined by Equation (A8). Substituting  $\lambda_2 = 0$  in (A8) and simplifying, we get,

$$\begin{aligned} \frac{\partial \mathcal{L}_2}{\partial w} &= [T_h - w + f(s^*) + g(a)] + (1 - \mu_2)\left(\frac{w}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] \\ &\quad + \frac{\mu_2}{2}T_h - \frac{\mu_2}{2}[-w + f(s^*) + g(a)] + \mu_2(-T_l + 1 - l)g(a) = 0. \end{aligned}$$

By writing  $\frac{\mu_2}{2}T_h = -\frac{\mu_2}{2}T_h + \mu_2T_h$ ,

$$\frac{\partial \mathcal{L}_2}{\partial w} = \frac{1 - \mu_2}{2}[T_h - w + f(s^*) + g(a)] + (1 - \mu_2)\left(\frac{w}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] + \mu_2(T_h - T_l + 1 - l)g(a) = 0.$$

By dividing throughout by  $1 - \mu_2$ , we get the following equation defining  $w_1^{**}$ .

$$\left(\frac{1}{2}\right)[T_h - w_1^{**} + f(s^*) + g(a_1^{**})] + \left(\frac{w_1^{**}}{2}\right)[-1 + f'(s^*)\frac{\partial s^*}{\partial w}] + \frac{\mu_2}{(1 - \mu_2)}(T_h - T_l + (1 - l)g(a_1^{**})) = 0.$$

It can be shown that  $\frac{\partial[f'(s^*)\frac{\partial s^*}{\partial w}]}{\partial a} = \frac{(T_h - w + f(\cdot) + g(\cdot))f'(\cdot)g'(\cdot)(-3f''(\cdot)^2 + f'(\cdot)f'''(\cdot))}{[f'(\cdot)^2 + (T_h - w + f(\cdot) + g(\cdot))f''(\cdot)]^3} > 0$  when  $f'''(\cdot) \leq 0$ .

Therefore, the marginal benefits of increases  $w$  is higher in the equation defining  $w_1^{**}$  than in the equation defining  $w_1^*$ . As a result, the above two equations are not satisfied if  $a_1^{**} > a_1^*$  and  $w_1^{**} \leq w_1^*$ . That is,  $a_1^{**} > a_1^* \Rightarrow w_1^{**} > w_1^*$ .

We also know that  $\Pi_l(w_1^{**}, a_1^{**}, L_1^{**} = 0) = 0$  and  $\Pi_l(w_2^*, a_2^*, L_2^* = 0) > 0$ . Therefore, it must be true that either  $w_1^{**} > w_2^*$  or  $a_1^{**} > a_2^*$  or both  $w_1^{**} > w_2^*$  and  $a_1^{**} > a_2^*$ . Suppose  $w_1^{**} > w_2^*$  and  $a_1^{**} < a_2^*$ .  $\Pi_r(w_1^{**}, a_1^{**}, L_1^{**} = 0) > \underline{\pi}$  and  $\Pi_r(w_2^*, a_2^*, L_2^* = 0) = \underline{\pi}$ . Since  $\Pi_r(\cdot)$  is decreasing in  $w$  and

<sup>1</sup>The effects through  $\frac{\partial s^*}{\partial a}$  are third and higher order effects which are very small. We discuss them here for the sake of completeness.  $f'''(\cdot) \leq 0$  is a stronger-than-necessary sufficient condition, what is necessary is that  $f'''(\cdot)$  not be a large positive quantity in the relevant parameter space. Formally,  $3[f''(\cdot)]^2 - f'(\cdot)f'''(\cdot) > 0$ .

increasing in  $a$ ,  $w_1^{**} > w_2^*$  is possible only if  $a_1^{**} > a_2^*$ . Now consider the other possibility:  $a_1^{**} > a_2^*$  and  $w_1^{**} < w_2^*$ . We first consider the case where  $l = 1$ . From Equations (A4) and (A7),  $a_2^*$  is defined by:

$$\frac{w}{2}[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 + \lambda_1 d_h g'(a) = 0,$$

and  $a_1^{**}$  is defined by:

$$\frac{w}{2}[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 = 0.$$

Since  $\lambda_1 d_h g' > 0$ ,  $\frac{w}{2}[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1 < 0$  at  $\{w_2^*, a_2^*\}$ . Since  $a_2^* < a_1^{**}$ ,  $g'(\cdot)$  is higher at  $a_2^*$  than at  $a_1^{**}$ . It can be shown that  $f'(s^*)\frac{\partial s^*}{\partial a}$  is decreasing in  $a$  and increasing in  $w$  for  $f'''(\cdot) \leq 0$ . Therefore, under the assumption of  $a_1^{**} > a_2^*$  and  $w_1^{**} < w_2^*$ ,  $g'(\cdot)$  is also higher at  $\{w_2^*, a_2^*\}$  than at  $\{w_1^{**}, a_1^{**}\}$ . Therefore, if  $w_2^* > w_1^{**}$ ,  $\frac{w}{2}[f'(s^*)\frac{\partial s^*}{\partial a} + g'(a)] - 1$  is higher at  $\{w_2^*, a_2^*\}$  than at  $\{w_1^{**}, a_1^{**}\}$  giving a contradiction. Therefore,  $w_1^{**} > w_2^*$  and  $a_1^{**} > a_2^*$ . Now consider  $l \neq 1$ . Since  $w_2^*$  is unaffected by  $l$  and  $w_1^{**}$  decreases as  $l$  increases, it follows that if  $w_1^{**} > w_2^*$  is true for  $l = 1$ , it is also true for any  $l < 1$ .

### A.3 Proof of Proposition 2

When the voluntary participation constraint is binding, the manufacturer may or may not offer a slotting allowance.

Note that if  $\frac{\partial \mathcal{L}}{\partial L} = -1 + \lambda_2 + \mu_2$  is negative, then the optimal slotting allowance is zero; otherwise it is positive. We start with the case where  $l = 1$ . We simultaneously solve Equations (A7)-(A8) for  $\lambda_2$  and  $\mu_2$ .

$$\lambda_2 = \frac{(T_h - T_l)(-2 + wf'(s^*)\frac{\partial s^*}{\partial a} + wg'(a))}{d_h(-2 + 2d_l g'(a) + wf'(s^*)\frac{\partial s^*}{\partial a} + \frac{\partial s^*}{\partial w}g'(a))}. \quad (\text{A9})$$

$$\mu_2 = \frac{(-2 + 2d_h g'(a) + wf'(s^*)\frac{\partial s^*}{\partial a} + \frac{\partial s^*}{\partial w}g'(a))}{(-2 + 2d_l g'(a) + wf'(s^*)\frac{\partial s^*}{\partial a} + \frac{\partial s^*}{\partial w}g'(a))}. \quad (\text{A10})$$

$$y = -1 + \lambda_2 + \mu_2 = \frac{(T_h - T_l)(-2 + wf'(s^*)\frac{\partial s^*}{\partial a} + (w + 2d_h)g'(a))}{d_h(-2 + 2d_l g'(a) + wf'(s^*)\frac{\partial s^*}{\partial a} + \frac{\partial s^*}{\partial w}g'(a))}. \quad (\text{A11})$$

From Equation(A7),

$$(1 - \mu_2) \left( \frac{w}{2} \left( f'(s) \frac{\partial s^*}{\partial a} + g'(a) \right) - 1 \right) = -\lambda_2 d_h g'(\cdot) < 0.$$

Therefore,

$$(w f'(s) \frac{\partial s^*}{\partial a} + w g'(a)) - 2 < 0.$$

This implies that the numerator in (A9) is negative. Since  $\lambda_2 > 0$ , the denominator of (A9) should be negative. Ruling out negative demand  $d_h$ , we conclude that  $(-2 + 2d_l g'(a) + w f'(s^*) (\frac{\partial s^*}{\partial a} + \frac{\partial s^*}{\partial w} g'(a))) < 0$ . This shows that the denominator of  $y = -1 + \lambda_2 + \mu_2$  is negative. Therefore,  $y$  is negative when its numerator is positive. Since  $T_h - T_l > 0$ , the numerator of  $y$  is positive when  $(-2 + w f'(s^*) \frac{\partial s^*}{\partial a} + (w + 2d_h) g'(a)) > 0$ . We have already shown that  $[-2 + (w f'(s) \frac{\partial s^*}{\partial a} + w g'(a))] < 0$ . Therefore,  $(-2 + w f'(s^*) \frac{\partial s^*}{\partial a} + (w + 2d_h) g'(a))$  can be positive when  $g'(\cdot)$  is sufficiently high. When  $(-2 + w f'(s^*) \frac{\partial s^*}{\partial a} + (w + 2d_h) g'(a)) > 0$ ,  $y = -1 + \lambda_2 + \mu_2 < 0$  and the optimal slotting allowance is zero. When the optimal slotting allowance is positive,  $-1 + \lambda_2 + \mu_2 = 0$ , and  $(-2 + w f'(s^*) \frac{\partial s^*}{\partial a} + (w + 2d_h) g'(a)) = 0$ .

We now show that the range of parameters for which the slotting allowance is positive is smaller when  $l < 1$ . The proof is by contradiction. Suppose that the range of parameters for which the slotting allowance is positive is the same or larger with  $l < 1$ . Now consider the value of  $g'(a)$  for which the slotting allowance is a small positive value  $\varepsilon$ . Let the equilibrium solution be  $\{\vec{w}, \vec{a}, \vec{L}\}$ . Now consider a deviation such that  $L = \vec{L} - \varepsilon$  and  $a = \vec{a} + \Delta a$  such that  $\frac{\vec{w}}{2} (2l - 1) [g(\vec{a} + \Delta a) - g(\vec{a})] - \Delta a = -\varepsilon$ . At this deviation, the signaling constraint remains unchanged but the high-demand manufacturer's profit is changed by the amount  $-\varepsilon + \frac{\vec{w}}{2} [g(\vec{a} + \Delta a) - g(\vec{a})] - \Delta a > 0$ , contradicting the assumed equilibrium.

## Separating Equilibrium Without Slotting Allowance

Let the solution in this case be  $\{w_2^{**}, a_2^{**}, L_2^{**}\}$ , where  $L_2^{**} = 0$ . We first compare the current solution with  $\{w_2^*, a_2^*, L_2^*\}$ .

$\Pi_l(w_2^{**}, a_2^{**}, L_2^{**}) = 0 < \Pi_l(w_2^*, a_2^*, L_2^*)$ . Since  $L_2^{**} = L_2^* = 0$ , the difference in  $\Pi_l(w_2^{**}, a_2^{**}, L_2^{**})$  and  $\Pi_l(w_2^*, a_2^*, L_2^*)$  can come only from  $w$  and  $a$ . The function  $\Pi_l(\cdot)$  is decreasing in  $w$  for  $w > w_2^*$  and in  $a$  for  $a > a_2^*$ . Therefore, it follows that either  $w_2^{**} > w_2^*$  or  $a_2^{**} > a_2^*$ , or  $w_2^{**} > w_2^*$  and  $a_2^{**} > a_2^*$ . That is, the high-demand manufacturer either increases wholesale price alone or advertising alone or increases both wholesale price and advertising beyond their symmetric information levels.

Suppose  $a_2^{**} > a_2^*$ . Since  $\Pi_r(w_2^{**}, a_2^{**}, L_2^{**}) = \underline{\pi} = \Pi_r(w_2^*, a_2^*, L_2^*)$ ,  $a_2^{**} > a_2^*$  is possible only if  $w_2^{**} > w_2^*$ . Similarly,  $w_2^{**} > w_2^*$  is possible only if  $a_2^{**} > a_2^*$ . Therefore,  $w_2^{**} > w_2^*$  and  $a_2^{**} > a_2^*$ .

We cannot rule out the possibility that  $\underline{\pi}^{**} < \underline{\pi}^*$ . Therefore, we now compare  $\{w_2^{**}, a_2^{**}\}$  and  $\{w_1^*, a_1^*\}$ .

From the first-order conditions defining  $\{w_1^{**}, a_1^{**}\}$  and  $\{w_1^*, a_1^*\}$  we can conclude that these solutions do not change with  $\underline{\pi}$  so long as  $\underline{\pi} < \underline{\pi}^*$  and the other parameter values do not change.

$\Pi_r(w_2^{**}, a_2^{**}, L_2^{**}) = \underline{\pi} > \underline{\pi}^{**}$ . Consider a case where all other parameters are the same and  $\underline{\pi} = \hat{\underline{\pi}} < \underline{\pi}^{**}$ , so that the solution is  $(w_1^{**}, a_1^{**}, L_1^{**})$ . Since the maximum value of  $\Pi_r(w_1^{**}, a_1^{**}, L_1^{**})$  is  $\underline{\pi}^{**}$ ,

$$\Pi_r(w_2^{**}, a_2^{**}, L_2^{**}) > \Pi_r(w_1^{**}, a_1^{**}, L_1^{**}). \quad (\text{A12})$$

Also,

$$\Pi_l(w_2^{**}, a_2^{**}, L_2^{**}) = \Pi_l(w_1^{**}, a_1^{**}, L_1^{**}). \quad (\text{A13})$$

Now, if  $w_2^{**} > w_1^{**}$ , then Equation(A12) is satisfied only if  $a_2^{**} > a_1^{**}$ . If  $w_2^{**} < w_1^{**}$ , then Equation(A13) is satisfied only if  $a_2^{**} > a_1^{**}$ . Therefore,  $a_2^{**} > a_1^{**}$ . We have already established that  $a_1^{**} > a_1^*$  when  $\underline{\pi} = \hat{\underline{\pi}}$ . Therefore,  $a_2^{**} > a_1^*$ . (Recall that the solution  $\{w_1^{**}, a_1^{**}, L_1^{**}\}$  does not change for any  $\underline{\pi} \leq \underline{\pi}^{**}$ .)

Since  $\Pi_r(w_2^{**}, a_2^{**}, L_2^{**}) = \underline{\pi} < \Pi_r(w_1^*, a_1^*, L_1^*)$ ,  $a_2^{**} > a_1^*$  implies  $w_2^{**} > w_1^*$ .

## Separating Equilibrium With Slotting Allowance

Let the solution in this case be  $\{w_3^{**}, a_3^{**}, L_3^{**}\}$ . We first compare this solution with  $\{w_2^*, a_2^*, L_2^*\}$ .

Note that  $\{w_2^*, a_2^*, L_2^*\}$  is the solution to a maximization problem that has a constraint less than the maximization problem solved by  $\{w_3^{**}, a_3^{**}, L_3^{**}\}$ . In particular,  $\Pi_l(w_2^*, a_2^*, L_2^*) > 0 = \Pi_l(w_3^{**}, a_3^{**}, L_3^{**})$ . Therefore,  $w_3^{**} < w_2^*$  is possible if and only if the high-type manufacturer is hurt less than the low-type manufacturer by a decrease in wholesale price. However,

$$\frac{\partial \Pi_h}{\partial w} = \frac{w}{2}(-1 + f'(s)\frac{\partial s^*}{\partial w}) + d_h,$$

and

$$\frac{\partial \Pi_l}{\partial w} = \frac{w}{2}(-1 + f'(s) \frac{\partial s^*}{\partial w}) + d_l.$$

Since  $d_h - d_l = T_h - T_l + (1-l)g(\cdot) > 0$ , the high-type manufacturer gains more (or is hurt less) by an increase in wholesale price. Therefore,  $w_3^{**} \not\leq w_2^*$ . Similarly  $w_3^{**} = w_2^*$  can be ruled out, and  $w_3^{**} > w_2^*$ .

Substituting  $\lambda_1$  from Equation (A5) into Equation(A4), the first-order condition for  $a_2^*$  is simplified as:

$$-2 + wf'(s^*) \frac{\partial s^*}{\partial a} + 2d_h g'(a) + wf'(s^*) \frac{\partial s^*}{\partial w} g'(a) = 0. \quad (\text{A14})$$

The first-order condition for  $a_3^{**}$ , after substituting  $\lambda_2 = 1 - \mu_2$  is simplified as:

$$-2 + wf'(s^*) \frac{\partial s^*}{\partial a} + 2d_h g'(a) + wg'(a) + \frac{2\mu_2}{(1-\mu_2)}(1-l)wg'(a) = 0. \quad (\text{A15})$$

Since  $\frac{\partial s^*}{\partial w} < 0$ ,  $a_3^{***} > a_2^*$  for any  $w_3^{***} > w_2^*$ . Therefore,  $a_3^{**} > a_2^*$ .

We now compare  $\{w_3^{**}, a_3^{**}, L_3^{**}\}$  and  $\{w_1^*, a_1^*, L_1^*\}$ . The proof of  $w_3^{**} > w_1^*$  is similar to the proof of  $w_3^{**} > w_2^*$ , and is not repeated here. the first-order condition describing  $a_1^*$  is:

$$-2 + wf'(s^*) \frac{\partial s^*}{\partial a} + wg'(a) = 0. \quad (\text{A16})$$

Comparing Equations (A15) and (A16), it's clear that for any  $w_3^{**} > w_1^*$ ,  $a_3^{**} > a_1^*$ .

#### A.4 Proof of Proposition 3

We solve the two retailers' first-order conditions to derive their optimal prices and service levels, which are given by:

$$p_1^* = p_2^* = \frac{2T_h + (2 - \phi^2 + e\phi^2)w + 2\gamma\sqrt{a}}{4 - \phi^2 - e(2 - \phi^2)}, \text{ and}$$

$$s_1^* = s_2^* = \left[ \frac{\phi(T_h + \gamma\sqrt{a} - (1-e)w)}{4 - \phi^2 - e(2 - \phi^2)} \right]^2.$$

The manufacturer's profit, after substituting these optimal prices and service levels is

$\Pi_h = \frac{4w(T_h + \gamma\sqrt{a} - (1-e)w)}{4 - \phi^2 - e(2 - \phi^2)} - a - 2L$ . The manufacturer maximizes the above profit function subject to the voluntary participation constraint, i.e.,  $\frac{(4 - \phi^2)(T_h + \gamma\sqrt{a} - (1-e)w)^2}{(4 - \phi^2 - e(2 - \phi^2))^2} + L \leq \underline{\pi}$ .

When the stocking costs are such that the voluntary participation constraint does not bind, the optimal wholesale price and advertising levels (derived from the manufacturer's first-order conditions) are:

$$w_1^* = \frac{T_h(4 - \phi^2 - e(2 - \phi^2))}{2[4 - \phi^2 - 2e(3 - \phi^2) + e^2(2 - \phi^2) - \gamma^2]}, \quad a_1^* = \frac{T_h^2\gamma^2}{[4 - \phi^2 - 2e(3 - \phi^2) + e^2(2 - \phi^2) - \gamma^2]^2}.$$

When the stocking costs are such that the voluntary participation constraint binds, the optimal solution may or may not involve a positive slotting allowance. If the optimal slotting allowance is positive, we can get its value by setting the voluntary participation constraint to equality and solving it for  $L$ . This value is  $L_3^* = \underline{\pi} - \frac{(4 - \phi^2)(T_h + \gamma\sqrt{a} - (1-e)w)^2}{(4 - \phi^2 - e(2 - \phi^2))^2}$ . After substituting this in the manufacturer's profit function, we get,

$\Pi_h = \frac{4w(T_h + \gamma\sqrt{a} - (1-e)w)}{4 - \phi^2 - e(2 - \phi^2)} - a - 2[\underline{\pi} - \frac{(4 - \phi^2)(T_h + \gamma\sqrt{a} - (1-e)w)^2}{(4 - \phi^2 - e(2 - \phi^2))^2}]$ . Solving the first-order conditions, we get the optimal wholesale price and advertising as follows.

$w_3^* = \frac{2eT_h}{4 - 4e - \phi^2 - 2e\phi^2 - e^2\phi^2 - 2\gamma^2}$ ,  $a_1^* = \frac{4\gamma^2 T_h^2}{[4 - 4e - \phi^2 - 2e\phi^2 - e^2\phi^2 - 2\gamma^2]^2}$ . The value of  $\underline{\pi}$  at which the slotting allowance is positive is  $\underline{\pi} \geq \frac{(1-e)^2(4 - \phi^2)T_h^2}{4[4 - 4e - \phi^2 - 2e\phi^2 - e^2\phi^2 - 2\gamma^2]^2}$  and  $\frac{4(2e - 2e^2 - 4\gamma^2)T_h^2}{4[4 - 4e - \phi^2 - 2e\phi^2 - e^2\phi^2 - 2\gamma^2]^2} > L_3^*$  is needed to ensure that  $\Pi_h > 0$ .

When the optimal slotting allowance is zero, we solve the first-order conditions to derive  $w$  and  $\lambda$  (the Lagrangian multiplier) as follows.

$w_2^* = \frac{2\gamma T_h - \sqrt{a}[4 - \phi^2 - 2e(3 - \phi^2) + e^2(2 - \phi^2) - 2\gamma^2]}{2(1-e)\gamma}$ , and  $\lambda = \frac{4[\gamma T_h - \sqrt{a}[4 - \phi^2 - 2e(3 - \phi^2) + e^2(2 - \phi^2) - \gamma^2]]}{2(1-e)^2(4 - \phi^2)}$ . We substitute the value of  $w_2^*$  in the voluntary participation constraint and solve it to get the optimal advertising level,  $a_2^* = \frac{4\gamma^2 \underline{\pi}}{(1-e)^2(4 - \phi^2)}$ .

We also give an alternate proof of the result that the slotting allowance may be positive in the symmetric information equilibrium. This proof does not rely on the specific functional forms of  $f(s_1)$ ,  $f(s_2)$  and  $g(a)$ . For this proof, we let

$$d_{1j} = T_h - p_1 + f(s_1) + g(a) + e(p_2 - f(s_2)), \quad (\text{A17})$$

$$d_{2j} = T_h - p_2 + f(s_2) + g(a) + e(p_1 - f(s_1)). \quad (\text{A18})$$

The retailer 1's optimal price and service levels are given by:

$$p_1^* = \frac{(2-e^2)f(s_1)-ef(s_2)+(2+e)(T_h+w+g(a))}{4-e^2},$$

$$(p_1 - w)f'(s_1) - 1 = 0.$$

Consider a stocking cost level,  $\underline{\pi}^\circ$  at which the voluntary participation constraint is binding and the manufacturer is not offering any slotting allowance. Now consider a small increase in the stocking cost,  $\Delta\underline{\pi}$ . The manufacturer can reduce the wholesale price, increase the advertising or increase the slotting allowance in order to meet the voluntary participation constraint. We first compare the profitability of reducing the wholesale price with the profitability of increasing the slotting allowance. The cost of increasing the slotting allowance to the manufacturer is  $2\Delta\underline{\pi}$ . The cost of reducing the wholesale price is  $(\Delta\underline{\pi}/\frac{\partial\Pi_{r1}}{\partial w})\frac{\partial\Pi_h}{\partial w}$ . Therefore, increasing the slotting allowance is less costly than reducing the wholesale price if  $(\frac{\partial\Pi_h}{\partial w}/\frac{\partial\Pi_{r1}}{\partial w}) > 2$ . We have already shown that this is not the case when  $e = 0$ . It can be shown that as  $e \rightarrow 1$ ,  $\frac{\partial\Pi_{r1}}{\partial w} \rightarrow 0$ . That is,  $(\frac{\partial\Pi_h}{\partial w}/\frac{\partial\Pi_{r1}}{\partial w}) > 2$  is always true as  $e \rightarrow 1$ .

Now we compare the profitability of increasing the advertising with the profitability of increasing the slotting allowance. The cost of increasing the advertising in order to meet the higher stocking costs is  $(\Delta\underline{\pi}/\frac{\partial\Pi_{r1}}{\partial a})\frac{\partial\Pi_h}{\partial a}$ . As before, increasing the slotting allowance is better than increasing the advertising for the manufacturer if  $(\frac{\partial\Pi_h}{\partial a}/\frac{\partial\Pi_{r1}}{\partial a}) > 2$ . It can be shown that

$$\frac{\partial\Pi_{r1}}{\partial a} = \frac{2d_{1h}g'(a)}{(2-e^2)^2} + \frac{2ed_{1h}f'(s)^2 g'(a)}{(2-e)^2[(2-e^2)f'(s)^2 + (2+e)d_{1h}f''(s)]}.$$

It is easy to see that  $\frac{\partial\Pi_{r1}}{\partial a}$  is sufficiently low for a low-enough value of  $g'(a)$  so that  $(\frac{\partial\Pi_h}{\partial a}/\frac{\partial\Pi_{r1}}{\partial a}) > 2$  is satisfied by sufficiently low  $g'(a)$ .

Therefore, for sufficiently high values of  $e$  and sufficiently low values of  $g'(a)$ , the manufacturer would offer a positive slotting allowance in the symmetric information equilibrium.

## A.5 Proof of Proposition 4

We consider those parameter values for which the high-demand manufacturer offers a positive slotting allowance in both symmetric information and separating equilibrium solutions. We define  $V_h(w, a) = \Pi_h(\cdot)+2L$  and  $V_l(w, a) = \Pi_l(\cdot)+2L$ . The proof is by contradiction. As before, we denote the symmetric and separating equilibrium solutions by the superscripts \* and \*\* respectively. Sup-

pose  $L^{**} \leq L^*$ . There are two possibilities:  $V_l(w^{**}, a^{**}) \geq V_l(w^*, a^*)$  or  $V_l(w^{**}, a^{**}) < V_l(w^*, a^*)$ . We begin with the case of  $V_l(w^{**}, a^{**}) \geq V_l(w^*, a^*)$ .

$V_l(w^{**}, a^{**}) \geq V_l(w^*, a^*)$  and  $L^{**} \leq L^*$  imply that  $V_l(w^{**}, a^{**}) - 2L^{**} \geq V_l(w^{**}, a^{**}) - 2L^* \geq V_l(w^*, a^*) - 2L^*$ . But we know that the low-type manufacturer makes a strictly positive profit by offering the high-type manufacturer's symmetric information solution and it makes zero profit by offering the high-type manufacturer's separating equilibrium solution. That is,  $V_l(w^{**}, a^{**}) - 2L^{**} < V_l(w^*, a^*) - 2L^*$ , giving a contradiction.

Now consider the other possibility:  $V_l(w^{**}, a^{**}) < V_l(w^*, a^*)$ . Let  $V_l(w^*, a^*) - 2L^* = \Delta > 0$ ,  $V_l(w^*, a^*) - V_l(w^{**}, a^{**}) = \Delta V > 0$ ,  $L^* - L^{**} = \Delta L > 0$ . By definition,  $V_l(w^{**}, a^{**}) - 2L^{**} = 0$ . It is easy to see that  $\Delta V - 2\Delta L = \Delta$ . Since  $\Delta L > 0$ ,  $\Delta V > \Delta$ . Now consider a small change from  $\{w^{**}, a^{**}, L^{**}\}$  such that  $\tilde{w} = w^{**} - \epsilon$ ,  $\tilde{a} = a^{**}$  and  $\tilde{L} = L^{**} + \frac{1}{2}[V_l(\tilde{w}, \tilde{a}) - V_l(w^{**}, a^{**})]$ . Recall that  $\Pi_h(w^*, a^*, L^*) > \Pi_h(w^{**}, a^{**}, L^{**})$ .  $\{\tilde{w}, \tilde{a}, \tilde{L}\}$  is closer to  $\{w^*, a^*, L^*\}$  than  $\{w^{**}, a^{**}, L^{**}\}$  is. Therefore, given the monotonicity of  $\Pi_h$  in  $w$  and  $L$ , the high-demand manufacturer can satisfy the signaling constraint and increase its profit by the deviation under consideration; thus giving a contradiction.