Online Appendix: Pricing Information Goods: A Strategic Analysis of the Selling and Pay-per-use Mechanisms

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1. Model extensions: Price commitment, two-part tariffs and consumer risk aversion

1.1. Firms cannot credibly commit to prices

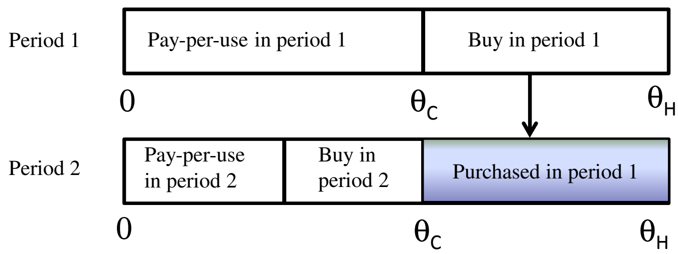
Proposition WA1: *Consider a monopoly without price commitment. If the firm adopts only selling, its profits are lower than when it can credibly commit to prices. If the firm adopts only pay-per-use, its profits are the same as when it can credibly commit to prices. If the firm adopts both mechanisms jointly and if $T > 0$, its profits are lower than when it could commit to prices.*

Proof of Proposition WA1: We assume: (i) there are two time periods in the horizon, (ii) firms choose their pricing mechanisms at the beginning of the horizon but choose prices at the beginning of each time period, (iii) consumers are strategic in timing the adoption of the information good. Let the usage frequency in each period be distributed U$[0, \bar{\theta}_H]$. Consumers discount the net utility obtained in period 2 by a discounting factor of δ.

In a monopoly, let the seller's prices be p_{S1} and p_{S2} respectively, and the pay-per-use provider's payments-per-use be p_{O1} and p_{O2} respectively. Under selling, consumers who buy the information good in period 1derive a net utility of $U_{S1}(p_{S1}) = (1+\delta)\phi\theta_i - p_{S1}$; if they buy the good in period 2, they derive a utility of $U_{S2}(p_{S2}) = \delta(\phi\theta_i - p_{S2})$. Under pay-per-use, consumers derive a net utility of $U_{O1}(p_{O1}) = \theta_i(\phi - T - p_{O1}) \geq 0$ if they use the good in period 1, and a net utility of $U_{O2}(p_{O2}) = \delta\theta_i(\phi - T - p_{O2}) \geq 0$ if they use the good in period 2. Figure 1 shows the resulting market structure.

The rest of the proof is structured as follows. We first derive the profits of the firm for the two-period model if the firm could commit to prices when it uses selling only, pay-per-use only,

Figure 1 Market shares from selling and pay-per-use pricing in monopoly



and a combination of the two mechanisms. Following that, we derive the profits of the firm in the three cases when it cannot commit to prices, and compare the profits under price commitment and no commitment.

Firm can commit to prices:

Let the superscript C denote that the firm can commit to prices, and the superscript NC denote that it cannot commit to prices. If the firm uses only the selling mechanism and could commit to its selling price, it would charge a price of p_S, and all consumers with $U_S(p_S) = (1+\delta)\phi\theta_i - p_S \geq 0$ would buy the information good.

Hence, $\Pi_S^C = \frac{Np_S}{\theta_H}[\bar{\theta}_H - \frac{p_S}{(1+\delta)\phi}]$. $\max_{p_S} \Pi_S^C = \frac{Np_S}{\theta_H}[\bar{\theta}_H - \frac{p_S}{(1+\delta)\phi}] \Rightarrow p_S = \frac{\bar{\theta}_H(1+\delta)\phi}{2} \Rightarrow \Pi_S^C = \frac{N\bar{\theta}_H(1+\delta)\phi}{4}$.

If the firm adopts only pay-per-use, $\Pi_O^C = \max_{p_{O1}} Np_O\frac{\bar{\theta}_H}{2}(1+\delta) \Rightarrow p_O = \phi - T \Rightarrow \Pi_O^C = \frac{N\bar{\theta}_H(1+\delta)(\phi-T)}{2}$.

If the firm adopts both mechanisms in a monopoly, then consumers who buy obtain a net utility of $U_S(p_S) = (1+\delta)\phi\theta_i - p_S \geq 0$, and consumers who adopt pay-per-use obtain a net utility of $U_O(p_O) = (1+\delta)\theta_i(\phi - p_O - T) \geq 0$. Therefore, consumers with a higher usage frequency than $\theta_C = \frac{p_S}{(1+\delta)(p_O+T)}$ buy the information good, and those with a lower usage frequency adopt pay-per-use. Here, the firm can leverage the threshold frequency θ_C by changing the ratio of $\frac{p_S}{p_O+T}$, which can be varied by altering the selling price and the payment-per-use. The total profits of the firm are given by:

$\Pi_{(S+O)}^C = \frac{N}{\theta_H}[\bar{\theta}_H - \theta_C] + \frac{N}{\theta_H}[\theta_C]^2 p_O(1+\delta)$. Solving for the FOC w.r.t. p_O yields $\frac{\partial \Pi_{(S+O)}^C}{\partial p_O} > 0 \Rightarrow p_O = \phi - T$. Substituting in $\Pi_{(S+O)}^C$ yields $p_S = \frac{\bar{\theta}_H \phi^2}{\phi+T} \Rightarrow \Pi_{(S+O)}^C = \frac{N\bar{\theta}_H}{2}\frac{(1+\delta)\phi^2}{(\phi+T)}$.

Firm cannot commit to prices:

(i) *Selling only*: If the firm only sells, rational consumers will anticipate that the firm will lower the price in period 2. Since $U_{S1}(p_{S1}) = (1+\delta)\phi\theta_i - p_{S1}$ and $U_{S2}(p_{S2}) = \delta(\phi\theta_i - p_{S2})$, all consumers with a higher usage frequency than $\theta_C = \frac{p_{S1} - \delta p_{S2}}{\phi}$ buy the good in period 1, and in period 2, all remaining consumers with a utility surplus of $U_{S2}(p_{S2}) = \phi\theta_i - p_{S2} \geq 0$ will buy the good.

Therefore, the profits of the firm in period 2 are: $\Pi_{S2} = \frac{N}{\theta_H}[\frac{p_{S1} - \delta p_{S2}}{\phi} - \frac{p_{S2}}{\phi}]p_{S2}$. The FOC yields: $p_{S2}^* = \frac{p_{S1}}{2(1+\delta)}$.

$\Rightarrow \Pi_{S2} = \frac{N}{\phi\theta_H}\frac{p_{S1}^2}{4(1+\delta)}$. Therefore, $\theta_C = \frac{p_{S1}-\delta p_{S2}}{\phi} = \frac{1}{\phi}p_{S1}\frac{2+\delta}{2(1+\delta)}$. To find p_{S1}, the firm solves:

$$\max_{p_{S1}} \frac{N}{\bar{\theta}_H}p_{S1}[\bar{\theta}_H - \frac{1}{\phi}p_{S1}\frac{2+\delta}{2(1+\delta)}] + \delta\Pi_{S2}$$
$$\Rightarrow \max_{p_{S1}} \frac{N}{\bar{\theta}_H}p_{S1}[\bar{\theta}_H - \frac{1}{\phi}p_{S1}\frac{2+\delta}{2(1+\delta)}] + \delta\frac{N}{\phi\bar{\theta}_H}\frac{p_{S1}^2}{4(1+\delta)}$$

$\Rightarrow p_{S1} = \frac{2\bar{\theta}_H\phi(1+\delta)}{4+\delta}$. Therefore, $\Pi_S^{NC} = \frac{N\bar{\theta}_H(1+\delta)\phi}{4+\delta}$.

(ii) If the monopolist uses only pay-per-use, all consumers with a utility surplus of $U_{O1}(p_{O1}) = \theta_i(\phi - T - p_{O1}) \geq 0$ use the good on a pay-per-use basis in period 1, and all consumers with a utility surplus of $U_{O2}(p_{O2}) = \delta\theta_i(\phi - T - p_{O2}) \geq 0$ use the good in period 2.

Then, $\Pi_{O2}^{NC} = Np_{O2}\frac{\bar{\theta}_H}{2} \Rightarrow p_{O2} = \phi - T \Rightarrow \Pi_{O2}^{NC} = N\frac{(\phi-T)\bar{\theta}_H}{2}$.

In period 1, the firm solves: $\max_{p_{O1}} \Pi_O^{NC} = Np_{O1}\frac{\bar{\theta}_H}{2} + \delta\Pi_{O2} = Np_{O1}\frac{\bar{\theta}_H}{2} + \delta N\frac{(\phi-T)\bar{\theta}_H}{2} \Rightarrow p_{O1} = \phi - T \Rightarrow \Pi_O^{NC} = \frac{N\bar{\theta}_H(1+\delta)(\phi-T)}{2}$

The lack of commitment on the part of the firm does not hurt the pay-per-use mechanism, as the decision to use the information good is decoupled over time.

(iii) If the monopolist uses both mechanisms in combination, in period 1, all consumers with a usage frequency above θ_C will buy the information good and exit the market. Consumers with a usage frequency below θ_C will use the information good on a pay-per-use basis. In period 2, all consumers with a usage frequency below θ_C will adopt the good. The utility surpluses from the two mechanisms are given by:

$U_{S2}(p_{S2}) = \delta(\phi\theta_i - p_{S2})$ and $U_{O2}(p_{O2}) = \delta\theta_i(\phi - T - p_{O2})$. Consumers with a usage frequency above $\frac{p_{S2}}{p_{O2}+T}$ will buy the good, and consumers with a usage frequency below $\frac{p_{S2}}{p_{O2}+T}$ will use it on a pay-per-use basis. The profits in period 2 from consumers who buy the good are $\frac{Np_{S2}}{\bar{\theta}_H}[\bar{\theta}_H - \frac{p_{S2}}{p_{O2}+T}]$, and the profits from pay-per-use are $\frac{N}{\bar{\theta}_H}[\frac{p_{S2}}{p_{O2}+T}]^2\frac{p_{O2}}{2}$, where the pay-per-use profits are the product of the market share ($\frac{N}{\bar{\theta}_H}\frac{p_{S2}}{p_{O2}+T}$), the payment-per-use ($p_{O2}$) and the average usage frequency ($\frac{1}{2}\frac{p_{S2}}{p_{O2}+T}$). The period 2 problem of the firm is as follows:

$$\max_{p_{S2},p_{O2}} \Pi_{(S+O)2}^{NC} = \frac{Np_{S2}}{\bar{\theta}_H}[\theta_C - \frac{p_{S2}}{p_{O2+T}}] + \frac{N}{\bar{\theta}_H}[\frac{p_{S2}}{p_{O2+T}}]^2\frac{p_{O2}}{2}$$
$$s.t.\ p_{O2} \leq \phi - T$$

$\Pi_{(S+O)2}^{NC} = \frac{Np_{S2}}{\bar{\theta}_H}[\theta_C - \frac{(p_{O2}+2T)p_{S2}}{2(p_{O2}+T)^2}] \Rightarrow \frac{\partial\Pi_{(S+O)2}^{NC}}{\partial p_{O2}} = \frac{p_{S2}(p_{O2}+3T)}{2(p_{O2}+T)^3} > 0 \Rightarrow p_{O2} = \phi - T \Rightarrow \max_{p_{S2}} \Pi_{(S+O)2}^{NC} = \frac{Np_{S2}}{\bar{\theta}_H}[\theta_C - \frac{p_{S2}(\phi+T)}{2\phi^2}] \Rightarrow p_{S2}^* = \theta_C\frac{\phi^2}{(\phi+T)}$.

Therefore, $\Pi_{(S+O)2}^{NC} = \frac{N\theta_C^2\phi^2}{2\bar{\theta}_H(\phi+T)}$. In period 1, marginal consumers with usage frequency θ_C satisfy: $\phi\theta_C(1+\delta) - p_{S1} = \theta_C(\phi - p_{O1} - T) + \delta(\phi\theta_C - p_{S2})$. Therefore, $\theta_C = \frac{p_{S1}-\delta p_{S2}}{p_{O1}+T}$. Since $p_{S2}^* = \theta_C\frac{\phi^2}{(\phi+T)}$, $\theta_C = \frac{p_{S1}}{p_{O1}+T+\frac{\delta\phi^2}{\phi+T}}$. Therefore, the total profits across the two periods are:

$\Pi^{NC}_{(S+O)} = \frac{N}{\theta_H} p_{S1}[\bar{\theta}_H - \theta_C] + \frac{N}{\theta_H}\frac{p_{O1}}{2}\theta_C^2 + \delta\Pi_2 = \frac{N}{\theta_H} p_{S1}[\bar{\theta}_H - \frac{p_{S1}}{p_{O1}+T+\frac{\delta\phi^2}{\phi+T}}] + \frac{N}{\theta_H}\frac{p_{O1}}{2}[\frac{p_{S1}}{p_{O1}+T+\frac{\delta\phi^2}{\phi+T}}]^2 + \frac{\delta N}{2\theta_H}[\frac{p_{S1}}{p_{O1}+T+\frac{\delta\phi^2}{\phi+T}}]^2 \frac{\phi^2}{\phi+T}$

On simplifying, $\Pi^{NC}_{(S+O)} = \frac{N}{\theta_H} p_{S1}[\bar{\theta}_H - \frac{p_{S1}\{(p_{O1}+2T)(\phi+T)+\delta\phi^2\}}{2(\phi+T)(p_{O1}+T+\frac{\delta\phi^2}{\phi+T})^2}]$

The derivative of $\Pi^{NC}_{(S+O)}$ w.r.t. p_{O1} gives:

$\frac{\partial\Pi^{NC}_{(S+O)1}}{\partial p_{O1}} = -\frac{N}{\theta_H} p_{S1}^2 [\frac{(p_{O1}+T+\frac{\delta\phi^2}{\phi+T})^2(\phi+T) - 2\{(p_{O1}+2T)(\phi+T)+\delta\phi^2\}(p_{O1}+T+\frac{\delta\phi^2}{\phi+T})}{2(\phi+T)(p_{O1}+T+\frac{\delta\phi^2}{\phi+T})^4}]$

$= -\frac{N}{\theta_H} p_{S1}^2 [\frac{-\delta\phi^2-(\phi+T)(p_{O1}+3T)}{2(\phi+T)(p_{O1}+T+\frac{\delta\phi^2}{\phi+T})^4}] \Rightarrow p_{O1} = \phi - T.$

Substituting the value of p_{O1}in $\Pi^{NC}_{(S+O)}$ yields $\Pi^{NC}_{(S+O)} = \frac{N}{\theta_H} p_{S1}[\bar{\theta}_H - \frac{p_{S1}\{(\phi+T)^2+\delta\phi^2\}}{2(\phi+T)(\phi+\frac{\delta\phi^2}{\phi+T})^2}]$. The FOC w.r.t. p_{S1}yields: $p_{S1} = \frac{\bar{\theta}_H\{\phi(\phi+T)^2+\delta\phi^2\}^2}{(\phi+T)\{(\phi+T)^2+\delta\phi^2\}}$.

$\Rightarrow \Pi^{NC}_{(S+O)} = \frac{N\bar{\theta}_H}{2}\frac{\{\phi(\phi+T)^2+\delta\phi^2\}^2}{(\phi+T)\{(\phi+T)^2+\delta\phi^2\}}$.

Comparison of profits in the commitment and no-commitment cases:

$\Pi^{NC}_S = \frac{N\bar{\theta}_H(1+\delta)\phi}{4+\delta} < \Pi^C_S = \frac{N\bar{\theta}_H(1+\delta)\phi}{4}$

$\Pi^{NC}_O = \frac{N\bar{\theta}_H(1+\delta)(\phi-T)}{2} = \Pi^C_O$

$\Pi^{NC}_{(S+O)} = \frac{N\bar{\theta}_H}{2}\frac{\{\phi(\phi+T)^2+\delta\phi^2\}^2}{(\phi+T)\{(\phi+T)^2+\delta\phi^2\}} < \Pi^C_{(S+O)} = \frac{N\bar{\theta}_H}{2}\frac{(1+\delta)\phi^2}{(\phi+T)}$ since $\delta T^2 > 0$.

The decision on the part of consumers to use pay-per-use in a monopoly are decoupled across periods – therefore, consumers do not gain from postponing their usage under pay-per-use. In contrast, if the firm sells, rational consumers expect that the firm will lower prices in period 2, as there are residual consumers who could potentially buy the information good at a lower price. This lack of commitment forces the firm to lower its selling price in period 1 to capture a higher market share, leading to lower profits. The firm can potentially leverage this threshold frequency by lowering its price in period 1 more strongly, thereby capturing higher market share. ■

Proposition WA2: *Consider a duopoly where the firms cannot credibly commit to prices: (i) If T is low, the firm that sells and the firm that employs pay-per-use make higher profits than when they can commit to prices. (ii) If T is high, then both firms make lower profits than when they can commit to prices.*

Proof of Proposition WA2: The proof is structured as follows. We first show in Part (I) of the proof that the only non-zero profit Nash equilibrium exists when each duopolist chooses a separate mechanism for the two periods. Following that, in Part (II), we compare the duopolists' profits under price commitment and no commitment.

Part (I): The proof of this part is very similar to the proof of Result 1, we only show the proof for the case where each duopolist employs both selling and pay-per-use, which is the most general case. In this case, let the two firms be denoted by A and B, let their selling prices in the two periods be p^A_{S1}, p^A_{S2} and p^B_{S1} and p^B_{S2} respectively, and their payments-per-use in the two periods be p^A_{O1}, p^A_{O2} and p^B_{O1} and p^B_{O2} respectively. Let the selling mechanisms of the two firms cover the market till $\bar{\theta}_C$ in the first period. Then in the second period, there exists no non-zero profit Nash equilibrium for the two firms from the proof of Case 4 in Result 1 (replacing θ_H by $\bar{\theta}_C$). Hence, $p^A_{S2} = p^B_{S2} = p^A_{O2} = p^B_{O2} = 0$.

In the first period, the net utilities of consumers from buying A or B or adopting A or B on a pay-per-use basis are given by: $U^A_{S1}(p^A_{S1}) = (1+\delta)\phi\theta_i - p^A_{S1}$, $U^B_{S1}(p^B_{S1}) = (1+\delta)\phi\theta_i - p^B_{S1}$, $U^A_{O1}(p^A_{O1}) = \theta_i(\phi - T - p^A_{O1}) \geq 0$, and $U^B_{O1}(p^B_{O1}) = \theta_i(\phi - T - p^B_{O1}) \geq 0$ respectively.

If the information good is not vertically differentiated, it is easy to see that if $p^{A*}_{S1} \neq p^{B*}_{S1}$, and $p^{A*}_{O1} \neq p^{B*}_{O1}$, the entire market share for the selling and pay-per-use sections go to the firm with the lower selling price and/or the lower payment-per-use, and the other firm is left with zero profits. Now, let us assume that $(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1})$ is a Nash equilibrium and let $p^{A*}_{S1} = p^{B*}_{S1} = p^*_S$, where $p^*_S > 0$, and $p^{A*}_{O1} = p^{B*}_{O1} = p^*_O$, where $p^*_O > 0$. In that case, the set of consumers who buy the information good are given by $(1+\delta)\theta_i\phi - p^*_S > \theta_i(\phi - p^*_O - T) + \delta(\theta_i\phi - p^*_{S2}) = \theta_i(\phi - p^*_O - T) + \delta\theta_i\phi$, since $p^*_{S2} = 0$. or the frequency of usage $\theta > \theta_C = \frac{p^*_S}{p^*_O + T}$, and consumers with $\theta < \theta_C$ use the information good on a pay-per-use basis. Because the net consumer surplus for consumers both firms is equal, we can assume that the market shares for both firms are equal (half of that of the monopolist who prices at p^*_S, p^*_O). Then:

$$\Pi_A(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1}) = \frac{Np^*_S}{2\theta_H}[\theta_H - \frac{p^*_S}{p^*_O + T}] + \frac{Np_O}{4\theta_H}[\frac{p^*_S}{p^*_O + T}]^2 \text{ and}$$
$$\Pi_B(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1}) = \frac{Np^*_S}{2\theta_H}[\theta_H - \frac{p^*_S}{p^*_O + T}] + \frac{Np_O}{4\theta_H}[\frac{p^*_S}{p^*_O + T}]^2$$

In this case, the Nash equilibrium conditions $\Pi_A(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1}) > \Pi_A(p^{A*}_{S1} - \epsilon, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1})$, $\Pi_B(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1}) > \Pi_B(p^{A*}_{S1}, p^{B*}_{S1} - \epsilon, p^{A*}_{O1}, p^{B*}_{O1})$, $\Pi_A(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1}) > \Pi_A(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1} - \epsilon, p^{B*}_{O1})$, and $\Pi_B(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1}) > \Pi_B(p^{A*}_{S1}, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1} - \epsilon)$ are not satisfied for all $p^*_S, p^*_O > 0$. If $\epsilon = \lim_{h\to 0} h$, for firm 1, then:

$$\Pi_A(p^{A*}_{S1} - \lim_{h\to 0} h, p^{B*}_{S1}, p^{A*}_{O1}, p^{B*}_{O1}) = \lim_{h\to 0} \frac{N(p^*_S - h)}{\theta_H}[\theta_H - \frac{p^*_S - h}{p^*_O + T}] + \frac{Np_O}{4\theta_H}[\frac{p^*_S - h}{p^*_O + T}]^2$$
$$> \frac{Np^*_S}{2\theta_H}[\theta_H - \frac{p^*_S}{p^*_O + T}] + \frac{Np_O}{4\theta_H}[\frac{p^*_S}{p^*_O + T}]^2.$$

A set of symmetric conditions holds for firm B's selling price and the payments-per-use of both firms. Therefore, there is a contradiction and a Nash equilibrium with non-zero prices and profits do not exist in this case. The other cases can be proven similarly.

Part (II): We now derive the profits of each duopolist for the two-period model if they could commit to prices when one of them uses selling, and the other adopts pay-per-use. Following that, we derive the profits of the duopolists when they cannot commit to prices, and compare the profits under price commitment and no commitment.

Firms can commit to prices:

We first analyze the two-period equilibrium between the firms using the selling and pay-per-use mechanisms when both of them can commit to prices. The net utilities of the consumer from the selling and pay-per-use mechanisms are given by: $U(S) = \phi\theta(1+\delta) - p_S$, and $U(O) = \theta(1+$

$\delta)(\phi - p_O - T)$ respectively. The marginal consumer satisfies: $\theta_C = \frac{p_S}{(1+\delta)(p_O+T)}$. Hence, if Π_S^C and Π_O^C represent the profits of the two firms with commitment, we have: $\Pi_S^C = \frac{Np_S}{\bar{\theta}_H}[\bar{\theta}_H - \frac{p_S}{(1+\delta)(p_O+T)}]$ and $\Pi_O^C = \frac{Np_O(1+\delta)}{2\bar{\theta}_H}[\frac{p_S}{(1+\delta)(p_O+T)}]^2$.

The FOC satisfying the Nash equilibrium are given by: (i) if $T \leq \frac{\phi}{2}$, $p_O^* = T$ and $p_S^* = \frac{\bar{\theta}_H(1+\delta)(p_O+T)}{2} = \bar{\theta}_H(1+\delta)T$, and the resulting profits of the two firms are given by: $\Pi_S^C = \frac{N\bar{\theta}_H T(1+\delta)}{2}$ and $\Pi_O^C = \frac{N\bar{\theta}_H T(1+\delta)}{8}$. (ii) if $T > \frac{\phi}{2}$, $p_O^* = \phi - T$ and $p_S^* = \frac{\bar{\theta}_H(1+\delta)(p_O+T)}{2} = \bar{\theta}_H(1+\delta)\frac{\phi}{2}$, and the resulting profits of the two firms are given by: $\Pi_S^C = \frac{N\bar{\theta}_H\phi(1+\delta)}{4}$ and $\Pi_O^C = \frac{N\bar{\theta}_H(\phi-T)(1+\delta)}{8}$.

Firms cannot commit to prices:

Next, we analyze the case where the firms cannot commit to prices. In the second period, the net consumer utilities of the information goods from the selling and pay-per-use mechanisms are given by: $U(S_2) = \delta(\theta\phi - p_{S2})$ and $U(O_2) = \delta\theta(\phi - p_{O2} - T)$ respectively. The marginal consumer satisfies: $\theta_2 = \frac{p_{S2}}{p_{O2}+T}$. Hence, $\Pi_{S2}^{NC} = \frac{N}{\bar{\theta}_H}p_{S2}[\theta_C - \frac{p_{S2}}{p_{O2}+T}]$ and $\Pi_{O2}^{NC} = \frac{N}{\bar{\theta}_H}\frac{p_{O2}}{2}[\frac{p_{S2}}{p_{O2}+T}]^2$. As shown in Proposition 3, the subgame perfect Nash equilibrium (SPNE) conditions in period 2 lead to: (i) if $\phi < \frac{T}{2}$, $p_{O2}^* = T$, $p_{S2}^* = \theta_C T$, $\Pi_{O2}^{NC} = \frac{N\theta_C^2 T}{8\bar{\theta}_H}$ and $\Pi_{S2}^{NC} = \frac{N\theta_C^2 T}{2\bar{\theta}_H}$. (ii) $\phi > \frac{T}{2}$, $p_{O2}^* = \phi - T$, $p_{S2}^* = \theta_C\frac{\phi}{2}$, $\Pi_{O2}^{NC} = \frac{N\theta_C^2(\phi-T)}{8\bar{\theta}_H}$ and $\Pi_{S2}^{NC} = \frac{N\theta_C^2\phi}{4\bar{\theta}_H}$.

In the first period, from Figure 1 in the Web Appendix, marginal consumers with usage frequency θ_C satisfy: $\phi\theta_C(1+\delta) - p_{S1} = \theta_C(\phi - p_{O1} - T) + \delta(\phi\theta_C - p_{S2})$. Hence, $\theta_C = \frac{p_{S1}-\delta p_{S2}}{p_{O1}+T}$.

Case (i): $T \leq \frac{\phi}{2}$. Since $p_{S2}^* = \theta_C T$, $\theta_C = \frac{p_{S1}}{p_{O1}+T(1+\delta)}$. Then, to find p_{S1}, the firm solves:

$$\max_{p_{S1}} \frac{N}{\bar{\theta}_H}p_{S1}[\bar{\theta}_H - \frac{p_{S1}}{p_{O1}+T(1+\delta)}] + \delta\Pi_{S2}$$
$$\Rightarrow \max_{p_{S1}} \frac{N}{\bar{\theta}_H}p_{S1}[\bar{\theta}_H - \frac{p_{S1}}{p_{O1}+T(1+\delta)}] + \delta\frac{NT}{2\bar{\theta}_H}[\frac{p_{S1}}{p_{O1}+T(1+\delta)}]^2$$

The FOC w.r.t. p_{S1}gives us the reaction function $p_{S1}^* = \frac{\bar{\theta}_H\{p_{o1}+T(1+\delta)\}^2}{2(p_{O1}+T)+\delta T}$.

Similarly, the firm using the pay-per-use mechanism solves: $\max_{p_{O1}} \Pi_O^{NC} = \frac{N}{\bar{\theta}_H}\frac{p_{O1}}{2}[\frac{p_{S1}}{p_{O1}+T(1+\delta)}]^2 + \delta\Pi_{O2} = \frac{N}{\bar{\theta}_H}\frac{p_{O1}}{2}[\frac{p_{S1}}{p_{O1}+T(1+\delta)}]^2 + \frac{\delta N}{\bar{\theta}_H}\frac{T}{8}[\frac{p_{S1}}{p_{O1}+T(1+\delta)}]^2$. The FOC gives us: (a) if $T \leq \frac{\phi}{2+\frac{\delta}{2}}$, $p_{O1}^* = T(1+\frac{\delta}{2})$, and (b) if $T > \frac{\phi}{2+\frac{\delta}{2}}$, $p_{O1}^* = \phi - T$.

Case (i)(a): If $0 < T \leq \frac{\phi}{2+\frac{\delta}{2}}$, $p_{O1}^* = T(1+\frac{\delta}{2}) \Rightarrow p_{S1}^* = \frac{\bar{\theta}_H\{p_{o1}+T(1+\delta)\}^2}{2(p_{O1}+T)+\delta T} = \frac{\bar{\theta}_H T(2+\frac{3\delta}{2})^2}{4+2\delta}$. Substituting these values into the profit functions yields:

$\Pi_S^{NC} = \frac{N\bar{\theta}_H T(4+3\delta)^2}{8(4+2\delta)}$ and $\Pi_O^{NC} = \frac{N\bar{\theta}_H T(4+3\delta)^3}{8(8+4\delta)^2}$.

Case (i)(b): If $\frac{\phi}{2+\frac{\delta}{2}} < T \leq \frac{\phi}{2}$, $p_{O1}^* = \phi - T \Rightarrow p_{S1}^* = \frac{\bar{\theta}_H\{p_{o1}+T(1+\delta)\}^2}{2(p_{O1}+T)+\delta T} = \frac{\bar{\theta}_H(\phi+\delta T)^2}{2\phi+\delta T}$. Substituting these values into the profit functions yields:

$\Pi_S^{NC} = \frac{N\bar{\theta}_H(\phi+\delta T)^2}{2(2\phi+\delta T)}$ and $\Pi_O^{NC} = \frac{N\bar{\theta}_H(\phi+\delta T)^2(4\phi-4T+\delta T)}{8(2\phi+\delta T)^2}$.

Case (ii): $T > \frac{\phi}{2}$. Since $p_{S2}^* = \theta_C\frac{\phi}{2}$, $\theta_C = \frac{p_{S1}}{p_{O1}+T+\delta\frac{\phi}{2}}$. Then, to find p_{S1}, the firm solves:

$$\max_{p_{S1}} \frac{N}{\bar{\theta}_H} p_{S1}[\bar{\theta}_H - \frac{p_{S1}}{p_{O1}+T+\delta\frac{\phi}{2}}] + \delta\Pi_{S2}$$
$$\Rightarrow \max_{p_{S1}} \frac{N}{\bar{\theta}_H} p_{S1}[\bar{\theta}_H - \frac{p_{S1}}{p_{O1}+T+\delta\frac{\phi}{2}}] + \delta\frac{N\phi}{4\bar{\theta}_H}[\frac{p_{S1}}{p_{O1}+T+\delta\frac{\phi}{2}}]^2$$

The FOC w.r.t. p_{S1} yields the reaction function $p^*_{S1} = \frac{2\bar{\theta}_H\{p_{o1}+T+\delta\frac{\phi}{2}\}^2}{4(p_{O1}+T)+\delta\phi}$.

Similarly, the firm using the pay-per-use mechanism solves: $\max_{p_{O1}} \Pi_O^{NC} = \frac{N}{\bar{\theta}_H}\frac{p_{O1}}{2}[\frac{p_{S1}}{p_{O1}+T+\delta\frac{\phi}{2}}]^2 + \delta\Pi_{O2} = \frac{N}{\bar{\theta}_H}\frac{p_{O1}}{2}[\frac{p_{S1}}{p_{O1}+T+\delta\frac{\phi}{2}}]^2 + \frac{\delta N}{\bar{\theta}_H}\frac{(\phi-T)}{8}[\frac{p_{S1}}{p_{O1}+T+\delta\frac{\phi}{2}}]^2$. The maximization is subject to $p_{O1} \leq \phi - T$. The FOC yields: $p^*_{O1} = T(1+\frac{\delta}{2})$, but since in this region, $T > \frac{\phi}{2}$, $p^*_{O1} = \phi - T$. Hence, $p^*_{S1} = \frac{2\bar{\theta}_H\{p_{o1}+T+\delta\frac{\phi}{2}\}^2}{4(p_{O1}+T)+\delta\phi} = \frac{2\bar{\theta}_H\phi(1+\frac{\delta}{2})^2}{4+\delta}$. Substituting these values into the profit functions yields:

$\Pi_S^{NC} = \frac{N\bar{\theta}_H\phi(1+\frac{\delta}{2})^2}{4+\delta}$ and $\Pi_O^{NC} = \frac{N\bar{\theta}_H(\phi-T)(2+\delta)^2}{8(4+\delta)}$.

Comparison of profits in the commitment and no-commitment cases:

We now compare $\Pi_S^{NC}$with Π_S^C and Π_O^{NC} with Π_O^C.

(i) In the region $0 < T \leq \frac{\phi}{2+\frac{\delta}{2}}$: $\Pi_S^{NC} = \frac{N\bar{\theta}_H T(4+3\delta)^2}{8(4+2\delta)} > \Pi_S^C = \frac{N\bar{\theta}_H T(1+\delta)}{2}$ as $\frac{(4+3\delta)^2}{8(4+2\delta)} > \frac{(1+\delta)}{2}$ as $\delta > 0$.

$\Pi_O^{NC} = \frac{N\bar{\theta}_H T(4+3\delta)^3}{8(8+4\delta)^2} > \Pi_O^C = \frac{N\bar{\theta}_H T(1+\delta)}{8}$ as $\frac{(4+3\delta)^3}{(8+4\delta)^2} > (1+\delta)$ as $11\delta^3 + 92\delta^2 + 16\delta > 0$.

(ii) In the region $\frac{\phi}{2+\frac{\delta}{2}} < T \leq \frac{\phi}{2}$: $\Pi_S^{NC} = \frac{N\bar{\theta}_H(\phi+\delta T)^2}{2(2\phi+\delta T)} > \Pi_S^C = \frac{N\bar{\theta}_H T(1+\delta)}{2}$ if $\phi^2 > 2\phi T + \delta T^2$ and $\Pi_S^{NC} = \frac{N\bar{\theta}_H(\phi+\delta T)^2}{2(2\phi+\delta T)} < \Pi_S^C = \frac{N\bar{\theta}_H T(1+\delta)}{2}$ if $\phi^2 < 2\phi T + \delta T^2$. If T is close to the lower limit ($T = \frac{\phi}{2+\frac{\delta}{2}} + \epsilon$, $\epsilon \to 0$), then $\Pi_S^{NC} > \Pi_S^C$ and if T is close to the upper limit ($T = \frac{\phi}{2} - \epsilon$, $\epsilon \to 0$), then $\Pi_S^{NC} < \Pi_S^C$.

$\Pi_O^{NC} = \frac{N\bar{\theta}_H(\phi+\delta T)^2(4\phi-4T+\delta T)}{8(2\phi+\delta T)^2} > \Pi_O^C = \frac{N\bar{\theta}_H T(1+\delta)}{8}$ if $(\phi+\delta T)^2(4\phi-4T+\delta T) > (1+\delta)T(2\phi+\delta T)^2$ and $\Pi_O^{NC} < \Pi_O^C$ if $(\phi+\delta T)^2(4\phi-4T+\delta T) > (1+\delta)T(2\phi+\delta T)^2$. If T is close to the lower limit ($T = \frac{\phi}{2+\frac{\delta}{2}} + \epsilon$, $\epsilon \to 0$), then $\Pi_S^{NC} > \Pi_S^C$ and if T is close to the upper limit ($T = \frac{\phi}{2} - \epsilon$, $\epsilon \to 0$), then $\Pi_S^{NC} < \Pi_S^C$.

(iii) In the region $T > \frac{\phi}{2}$: $\Pi_S^{NC} = \frac{N\bar{\theta}_H\phi(1+\frac{\delta}{2})^2}{4+\delta} < \Pi_S^C = \frac{N\bar{\theta}_H\phi(1+\delta)}{4}$ as $4(1+\frac{\delta}{2})^2 < (4+\delta)(1+\delta)$ as $\delta > 0$.

$\Pi_O^{NC} = \frac{N\bar{\theta}_H(\phi-T)(2+\delta)^2}{8(4+\delta)} < \Pi_O^C = \frac{N\bar{\theta}_H(1+\delta)(\phi-T)}{8}$ as $(2+\delta)^2 < (4+\delta)(1+\delta)$ because $\delta > 0$. ■

1.2. The pay-per-use provider uses a two-part tariff

Proposition WA3: *Consider a monopoly where the firm uses only the pay-per-use mechanism which could also incorporate a fixed fee. Here, (i) when* $C \geq \frac{N\theta_H}{2}$*, representing unconstrained capacity, the firm sets the fixed fee to zero, and charges only a payment-per-use, equal to* $\phi - T$*. (ii) When* $0 < C < \frac{N\theta_H}{2}$*, representing a capacity constraint, the firm uses a two-part tariff, and obtains a profit of* $\Pi_O = (\phi - T)C$*.*

Consider a monopoly where the firm uses only the selling mechanism. Here, if $C < \frac{3N\theta_H}{8}$*, representing a capacity constraint, the firm sets a higher selling price than that when capacity was unconstrained. The profits of the pay-per-use provider are higher than that of the seller for any level of capacity if the psychological cost is low.*

Proof of Proposition WA3: With no capacity constraints, the pay-per-use provider extracts all consumer surplus. The firm uses a two-part tariff only when it is capacity constrained (Cachon and Feldman 2011; Essegaier et al. 2002).

Pay-per-use: If $U_{iO}(p_O,\bar{p}) = \theta_i(\phi - T) - \bar{p} - (\theta_i - \bar{\theta})^+ p_O$, only consumers with a positive utility surplus, i.e., those with a higher usage frequency than the cutoff value of $\theta_C > \frac{\bar{p}}{\phi - T}$, will adopt the information good. The profit of the pay-per-use provider is given by: $\Pi_O = \frac{N}{\theta_H}[(\theta_H - \theta_C)\bar{p} + \int_{\bar{\theta}}^{\theta_H} p_O(\theta - \bar{\theta})d\theta] = \frac{N}{\theta_H}[(\theta_H - \theta_C)\bar{p} + \frac{p_O}{2}(\theta_H - \bar{\theta})^2]$. Since Π_O is strictly decreasing in $\bar{\theta}$, $\bar{\theta}$ is set equal to its lower limit θ_C. The threshold frequency θ_C is dependent on the ratio of the fixed fee $\bar{p}$ only, and the fixed fee can be changed to determine the number of consumers who adopt the information good. The fixed fee should be set by the following tradeoff: a high fixed fee results in a higher revenue per consumer, but less consumers who adopt the good. A low fixed fee has the opposite effect: the revenue per consumer who adopts is lower, but the number of consumers adopting is larger. Note that $\bar{\theta}$ cannot be lower than θ_C as that would lead to negative utility surplus for the consumer with a usage frequency of θ_C. Hence, $U_{iO}(p_O) = \theta_i(\phi - T) - \bar{p} - (\theta - \theta_C)p_O$. Setting $U_{iO}(p_O) = 0$ yields $\theta_C = \frac{\bar{p}}{\phi - T}$.

Seller: The seller uses just one tariff (p_S) as before.

Unconstrained capacity:

Pay-per-use: Hence, the pay-per-use provider in a monopoly solves: $\max_{p_O,\bar{p}} \frac{N}{\theta_H}[(\theta_H - \frac{\bar{p}}{\phi - T})\bar{p} + \frac{p_O}{2}(\theta_H - \frac{\bar{p}}{\phi - T})^2]$. Since the profit function is quadratic in $\bar{p}$ with the second derivative being negative, the FOC of Π_O w.r.t. $\bar{p}$ yields $\bar{p} = \frac{\theta_H(\phi - T)(\phi - p_O - T)}{(2\phi - 2T - p_O)}$. Substituting this value in Π_O yields $\Pi_O = N\frac{\theta_H(\phi - T)^2}{2(2\phi - 2T - p_O)}$. This is maximized when $(2\phi - 2T - p_O)$ is minimized, hence, p_O is set to its maximum value of $\phi - T$. Substituting this optimal value in the equation for $\bar{p}$ yields $\bar{p} = 0$. Hence, when unconstrained by capacity, the pay-per-use provider sets $p_O = \phi - T$ and $\bar{p} = 0$. Its profits are given by $\Pi_O = \frac{N\theta_H(\phi - T)}{2}$, which is the same as in Proposition 1.

Seller: If the seller is unconstrained by capacity, it sets $p_S = \frac{\phi\theta_H}{2}$, and its profits are $\Pi_S = \frac{N\phi\theta_H}{4}$ from Proposition 1.

Constrained capacity:

Pay-per-use: To find the boundary when the pay-per-use provider is constrained for capacity, we set the constraint $TU \leq C$ to be tight. The total usage TU can be found in one of two ways: $TU = \frac{N}{\theta_H}\int_{\theta_C}^{\theta_H} \theta d\theta = \frac{N}{2\theta_H}[\theta_H^2 - \theta_C^2]$, which is the total usage integrated across all market participants, or by $TU = \frac{N}{\theta_H}(\theta_H - \theta_C)\frac{\theta_H + \theta_C}{2}$, which is the product of the market size and average frequency, both yield the same results. Since the unconstrained solution is $\bar{p} = 0$, the constraint is tight when $C = \frac{N\theta_H}{2}$.

When the constraint is tight, i.e., when $C < \frac{N\theta_H}{2}$, the pay-per-use provider needs to control the usage of the information good by using the fixed fee $\bar{p}$. Since $\theta_C = \frac{\bar{p}}{\phi - T}$, this yields $\bar{p} = (\phi -$

$T)\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}$ and $p_O = \phi - T$ as the elements of the two-part tariff. The profits of the pay-per-use provider are then given by: $\Pi_O = \frac{N}{\theta_H}[(\theta_H - \frac{\bar{p}}{\phi-T})\bar{p} + \frac{p_O}{2}(\theta_H - \frac{\bar{p}}{\phi-T})^2] = (\phi - T)C$.

Seller: To find the boundary when the seller is constrained for capacity, we set the constraint $TU = \frac{N}{\theta_H}\int_{\theta_C}^{\theta_H}\theta d\theta = \frac{N}{2\theta_H}[\theta_H^2 - (\frac{p_S}{\phi})^2] \leq C$ to be tight when $p_S = \frac{\phi\theta_H}{2}$. The constraint is tight when $C = \frac{3N\theta_H}{8}$.

When the constraint is tight, i.e., when $C < \frac{3N\theta_H}{8}$, the seller needs to control the usage of the information good by increasing p_S. This yields $p_S = \phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}$. The profits of the seller are then given by: $\Pi_S = \frac{Np_S}{\theta_H}[\theta_H - \frac{p_S}{\phi}] = \frac{N_S}{\theta_H}\phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}[\theta_H - \sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}]$. We now compare the profits of the seller and the pay-per-use provider.

Comparison of profits in the unconstrained and constrained cases:

Case 1: $C > \frac{N\theta_H}{2}$: As shown in Proposition 1, when both the seller and pay-per-use provider are unconstrained, $\Pi_O = \frac{N\theta_H(\phi-T)}{2} > \Pi_S = \frac{N\phi\theta_H}{4}$ when T is low.

Case 2: $\frac{3N\theta_H}{8} < C < \frac{N\theta_H}{2}$: In this region, the seller is not constrained by capacity, but the pay-per-use provider is constrained by capacity. Hence, if $T = 0$, the profits of the seller and pay-per-use provider in this region are given by: $\Pi_O = (\phi - T)C = \phi C > \Pi_S = \frac{N\phi\theta_H}{4}$ as $C > \frac{3N\theta_H}{8} > \frac{N\theta_H}{4}$ in this region.

Case 3:$0 < C < \frac{3N\theta_H}{8}$: In this region, both firms are constrained by capacity. We will show that if $T = 0$, $\Pi_O = (\phi - T)C = \phi C > \Pi_S = \frac{N_S}{\theta_H}\phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}[\theta_H - \sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}]$.

On simplification, this reduces to showing that: $\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}} < \theta_H - \frac{2C}{N} + \frac{\theta_H}{2}$. Squaring both sides and simplifying, we need to show that: $\frac{5\theta_H^2}{4} + \frac{4C^2}{N^2} - \frac{4\theta_H C}{N} > 0$. This is true as $\frac{5\theta_H^2}{4} + \frac{4C^2}{N^2} - \frac{4\theta_H C}{N} = \frac{\theta_H^2}{4} + (\theta_H - \frac{2C}{N})^2 > 0$. ■

PropositionWA4: *Consider a duopoly where the firms are not capacity-constrained ($C \geq \frac{3N\theta_H}{8}$) and the pay-per-use provider potentially imposes a fixed fee. Here: (i) If $T \leq \frac{\phi}{2}$, there is no equilibrium with positive profits for each firm. (ii) If $T > \frac{\phi}{2}$, the pay-per-use provider sets $p_O = \phi - T$, and $\bar{p} = 0$. Further, the seller has higher profits than the pay-per-use provider.*

Consider a duopoly where both the firms are capacity-constrained ($C < \frac{N\theta_H}{4}$) and the pay-per-use provider potentially imposes a fixed fee.. Here: (i) if $T \leq \frac{\phi}{2}$, there is no equilibrium with positive profits for each firm. (ii) If $T > \frac{\phi}{2}$, the pay-per-use provider uses a two-part tariff and sets $p_O = \phi - T$, and $\bar{p} = (\phi - T)\sqrt{\theta_H^2 - \frac{4\theta_H C}{N}}$. The seller has higher profits than the pay-per-use provider.

Proof of Proposition WA4: The proof is divided into two parts: we first show in Part (I) that two firms cannot have a non-zero profits equilibrium when both of them use the pay-per-use mechanism (the proofs for other combinations are analogous to those in Result 1). We then find the equilibrium pricing outcomes in Part (II) and compare the profits of the two firms when one of them sells the product and the other adopts pay-per-use.

Part (I): Denoting the two firms as 1 and 2, the only asymmetrical pricing scheme that can result in a Nash equilibrium with positive profits is if $\bar{p}_1 \neq \bar{p}_2$ and $p_{O1} \neq p_{O2}$. Without loss of generality, let $\bar{p}_1 < \bar{p}_2$ and $p_{O1} > p_{O2}$ (the reverse inequalities are a necessary condition for the existence of an equilibrium, if one firm prices higher on both dimensions, it trivially follows that it has zero market share).

As in the previous section,$U_{iO1}(p_{O1}, \bar{p}_1) = \theta_i(\phi - T) - \bar{p}_1 - (\theta_i - \bar{\theta}_1)^+ p_{O1}$ and $U_{iO2}(p_{O2}, \bar{p}_2) = \theta_i(\phi - T) - \bar{p}_2 - (\theta_i - \bar{\theta}_2)^+ p_{O2}$. Since $p_{O1} > p_{O2}$, consumers with a usage frequency above θ_C choose good 2 and below θ_C choose good 1. Hence, for good 1, $\theta_{C1} = \bar{\theta}_1 = \frac{p_1}{\phi - T}$. To find $\theta_C = \bar{\theta}_2$, we set $U_{iO1}(p_{O1}, \bar{p}_1) = U_{iO2}(p_{O2}, \bar{p}_2)$ at θ_C to obtain $\theta_C = \frac{\bar{p}_2 - \bar{p}_1}{p_{O1}} + \frac{\bar{p}_1}{\phi - T}$. Hence, $\Pi_1 = (\theta_C - \bar{\theta}_1)[\bar{p}_1 + \frac{p_{O1}}{2}(\theta_C - \bar{\theta}_1)] = \frac{\bar{p}_2 - \bar{p}_1}{p_{O1}}[\frac{\bar{p}_1}{2} + \frac{\bar{p}_2}{2}]$ and $\Pi_2 = (\theta_H - \theta_C)[\bar{p}_2 + \frac{p_{O2}}{2}(\theta_H - \theta_C)] = (\theta_H - \frac{\bar{p}_2 - \bar{p}_1}{p_{O1}} - \frac{\bar{p}_1}{\phi - T})[\bar{p}_2 + \frac{p_{O2}}{2}(\theta_H - \frac{\bar{p}_2 - \bar{p}_1}{p_{O1}} - \frac{\bar{p}_1}{\phi - T})]$.

The FOC of Π_1 w.r.t. p_{O1} yields $p_{O1} \to 0$. The FOC of Π_2 w.r.t. $\bar{p}_2$ yields: $\bar{p}_2 = \frac{p_{O1}^2\theta_H + \frac{\bar{p}_1 p_{O1}(\phi - p_{O1} - T)}{\phi - T} - p_{O2}p_{O1}\theta_H - \frac{\bar{p}_1 p_{O2}(\phi - p_{O1} - T)}{\phi - T}}{2p_{O1} - p_{O2}}$. On substituting in Π_2, $\Pi_2 = [\theta_H + \frac{\bar{p}_1(\phi - p_{O1} - T)}{p_{O1}(\phi - T)}]^2 \frac{p_{O1}^2}{2(2p_{O1} - p_{O2})}$. Maximizing $\Pi_2 \Rightarrow p_{O2}$ is set to its maximum value, which is equal to p_{O1}, since w.l.o.g., $p_{O1} > p_{O2}$ has been assumed earlier. Hence, the two firms use fixed fees only, and set $p_{O1} = p_{O2} = 0$. This implies both firms sell the information good, and from Case 1 in Proposition 3, there in no non-zero profit equilibrium in this case.

Part (II): As before, $U_{iS} = \phi\theta - p_S$ and $U_{iO}(p_O) = \theta_i(\phi - T) - \bar{p} - (\theta_i - \bar{\theta})^+ p_O$. Similar to the proof of Proposition 7, for the pay-per-use provider, since Π_O is strictly decreasing in $\bar{\theta}$, $\bar{\theta}$is set equal to its lower limit θ_C. Setting $U_{iO}(p_O) = 0$ yields $\theta_C = \frac{\bar{p}}{\phi - T}$. Hence, $U_{iO}(p_O) = \theta_i(\phi - T) - \bar{p} - (\theta_i - \theta_C)p_O$. If the marginal consumer who is indifferent between the selling and pay-per-use mechanisms has a usage frequency of θ_1, $\theta_1 = \frac{p_S - \bar{p} + \frac{p_O \bar{p}}{\phi - T}}{p_O + T}$. Let $k = 1 - \frac{p_O}{\phi - T}$. Then, $\theta_1 = \frac{p_S - k\bar{p}}{p_O + T}$.

Then, $\Pi_S = \frac{Np_S}{\theta_H}[\theta_H - \frac{p_S - k\bar{p}}{p_O + T}]$ and $\Pi_O = \frac{N}{\theta_H}\int_{\theta_C}^{\theta_1}[\bar{p} + (\theta - \theta_C)]d\theta = \frac{N}{\theta_H}[\theta_1 - \theta_C][\bar{p} + \frac{p_O}{2}(\theta_1 - \theta_C)]$.

$\Rightarrow \Pi_O = \frac{N}{\theta_H}[\frac{p_S - k\bar{p}}{p_O + T} - \frac{\bar{p}}{\phi - T}][\bar{p} + \frac{p_O}{2}(\frac{p_S - k\bar{p}}{p_O + T} - \frac{\bar{p}}{\phi - T})]$

Unconstrained capacity: We first find the reaction function of the pay-per-use firm in equilibrium without the capacity constraint. Since $\frac{\partial^2 \Pi_O}{\partial \bar{p}^2} < 0$, the FOC of Π_O w.r.t. $\bar{p}$ yields: $\bar{p} = \frac{p_S T(\phi - T)(\phi - p_O - T)}{\phi[p_O(\phi - 2T) + 2T^2]}$. Substituting this value back into Π_O yields: $\Pi_O = \frac{Np_S^2(\phi - T)^2}{2\theta_H \phi[2(\phi - T)(p_O + T) - p_O\phi]}$. To maximize Π_O, we have to minimize $\Gamma = 2(\phi - T)(p_O + T) - p_O\phi = p_O(\phi - 2T) + 2T(\phi - T)$. When $\phi \geq 2T$, Γ is minimized when $p_O = 0$. When $\phi < 2T$, Γ is minimized when $p_O = \phi - T$.

Case (i): $\phi \geq 2T \Rightarrow T \leq \frac{\phi}{2}$. In this case, the unconstrained profit function is minimized when $p_O = 0$. However, this implies that $T = 0$ as the psychological cost (associated with the ticking meter effect, insurance effect and overestimation effect) is zero when the payment-per-use is zero. Hence, in this case, there is no non-zero profit equilibrium, as the two firms use fixed fees without any differentiation (see the proof of Result 1). In the capacity-constrained cases as well, there is no equilibrium, as the capacity-constrained pay-per-use provider can only increase the fixed fee to control the total usage frequency, and there is no differentiation between the firms' pricing mechanisms.

Case (ii): $\phi < 2T \Rightarrow T > \frac{\phi}{2}$. In this case, the unconstrained profit function is minimized when $p_O^* = \phi - T$. Substituting this back in the reaction function of $\bar{p}$ implies the pay-per-use provider sets $p_O = \phi - T$ and $\bar{p}^* = 0$ in the reaction function.

Next, we find the seller's reaction function in equilibrium without the capacity constraint. Since $\frac{\partial^2 \Pi_S}{\partial p_S^2} < 0$, the reaction function of the seller is given by the FOC.

$p_S = \frac{\theta_H(p_O+T)}{2} + \frac{k\bar{p}}{2}$.

When $T > \frac{\phi}{2}$, the seller sets $p_S^* = \frac{\theta_H(p_O+T)}{2} + \frac{k\bar{p}}{2} = \frac{\theta_H \phi}{2}$. This gives us the same equilibrium in Proposition 3. Hence, in this case, the seller's profits are higher than the pay-per-use provider.

Constrained capacity: If the capacity constraint is included, the total usage of the seller is higher than the pay-per-use provider (for the seller, $TU_S = \frac{N}{\theta_H}\int_{\theta_1}^{\theta_H} \theta d\theta = \frac{3N\theta_H}{4}$, while for the pay-per-use provider, $TU_O = \frac{N}{\theta_H}\int_{\theta_C}^{\theta_1} \theta d\theta = \frac{N\theta_H}{4}$). If the capacity of the firms $C < \frac{3N\theta_H}{4}$, the seller sets p_S by $\frac{N}{\theta_H}\int_{\theta_1}^{\theta_H} \theta d\theta = C$, where $\theta_1$is set by the reaction function of the seller $\theta_1 = \frac{p_S - k\bar{p}}{p_O+T} = \frac{p_S}{\phi}$, since $p_O = \phi - T$ and $\bar{p}^* = 0$ is the pay-per-use provider's reaction function. This yields $p_S = \phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}$. The profits of the seller are then given by: $\Pi_S = \frac{Np_S}{\theta_H}[\theta_H - \frac{p_S - k\bar{p}}{p_O+T}] = \frac{N_S}{\theta_H}\phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}[\theta_H - \sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}]$.

The pay-per-use provider's profits are given by: $\Pi_O = \frac{N}{\theta_H}[\frac{p_S - k\bar{p}}{p_O+T} - \frac{\bar{p}}{\phi - T}][\bar{p} + \frac{p_O}{2}(\frac{p_S - k\bar{p}}{p_O+T} - \frac{\bar{p}}{\phi - T})] = \frac{N}{\theta_H}(\phi - T)(\theta_H^2 - \frac{2\theta_H C}{N})$. In this range, the total usage of the pay-per-use provider is given by $TU_O = \frac{N}{\theta_H}\int_{\theta_C}^{\theta_1} \theta d\theta = \frac{N\theta_H}{4}$. When $C < \frac{N\theta_H}{4}$, both firms are constrained by capacity. The seller's reaction function is given by: $p_S = \phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}$. The pay-per-use provider's reaction function is given by: $p_O^* = \phi - T$ and $\bar{p}$ is set by $\frac{N}{\theta_H}\int_{\theta_C}^{\theta_1} \theta d\theta = C$. Since the value of n, $\theta_1 = \frac{p_S - k\bar{p}}{p_O+T}$ and $k = 1 - \frac{p_O}{\phi - T}$, $k = 0$. Hence, $\theta_1 = \frac{p_S}{p_O+T} = \frac{p_S}{\phi}.TU_S = C \Rightarrow p_S = \phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}$. This yields $\theta_C = \frac{\bar{p}}{\phi - T} = \sqrt{\theta_H^2 - \frac{4\theta_H C}{N}}$.

The profits of the two firms are given by: $\Pi_S = \frac{N_S}{\theta_H}\phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}[\theta_H - \sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}]$.

$\Pi_O = \frac{N}{\theta_H}[\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}} - \sqrt{\theta_H^2 - \frac{4\theta_H C}{N}}][(\phi - T)\sqrt{\theta_H^2 - \frac{4\theta_H C}{N}} + \frac{\phi - T}{2}(\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}} - \sqrt{\theta_H^2 - \frac{4\theta_H C}{N}})] = C(\phi - T)$.

Since $T > \frac{\phi}{2}$, $\Pi_O < C\frac{\phi}{2} < \Pi_S = \frac{N_S}{\theta_H}\phi\sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}[\theta_H - \sqrt{\theta_H^2 - \frac{2\theta_H C}{N}}]$ as on simplification, this condition gives $C < \frac{4N\theta_H}{9}$ as the sufficient condition, and in this region, $C < \frac{N\theta_H}{4}$. ■

1.3. Effect of consumer risk aversion

Proposition WA5: *If the monopolist uses only pay-per-use pricing or selling, the profits from pay-per-use pricing are higher than those from selling if the psychological cost T and the coefficient of absolute risk aversion (r) are such that $rk^2\theta_H(\phi + 2T)^2 + 8T < 4\phi$. If $T = 0$, the profits from pay-per-use pricing are higher than those from selling if the coefficient of absolute risk aversion (r) is below a threshold value $\frac{4}{k^2\phi\theta_H}$. In contrast, in a duopoly, as before, there exists no Nash equilibrium with positive firm profits where either firm employs both mechanisms, or both firms employ the* *same mechanism. There are two asymmetric pure-strategy Nash equilibria and one unique mixed-strategy Nash equilibrium in the game where two firms offer identical information goods, with one firm employing selling and the other employing pay-per-use. The profits of the seller are always higher than those of the firm that offers pay-per-use pricing. The pay-per-use provider's profits have an inverted-U shape with respect to the psychological cost T, and the pay-per-use provider's profits peak earlier as the coefficient of absolute risk aversion (r) increases. The seller's profits reach the monopoly level for lower levels of the psychological cost T as consumer risk aversion (r) increases.*

Proof of Proposition WA5: We make the following assumptions:

(1) The consumer utility function is characterized by constant absolute risk aversion (CARA), with the coefficient of absolute risk aversion r. Therefore, $U(z) = 1 - e^{-rz}$.

(2) The usage frequency of consumer i, denoted as κ_i, is distributed normally with a mean of θ_i and a variance of $k^2\theta_i^2$. The assumption of a constant coefficient of variation for the usage frequency is useful on multiple fronts. This assumption: (a) allows the moment-generating function of the normal distribution to form a conjugate pair with the exponential utility function, which enables us to get an analytically tractable certainty equivalent of utility; (b) allows us to consider cases where the variance of the usage frequency is high or low; and (c) ensures that consumers with a higher frequency of usage are also associated with a higher variability of usage. We first estimate the effect of risk aversion on the utility from the payments under the selling and pay-per-use mechanisms and derive the corresponding certainty equivalents for the consumer. We then derive and compare the profits of the seller and the pay-per-use provider under the two mechanisms in the monopoly and duopoly contexts.

Derivation of certainty equivalents of consumer utility:

We denote the certainty equivalent of the payments made by consumer i by $CE_{Oi}(p_O, \kappa_i)$. We first estimate the effect of risk aversion on payment-related disutility under the selling and pay-per-use mechanisms and derive the corresponding certainty equivalents. Under pay-per-use, consumers obtain a total disutility of $\kappa_i(p_O + T)$. To obtain the certainty equivalent of the payments made by the consumer ($CE_{Oi}(p_O)$), we note that:

$1 - e^{-rCE_{Oi}(p_O)} = \int_{-\infty}^{\infty}[1 - e^{-\{-r\kappa_i(p_O+T)\}}]f(\kappa_i)d\kappa_i = \int_{-\infty}^{\infty} f(\kappa_i)d\kappa_i - \int_{-\infty}^{\infty} e^{r\kappa_i(p_O+T)}f(\kappa_i)d\kappa_i$

$e^{-rCE_{Oi}(p_O)} = \int_{-\infty}^{\infty} e^{r\kappa_i(p_O+T)}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\kappa_i-\mu)^2}{2\sigma^2}}d\kappa_i$ where $\mu = \theta_i$ and $\sigma = k\theta_i$.

Here, $r\kappa_i - \frac{(\kappa_i-\mu)^2}{2\sigma^2} = -\{\kappa_i - [\mu + r\sigma^2(p_O + T)]\}^2 - \frac{2r\sigma^2(p_O+T)\mu + r^2\sigma^4(p_O+T)^2}{2\sigma^2}$

Therefore, $e^{-rCE_{Oi}(p_O)} = \int_{-\infty}^{\infty} e^{r\mu(p_O+T)+\frac{r^2\sigma^2(p_O+T)^2}{2}}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{\{\kappa_i-(\mu+r\sigma^2(p_O+T))\}^2}{2\sigma^2}}d\kappa_i = e^{r\mu(p_O+T)+\frac{r^2\sigma^2(p_O+T)^2}{2}}$

$\implies -rCE_{Oi}(p_O) = -(p_O + T)\mu - \frac{r\sigma^2(p_O+T)^2}{2}$

Therefore, the certainty equivalent for consumer i under pay-per-use is:

$CE_{Oi}(p_O,\kappa_i) = \phi\theta_i - (p_O+T)\theta_i - \frac{rk^2}{2}(p_O+T)^2\theta_i^2$.

Note that the certainty equivalent that we have derived is similar to the common form $(\mu - r\sigma^2/2)$ which is generalizable across distributions (Pratt 1964).

The selling price is fixed. Therefore, the certainty equivalent for consumer i under selling is:

$CE_{Si}(p_S,\kappa_i) = \phi\theta_i - p_S$

Derivation of firm profits from each mechanism:

Monopoly:

(i) Pay-per-use: As before, we assume that θ_i is uniformly distributed between 0 and θ_H. Since the certainty equivalent of the consumer's utility $CE_{Oi}(p_O,\kappa_i) = \phi\theta_i - (p_O+T)\theta_i - \frac{rk^2}{2}(p_O+T)^2\theta_i^2$ has to be non-negative for consumers to adopt, $CE_{Oi}(p_O,\kappa_i) \geq 0$. This yields the marginal consumer's utility to be:

$\theta_C = \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}$, $CE_{Oi}(p_O,\kappa_i) \geq 0 \forall\theta_i \in [0,\theta_C]$. As before, the firm's profits are denoted by: $\Pi_O(p_O) = \frac{N}{2\theta_H}\theta_C^2 p_O$, where $\frac{\theta_C}{\theta_H}$ is the market share, and the average payment of consumers is given by $\frac{\theta_C p_O}{2}$. The firm solves:

$$\max_{p_O} \frac{N}{2\theta_H}\{\frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}\}^2 p_O$$
$$s.t.\ \ \theta_C = \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2} \leq \theta_H$$

The Lagrangean of this constrained optimization problem is denoted by: $\Lambda = \frac{N}{2\theta_H}\{\frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}\}^2 p_O - \lambda(\frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2} - \theta_H)$.

The Karush-Kuhn-Tucker (KKT) conditions imply that:

$\frac{\partial\Lambda}{\partial p_O} = \frac{2N(\phi-p_O-T)}{\theta_H r^2 k^4}[\{\frac{-(3p_O-T)(\phi-p_O-T)-2p_O(p_O+T)}{(p_O+T)^5}\} + \lambda\{\frac{2(p_O+T)^2+4(p_O+T)(\phi-p_O-T)}{rk^2(p_O+T)^4}\}] = 0$; $\lambda(\theta_H - \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}) = 0$. From the first condition, since $\frac{-(3p_O-T)(\phi-p_O-T)-2p_O(p_O+T)}{(p_O+T)^5}$ is strictly negative, $\lambda \neq 0$, $\theta_C = \theta_H$. Hence, the payment-per-use is given by the following condition: $\theta_H\frac{rk^2}{2}(p_O+T)^2 + p_O + T - \phi = 0$. This yields $p_O^* = \frac{\sqrt{1+2rk^2\phi\theta_H}-1}{rk^2\theta_H} - T$. Since $\theta_C = \theta_H$, the expected profits for the firm are:

$\Pi_O = N\frac{\theta_H}{2}p_O^* = \frac{N\theta_H}{2}(\frac{\sqrt{1+2rk^2\phi\theta_H}-1}{rk^2\theta_H} - T)$

Note: These results with risk aversion are similar to those in the model with only the psychological cost (Table 2 in the paper). The results in Table 2 can be obtained by evaluating the market share and other outcomes described above as $r \to 0$. Specifically, the market share in this case $MS_O^* = 1$ as $r \to 0$, $p_O^* = lim_{r\to 0}\frac{\sqrt{1+2rk^2\phi\theta_H}-1}{rk^2\theta_H} - T \to \frac{1+rk^2\phi\theta_H-1}{rk^2\theta_H} = \phi - T$. Further, this price is lower than $\phi - T$ for any positive r, as detailed below.

To show $\frac{\sqrt{1+2rk^2\phi\theta_H}-1}{rk^2\theta_H} - T < \phi - T \implies$ to show $\sqrt{1+2rk^2\phi\theta_H} < 1 + rk^2\phi\theta_H \implies$ to show $r^2k^4\phi^2\theta_H^2 > 0$, which is true.

Similarly, as $r \to 0$, $\Pi_O = \frac{N\theta_H}{2}(\frac{\sqrt{1+2rk^2\phi\theta_H}-1}{rk^2\theta_H} - T) \to N\frac{(\phi-T)\theta_H}{2}$. Further, these profits are lower than those in Table 2 for any positive r.

(ii) Selling: Under selling, as before, at any given usage utility ϕ, consumers with usage frequencies in the range $\theta \in [\frac{p_S}{\phi D}, \theta_H]$ derive (weakly) positive utility from the purchase.

$MS_S = \frac{1}{\theta_H}[\theta_H - \frac{p_S}{\phi}]$

$\Pi_S = \frac{Np_S}{\theta_H}[\theta_H - \frac{p_S}{\phi}]$

The FOC with respect to p_S yields $p_S = \frac{\phi\theta_H}{2}$, and $\Pi_S = \frac{N\phi\theta_H}{4}$. Setting $\Pi_O > \Pi_S$ yields the condition $rk^2\theta_H(\phi+2T)^2 + 8T < 4\phi$ and setting $T=0$ yields the condition $r < \frac{4}{k^2\phi\theta_H}$. ■

Duopoly:

We first demarcate the market shares of the firms offering the selling and pay-per-use mechanisms. The certainty equivalents of consumers under the two mechanisms are denoted by:

$CE_{Oi}(p_O,\kappa_i) = \phi\theta_i - (p_O+T)\theta_i - \frac{rk^2}{2}(p_O+T)^2\theta_i^2$ and $CE_{Si}(p_S,\kappa_i) = \phi\theta_i - p_S$ respectively. Note that for higher values of θ_i, $CE_{Si}(p_S,\kappa_i) = \phi\theta_i - p_S$ is the larger of the two certainty equivalents.

Therefore, consumers with a lower frequency of usage than the marginal consumer will prefer pay-per-use and consumers with a higher frequency of usage will prefer selling. Equating the two certainty equivalents gives us the average usage frequency of the marginal consumer: $\theta_C = \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)}$.

Seller: The seller's problem is denoted by:

$\underset{p_S}{Max}\ \Pi_S(p_S,p_O) = \frac{Np_S}{\theta_H}[\theta_H - \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)}]$

The FOC for the seller with respect to p_s yields $\theta_H rk^2(p_O+T)+1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}}$.

Pay-per-use: Since $CE_{Oi}(p_O,\kappa_i) = \phi\theta_i - (p_O+T)\theta_i - \frac{rk^2}{2}(p_O+T)^2\theta_i^2 \geq 0$, for consumers to obtain a positive certainty equivalent, the higher limit of θ_C is derived by setting $CE_{Oi}(p_O,\kappa_i) = 0$ which yields $\theta_C \leq \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}$. The firm adopting the pay-per-use mechanism solves the following problem:

$$\max_{p_O} \frac{Np_O}{2\theta_H}[\frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)}]^2$$
$$s.t.\ \ \theta_C = \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)} \leq \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}$$

The Lagrangean of this constrained optimization problem is denoted by: $\Lambda = \frac{Np_O}{2\theta_H}[\frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)}]^2 - \lambda(\frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)} - \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}$.

The Karush-Kuhn-Tucker (KKT) conditions imply that:

$\frac{\partial\Lambda}{\partial p_O} = \frac{N}{2\theta_H r^2k^4}[\frac{\{\sqrt{1+2rk^2p_S}-1\}^2\{(p_O+T)^2-2p_O(p_O+T)\}}{(p_O+T)^4}] - \lambda(\frac{-\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)^2} + \frac{2(p_O+T)^2+4(p_O+T)(\phi-p_O-T)}{rk^2(p_O+T)^4}) = 0$; $\lambda(\frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)} - \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}) = 0$.

From the KKT conditions, since the FOC has a unique solution ($p_O = T$) , when the constraint is non-binding, $\lambda = 0$. The SOC of Π_O with respect to p_O yields a negative value, meaning that $p_O = T$ gives a maximum. When the value of T is high, the constraint becomes tight, and $\frac{2(\phi - p_O - T)}{rk^2(p_O+T)^2} - \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)} = 0$.

Case 1: *T is low* ($0 < T < \frac{(\phi-2T)(6\phi-4T)}{8\theta_H rk^2 T(\phi-T)}$): When T is low, $p_O = T$. Therefore, in this case, from the reaction function of the selling firm, $2\theta_H rk^2 T + 1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}}$.

Comparison of profits:

Note that the two reaction functions of the firms are $\theta_H rk^2(p_O+T)+1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}}$ and $p_O = T$.

We want to prove that:

$\Pi_O = \frac{N}{2\theta_H}\theta_C^2 p_O < \Pi_S = \frac{N}{\theta_H}(\theta_H - \theta_C)p_S$ or to show that
$\{\frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)}\}^2 p_O < 2[\theta_H - \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)}]p_S$. Since $p_O = T$, we have to show that
$\{\frac{\sqrt{1+2rk^2p_S}-1}{2rk^2T}\}^2 T < \{\frac{1+rk^2p_S-1}{2rk^2T}\}^2 T < 2[\theta_H - \frac{\sqrt{1+2rk^2p_S}-1}{2rk^2T}]p_S$, since $\sqrt{1+2rk^2p_S} < 1 + rk^2p_S$. We have to show that

$p_S < 8T[\theta_H - \frac{\sqrt{1+2rk^2p_S}-1}{2rk^2T}]$ or to show that:
$\frac{\sqrt{1+2rk^2p_S}}{2rk^2T} + \frac{p_S}{8T} < \theta_H + \frac{1}{2rk^2T}$, or to show that $\frac{\sqrt{1+2rk^2p_S}}{2rk^2T} + \frac{p_S}{8T} < \frac{1+rk^2p_S}{2rk^2T} + \frac{p_S}{8T} < \theta_H + \frac{1}{2rk^2T}$or to show that

$p_S < \frac{8T\theta_H}{5}$.

Now from the selling firm's reaction function, $2\theta_H rk^2T + 1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}} \implies 1 + 3rk^2p_S < (2\theta_H rk^2T+1)(1+rk^2p_S)$.

$p_S < T\theta_H < \frac{8T\theta_H}{5}$. Therefore, $\Pi_O < \Pi_S$. ■

As can be seen, if only risk aversion is present and the psychological cost (T) is zero, $p_O = p_S = 0$, and there is no non-zero profit equilibrium in the game.

Case 2: *T is high* ($T > \frac{(\phi-2T)(6\phi-4T)}{8\theta_H rk^2T(\phi-T)}$): In this case, the constraint $\frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2} - \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)} = 0$ becomes binding. Hence, the payment-per-use is given by the following condition: $p_O + T = \frac{2\phi}{1+\sqrt{1+2rk^2p_S}}$, which on simplification reduces to $p_S = \frac{2\phi(\phi-p_O-T)}{rk^2(p_O+T)^2}$. As before, the reaction function of the seller is given by: $\theta_H rk^2(p_O+T)+1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}}$.

Comparison of profits:

Note that the two reaction functions $\theta_H rk^2(p_O+T)+1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}}$ and $p_S = \frac{2\phi(\phi-p_O-T)}{rk^2(p_O+T)^2}$ give us a cubic function for the solution of p_O, $\theta_H rk^2(p_o+T) = \frac{(\phi-p_O-T)(6\phi-2p_O-2T)}{p_O(2\phi-p_O-T)}$, which is not possible to solve analytically (to prove this identity, we substitute $p_S = \frac{2\phi(\phi-p_O-T)}{rk^2(p_O+T)^2}$ in the reaction function of the seller $\theta_H rk^2(p_O+T)+1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}}$). The condition $\theta_H rk^2(p_o+T) = \frac{(\phi-p_O-T)(6\phi-2p_O-2T)}{p_O(2\phi-p_O-T)}$ gives us the cutoff psychological cost T between Cases 1 and 2. However, using the two reaction functions, we can show that the selling firm's profits are always higher than that of the firm using the pay-per-use mechanism. Note that $\theta_C = \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)}$. Substituting $p_S = \frac{2\phi(\phi-p_O-T)}{rk^2(p_O+T)^2}$ in this equation yields:

$\theta_C = \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)} = \frac{\sqrt{1+\frac{4\phi(\phi-p_O-T)}{(p_O+T)^2}}-1}{rk^2(p_O+T)} = \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}$

We want to prove that:

$\Pi_O = \frac{N}{2\theta_H}\theta_C^2 p_O < \Pi_S = \frac{N}{\theta_H}(\theta_H - \theta_C)p_S$ or to show that

$\frac{2(\phi-p_O-T)^2p_O}{r^2k^4(p_O+T)^4} < [\theta_H - \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}]\frac{2\phi(\phi-p_O-T)}{rk^2(p_O+T)^2}$ or to show that

$\frac{2(\phi-p_O-T)^2p_O}{r^2k^4(p_O+T)^4} < \frac{2(\phi-p_O-T)^2(p_O+T)}{r^2k^4(p_O+T)^4} < [\theta_H - \frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2}]\frac{2\phi(\phi-p_O-T)}{rk^2(p_O+T)^2}$ or to show that

$\frac{\phi-p_O-T}{rk^2(p_O+T)} < [\frac{\theta_H rk^2(p_O+T)^2-2(\phi-p_O-T)}{rk^2(p_O+T)^2}]\phi$. Note that $\theta_H rk^2(p_o+T) = \frac{(\phi-p_O-T)(6\phi-2p_O-2T)}{p_O(2\phi-p_O-T)}$. Substituting this equation, we want to show that:

$(\phi-p_O-T)(p_O+T) < [\frac{(\phi-p_O-T)(6\phi-2p_O-2T)}{(2\phi-p_O-T)} - 2(\phi-p_O-T)]\phi$ or to show that

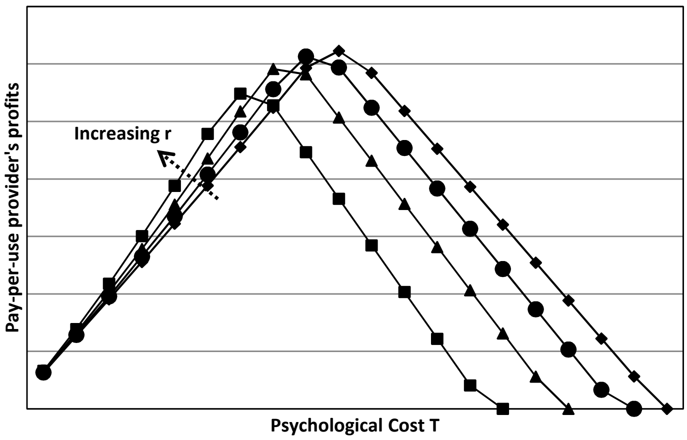
$p_O + T < [\frac{(6\phi-2p_O-2T)}{(2\phi-p_O-T)} - 2]\phi$ or to show that

$p_O + T < \phi$ which is true because $CE_{OC}(p_O,\theta_C) = \phi\theta_C - (p_O+T)\theta_C - rk^2(p_O+T)^2\theta_C^2 > 0$.

■

We have plotted the profits of the pay-per-use provider and the seller in the Figures below (Figures 2 and 3). As in the paper, we have used the following data for the figures: $\theta_H = 5$, $\phi = 1$, and we use $k = 0.5$ (please compare with Figure 5 in the main paper).

Figure 2 Profits for pay-per-use provider in a duopoly



Peak of pay-per-use profits with respect to psychological cost T:

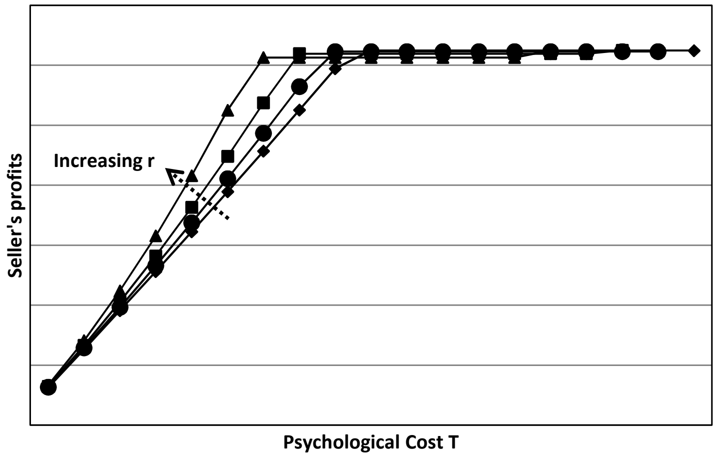
We first show that the pay-per-use provider's providers increase in T when p_O is unconstrained, and decrease in T when p_O is constrained.

Case 1: *T is low* $(0 < T < \frac{(\phi-2T)(6\phi-4T)}{8\theta_H rk^2T(\phi-T)})$: When T is low, $p_O = T$. In this case, $\Pi_O = \frac{N}{2\theta_H}\theta_C^2 p_O$. We have to show $\frac{d\Pi_O}{dT}$ is positive when $p_O = T$.

$\frac{d\Pi_O}{dT} = \frac{N}{2\theta_H}[\theta_C^2 + 2p_O\theta_C\frac{d\theta_C}{dT}] > 0$ iff $\theta_C > -2p_O\frac{d\theta_C}{dT}]$. Note that $\theta_H rk^2(p_O+T)+1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}} \implies \frac{dp_S}{dT} = \frac{2\theta_H(1+2rk^2p_S)^{3/2}}{2+3rk^2p_S}$.

$\theta_C = \frac{\sqrt{1+2rk^2p_S}-1}{rk^2(p_O+T)} \implies \frac{d\theta_C}{dT} = \frac{1}{2rk^2}\frac{\frac{2\theta_H rk^2T(1+2rk^2p_S)}{(2+3rk^2p_S)} - (\sqrt{1+2rk^2p_S}-1)}{T^2}$

Figure 3 Profits for selling in a duopoly



We have to show $\theta_C > -2p_O\frac{d\theta_C}{dT} \implies \sqrt{1+2rk^2p_S}-1 > 2(\sqrt{1+2rk^2p_S}-1) - 4\frac{\theta_H rk^2T(1+2rk^2p_S)}{(2+3rk^2p_S)}$

To show $\sqrt{1+2rk^2p_S} - 1 < 4\frac{\theta_H rk^2T(1+2rk^2p_S)}{(2+3rk^2p_S)}$, or $\sqrt{1+2rk^2p_S} - 1 < 1 + rk^2p_S - 1 < 4\frac{\theta_H rk^2T(1+2rk^2p_S)}{(2+3rk^2p_S)}$

To show $4\theta_H T(1+2rk^2p_S) > p_S(2+3rk^2p_S)$. It is sufficient to show $p_S < 2\theta_H T$.

The seller's reaction function is $\theta_H rk^2(p_O+T)+1 = \frac{1+3rk^2p_S}{\sqrt{1+2rk^2p_S}} > \frac{1+3rk^2p_S}{1+rk^2p_S} = 1 + \frac{2rk^2p_S}{1+rk^2p_S}$

$\implies 2\theta_H rk^2T > \frac{2rk^2p_S}{1+rk^2p_S} > 2rk^2p_S \implies p_S < \theta_H T$ which is stronger than the sufficient condition needed.

Case 2: *T is high* ($T > \frac{(\phi-2T)(6\phi-4T)}{8\theta_H rk^2T(\phi-T)}$): In this case, the constraint $\frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2} = \theta_C$ becomes binding. We have to show $\frac{d\Pi_O}{dT}$ is negative when the constraint is binding.

We have to show $\theta_C < -2p_O\frac{d\theta_C}{dT}\frac{2(\phi-p_O-T)}{rk^2(p_O+T)^2} = \theta_C \implies \frac{d\theta_C}{dT} = \frac{2}{rk^2}\frac{\{-(p_O+T)^2-2(p_O+T)(\phi-p_O-T)\}}{(p_O+T)^4}$.

This reduces to $(3p_O-T)\phi + (p_O+T)(T-p_O) > 0$, which is true as $p_O < T$ in the constrained case.

Hence, the peak of the pay-per-use profits is at $T = \frac{(\phi-2T)(6\phi-4T)}{8\theta_H rk^2T(\phi-T)}$ when the constraint is just reached. At this point, $CE_{Oi}(p_O,\kappa_i) = \phi\theta_i - (p_O+T)\theta_i - \frac{rk^2}{2}(p_O+T)^2\theta_i^2 = 0$ and $CE_{Si}(p_S,\kappa_i) = \phi\theta_i - p_S = 0$. Hence, as r increases, the value of the cutoff psychological cost T ($T = \frac{(\phi-2T)(6\phi-4T)}{8\theta_H rk^2T(\phi-T)}$) decreases. Hence the peak pay-per-use profits Π_O decreases in r, and is reached at lower values of T.

Seller profits with respect to psychological cost T:

When $T = \frac{(\phi-2T)(6\phi-4T)}{8\theta_H rk^2T(\phi-T)}$, $CE_{Si}(p_S,\kappa_i) = \phi\theta_i - p_S = 0$, the seller has access to a monopoly market, and prices at $p_S = \frac{\phi\theta_H}{2}$. As the cutoff value of the psychological cost T is decreasing in r, the seller reaches its monopoly profits at lower values of T as r increases.

Note 2: We have applied the model of risk aversion only for the payments (disutility), and not for the overall utility function for the following reasons. If we derive the certainty equivalent for the pay-per-use mechanism for the entire utility stream of the consumer ($U_O(p_O,\kappa_i) = \kappa_i(\phi - p_O - T)$), we find that:$1 - e^{-rCE_{Oi}(p_O)} = \int_{-\infty}^{\infty}[1 - e^{\{-r\kappa_i(\phi - p_O - T)\}}]f(\kappa_i)d\kappa_i = \int_{-\infty}^{\infty} f(\kappa_i)d\kappa_i - \int_{-\infty}^{\infty} e^{-r\kappa_i(\phi - p_O - T)} f(\kappa_i)d\kappa_i$

$e^{-rCE_{Oi}(p_O)} = \int_{-\infty}^{\infty} e^{-r\kappa_i(\phi - p_O - T)} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\kappa_i - \mu)^2}{2\sigma^2}} d\kappa_i$ where $\mu = \theta_i$ and $\sigma = k\theta_i$.

$\implies rCE_{Oi}(p_O) = \phi\mu - (p_O + T)\mu - \frac{r\sigma^2(\phi - p_O - T)^2}{2}$

Therefore, the certainty equivalent of the utility of the consumer from the pay-per-use mechanism is given by:

$CE_{Oi}(p_O,\kappa_i) = (\phi - p_O - T)\theta_i - \frac{rk^2}{2}(\phi - p_O - T)^2\theta_i^2$. The pay-per-use provider can efficiently extract all surplus by setting $p_O = \phi - T$, which means that the pay-per-use provider is not affected by consumer risk aversion. Conversely, for the seller, ($U_S(p_S,\kappa_i) = \kappa_i\phi - p_S$), we find that $CE_{Si}(p_S,\kappa_i) = \phi\theta_i - \frac{rk^2\phi^2}{2}\theta_i^2 - p_S$, hence the seller is adversely affected by risk aversion. This is conceptually incorrect because it renders selling worse off than the pay-per-use. In our surveys, and as reflected in the literature on the flat rate bias, consumers are risk-averse to variable payments.

References not in the paper

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