Simultaneous or Sequential?
Search Strategies in the U.S. Auto Insurance Industry

- ONLINE APPENDIX -

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Online Appendix A: Sequential Search Model Estimation

We use simulated maximum likelihood (SMLE) to estimate our model. The probability of observing a consumer search a set of companies Υ and purchase from company j is given by

$$P_{ij\Upsilon|\epsilon} = P(\min_{j\in S_i} r_{ij} \geq \max_{j'\notin S_i} r_{ij'} \quad \cap \quad \max_{j\in S_i} u_{ij} \geq u_{ij''}, \max_{j'\notin S_i} r_{ij'} \quad \cap \quad \bigcap_{l=2}^{k} \max_{t<l} \hat{u}_{it} < \hat{r}_{it=l} \quad \forall j'' \in S_i \backslash \{j\}, t = 2, \ldots, k)$$

This probability does not have a closed-form solution and is non-smooth. Since common optimization routines require smoothness, the non-smooth probabilities would either require using non-gradient based optimization methods or taking a very large number of draws (simple frequency simulator, McFadden 1989). Instead, we chose to smooth the probabilities using a scaled multivariate logistic CDF (Gumbel 1961)

$$F\left(w_1, \ldots, w_M; s_1, \ldots, s_M\right) = \frac{1}{1 + \sum_{m=1}^{M} \exp\left(-s_m w_m\right)} \qquad \forall\, m = 1, \ldots, M \tag{A1}$$

where $s_1, \ldots, s_M$ are scaling parameters. McFadden (1989) suggests this kernel-smoothed frequency simulator which satisfies the summing-up condition, i.e. that probabilities sum up to one, and is asymptotically unbiased.

We now describe the step-by-step implementation of the kernel-smoothed frequency simulator.

1. Take $q = 1, \ldots, Q$ draws from ϵ_{ij} (for each consumer/company combination)
2. For each ϵ_{ij} draw, calculate ω_m^q
   (a) $\omega_{1|\epsilon}^q = \min_{j\in S_i} r_{ij} - \max_{j'\notin S_i} r_{ij'}$
   (b) $\omega_{2|\epsilon}^q = \max_{j\in S_i} u_{ij} - u_{ij''}, \max_{j'\notin S_i} r_{ij'}$
   (c) $\omega_{3\ldots M|\epsilon}^q = \bigcap_{l=2}^{T} \max_{t<l} \hat{u}_{it} - \hat{r}_{it=l}$
3. Calculate smoothed search and purchase probabilities using equation (A1)
   $P_{ij\Upsilon|\epsilon}^q = \frac{1}{1+\sum_{m=1}^{M} \exp\left(-s_m w_{m|\epsilon}^q\right)}$
4. Integrate over the distribution of the ϵ_{ij} by averaging the search and purchase probabilities across all Q draws
   $P_{ij|\Upsilon} = \frac{1}{Q} \sum_{q=1}^{Q} P_{ij\Upsilon|\epsilon}^q$

In the estimation, we use a scaling factor of $s_1 = ... = s_M = 5$ and take 100 draws from the error distribution.

Online Appendix B: Proof of Search Method Identification

In Section 3.2, we showed that under simultaneous search, the proportion of below-price expectation price draws in consumers' consideration sets is constant and always equals λ (the probability of getting a below-price expectation price draw) for all consideration set sizes. The search method is identified if it is not possible to observe the same pattern under sequential search. The goal of this appendix is to see whether it is possible that, under sequential search, the proportion of below-price expectation price draws among consumers searching once equals λ - the same pattern as under simultaneous search.

Let r (with some abuse of notation) denote consumers' reservation price for homogeneous goods and consumers' reservation utility for differentiated goods. The proof of identification of the sequential search method depends on whether reservation prices/utilities are constant across consumers and companies, consumer-specific, company-specific or both. Note that all of the commonly used demand specifications in search models fall into one of these cases. More specifically, consumers' reservation prices/utilities are

(1) constant across consumers and companies in models for

- homogeneous goods with a market-wide price distribution and constant search costs

(2) company-specific (but constant across consumers) in models for

- homogeneous goods with company-specific price distributions and constant or company-specific search costs
- homogeneous goods with a market-wide price distribution and company-specific search costs

(3) consumer-specific (but constant across companies) in models for

- homogenous goods with a market-wide price distribution and consumer-specific search costs
- homogeneous goods with consumer-specific price distributions and constant or consumer-specific search costs

(4) consumer- and company-specific in models for

- homogeneous goods with consumer- and company-specific search costs (no matter whether the price distribution is market-wide, consumer-, company-specific or both)
- homogeneous goods with consumer- and company-specific price distributions (no matter whether search costs are constant, consumer-, company-specific or both)
- homogeneous goods with company-specific price distributions and consumer-specific search costs
- homogeneous goods with consumer-specific price distributions and company-specific search costs

- differentiated goods with any price distribution and any search cost specification.

We note two important things: First, while price distributions can be consumer- and/or company-specific, our proof relies on the assumption that the probability of getting a below-price expectation price draw is constant across consumers and companies, i.e. λ *cannot* have an i and/or j subscript. And second, we do not take a stand on whether these price distribution specifications can be an equilibrium outcome. We simply show that the search method is identified should they occur. We first show proof of search method identification for the (homogeneous goods) case where reservation prices are consumer-specific, i.e. case 3. We do not separately show proof for case 1 as it is a straightforward simplification of case 3. Next, we show proof of search method identification for case 4 in which reservation prices/utilities are consumer- and company-specific. We do not separately show proof for case 2 as it is a straightforward simplification of case 4.

Case 3: Consumer-Specific Reservation Prices r_i

Suppose there are $i = 1, ..., N$ consumers and $j = 1, ..., J$ companies in the market. For homogeneous goods, the sequential search model (with perfect recall) is a pure price search model, i.e. $u_{ij} = -p_{ij}$. Prices follow a well-defined market-wide distribution with mean μ^p and with the probability of getting a below-price expectation price draw being λ, i.e. $P(p < \mu^p) = \lambda$. Note that $0 < \lambda < 1$. Consumers have search costs $c_i > 0$ and make k_i searches. We assume that the distribution of search costs across consumers is well-defined and unobserved by the researcher. Consumers have a reservation price r_i. Note that reservation prices r_i vary across consumers due to the (unobserved) heterogeneity in search costs.[1] The researcher does not observe reservation prices r_i. The stopping rule under sequential search requires consumers to stop searching *when and only when* they get a price draw below their reservation price r_i. We classify consumers into one of two types - type A and type B - with $N = N^A + N^B$.[2] Type A (B) consumers have a reservation price that is larger (smaller) than the expected price, i.e. $r^A > \mu$ ($r^B \leq \mu$). Note that consumers do *not* change their type throughout the whole search process as their reservation prices r_i are constant across searches.

[1] The set-up for homogeneous goods with consumer-specific price distributions and constant or consumer-specific search costs is very similar: Prices follow some well-defined distribution with mean μ_i^p and with $P(p < \mu_i) = \lambda$. Search costs are constant or consumer-specific. This results in reservation prices that are consumer-specific. As long as μ_i and λ are observed (which we assume in all specifications), the proof proceeds as above and holds under the same condition.

[2] Our search method identification does not rely on the existence of both consumer types. We proceed through the proof as if both consumer types exist and then discuss at the end what happens if either only type A or only type B consumers exist.

We now derive the proportion of below-price expectation price draws among consumers searching once X_1:[3] Before the 1st search, there are N consumers ready to search.[4] Since prices are randomly and independently drawn, $\lambda\left(N^A+N^B\right)$ consumers draw a below-price expectation price draw:

- All type A consumers who got a below-price expectation price draw, stop searching after the 1st search since any below-price expectation price draw is always smaller than their reservation prices which are larger than the expected price.

- Among the type B consumers who got a below-price expectation price draw, a proportion $0 < \delta_1 \leq 1$ stops searching since their price draws were below their reservation prices which are smaller than the expected price and a proportion $(1-\delta_1)$ continues to search since their price draws were above their reservation prices which are smaller than the expected price.[5]

$(1-\lambda)\left(N^A+N^B\right)$ consumers draw an above-price expectation price draw:

- Among the type A consumers who got an above-price expectation price draw, a proportion $0 \leq \gamma_1 \leq 1$ stops searching since their price draws were below their reservation prices which are larger than the expected price and a proportion $(1-\gamma_1)$ continues to search since their price draws were above their reservation prices which are larger than the expected price.

- Among the type B consumers who got an above-price expectation price draw, all continue searching since any above-price expectation price draw is always larger than their reservation prices which are below the expected price.

Thus the number of people who stop searching after the 1st search is $N_1 = \lambda N^A + (1-\lambda)\gamma_1 N^A + \lambda\delta_1 N^B$. Let us define $w_1^A = \lambda + (1-\lambda)\gamma_1$ and $w_1^B = \lambda\delta_1$. Note that w_1^A and w_1^B denote the proportion of type A and type B consumers who stop searching after the 1st search among all type A and all type B consumers, respectively. Then

$$N_1 = w_1^A N^A + w_1^B N^B \tag{B1}$$

with $0 < w_1^B \leq \lambda \leq w_1^A \leq 1$ following from the assumptions for λ, δ_1, and γ_1.

[3] X_1 always exists in this setting under sequential search, i.e. a positive number of consumers stops searching after the first search. This is so because $c_i > 0\ \forall i$ and thus reservation price $r_i > p_{min}\ \forall i$ for the companies to be searched first (where p_{min} is the minimum price of the market-wide price distribution) and thus the probability that a consumer stops the search process after the 1st search is larger than $0\ \forall i$.

[4] Without loss of generality we assume that all consumers search at least once.

[5] Note that $\delta_1 > 0$. This is so because $c_i > 0\ \forall i$ and thus reservation price $r_i > p_{min}\ \forall i$ for the companies to be searched first (where p_{min} is the minimum price of the market-wide price distribution) and thus the probability that a consumer stops the search process after the 1st search is larger than $0\ \forall i$.

We can write the proportion of below-price expectation price draws among consumers who stop searching after the 1st search, X_1, as a weighted average of the proportions of below-price expectation price draws among type A and among type B consumers weighted by the proportion of type A and type B consumers among those searching once. The proportion of type A consumers among all consumers who search once is $\rho_1 = \frac{N_1^A}{N_1} = \frac{w_1^A N^A}{w_1^A N^A + w_1^B N^B}$. Then the proportion of below-price expectation price draws among consumers searching once is

$$X_1 = \frac{\lambda}{w_1^A}\rho_1 + (1 - \rho_1) \quad \text{(B2)}$$

Note that $\lambda \leq \frac{\lambda}{w_1^A} \leq 1$ of type A consumers who stop after the first search have a below-price expectation price draw and all of type B consumers who stop after the 1st search have a below-price expectation price draw.

Is the Search Method Identified?

Recall that the search method is identified if we can show for at least one consideration set size that we cannot get the same pattern under both simultaneous and sequential search. We showed that, under simultaneous search, the proportion of below-price expectation price draws in consumers' consideration sets is constant and equals λ for all k. Thus the search method is identified if we can show that

$$X_1 \neq \lambda \quad \text{(B3)}$$

Note that $X_1 = \lambda$ *if and only if* $\rho_1 = 1$ <u>and</u> $w_1^A = 1$. If both type A and type B consumers exist, then $\rho_1 < 1$ since $\delta_1 > 0$ and thus $w_1^B > 0$. More intuitively, if both type A and type B consumers exist, a positive number of type B consumers stops searching after the first search and thus $\rho_1 < 1$. If *only* type B consumers exist, then $\rho_1 = 0$ and $X_1 = 1$. If *only* type A consumers exist, then it is possible that $\rho_1 = 1$. In such a case, if only type A consumers exist, $X_1 = \lambda$ can only occur if all type A consumers stop searching after the 1st search, i.e. $\gamma_1 = 1$ (and so $w_1^A = 1$). Thus, since the researcher does not observe whether only type A or only type B or both types of consumers exist, the search method is identified under the necessary condition that there is a positive number of consumers in the data searching more than once. If this condition is satisfied, the proportion of below-price expectation price draws in consumers' consideration sets of size one is always larger than λ under sequential search.

Case 4: Consumer- and Company-Specific Reservation Prices r_{ij}

Since differentiated goods are of particular interest to researchers, we will show search method identification for them. An analogue proof can be derived for the various homogeneous goods specifications discussed under case 4 at the beginning of Online Appendix B.

Suppose there are $i = 1, ..., N$ consumers and $j = 1, ..., J$ companies in the market. Consumers have a utility function such as $u_{ij} = \alpha_{ij} + \beta p_{ij} + X_{ij}\gamma + \epsilon_{ij}$ with $\beta < 0$ and $\epsilon_{ij} \sim iid$. As for homogeneous goods, this is a sequential search model with recall. Prices follow some well-defined distributions with mean μ^p_{ij} and with the probability of getting a below-mean price price draw being λ, i.e. $P\left(p < \mu^p_{ij}\right) = \lambda$. Note that $0 < \lambda < 1$. Consumers have search costs $c_{ij} > 0$ and make k_i searches. In this example, reservation utilities r_{ij} vary across consumers and companies due to consumers' utilities varying across consumers and companies and also due to consumer- and company-specific search costs. Given the assumptions on the price distributions, consumers' utility also follows some well-defined distribution with mean $\mu_{ij} = \alpha_{ij} + \beta\mu^p_{ij} + X_{ij}\gamma + \epsilon_{ij}$. Further, since $P\left(p < \mu^p_{ij}\right) = \lambda$, it must be that $P\left(u > \mu_{ij}\right) = \lambda$, i.e. a below-price expectation price draw always results in an above-mean utility utility.

Using Weitzman's (1979) selection rule, we know that consumers order all alternatives in a decreasing order of their reservation utilities r_{ij}. Consumers first search the alternative with the highest, then the alternative with the second highest etc. reservation utility. To express the ranking according to the reservation utilities r_{ij}, let us define $r_{i,t=1}$ as the company with the highest reservation utility for consumer i, $r_{i,t=2}$ as the company with the second-highest reservation utility for consumer i etc. Using Weitzman's (1979) stopping rule, we know that consumers stop searching when the maximum utility among the searched alternatives is larger than the maximum reservation utility among the non-searched companies.

We now derive the proportion of below-price expectation price draws among consumers searching once X_1:[6] Before the 1st search we can classify consumers into two one of types - type A and type B - with $N = N^{b1,A} + N^{b1,B}$.[7] For before 1st search type A (B) consumers, the reservation utility

[6] Note that X_1 always exists in this setting under sequential search, i.e. a positive number of consumers stops searching after the first search. This is so because $c_{ij} > 0$ $\forall i, \forall j$ and thus reservation utility $r_{i,t=2} < u_{max,t=1}$ $\forall i$ (where $u_{max,t=1}$ is the maximum utility consumer i can get from company $t = 1$) and thus the probability that a consumer stops the search process after the 1st search is larger than 0 $\forall i$.

[7] Note that our search method identification does not rely on the existence of both consumer types. We proceed through the proof as if both consumer types exist and then discuss at the end what happens if either only before 1st search type A or before 1st search type B consumers exist.

of the potentially second-to-be-searched company is smaller (larger) than the expected utility of the company searched first, i.e. $r_{i,t=2} < \mu_{i,t=1}$ ($r_{i,t=2} \geq \mu_{i,t=1}$). Note an important difference compared to the consumer type definitions used in the proof of case 3: In case 3, consumers remained the same type throughout their whole search process. Here, consumers can change their type, i.e. change from being type B to being type A consumers (but not vice versa!). Therefore "$b1$" in $N^{b1,A}$ and $N^{b1,B}$ stands for the number of consumers belonging to type A and type B, respectively, before the 1st search - hence $b1$.

Before the 1st search there are N consumers ready to search.[8] Since prices are randomly and independently drawn, $\lambda\left(N^{b1,A}+N^{b1,B}\right)$ consumers get a utility draw that is above the expected utility of the company searched first (due to a below-price expectation price draw):

- All before 1st search type A consumers who got a utility draw that is above the expected utility of the company searched first, stop searching after the 1st search since any utility draw that is above the expected utility of the company searched first is always larger than the reservation utility of the potentially second-to-be-searched company $r_{i,t=2}$.

- Among the before 1st search type B consumers who got a utility draw that is above the expected utility of the company searched first, a proportion $0 < \delta_1 \leq 1$ stops searching since their utility draws were larger than the reservation utilities of the potentially second-to-be-searched companies $r_{i,t=2}$ and a proportion $(1-\delta_1)$ continues to search since their utility draws were below the reservation prices of the potentially second-to-be-searched companies.[9]

$(1-\lambda)\left(N^{b1,A}+N^{b1,B}\right)$ consumers get a utility draw that is below the expected utility of the company searched first (due to an above-price expectation price draw):

- Among before 1st search the type A consumers who got a utility draw that is below the expected utility of the company searched first, a proportion $0 \leq \gamma_1 \leq 1$ stops searching since the utility draws were larger than the reservation utilities of the potentially second-to-be-searched company $r_{i,t=2}$ and a proportion $(1-\gamma_1)$ continues to search since their utility draws were below the reservation utilities of the potentially second-to-be-searched company.

[8] Without loss of generality we assume that all consumers search at least once.

[9] Note that $\delta_1 > 0$. This is so because $c_{ij} > 0\ \forall i, \forall j$ and thus reservation utility $r_{i,t=2} < u_{max,t=1}\ \forall i$ (where $u_{max,t=1}$ is the maximum utility consumer i can get from company $t=1$) and thus the probability that a consumer stops the search process after the 1st search is larger than 0 $\forall i$.

- Among before 1st search the type B consumers who got a utility draw that is below the expected utility of the company searched first, all continue searching since any utility draw that is below the expected utility of the company searched first is always smaller than the reservation utilities of the potentially second-to-be-searched companies.

Thus the number of consumers who stop searching after the 1st search is $N_1 = \lambda N^{b1,A} + (1-\lambda)\gamma_1 N^{b1,A} + \lambda\delta_1 N^{b1,B}$. Let us define $w_1^A = \lambda + (1-\lambda)\gamma_1$ and $w_1^B = \lambda\delta_1$. Note that w_1^A and w_1^B denote the proportion of before 1st search type A and before 1st search type B consumers who stop searching after the 1st search among all before 1st search type A and before 1st search type B consumers, respectively. Then

$$N_1 = w_1^A N^{b1,A} + w_1^B N^{b1,B} \tag{B4}$$

with $0 < w_1^B \leq \lambda \leq w_1^A \leq 1$ following from the assumptions for λ, δ_1, and γ_1.

We can write the proportion of below-price expectation price draws among consumers who stop searching after the 1st search, X_1, as a weighted average of the proportions of below-price expectation price draws among before 1st search type A and among before 1st search type B consumers weighted by the proportion of before 1st search type A and before 1st search type B consumers among those searching once. The proportion of before 1st search type A consumers among all consumers who search once is $\rho_1 = \frac{N_1^{b1,A}}{N_1} = \frac{w_1^A N^{b1,A}}{w_1^A N^{b1,A} + w_1^B N^{b1,B}}$. Then the proportion of below-price expectation price draws among consumers searching once is

$$X_1 = \frac{\lambda}{w_1^A}\rho_1 + (1-\rho_1) \tag{B5}$$

Note that $\lambda \leq \frac{\lambda}{w_1^A} \leq 1$ of before 1st search type A consumers who stop after the first search have a below-price expectation price draw and all of before 1st search type B consumers who stop after the 1st search have a below-price expectation price draw.

Is the Search Method Identified?

Note that equation (B5) is identical to equation (B2). The same arguments for search method identification can be made and the same condition under which it holds apply for case 4 as for case 3. Note that $X_1 = \lambda$ *if and only if* $\rho_1 = 1$ and $w_1^A = 1$. If both before 1st search type A and before 1st search type B consumers exist, then $\rho_1 < 1$ since $c_{ij} > 0\ \forall i, \forall j$ and thus $w_1^B > 0$. More intuitively, if both before 1st search type A and before 1st search type B consumers exist, a positive number of before

1st search type B consumers stop searching after the first search and thus $\rho_1 < 1$. If *only* (before 1st search) type B consumers exist, then $\rho_1 = 0$ and $X_1 = 1$. If *only* before 1st search type A consumers exist, then it is possible that $\rho_1 = 1$. Then, if only (before 1st search) type A consumers exist, $X_1 = \lambda$ can only occur if all before 1st search type A consumers stop searching after the 1st search, i.e. $\gamma_1 = 1$ (and thus $w_1^A = 1$). Thus the search method is identified under the necessary condition that there is a positive number of consumers in the data searching more than once. If this condition is satisfied, the proportion of below-price expectation price draws in consumers' consideration sets of size one is always larger than λ under sequential search.

Online Appendix C: Proof of Search Method Identification When Some Search Costs are Zero

In this appendix, we relax assumption (f), i.e. that consumers have non-zero search costs, from Section 3.1 of the paper. Here, we show the conditions under which the search method is identified when some consumers have zero search costs for some companies (when search costs are consumer- and company-specific) or some consumers have zero search costs for all companies (when search costs are consumer-specific).

In Section 3.2, we showed that under simultaneous search the proportion of below-price expectation price draws in consumers' consideration sets is constant and always equals λ (the probability of getting a below-price expectation price draw) for all consideration set sizes. This result holds when some (consumer-specific or consumer- and company-specific) search costs are zero. Then the search method is identified if it is not possible to observe the same pattern under sequential search. The goal of this appendix is to see whether it is possible that, under sequential search, the proportion of below-price expectation price draws among consumers searching once equals λ - the same pattern as under simultaneous search.

Let r (with some abuse of notation) denote consumers' reservation price for homogeneous goods and consumers' reservation utility for differentiated goods. The search method identification proof depends on two things: (i) whether reservation prices/utilities are constant across consumers and companies, consumer-specific, company-specific or both and (ii) whether search costs are constant, consumer-specific, company-specific or both. Note that all of the commonly used demand specifications in search models fall into one of these cases. More specifically, consumers' (i) reservation prices/utilities and (ii) search costs are:

(1) both (i) and (ii) are constant across consumers and companies e.g. in models for

- homogeneous goods with a market-wide price distribution and constant search costs

(2a) (i) are company-specific (but constant across consumers) and (ii) are constant across consumers and companies e.g. in models for

- homogeneous goods with company-specific price distributions and constant search costs

(2b) both (i) and (ii) are company-specific (but constant across consumers) e.g. in models for

- homogeneous goods with a market-wide or company-specific price distribution and company-specific search costs

(3) (i) are consumer-specific (but constant across companies) and (ii) are constant or consumer-specific e.g. in models for

- homogenous goods with a market-wide price distribution and consumer-specific search costs
- homogeneous goods with consumer-specific price distributions and constant or consumer-specific search costs

(4a) (i) are consumer- and company-specific and (ii) are constant or consumer-specific e.g. in models

- homogeneous goods with consumer- and company-specific price distributions and constant or consumer-specific search costs
- homogeneous goods with company-specific price distr. and consumer-specific search costs
- differentiated goods with any price distribution and constant or consumer-specific search costs

(4b) (i) are consumer- and company-specific and (ii) are company-specific e.g. in models for

- homogeneous goods with consumer- or consumer- and company-specific price distributions and company-specific search costs
- differentiated goods with any price distribution and company-specific search costs

(5) (i) are consumer- and company-specific and (ii) are consumer- and company-specific e.g. in models for

- homogeneous goods with consumer- and company-specific search costs (no matter whether the price distribution is market-wide, consumer-, company-specific or both)
- differentiated goods with any price distribution and consumer- and company-specific search costs

We note two important things: First, while price distributions can be consumer- and/or company-specific, our proof relies on the assumption that the probability of getting a below-price expectation price draw is constant across consumers and companies, i.e. λ *cannot* have an i and/or j subscript. And second, we do not take a stand on whether these price distribution specifications can be an equilibrium outcome. We simply show that the search method is identified should they occur.

We first show a summary of our search method identification results and the conditions under which they apply in the next section. Then we show detailed proof of search method identification for all cases 1 through 5 in the following sections. For the detailed proofs, we start with the cases where search costs are constant or consumer-specific, i.e. first for case 3, then for case 1 (as it is a simplified version of case 3) and, finally, for case 4a. Then we show proof of search method identification for the cases where search costs are company-specific or consumer- and company-specific, i.e. for cases 2a, 2b, 4b, and 5.

Search Method Identification Summary

Table C-1 summarizes our results for all cases discussed in this appendix. The table includes the conditions necessary for search method identification. The conditions for search method identification can be summarized as follows:

1. The distribution of consideration set sizes has to have at least two mass points i.e. the researcher needs to observe consumers picking two different consideration set sizes in the data (necessary condition).

2. The distribution of consideration set sizes cannot only have two mass points at k=1 and k=J, i.e. consumers cannot only search one or all companies in the market (sufficient but not necessary condition).

Note that the necessary condition that the distribution of consideration set sizes has to have at least two mass points is a straightforward generalization of the necessary condition that the researcher has to observe a positive number of consumers searching more than once (from Section 3.2 and Online Appendix B) to achieve search method identification after accounting for the possibility that some search costs might be zero and thus X_1 might not exist.

If the researcher believes that the data falls under case 5 (consumer- and company-specific reservation prices/utilities and consumer- and company-specific search costs), he needs to be willing to make the additional assumption that a positive number of (before 1st search) type B consumers has non-zero search costs for the companies they search second to be able to identify the search method consumers use. This assumption can be implemented by assuming that the search cost distribution is continuous, i.e. has support, between 0 and A with $A > 0$. Note that we do not require the search cost distribution to be continuous over its whole range. We only require it to be continuous for the interval from 0 to $A > 0$.

	Price Pattern Under Sim. Search	Seq. Search	Identification Condition(s)
Case 1: r, c	$X_k = \lambda$ All consumers pick same cons. set size	$X_{\min(k)} = X_1 > \lambda$	*Necessary condition:* pos. number of consumers searches more than once
Case 2a: r_j, c	$X_k = \lambda$ All consumers pick same cons. set size	$X_{\min(k)} = X_1 > \lambda$	*Necessary condition:* pos. number of consumers searches more than once
Case 2b: r_j, c_j	$X_k = \lambda$ All consumers pick same cons. set size	$X_{\min(k)} > \lambda$	*Necessary condition:* pos. number of consumers searches more than $\min(k)$ times
Case 3: r_j, c or c_i	$X_k = \lambda$	$X_{\min(k)} = X_1 > \lambda$	*Necessary condition:* pos. number of consumers searches more than once *Sufficient but not necessary condition:* cons. set size distribution does not only have mass points at $k = 1$ and $k = J$
Case 4a: r_{ij}, c or c_i	$X_k = \lambda$	$X_{\min(k)} = X_1 > \lambda$	*Necessary condition:* pos. number of consumers searches more than once *Sufficient but not necessary condition:* cons. set size distribution does not only have mass points at $k = 1$ and $k = J$
Case 4b: r_{ij}, c_j	$X_k = \lambda$	$X_{\min(k)} > \lambda$	*Necessary condition:* pos. number of consumers searches more than $\min(k)$ times
Case 5: r_{ij}, c_{ij}	$X_k = \lambda$	$X_{\min(k)} > \lambda$	*Necessary condition:* pos. number of consumers searches more than $\min(k)$ times *Sufficient but not necessary condition:* cons. set size distribution does not only have mass points at $k = \min(k)$ and $k = J$ *Additional assumption:* The search cost distribution is continuous between 0 and $A > 0$.

Table C-1: Search Method Identification Summary

r: Reservation price/utility
c: Consumer search cost
X_l: Proportion of consumers with below-price expectation actual prices in consideration sets of size l
$\min(k)$: Minimum number of searches consumers make
λ: Probability of getting a below-price expectation actual price draw
i denotes consumers
j denotes companies

Case 3: Consumer-Specific Reservation Prices r_i and Consumer-Specific Search Costs c_i

Suppose there are $i = 1, ..., N$ consumers and $j = 1, ..., J$ companies in the market. For homogeneous goods, the sequential search model (with perfect recall) is a pure price search model, i.e. $u_{ij} = -p_{ij}$. Prices follow a well-defined market-wide distribution with mean μ^p and with the probability of getting a below-price expectation price draw being λ, i.e. $P(p < \mu^p) = \lambda$. Note that $0 < \lambda < 1$. Consumers have search costs c_i and make k_i searches. We assume that the distribution of search costs across consumers is well-defined and unobserved by the researcher. Consumers have a reservation price r_i. Note that reservation prices r_i vary across consumers due to the (unobserved) heterogeneity in search costs.[10] The researcher does not observe reservation prices r_i. The stopping rule under sequential search defines that consumers stop searching *when and only when* they get a price draw below their reservation price r_i. We classify consumers into one of two types - type A and type B - with $N = N^A + N^B$.[11] Type A (B) consumers have a reservation price that is larger (smaller) than the expected price, i.e. $r^A > \mu$ ($r^B \leq \mu$). Note that consumers do *not* change their type throughout the whole search process as their reservation prices r_i are constant across searches.

We now derive the proportion of below-price expectation price draws among consumers searching once X_1:[12] Before the 1st search, there are N consumers ready to search.[13] Since prices are randomly and independently drawn, $\lambda\left(N^A + N^B\right)$ consumers draw a below-price expectation price.

- All type A consumers who got a below-price expectation price draw, stop searching after the 1st search since any below-price expectation price draw is always smaller than their reservation prices which are larger than the expected price.

- Among the type B consumers who got a below-price expectation price draw, a proportion $0 \leq \delta_1 \leq 1$ stops searching since their price draws were below their reservation prices which are smaller than the expected price and a proportion $(1 - \delta_1)$ continues to search since their price

[10] Note that the set-up for homogeneous goods with consumer-specific price distributions and constant or consumer-specific search costs is very similar: Prices follow some well-defined distribution with mean μ_i^p and with $P(p < \mu_i) = \lambda$. Search costs are constant or consumer-specific. This results in reservation prices that are consumer-specific. As long as μ_i and λ are observed (which we assume in all specifications), the proof proceeds as above and holds under the same conditions.

[11] Note that our search method identification does not rely on the existence of both consumer types. We proceed through the proof as if both consumer types exist and then discuss at the end what happens if either only type A or only type B consumers exist.

[12] Note that X_1 always exists in this setting under sequential search, i.e. a positive number of consumers stops searching after the first search, unless all consumers have zero search costs. However, the latter, i.e. all consumers having zero search costs, is equivalent to the full information case which is not the subject of study in this paper.

[13] Without loss of generality we assume that all consumers search at least once.

draws were above their reservation prices which are smaller than the expected price.

$(1-\lambda)\left(N^A+N^B\right)$ consumers draw an above-price expectation price:

- Among the type A consumers who got an above-price expectation price draw, a proportion $0 \leq \gamma_1 \leq 1$ stops searching since their price draws were below their reservation prices which are larger than the expected price and a proportion $(1-\gamma_1)$ continues to search since their price draws were above their reservation prices which are larger than the expected price.

- Among the type B consumers who got an above-price expectation price draw, all continue searching since any above-price expectation price draw is always larger than their reservation prices which are below the expected price.

Thus the number of people who stop searching after the 1st search is $N_1 = \lambda N^A + (1-\lambda)\gamma_1 N^A + \lambda\delta_1 N^B$. Let us define $w_1^A = \lambda + (1-\lambda)\gamma_1$ and $w_1^B = \lambda\delta_1$. Note that w_1^A and w_1^B denote the proportion of type A and type B consumers who stop searching after the 1st search among all type A and type B consumers, respectively. Then

$$N_1 = w_1^A N^A + w_1^B N^B \tag{C1}$$

with $0 \leq w_1^B \leq \lambda \leq w_1^A \leq 1$ following from the assumptions for λ, δ_1, and γ_1.

We can write the proportion of below-price expectation price draws among consumers who stop searching after the 1st search, X_1, as a weighted average of the proportions of below-price expectation price draws among type A and among type B consumers weighted by the proportion of type A and type B consumers among those searching once. The proportion of type A consumers among all consumers who search once is $\rho_1 = \frac{N_1^A}{N_1} = \frac{w_1^A N^A}{w_1^A N^A + w_1^B N^B}$. Then the proportion of below-price expectation price draws among consumers searching once is

$$X_1 = \frac{\lambda}{w_1^A}\rho_1 + (1-\rho_1) \tag{C2}$$

Intuitively speaking, $\lambda \leq \frac{\lambda}{w_1^A} \leq 1$ of type A consumers who stop after the first search have a below-price expectation price draw and all of type B consumers who stop after the 1st search have a below-price expectation price draw.

Is the Search Method Identified?

Recall that the search method is identified if we can show for at least one consideration set size that we cannot get the same pattern under both simultaneous and sequential search. We showed that, under simultaneous search, the proportion of below-price expectation price draws in consumers' consideration sets is constant and equals λ for all k. Thus the search method is identified if we can show that

$$X_1 \neq \lambda \tag{C3}$$

Note that $X_1 = \lambda$ *if and only if* $\rho_1 = 1$ and $w_1^A = 1$. Recall that ρ_1 denotes the proportion of type A consumers among all consumers searching once and w_1^A denotes the proportion of type A consumers who stop searching after the 1st search among all type A consumers. Then $X_1 = \lambda$ can only occur when there are only type A consumers among those who search once. This can only happen when (i) there are only type A consumers in the data in general and all type A consumers stop searching after the 1st search, i.e. $N^B = 0$ and $\gamma_1 = 1$ or (ii) all type B consumers search more than once and all type A consumers search exactly once, i.e. $\delta_1 = 0$ and $\gamma_1 = 1$.

To avoid the situation described under (i), the first necessary condition for search method identification is that there is a positive number of consumers in the data searching more than once. To avoid the situation described under (ii), note that (ii) can only happen when search costs for all type B consumers are zero since then $\delta_1 = 0$.[14] When consumers have zero search costs, they search all companies in the market. Thus (ii) can be excluded if we do not observe any consumer searching all the companies in the market, i.e. $k_i \neq J \; \forall j$. Now, for small J, it is quite possible to observe consumers to search all companies in the market. However, note that (ii) requires that both $\delta_1 = 0$ and $\gamma_1 = 1$. This means that, if we observe a positive number of consumers searching all companies in the market, we also need to observe that all consumers who do not search all companies in the market, only search once. To put it differently, the distribution of consideration set sizes cannot only have mass points at $k = 1$ and $k = J \;\; \forall k > 0$.

To summarize, the two conditions for search method identification are as follows:

1. There is a positive number of consumers who search more than once (necessary condition).

2. The distribution of consideration set size cannot only have mass points at $k = 1$ and $k = J$

[14] Reservation price r equals the minimum price of the price distribution p_{min} if search costs are zero. Thus $r > p_{min} \; \forall c_i > 0$ and consequently some consumers will get a price draw below their reservation price r in the first search and stop searching. Thus $\delta_1 > 0$ if $c_i > 0$ for some type B consumers.

$\forall k > 0$ (sufficient but not necessary condition).

If these two conditions are satisfied, the proportion of below-price expectation price draws in consumers' consideration sets of size one is always larger than λ under sequential search.

Case 1: Constant Reservation Prices r and Constant Search Costs c

Suppose prices follow a well-defined market-wide distribution with mean μ^p and with the probability of getting a below-price expectation price draw being λ, i.e. $P(p < \mu^p) = \lambda$. Note that $0 < \lambda < 1$. All consumers have the same search cost c. Then all consumers also have the same reservation price r. Since prices are randomly and independently drawn across consumers and searches, the proportion of consumers who get a below-reservation price price draw is constant across searches, i.e. $\gamma_1 = \gamma_2$ and $\delta_1 = \delta_2$ and following from that $w_1^A = w_2^A = w^A$ and $w_1^B = w_2^B = w^B$. All consumers belong to one type since all consumers have the same reservation price.

Suppose there are only type B consumers in the market, i.e. $N = N^B$ and $\rho_1 = \rho_2 = 0$. Then the proportion of below-price expectation price draws in consumers' consideration sets of size one[15] is

$$X_1 = 1 > \lambda. \tag{C4}$$

Suppose there are only type A consumers in the market, i.e. $N = N^A$ and $\rho_1 = \rho_2 = 1$. Then

$$X_1 = \frac{\lambda}{w^A} \geq \lambda \tag{C5}$$

since $0 \leq w_1^A \leq 1$. Since there can only be one type of consumers in the market - either type A or type B, only the first neccesary condition for search method identification (see case 3) needs to be satisfied, i.e. that there is a positive number of consumers who search more than once.[16] Then, under sequential search, the proportion of below-price expectation price draws in consideration sets of size one is always larger than λ and the sequential search method is identified.

[15] Note that X_1 always exists under sequential search, i.e. a positive number of consumers stops searching after the first search, unless all consumers have zero search costs. However, the latter, i.e. all consumers having zero search costs, is equivalent to the full information case which is not the subject of study in this paper.

[16] The second sufficient but not necessary condition requires that $\delta_1 = 0$, i.e. all type B consumers have zero search costs and thus search all companies in the market. Since type A consumers do not exist, this is the full information case where all consumers have full information on all companies in the market and not the subject of study in this paper.

Case 4a: Consumer- and Company-Specific Reservation Prices r_{ij} and Consumer-Specific Search Costs c_i

Since differentiated goods are of particular interest to researchers, we will show search method identification for them. An analogue proof can be derived for the various homogeneous goods specifications discussed under case 4a at the beginning of Online Appendix C.

Suppose there are $i = 1, ..., N$ consumers and $j = 1, ..., J$ companies in the market. Consumers have a utility function such as $u_{ij} = \alpha_{ij} + \beta p_{ij} + X_{ij}\gamma + \epsilon_{ij}$ with $\beta < 0$ and $\epsilon_{ij} \sim iid$. As for homogeneous goods, this is a sequential search model with recall. Prices follow some well-defined distributions with mean μ^p_{ij} and with the probability of getting a below-mean price price draw being λ, i.e. $P\left(p < \mu^p_{ij}\right) = \lambda$. Note that $0 < \lambda < 1$. Consumers have search costs c_i and make k_i searches. In this example, reservation utilities r_{ij} vary across consumers and companies due to consumers' utilities varying across consumers and companies and also due to consumer-specific search costs. Given the assumptions on the price distributions, consumers' utility also follows some well-defined distribution with mean $\mu_{ij} = \alpha_{ij}+\beta\mu^p_{ij}+X_{ij}\gamma+\epsilon_{ij}$. Further, since $P\left(p < \mu^p_{ij}\right) = \lambda$, it must be that $P\left(u > \mu_{ij}\right) = \lambda$, i.e. a below-price expectation price draw always results in an above-mean utility utility.

Using Weitzman's (1979) selection rule, we know that consumers order all alternatives in a decreasing order of their reservation utilities r_{ij}. Consumers first search the alternative with the highest, then the alternative with the second highest etc. reservation utility. To express the ranking according to the reservation utilities r_{ij}, let us define $r_{i,t=1}$ as the company with the highest reservation utility for consumer i, $r_{i,t=2}$ as the company with the second-highest reservation utility for consumer i etc. Using Weitzman's (1979) stopping rule, we know that consumers stop searching when the maximum utility among the searched alternatives is larger than the maximum reservation utility among the non-searched companies.

Before the 1st search we can classify consumers into two one of types - type A and type B - with $N = N^{b1,A} + N^{b1,B}$.[17] For before 1st search type A (B) consumers, the reservation utility of the potentially second-to-be-searched company is smaller (larger) than the expected utility of the company searched first, i.e. $r_{i,t=2} < \mu_{i,t=1}$ ($r_{i,t=2} \geq \mu_{i,t=1}$). Note an important difference compared to the consumer type definitions used in the proof of cases 1 and 3: In cases 1 and 3, consumers remained

[17] Note that our search method identification does not rely on the existence of both consumer types. We proceed through the proof as if both consumer types exist and then discuss at the end what happens if either only before 1st search type A or only before 1st search type B consumers exist.

the same type throughout their whole search process. Here, as well as later in cases 2a, 2b, 4b, and 5, consumers can change their type, i.e. change from being type B to being type A consumers (but not vice versa!). Therefore "$b1$" in $N^{b1,A}$ and $N^{b1,B}$ stands for the number of consumers belonging to type A and type B, respectively, before the 1st search - hence $b1$.

Before the 1st search there are N consumers ready to search.[18] Since prices are randomly and independently drawn, $\lambda \left(N^{b1,A} + N^{b1,B}\right)$ consumers get a utility draw that is above the expected utility of the company searched first (due to a below-price expectation price draw):

- All before 1st search type A consumers who got a utility draw that is above the expected utility of the company searched first, stop searching after the 1st search since any utility draw that is above the expected utility of the company searched first is always larger than the reservation utility of the potentially second-to-be-searched company $r_{i,t=2}$.

- Among the before 1st search type B consumers who got a utility draw that is above the expected utility of the company searched first, a proportion $0 \leq \delta_1 \leq 1$ stops searching since their utility draws were larger than the reservation utilities of the potentially second-to-be-searched companies $r_{i,t=2}$ and a proportion $(1 - \delta_1)$ continues to search since their utility draws were below the reservation prices of the potentially second-to-be-searched companies.

$(1 - \lambda) \left(N^{b1,A} + N^{b1,B}\right)$ consumers get a utility draw that is below the expected utility of the company searched first (due to an above-price expectation price draw):

- Among the before 1st search type A consumers who got a utility draw that is below the expected utility of the company searched first, a proportion $0 \leq \gamma_1 \leq 1$ stops searching since the utility draws were larger than the reservation utilities of the potentially second-to-be-searched company $r_{i,t=2}$ and a proportion $(1 - \gamma_1)$ continues to search since their utility draws were below the reservation utilities of the potentially second-to-be-searched company.

- Among the before 1st search type B consumers who got a utility draw that is below the expected utility of the company searched first, all continue searching since any utility draw that is below the expected utility of the company searched first is always smaller than the reservation utilities of the potentially second-to-be-searched companies.

[18] Without loss of generality we assume that all consumers search at least once.

Thus the number of consumers who stop searching after the 1st search is $N_1 = \lambda N^{b1,A} + (1-\lambda)\gamma_1 N^{b1,A} + \lambda\delta_1 N^{b1,B}$. Let us define $w_1^A = \lambda + (1-\lambda)\gamma_1$ and $w_1^B = \lambda\delta_1$. Note that w_1^A and w_1^B denote the proportion of before 1st search type A and before 1st search type B consumers who stop searching after the 1st search among all before 1st search type A and before 1st search type B consumers, respectively. Then

$$N_1 = w_1^A N^{b1,A} + w_1^B N^{b1,B} \tag{C6}$$

with $0 \leq w_1^B \leq \lambda \leq w_1^A \leq 1$ following from the assumptions for λ, δ_1,and γ_1.

We can write the proportion of below-price expectation price draws among consumers who stop searching after the 1st search, X_1, as a weighted average of the proportions of below-price expectation price draws among before 1st search type A and among before 1st search type B consumers weighted by the proportion of before 1st search type A and before 1st search type B consumers among those searching once. The proportion of before 1st search type A consumers among all consumers who search once is $\rho_1 = \frac{N_1^{b1,A}}{N_1} = \frac{w_1^A N^{b1,A}}{w_1^A N^{b1,A} + w_1^B N^{b1,B}}$. Then the proportion of below-price expectation price draws among consumers searching once[19] is

$$X_1 = \frac{\lambda}{w_1^A}\rho_1 + (1-\rho_1) \tag{C7}$$

Intuitively speaking, $\lambda \leq \frac{\lambda}{w_1^A} \leq 1$ of before 1st search type A consumers who stop after the first search have a below-price expectation price draw and all of before 1st search type B consumers who stop after the 1st search have a below-price expectation price draw.

Is the Search Method Identified?

Note that equation (C7) is identical to equation (C2). Similar arguments for search method identification and the conditions under which it holds can be made for case 4a as for case 3. Thus the search method is identified under the same two conditions, namely, (1) that there is a positive number of consumers who search more than once (necessary condition) and (2) that the distribution of consideration set sizes does not only have mass points at $k = 1$ and $k = J \ \forall k > 0$ (sufficient but not necessary condition). Under simultaneous search, the proportion of below-price expectation price draws among consumers searching once equals the probability of getting a below-price expectation price draw. Under sequential search, the proportion of below-price expectation price draws among

[19] Note that X_1 always exists in this setting under sequential search, i.e. a positive number of consumers stops searching after the first search, unless all consumers have zero search costs. However, the latter, i.e. all consumers having zero search costs, is equivalent to the full information case which is not the subject of study in this paper.

consumers searching once is larger than the probability of getting a below-price expectation price draw.

Case 2a: Company-Specific Reservation Prices r_j and Constant Search Costs c

Suppose goods are homogenous and prices follow well-defined company-specific distribution with mean μ_j^p and search costs are constant. This results in consumers having company-specific reservation prices r_j. Consumers rank companies in an increasing order of the reservation prices r_j (Weitzman 1979). Since reservation prices are not consumer-specific, all consumers search the same company (with the lowest reservation price) first and all consumers (before the 1st search) belong to either type A or type B. Recall that before 1st search type A (B) consumers have a reservation price for the potentially second-to-be searched company that is larger (smaller) than the expected price of the company to be searched first, i.e. $r_{t=2} > \mu_{t=1}$ ($r_{t=2} \leq \mu_{t=1}$).[20]

Suppose all consumers belong to type A before the 1st search. Then the proportion of below-price expectation price draws in consideration sets of size one is given by equation (C5) and the search method is identified under the necessary condition that there is a positive number of consumers searching more than once.[21]

Suppose all consumers belong to type B before the 1st search.[22] Then the proportion of below-price expectation price draws in consideration sets of size one is given by equation (C4) and is always larger than λ.[23]

To summarize, under the necessary condition that we observe a positive number of consumers searching more than once, under sequential search, the proportion of below-price expectation price draws in consideration sets of size one is always larger than λ and the search method is identified.

Case 2b: Company-Specific Reservation Prices r_j and Company-Specific Search Costs c_j

Suppose goods are homogenous and prices follow a well-defined market-wide distribution with mean μ^p and search costs are company-specific c_j. This results in consumers having company-specific reser-

[20] Note that the inequality signs are flipped compared to case 4a because we talk about reservation prices in case 2a and not about reservation utilities as in case 4a.

[21] Note that all before 1st search type A consumers by definition have search costs larger than zero. Thus X_1 always exists.

[22] Before 1st search type B consumers cannot have search costs of zero since that is equivalent to the full information case in this setting where only one type of consumers exist and not the subject of study in this paper.

[23] Note that X_1 always exists in this setting if search costs are larger than zero.

vation prices r_j.[24] Consumers rank companies in an increasing order of the reservation prices r_j (Weitzman 1979). Since reservation prices are not consumer-specific, all consumers search the same company (with the lowest reservation price) first and all consumers (before the 1st search) belong to either type A and type B. Recall that before 1st search type A (B) consumers have a reservation price for the potentially second-to-be searched company that is larger (smaller) than the expected price of the company to be searched first, i.e. $r_{t=2} > \mu_{t=1}$ ($r_{t=2} \leq \mu_{t=1}$).

Suppose all consumers belong to type A before the 1st search. Then the proportion of below-price expectation price draws among consumers searching once is given by (C5) and the search method is identified under the necessary condition that there is a positive number of consumers searching more than once.[25]

Suppose all consumers belong to type B before the 1st search[26] and suppose before 1st search type B consumers have zero search costs for $\min(k)$ companies where $\min(k)$ denotes the smallest consideration set size chosen by consumers. Then for those $\min(k)$ searches, consumers always continue searching no matter which price draw they get and the proportion of below-price expectation actual prices is λ among those $\min(k)$ searches. Before 1st search type B consumers have positive search costs for the $\min\left((k+1)^{th}\right)$ search and we can classify them as before $\min\left((k+1)^{th}\right)$ search type A or before $\min\left((k+1)^{th}\right)$ search type B consumers. This is so because type B consumers can become type A consumers during the search process, i.e. across searches, but not vice versa (see also discussion of this in case 4a on page 59). Since search costs are not consumer-specific, all consumers are either type A or type B before the $\min\left((k+1)^{th}\right)$ search.

Suppose all consumers are type B before the $\min\left((k+1)^{th}\right)$ search. Then the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is

$$X_{\min(k)} = \frac{1}{\min(k)}\left[\left(\min(k) - 1\right)\lambda + 1\right] \tag{C8}$$

and it is easy to show that $X_{\min(k)} > \lambda$. Note that $\min(k) = 1$ unless consumer search costs for the first *and* second company to be searched are zero for all before 1st search type B consumers.

Suppose all consumers are type A before the $\min\left((k+1)^{th}\right)$ search. Then the proportion of

[24] Note that the set-up for homogeneous goods with company-specific price distributions and company-specific search costs is very similar: Prices follows some distribution $p \sim D_j(\mu_j, \sigma_j)$ with $P(p < \mu_j) = \lambda$. Search costs are company-specific. This results is reservation prices that are company-specific. As long as μ_j and λ are observed by the researcher (which we assume in all specifications), the proof proceeds as above and holds under the same conditions.

[25] Note that all before 1st search type A consumers by definition have search costs larger than zero. Thus X_1 always exists.

[26] Before 1st search type B consumers cannot have search costs of zero for *all* companies in the market since that is equivalent to the full information case and not the subject of study in this paper. However, search costs can be zero for up to $J-1$ companies.

below-price expectation price draws in consideration sets of size $\min(k)$ is

$$X_{\min(k)} = \frac{1}{\min(k)} \left[(\min(k) - 1)\lambda + \frac{\lambda}{w^A_{min(k)}} \right] \tag{C9}$$

and the search method is identified under the necessary condition that a positive number of consumers searches more than $\min(k)$ times.

To summarize, under the necessary condition that we observe a positive number of consumers to search more than $\min(k)$ times, under sequential search, the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is always larger than λ and the search method is identified.

Case 4b: Consumer- and Company-Specific Reservation Prices r_{ij} and Company-Specific Search Costs c_j

Since differentiated goods are of particular interest to researchers, we will show search method identification for them. An analogue proof can be derived for the homogeneous goods specification discussed under case 4b at the beginning of Appendix C.

Note that even though search costs are company-specific, there is variation across consumers in which company is being searched first (as long as $c_j > 0\ \forall j$) because reservation utilities are consumer- and company-specific.[27]

1. Suppose all consumers belong to type A before the 1st search. Then the proportion of below-price expectation price draws among consumers searching once is given by (C5) and the search method is identified under the necessary condition that there is a positive number of consumers searching more than once.[28]

2. Suppose all consumers belong to type B before the 1st search[29] and suppose before 1st search type B consumers have zero search costs for $\min(k)$ companies where $\min(k)$ denotes the smallest consideration set size chosen by consumers.[30] Then for those $\min(k)$ searches, consumers always continue searching no matter which price draw they get and the proportion of below-price expectation actual prices is λ among those $\min(k)$ searches. Before 1st search type B consumers have positive

[27] We do not require $c_j > 0\ \forall j$ for search method identification. We just point to this data pattern.

[28] Note that all before 1st search type A consumers by definition have search costs larger than zero. Thus X_1 always exists.

[29] Before 1st search type B consumers cannot have search costs of zero for *all* companies in the market since that is equivalent to the full information case and not the subject of study in this paper. However, search costs can be zero for up to $J-1$ companies.

[30] Note that $\min(k) = 1$ unless consumer search costs for the first *and* second company to be searched are zero for all before 1st search type B consumers.

search costs $c_j > 0$ for the $\min\left((k+1)^{th}\right)$ search and we can classify them as before $\min\left((k+1)^{th}\right)$ search type A or before $\min\left((k+1)^{th}\right)$ search type B consumers.[31] This is so because type B consumers can become type A consumers during the search process, i.e. across searches, but not vice versa (see also discussion of this in case 2b). Even though search costs are not consumer-specific but because reservations utilities are consumer- and company-specific, both before $\min\left((k+1)^{th}\right)$ search type A and before $\min\left((k+1)^{th}\right)$ search type B consumers can exist.

2a. Suppose all consumers are type B before the $\min\left((k+1)^{th}\right)$ search. Then the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is given by (C8) and it is easy to show that $X_{min(k)} > \lambda$.

2b. Suppose all consumers are type A before the $\min\left((k+1)^{th}\right)$ search. Then the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is given by (C9) and the search method is identified under the necessary condition that a positive number of consumers searches more than $\min(k)$ times.

2c. Suppose both before $\min\left((k+1)^{th}\right)$ search type A and before $\min\left((k+1)^{th}\right)$ search type B consumers exist. Then the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is

$$X_{\min(k)} = \frac{1}{\min(k)}\left[\left(\min(k)-1\right)\lambda + \frac{\lambda}{w^A_{min(k)}}\rho_{\min(k)} + \left(1-\rho_{\min(k)}\right)\right] \tag{C10}$$

Then the search method is identified if there is a positive number of consumers who search more than $\min(k)$ times (necessary condition). Note that we do not need the sufficient but not necessary condition that the distribution of consideration set sizes does not only have mass points at $\min(k)$ and J. This is so because search costs are company-, but not consumer-specific. Thus it must be that search costs $c_j > 0$ if both before $\min\left((k+1)^{th}\right)$ search type A and before $\min\left((k+1)^{th}\right)$ search type B consumers exist. If search cost $c_j = 0$, all consumers search company j and are – by definition – before $\min\left((k+1)^{th}\right)$ search type B consumers.

3. Suppose both before 1st search type A and before 1st search type B consumers exist. Then it must be that $c_j > 0$ because if $c_j = 0$, then all consumers would search company j and would be – by definition – before 1st search type B consumers. If both before 1st search type A and before 1st search type B consumers exist, the proportion of below-price expectation price draws among consumers

[31] Search cost $c_j > 0$ because if $c_j = 0$, then all consumers would search company j and would be – by definition – type B consumers. Search cost must be larger than 0 for at least one company in the market; otherwise we would be in the full information case which is not the subject of this paper.

searching once is given by (C7). Then the search method is identified under the necessary condition that a positive number of consumers searches more than once. Since $c_j > 0$, we do *not* need the sufficient but not necessary condition that the distribution of consideration set sizes does not only have mass points at $k = 1$ and $k = J$.

Putting these different section parts together, we conclude that the search method is identified under the necessary condition that we observe a positive number of consumers to search more than $\min(k)$ times. Then, under sequential search, the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is always larger than λ and the search method is identified. Note that $\min(k) = 1$ unless only before 1st search type B consumers exist and all before 1st search type B consumers have zero search costs for the first *and* second company to be searched.

Case 5: Consumer- and Company-Specific Reservation Prices r_{ij} and Consumer- and Company-Specific Search Costs c_{ij}

Since differentiated goods are of particular interest to researchers, we will show search method identification for them. An analogue proof can be derived for the homogeneous goods specification discussed under case 5 at the beginning of Appendix C.

1. Suppose all consumers belong to type A before the 1st search. Then the proportion of below-price expectation price draws among consumers searching once is given by (C5) and the search method is identified under the necessary condition that there is a positive number of consumers searching more than once.[32]

2. Suppose all consumers belong to type B before the 1st search[33] and suppose before 1st search type B consumers have zero search costs for $\min(k)$ companies where $\min(k)$ denotes the smallest consideration set size chosen by consumers.[34] Then for those $\min(k)$ searches, consumers always continue searching no matter which price draw they get and the proportion of below-price expectation actual prices is λ among those $\min(k)$ searches. A positive number of before 1st search type B consumers have positive search costs for the $\min\left((k+1)^{th}\right)$ search and we can classify them as before $\min\left((k+1)^{th}\right)$ search type A or before $\min\left((k+1)^{th}\right)$ search type B consumers. This is so because type B consumers can become type A consumers during the search process, i.e. across searches, but

[32] Note that all before 1st search type A consumers by definition have search costs larger than zero. Thus X_1 always exists.

[33] Before 1st search type B consumers cannot have search costs of zero for *all* companies in the market since that is equivalent to the full information case and not the subject of study in this paper. However, search costs can be zero for up to $J-1$ companies.

[34] Note that $\min(k) = 1$ unless consumer search costs for the first *and* second company to be searched are zero for all before 1st search type B consumers.

not vice versa (see also discussion of this in case 4b). Because both search costs and reservations utilities are consumer- and company-specific, both before $\min\left((k+1)^{th}\right)$ search type A and before $\min\left((k+1)^{th}\right)$ search type B consumers can exist.

2a. Suppose all consumers are type B before the $\min\left((k+1)^{th}\right)$ search. It must be that a positive number of before $\min\left((k+1)^{th}\right)$ search type B consumers have positive search costs; otherwise all before $\min\left((k+1)^{th}\right)$ search type B consumers and thus all consumers would make the $\min\left((k+1)^{th}\right)$ search. Then the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is given by (C8) and it is easy to show that $X_{min(k)} > \lambda$.

2b. Suppose all consumers are type A before the $\min\left((k+1)^{th}\right)$ search. Then the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is given by (C9) and the search method is identified under the necessary condition that a positive number of consumers searches more than $\min(k)$ times.

2c. Suppose both before $\min\left((k+1)^{th}\right)$ search type A and before $\min\left((k+1)^{th}\right)$ search type B consumers exist. Then the proportion of below-price expectation price draws in consideration sets of size $\min(k)$ is

$$X_{\min(k)} = \frac{1}{\min(k)}\left[(\min(k)-1)\,\lambda + \frac{\lambda}{w^A_{min(k)}}\rho_{\min(k)} + \left(1-\rho_{\min(k)}\right)\right] \tag{C11}$$

To identify the search method, we need the necessary condition that a positive number of consumers searches more than $\min(k)$ times and the sufficient but not necessary condition that the distribution of consideration set sizes does not only have mass points at $k = \min(k)$ and $k = J$.[35]. However, this is not enough. We describe in detail in the "Counterexample" below why this is the case.

3. Suppose before 1st search type A and before 1st search type B consumers exist. Then the proportion of below-price expectation price draws among consumers searching once is given by (C7). To identify the search method, we need the necessary condition that a positive number of consumers searches more than once and the sufficient but not necessary condition that the distribution of consideration set sizes does not only have mass points at $k = 1$ and $k = J$.[36] However, this is not enough. We describe in detail in the next paragraph why this is the case.

[35] The sufficient but not necessary condition rules out a situation where all before $\min\left((k+1)^{th}\right)$ search type A consumers have such high search costs that they all only search $\min(k)$ companies and all before $\min\left((k+1)^{th}\right)$ search type B consumers have zero search costs so that they search all companies in the market.

[36] The sufficient but not necessary condition rules out a situation where all before 1st search type A consumers have such high search costs that they all only search one company and all before 1st search type B consumers have zero search costs so that they search all companies in the market.

Counterexample

Here, we give a simple example why the necessary condition and the sufficient but not necessary condition are not enough to achieve search method identification for case 5:

- Suppose there are three groups of consumers labelled I, II, and III and three companies in the market.

- Consumers in group I have such high search costs for all companies in the market that it is optimal for them to search one company (Note that it is not the same company across consumers within this consumer group. There is variation in the company consumers in group I search due to consumer- and company-specific reservation utilities and consumer- and company-specific search costs.) and stop searching after that first price draw no matter what that price draw is.

- Consumers in group II have zero search costs for the companies they search first and second (Note that these are different companies across consumers within this consumer group. There is variation in the companies consumers in group II search due to consumer- and company-specific reservation utilities and consumer- and company-specific search costs.), but extremely highly search costs for the companies to be searched third so that all consumers in this group stop searching after the second search - no matter which price draw they get in the second search.

- Consumers in group III have zero search costs for all companies in the market. They always search all companies in the market.

Outcome:

- Consumers searching once are all consumers from group I (and only group I). The proportion of below-price expectation price draws among consumers searching once is .5. This is so because consumers from group I stop searching after the first search no matter which price they got in the first search.

- Consumers searching twice are all consumers from group II (and only group II). The proportion of below-price expectation price draws among consumers searching twice is also .5. This is because consumers from group II continue searching after the first search no matter what price they got there and they stop searching after the second search no matter which price they got there.

- Consumers searching three times are all consumers from group III (and only group III). The proportion of below-price expectation price draws among consumers searching three times is also .5. This is because consumers from group III continue searching after the first and second search no matter what price they got there.

Bottom line:

Note that both the necessary condition that there is a positive number of consumers searching more than once and the sufficient but not necessary condition that the distribution of consideration set sizes does not only have mass points at one and three are satisfied in this counterexample. Nevertheless, even though consumers search sequentially, the proportion of below-price expectation price draws equals .5 in all three consideration set sizes - the same pattern that we would expect under simultaneous search.

The basic problem is that the search cost distribution is so "extreme", i.e. consumers in group I have very high search costs, consumers in group III have zero search costs, and consumers in group II have zero search costs for the companies they search first and second, but very high search costs for the company potentially to be searched third, that none of the consumers is groups I, II, and III **react** to price draws. They all either stop searching - no matter what the price draw is - or continue searching - no matter what the price draw is - depending on whether they have extremely high or zero search costs.

In the model presented in Section 3.1 of the paper, we rule out such a situation as described in the Counterexample by assuming that consumers have non-zero search costs. Non-zero search costs prevent such an outcome from happening because if search costs are (marginally) larger than 0, the reservation utilities for the companies to be searched second for consumers in groups II and III are smaller than the maximum utilities for the companies searched first and thus the probability of stopping after the first search is larger than 0 for consumers in groups II and III. Since consumers who have search costs marginally larger than 0 are type B consumers who only stop searching after a below-price expectation price draw, the proportion of below-price expectation price draws among consumers searching once will be larger than .5 (in the Counterexample) when consumers have non-zero search costs.

Here, we want to allow some consumers to have zero search costs. Then, to rule a situation as described in the Counterexample out when some consumers have zero search costs, we need to make the additional assumption that a positive number of (before 1st search) type B consumers has non-zero search costs for the companies they search second to be able to identify the search method consumers use. This assumption can be implemented by assuming that the search cost distribution is continuous, i.e. has support, between 0 and A with $A > 0$. Note that we do not require the search cost distribution to be continuous over its whole range. We only require it to be continuous for the interval from 0 to $A > 0$.

Online Appendix D: Monte Carlo Simulations

We present three sets of results from simulation studies. In the first set, we describe how our analytical identification results discussed in Section 3.2 of the paper allow us to determine the search method employed by consumers. We have two objectives for the second set of simulation studies. First, as a check, we would like to ensure that our estimation algorithms are able to recover the parameters corresponding to the true data generating process (DGP). And second, we want to understand the consequences on estimates, model fit etc. of assuming an incorrect search strategy when estimating the model. In the last set of simulation studies, we evaluate the consequences on estimates, model fit etc. of making incorrect assumptions on the variance(s) of the price distributions and on search costs. Before discussing the first set of simulation results, we provide details on the generated data.

In all simulation studies, we generate data for 1,000 consumers (to mimic the size of the sample in our data). For homogeneous goods, we use the utility function $u_{ij} = -p_{ij}$ and search costs of .6. Prices follow a normal distribution with a mean of 3 and a standard deviation of .5. Data are generated under both search methods. For differentiated goods, consumer i receives utility for company j in the following form

$$u_{ij} = \alpha_j + \beta_1 p_{ij} + \beta_2 adv_{ij} + \epsilon_{ij}$$

The independent variables price and advertising were generated to largely mimic the characteristics of the data in our empirical application with the difference being that we focus on a smaller set of six brands. We pick the true values of the six brand intercepts (-2.0, -1.6, -2.1, -2.4, -1.4, -1.8) to be similar to the brand intercepts of the six largest insurance companies in our empirical application. Prices in our simulation studies are normally distributed with mean prices of .45, .55, .10, .07, -.10, .43 and a standard deviation of 2.00. The choice of mean prices and the standard deviation is again driven by the empirical application. After regressing prices (measured in $100) on a large set of consumer characteristics (see left column in Table **??** in the paper plus the chosen coverage, state and make/class dummies), we find the company-specific price residuals to have means and a standard deviation similar to the one we use in these simulation studies.

The distributions of the advertising data are company-specific with mean advertising levels of 2.7, 4.6, 3.0, 1.9, .5, .6 and standard deviations of 1.2, .75, .25, .65, .2, .5, respectively.[37] When

[37] Note that the advertising spending data in our empirical application was scaled by $10,000,000, i.e. 2.7 reflects a monthly advertising spending level of $27,000,000.

simulating advertising data for consumers, we take consumer-specific draws from the company-specific advertising distributions. We generate this advertising data that is both consumer- and company-specific to mimic the advertising data in our empirical application. There, advertising is also consumer- and company-specific measured through an interaction effect of company-specific advertising spending and consumer-specific advertising recall. We assume that the first search is free to ensure that all consumers participate in the market. The true search cost for all consumers is .3 in terms of utility. We chose the true price coefficient to equal -1.0 so that search costs in terms of dollars are $30 - a similar magnitude to the one found by Honka (2014) for auto insurance. The advertising coefficient was set to .5 in the simulations.

We simulate data under the two assumptions: (1) all consumers search simultaneously and (2) all consumers search sequentially. We generate the data using the following three steps: First, we fix all parameters to their true values, generate the independent variables (consumer- and company-specific prices and advertising) and draws from the error distribution for all 1,000 consumers. Second, for the case when all consumers search simultaneously only, we generate 100 draws from the price distributions (for each consumer/ company combination) to numerically approximate the expected maximum utility among the search companies (see equation (2)) which does not have a closed-form solution. And finally, using the true parameter values, the generated independent variables and error draws, we calculate the optimal behavior for each consumer, i.e. the optimal number of searches, the companies to search and the company to purchase from.

In the estimations, we run 50 replications of each experiment described above, each time using a different set of draws from the error distribution ϵ_{ij}. The reported parameter estimates are the means and standard deviations of the parameter estimates across these 50 replications.

Search Method Identification

In Section 3.2 of the paper, we discuss that the search method consumers are using is identified if the researcher observes a non-zero number of consumers making more than one search in the data. We show here how this condition can be evaluated for differentiated goods.[38] Figure D-1 shows a histogram of the number of search consumers make under simultaneous and sequential search. In the data set generated under simultaneous search, nearly 90 percent of consumers make more than one

[38] A similar histogram of the number of searches consumers are making in the homogeneous goods case reveals that the search method is identified in those cases as well.

search. In the data set generated under sequential search, over 50 percent of consumers make more than one search. Thus the search method is identified.

Next, we show the different data patterns in actual prices in consumers' consideration sets generated by the two search methods. The upper half of Table D-1 shows the results for homogeneous goods. As expected based on our discussion in section 4 of the paper, under simultaneous search, the percentage of below-expected price actual prices in consumers' consideration sets is constant across all consideration set sizes. Since prices follow a normal distribution in our simulation studies, the proportion of below-expected price actual prices in consumers' consideration sets is about 50 percent. By contrast, under sequential search, the percentage of below-expected price actual prices in consumers' consideration sets of size one is 100 percent - clearly much larger than 50 percent which would be expected under simultaneous search. Further, we see that the proportion of below-price expectation actual prices in consumers' consideration sets declines as consideration sets increase in size - a common pattern under sequential search.

The lower half of Table D-1 shows the results for differentiated goods. Under simultaneous search, the percentage of below-expected price actual prices in consumers' consideration sets is again constant and around 50 percent across all consideration set sizes. Under sequential search, the percentage of below-expected price actual prices in consumers' consideration sets of size one is 87 percent - clearly much larger than 50 percent which would be expected under simultaneous search. Further, we again find the common pattern that the proportion of below-price expectation actual prices in consumers' consideration sets declines as consideration sets increase in size.

Other Data Patterns

Next, we briefly describe different data patterns (beyond those described in the previous section) that arise as a consequence of different search methods. We first discuss how consumers' search decisions change when their search method changes and then move on to discuss changes in purchase patterns as a consequence of changes in the search method. 64.50 percent of consumers search a different number of times across the two search methods. The average number of searches is 2.27 under simultaneous and 1.94 under sequential search. This represents a decrease of 14.5 percent in the average number of searches. Figure D-1 shows a histogram of the distributions of searches. Note that the distribution of the number of searches has a longer tail under sequential than under simultaneous search.[39] While at the individual level, a consumer might search the same, more or fewer companies when switching from simultaneous to sequential search, we find that 35.5 percent of consumers search the same number of companies and 47.2 percent (17.3 percent) of consumers search fewer (more) companies under sequential versus simultaneous search. Now, we discuss how the search method influences consumers' purchase decisions. 15.70 percent of consumers change the company they purchase from solely as a consequence of the search method they use (since all the parameters and all generated data are held fixed). Among consumers who consider different sets of companies,[40] 24.3 percent purchase from different companies, while the remainder signs up with the same company under both search methods.

To summarize, keeping all utility parameters, independent variables and error terms the same, the type of search alone has a significant influence on the resulting data patterns in terms of which and how many companies consumers search and which company consumers purchase from.

Known Search Method

Table D-2 displays the estimation results when the true DGP is known to the researcher. Column (i) shows the results when all consumers search simultaneously and column (ii) shows the results when all consumers search sequentially. Generally, our estimation approaches are able to recover consumer preferences and search costs well. In the estimation, we also tried a variety of starting values to assess the sensitivity of our results to starting far away from the true values. We find that starting with values that are e.g. all zero or are several times the magnitude of the true values does not result in different sets of converged values.

Unknown Search Method

As discussed in the Introduction section of the paper, most previous empirical research has made an assumption for the search method consumers use. In Table D-3, we investigate the consequences of assuming the wrong search method on estimates and model fit. In column (i), the true search method consumers use is simultaneous, but the researcher assumes that it is sequential, while in column (ii) the true search method consumers use is sequential, but the researcher assumes that it is simultaneous. In

[39] The maximum number of searches under simultaneous and sequential search is 4 and 6, respectively.

[40] Consumers who consider the same set of companies under both search methods always purchase from the same utility-maximizing company (see equation (**??**) for the simultaneous and equation (**??**) for the sequential search model).

both cases, true consumer preferences are no longer recovered (especially the price coefficient) and the search cost estimates are very close to zero. Comparing the loglikelihoods from column (i) in Table D-2 and column (i) in Table D-3, the loglikelihood is worse on average across all 50 replications as well as *for every individual replication* when the wrong search method is assumed. A similar picture emerges by comparing the loglikelihoods from column (ii) in Table D-2 and column (ii) in Table D-3. These results indicate that imposing the incorrect search method when estimating the model parameters leads to a model fit that is worse than that corresponding to the correct model across all replications. We take this as evidence that the fit statistic is an additional suggestive predictor of the correct search method.[41]

The insights from our discussion on the identification of the search method in section 4.1 of the paper are crucial to understand why the price and search cost coefficients are severely biased downwards under the incorrect search method assumption. In essence, when the data are generated under sequential search, the distribution of consideration set sizes has a longer right tail than it would have under simultaneous search as the true DGP. To accommodate this longer right tail when the model is estimated under simultaneous search, the net benefit of search Γ_{ik} (equation (**??**)) must be larger (for some consumers) to rationalize them having such larger consideration sets. In the estimation, a larger net benefit of search is achieved by a smaller price coefficient which increases the expected maximum utility among the searched companies (and thereby increases the net benefit of search) and by a smaller search cost coefficient which also increases the net benefit of search. This explains the downward bias of both the search cost and price coefficient when the true DGP is sequential search, but the model is estimated under the simultaneous search assumption.

Suppose the true DGP is simultaneous search, but a sequential search model is estimated. Under sequential search, a realtively large proportion of below-price expectation prices would be expected among consumers searching once. To accommodate the relatively small proportion of below-price expectation prices among consumers searching once due to the data being generated under simultaneous search, there must be many consumers with low reservation utilities who are willing to stop searching after getting relatively high prices. This can be rationalized by consumers either having very small search costs or being very insensitive to prices and explains the downward bias of the search cost and price coefficient when the true DGP is simultaneous search, but the model is estimated under the

[41]This fit statistics is only a suggestive predictor of the correct search method since simulations cannot cover the entire space of variables, parameters and all possible functional forms of utilities.

sequential search assumption.

Unequal Variances in Price Distributions

In the empirical sections of the paper, we assume that the variances of the price distributions are constant across companies. Here we explore two issues related to the equal variance assumption: First, we show that we can relax this assumption, i.e. assume company-specific price variances, in the sequential search model and our estimation method is able to recover the true values. And second, we investigate the consequences of the equal variance assumption on the estimates when the data are generated with company-specific price variances under both search methods.

To study the effects of the equal variance assumption, we modify the data generating process described at the beginning of this Appendix C: Instead of assuming that the variance of the price distributions is constant across companies and equals $2^2 = 4$, we assume that the standard deviations of the company-specific price distributions are 1.0, 1.5, 2.0, 2.0, 2.5, 3.0. Note that the price variance across all companies remains 4. Further, to generate the data under simultaneous search, we can no longer rely on the ranking according to the expected indirected utilities. Instead, we simulate all possible consideration sets varying by their size and composition and let the consumer choose the one with the highest benefit net of search costs.

Column (i) in Table D-4 shows that, in a sequential search model, we can recover the true parameter values when the data are generated with company-specific price variances and we assume this as well in the estimation. Columns (ii) and (iii) explore the effects of the equal variance assumption on the estimates when the data was generated with company-specific price variances under both simultaneous and sequential search, respectively. In general, true consumer preferences cannot be recovered in these cases. Under simultaneous search (column (ii)), we find the search cost estimate to be severely downward biased. The search cost estimate is .01 (std. err. .00), while the true search cost parameter is .30. This also holds for search costs in dollars where the true value is $30, while the estimated search costs are $4. Under sequential search (column (iii)), the search cost estimate and the estimates for price and advertising effects also show a downward bias. The search cost estimate is .13 (std. err. .05) or $20.63. At the same time, the model does a reasonable job of recovering the preferences. The bottom line on our results here is that it is critical that the equal variance assumption holds in the data, especially for the simultaneous search model, to have some reassurance regarding the results.

Identical Search Costs Across Companies

In the empirical sections of the paper, we assume that search costs are identical across companies. Here we explore two issues related to the identical search cost assumption: First, we show that we can relax this assumption, i.e. allow for company-specific search costs, in the sequential search model and our estimation method is able to recover the true values. And second, we investigate the consequences of the identical search cost assumption on the estimates when the data was generated with company-specific search costs under both simultaneous and sequential search.

To study the effects of the identical search cost assumption, we modify the data generating process described at the beginning of this Appendix C: Instead of assuming that search costs are constant across companies and equal .3, we assume that the company-specific search costs are .3, .2, .1, .4, .5, .3. Note that the average search costs across all companies remain .3; the average *actual* search costs that consumers incur in the data we generate are .23. Furthermore, to generate the data under simultaneous search, we can no longer rely on the ranking according to the expected indirected utilities. Instead, we simulate all possible consideration sets varying by their size and composition and let the consumer choose the one with the highest benefit net of search costs.

Column (i) in Table D-5 shows that, in a sequential search model, we can recover the true parameter values when the data was generated with company-specific search costs and we assume this as well in the estimation. Columns (ii) and (iii) explore the effects of the identical search cost assumption on the estimates when the data was generated with company-specific search costs under both simultaneous and sequential search, respectively. Under simultaneous search (column (ii)), consumer preferences are incorrectly estimated. The search cost estimate exhibits a severe downward bias. While the true search cost coefficient is .3, the estimated search cost coefficient is .03 (std. err. .00). This also holds for search costs in dollars where the true value is $30, while the estimated search costs are $7.50. Under sequential search (column (iii)), the search cost estimate and the effects of price and advertising exhibit a downward bias, although not as severe as when the incorrect search method is assumed or even when compared to the unequal variances case above. The estimated search cost coefficient is .15 (std. err. .04) or, in terms of dollars, $20.27. At the same time, the model does a reasonable job of recovering consumer preferences. Our results indicate that it is important that the assumption of identical search costs holds in the data, especially for the simultaneous search model, to have some reassurance regarding our results.

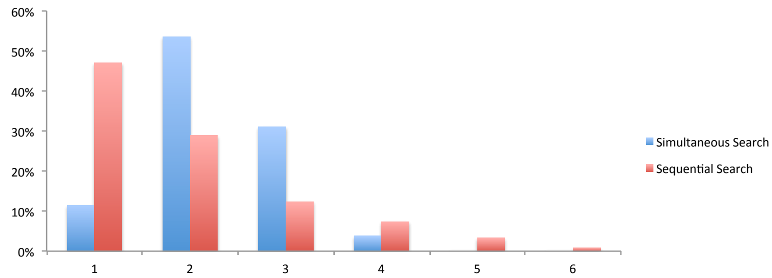

Figure D-1: Number of Searches

	Percentage of Below-Expected Price Actual Prices in Consideration Sets of Size					
	1	2	3	4	5	6
Homogeneous Goods						
Simultaneous Search		50.13	47.47			
Sequential Search	100.00	57.76	38.86	21.72	12.73	5.93
Differentiated Goods						
Simultaneous Search	50.40	50.90	49.80	51.30		
Sequential Search	86.60	49.90	39.80	30.40	27.90	25.90

Table D-1: Results on Search Method Identification

Assumption for ...		(i)		(ii)	
... Data Generation		Simultaneous Search		Sequential Search	
... Estimation		Simultaneous Search		Sequential Search	
	True Values	Means	Std. Error	Means	Std. Error
Brand 1	-2.0	-1.99	0.14	-1.96	0.14
Brand 2	-1.6	-1.72	0.14	-1.61	0.14
Brand 3	-2.1	-2.07	0.13	-2.05	0.13
Brand 4	-2.4	-2.27	0.13	-2.31	0.17
Brand 5	-1.4	-1.59	0.15	-1.44	0.18
Brand 6	-1.8	-1.77	0.15	-1.75	0.15
Advertising	0.5	0.47	0.03	0.45	0.06
Price	-1.0	-0.96	0.04	-0.94	0.08
Search Cost	0.3	0.28	0.03	0.28	0.05
Loglikelihood		-3,545.06		-3,213.38	

Table D-2: Results with Known Search Method

Assumption for ...		(i)		(ii)	
... Data Generation		Simultaneous Search		Sequential Search	
... Estimation		Sequential Search		Simultaneous Search	
	True Values	Means	Std. Error	Means	Std. Error
Brand 1	-2.0	-2.34	1.58	-2.05	0.04
Brand 2	-1.6	-2.19	1.58	-1.85	0.07
Brand 3	-2.1	-2.33	1.58	-1.84	0.04
Brand 4	-2.4	-2.49	1.60	-2.18	0.04
Brand 5	-1.4	-1.73	1.59	-1.32	0.07
Brand 6	-1.8	-2.15	1.60	-2.04	0.07
Advertising	0.5	0.38	0.08	0.35	0.03
Price	-1.0	-0.35	0.03	-0.10	0.00
Search Cost	0.3	0.02	0.01	0.00	0.00
Loglikelihood		-4,452.56		-3,555.97	

Table D-3: Results Under the Wrong Search Method Assumption

Assumptions for ...		(i)		(ii)		(iii)	
... Data Generation							
Price Variances		Unequal		Unequal		Unequal	
Search Method		Sequential		Simultaneous		Sequential	
... Estimation							
Price Variances		Unequal		Equal		Equal	
Search Method		Sequential		Simultaneous		Sequential	
	True Values	Means	Std. Error	Means	Std. Error	Means	Std. Error
Brand Intercept 1	-2.0	-1.88	.17	-2.69	.11	-2.66	.29
Brand Intercept 2	-1.6	-1.62	.13	-2.09	.11	-1.79	.29
Brand Intercept 3	-2.1	-2.05	.14	-2.03	.09	-1.98	.27
Brand Intercept 4	-2.4	-2.27	.15	-2.08	.09	-2.36	.25
Brand Intercept 5	-1.4	-1.53	.14	-1.13	.11	-1.43	.26
Brand Intercept 6	-1.8	-1.83	.13	-1.19	.11	-1.40	.25
Advertising	0.5	.47	.05	.35	.03	.20	.08
Price	-1.0	-.91	.07	-.25	.01	-.62	.07
Search Cost	0.3	.23	.04	.01	.00	.13	.04
Loglikelihood		-3,338.65		-3,792.12		-3,570.45	

Table D-4: Results with Regard to Different Assumptions for Price Variances

Assumptions for ...		(i)		(ii)		(iii)	
... Data Generation							
Search Costs		Company-Specific		Company-Specific		Company-Specific	
Search Method		Sequential Search		Simultaneous Search		Sequential Search	
... Estimation							
Search Costs		Company-Specific		Identical		Identical	
Search Method		Sequential Search		Simultaneous Search		Sequential Search	
	True Values	Means	S.E.	Means	S.E.	Means	S.E.
Brand Intercept 1	-2.0	-2.01	.07	-2.03	.05	-2.04	.17
Brand Intercept 2	-1.6	-1.65	.10	-1.51	.07	-1.68	.24
Brand Intercept 3	-2.1	-2.11	.07	-1.34	.04	-1.54	.19
Brand Intercept 4	-2.4	-2.39	.07	-2.51	.05	-2.47	.19
Brand Intercept 5	-1.4	-1.40	.05	-2.08	.08	-1.70	.26
Brand Intercept 6	-1.8	-1.81	.05	-1.88	.08	-1.75	.19
Advertising	0.5	.38	.05	.35	.03	.47	.07
Price	-1.0	-.90	.05	-.41	.02	-.73	.08
Search Cost 1	0.3	.28	.03	.03	.00	.15	.04
Search Cost 2	0.2	.21	.04				
Search Cost 3	0.1	.11	.02				
Search Cost 4	0.4	.38	.04				
Search Cost 5	0.5	.53	.05				
Search Cost 6	0.3	.30	.03				
Loglikelihood		-3,015.95		-3,345.06		-3,165.00	
BIC		6,128.61		6,752.29		6,386.24	

Table D-5: Results with Regard to Different Assumption for Search Costs

Online Appendix E: Sequential Search Model Estimation Approach for Auto Insurance Data

We present a general estimation approach for the sequential search model in Section 4.2 of the paper. To estimate the sequential search model with insurance data, we need to adapt our estimation approach to a specific setting in the auto insurance industry: Insurance companies send their customers "free" renewal offers. "Free" in this context means that the consumer does not incur any (search) cost to learn the exact price he would have to pay to renew his insurance policy with his previous insurance provider. We incorporate this industry practice in our model by assuming that consumers receive the renewal notice before starting their search and therefore know the price their previous insurance provider is going to charge them to renew the insurance policy beforehand.

Let us define $u_{ij_{PI}}$ as the utility a consumer receives from their previous insurer's renewal offer and C_i as consumer $i's$ consideration set of size $k+1$ containing his previous insurer and the set of companies he searched. Equation (**??**) from the paper can remain as it is. We then need to adapt equations (**??**) - (**??**): First, to reflect the choice and stopping rules, the maximum utility among the searched companies and the renewal offer from the previous insurer have to be larger than any other utility among the considered companies and the maximum reservation utility among the non-considered companies, i.e.

$$\max_{j \in C_i} u_{ij} \geq u_{ij'}, \max_{j'' \notin S_i} r_{ij''} \qquad \forall j' \in C_i \setminus \{j\} \tag{E1}$$

Second, the equations illustrating why it must have been optimal for the consumer *not* to stop searching and purchase earlier given Weitzman's (1979) rules (equation (**??**) in the paper) must be adapted to

$$\bigcap_{l=2}^{k+1} \max_{t<l-1} \left(\hat{u}_{ij_{PI}}, \hat{r}_{it}\right) < \hat{r}_{it=l-1} \tag{E2}$$

The probability of observing a consumer search a set of companies Υ and purchase from company j under sequential search is then given by

$$P_{ij\Upsilon} = P(\min_{j \in S_i} r_{ij} \geq \max_{j' \notin S_i} r_{ij'} \cap \max_{j \in C_i} u_{ij} \geq u_{ij'}, \max_{j'' \notin S_i} r_{ij''} \bigcap_{l=2}^{k+1} \max_{t<l-1} \left(\hat{u}_{ij_{PI}}, \hat{r}_{it}\right) < \hat{r}_{it=l-1} \quad \forall j' \in C_i \backslash \{j\}, t = 2, \ldots, k) \tag{E3}$$

and the loglikelihood of the model is shown in equation (**??**).