Online Appendix: "Is Advance Selling Desirable with Competition?"

Gérard P. Cachon and Pnina Feldman

In this document, we analyze an additional version of Model I in which we allow consumers to purchase from both firms in advance. Allowing the purchase of two units decreases the competition between the two firms in advance–if firms charge a low enough price customers are going to buy from both of them. This happens when consumers are relatively homogeneous. Therefore, it is possible that advance selling dominates spot selling not only when customers are very heterogeneous (large δ) as in Model 1, but also when they are homogeneous or the degree of heterogeneity is low (small δ).

A Model I: Consumers may purchase two units in the advance period

Allowing consumers to purchase two units in advance expands their choice set. A consumer that attaches a probability α for preferring firm 1, now evaluates the expected utility of four different strategies:

1. Buy in advance from both firms, which yields an expected utility of $\mathbb{E}[V] - p_1 - p_2$
2. Buy in advance from firm 1, which yields an expected utility of $\alpha\mathbb{E}[V] - p_1$
3. Buy in advance from firm 2, which yields an expected utility of $(1-\alpha)\,\mathbb{E}[V] - p_2$
4. Wait for the spot and then, if $V = v_h$, buy from the preferred firm, which yields an expected utility of zero.

Comparing the different strategies, we get that consumers purchase from both firms in advance if $p_1 + p_2 \leq \mathbb{E}[V]$ and $\alpha \in (p_1/\mathbb{E}[V], 1 - p_2/\mathbb{E}[V])$. Consumers whose $\alpha > 1 - p_2/\mathbb{E}[V]$, purchase only from firm 1 in advance and those with $\alpha < p_1/\mathbb{E}[V]$ purchase only from firm 2 in advance. That is, compared to the model where customers are restricted to purchase only once, consumer behavior differs when $p_1 + p_2 \leq \mathbb{E}[V]$. It is the same when $p_1 + p_2 > \mathbb{E}[V]$.

Suppose first that customers are homogeneous in advance ($\delta = 0$). The next theorem describes the equilibrium in this case.

Theorem 1. *When $\delta = 0$, the unique set of advance period price equilibrium is $p_1^* = p_2^* = \mathbb{E}[V]/2$ and all consumers buy from both firms in advance. The corresponding revenues, $\Pi_1^* = \Pi_2^* = \mathbb{E}[V]/2$, are greater than the revenues from selling only on the spot.*

Proof. Customers are going to buy from both firms if $p_i \leq \mathbb{E}[V]/2\ \forall i = 1, 2$, buy only from firm i if $p_i \leq \mathbb{E}[V]/2$ and $p_j > \mathbb{E}[V]/2$ and wait if $p_i > \mathbb{E}[V]/2\ \forall i$. Therefore, in equilibrium each firm charges $\mathbb{E}[V]/2$

and sells to all consumers: Charging a lower price will decrease revenues per sale without increasing the total sales. Increasing the price will eliminate sales altogether. The revenue obtained from selling only on the spot is $\Pi_S^* = v_h/4$. Therefore, selling in advance is superior. $\square$

If consumers are completely homogeneous in advance and may purchase from both firms, there is no competition between firms in advance. Each firm is able to charge the monopolist advance period price and sell to all consumers. Therefore, the advance selling result holds in this case and advance selling dominates spot selling.

Next, assume that $\delta > 0$. The revenues listed in Table 1 of the main text remain the same, aside for the revenues in range (a_1, a_2), which, when allowing consumers to purchase from both firms in advance, become $\Pi_i = \frac{p_i}{\delta}\left(\frac{1+\delta}{2} - \frac{p_i}{\mathbb{E}[V]}\right)$ $\forall i$. The next theorem describes the equilibria of the game.

Theorem 2. *Assume $\delta > 0$. Let $\delta_1(\beta) = 4\sqrt{2\,(1+\beta)\,(1+2\beta)} - 5 - 8\beta$ and $\delta_2\,(\beta) = 3 + 8\beta - 2\sqrt{\frac{2(1+2\beta)^3}{1+\beta}}$. Two symmetric price equilibria are possible:*

1. *If $\delta \leq \delta_1\,(\beta)$, the firms charge $p_1^l = p_2^l = (1+\delta)\,\mathbb{E}[V]/4$ in advance and all customers purchase in advance. If $\delta < 1/3$, all customers purchase from both firms. Otherwise, customers with $\alpha > (3-\delta)/4$ purchase only from firm one, customers with $\alpha < (1+\delta)/4$ purchase only from firm 2, and the rest purchase from both firms in advance.*

2. *If $\delta \geq \delta_2(\beta)$, the firms charge*
$$p_1^h = p_2^h = \frac{(1+\delta)\,(1+\beta)\,\mathbb{E}[V]}{2\,(1+2\beta)}.$$
Furthermore,
$$\bar{\alpha} = 1 - \bar{\alpha} = \frac{(1+\delta)\,(1+\beta)}{2\,(1+2\beta)} < 1.$$
Customers with $\alpha > \bar{\alpha}$ buy from in advance from firm 1, those with $\alpha < \underline{\alpha}$ buy in advance from firm 2 and the rest wait for the spot.

Proof. The steps for the proof are similar to the proof of Theorem 1 of the main text. There are four candidates for a symmetric price equilibrium. The first candidate is a set of prices which results in all customers buying in advance, some from both firms. The second candidate is a set of prices which results in only a fraction of consumers buying in advance from one of the two firms and the rest wait for the spot. The third candidate falls in the boundary: half of the customers buys in advance from firm 1 and the other half buys from firm 2. None buys from both firms and none waits for the spot. The fourth candidate is the spot selling equilibrium where both firms charge a high enough advance price, so that all customers wait. We check under which conditions these prices are sustainable in equilibrium. In checking for profitable deviations, we will focus on firm 1. The behavior of firm 2 is identical due to the symmetry of the game.

(i) The (a_1, a_2) range: in this range, $\Pi_1\,(p_1) = \frac{p_1}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1}{\mathbb{E}[V]}\right)$, which is strictly concave and is maximized at p_1^l. Thus, the only interior equilibrium candidate in this range is (p_1^l, p_2^l). The equilibrium profits in this range are: $\Pi_i^*\left(p_i^l\right) = (1+\delta)^2\,\mathbb{E}[V]/(16\delta)$. To show that it is an equilibrium, it remains to check for which parameter values these prices fall in the range and whether there are no profitable deviations. If $\delta \leq 1/3$, p_1^h fall below $(1-\delta)\mathbb{E}[V]/2$. Given p_2^l, the only possible deviation is to increase the price so that only firm 2 sells, which is definitely not profitable. Therefore, (p_1^l, p_2^l) is an equilibrium in this range. If $\delta > 1/3$, p_1^l falls in $((1-\delta)\,\mathbb{E}[V]/2, (1+\delta)\,\mathbb{E}[V]/2)$. The only possible profitable deviation is to increase the price to p_1^h, if it falls in (a_1, a_2, a_s) range and earns higher profits. p_1^h falls in $\left(\mathbb{E}[V] - p_2^l, (1+\delta)\,\mathbb{E}[V]/2\right)$ if $\delta > (v_h + 4v_l)\,/\,(3v_h/4v_l)$. In this case, firm 1's revenue from deviating is:

$$\Pi_1\left(p_1^h; p_2^l\right) = \frac{p_1^h}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1^h}{\mathbb{E}[V]}\right) + \frac{v_h}{2\delta}\int_{1-\frac{p_2^l}{\mathbb{E}[V]}}^{\frac{p_1^h}{\mathbb{E}[V]}} \alpha d\alpha.$$

It dominates $\Pi_1^*\left(p_1^l\right)$ if $\delta > \delta'(\beta)$. Therefore,(p_1^l, p_2^l) is an equilibrium otherwise.

(ii) The (a_1, a_2, a_s) range: As in the base model, in this range, the revenue function is maximized at p_1^h. For the high price to be an equilibrium it must fall in $\left(\mathbb{E}[V] - p_2^h, (1+\delta)\,\mathbb{E}[V]/2\right)$. It does $\forall\delta > v_l/\,(v_l + v_h)$. In addition, for the high price to be an equilibrium, there shouldn't be any profitable deviation. The only possible profitable deviation may be to p_1^l. This is only possible if the low price falls in (a_1, a_2), i.e., if $p_1^l < \mathbb{E}[V] - p_2^h$. This happens if $\delta \in \left(\frac{v_l}{v_l+v_h}, \frac{v_h+4v_l}{3v_h+4v_l}\right)$. Furthermore, for the deviation to be profitable, the revenue obtained from deviating should be higher. This revenue is given by:

$$\Pi_1\left(p_1^l\right) = \frac{(1+\delta)^2\,\mathbb{E}[V]}{16\delta},$$

which is higher than the high-price equilibrium profit if $\delta \in \left(\frac{v_l}{v_l+v_h}, \delta_2(\beta)\right)$. Therefore, $\left(p_1^h, p_2^h\right)$ is an equilibrium if $\delta > \delta_2(\beta)$.

(iii) $p_1 = p_2 = \mathbb{E}[V]/2$: The revenue in this case is $\mathbb{E}[V]/4$. For this to be an equilibrium, we must have $(1+\delta)\,\mathbb{E}^2[V]/\,(v_h + 2v_l) < \mathbb{E}[V]/2 < (1+\delta)\,\mathbb{E}[V]/4$, which never holds. Therefore $p_1 = p_2 = \mathbb{E}[V]/2$ is never an equilibrium.

(iv) Spot selling: For both firms to sell in the spot the candidate symmetric equilibrium is every pair of prices that satisfy $p_1 = p_2 > (1+\delta)\,\mathbb{E}[V]/2$. This yields revenue of $v_h/4$ for each firm. This, however is not an equilibrium. Given that sets a high advance price, the other benefits from deviating by setting a lower price, p', such that:

$$p' = \max\left\{\frac{1+\delta}{2}\mathbb{E}[V], \frac{1+\delta}{(v_h + 2v_l)}\mathbb{E}^2[V]\right\}.$$

□

Similarly to the original model, here too there are two types of symmetric equilibria. One in which the firms sell to all consumers in advance with some consumers purchasing from both firms and the other in which some consumers purchase in advance from either firm 1 or firm 2 and others wait for the spot period. The latter equilibrium is the same as the one in the original model.

As in the original model, $\delta_1(\beta) > \delta_2(\beta)$, implying that an equilibrium always exists, but is not necessarily unique. Comparing the profit functions in the range $\delta \in (\delta_2(\beta), \delta_1(\beta))$, we find that equilibrium (ii) Pareto dominates equilibrium (i) in that range. Finally, Corollary 1 shows that firms, in most cases, would benefit if they could commit not to sell in advance.

Corollary 1. *Firms' revenues from selling only on the spot, $v_h/4$, are strictly higher than the revenue obtained from selling at least partly in advance if $\delta'(\beta) < \delta < \delta''(\beta)$, where*

$$\delta' = \frac{3 - \beta - 2\sqrt{2\,(1-\beta)}}{1+\beta}$$

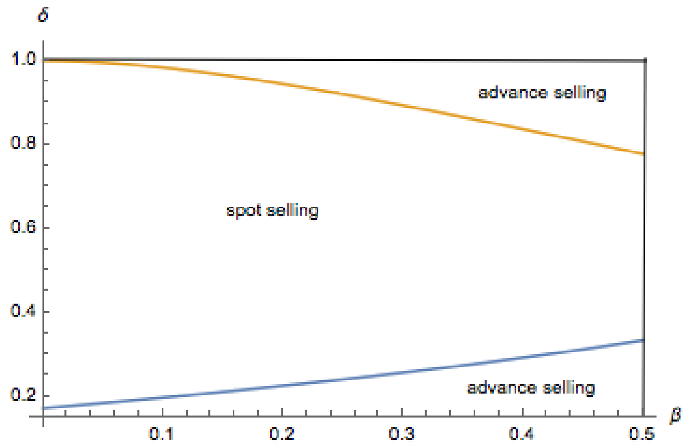

Figure 1. Areas in the (β, δ) parameter space for which selling only on the spot is preferred to the revenue obtained in the advance selling equilibrium.

and

$$\delta'' = \frac{1 + 2\beta - \beta^2}{(1+\beta)^2}$$

Corollary 1 demonstrates that in most cases the possibility of advance selling ends up hurting firms under competition, even when consumers may purchase from both firms in advance. Figure 1 illustrates the ranges where advance selling / spot selling dominates. The range $\delta > \delta''$ is similar to the range in the original model, where because of the high level of heterogeneity among customers, advance selling is better, because there is essentially no competition between firms. The range $\delta < \delta'$ is new. Here, advance selling is better because consumers end up buying from both firms in advance, again, limiting the level of competition. Still, in most cases, $\delta' < \delta < \delta''$ implying that firms would benefit if they were able to commit to sell only on the spot, even though spot selling alone is never an equilibrium.