Supplementary Appendix A (Backward Induction Analysis)

In this supplementary appendix, we present the detailed two-period backward induction analysis of the game.

Define

$p_0 := k_C + \frac{\lambda}{2}(1-\alpha)$

$\pi_0 := k_C + \frac{\lambda}{2}(1-\alpha) - k_I$

$p_1 := \frac{k_I + 2(1-q) + k_C + \frac{\lambda}{2}(1-\alpha)}{2}$

$\pi_1 := \frac{1}{2(1-q)}(\frac{2(1-q)+k_C+\frac{\lambda}{2}(1-\alpha)-k_I}{2})^2$

$p_2 := 2(1-q) + k_C + \frac{\lambda}{2}(1-\alpha)$

$p_5 := 1 - q + \frac{\lambda}{2}$

$p_3 := \frac{\lambda}{2}(1+\alpha) - k_C$

$\pi_3 := \frac{1}{1-q}(1 - q - \frac{\lambda\alpha}{2} + k_C)(\frac{\lambda}{2}(1+\alpha) - k_C - k_I)$

$p_4 := \frac{k_I + (1-q) + \frac{\lambda}{2}}{2}$

$\pi_4 := \frac{1}{1-q}(\frac{1-q+\frac{\lambda}{2}-k_I}{2})^2$

$p_6 := \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)$

$\pi_6 := \frac{1}{1-q}(1 - q - \frac{1}{q}(\frac{\lambda\alpha}{2} - k_C))(\frac{1}{q}(\frac{\lambda}{2}(q+\alpha) - k_C) - k_I)$

$p_7 := \frac{k_I + \frac{2(1-q)-\frac{\lambda\alpha}{2}+k_C}{2-q} + \frac{\lambda}{2}}{2}$

$\pi_7 := \frac{2-q}{2(1-q)}(\frac{\frac{2(1-q)-\frac{\lambda\alpha}{2}+k_C}{2-q}+\frac{\lambda}{2}-k_I}{2})^2$

$p_8 := \frac{1}{2-q}(2(1-q) + k_C - \frac{\lambda}{2}(\alpha+q) + \lambda)$

$p'_0 := \frac{1}{q}(k_C - \frac{\lambda}{2}(\alpha - q))$

$l_1 := k_C + \frac{\lambda}{2}(1-\alpha) - 2(1-q)$

$l_2 := 2(1-q) + k_C + \frac{\lambda}{2}(1-\alpha)$

$l_3 := -3k_C + \frac{3\lambda\alpha}{2} + \frac{\lambda}{2} - 2(1-q)$

$l_4 := \frac{\lambda\alpha}{2} - (1-q)$

$l_5 := \frac{\lambda}{2} + \lambda\alpha - 2k_C - (1-q)$

$l_6 := 1 - q + \frac{\lambda}{2}$

$l_7 := \frac{\lambda\alpha}{q} - \frac{2k_C}{q} + \frac{\lambda}{2} - (1-q)$

$l_8 := \frac{\lambda\alpha}{2} - q(1-q)$

$l_9 := \frac{\lambda}{2} + \frac{1}{2-q}(\frac{4-q}{q}(\frac{\lambda\alpha}{2} - k_C) - 2(1-q))$

$l_{10} := \frac{1}{2-q}(2(1-q) - \frac{\lambda\alpha}{2} + k_C) + \frac{\lambda}{2}$

$l'_{10} := \frac{\lambda}{2} + \frac{1}{2-q}(\frac{4-3q}{q}(\frac{\lambda\alpha}{2} - k_C) - 2(1-q))$

Case 1. $\underline{k_C \le \frac{\lambda\alpha}{2}}$

Applying backward induction, we first consider Period 2.

Period 2

In Appendix section A, we already derived the result when for Subcase 1 ($\tau_C \le 0$).

Subcase 2: $\tau_C \le 0$. $\tau_C \le 0$ is equivalent to $p_C \ge \frac{\lambda\alpha}{2}$. In order for both I and C staying in the market, they should both have nonnegative demands, $\tau_C \le \tau_I \le 1$. In Period 2, C must ensure $\tau_I \ge \tau_C$ in order to enter the market, which is equivalent to $p_C \le qp_I - \frac{\lambda}{2}(q-\alpha)$. C maximizes its profit with respect to price p_C, given p_I.

$$\begin{aligned} \max \quad & \pi_C(p_I) = (p_C - k_C))(\tau_I - \tau_C) \\ \text{s. t.} \quad & p_C \ge \frac{\lambda\alpha}{2} \\ & p_C \le qp_I - \frac{\lambda}{2}(q-\alpha) \end{aligned}$$

The feasible region is not empty when $qp_I - \frac{\lambda}{2}(q-\alpha) \ge \frac{\lambda\alpha}{2}$, which is equivalent to $p_I \ge \frac{\lambda}{2}$. We have the optimal price

$$p_C^*(p_I) = \begin{cases} \frac{\lambda\alpha}{2}, & \text{if } p_I \in [\frac{\lambda}{2}, \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)] \\ \frac{k_C + qp_I - \frac{\lambda}{2}(q-\alpha)}{2}, & \text{if } p_I \ge \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C) \end{cases}$$

Combine the two subcases, we have the optimal response price for C and profits as follows:

$$p_C^*(p_I) = \begin{cases} p_I - \frac{\lambda}{2}(1-\alpha), & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)] \\ \frac{k_C + p_I - \frac{\lambda}{2}(1-\alpha)}{2}, & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{\lambda}{2}(1+\alpha) - k_C] \\ \frac{\lambda\alpha}{2}, & \text{if } p_I \in [\frac{\lambda}{2}(1+\alpha) - k_C, \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)] \\ \frac{k_C + qp_I - \frac{\lambda}{2}(q-\alpha)}{2}, & \text{if } p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C) \end{cases}$$

$$\pi_C^*(p_I) = \begin{cases} 0, & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)] \\ \frac{1}{4(1-q)}(p_I - \frac{\lambda}{2}(1-\alpha) - k_C)^2, & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{\lambda}{2}(1+\alpha) - k_C] \\ \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)(p_I - \frac{\lambda}{2}), & \text{if } p_I \in [\frac{\lambda}{2}(1+\alpha) - k_C, \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)] \\ \frac{1}{4q(1-q)}(qp_I - \frac{\lambda}{2}(q-\alpha) - k_C)^2, & \text{if } p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C) \end{cases}$$

Hence, there are four scenarios depending on I's price in Period 1.

Period 1 In Period 1, I maximizes its profit with respect to price p_I. I should also ensure $\tau_I \leq 1$ to have nonnegative demand. This condition is equivalent to $p_I \leq 1 - q + p_C^*(p_I) + \frac{\lambda}{2}(1-\alpha)$

Analysis of Scenario 1.1 and Scenario 1.2 are given in Appendix A.

Scenario 1.3**:** $p_I \in [\frac{\lambda}{2}(1+\alpha) - k_C, \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)]$. In this scenario, $\tau_C = 0$ and $\tau_I > 0$. $p_C^*(p_I) = \frac{\lambda\alpha}{2}$. $\tau_I \leq 1$ implies $p_I \leq (1-q) + \frac{\lambda}{2}$. I maximizes its own profit function,

$$\begin{aligned} \max \quad & \pi_I = (p_I - k_I)(1 - \tau_I(p_I)) \\ \text{s. t.} \quad & p_I \leq (1-q) + \frac{\lambda}{2} \\ & p_I \leq \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C) \\ & p_I \geq \frac{\lambda}{2}(1+\alpha) - k_C \end{aligned}$$

We have the optimal price for I, $p_I^* = \min(p_4, p_5, p_6)$ and $p_I^* \geq p_3$

1.3.1 When $l_5 \leq k_I \leq \min(l_6, l_7)$, $p_I^* = p_4$, $\pi_I^* = \pi_4$.

1.3.2 When $l_4 \leq k_C \leq l_8, k_I \geq l_6$, $p_I^* = p_5, \pi_I^* = 0$.

1.3.3 When $k_C \geq l_8, k_I \geq l_7$, $p_I^* = p_6$, $\pi_I^* = \pi_6$.

1.3.4 When $k_I \leq l_5$, $p_I^* = p_3, \pi_I^* = \pi_3$

Scenario 1.4**:** $p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C)$. In this scenario, $\tau_C > 0$ and $\tau_I > \tau_C$. $p_C^*(p_I) = \frac{k_C + qp_I + \frac{\lambda}{2}(\alpha-q)}{2}$. $\tau_I \leq 1$ implies $p_I \leq \frac{1}{2-q}(2(1-q) + k_C - \frac{\lambda}{2}(\alpha+q) + \lambda)$. I maximizes its own profit function,

$$\begin{aligned} \max \quad & \pi_I = (p_I - k_I)(1 - \tau_I(p_I)) \\ \text{s. t.} \quad & p_I \leq \frac{1}{2-q}(2(1-q) + k_C - \frac{\lambda}{2}(\alpha+q) + \lambda) \\ & p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha+q) - k_C) \end{aligned}$$

We have the optimal price for I, $p_I^* = \min(p_7, p_8)$ and $p_I^* \geq p_6$.

1.4.1 When $l_9 \le k_I \le l_{10}$, $p_I^* = p_7$, $\pi_I^* = \pi_7$.

1.4.2 When $k_I \ge l_{10}$,$p_I^* = p_8$, $\pi_I^* = 0$.

1.4.3 When $k_I \le l_{10}$,$p_I^* = p_6$, $\pi_I^* = \pi_6$.

Combine those 4 scenarios, profits sharing the same valid regions are compared such that the price with the largest profit is selected as the optimal price in that valid region. We obtain following result:

1. When $k_I \in [k_C, l_1]$, $k_C \le \frac{\lambda\alpha}{2}$. $p_I = p_0$, $\pi_I = \pi_0$. C will not enter the market.
2. When $k_I \in [l_1, \min(l_2, l_3)]$, C enters the market. $p_I = p_1$, $\pi_I = \pi_1$. $p_C = \frac{k_I + 3k_C - \frac{\lambda}{2}(1-\alpha) + 2(1-q)}{4}$, $\pi_C = \frac{1}{1-q}(\frac{k_I - k_C - \frac{\lambda}{2}(1-\alpha)+2(1-q)}{4})^2$.
3. When $k_I \in [l_3, l_5]$, $k_C \in [l_4, \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = p_3$, $\pi_I = \pi_3$.$p_C = \frac{\lambda\alpha}{2}$, $\pi_C = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)^2$.
4. When $k_I \in [l_5, \min(l_6, l_9)]$, $k_C \in [l_4, \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = p_4$, $\pi_I = \pi_4$. $p_C = \frac{\lambda\alpha}{2}$, $\pi_C = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)(\frac{k_I - \frac{\lambda}{2} + (1-q)}{2})$.
5. When $k_I \in [\max(l_5, l_9), l_{10}]$, $k_C \in [l_8, \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = p_7$, $\pi_I = \pi_7$. $p_C = \frac{q}{4}k_I - \frac{\lambda q}{8} + \frac{2q(1-q)+(4-q)k_C+(4-3q)\frac{\lambda\alpha}{2}}{4(2-q)}$, $\pi_C = \frac{q}{1-q}(\frac{k_I - \frac{\lambda}{2} + \frac{2(1-q)}{2-q} + \frac{4-3q}{q(2-q)}(\frac{\lambda\alpha}{2} - k_C)}{4})^2$.

Case 2. $\underline{k_C > \frac{\lambda\alpha}{2}}$

We have presented the whole analysis of the case $k_C > \frac{\lambda\alpha}{2}$. in Appendix A. The result is as follows.

When $k_C \le k_I \le l_{10}$, $1 \ge k_C > \frac{\lambda\alpha}{2}$, C enters the market. $p_I^* = p_7$, $\pi_I^* = \pi_7$, $p_C = \frac{q}{4}k_I - \frac{\lambda q}{8} + \frac{2q(1-q)+(4-q)k_C+(4-3q)\frac{\lambda\alpha}{2}}{4(2-q)}$, $\pi_C = \frac{q}{1-q}(\frac{k_I - \frac{\lambda}{2} + \frac{2(1-q)}{2-q} + \frac{4-3q}{q(2-q)}(\frac{\lambda\alpha}{2} - k_C)}{4})^2$.

Now putting the cases $k_C \le \frac{\lambda\alpha}{2}$ and $k_C > \frac{\lambda\alpha}{2}$ together, we have,

(Equilibrium Case 1) When $k_C \le \frac{\lambda\alpha}{2}$ and $k_I \in [k_C, l_1]$, the copycat C will not enter the market. As incumbent I operates as a monopoly, it will set its price $p_{I,1} = p_0$ and earns a profit $\pi_{I,1} = \pi_0$ in equilibrium. Also, $\tau_{I,1} = 0$ and $p^*_{C,1}(p_{I,1}) = k_C$ so that the incumbent will capture the entire market.

(Equilibrium Case 2) When $k_C \le \frac{\lambda\alpha}{2}$ and $k_I \in [l_1, \min(l_2, l_3)]$, copycat C will enter the market so that $p_{C,2} = \frac{1}{4}(k_I + 3k_C - \frac{\lambda}{2}(1-\alpha) + 2(1-q))$, $\tau_{C,2} = \frac{1}{4q}(k_I + 3k_C - \frac{\lambda}{2}(1+3\alpha) + 2(1-q)) < 0$, $\pi_{C,2} = \frac{1}{16(1-q)}(k_I - k_C - \frac{\lambda}{2}(1-\alpha) + 2(1-q))^2$. Also, incumbent I will set $p_{I,2} = p_1$ and $\tau_{I,2} = \frac{1}{4(1-q)}(2(1-q) + k_I - k_C - \frac{\lambda}{2}(1-\alpha))$ so that $\pi_{I,2} = \pi_1$.

(Equilibrium Case 3) When $k_I \in [l_3, l_5]$, C enters the market so that $p_{C,3} = \frac{\lambda\alpha}{2}$, $\tau_{C,3} = 0$, and $\pi_{C,3} = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)^2$. Also, the incumbent will set $p_{I,3} = p_3$ and $\tau_{I,3} = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)$ so that $\pi_{I,3} = \pi_3$.

(Equilibrium Case 4) When $k_I \in [l_5, \min(l_6, l_9)]$, C enters the market so that $p_{C,4} = \frac{\lambda\alpha}{2}$, $\tau_{C,4} = 0$, and $\pi_{C,4} = \frac{1}{2(1-q)}(\frac{\lambda\alpha}{2} - k_C)(k_I - \frac{\lambda}{2} + (1-q))$. Also, the incumbent will set $p_{I,4} = p_4$ and $\tau_{I,4} = \frac{1}{2(1-q)}(1 - q + k_I - \frac{\lambda}{2})$ so that $\pi_{I,4} = \pi_4$.

(Equilibrium Case 5) When $k_I \in [\min\{\max(l_5, l_9), \frac{\lambda\alpha}{2}\}, l_{10}]$, $k_C \in [l_8, 1]$, C enters the market and sets $p_{C,5} = \frac{q}{4}k_I - \frac{\lambda q}{8} + \frac{2q(1-q)+(4-q)k_C+(4-3q)\frac{\lambda\alpha}{2}}{4(2-q)}$ and $\tau_{C,5} = \tau_I^5 - \frac{1}{4q(1-q)}(qk_I + \frac{2q(1-q)}{2-q} + \frac{\lambda q}{2})$ so that $\pi_{C,5} = \frac{q}{16(1-q)}(k_I - \frac{\lambda}{2} + \frac{2(1-q)}{2-q} + \frac{4-3q}{q(2-q)}(\frac{\lambda\alpha}{2} - k_C))^2$. Also, the incumbent sets $p_{I,5} = p_7$ and $\tau_{I,5} = \frac{2-q}{4(1-q)}(\frac{2(1-q)+\frac{\lambda\alpha}{2}-k_C}{2-q} - \frac{\lambda}{2} + k_I)$ so that $\pi_{I,5} = \pi_7$.

Supplementary Appendix B (Backward Induction Analysis of Extension 1: Sequential Sales of I and C)

Define

$p^d_0 := \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha)$

$p^d_1 := \frac{k_I + 2(1-\delta q) + \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2}}{2}$

$p^d_2 := 2(1-\delta q) + \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2}$

$p^d_3 := \frac{\lambda\delta}{2}(1+\alpha) + \frac{\lambda}{2} - \delta k_C$

$p^d_4 := \frac{k_I + (1-\delta q) + \frac{\lambda}{2}(1+\delta)}{2}$

$p^d_6 := \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C)$

$p^d_7 := \frac{k_I + \frac{2(1-\delta q) - \frac{\delta\lambda\alpha}{2} + \delta k_C}{2-\delta q} + \frac{\lambda}{2}(1+\delta)}{2}$

$p^d_8 := \frac{1}{2-\delta q}(2(1-\delta q) + \delta k_C - \frac{\lambda\delta}{2}(\alpha + q(1+\delta)) + \lambda(1+\delta))$

$l^d_1 := \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) - 2(1-\delta q)$

$l^d_3 := -3\delta k_C + \frac{3\delta\lambda\alpha}{2} + \frac{\lambda}{2}(1+\delta) - 2(1-\delta q)$

$l^d_5 := \frac{\lambda}{2}(1+\delta) + \delta\lambda\alpha - 2\delta k_C - (1-\delta q)$

$l^d_7 := \frac{\lambda\alpha}{q} - \frac{2k_C}{q} + \frac{\lambda}{2}(1+\delta) - (1-\delta q)$

$l^d_9 := \frac{\lambda}{2}(1+\delta) + \frac{1}{2-\delta q}(\frac{4-\delta q}{q}(\frac{\lambda\alpha}{2} - k_C) - 2(1-\delta q))$

$\pi^d_0 := k_{\frac{\lambda}{2}} + \frac{\lambda\delta}{2}(1-\alpha) - k_I$

$\pi^d_1 := \frac{1}{2(1-\delta q)}(\frac{2(1-\delta q) + \delta k_C + \frac{\lambda}{2}(1+\delta-\delta\alpha) - k_I}{2})^2$

$p^d_5 := 1 - \delta q + \frac{\lambda}{2}(1+\delta)$

$\pi^d_3 := \frac{1}{1-\delta q}(1 - \delta q - \frac{\lambda\alpha\delta}{2} + \delta k_C)(\frac{\lambda}{2}(1+\delta+\delta\alpha) - \delta k_C - k_I)$

$\pi^d_4 := \frac{1}{1-\delta q}(\frac{1-\delta q + \frac{\lambda}{2}(1+\delta) - k_I}{2})^2$

$\pi^d_6 := \frac{1}{1-\delta q}(1 - \delta q - \frac{1}{q}(\frac{\lambda\alpha}{2} - k_C))(\frac{1}{q}(\frac{\lambda}{2}(q(1+\delta)+\alpha) - k_C) - k_I)$

$\pi^d_7 := \frac{2-\delta q}{2(1-\delta q)}(\frac{\frac{2(1-\delta q) - \frac{\delta\lambda\alpha}{2} + \delta k_C}{2-\delta q} + \frac{\lambda}{2}(1+\delta) - k_I}{2})^2$

$l^d_2 := 2(1-\delta q) + \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2}$

$l^d_4 := \frac{\lambda\alpha}{2} - (1-\delta q)$

$l^d_6 := 1 - \delta q + \frac{\lambda}{2}(1+\delta)$

$l^d_8 := \frac{\lambda\alpha}{2} - q(1-\delta q)$

$l^d_{10} := \frac{1}{2-\delta q}(2(1-\delta q) - \frac{\delta\lambda\alpha}{2} + \delta k_C) + \frac{\lambda}{2}(1+\delta)$

We here gives the full detailed analysis of the game under this extension.

Consumer purchasing decisions

We apply backward induction.

Period 2 We first establish how consumers choose between I's product and C's product in Period 2 given $v \in [\tau^1_I, 1]$ purchase I's product in Period 1. The indifference point τ^2_{IC} is obtained as below $\lambda\frac{\tau^1_I}{2} + \delta[\tau^2_{IC} - p_I + \lambda(\alpha\frac{1+\tau_{CN}}{2} + (1-\alpha)\frac{1+\tau^2_{IC}}{2})] = \lambda\frac{\tau^1_I}{2} + \delta[\tau^2_{IC}q - p_C + \lambda(\alpha\frac{1+\tau_{CN}}{2} + (1-\alpha)\frac{\tau^2_{IC}}{2})]$. We have $\tau^{2*}_{IC} = \frac{p_I - p_C}{1-q} - \frac{\lambda(1-\alpha)}{2(1-q)}$ Consumers lying in $[\tau^{2*}_{IC}, 1]$ if have not purchased I in Period 1 would prefer I's product to C's product in Period 2; whereas those in $[0, \tau^{2*}_{IC}]$ if have not purchased I in Period 1 would prefer C's product in Period 2. These consumers also need to be individual rational. Hence, comparing utility of buyers of I or C with the utility of nonbuyers, indifference points τ^2_{IN} and τ^2_{CN} can be found by $\lambda\frac{\tau^1_I}{2} + \delta[\tau^2_{IN} - p_I + \lambda\frac{1+\tau^2_{IN}}{2}] = \lambda\frac{\tau^1_I}{2} + \delta\lambda\frac{\tau^2_{IN}}{2}$; and $\lambda\frac{\tau^1_I}{2} + \delta[\tau^2_{CN}q - p_C + \lambda(\alpha\frac{1+\tau^2_{CN}}{2} + (1-\alpha)\frac{\tau^2_{IC}}{2})] = \lambda\frac{\tau^1_I}{2} + \delta\lambda(\alpha\frac{\tau^2_{CN}}{2} + (1-\alpha)\frac{\tau^2_{IC}}{2})$. We obtain $\tau^{2*}_{IN} = p_I - \frac{\lambda}{2}$, and $\tau^{2*}_{CN} = \frac{1}{q}(p_C - \frac{\lambda\alpha}{2})$ Since C can choose the level of p_C so that its demand is positive, $\tau^{2*}_{CN} < \tau^{2*}_{IC}$ is assumed to be a condition held and a constraint to obtain optimal p_C. It can be shown that $\tau^{2*}_{CN} < \tau^{2*}_{IC}$ is equivalent to $\tau^{2*}_{CN} < \tau^{2*}_{IN}$. Hence, for all $v \in [\tau^{2*}_{IC}, 1]$, they are better off purchasing I's product; for all $v \in [\tau^{2*}_{CN}, \tau^{2*}_{IC}]$, they are better off purchasing C's product and the rest are nonbuyers. In summary, in Period 2, consumers in $[\tau^{2*}_{IC}, 1]$ prefer purchasing I's product in Period 2 if they do not make purchase in Period 1; consumers in $[\tau^{2*}_{CN}, \tau^{2*}_{IC}]$ prefer purchasing C's product in Period 2 if they do not make purchase in Period 1; and consumers in $[0.\tau^{2*}_{CN}]$ prefer not purchasing any product in Period 2 if they do not make purchase in Period 1.

Period 1

Whether those buyers and nonbuyers in Period 2 would have purchased in Period 1 is determined by comparing total utilities of making purchase in Period 2 and making purchasing in Period 1. Firstly, for those consumers in $[\tau_{IC}^{2*}, 1]$, the indifference point between purchasing I in Period 1 and doing so in Period 2, τ_I^1, is obtained as follows. $\tau_I^1 - p_I + \lambda\frac{1+\tau_I^1}{2} + \delta\big[\lambda(\alpha\frac{1+\tau_{CN}^2}{2} + (1-\alpha)\frac{1+\tau_{IC}^2}{2})\big] = \lambda\frac{\tau_I^1}{2} + \delta\big[\tau_I^1 - p_I + \lambda(\alpha\frac{1+\tau_{CN}^2}{2} + (1-\alpha)\frac{1+\tau_{IC}^2}{2})\big]$ We obtain $\tau_I^{1*} = p_I - \frac{\lambda}{2(1-\delta)}$ It is observed $\tau_I^{1*} < \tau_{IN}^{2*}$. Since we have already established $\tau_{IN}^{2*} < \tau_{IC}^{2*}$, $\tau_I^{1*} < \tau_{IC}^{2*}$ always holds, meaning for all consumers in $[\tau_{IC}^{2*}, 1]$, they will make purchase of I's product in Period 1. Similarly, we can determine the indifference point of purchasing C's product in Period 2 and purchasing I's product in Period 1, and indifference point of not buying any product in Period 2 and purchasing I's product in Period 1, τ_{IC}^1 and τ_{IN}^1, respectively. $\tau_{IC}^1 - p_I + \lambda\frac{1+\tau_{IC}^1}{2} + \delta\big[\lambda(\alpha\frac{1+\tau_{CN}^2}{2} + (1-\alpha)\frac{1+\tau_{IC}^1}{2})\big] = \lambda\frac{\tau_{IC}^1}{2} + \delta\big[\tau_{IC}^1 q - p_C + \lambda(\alpha\frac{1+\tau_{CN}^2}{2} + (1-\alpha)\frac{\tau_{IC}^1}{2})\big]$, and $\tau_{IN}^1 - p_I + \lambda\frac{1+\tau_{IN}^1}{2} + \delta\lambda\frac{1+\tau_{IN}^1}{2} = \lambda\frac{\tau_{IN}^1}{2} + \delta\lambda\frac{\tau_{IN}^1}{2}$. We have $\tau_{IC}^{1*} = \frac{1}{1-\delta q}(p_I - \delta p_C - \frac{\lambda}{2} - \frac{\delta\lambda(1-\alpha)}{2})$, and $\tau_{IN}^{1*} = p_I - \frac{\lambda}{2} - \frac{\delta\lambda}{2}$. In order for C to have positive demand, we also assume the condition $\tau_{CN}^{2*} < \tau_{IC}^{1*}$ holds here and also as a constraint to determine optimal p_C. It can be shown this condition is equivalent to $\tau_{IN}^{1*} > \tau_{CN}^{2*}$. Additionally, $\tau_{IC}^{1*} < \tau_{IC}^{2*}$ always holds. These conditions imply all consumers in $[\tau_{IC}^{1*}, 1]$ will purchase I's product in Period 1, whereas those in $[\tau_{CN}^{2*}, \tau_{IC}^{1*}]$ will purchase C's product in Period 2, and for all consumers in $[0, \tau_{CN}^{2*}]$, they will not purchase any product. We summarize the two-period equilibrium for consumer purchasing decision as below. *In Period 1, consumers whose $v \in [\tau_{IC}^{1*}, 1]$ purchase I's product in Period 1, whose $v \in [\tau_{CN}^{2*}, \tau_{IC}^{1*}]$ purchase C's product in Period 2, and whose $v \in [0, \tau_{CN}^{2*}]$ do not purchase any product in Period 1.In Period 2, if the consumers whose $v \in [\tau_{IC}^{1*}, 1]$ missed the chance to purchase I's product in Period 1, then those $v \in [\tau_{IC}^{2*}, 1]$ have to purchase I's product in Period 2, and those $v \in [\tau_{IC}^{1*}, \tau_{IC}^{2*}]$ will purchase C's product in Period 2.*

Firm pricing decision Applying backward induction, we first consider Period 2. For simplicity, let $\tau_C = \tau_{CN}^{2*}$, and $\tau_I = \tau_{IC}^{1*}$.

Period 2 **Subcase 1: $\tau_C \le 0$** First, we consider the case when $\tau_C \le 0$, which is equivalent to $p_C \le \frac{\lambda\alpha}{2}$. In order for both I and C staying in the market, they should both have nonnegative demands, $0 \le \tau_I \le 1$. In Period 2, C must ensure $\tau_I \ge 0$ in order to enter the market, which is equivalent to $p_C \le \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))$. In Period 2, C maximizes its profit with respect to price p_C, given p_I.

$$
\begin{aligned}
\max \quad & \pi_C(p_I) = (p_C - k_C)\tau_I \\
\text{s. t.} \quad & p_C \le \frac{\lambda\alpha}{2} \\
& p_C \le \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))
\end{aligned}
$$

We have the optimal price $p_C^*(p_I) = \min(\frac{\lambda\alpha}{2}, \frac{k_C + \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))}{2}, \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha)))$. Always assume $k_C \le \frac{\lambda\alpha}{2}$, comparing these three terms, we have

$$
p_C^*(p_I) = \begin{cases}
\frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))), & \text{if } p_I \in [0, \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha)] \\
\frac{k_C + \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))}{2}, & \text{if } p_I \in [\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha), -\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha)] \\
\frac{\lambda\alpha}{2}, & \text{if } p_I > -\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha)
\end{cases}
$$

Subcase 2: $\tau_C \geq 0$ Next, we consider the case when $\tau_C \geq 0$, which is equivalent to $p_C \geq \frac{\lambda\alpha}{2}$. In order for both I and C staying in the market, they should both have nonnegative demands, $\tau_C \leq \tau_I \leq 1$. In Period 2, C must ensure $\tau_I \geq \tau_C$ in order to enter the market, which is equivalent to $p_C \leq qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)$ C maximizes its profit with respect to price p_C, given p_I.

$$\begin{aligned} \max \quad & \pi_C(p_I) = (p_C - k_C))(\tau_I - \tau_C) \\ \text{s. t.} \quad & p_C \geq \frac{\lambda\alpha}{2} \\ & p_C \leq qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha) \end{aligned}$$

The feasible region is not empty when $qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha) \geq \frac{\lambda\alpha}{2}$, which is equivalent to $p_I \geq \frac{\lambda(1+\delta)}{2}$. This condition must hold here. We have the optimal price

$$p_C^*(p_I) = \begin{cases} \frac{\lambda\alpha}{2}, & \text{if } qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha) \geq \frac{\lambda\alpha}{2} \geq \frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)}{2} \\ \frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)}{2}, & \text{if } qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha) \geq \frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)}{2} \geq \frac{\lambda\alpha}{2} \\ qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha) & \text{if } \frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)}{2} \geq qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha) \geq \frac{\lambda\alpha}{2} \end{cases}$$

Still assume $k_C \leq \frac{\lambda\alpha}{2}$, comparing these three terms $\frac{\lambda\alpha}{2}$, $\frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)}{2}$, and $qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)$, we have

$$p_C^*(p_I) = \begin{cases} \frac{\lambda\alpha}{2}, & \text{if } p_I \in [\frac{\lambda}{2}(1+\delta), \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C)] \\ \frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)}{2}, & \text{if } p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C) \end{cases}$$

Combine the two subcases, we have the optimal response price for C and profits as follows:

$$p_C^*(p_I) = \begin{cases} \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))), & \text{if } p_I \in [0, \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha)] \\ \frac{k_C + \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))}{2}, & \text{if } p_I \in [\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha), -\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha)] \\ \frac{\lambda\alpha}{2}, & \text{if } p_I \in [-\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha), \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C)] \\ \frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha)}{2}, & \text{if } p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C) \end{cases}$$

$$\pi_C^*(p_I) = \begin{cases} 0, & \text{if } p_I \in [0, \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha)] \\ \frac{\delta}{4(1-\delta q)}(\frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha)) - k_C)^2, & \text{if } p_I \in [\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha), -\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha)] \\ \frac{1}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)(p_I - \frac{\lambda}{2}(1+\delta)), & \text{if } p_I \in [\frac{\lambda}{2}(1+\delta), \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C)] \\ \frac{1}{4q(1-\delta q)}(qp_I - \frac{\lambda}{2}(q(1+\delta)-\alpha) - k_C)^2, & \text{if } p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C) \end{cases}$$

Hence, there are four scenarios depending on I's price in Period 1.

Period 1 In Period 1, I maximizes its profit with respect to price p_I. I should also ensure $\tau_I \leq 1$ to have nonnegative demand. This condition is equivalent to $p_I \leq 1 - \delta q + \delta p_C^*(p_I) + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2}$.

Scenario 1: $p_I \in [0, \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha)]$. In this scenario, $\tau_I = 0$ which satisfies the condition $\tau_I \leq 1$ automatically. $p_C^*(p_I) = \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha)))$. I maximizes its own profit function,

$$\begin{aligned} \max \quad & \pi_I = p_I - k_I \\ \text{s. t.} \quad & p_I \leq \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha) \end{aligned}$$

We have the optimal price and optimal profit for I, $p_I^* = p_0^d; \pi_I^* = \pi_0^d$. C will not enter the market.

Scenario 2: $p_I \in [\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha), -\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha)]$. In this scenario, $\tau_C < 0$ and $\tau_I > 0$. $p_C^*(p_I) = \frac{k_C + \frac{1}{\delta}(p_I - \frac{\lambda}{2} - \frac{\lambda\delta}{2}(1-\alpha))}{2}$. $\tau_I \leq 1$ implies $p_I \leq 2(1-\delta q) + \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2}$. I maximizes its own profit function,

$$\begin{aligned}
\max \quad & \pi_I = (p_I - k_I)(1 - \tau_I(p_I)) \\
\text{s. t.} \quad & p_I \leq 2(1-\delta q) + \delta k_C + \frac{\lambda\delta}{2}(1-\alpha) + \frac{\lambda}{2} \\
& p_I \leq -\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha) \\
& p_I \geq \delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1-\alpha)
\end{aligned}$$

We have the optimal price for I, $p_I^* = \min(p_1^d, p_2^d, p_3^d)$, and $p_I^* \geq p_0^d$.

1. When $l_1^d \leq k_I \leq \min(l_2^d, l_3^d)$, $p_I^* = p_1^d$, $\pi_I^* = \pi_1^d$.
2. When $l_4^d, k_I \geq l_2^d$, $p_I^* = p_2^d$, $\pi_I^* = 0$.
3. When $k_C \geq l_4^d, k_I \geq l_3^d$, $p_I^* = p_3^d$, $\pi_I^* = \pi_3^d$.
4. When $k_C \leq l_1^d$, $p_I^* = p_0^d$, $\pi_I^* = \pi_0^d$

Scenario 3: $p_I \in [-\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha), \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C)]$. In this scenario, $\tau_C = 0$ and $\tau_I > 0$. $p_C^*(p_I) = \frac{\lambda\alpha}{2}$. $\tau_I \leq 1$ implies $p_I \leq (1-\delta q) + \frac{\lambda}{2}(1+\delta)$ I maximizes its own profit function,

$$\begin{aligned}
\max \quad & \pi_I = (p_I - k_I)(1 - \tau_I(p_I)) \\
\text{s. t.} \quad & p_I \leq (1-\delta q) + \frac{\lambda}{2}(1+\delta) \\
& p_I \leq \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C) \\
& p_I \geq -\delta k_C + \frac{\lambda}{2} + \frac{\lambda\delta}{2}(1+\alpha)
\end{aligned}$$

We have the optimal price for I, $p_I^* = \min(p_4^d m p_5^d, p_6^d)$ and $p_I^* \geq p_3^d$.

1. When $l_5^d \leq k_I \leq \min(l_6^d, l_7^d)$, $p_I^* = p_4^d$, $\pi_I^* = \pi_4^d$.
2. When $l_4^d \leq k_C \leq l_8^d, k_I \geq l_6^d$, $p_I^* = p_5^d$, $\pi_I^* = 0$.
3. When $k_C \geq l_8^d, k_I \geq l_7^d$, $p_I^* = p_6^d$, $\pi_I^* = \pi_6^d$.
4. When $k_I \leq l_5^d$, $p_I^* = p_3^d$, $\pi_I^* = \pi_3^d$.

Scenario 4: $p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C)$. In this scenario, $\tau_C > 0$ and $\tau_I > \tau_C$. $p_C^*(p_I) = \frac{k_C + qp_I - \frac{\lambda}{2}(q(1+\delta) - \alpha)}{2}$. $\tau_I \leq 1$ implies $p_I \leq \frac{1}{2-\delta q}(2(1-\delta q) + \delta k_C - \frac{\lambda\delta}{2}(\alpha + q(1+\delta)) + \lambda(1+\delta)$ I maximizes its own profit function,

$$\begin{aligned}
\max \quad & \pi_I = (p_I - k_I)(1 - \tau_I(p_I)) \\
\text{s. t.} \quad & p_I \leq \frac{1}{2-\delta q}(2(1-\delta q) + \delta k_C - \frac{\lambda\delta}{2}(\alpha + q(1+\delta)) + \lambda(1+\delta) \\
& p_I \geq \frac{1}{q}(\frac{\lambda}{2}(\alpha + q(1+\delta)) - k_C)
\end{aligned}$$

We have the optimal price for I, $p_I^* = \min(p_7^d, p_8^d)$ and $p_I^* \geq p_6^d$.

1. When $l_9^d \leq k_I \leq l_{10}^d$, $p_I^* = p_7^d$, $\pi_I^* = \pi_7^d$
2. When $k_I \geq l_{10}^d$, $p_I^* = p_8^d$, $\pi_I^* = 0$.
3. When , which is equivalent to $k_I \leq l_{10}^d$, $p_I^* = p_6^d$, $\pi_I^* = \pi_6^d$.

Combine those 4 scenarios, profits sharing the same valid regions are compared such that the price with the largest profit is selected as the optimal price in that valid region. We have the following result

1. When $k_I \in [k_C, l_1^d]$, $k_C \leq \frac{\lambda\alpha}{2}$. $p_I = p_0^d$, $\pi_I = \pi_0^d$. C will not enter the market.
2. When $k_I \in [l_1^d, \min(l_2^d, l_3^d)]$, C enters the market. $p_I = p_1^d$, $\pi_I = \pi_1^d$. ,$p_C = \frac{k_I + 2(1-\delta q) + 3\delta k_C - \frac{\lambda}{2}(1+\delta-\delta\alpha)}{4\delta}$ $\pi_C = \frac{1}{16\delta(1-\delta q)}(k_I + 2(1-\delta q) - \delta k_C - \frac{\lambda}{2}(1+\delta-\delta\alpha))^2$.
3. When $k_I \in [l_3^d, l_5^d]$, $k_C \in [l_4^d, \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = p_3^d$, $\pi_I = \pi_3^d . p_C = \frac{\lambda\alpha}{2}$. $\pi_C = \frac{\delta}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)^2$.
4. When $k_I \in [l_5^d, \min(l_6^d, l_9^d)]$, $k_C \in [l_4^d, \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = p_4^d$, $\pi_I = \pi_4^d$. $p_C = \frac{\lambda\alpha}{2}$. $\pi_C = \frac{1}{1-\delta q}(\frac{\lambda\alpha}{2} - k_C)(\frac{k_I + 1 - \delta q - \frac{\lambda(1+\delta)}{2}}{2})$.
5. When $k_I \in [\max(l_5^d, l_9^d), l_{10}^d]$, $k_C \in [l_8^d, \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = p_7^d$, $\pi_I = \pi_7^d$. $p_{C9} = \frac{2q(1-\delta q) + (4-\delta q)k_C + (4-3\delta q)\frac{\lambda\alpha}{2}}{4(2-\delta q)} - \frac{\lambda q(1+\delta)}{8} + \frac{qk_I}{4}$. $\pi_{C9} = \frac{q}{16(1-\delta q)}(k_I - \frac{\lambda(1+\delta)}{2} + \frac{2(1-\delta q)}{2-\delta q} + \frac{4-3\delta q}{q(2-\delta q)}(\frac{\lambda\alpha}{2} - k_C))^2$

Case 2. $\underline{k_C > \frac{\lambda\alpha}{2}.}$
We can do similar analysis under the case when $k_C > \frac{\lambda\alpha}{2}$. We omit the details and the equilibrium results are as follows
When $\max\{l_8^d, k_C\} \leq k_I \leq l^d 10$, $1 \geq k_C > \frac{\lambda\alpha}{2}$, C enters the market. $p_I^* = p_7^d$, $\pi_I^* = \pi_7^d$. Now putting these two cases together we have exactly the same form of equilibrium as in the original game (replacing p_k by p_k^d), π_k by π_k^d; and l_k by l_k^d.

Supplementary Appendix C (Backward Induction Analysis of Extension 2: Alternative Formulation of Status Utility)

We here gives the full detailed analysis of the game under this extension.

Consumer purchasing decisions

τ_1 is solved by $\tau_I \cdot 1 - p_I + \lambda(\alpha \frac{1+\tau_C}{2} + (1-\alpha)\frac{1+\tau_I}{2}) = \tau_I \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I+\tau_C}{2})$. We get $\tau_I = \frac{p_I - p_C - (1-\alpha)\frac{\lambda}{2}(1-\max(0,\tau_C))}{1-q}$. τ_C can be solved similarly. $\tau_C \cdot q - p_C + \lambda(\alpha\frac{1+\tau_C}{2} + (1-\alpha)\frac{\tau_I+\tau_C}{2}) = \lambda(\alpha\frac{\tau_C}{2} + (1-\alpha)\frac{\tau_I}{2})$. We have $\tau_C = \frac{p_C - \alpha\frac{\lambda}{2}}{q+(1-\alpha)\frac{\lambda}{2}}$.

Firm pricing decision

Case 1. $k_C \le \frac{\lambda\alpha}{2}$

Applying backward induction, we first consider Period 2.

Period 2

Subcase 1*:* $\tau_C \le 0$. First, we consider the case when $\tau_C \le 0$, which is equivalent to $p_C \le \frac{\lambda\alpha}{2}$. In this case, $\tau_I = \frac{p_I - p_C - (1-\alpha)\frac{\lambda}{2}}{1-q}$. This analysis is the exactly same as that for the original game. We omit it here.

Subcase 2*:* $\tau_C \ge 0$. $\tau_C \ge 0$ is equivalent to $p_C \ge \frac{\lambda\alpha}{2}$. In order for both I and C staying in the market, they should both have nonnegative demands, $\tau_C \le \tau_I \le 1$. In Period 2, C must ensure $\tau_I \ge \tau_C$ in order to enter the market, which is equivalent to $p_C \le (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}$. C maximizes its profit with respect to price p_C, given p_I.

$$\begin{aligned} \max \quad & \pi_C(p_I) = (p_C - k_C))(\tau_I - \tau_C) \\ \text{s. t.} \quad & p_C \ge \frac{\lambda\alpha}{2} \\ & p_C \le (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} \end{aligned}$$

The feasible region is not empty when $(p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} \ge \frac{\lambda\alpha}{2}$, which is equivalent to $p_I \ge \frac{\lambda}{2}$. We have the optimal price

$$p_C^*(p_I) = \begin{cases} \frac{\lambda\alpha}{2}, & \text{if } (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} \ge \frac{\lambda\alpha}{2} \ge \frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2} \\ \frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2}, & \text{if } (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} \ge \frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2} \ge \frac{\lambda\alpha}{2} \\ (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} & \text{if } \frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2} \ge (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} \ge \frac{\lambda\alpha}{2} \end{cases}$$

Since $k_C \le \frac{\lambda\alpha}{2}$, comparing these three terms $\frac{\lambda\alpha}{2}$, $\frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2} \ge (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}$, and $(p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}$, we have

$$p_C^*(p_I) = \begin{cases} \frac{\lambda\alpha}{2}, & \text{if } p_I \in [\frac{\lambda}{2}, \frac{1}{q + \frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2}] \\ \frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2} & \text{if } p_I \ge \frac{1}{q + \frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2} \end{cases}$$

Combine the two subcases, we have the optimal response price for C and profits as follows:

When $q + \frac{\lambda(1-\alpha)}{2} \le 1$

$$p_C^*(p_I) = \begin{cases} p_I - \frac{\lambda}{2}(1-\alpha), & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)] \\ \frac{k_C + p_I - \frac{\lambda}{2}(1-\alpha)}{2}, & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{\lambda}{2}(1+\alpha) - k_C] \\ \frac{\lambda\alpha}{2}, & \text{if } p_I \in [\frac{\lambda}{2}(1+\alpha) - k_C, \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2} \\ \frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2}, & \text{if } p_I \ge \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2} \end{cases}$$

$$\pi_C^*(p_I) = \begin{cases} 0, & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)] \\ \frac{1}{4(1-q)}(p_I - \frac{\lambda}{2}(1-\alpha) - k_C)^2, & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{\lambda}{2}(1+\alpha) - k_C] \\ \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)(p_I - \frac{\lambda}{2}), & \text{if } p_I \in [\frac{\lambda}{2}(1+\alpha) - k_C, \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2}] \\ \frac{1}{4(q+\frac{\lambda(1-\alpha)}{2})(1-q)}((p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} - k_C)^2, & \text{if } p_I \ge \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2} \end{cases}$$

When $q + \frac{\lambda(1-\alpha)}{2} > 1$

$$p_C^*(p_I) = \begin{cases} p_I - \frac{\lambda}{2}(1-\alpha), & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)] \\ \frac{k_C + p_I - \frac{\lambda}{2}(1-\alpha)}{2}, & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2}] \\ \frac{k_C + (p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2}}{2}, & \text{if } p_I \ge \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2} \end{cases}$$

$$\pi_C^*(p_I) = \begin{cases} 0, & \text{if } p_I \in [0, k_C + \frac{\lambda}{2}(1-\alpha)] \\ \frac{1}{4(1-q)}(p_I - \frac{\lambda}{2}(1-\alpha) - k_C)^2, & \text{if } p_I \in [k_C + \frac{\lambda}{2}(1-\alpha), \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2}] \\ \frac{1}{4(q+\frac{\lambda(1-\alpha)}{2})(1-q)}((p_I - \frac{\lambda}{2})(q + \frac{\lambda(1-\alpha)}{2}) + \frac{\lambda\alpha}{2} - k_C)^2, & \text{if } p_I \ge \frac{1}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2} - k_C) + \frac{\lambda}{2} \end{cases}$$

Comparing with the result of the original game, this extension gives the same result for the cases when $\tau_C \le 0$. Whereas when $\tau_C > 0$, every term q in the ordinal analysis is amplified to $q + \frac{\lambda(1-\alpha)}{2}$.

We assume $q + \frac{\lambda(1-\alpha)}{2} \le 1$ since copycat usually has low quality, q, and high resemblance, α.

Period 1 Hence, there are four scenarios depending on I's price in Period 1. We then conduct the same analysis procedure for Period 1 as for the original game. We have the equilibrium result under the case $k_C \le \frac{\lambda\alpha}{2}$ as below:

1. When $k_I \in (k_C, k_C + \frac{\lambda}{2}(1-\alpha) - 2(1-q))$, $k_C \le \frac{\lambda\alpha}{2}$. $p_I = k_C + \frac{\lambda}{2}(1-\alpha)$, $\pi_I = k_C + \frac{\lambda}{2}(1-\alpha) - k_I$. C will not enter the market.
2. When $k_I \in (k_C + \frac{\lambda}{2}(1-\alpha) - 2(1-q), min(k_C + \frac{\lambda}{2}(1-\alpha) + 2(1-q), -3k_C + \frac{\lambda}{2}(1+3\alpha) - 2(1-q)))$, C enters the market. $p_I = \frac{k_I + k_C + \frac{\lambda}{2}(1-\alpha) + 2(1-q)}{2}$, $\pi_I = \frac{1}{2(1-q)}(\frac{-k_I + k_C + \frac{\lambda}{2}(1-\alpha) + 2(1-q)}{2})^2$. $p_C = \frac{k_I + 3k_C - \frac{\lambda}{2}(1-\alpha) + 2(1-q)}{4}$, $\pi_C = \frac{1}{1-q}(\frac{k_I - k_C - \frac{\lambda}{2}(1-\alpha) + 2(1-q)}{4})^2$.
3. When $k_I \in (-3k_C + \frac{\lambda}{2}(1+3\alpha) - 2(1-q), -2k_C + \frac{\lambda}{2}(1+2\alpha) - (1-q))$, $k_C \in [\frac{\lambda\alpha}{2} - (1-q), \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = \frac{\lambda}{2}(1+\alpha) - k_C$, $\pi_I = \frac{1}{(1-q)}(k_C - \frac{\lambda\alpha}{2} + 1 - q)(\frac{\lambda}{2}(1+\alpha) - k_C - k_I).p_C = \frac{\lambda\alpha}{2}$, $\pi_C = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)^2$, $p_{I,3} = \frac{\lambda}{2}(1+\alpha) - k_C$.
4. When $k_I \in [-2k_C + \frac{\lambda}{2}(1+2\alpha) - (1-q), min(\frac{\lambda}{2} + (1-q), \frac{\lambda}{2} - \frac{2(1-q)}{2-q} + \frac{4-q}{(q+\frac{\lambda(1-\alpha)}{2})(2-q)}(-k_C + \frac{\lambda\alpha}{2})]$, $k_C \in [\frac{\lambda\alpha}{2} - (1-q), \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = \frac{k_I + \frac{\lambda}{2} + (1-q)}{2}$, $\pi_I = \frac{1}{1-q}(\frac{-k_I + \frac{\lambda}{2} + (1-q)}{2})^2$. $p_C = \frac{\lambda\alpha}{2}$, $\pi_C = \frac{1}{1-q}(\frac{\lambda\alpha}{2} - k_C)(\frac{k_I - \frac{\lambda}{2} + (1-q)}{2})$.

5. When $k_I \in [\max(-2k_C+\frac{\lambda}{2}(1+2\alpha)-(1-q), \frac{\lambda}{2}-\frac{2(1-q)}{2-q}+\frac{4-q}{(q+\frac{\lambda(1-\alpha)}{2})(2-q)}(-k_C+\frac{\lambda\alpha}{2})), \frac{1}{2-q}(k_C-\frac{\lambda\alpha}{2})+\frac{\lambda}{2}+\frac{2(1-q)}{2-q}]$, $k_C \in [\frac{\lambda\alpha}{2}-(q+\frac{\lambda(1-\alpha)}{2})(1-q), \frac{\lambda\alpha}{2}]$, C enters the market. $p_I = \frac{k_I+\frac{2(1-q)+\frac{q}{q+\frac{\lambda(1-\alpha)}{2}}(k_C-\frac{\lambda\alpha}{2})}{2-q}+\frac{\lambda}{2}}{2}$, $\pi_I = \frac{2-q}{2(1-q)}(\frac{1}{2}(-k_I+\frac{\lambda}{2}+\frac{\frac{q}{q+\frac{\lambda(1-\alpha)}{2}}(k_C-\frac{\lambda\alpha}{2})+2(1-q)}{2-q}))^2$.

The first three cases are exact identical to those in the original game. In the last two case, compared with the the original results, only q is replaced by $q+\frac{\lambda(1-\alpha)}{2}$.

Case 2. $k_C > \frac{\lambda\alpha}{2}$.

We can do similar analysis under the case when $k_C > \frac{\lambda\alpha}{2}$. We omit the details and the equilibrium results are as follows

When $\max\{\frac{\lambda}{2}-(1-q), k_C\} \le k_I \le \frac{1}{2-q}(k_C-\frac{\lambda\alpha}{2})+\frac{\lambda}{2}+\frac{2(1-q)}{2-q}$, $1 \ge k_C > \frac{\lambda\alpha}{2}$, C enters the market. $p_I^* = \frac{k_I+\frac{2(1-q)+\frac{q}{q+\frac{\lambda(1-\alpha)}{2}}(k_C-\frac{\lambda\alpha}{2})}{2-q}+\frac{\lambda}{2}}{2}$, $\pi_I^* = \frac{2-q}{2(1-q)}(\frac{\frac{2(1-q)-\frac{q}{q+\frac{\lambda(1-\alpha)}{2}}(\frac{\lambda\alpha}{2}-k_C)}{2-q}+\frac{\lambda}{2}-k_I}{2})^2$. Now putting these two cases together we have,

(Equilibrium Case 1-3) The same as in Equilibrium Case 1-3 in the original game results.

(Equilibrium Case 4) When $k_I \in [l_5, \min(l_6, \frac{\lambda}{2}-\frac{2(1-q)}{2-q}+\frac{4-q}{(q+\frac{\lambda(1-\alpha)}{2}(2-q))}(-k_C+\frac{\lambda\alpha}{2})]$, C enters the market so that $p_{C,4} = \frac{\lambda\alpha}{2}$, $\tau_{C,4} = 0$, and $\pi_{C,4} = \frac{1}{2(1-q)}(\frac{\lambda\alpha}{2}-k_C)(k_I-\frac{\lambda}{2}+(1-q))$. Also, the incumbent will set $p_{I,4} = p_4$ and $\tau_{I,4} = \frac{1}{2(1-q)}(1-q+k_I-\frac{\lambda}{2})$ so that $\pi_{I,5} = \pi_5$.

(Equilibrium Case 5) When $k_I \in [\min\big\{\max(l_5, \frac{\lambda}{2}-\frac{2(1-q)}{2-q}+\frac{4-q}{(q+\frac{\lambda(1-\alpha)}{2}(2-q))}), \frac{\lambda\alpha}{2}\big\}, l_{10}]$, $k_C \in [l_8, 1]$, C enters the market and sets C enters the market. $p_I = \frac{k_I+\frac{2(1-q)+\frac{q}{q+\frac{\lambda(1-\alpha)}{2}}(k_C-\frac{\lambda\alpha}{2})}{2-q}+\frac{\lambda}{2}}{2}$, $\pi_I = \frac{2-q}{2(1-q)}(\frac{1}{2}(-k_I+\frac{\lambda}{2}+\frac{\frac{q}{q+\frac{\lambda(1-\alpha)}{2}}(k_C-\frac{\lambda\alpha}{2})+2(1-q)}{2-q}))^2$.

Since Equilibrium Case 1 is exactly same as the our original case, Proposition 1 and Corollary 1 preserve in the extension.

For Equilibrium case 1-3, results are exactly the same the original case, and for both Equilibrium Case 4 and 5, the only change is factor q in the original results is replaced by $q+\frac{\lambda(1-\alpha)}{2}$. The union of feasible regions of case 4 and 5 are the same as that in the original results. Therefore, Proposition 2 preserves.

Note that for Equilibrium Case 4, the equilibrium price is identical to the one in the Equilibrium Case 4 in the original analysis. Only the condition of k_I changes. So the consumer surplus and social surplus results for this case is the same as those the original analysis. We still have the same conclusion that when q is sufficiently small, both consumer surplus and social surplus are less than the benchmark case where there is no potential copycat in the market. That is to say, Proposition 3 also preserves.