

# Online Appendix

to

Add-on Policies under Vertical Differentiation:  
Why Do Luxury Hotels Charge for Internet  
While Economy Hotels Do Not?

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# Online Appendix

to

## Add-on Policies under Vertical Differentiation: Why Do Luxury Hotels Charge for Internet While Economy Hotels Do Not?

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## A Proof of Proposition 1

### A-1 A Lemma for Firm $L$

To prove the proposition, the following lemma for firm  $L$  will be useful.

**Lemma A-1.** *For firm  $L$ , not selling the add-on is strictly dominated by selling it, if: (1)  $P_h > \alpha\Delta V$  when firm  $H$  sells optional add-on, or (2)  $P_h^+ > \alpha(\Delta V + w)$  when firm  $H$  sells standard add-on.*

The proof of the lemma follows the similar logic of the proof of Lemma 1 in the main text: (1) assume that firm  $L$  sells only the base; (2) find the equivalent total price and add-on price, (3) consider a small deviation by lowering the total price and the add-on price; (4) show that the deviation is profitable. In what follows, I prove the first case when  $H$  sells the add-on optionally. The proof of the second case follows the same argument.

Suppose that firm  $L$  sells only the base to consumers at price  $P_l$ . Then the marginal consumer indifferent between the two firms becomes  $\theta_{hl} = (P_h - P_l)/\Delta V$ . Firm  $L$ 's profit is  $\Pi_l = (\theta_{hl} - \underline{\theta})P_l$ . This strategy is exactly equivalent to charging a bundle price  $P_l^+$  and an add-on price  $p_l$  such that  $P_l^+ = P_l + p_l$ , and that

$$\frac{P_h - P_l^+}{\Delta V - w} = \frac{P_h - P_l}{\Delta V} = \frac{p_l}{w}. \quad (\text{A-1})$$

Consider a small decrease of the bundle price to  $P_l^{+'} = P_l^+ - \epsilon$  with  $\epsilon > 0$ , and a small decrease of the add-on price to  $p_l' = p_l - \epsilon$  by the same amount. The base price  $P_l$  remains unchanged. This results in a segmentation of consumers because

$$\frac{P_h - P_l^{+'}}{\Delta V - w} > \frac{P_h - P_l}{\Delta V} > \frac{p_l'}{w}.$$

Firm  $L$ 's profit now becomes

$$\Pi_l(P_l^{+'}, p_l') = \left(\frac{P_h - P_l^{+'}}{\Delta V - w} - \underline{\theta}\right)(P_l^{+'} - c) - \left(\frac{p_l'}{w} - \underline{\theta}\right)(p_l' - c).$$

Evaluating this function around the original prices  $(P_l^+, p_l)$  by Taylor series yields:

$$\begin{aligned} \Pi_l(P_l^{+'}, p_l') &= \Pi_l(P_l^+, p_l) - \epsilon \cdot \Pi_{P_l^+}(P_l^+, p_l) - \epsilon \cdot \Pi_{p_l}(P_l^+, p_l) \\ &\quad + \frac{1}{2} \left[ \epsilon^2 \cdot \Pi_{P_l^+ P_l^+}(P_l^+, p_l) + 2\epsilon^2 \cdot \Pi_{P_l^+ p_l}(P_l^+, p_l) + \epsilon^2 \cdot \Pi_{p_l p_l}(P_l^+, p_l) \right] + R_2 \end{aligned}$$

$$\begin{aligned}
&= \Pi_l(P_l^+, p_l) - \epsilon \cdot \left[ \frac{P_h - P_l^+}{\Delta V - w} - \underline{\theta} - \frac{P_l^+ - c}{\Delta V - w} \right] + \epsilon \cdot \left[ \frac{p_l}{w} - \underline{\theta} + \frac{p_l - c}{w} \right] \\
&\quad + \frac{\epsilon^2}{2} \left[ -\frac{2}{\Delta V - w} - \frac{2}{w} \right] \\
&= \Pi_l(P_l^+, p_l) + \underbrace{\frac{\epsilon}{\Delta V - w} (P_h - \alpha \Delta V) - \frac{\epsilon^2}{w(\Delta V - w)} \Delta V}_M,
\end{aligned}$$

where the second equality follows from the remainder term  $R_2 = 0$  because the higher order derivatives are all zero, and the last equality follows from Equation (A-1). Therefore,  $M > 0$  for small positive  $\epsilon$  when  $P_h > \alpha \Delta V$ .

## A-2 Equilibrium When $\alpha \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta})$

To derive the equilibrium profile under this case, note that the indifferent consumer becomes  $\theta_{hl} = (P_h - P_l)/\Delta V$ . The two firms' profits are given by

$$\Pi_h = (\bar{\theta} - \frac{P_h - P_l}{\Delta V})P_h + (\bar{\theta} - \frac{p_h}{w})(p_h - c); \quad \Pi_l = (\frac{P_h - P_l}{\Delta V} - \underline{\theta})P_l.$$

It is straightforward to find the following equilibrium prices using the first-order conditions:

$$P_h^* = \frac{1}{3}(2\bar{\theta} - \underline{\theta})\Delta V, \quad p_h^* = w\theta_h^*; \quad P_l^* = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})\Delta V, \quad p_l^* > w\theta_{hl}^*,$$

where the thresholds are given by  $\theta_{hl}^* = \frac{1}{3}(\bar{\theta} + \underline{\theta})$  and  $\theta_h^* = \frac{1}{2}(\bar{\theta} + \alpha)$ . Next I first establish that there is no profitable deviation for either firm, and then show that this is the unique equilibrium in this parameter range.

### A-2.1 No Profitable Deviation for Firm $L$

This is established by considering three possible non-local deviations.

*Case (a): firm  $L$  does not sell the add-on and all consumers of firm  $H$  buy the add-on.* The marginal consumer indifferent between the two firms becomes  $\theta_{hl} = (P_h^{+*} - P_l)/(\Delta V + w)$ . For this case to arise, we need that  $\theta_{hl} \geq p_h^*/w$  so that no consumer buys the base only from  $H$ . But note that in equilibrium the incentive constraint has to satisfy:

$$\theta_{hl}^* < \theta_h^* \Leftrightarrow \frac{P_h^* - P_l^*}{\Delta V} < \frac{p_h^*}{w} \implies \frac{P_h^{+*} - P_l^*}{\Delta V + w} < \frac{p_h^*}{w}. \quad (\text{A-2})$$

Firm  $L$ 's deviation profit is given by  $\Pi_l = (\theta_{hl} - \underline{\theta})P_l$ . It turns out that the optimal profit under this deviation case is achieved by the corner solution  $\theta_{hl} = p_h^*/w$ . To see this, note

that the first-order condition of the Lagrangian is given by  $P_l = (P_h^{+*} - (\Delta V + w)\underline{\theta} - \lambda)/2$ . Then,

$$\theta_{hl} - \frac{p_h^*}{w} = \frac{P_h^{+*} - P_l}{\Delta V + w} - \frac{p_h^*}{w} < \frac{P_l^* - P_l}{\Delta V + w} = \frac{1}{2(\Delta V + w)}(-p_h^* + w\underline{\theta} + \lambda),$$

where the inequality follows from Equation (A-2) and the last equality is obtained by substituting the best responses of firm  $L$ . Because  $p_h^* > w\underline{\theta}$ , to guarantee that the constraint is nonnegative, we need  $\lambda > 0$ . This implies that the constraint has to be binding by complementary slackness. This corner solution then coincides with the corner solution for the problem under the proposed equilibrium (with  $L$  not selling the add-on and  $H$  selling it optionally) because:

$$\frac{P_h^{+*} - P_l}{\Delta V + w} = \frac{p_h^*}{w} \implies \frac{P_h^* - P_l}{\Delta V} = \frac{p_h^*}{w}.$$

Therefore, the deviation does not improve firm  $L$ 's profit.

*Case (b): firm  $L$  sells the add-on and some consumers of firm  $H$  do not buy the add-on.* First, let us consider the scenario whereby firm  $L$  sells the add-on optionally. The market is divided into four consumer segments as in Figure 2 in the main text. The thresholds are given by  $\theta_h = p_h^*/w$ ,  $\theta_{hl} = (P_h^* - P_l - p_l)/(\Delta V - w)$ , and  $\theta_l = p_l/w$ . It will later become evident that when firm  $L$  sells the add-on to all of its consumers (i.e.,  $\theta_l < \underline{\theta}$ ) the profit is not improved. Given firm  $H$ 's equilibrium pricing, firm  $L$  maximizes its profit  $\Pi_l = (\theta_{hl} - \underline{\theta})(P_l^+ - c) - (\theta_l - \underline{\theta})(p_l - c)$  subject to the constraints: (1)  $\theta_{hl} \geq \theta_l$ , (2)  $\theta_l \geq \underline{\theta}$ , and (3)  $\theta_h \geq \theta_{hl}$ .

The objective function is concave and thus the *necessary* and *sufficient* condition for the optimization program is the Karush-Kuhn-Tucker conditions. Let  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  be the Lagrangian multipliers for the three constraints. The first-order conditions lead to  $P_l^+ = (P_h^* - (\Delta V - w)\underline{\theta} + c - \lambda_1 + \lambda_3)/2$  and  $p_l = (w\underline{\theta} + c - \lambda_1 + \lambda_2)/2$ . Therefore, Constraint (1) can be rewritten as

$$\theta_{hl} - \theta_l = \frac{1}{2w(\Delta V - w)} \underbrace{\left[ w\Delta V \left( \frac{1}{3}(2\bar{\theta} - \underline{\theta}) - \alpha \right) - \lambda_2(\Delta V - w) - \lambda_3w + \lambda_1\Delta V \right]}_M \geq 0.$$

Because  $\alpha > (2\bar{\theta} - \underline{\theta})/3$  and both  $\lambda_2$  and  $\lambda_3$  are nonnegative, the term  $M$  in the bracket is strictly negative ( $M < 0$ ). To guarantee that the constraint is nonnegative, we need  $\lambda_1 > 0$ . This implies that Constraint (1) has to be binding. Therefore, the profit is no greater than that obtained from not selling the add-on (i.e., the equilibrium strategy).

It remains to verify that firm  $L$  will not improve the deviation profit if it were to sell the

add-on to all of its consumers, in which case the profit is  $\Pi_l = (\theta_{hl} - \underline{\theta})(P_l^+ - c)$ . This profit can be strictly improved if the firm sets the add-on price  $p_l$  to be anywhere between  $[\underline{\theta}w, c]$  so that some consumers decide not to buy the add-on. The incremental benefit is equal to  $-(\theta_l - \underline{\theta})(p_l - c)$  which is strictly positive.

*Case (c): firm L sells the add-on and all consumers of firm H buy the add-on..* The marginal consumer indifferent between the two firms becomes  $\theta_{hl} = (P_h^{+*} - P_l^+)/\Delta V$ . To ensure that all consumers buy the add-on from H, it is necessary that  $\theta_{hl} \geq p_h^*/w$ . Consider that firm L maximizes its profit  $\Pi_l = (\theta_{hl} - \underline{\theta})(P_l^+ - c) - (\theta_l - \underline{\theta})(p_l - c)$ . The profit differs from Case (b) only in terms of the profit of selling the bundle,  $(\theta_{hl} - \underline{\theta})(P_l^+ - c)$ . Maximizing this bundling profit under the constraint yields  $P_l^+ = (P_h^{+*} - \Delta V \underline{\theta} + c - \lambda)/2$ . Substituting this best response back to the constraint gives

$$\theta_{hl} - \frac{p_h^*}{w} = \frac{P_h^{+*} - P_l^+}{\Delta V} - \frac{p_h^*}{w} = \frac{1}{2\Delta V} \underbrace{\left[ -\frac{1}{3}(\bar{\theta} - 2\underline{\theta} + 3\alpha)\Delta V + \frac{1}{2}(\bar{\theta} - \alpha)w + \lambda \right]}_M \geq 0.$$

Note further that,

$$\begin{aligned} M &\leq -\frac{1}{3}(\bar{\theta} - 2\underline{\theta} + 2\bar{\theta} - \underline{\theta})\Delta V + \frac{1}{6}(3\bar{\theta} - (2\bar{\theta} - \underline{\theta}))w \\ &< -(\bar{\theta} - \underline{\theta})w + \frac{1}{6}(\bar{\theta} + \underline{\theta})w \\ &< -\frac{w}{2}, \end{aligned}$$

where the first inequality follows from  $\alpha \geq (2\bar{\theta} - \underline{\theta})/3$ , the second from  $\Delta V > w$ , and the last from  $\bar{\theta} > 2\underline{\theta}$ . Since  $M < 0$ , for the constraint to be nonnegative, it is necessary that  $\lambda > 0$ . This implies that the constraint has to be binding by complementary slackness. Therefore the profit is no greater than that obtained from the previous Case (b), which is proved to be not profitable.

## A-2.2 No Profitable Deviation for Firm H

*Case (a): firm H sells the add-on as optional and some consumers buy L<sup>+</sup>.* The marginal consumer indifferent between the two firms becomes  $\theta_{hl} = (P_h - P_l^* - p_l^*)/(\Delta V - w)$ . Consider firm H's maximization of profit,  $\Pi_h = (\bar{\theta} - \theta_{hl})P_h + (\bar{\theta} - \theta_h)(p_h - c)$ , subject to the constraint  $\theta_{hl} \geq p_l^*/w$ . Notice that the profit function is the same as the one of the proposed equilibrium except the threshold  $\theta_{hl}$ . It then suffices to compare the profits from selling the base only. The first-order condition of the constrained problem is given by  $P_h = (P_l^* + p_l^* + (\Delta V - w)\bar{\theta} + \lambda)/2$ .

Therefore,

$$\begin{aligned}
\theta_{hl} - \frac{P_l^*}{w} &= \frac{1}{2(\Delta V - w)} \left[ \frac{2}{3}(\bar{\theta} + \underline{\theta})\Delta V - \bar{\theta}w - \frac{P_l^*}{w}(2\Delta V - w) + \lambda \right] \\
&\leq \frac{1}{2(\Delta V - w)} \left[ \frac{2}{3}(\bar{\theta} + \underline{\theta})\Delta V - \bar{\theta}w - \frac{1}{3}(\bar{\theta} + \underline{\theta})(2\Delta V - w) + \lambda \right] \\
&= \frac{1}{2(\Delta V - w)} \left( -\frac{1}{3}(2\bar{\theta} - \underline{\theta})w + \lambda \right),
\end{aligned}$$

where the inequality holds because  $p_l^* = \frac{1}{2}(\underline{\theta} + \alpha)w \geq (\bar{\theta} + \underline{\theta})w/3$ . Because  $(2\bar{\theta} - \underline{\theta})$  is positive, to ensure that the constraint is nonnegative it is necessary that  $\lambda > 0$ . By complementary slackness the constraint must be binding, implying that firm  $H$  should leave no demand for  $L^+$ . Therefore, the optimal profit obtained in this case is no greater than the equilibrium profit.

*Case (b): firm  $H$  sells the add-on as standard and some consumers buy  $L^+$ .* Because  $P_l^{+*} > 0$ , by Lemma 1 this strategy is strictly dominated by a local deviation whereby firm  $H$  sells the add-on optionally by increasing the add-on price while lowering the base price. This deviation is exactly in the form of Case (a), which yields no greater profit than the equilibrium one.

*Case (c): firm  $H$  sells the add-on as standard but no consumer buys  $L^+$ .* Because  $P_l^* > 0$ , by Lemma 1 this strategy is strictly dominated by a local deviation whereby firm  $H$  sells the add-on optionally by increasing the add-on price while lowering the base price. This local deviation is exactly in the form of the maximization problem in the proposed equilibrium. Therefore, there is no profitable deviation.

*Case (d): firm  $H$  does not sell the add-on at all.* This deviation can take two forms. First, firm  $H$  can leave no demand for the bundle of  $L^+$ . The optimal base price is then the same as the equilibrium strategy  $P_h^*$ . However, the deviation forgoes the profit from selling the add-on, leading to reduced total profit. Second, firm  $H$  can leave some demand for the bundle of  $L^+$ . The profit is less than that in Case (a) because some positive sales of the add-on can improve the profit without affecting the profit of selling the base good. Therefore, in either case, the deviation is not profitable.

### A-2.3 No Other Equilibrium Exists

The only possible alternative equilibrium profile is such that firm  $L$  sells the add-on to some consumers, leading to the following pricing strategies:

$$P_h^* = \frac{1}{3}(2\bar{\theta} - \underline{\theta})(\Delta V - w) + \frac{1}{3}c, \quad p_h^* = w\theta_h^*; \quad P_l^{+*} = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})(\Delta V - w) + \frac{2}{3}c,$$

and  $p_l^* = w\theta_l^*$  if  $\alpha > \underline{\theta}$  but  $p_l^* \leq w\theta_l^*$  if  $\alpha \leq \underline{\theta}$ . The thresholds are given by  $\theta_{hl}^* = (\bar{\theta} + \underline{\theta})/3 - c/(3\Delta V - 3w)$ ,  $\theta_h^* = (\bar{\theta} + \alpha)/2$ , and  $\theta_l^* = (\underline{\theta} + \alpha)/2$ . To verify that this equilibrium does not sustain, it suffices to notice that  $\theta_{hl}^* < \theta_l^*$  when  $\alpha \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ .

### A-3 Equilibrium When $\alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$

Recall that the equilibrium profile where firm  $L$  sells the add-on optionally is summarized in Section 2.3 of the main text. The next two subsections establish the second and third statements of the proposition by showing that neither firm will deviate when  $\Delta V > \max\{\Delta_1, \Delta_2\}$ . The last subsection shows that no pure strategy equilibrium exists when  $\Delta V \leq \max\{\Delta_1, \Delta_2\}$ .

#### A-3.1 No Profitable Deviation for Firm $L$

*Case (a): firm  $L$  does not sell the add-on and some consumers of firm  $H$  buy the add-on.*

By Lemma A-1 this is not a profitable deviation as long as  $P_h^* > \alpha\Delta V$ . Notice that

$$\begin{aligned} \theta_{hl}^* - \theta_l^* &= \frac{P_h^* - P_l^{+*}}{\Delta V - w} - \frac{p_l^*}{w} = \frac{1}{\Delta V - w} [P_h^* - \frac{1}{2}(P_h^* + c - \underline{\theta}(\Delta V - w))] - \frac{1}{2}(\underline{\theta} + \alpha) \\ &= \frac{1}{2(\Delta V - w)}(P_h^* - \alpha\Delta V), \end{aligned}$$

where the second equality results from substituting the best-response prices of firm  $L$ . Therefore, as long as  $\alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$  and  $\Delta V > \Delta_1$  (the conditions for  $\theta_{hl}^* > \theta_l^*$ ), we have  $P_h^* > \alpha\Delta V$  and thus the deviation is not profitable.

*Case (b): firm  $L$  does not sell the add-on and all consumers of firm  $H$  buy the add-on.*

By Lemma A-1 this is not a profitable deviation as long as  $P_h^{+*} > \alpha(\Delta V + w)$ . Notice that

$$P_h^{+*} - \alpha(\Delta V + w) = P_h^* - \alpha\Delta V + p_h^* - c > P_h^* - \alpha\Delta V > 0,$$

where the first inequality follows from  $p_h^* > c$ , and the second inequality follows from the result of Case (a). Hence, any deviation in this case is not profitable.

*Case (c): firm  $L$  sells the add-on and all consumers of firm  $H$  buy the add-on.* The marginal consumer indifferent between the two firms now becomes  $\theta_{hl} = (P_h^{+*} - P_l^+)/\Delta V$ .

To ensure that all consumers buy the add-on from  $H$ , firm  $L$  has to set the total price  $P_l^+$  sufficiently low so that  $\theta_{hl} \geq p_h^*/w$ . But recall that the marginal consumer in the equilibrium profile satisfies:

$$\theta_{hl}^* < \theta_h^* \Leftrightarrow \frac{P_h^* - P_l^{+*}}{\Delta V - w} < \frac{p_h^*}{w} \implies \frac{P_h^{+*} - P_l^{+*}}{\Delta V} < \frac{p_h^*}{w}. \quad (\text{A-3})$$

Firm  $L$ 's deviation profit is given by  $\Pi_l = (\theta_{hl} - \underline{\theta})(P_l^+ - c) - (\theta_l - \underline{\theta})(p_l - c)$ , which is different from the equilibrium profit only in terms of the bundle component  $(\theta_{hl} - \underline{\theta})(P_l^+ - c)$ . The first-order condition of the Lagrangian for the constrained maximization of this bundle profit is given by  $P_l^+ = (P_h^{+*} - \Delta V \underline{\theta} + c - \lambda)/2$ . The constraint becomes

$$\theta_{hl} - \theta_h^* = \frac{P_h^{+*} - P_l^+}{\Delta V} - \frac{p_h^*}{w} < \frac{P_l^{+*} - P_l^+}{\Delta V} = \frac{1}{2\Delta V}(-p_h^* + w\underline{\theta} + \lambda),$$

where the inequality follows from Equation (A-3) and the last equality is obtained by substituting the best responses of firm  $L$ . Because  $p_h^* > w\underline{\theta}$ , to guarantee that the constraint is nonnegative, we need  $\lambda > 0$ . Thus, the constraint has to be binding. The corner solution exactly coincides with the one for the problem under the equilibrium because

$$\frac{P_h^{+*} - P_l^+}{\Delta V} = \frac{p_h^*}{w} \implies \frac{P_h^* - P_l^+}{\Delta V - w} = \frac{p_h^*}{w}.$$

Hence, the deviation profit is no greater than that obtained from the equilibrium strategy.

### A-3.2 No Profitable Deviation for Firm $H$

*Case (a):* firm  $H$  sells the add-on as optional but no consumer buys  $L^+$ . The marginal consumer indifferent between the two firms is  $\theta_{hl} = (P_h - P_l^*)/\Delta V$ . Firm  $H$  maximizes the profit  $\Pi_h = (\bar{\theta} - \theta_{hl})P_h + (\bar{\theta} - \theta_h)(p_h - c)$  subject to the constraint,  $\theta_{hl} \leq p_l^*/w$ , so that there is no demand for consumers who buy  $L^+$ . Note that the profit is also separable into a component for the base good and a component for the add-on price. It suffices to examine the profit component of the base good which is different from the equilibrium one. The first-order condition for the constrained optimization problem of the base profit is  $P_h = (P_l^* + \Delta V \bar{\theta} - \lambda)/2$ . There are two cases to consider. First, if  $\alpha > \underline{\theta}$ , then the equilibrium add-on price is  $p_l^* = (\underline{\theta}w + c)/2$  and satisfies  $\theta_{hl}^* > \theta_l^*$ . The constraint becomes

$$\theta_l^* - \theta_{hl} = \frac{p_l^*}{w} - \frac{P_h - P_l^*}{\Delta V} < \frac{P_h^* - P_h}{\Delta V} = \frac{1}{2\Delta V}(p_l^* - \bar{\theta}w + \lambda),$$

where the inequality follows from  $\theta_{hl}^* > \theta_l^*$ , and the last equality is obtained by substituting the best responses of firm  $H$ . Because  $p_l^* < \bar{\theta}w$ , for the constraint to be nonnegative we need  $\lambda > 0$ , implying that the constraint must be binding. The corner solution is the same as the one under the equilibrium problem because:

$$\frac{P_h - P_l^*}{\Delta V} = \frac{p_l^*}{w} \implies \frac{P_h - P_l^{+*}}{\Delta V - w} = \frac{p_l^*}{w}.$$

Therefore the deviation profit is no greater than the equilibrium profit.

Second, if  $\alpha \leq \underline{\theta}$ , then  $p_l^* \leq \underline{\theta}w$ . There is no demand for firm  $L$  at all. The equilibrium strategy is optimal.

*Case (b): firm  $H$  sells the add-on as standard and some consumers buy  $L^+$ .* Because  $P_l^{+*} > 0$ , by Lemma 1 this strategy is strictly dominated by selling the add-on optionally by increasing the add-on price and lowering the base price. This is exactly the equilibrium strategy. Therefore, deviation in this case is not profitable.

*Case (c): firm  $H$  sells the add-on as standard but no consumer buys  $L^+$ .* Firm  $H$ 's profit becomes  $\Pi_h = (\bar{\theta} - \theta_{hl})(P_h - c)$  subject to the constraint,  $\theta_{hl} \leq p_l^*/w$ , where the indifferent consumer becomes  $\theta_{hl} = (P_h^+ - P_l^*)/(\Delta V + w)$ . The first-order condition for the constrained optimization problem of the base profit is  $P_h^+ = (P_l^* + c + (\Delta V + w)\bar{\theta} - \lambda)/2$ . There are two cases to consider. First, if  $\alpha > \underline{\theta}$ , then the equilibrium add-on price is  $p_l^* = (\underline{\theta}w + c)/2$  and satisfies  $\theta_{hl}^* > \theta_l^*$ . The constraint under deviation becomes

$$\theta_l^* - \theta_{hl} = \frac{p_l^*}{w} - \frac{P_h^+ - P_l^*}{\Delta V + w} = \frac{1}{2(\Delta V + w)} \underbrace{[P_l^* - (\bar{\theta} - \underline{\theta} - \alpha)(\Delta V + w) - c + \lambda]}_M,$$

It can be shown that  $M < 0$  using  $\alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ . For the constraint to be nonnegative we need  $\lambda > 0$ , implying that the constraint must be binding. The corner solution is the same as the one under the deviation in Case (b) because:

$$\frac{P_h^+ - P_l^*}{\Delta V + w} = \frac{p_l^*}{w} \implies \frac{P_h^+ - P_l^{+*}}{\Delta V} = \frac{p_l^*}{w},$$

implying that the deviation is not profitable. Second, if  $\alpha \leq \underline{\theta}$ , then  $p_l^* \leq \underline{\theta}w$ . There is no demand for firm  $L$  at all. The equilibrium strategy is optimal.

*Case (d): firm  $H$  does not sell the add-on at all.* This deviation can take two forms. First, firm  $H$  can leave no demand for buying the add-on from firm  $L$ . The profit is smaller than that in Case (a) because a small positive sales of the add-on can improve profit without

affecting profit from the base good. Second, firm  $H$  can leave some demand for the low-quality bundle  $L^+$ . The optimal base price is then the same as the equilibrium strategy  $P_h^*$ . However, the deviation forgoes the profit from selling the add-on, resulting in a lower profit than the equilibrium profit. Hence, in either case, the deviation is not profitable.

### A-3.3 No Other Equilibrium Exists When $\Delta V > \max\{\Delta_1, \Delta_2\}$

The only possible alternative equilibrium is such that firm  $L$  does not sell the add-on, resulting in a pricing profile described in Section A-2 in this appendix. Firm  $H$ 's base price is  $P_h^* = (2\bar{\theta} - \underline{\theta})\Delta V/3$ . Since  $\alpha < (2\bar{\theta} - \underline{\theta})/3$ , we have  $P_h^* > \alpha\Delta V$ . Therefore, by Lemma A-1 firm  $L$  can profitably deviate by selling the add-on to some consumers.

### A-3.4 No Pure-Strategy Equilibrium Exists When $\Delta V \leq \max\{\Delta_1, \Delta_2\}$

Because in any possible equilibrium firm  $H$  will always sell the add-on as optional by Observation 1 in the main text, there are two possible equilibrium outcomes depending on firm  $L$ 's implementation:

*Case (a): firm  $L$  sells the add-on.* The equilibrium profiles are the same as those in the second and the third statements of the proposition. When  $\alpha > \underline{\theta}$ , both firms sell the add-on optionally. The fraction of consumers who buy the bundle from firm  $L$  is given by  $\theta_{hl}^* - \theta_l^*$ , which is positive only if  $\Delta V > \Delta_1$ .<sup>1</sup> When  $\alpha \leq \underline{\theta}$ , firm  $L$  will sell the add-on as standard, and the fraction of consumers who buy the bundle from the firm becomes  $\theta_{hl}^* - \underline{\theta}$  which is positive only if  $\Delta V > \Delta_2$ . Hence, no equilibrium is sustainable in either case if  $\Delta V \leq \max\{\Delta_1, \Delta_2\}$ .

*Case (b): firm  $L$  does not sell the add-on.* Following the same proof of A-3.3, firm  $L$  can profitably deviate by selling the add-on to some consumers.

## B Extension 1: Partial Market Coverage and Positive Base Cost

Let  $C > 0$  be the marginal cost of the base good. Further, the market is not fully covered and assume that  $\bar{\theta} > 2C/V$  to ensure that the market can accommodate both firms. The first observation is that firm  $H$ 's incentive to price discriminate is reinforced under vertical differentiation, just as in Lemma 1 and in Observation 1 in the main text.

**Lemma B-1.** *For firm  $H$ , selling the add-on as standard is strictly dominated by selling it as optional, if: (1)  $P_l - C > -\alpha\Delta V$  when firm  $L$  does not sell the add-on, or (2)  $P_l^+ - C > -\alpha(\Delta V - w)$  when firm  $L$  sells the add-on.*

**Observation B-1.** *In any pure-strategy equilibrium, firm  $H$  sells the add-on as optional.*

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<sup>1</sup>Note that  $\Delta_1 > \Delta_2$  when  $\alpha > \underline{\theta}$ , and  $\Delta_2 > \Delta_1$  when  $\alpha < \underline{\theta}$ .

The equilibrium outcomes then rest on firm  $L$ 's behaviors. Define  $\alpha_1, \alpha_2, \alpha_3$  as follows:

$$\alpha_1 = \frac{2(V + \Delta V)\bar{\theta} - C}{3V + 4\Delta V}, \quad \alpha_2 = \frac{2(V + \Delta V)(\Delta V - w)\bar{\theta} - (\Delta V - w)C}{(V + \Delta V)(3\Delta V - w) + (\Delta V - w)\Delta V}, \quad (\text{B-1})$$

$$\alpha_3 = \frac{\Delta V - w}{(V + \Delta V)w} \left[ (V + 2\Delta V - w)\bar{\theta} - (2V + 4\Delta V - w)\frac{C}{V} \right]. \quad (\text{B-2})$$

It can be shown that  $\alpha_1 > \alpha_2 > \min\{\frac{C}{V}, \alpha_3\}$ . The following proposition summarizes all the pure-strategy equilibria of the game.

**Proposition B-1.** (1) If  $\alpha \geq \alpha_1$ , there exists a unique equilibrium in which firm  $H$  sells the add-on as optional whereas firm  $L$  does not sell it; (2) If  $\frac{C}{V} < \alpha < \alpha_2$ , there exists a unique equilibrium in which both firms sell the add-on as optional; (3) If  $\alpha \leq \min\{\frac{C}{V}, \alpha_3\}$ , there exists a unique equilibrium in which firm  $H$  sells the add-on as optional whereas firm  $L$  sells it as standard; (4) No pure-strategy equilibrium exists in other parameter regions.

The proof follows exactly the same line of proof in Online Appendix A. First, a lemma for firm  $L$  is obtained:

**Lemma B-2.** For firm  $L$ , not selling the add-on is strictly dominated by selling it, if: (1)  $P_h - C > \alpha\Delta V$  when firm  $H$  sells optional add-on, or (2)  $P_h^+ - C > \alpha(\Delta V + w)$  when firm  $H$  sells standard add-on.

Next I derive the equilibria and some of the tedious proof that no profitable deviation exists will be omitted as the proof is very much similar to that in Online Appendix A.

### B-1 Equilibrium When $\alpha \geq \alpha_1$

The best response prices in this case are:

$$P_h = \frac{1}{2}[P_l + \Delta V\bar{\theta} + C]; \quad P_l = \frac{V}{2(V + \Delta V)}P_h + \frac{1}{2}C.$$

The resulting equilibrium profile becomes:

$$P_h^* = \frac{1}{3V + 4\Delta V}[2(V + \Delta V)\Delta V\bar{\theta} + 3(V + \Delta V)C], \quad p_h^* = w\theta_h^*;$$

$$P_l^* = \frac{1}{3V + 4\Delta V}[V\Delta V\bar{\theta} + (3V + 2\Delta V)C], \quad p_l^* > w\theta_{hl}^*,$$

where the thresholds are given by  $\theta_{hl}^* = \frac{(V+2\Delta V)\bar{\theta}+C}{3V+4\Delta V}$  and  $\theta_h^* = \frac{1}{2}(\bar{\theta} + \alpha)$ . It can be verified that there is no profitable deviation for either firm under this equilibrium.

The only possible alternative equilibrium profile is where firm  $L$  sells the add-on to some consumers. The equilibrium profile consists of the following pricing strategies:

$$P_h^* = \frac{V + \Delta V}{3V + 4\Delta V - w} [2(\Delta V - w)\bar{\theta} + 3C + c], \quad p_h^* = w\theta_h^*;$$

$$P_l^{+*} = \frac{1}{3V + 4\Delta V - w} [(V + w)(\Delta V - w)\bar{\theta} + (3V + 2\Delta V + w)C + 2(V + \Delta V)c],$$

and  $p_l^* = w\theta_l^*$  if  $\alpha > \underline{\theta}$  but  $p_l^* \leq w\theta_l^*$  if  $\alpha \leq \underline{\theta}$ . To verify that this equilibrium does not sustain, it suffices to notice that  $\theta_{hl}^* < \theta_l^*$  when  $\alpha \geq \frac{2(V+\Delta V)}{3V+4\Delta V-w}\bar{\theta}$ .

## B-2 Equilibrium When $\alpha < \alpha_2$

Under this equilibrium, firm  $H$  sells the add-on as optional while firm  $L$  sells it either as optional or as standard. Firm  $L$  maximizes its profit:

$$\Pi_l = (\theta_{hl} - \underline{\theta})(P_l^+ - C^+) - (\theta_l - \underline{\theta})(p_l - c) - (\theta_0 - \underline{\theta})(P_l^+ - p_l - C).$$

The first-order conditions are  $P_l^+ = \frac{1}{2}[\frac{V+w}{V+\Delta V}P_h + C^+]$  and  $p_l = \frac{1}{2}[\frac{w}{V+\Delta V}P_h + c]$ . Notice that, regardless of firm  $H$ 's prices, the demand of consumers who buy the base without the add-on from firm  $L$  is given by

$$\theta_l - \theta_0 = \frac{p_l}{w} - \frac{P_l}{V} = \frac{1}{2}\left(\alpha - \frac{C}{V}\right). \quad (\text{B-3})$$

Therefore, if  $\alpha > \frac{C}{V}$ , firm  $L$  sells the add-on as optional, but as standard if  $\alpha \leq \frac{C}{V}$ . Now consider firm  $H$ 's profit maximization problem:

$$\Pi_h = (\bar{\theta} - \theta_{hl})(P_h - C) + (\bar{\theta} - \theta_h)(p_h - c).$$

The first-order condition for  $P_h$  is  $P_h = (P_l^+ + (\Delta V - w)\bar{\theta} + C)/2$ . The best-response prices lead to the following equilibrium strategies:

$$P_h^* = \frac{V + \Delta V}{3V + 4\Delta V - w} [2(\Delta V - w)\bar{\theta} + 3C + c], \quad p_h^* = w\theta_h^*;$$

$$P_l^{+*} = \frac{1}{3V + 4\Delta V - w} [(V + w)(\Delta V - w)\bar{\theta} + (3V + 2\Delta V + w)C + 2(V + \Delta V)c],$$

and  $p_l^* = w\theta_l^*$  if  $\alpha > \underline{\theta}$  but  $p_l^* \leq w\theta_l^*$  if  $\alpha \leq \underline{\theta}$ . The constraint  $\theta_{hl}^* > \theta_l^*$  is equivalent to  $P_h^* - C > \alpha\Delta V$ , which holds if and only if  $\alpha < \alpha_2$ . When  $\alpha \leq \frac{C}{V}$ , firm  $L$  sells the add-on as standard. To guarantee equilibrium existence, the constraint becomes  $\theta_{hl}^* \geq C/V$ , which leads to  $\alpha \leq \alpha_3$ . Hence, the equilibrium exists if and only if  $\alpha \leq \min\{\frac{C}{V}, \alpha_3\}$ . It can be

verified that there is no profitable deviation for either firm under this equilibrium.

To show that the above equilibrium is unique when  $\alpha \in [0, \min\{\frac{C}{V}, \alpha_3\}] \cup (\frac{C}{V}, \alpha_2)$ , it suffices to consider the only alternative equilibrium profile in which firm  $L$  does not sell the add-on. The equilibrium profile is summarized in Section B-1. Firm  $H$ 's base price is then  $P_h^* = (2\bar{\theta}\Delta V + 3C)(V + \Delta V)/(3V + 4\Delta V)$ . It follows that  $P_h^* - C > \alpha\Delta V$  if and only if  $\alpha < \alpha_1$ . Hence, by Lemma B-2 firm  $L$  can profitably deviate by selling the add-on.

### B-3 No Other Pure-Strategy Equilibrium Exists

The only parameter region left to consider is  $\alpha \in [\alpha_2, \alpha_1)$  and  $\alpha \in (\alpha_3, \frac{C}{V})$  if  $\alpha_3 < \frac{C}{V}$ . Because in equilibrium firm  $H$  will always sell the add-on as optional by Observation B-1, there are two possible equilibrium outcomes depending on firm  $L$ 's implementation.

*Case (a): firm  $L$  sells the add-on.* The equilibrium profiles are the same as those in the second and the third statements of the proposition. When  $\alpha > C/V$ , both firms sell optional add-on. The fraction of consumers who buy the bundle from firm  $L$  is given by  $\theta_{hl}^* - \theta_l^*$  which is positive only if  $\alpha < \alpha_2$ . When  $\alpha \leq C/V$ , firm  $L$  will sell the add-on as standard, and the fraction of consumers who buy the bundle from firm  $L$  becomes  $\theta_{hl}^* - \underline{\theta}$  which is positive only if  $\alpha < \alpha_3$ . In either case the equilibrium is not sustainable.

*Case (b): firm  $L$  does not sell the add-on at all.* Following the same proof above, firm  $L$  can profitably deviate by selling the add-on.

## C Extension 2: Asymmetric Add-on

Consider the extension that the cost and value of the add-on are different for the two firms:  $(c_h, w_h)$  for firm  $H$  and  $(c_l, w_l)$  for firm  $L$ . Then the cost-to-value ratio also varies across the firms:  $\alpha_h = c_h/w_h$  and  $\alpha_l = c_l/w_l$ . No restriction is placed on whether the cost and/or value should be higher for firm  $H$  than for firm  $L$ , and thus the cost-to-value ratio can be either higher or lower for firm  $L$ . It is, however, assumed that the quality premium  $\Delta V$  remains higher than the maximum value of the add-ons,  $\Delta V > \max\{w_h, w_l\}$ . In addition, the assumption that the cost of the add-on is not unreasonably large,  $\alpha_h < \bar{\theta}$ , still applies so that firm  $H$  always sells the add-on in equilibrium. Other assumptions follow from the main analysis. With this specification, the conclusions in the preceding analysis are qualitatively unchanged. Let

$$\Delta_{1,l} = \frac{2\bar{\theta} - \underline{\theta} - \alpha_l}{2\bar{\theta} - \underline{\theta} - 3\alpha_l} \cdot w_l, \quad \text{and} \quad \Delta_{2,l} = \frac{\bar{\theta} - 2\underline{\theta} + \alpha_l}{\bar{\theta} - 2\underline{\theta}} \cdot w_l.$$

**Proposition C-1.** (1) If  $\alpha_l \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ , there exists a unique equilibrium in which  $H$  sells

the add-on as optional whereas  $L$  does not sell it; (2) If  $\underline{\theta} < \alpha_l < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ , there exists a unique equilibrium when  $\Delta V > \Delta_{1,l}$ , in which both firms sell the add-on as optional; (3) If  $\alpha_l \leq \underline{\theta}$ , there exists a unique equilibrium when  $\Delta V > \Delta_{2,l}$ , in which  $H$  sells the add-on as optional whereas  $L$  sells it as standard; (4) No pure-strategy equilibrium exists in other parameter regions.

The proof of the proposition largely follows the one for Proposition 1 of the main model. Thus, I will just derive the equilibrium pricing in what follows and omit the tedious proof that no deviation can improve equilibrium profit.

### C-1 Equilibrium When the Add-on is Too Costly ( $\alpha_l \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ )

In this case, the indifferent consumer becomes  $\theta_{hl} = (P_h - P_l)/\Delta V$ . The two firms' profits are given by

$$\Pi_h = (\bar{\theta} - \frac{P_h - P_l}{\Delta V})P_h + (\bar{\theta} - \frac{p_h}{w_h})(p_h - c_h); \quad \Pi_l = (\frac{P_h - P_l}{\Delta V} - \underline{\theta})P_l.$$

It is straightforward to find the following equilibrium prices using the first-order conditions:

$$P_h^* = \frac{1}{3}(2\bar{\theta} - \underline{\theta})\Delta V, \quad p_h^* = w_h\theta_h^*; \quad P_l^* = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})\Delta V, \quad p_l^* > w_l\theta_{hl}^*,$$

where the thresholds are given by  $\theta_{hl}^* = \frac{1}{3}(\bar{\theta} + \underline{\theta})$  and  $\theta_h^* = \frac{1}{2}(\bar{\theta} + \alpha_h)$ .

### C-2 Equilibrium When the Add-on is Not Too Costly ( $\alpha_l < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ )

In this case, the indifferent consumers become  $\theta_{hl} = (P_h - P_l^+)/(\Delta V - w_l)$ . The two firms' profits are given by

$$\Pi_h = (\bar{\theta} - \frac{P_h - P_l^+}{\Delta V - w_l})P_h + (\bar{\theta} - \frac{p_h}{w_h})(p_h - c_h); \quad \Pi_l = (\frac{P_h - P_l^+}{\Delta V - w_l} - \underline{\theta})(P_l^+ - c_l) - (\frac{p_l}{w_l} - \underline{\theta})(p_l - c_l).$$

It is straightforward to derive the solution using first-order conditions:

$$P_h^* = \frac{1}{3}(2\bar{\theta} - \underline{\theta})(\Delta V - w_l) + \frac{1}{3}c_l, \quad p_h^* = w_h\theta_h^*,$$

$$P_l^{+*} = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})(\Delta V - w_l) + \frac{2}{3}c_l, \quad p_l^* \begin{cases} = w_l\theta_l^* & \text{if } \alpha > \underline{\theta} \\ \leq w_l\underline{\theta} & \text{if } \alpha \leq \underline{\theta} \end{cases}$$

where the thresholds are given by  $\theta_{hl}^* = (\bar{\theta} + \underline{\theta})/3 - c_l/(3\Delta V - 3w_l)$ ,  $\theta_h^* = (\bar{\theta} + \alpha_h)/2$ , and  $\theta_l^* = (\underline{\theta} + \alpha_l)/2$ .

## D Extension 3: Asymmetric Marginal Cost of the Base Good

Consider next the extension that the marginal cost for firm  $H$  is higher. The marginal cost of firm  $L$  remains zero by normalization, but the cost for firm  $H$  now becomes  $\Delta C \geq 0$ . Define the cost-to-value ratio for the base good as  $A = \Delta C/\Delta V$ , which measures the unit cost of the quality premium. Let

$$\bar{A} = \frac{(\bar{\theta} - 2\underline{\theta} + 3\alpha)\Delta V - (\bar{\theta} - 2\underline{\theta} + \alpha)w}{2\Delta V}.$$

This is the bound above which it is costly for firm  $H$  to serve additional consumers who buy only the base by lowering the base price. Therefore we shall focus on the case  $A < \bar{A}$ .<sup>2</sup> Other assumptions follow from the main analysis. Let

$$\Delta'_1 = \max\left\{w, \frac{2\bar{\theta} - \underline{\theta} - \alpha}{2A + 2\bar{\theta} - \underline{\theta} - 3\alpha} \cdot w\right\}, \quad \text{and} \quad \Delta'_2 = \max\left\{w, \frac{\bar{\theta} - 2\underline{\theta} + \alpha}{A + \bar{\theta} - 2\underline{\theta}} \cdot w\right\}.$$

**Proposition D-1.** (1) If  $\alpha \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta}) + \frac{2}{3}A$ , there exists a unique equilibrium in which  $H$  sells the add-on as optional whereas  $L$  does not sell it; (2) If  $\underline{\theta} < \alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta}) + \frac{2}{3}A$ , there exists a unique equilibrium when  $\Delta V > \Delta'_1$ , in which both firms sell the add-on as optional; (3) If  $\alpha \leq \underline{\theta}$ , there exists a unique equilibrium when  $\Delta V > \Delta'_2$ , in which  $H$  sells the add-on as optional whereas  $L$  sells it as standard; (4) No pure-strategy equilibrium exists in other parameter regions.

Again, the proof largely follows the proof of Proposition 1 of the main model. The equilibrium pricing will be summarized below with the detailed proof omitted.

### D-1 Equilibrium When the Add-on is Too Costly ( $\alpha \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta}) + \frac{2}{3}A$ )

To derive the equilibrium profile under this case, note that the indifferent consumer becomes  $\theta_{hl} = (P_h - P_l)/\Delta V$ . The two firms' profits are given by

$$\Pi_h = \left(\bar{\theta} - \frac{P_h - P_l}{\Delta V}\right)(P_h - \Delta C) + \left(\bar{\theta} - \frac{p_h}{w}\right)(p_h - c); \quad \Pi_l = \left(\frac{P_h - P_l}{\Delta V} - \underline{\theta}\right)P_l,$$

The first-order conditions lead to the following equilibrium profile:

$$P_h^* = \frac{1}{3}(2\bar{\theta} - \underline{\theta})\Delta V + \frac{2}{3}\Delta C, \quad p_h^* = w\theta_h^*, \quad (\text{D-1})$$

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<sup>2</sup> When  $A \geq \bar{A}$  there exists equilibrium where firm  $H$  sells the add-on as standard.

$$P_l^* = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})\Delta V + \frac{1}{3}\Delta C, \quad p_l^* > w\theta_{hl}^*, \quad (\text{D-2})$$

where the thresholds are given by  $\theta_{hl}^* = (\bar{\theta} + \underline{\theta} + A)/3$  and  $\theta_h^* = (\bar{\theta} + \alpha)/2$ .

## D-2 Equilibrium When the Add-on is Not Too Costly ( $\alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta}) + \frac{2}{3}A$ )

In this case, the indifferent consumer becomes  $\theta_{hl} = (P_h - P_l)/\Delta V$ . The two firms' profits are given by

$$\Pi_h = \left(\bar{\theta} - \frac{P_h - P_l^+}{\Delta V - w}\right)(P_h - \Delta C) + \left(\bar{\theta} - \frac{p_h}{w}\right)(p_h - c); \quad \Pi_l = \left(\frac{P_h - P_l^+}{\Delta V - w} - \underline{\theta}\right)P_l - \left(\frac{p_l}{w} - \underline{\theta}\right)(p_l - c),$$

The equilibrium profile now becomes:

$$\begin{aligned} P_h^* &= \frac{1}{3}(2\bar{\theta} - \underline{\theta})(\Delta V - w) + \frac{1}{3}c + \frac{2}{3}\Delta C, & p_h^* &= w\theta_h^*, \\ P_l^{*+} &= \frac{1}{3}(\bar{\theta} - 2\underline{\theta})(\Delta V - w) + \frac{2}{3}c + \frac{1}{3}\Delta C, & p_l^* &\begin{cases} = w\theta_l^* & \text{if } \alpha > \underline{\theta} \\ \leq w\underline{\theta} & \text{if } \alpha \leq \underline{\theta} \end{cases} \end{aligned}$$

where the thresholds are given by

$$\theta_{hl}^* = \frac{1}{3}(\bar{\theta} + \underline{\theta}) - \frac{c - \Delta C}{3(\Delta V - w)}, \quad \theta_h^* = \frac{1}{2}(\bar{\theta} + \alpha), \quad \theta_l^* = \frac{1}{2}(\underline{\theta} + \alpha).$$

## E Extension 4: Imperfectly Correlated Tastes

In this extension, I relax the assumption that the tastes for the base and the add-on are perfectly correlated. For tractability, I focus on the case where there is at least some correlation between the two tastes. Suppose the taste for the base good remains to be  $\theta$ , but the taste for the add-on is given by  $\lambda = \beta\theta + e$ , where the constant  $\beta > 0$  and  $e \sim U[\underline{e}, \bar{e}]$ . Without loss of generality, assume that  $\beta = 1$ . Therefore, each consumer is summarized by a pair of taste parameters  $(\theta, e)$ . The utility of buying from firm  $j$  for type- $(\theta, e)$  consumer is then

$$U_j = \begin{cases} \theta V_j - P_j & \text{if only the base good is purchased;} \\ \theta V_j + (\theta + e)w - P_j - p_j & \text{if both the base good and the add-on are purchased.} \end{cases}$$

Let  $\Delta e = \bar{e} - \underline{e}$ . To reduce the complexity of the solution, let us assume that  $\bar{e} > \alpha - \bar{\theta}$  and  $\underline{e} < \frac{1}{3}(2\alpha - 2\bar{\theta} + \bar{e})$ . The implication is that type- $(\bar{\theta}, \underline{e})$  consumer does not buy the add-on in

equilibrium. Define the following quantities

$$\Delta_1'' = \max\left\{\frac{\bar{\theta} - \alpha - \underline{e}}{\bar{\theta} - 2\alpha + 2\underline{e}}w, \frac{\alpha + \bar{e} - \underline{\theta}}{2\alpha - 2\bar{e} - \underline{\theta}}w\right\}, \quad \Delta_2'' = \max\left\{\frac{\bar{\theta} - 2\underline{\theta} + \alpha - 3\underline{e}}{\bar{\theta} - 2\underline{\theta}}w, \frac{\alpha + \bar{e} - \underline{\theta}}{2\alpha - 2\bar{e} - \underline{\theta}}w\right\}$$

**Proposition E-1.** (1) If  $\alpha \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta}) + \bar{e}$ , there exists a unique equilibrium in which  $H$  sells the add-on as optional whereas  $L$  does not sell it; (2) If  $\underline{\theta} + \underline{e} < \alpha < \frac{1}{2}\bar{\theta} + \underline{e}$ , there exists a unique equilibrium when  $\Delta V > \Delta_1''$ , in which both firms sell the add-on as optional; (3) If  $\alpha \leq \underline{\theta} + \underline{e}$ , there exists a unique equilibrium when  $\Delta V > \Delta_2''$ , in which  $H$  sells the add-on as optional whereas  $L$  sells it as standard; (4) No pure-strategy equilibrium exists in other parameter regions.

### E-1 Equilibrium When the Add-on is Too Costly ( $\alpha \geq \frac{1}{3}(2\bar{\theta} - \underline{\theta}) + \bar{e}$ )

To derive the equilibrium profile under this case, note that the indifferent consumers between the two firms are defined by  $\theta_{hl} = (P_h - P_l)/\Delta V$ . The intra-marginal consumers for firm  $H$  are defined by  $e + \theta = p_h/w$ , which intersect with the upper bounds of  $e$  and  $\theta$  at  $e_h = \frac{p_h}{w} - \bar{\theta}$  and  $\theta_h = (\frac{p_h}{w} - \bar{e})$ . The two firms' profits are then

$$\Pi_h = \Delta e \cdot \left(\bar{\theta} - \frac{P_h - P_l}{\Delta V}\right)P_h + \pi_h(p_h); \quad \Pi_l = \left(\frac{P_h - P_l}{\Delta V} - \underline{\theta}\right)P_l,$$

where  $\pi_h(p_h) = \frac{1}{2}(\bar{\theta} - \theta_h)(\bar{e} - e_h)(p_h - c) = \frac{1}{2}(\bar{\theta} - \frac{p_h}{w} + \bar{e})^2(p_h - c)$ . Solving for the equilibrium prices yield

$$P_h^* = \frac{1}{3}(2\bar{\theta} - \underline{\theta})\Delta V, \quad p_h^* = \frac{w}{3}(\bar{\theta} + 2\alpha + \bar{e}); \quad P_l^* = \frac{1}{3}(\bar{\theta} - 2\underline{\theta})\Delta V, \quad p_l^* > w\theta_{hl}^*,$$

where the thresholds are given by  $\theta_{hl}^* = \frac{1}{3}(\bar{\theta} + \underline{\theta})$  and  $\theta_h^* = \frac{1}{3}(\bar{\theta} + 2\alpha - 2\bar{e})$ .

### E-2 Equilibrium When the Add-on is Not Too Costly ( $\alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta}) + \bar{e}$ )

In this case, the indifferent consumers are defined by the condition  $\theta(\Delta V - w) - ew = P_h - P_l^+$ . To determine the market shares, it is sufficient to know the upper and lower bounds of the marginal consumers

$$\theta_{hl,u} = \frac{\bar{e}w + P_h - P_l^+}{\Delta V - w}, \quad \text{and} \quad \theta_{hl,l} = \frac{\underline{e}w + P_h - P_l^+}{\Delta V - w}.$$

Therefore, firm  $H$ 's profit is

$$\Pi_h = \Delta e \cdot \left[ \bar{\theta} - \frac{1}{2}(\theta_{hl,u} + \theta_{hl,l}) \right] P_h + \underbrace{\frac{1}{2}(\bar{\theta} - \theta_h)(\bar{e} - e_h)(p_h - c)}_{\pi_h(p_h)}, \quad (\text{E-1})$$

where  $\theta_h = (\frac{p_h}{w} - \bar{e})$  and  $e_h = \frac{p_h}{w} - \bar{\theta}$ . Similar to the case of perfect correlation, the profit maximization problem reduces to maximizing two profit components separately using the base price  $P_h$  and the add-on price  $p_h$ .

Firm  $L$ 's profit is

$$\Pi_l = \Delta e \cdot \left[ \frac{1}{2}(\theta_{hl,u} + \theta_{hl,l}) - \underline{\theta} \right] (P_l^+ - c) - \underbrace{\frac{1}{2}(\theta_l - \underline{\theta})(e_l - \underline{e})(p_l - c)}_{\pi_l(p_l)}. \quad (\text{E-2})$$

where  $\theta_l = (\frac{p_l}{w} - \underline{e})$  and  $e_l = \frac{p_l}{w} - \underline{\theta}$ . Similar to the case of perfect correlation, maximizing profit reduces to maximizing the two components of the profit separately using the bundle price  $P_l^+$  and the add-on price  $p_l$ .

The price competition between the two firms pins down the equilibrium prices

$$P_h^* = \frac{1}{3}(\Delta V - w)(2\bar{\theta} - \underline{\theta}) + \frac{1}{3}c + \frac{1}{2}(\bar{e} + \underline{e})w, \quad \text{and} \quad P_l^{+*} = \frac{1}{3}(\Delta V - w)(\bar{\theta} - 2\underline{\theta}) + \frac{2}{3}c + \frac{1}{2}(\bar{e} + \underline{e})w,$$

Independent from the strategic interaction that determines equilibrium market shares, the firms set the optimal add-on prices. Add-on prices  $p_h$  and  $p_l$  are chosen to maximize  $\pi_h(p_h)$  and  $\pi_l(p_l)$  in Equations (E-1) and (E-2) respectively, which lead to

$$p_h^* = \frac{1}{3}(\bar{\theta} + \bar{e} + 2\alpha)w, \quad \text{and} \quad p_l^* = \frac{1}{3}(\underline{\theta} + \underline{e} + 2\alpha)w.$$

## F Proof of Proposition 2 and Two Additional Results on Commitment

Before proving Proposition 2, I first analyze two benchmark cases assuming partial commitment ability: (1) both firms can only make commitment to the no-add-on policy, and (2) both can only make commitment to the standard-add-on policy. Building on these two results, in the last subsection, I provide the proof of Proposition 2 assuming both firms can commit to either policy (full commitment ability).

## F-1 Firms Can Only Commit to Not Selling the Add-on

**Proposition F-1.** *Suppose that firms can commit to the no-add-on policy. In any pure-strategy equilibrium, (a) firm  $H$  does not commit; (b) firm  $L$  commits if  $\alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ , and is indifferent between committing and not committing if otherwise.*

First observe that firm  $H$  will never commit to the no-add-on policy. By committing to not selling the add-on, firm  $H$  always uses the base good to compete with firm  $L$  in the second stage. Breaking the commitment allows it to sell the add-on to a fraction of consumers who have the highest taste for quality. This does not affect the profit from selling the base good but can generate additional profit from selling the add-on. The total profit is then strictly improved.

Given this observation, firm  $H$  will always sell the add-on as optional in any equilibrium, applying Observation 1. It remains to consider firm  $L$ 's strategies. If firm  $L$  commits to not selling the add-on, then its equilibrium profit is given by  $\Pi_l^{(opt,no)} = \frac{1}{9}(\bar{\theta} - 2\underline{\theta})^2 \Delta V$ .

If firm  $L$  does not commit, then the equilibrium prices and profits are exactly the same as in Proposition 1. Whether or not it is optimal to commit depends on the size of  $\alpha$ . When  $\alpha > \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ , it will not sell the add-on in the second stage and thus obtains the same profit  $\Pi_l^{(opt,no)}$  as in the commitment case. Hence, committing or not does not change the equilibrium profit. When  $\underline{\theta} < \alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ , the firm chooses to commit if and only if

$$\Pi_l^{(opt,no)} - \Pi_l^{(opt,opt)} = \frac{w}{9} \left[ \underbrace{(\bar{\theta} - 2\underline{\theta})^2 + 2\alpha(\bar{\theta} - 2\underline{\theta}) - \frac{9}{4}(\underline{\theta} - \alpha)^2}_Y - \frac{\alpha^2 w}{\Delta V - w} \right] > 0. \quad (\text{F-1})$$

It can be shown that  $Y > 0$  when  $\underline{\theta} < \alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ . Therefore, commitment is optimal if  $\Delta V > (1 + \alpha^2/Y)w \equiv \Delta_l$ . If  $\Delta V < \Delta_l$ , then the firm does not commit, leading to the possibility of an optional add-on in equilibrium as in Proposition 1. However, this equilibrium exists only when  $\Delta V > \Delta_1$ . It can be shown that  $\Delta_l < \Delta_1$ , so this equilibrium is not sustainable.

When  $\alpha \leq \underline{\theta}$ , the firm chooses to commit if and only if

$$\Pi_l^{(opt,no)} - \Pi_l^{(opt,std)} = \frac{w}{9} \left[ \underbrace{(\bar{\theta} - 2\underline{\theta})^2 + 2\alpha(\bar{\theta} - 2\underline{\theta})}_{Y'} - \frac{\alpha^2 w}{\Delta V - w} \right] > 0.$$

Since  $Y' > 0$ , committing not to offer the add-on is optimal if  $\Delta V > (1 + \alpha^2/Y')w \equiv \Delta'_l$ . If  $\Delta V < \Delta'_l$ , then the firm does not commit, leading to the possibility of a standard add-on in

equilibrium as in Proposition 1. However, this equilibrium exists only when  $\Delta V > \Delta'_1$ . It can be shown that  $\Delta'_1 < \Delta_2$ , so this equilibrium is not sustainable.

## F-2 Firms Can Only Commit to Standard Add-on

**Proposition F-2.** *Suppose that firms can commit to the standard-add-on policy (bundling). There exists a threshold  $\hat{\alpha} > \frac{1}{3}(2\bar{\theta} - \underline{\theta})$  such that, in any pure-strategy equilibrium: (a) firm  $H$  commits if  $\alpha < \hat{\alpha}$ , but does not (and sells the add-on as optional) if otherwise; (b) firm  $L$  does not commit if  $\alpha > \underline{\theta}$ , and is indifferent between committing and not if otherwise.*

*Case (a):  $\alpha > \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ .* First it is easy to show that firm  $L$  has no incentive to sell the add-on in the second stage, regardless of  $H$ 's commitment choices, because of its high cost. Therefore, it remains to consider firm  $H$ 's incentive to commit, given firm  $L$  doesn't sell the add-on. The equilibrium profit of committing to bundling is given by

$$\Pi_h^{(std,no)} = \frac{1}{9(\Delta V + w)} \left[ (2\bar{\theta} - \underline{\theta})(\Delta V + w) - c \right]^2.$$

The equilibrium profit of not committing is given in Table 1 in the main text. Therefore, firm  $H$  commits if and only if

$$\Pi_h^{(std,no)} - \Pi_h^{(opt,no)} = \frac{w}{9} \underbrace{\left[ (2\bar{\theta} - \underline{\theta})^2 - 2\alpha(2\bar{\theta} - \underline{\theta}) - \frac{9}{4}(\bar{\theta} - \alpha)^2 \right]}_X + \frac{\alpha^2 w}{\Delta V + w} > 0. \quad (\text{F-2})$$

There exists a threshold  $\hat{\alpha} > \frac{1}{3}(2\bar{\theta} - \underline{\theta})$  such that when  $\alpha < \hat{\alpha}$ , the above inequality holds. When  $\alpha > \hat{\alpha}$ , it is optimal for  $H$  to not commit and sell the add-on optionally.

*Case (b):  $\underline{\theta} < \alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ .* First note that firm  $L$  has no incentive to commit to bundling, because not committing allows the firm to charge some consumers for a fee to recover some cost. Therefore, it remains to consider firm  $H$ 's incentive to commit, given that firm  $L$  sells the add-on to some consumers. The equilibrium profit of committing to bundling is given by  $\Pi_h^{(std,opt)} = \frac{1}{9}(2\bar{\theta} - \underline{\theta})^2 \Delta V$ . The equilibrium profit of not committing is given in Table 1 in the main text. Therefore, firm  $H$  commits if and only if

$$\Pi_h^{(std,opt)} - \Pi_h^{(opt,opt)} = \frac{w}{9} \left[ (2\bar{\theta} - \underline{\theta})^2 - 2\alpha(2\bar{\theta} - \underline{\theta}) - \frac{9}{4}(\bar{\theta} - \alpha)^2 - \frac{\alpha^2 w}{\Delta V - w} \right] > 0.$$

There exists a threshold  $\hat{\alpha}_h$  such that when  $\alpha < \hat{\alpha}_h$ , the above inequality holds. When  $\alpha > \hat{\alpha}_h$ , it is optimal for the firm to not commit. However, for such equilibrium to exist,

we need  $\alpha < (2\bar{\theta} - \underline{\theta})(\Delta V - w)/(3\Delta V - w) \equiv \hat{\alpha}_1$ .<sup>3</sup> With some algebra, one can show that  $\hat{\alpha}_h > \hat{\alpha}_1$ . Therefore, not committing does not constitute an equilibrium in this case.

*Case (c):*  $\alpha < \underline{\theta}$ . First note that firm  $L$  has no strict incentive to commit to bundling because, even if it doesn't commit, it will sell the add-on to all consumers anyway. It then remains to consider firm  $H$ 's incentive to commit, which is exactly the same condition as in Case (b): it will commit if and only if  $\Pi_h^{(std,opt)} - \Pi_h^{(opt,opt)} > 0$ . This is because firm  $L$ 's commitment choice does not affect firm  $H$ 's problem as long as  $L$  sells the add-on to some consumers. The condition holds when  $\alpha < \hat{\alpha}_h$ . When  $\alpha > \hat{\alpha}_h$ , it is optimal for the firm to not commit. However, for such equilibrium to exist, we need  $\alpha < (\bar{\theta} - 2\underline{\theta})(\Delta V - w)/w \equiv \hat{\alpha}_2$ .<sup>4</sup> With some algebra, one can show that  $\hat{\alpha}_h > \hat{\alpha}_2$ . Therefore, not committing does not constitute an equilibrium in this case.

### F-3 Full Commitment Ability (Proposition 2)

*Case (a):*  $\alpha > \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ . Following the same line of proof of the proposition in Section F-2, we have: when  $\alpha < \hat{\alpha}$ , there exists an equilibrium in which firm  $H$  commits to bundling; When  $\alpha > \hat{\alpha}$ , there exists an equilibrium in which firm  $H$  does not commit and sells the add-on as optional. In both cases, firm  $L$  does not sell the add-on (and is indifferent between committing to no-add-on and not).

*Case (b):*  $\underline{\theta} < \alpha < \frac{1}{3}(2\bar{\theta} - \underline{\theta})$ . First note that if firm  $L$  commits to the no-add-on policy, then  $\Pi_h^{(std,no)} > \Pi_h^{(opt,no)}$  given that  $X > 0$  (defined in Equation F-2). If firm  $H$  commits to bundling, then

$$\Pi_l^{(std,no)} - \Pi_l^{(std,opt)} = \frac{w}{9} \left[ \underbrace{(\bar{\theta} - 2\underline{\theta})^2 + 2\alpha(\bar{\theta} - 2\underline{\theta}) - \frac{9}{4}(\underline{\theta} - \alpha)^2}_Y + \frac{\alpha^2 w}{\Delta V + w} \right] > 0,$$

always holds given that  $Y > 0$ . Therefore, the profile that firm  $L$  commits to the no-add-on policy whereas firm  $H$  commits to the standard-add-on policy constitutes an equilibrium. The other possible equilibrium would be that neither firm commit and thus they both sell it as optional in the second-stage game. However, as shown in the proof of Proposition F-1, this equilibrium is not sustainable because  $\Delta_l < \Delta_1$ .

*Case (c):*  $\alpha < \underline{\theta}$ . Following the same argument of Case (b), the profile that firm  $L$  commits to the no-add-on policy whereas firm  $H$  commits to the standard-add-on policy constitutes an equilibrium. It remains to verify that this is the only possible equilibrium

<sup>3</sup> This condition follows from  $\Delta V > w(2\bar{\theta} - \underline{\theta} - \alpha)/(2\bar{\theta} - \underline{\theta} - 3\alpha) \equiv \Delta_1$ .

<sup>4</sup> This condition follows from  $\Delta V > w(\bar{\theta} - 2\underline{\theta} + \alpha)/(\bar{\theta} - 2\underline{\theta}) \equiv \Delta_2$ .

under this parameter region. The alternative equilibrium would be that neither firm commits and thus  $H$  sells the add-on as optional whereas  $L$  sells it to all consumers. However, as shown in the proof of Proposition F-1, this equilibrium is not sustainable because  $\Delta'_l < \Delta_2$ .

## G Proof of Proposition 3

### G-1 When $\alpha < (\bar{\theta} + \underline{\theta})/3$

For the equilibrium in which firm  $L$  sells the add-on as *optional* to exist, the following incentive constraints have to be satisfied: (a)  $\theta_{hl}^* > \theta_l^*$ ; (b)  $\theta_l^* > \underline{\theta} \Rightarrow \theta_{hl}^* > 2\underline{\theta} - \alpha$ . For the equilibrium in which firm  $L$  sells the add-on as *standard* to exist, it requires that (c)  $\theta_{hl}^* > \underline{\theta}$ ; (d)  $\theta_l^* < \underline{\theta}$ . Note that (a) and (b) imply (c). Define the following quantities:

$$\Delta_0^{(u)} = \frac{\bar{\theta} - 2\underline{\theta} + \alpha}{\bar{\theta} - 2\underline{\theta}} \cdot w, \quad \Delta_1^{(u)} = \frac{2\bar{\theta} + \underline{\theta} - 3\alpha}{2(\bar{\theta} + \underline{\theta} - 3\alpha)} \cdot w, \quad \text{and} \quad \Delta_2^{(u)} = \frac{2\bar{\theta} - 9\underline{\theta} + 7\alpha}{2(\bar{\theta} - 5\underline{\theta} + 3\alpha)} \cdot w.$$

Constraint (a) is equivalent to  $\theta_{hl}^* > \alpha$  since  $\theta_l^* = \frac{1}{2}(\theta_{hl}^* + c)$ . Note that  $\theta_{hl}^* - \alpha$  is positive as long as  $\Delta V > \Delta_1^{(u)}$ .<sup>5</sup> Constraint (b) is equivalent to  $\theta_{hl}^* > 2\underline{\theta} - \alpha$  since  $\theta_l^* = \frac{1}{2}(\theta_{hl}^* + \alpha)$ . Note that  $\theta_{hl}^* - (2\underline{\theta} - \alpha)$  is positive as long as  $\Delta V > \Delta_2^{(u)}$ . Note further that  $\Delta_1^{(u)} > \Delta_2^{(u)}$  is equivalent to  $2(\alpha - \underline{\theta})(\bar{\theta} - 2\underline{\theta} + 6\alpha) > 0$ , which holds if and only if  $\alpha > \underline{\theta}$ . Therefore, if firm  $L$  sells optional add-on in equilibrium, then either one of the following must hold: (1)  $\alpha > \underline{\theta}$  and  $\Delta V > \Delta_1^{(u)}$ , or (2)  $\alpha < \underline{\theta}$  and  $\Delta V > \Delta_2^{(u)}$ .

Finally, if firm  $L$  sells standard add-on in equilibrium, Constraint (c) gives

$$\theta_{hl}^{*(u)} - \underline{\theta} = \frac{2[(\bar{\theta} - 2\underline{\theta})\Delta V - (\bar{\theta} - 2\underline{\theta} + \alpha)w]}{6\Delta V - 5w} > 0 \iff \Delta V > \Delta_0^{(u)}.$$

To satisfy Constraint (d), it suffices that  $\Delta V < \Delta_2^{(u)}$ . Note that  $\Delta_2^{(u)} > \Delta_0^{(u)}$  is equivalent to  $(\underline{\theta} - \alpha)(\bar{\theta} - 2\underline{\theta} + 6\alpha) > 0$ , which always holds when  $\alpha < \underline{\theta}$ . It remains to verify that there is no profitable deviation from either firm.

#### G-1.1 No Profitable Deviation for Firm $L$

The only non-local deviation is that firm  $L$  does not leave demand for  $H$ . Fixing firm  $H$ 's equilibrium prices,  $(P_h^*, p_h^{e*})$ , and given firm  $L$ 's total price,  $P_l^+$ , the consumers with  $\theta \in [\underline{\theta}, \theta_{hl}]$  choose to pay the base price to visit firm  $L$  at time  $t = 1$ . The marginal consumer is given by  $\theta_{hl} = (P_h^{+*} - P_l^+)/\Delta V$ . At time  $t = 2$ , these consumers observe  $p_l$  and decide whether to buy it or not. The firm maximizes its *ex post* profit,  $\pi_l = (\theta_{hl} - \theta_l)(p_l^e - c)$ , with

<sup>5</sup> Note also that  $\Delta V > \Delta_1^{(u)}$  occurs only if  $\alpha < (\bar{\theta} + \underline{\theta})/3$ . If, however,  $\alpha > (\bar{\theta} + \underline{\theta})/3$ , then  $\Delta V < \Delta_1^{(u)}$ .

$\theta_l = p_l^e/w$ . Again, the optimal strategy is to set  $\theta_l$  as a function of the marginal consumer  $\theta_{hl}$ :  $\theta_l(\theta_{hl}) = (\theta_{hl} + \alpha)/2$ .

The firm's problem at time  $t = 0$  is then finding the optimal  $\theta_{hl}$  that maximizes the total profit taking into account the second-period problem

$$\max_{\theta_{hl}} (\theta_{hl} - \underline{\theta}) \underbrace{(P_h^{+*} - \theta_{hl}\Delta V - c)}_{=P_l + p_l^e} - (\theta_l(\theta_{hl}) - \underline{\theta})(w \cdot \theta_l(\theta_{hl}) - c).$$

The (unconstrained) optimal choice of the marginal consumer is  $\theta_{hl}^d = [2P_h^{+*} - 2c + (2\Delta V + w)\underline{\theta}]/(4\Delta V + w)$ . This deviation requires that no consumer would buy only the base good from firm  $H$ , that is,  $\theta_{hl}^d \geq \theta_h^*$ . However, notice that

$$\theta_{hl}^d - \theta_h^* = \frac{-4(\bar{\theta} - 2\underline{\theta} + 3\alpha)\Delta V^2 - (\bar{\theta} - 2\underline{\theta} - 5\alpha)w\Delta V + \frac{7}{2}(\bar{\theta} - 2\underline{\theta} + \alpha)w^2}{(4\Delta V + w)(6\Delta V - 5w)},$$

which is negative because  $\Delta V > w$ . Therefore, the optimal solution is binding such that  $\theta_{hl} = \theta_h^*$ . Hence, firm  $L$  has no incentive to deviate.

### G-1.2 No Profitable Deviation for Firm $H$

The only non-local deviation that could be profitable is that firm  $H$  implements the standard-add-on policy. Since it is always optimal for firm  $H$  to set the add-on price,  $p_h^{e*} = (\bar{\theta}w + c)/2$ , that maximizes *ex post* profit, the deviation is achieved by increasing the base price so that no consumer will buy the base good only. In the first case, the firm deviates by charging the total price  $P_h^{+d} = P_h^d + p_h^{e*}$  at  $t = 0$  while leaving some demand for  $L^+$ . The optimal deviation is  $P_h^{+d} = \frac{1}{2}(P_l^{+*} + \Delta V\bar{\theta} + c)$ . The profit from this deviation,  $\Pi_h^d$ , is no greater than the equilibrium profit because

$$\Pi_h^* - \Pi_h^d = \frac{\left[ w[P_l^{+*} + \bar{\theta}(\Delta V - w)] - (\Delta V - w)(\bar{\theta}w - c) \right]^2}{4(\Delta V - w)w\Delta V} \geq 0$$

In the second case, the firm charges  $P_h^{+d'}$  such that there is no demand for  $L^+$ . The optimal deviation is  $P_h^{+d'} = (P_l^* + \Delta V(\bar{\theta} + w) + c)/2$ . The profit from this deviation,  $\Pi_h^{d'}$  is no greater than the deviation profit in the first case, because

$$\Pi_h^d - \Pi_h^{d'} = \frac{1}{4\Delta V} \underbrace{[P_l^{+*} + \bar{\theta}\Delta V - c]^2}_{=A+B} - \frac{1}{4(\Delta V + w)} \underbrace{[P_l^* + \bar{\theta}(\Delta V + w) - c]^2}_{=A}$$

$$= \frac{B^2(\Delta V + w) + 2AB(\Delta V + w) + A^2w}{4(\Delta V + w)\Delta V} \geq 0$$

given that  $A + B > 0$  and  $B < 0$ . Therefore, there is no profitable deviation for firm  $H$ .

## G-2 When $\alpha \geq (\bar{\theta} + \underline{\theta})/3$

In this case, the equilibrium prices are exactly the same as case (1) in Proposition 1 (see subsection A-2), except that firm  $H$ 's add-on price is unobserved but correctly expected by rational consumers. Therefore, as long as  $\alpha \geq (2\bar{\theta} - \underline{\theta})/3$ , there is no profitable deviation from either firm. However, when  $\alpha \in ((\bar{\theta} + \underline{\theta})/3, (2\bar{\theta} - \underline{\theta})/3)$ , firm  $L$  can profitably deviate by selling the add-on to some consumers.

## H Proof of Proposition 4

Let superscripts “ $o$ ” and “ $u$ ” denote the observed-price and the unobserved-price cases, and

$$B = \frac{[(\bar{\theta} - 2\underline{\theta})\Delta V - (\bar{\theta} - 2\underline{\theta} + \alpha)w]w}{3(\Delta V - w)(6\Delta V - 5w)}.$$

Note that  $B > 0$  if and only if  $\Delta V > \Delta_0^{(u)}$ .

### H-1 Market Shares

The difference in the marginal consumers indifferent between the two firms is  $\theta_{hl}^{*(u)} - \theta_{hl}^{*(o)} = -B < 0$ , where the inequality follows from  $B > 0$ . The difference in the marginal consumers indifference between buying the add-on and not buying is

$$\theta_l^{*(u)} - \theta_l^{*(o)} = \frac{1}{2}(\theta_{hl}^{*(u)} - \underline{\theta}) = \frac{3(\Delta V - w)}{w}B > 0,$$

where the inequality follows from  $B > 0$ .

### H-2 Prices

The difference in the base prices of firm  $H$  is given by:

$$P_h^{*(u)} - P_h^{*(o)} = \frac{w(\Delta V - w)(\bar{\theta} - 2\underline{\theta} + 6\alpha)}{3(6\Delta V - 5w)} > 0.$$

The difference in the add-on prices for firm  $L$  is given by:

$$p_l^{*(u)} - p_l^{*(o)} = \frac{1}{2}(\theta_{hl}^{*(u)} - \underline{\theta})w = 3(\Delta V - w)B > 0,$$

where the inequality follows from  $B > 0$ .

Note that under either unobserved-price or observed-price equilibrium, the indifference condition and firm  $H$ 's best response satisfy:

$$\begin{cases} P_l^{+*} = P_h^* - \theta_{hl}^*(\Delta V - w) \\ P_h^* = \frac{1}{2}(P_l^{+*} + \bar{\theta}(\Delta V - w)) \end{cases} \Rightarrow P_l^{+*} = (\Delta V - w)(\bar{\theta} - 2\theta_{hl}^*). \quad (\text{H-1})$$

With some algebras, one can show that the difference in the total prices is positive:  $P_l^{+*(u)} - P_l^{+*(o)} = 2(\Delta V - w)(\theta_{hl}^{*(o)} - \theta_{hl}^{*(u)}) > 0$ . The difference in the base price is given by

$$P_l^{*(u)} - P_l^{*(o)} = 2(\Delta V - w)(\theta_{hl}^{*(o)} - \theta_{hl}^{*(u)}) - \frac{1}{2}(\theta_{hl}^{*(u)} - \underline{\theta})w = -(\Delta V - w)B,$$

which is negative given that  $B > 0$ .

### H-3 Profits

Firm  $H$ 's profit is clearly increased because it sells the base to more consumers at a higher price while keeping the add-on profit unchanged. For firm  $L$ , the profit difference is

$$\Delta \Pi_l = (\theta_{hl}^{*(u)} - \underline{\theta})(P_l^{+*(u)} - c) - (\theta_{hl}^{*(o)} - \underline{\theta})(p_l^{*(u)} - c) - (\theta_{hl}^{*(o)} - \underline{\theta})(P_l^{+*(o)} - c) + (\theta_l^{*(o)} - \underline{\theta})(p_l^{*(o)} - c).$$

Note that  $\theta_{hl}^{*(u)} = \theta_{hl}^{*(o)} - B$ ,  $\theta_l^{*(u)} = \theta_l^{*(o)} + 3B(\Delta V - w)/w$ ,  $P_l^{+*(u)} = P_l^{+*(o)} + 2(\Delta V - w)B$ , and  $p_l^{*(u)} = p_l^{*(o)} + 3(\Delta V - w)B$ . Substituting these quantities into the profit difference yields  $\Delta \Pi_l = -(3\Delta V - 2w)(\Delta V - w)B^2/w < 0$ , where the inequality follows from  $3\Delta V > 2w$ .