

## Online Appendix

**Derivation for Results with Residents of Mass  $k$  at the Middle Location:** The condition

$k < \frac{V-2s}{V-t}$  ensures that under uniform pricing, a firm has no incentive to lower its price to attract

residents in the middle or at the competitor's home base. Similar to the derivation in the proof of

Proposition 1, this condition guarantees that  $V-s > \max_{0 \leq \lambda \leq 1} (V-t-\lambda s)(1+k\lambda)$  and  $V-$

$s > \max_{0 \leq \lambda \leq 1} (V-2t-2\lambda s)(1+k+\lambda)$ . The condition automatically holds when  $k \leq 1$  and it implies that

$k < 3/2$  given  $2s < t < 4s$  and  $2t < V < 2t+s$ .

Equilibrium prices are obtained similarly as in the proof of Proposition 2 except for two major modifications. First, when checking if a firm has an incentive to lower the price at distance  $1/2$  in scenario B), the deviating firm now maximizes  $k \cdot p_D(p_{1/2}-p_D+s)/(2s)-(p_{1/2}-p_D)$ , where  $p_D \leq p_{1/2}$ .

To keep the firm from deviating, we need  $p_{1/2} \leq (2/k+1)s$ . Therefore, the equilibrium holds when  $t-s < (2/k+1)s$ , i.e.,  $t < \left(\frac{2}{k}+2\right)s$ .

Second, the firm's deviation profit from uniform pricing is now maximized when it sells only to local residents if  $t \leq 2s/k$ , which is possible if  $k < 1$  and guaranteed if  $k \leq 1/2$ . In this case, the deviation profit  $2t-s$  is always lower than the firm's profit in the mobile geo-targeting

equilibrium, which is  $2t-s + \frac{k}{2}(t-s)$ . When  $t > 2s/k$ , which is possible only if  $k > 1/2$ , the firm's

deviation profit is maximized by selling also to some residents at  $1/2$ . In particular, the maximum

deviation profit, at deviation price  $\frac{t}{2} + \frac{s}{k}$ , is  $\frac{k}{2s} \left(\frac{t}{2} + \frac{s}{k}\right) \left(\frac{t}{2} - \frac{s}{k}\right) + \frac{t}{2} + \frac{s}{k}$ , which is always lower than

$2t-s$  (smaller than firms' profit under mobile geo-targeting) given  $1/2 < k < 3/2$ .

In sum, when  $0 < k \leq 1/2$ , firms deviate to selling to all local residents and make less profit.

When  $1/2 < k < 1$ , firms deviate to either selling to all local residents or selling also to some

residents at  $\frac{1}{2}$ . In either case, profit is lower than under mobile geo-targeting. When  $1 \leq k < 3/2$ , firms deviate to selling to all local residents and some residents at  $\frac{1}{2}$  and deviation profit is also lower than under mobile geo-targeting. Therefore, there is no profitable deviation for any possible  $0 < k < 3/2$ .

It can be easily seen that the profit increases with  $k$ . The firms' equilibrium profit under uniform pricing is  $V-s$  and their equilibrium price under mobile geo-targeting is  $2t-s + \frac{k}{2}(t-s)$ .

The comparison of these two profits suggests that the former is higher if  $k > \frac{2(V-2t)}{t-s}$ . To see what happens if we fix the total market size, we normalize the firms' profit by the total mass of consumers  $2+k$ . The firm's normalized profit can be written as  $[(2t-s)+k(t-s)/2]/(2+k)$  and the first order derivative of this profit with respect to  $k$  is negative.

**Derivation for Equilibrium Outcomes Outside the Main Parameter Range:** Starting with the basic assumption  $t > s$ , we obtain results in other parameter regions in three steps below.

1. If consumers cannot get deals outside of their home locations, the price equilibrium is  $\{2t-s, s, 0\}$  if  $V > 2t$  and  $\{V-s, s, 0\}$  if  $V \leq 2t$ . A consumer has no incentive to get an offer outside of his home location if  $s \geq t/2$  or  $V < t+2s$ . Essentially, if  $s \geq t/2$  or  $V \leq t+2s$ , mobile geo-targeting degenerates to traditional targeting. We thus focus on the region where  $s < t/2$  and  $V > t+2s$ .
2. Given  $s < t/2$  and  $V > t+2s$ , we can make two observations under Uniform Pricing (UP henceforth), assuming that mobile geo-targeting is not available to firms.
  - a. There is no equilibrium where middle segment is served in equilibrium
    - i. If the middle segment is fully covered, we have  $p=3t$  and  $V \geq t+7s/2$  and we can show that the firm always has incentive to deviate to  $p=\text{Min}[V-s, 2t+2s]$

1. If  $V \geq 2t + 3s$ , the deviation profit  $2t + 3s$  is greater than the equilibrium profit  $9s/2$  given  $t > 2s$ ;
  2. If  $V < 2t + 3s$ , the deviation profit  $V - s$  is greater than the equilibrium profit  $9s/2$  given  $V > t + 7s/2$  and  $t > 2s$ .
- ii. If the middle segment is partially covered, the optimal price is  $p = \frac{1}{2}(s - t + V)$ , but under this optimal price, the middle segment is in fact completely covered, given  $V \geq t + 2s$ . Therefore, this equilibrium cannot hold.
- b. The equilibrium where both firms sell only to the home segment exists only if  $V < 2t + s$ , and the equilibrium price and profit are both  $V - s$  as shown in Proof of Proposition 1.

We thus focus on the region where  $s < t/2$  and  $t + 2s < V < 2t + s$ .

3. Given  $s < t/2$  and  $t + 2s < V < 2t + s$ , we can solve the equilibrium under mobile geo-targeting (MT henceforth).
  - a. If  $V > 2t$  and  $s > t/4$ , we are back to the main model.
  - b. If  $V > 2t$  and  $s \leq t/4$ , we can follow the same process for Lemma 2 and Proposition 2 to show that the equilibrium prices are  $\{t + 3s, 3s, 0\}$  and the equilibrium profit is  $t + 9s/2$ .
    - i. The equilibrium holds (i.e., no deviation to UP) if  $s > 2t/11$ : if deviating to UP, deviation profit is  $2t - s$  which is smaller than  $t + 9s/2$  if  $s > 2t/11$
    - ii. Compare MT to UP:  $t + 9s/2 > V - s$  if  $V < t + 11s/2$
  - c. If  $V \leq 2t$  and  $s > t/4$ , we can follow the same process for Lemma 2 and Proposition 2 to show that the equilibrium prices are  $\{V - s, V - t - s, 0\}$  and profit is  $3V/2 - t/2 - 3s/2$ .

- i. The equilibrium holds (i.e., no deviation to UP): if deviating to UP, deviation profit is  $V-s$  which is smaller than  $3V/2-t/2-3s/2$  given  $V>t+2s$
  - ii. Compare MT to UP:  $MT>UP$
- d. If  $V\leq 2t$  and  $s\leq t/4$ , we can follow the same process for Lemma 2 and Proposition 2 to show that the equilibrium prices are:
- i. If  $V\geq t+4s$ ,  $\{t+3s, 3s, 0\}$  and profit is  $t+9s/2$ .
  - ii. If  $V<t+4s$ ,  $\{V-s, V-t-s, 0\}$  and profit is  $3V/2-t/2-3s/2$ .
  - iii. The equilibrium holds (i.e., no deviation to UP) if  $V<t+11s/2$ :
    - 1. If  $V\geq t+4s$ , by deviating to UP, deviation profit is  $V-s$  which is smaller than  $t+9s/2$  if  $V<t+11s/2$
    - 2. If  $V<t+4s$ , by deviating to UP, deviation profit is  $V-s$  which is smaller than  $3V/2-t/2-3s/2$  given  $V>t+2s$
  - iv. Compare MT to UP:  $MT>UP$

We thus show that given  $s<t/2$  and  $t+2s<V<2t+s$  the equilibrium under MT holds only if  $s>2t/11$  or  $V<t+11s/2$ .

In sum, the parameter region where equilibrium exists under both UT and MT and mobile geo-targeting does not degenerate to traditional targeting is:

$$s<t/2 \text{ and } t + 2s < V < \hat{V}, \text{ where } \hat{V} = \begin{cases} 2t + s & \text{if } s > 2t/11 \\ t + 11s/2 & \text{if } s \leq 2t/11 \end{cases}$$

In this region, the profit ranking is  $MT>UP$  if  $V<\text{Min}[t+11s/2, 5t/2-s/2]$ .