Cooperative Search Advertising
Online Appendix

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Online Appendix

OA.1. Analysis of the Basic Model when v_A is between $\theta_1 r_1$ and $\theta_2 r_2$:

In the following analysis, we do not presume that $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2)$. Without loss of generality, suppose $\theta_1 r_1 \geq v_A \geq \theta_2 r_2$. Without any support from the manufacturer, retailer 1 will get position 1, the outside advertiser A will get position 2, and retailer 2 will get position 3. The manufacturer's profit only comes from retailer 1, and it equals to,

$$\pi_M(0,0) = d_1\theta_1 m_1.$$

Obviously, the manufacturer has no incentive to give retailer 1 any support, since retailer 1 can already get the first position and thus the manufacturer can get the maximum demand from retailer 1 without paying anything for the clicks. Now let's see whether the manufacturer would like to support retailer 2 to move up.

If the manufacturer moves retailer 2 up to position 2, it needs to provide participation rate α_2 to retailer 2 such that

$$\theta_1 r_1 \geq \frac{\theta_2 r_2}{1-\alpha_2} \geq v_A. \tag{OA-1}$$

The outside advertiser's bid at position 3 is v_A, and retailer 2's bid at position 2 is $(d_1-d_2)/d_1 \cdot \theta_2 r_2/(1-\alpha_2) + d_2/d_1 \cdot v_A$, which is the price per click for retailer 1. The manufacturer's profit is then,

$$\pi_M(0,\alpha_2) = d_1\theta_1 m_1 + d_2(\theta_2 m_2 - \alpha_2 v_A),$$

which decreases in α_2, so the manufacturer will choose the smallest α_2 that satisfies (OA-1), $\alpha_2^* = 1-\theta_2 r_2/v_A$. Correspondingly, the manufacturer's profit is

$$\pi_M(0,\alpha_2^*) = d_1\theta_1 m_1 + d_2[\theta_2(m_2+r_2) - v_A].$$

If the manufacturer further supports retailer 2 to move up to position 1, it needs to provide participation rate α_2 to retailer 2 such that

$$\frac{\theta_2 r_2}{1-\alpha_2} > \theta_1 r_1 \geq v_A. \tag{OA-2}$$

The outside advertiser's bid at position 3 is v_A, and retailer 1's bid at position 2 is $(d_1-d_2)/d_1 \cdot \theta_1 r_1 + d_2/d_1 \cdot v_A$. Similarly, the manufacturer will choose the smallest α_2 that satisfies (OA-2), so

$\alpha_2^* = 1 - \theta_2 r_2/(\theta_1 r_1)$. The manufacturer's profit is then,

$$\begin{aligned}\pi_M(0,\alpha_2^*) &= d_1\left[\theta_2 m_2 - \alpha_2\left(\frac{d_1-d_2}{d_1}\theta_1 r_1 + \frac{d_2}{d_1}v_A\right)\right] + d_2\theta_1 m_1 \\ &= d_1\left[\theta_2(m_2+r_2) - \theta_1 r_1\right] + d_2\left[\theta_1(m_1+r_1) - \theta_2 r_2 - v_A\left(1-\frac{\theta_2 r_2}{\theta_1 r_1}\right)\right].\end{aligned}$$

Comparing the manufacturer's profits in the three scenarios, we can see that the manufacturer will support retailer 2 to get position 2 if and only if,

$$\theta_1(m_1+r_1) + \frac{d_2}{d_1-d_2}\frac{\theta_1 r_1 - v_A}{\theta_1 r_1}\theta_2 r_2 \geq \theta_2(m_2+r_2) \geq v_A,$$

and the manufacturer will support retailer 2 to get position 1 if and only if,

$$\theta_2(m_2+r_2) > \theta_1(m_1+r_1) + \frac{d_2}{d_1-d_2}\frac{\theta_1 r_1 - v_A}{\theta_1 r_1}\theta_2 r_2.$$

To summarize, when v_A is between $\theta_1 r_1$ and $\theta_2 r_2$, the retailer with higher total channel profit does not necessarily get a higher position. Specifically, given $\theta_1 r_1 \geq v_A \geq \theta_2 r_2$, retailer 1 will get a higher position when $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2)$; but retailer 2 may not get a higher position when $\theta_2(m_2+r_2) > \theta_1(m_1+r_1)$. In other words, we need a stricter condition than $\theta_2(m_2+r_2) > \theta_1(m_1+r_1)$, as shown above, to grant retailer 2 a higher position than retailer 1.

The reason is that, in the case that $v_A > \theta_i r_i$ $(i = 1, 2)$, the manufacturer can choose the participation rates such that the bid at position 2 equals to v_A and both retailers pay v_A per click, when the two retailers take the top two positions. Now, consider the case that $\theta_1 r_1 \geq v_A$. When retailer 1's takes position 2 and retailer 2 takes position 1, retailer 1's bid is a linear combination of v_A and $\theta_1 r_1$, which is greater than v_A. As a result, the price per click at position 1 is higher than v_A. On the other hand, when retailer 1 takes position 1 and retailer 2 takes position 2, the manufacturer can choose retailer 2's participation rate such that the bid at position 2 equals to v_A and both retailers pay v_A per click. Therefore, having retailer 2 at position 1 is more costly and thus demand a stronger condition to ensure it as the equilibrium.

OA.2. Manufacturer's Direct Participation: Analyzing 12 Position Configurations

We first consider the four position configurations in which the manufacturer gets a higher position than both retailers: (M, R_1), (M, R_2), (M, A), (A, M).

- Consider the position configuration (M, R_1).

For this position configuration to be the equilibrium, we need to have

$$b_M \geq b_1 \geq b_A, b_2. \tag{OA-3}$$

The SNE condition and the LB selection rule that guard against the outside advertiser A and R_2's deviation from position 3 to position 2 imply that,

$$b_A = v_A, b_2 = \frac{\theta_2 r_2}{1-\alpha_2}. \tag{OA-4}$$

Given $b_1 > b_A, b_2$ and the equation above, we can show that the SNE condition that guards against the retailer's deviation to position 1 is has been satisfied.

The SNE condition and the LB selection rule that guard against R_1's deviation from position 2 to position 1 imply that,

$$b_1 = \frac{d_1 - d_2}{d_1}\frac{\theta_1 r_1}{1-\alpha_1} + \frac{d_2}{d_1}\max\{b_A, b_2\}. \tag{OA-5}$$

The SNE condition that guards against the R_1's deviation to position 3 is automatically satisfied given $b_1 > b_A, b_2$ and the equation above.

The manufacturer chooses α_1, α_2 to maximize its profit $\pi_M(\alpha_1, \alpha_2)$, subject to (OA-3)-(OA-5), where,

$$\begin{aligned}
\pi_M(\alpha_1, \alpha_2) &= d_1(\theta_0 m_0 - b_1) + d_2(\theta_1 m_1 - \alpha_1 \max\{b_A, b_2\}) \\
&= d_1\left[\theta_0 m_0 - \left(\frac{d_1-d_2}{d_1}\frac{\theta_1 r_1}{1-\alpha_1} + \frac{d_2}{d_1}\max\left\{v_A, \frac{\theta_2 r_2}{1-\alpha_2}\right\}\right)\right] \\
&\quad + d_2\left(\theta_1 m_1 - \alpha_1 \max\left\{v_A, \frac{\theta_2 r_2}{1-\alpha_2}\right\}\right),
\end{aligned}$$

which decreases in α_1 and α_2. Therefore, the manufacturer will provide no support to R_2 ($\alpha_2^* = 0$) and choose the smallest α_1 that satisfies $\frac{\theta_1 r_1}{1-\alpha_1} > v_A$, i.e., $\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A}$.

To prevent the manufacturer from deviating to position 2, we need to have

$$d_1(\theta_0 m_0 - b_1) + d_2(\theta_1 m_1 - \alpha_1 b_A) \geq d_1(\theta_1 m_1 - \alpha_1 b_A) + d_2(\theta_0 m_0 - b_A),$$

which is equivalent to $\theta_0 m_0 \geq \theta_1(m_1 + r_1)$ given the choice of α_1.

To prevent the manufacturer from deviating to position 3, we need to have

$$d_1(\theta_0 m_0 - b_1) + d_2(\theta_1 m_1 - \alpha_1 b_A) \geq 0,$$

which is equivalent to $d_1(\theta_0 m_0 - v_A) + d_2(\theta_1(m_1 + r_1) - v_A) \geq 0$ given the choice of α_1.

To summarize, the optimal participation rates and manufacturer's profit in the case of (M, R_1) are

$$\begin{aligned} \alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\ \alpha_2^* &= 0, \\ \pi_M(\alpha_1^*, \alpha_2^*) &= d_1(\theta_0 m_0 - v_A) + d_2[\theta_1(m_1 + r_1) - v_A], \end{aligned} \tag{OA-6}$$

and the equilibrium exists when the parameters satisfy $\theta_0 m_0 \geq \theta_1(m_1 + r_1)$ and $d_1(\theta_0 m_0 - v_A) + d_2(\theta_1(m_1 + r_1) - v_A) \geq 0$.

The manufacturer's optimal profit in the case of (M, R_2) can be obtained by exchanging R_1 and R_2 in (OA-6), i.e., $\pi_M(\alpha_1^*, \alpha_2^*) = d_1(\theta_0 m_0 - v_A) + d_2[\theta_2(m_2 + r_2) - v_A]$, which is smaller than its optimal profit under (M, R_1) given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$. Therefore, (M, R_2) cannot be the position configuration in equilibrium.

- Consider the position configuration (M, A).

For the position configuration to be the equilibrium, we need to have

$$b_M \geq b_A > \max\{b_1, b_2\}. \tag{OA-7}$$

Similarly as above, the SNE and LB conditions that guard against R_1 and R_2's deviations from position 3 to a higher position will lead to $b_1 = \frac{\theta_1 r_1}{1-\alpha_1}, b_2 = \frac{\theta_2 r_2}{1-\alpha_2}$.

The SNE condition and the LB selection rule that guard against the outside advertiser A's deviation from position 2 to position 1 imply that,

$$b_A = \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \max\left\{\frac{\theta_1 r_1}{1 - \alpha_1}, \frac{\theta_2 r_2}{1 - \alpha_2}\right\}. \tag{OA-8}$$

Given this equation and $b_A > \max\left\{\frac{\theta_1 r_1}{1-\alpha_1}, \frac{\theta_2 r_2}{1-\alpha_2}\right\}$, the SNE condition that guards against the outside advertiser A's deviation to position 3 has been satisfied.

To prevent the manufacturer from deviating to position 2, we need to have

$$d_1(\theta_0 m_0 - b_A) \geq d_2(\theta_0 m_0 - \max\{b_1, b_2\}).$$

Given the expression of b_A as in (OA-8), this is equivalent to $\theta_0 m_0 \geq v_A$.[1]

[1]This requirement is independent of the choice α_1, α_2.

The manufacturer's profit is

$$\begin{aligned}\pi_M(\alpha_1, \alpha_2) &= d_1(\theta_0 m_0 - b_A) \\ &= d_1(\theta_0 m_0 - v_A) + d_2\left(v_A - \max\left\{\frac{\theta_1 r_1}{1-\alpha_1}, \frac{\theta_2 r_2}{1-\alpha_2}\right\}\right),\end{aligned}$$

which decreases in α_1, α_2. Therefore, the manufacturer will choose $\alpha_1^* = 0, \alpha_2^* = 0$, which satisfies (OA-7) and maximizes its profit.

To prevent the manufacturer from deviating to position 3, we need to have

$$d_1(\theta_0 m_0 - b_A) \geq 0,$$

which is equivalent to $\theta_0 m_0 \geq \frac{d_1-d_2}{d_1} v_A + \frac{d_2}{d_1}\max\{\theta_1 r_1, \theta_2 r_2\}$ and is satisfied given $\theta_0 m_0 \geq v_A$ and $v_A > \theta_1 r_1, \theta_2 r_2$.

To summarize, the optimal participation rates and manufacturer's profit in the case of (M, A) are

$$\begin{aligned}\alpha_1^* &= 0, \\ \alpha_2^* &= 0, \\ \pi_M(\alpha_1^*, \alpha_2^*) &= d_1(\theta_0 m_0 - v_A) + d_2(v_A - \max\{\theta_1 r_1, \theta_2 r_2\}),\end{aligned}$$

and the equilibrium exists when the parameters satisfy $\theta_0 m_0 \geq v_A$.

- Consider the position configuration (A, M).

For the position configuration to be the equilibrium, we need to have

$$b_A > b_M \geq \max\{b_1, b_2\}. \quad \text{(OA-9)}$$

Similarly as above, the SNE and LB conditions that guard against R_1 and R_2's deviations from position 3 to a higher position will lead to $b_1 = \frac{\theta_1 r_1}{1-\alpha_1}, b_2 = \frac{\theta_2 r_2}{1-\alpha_2}$.

The SNE condition and the LB selection rule that guard against the outside advertiser M's deviation from position 2 to position 1 imply that,

$$b_M = \frac{d_1 - d_2}{d_1}\theta_0 m_0 + \frac{d_2}{d_1}\max\left\{\frac{\theta_1 r_1}{1-\alpha_1}, \frac{\theta_2 r_2}{1-\alpha_2}\right\}. \quad \text{(OA-10)}$$

Given this equation and $b_M > \max\left\{\frac{\theta_1 r_1}{1-\alpha_1}, \frac{\theta_2 r_2}{1-\alpha_2}\right\}$, the SNE condition that guards against the manufacturer's deviation to position 3 has been satisfied.

To prevent advertiser A from deviating to the second position, we need to have

$$d_1(v_A - b_M) > d_2(v_A - \max\{b_1, b_2\}),$$

which is equivalent to $v_A > \theta_0 m_0$. Similarly as above, the condition of preventing A from deviating to the third position is automatically satisfied given this and $v_A > \theta_1 r_1, \theta_2 r_2$.

The manufacturer's profit is

$$\pi_M(\alpha_1, \alpha_2) = d_2 \left(\theta_0 m_0 - \max\left\{ \frac{\theta_1 r_1}{1-\alpha_1}, \frac{\theta_2 r_2}{1-\alpha_2} \right\} \right)$$

which decreases in α_1, α_2. Therefore, the manufacturer will choose $\alpha_1^* = 0, \alpha_2^* = 0$, which satisfies (OA-9) and maximizes its profit.

To summarize, the optimal participation rates and manufacturer's profit in the case of (A, M) are

$$\begin{aligned}
\alpha_1^* &= 0, \\
\alpha_2^* &= 0, \\
\pi_M(\alpha_1^*, \alpha_2^*) &= d_2(\theta_0 m_0 - \max\{\theta_1 r_1, \theta_2 r_2\}),
\end{aligned}$$

and the equilibrium exists when the parameters satisfy $v_A > \theta_0 m_0$.

Now we consider the eight position configurations in which the manufacturer gets a lower position than one or both retailers: (R_1, M), (R_2, M), (R_1, R_2), (R_2, R_1), (R_1, A), (R_2, A), (A, R_1), (A, R_2).

• Consider the position configuration (R_1, M).

For this position configuration to be the equilibrium, we need to have

$$b_1 > b_M > b_A, b_2. \tag{OA-11}$$

The SNE condition and the LB selection rule that guard against the outside advertiser A and R_2's deviation from position 3 to position 2 imply that,

$$b_A = v_A, b_2 = \frac{\theta_2 r_2}{1-\alpha_2} \tag{OA-12}$$

Given $b_M > b_A, b_2$ and the equation above, we can show that the SNE condition that guards against advertiser A and R_2's deviation to position 1 is has been satisfied.

Given b_A and b_M, the NE condition that guards against R_1's deviation from position 1 to position 2 imply that,

$$d_1[\theta_1 r_1 - (1-\alpha_1)b_M] > d_2[\theta_1 r_1 - (1-\alpha_1)b_A]. \tag{OA-13}$$

Given $b_M > b_A$ and the inequality above, the SNE condition that guards against R_1's deviation to position 3 must be satisfied.

The manufacturer chooses α_1, α_2 and b_M to maximize its profit $\pi_M(\alpha_1, \alpha_2, b_M)$, subject to conditions (OA-11)-(OA-13), where,

$$\begin{aligned}\pi_M(\alpha_1,\alpha_2,b_M) &= d_1(\theta_1 m_1 - \alpha_1 b_M) + d_2(\theta_0 m_0 - \max\{b_A, b_2\}) \\ &= d_1(\theta_1 m_1 - \alpha_1 b_M) + d_2\left(\theta_0 m_0 - \max\left\{v_A, \frac{\theta_2 r_2}{1-\alpha_2}\right\}\right),\end{aligned} \tag{OA-14}$$

which decreases with α_1, α_2 and b_M. Thus the manufacturer will choose $\alpha_2^* = 0$. The smallest bid b_M that satisfies (OA-11) is $b_M = v_A$. Notice that the bound of α_1 given by (OA-13) decreases with b_M, so the smallest possible α_1 that satisfies (OA-13) is $\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A}$.

To prevent the manufacturer from deviating to the first position, we need to have $d_1(\theta_1 m_1 - \alpha_1 b_M) + d_2(\theta_0 m_0 - v_A) > d_1(\theta_0 m_0 - b_1) + d_2(\theta_1 m_1 - \alpha_1 v_A)$, which can be satisfied as long as b_1 is large enough. [2]

To prevent the manufacturer from deviating to the third position, we need to have $d_1(\theta_1 m_1 - \alpha_1 b_M) + d_2(\theta_0 m_0 - v_A) \geq d_1(\theta_1 m_1 - \alpha_1 b_A)$, which is equivalent to $\theta_0 m_0 \geq v_A$.

To summarize, the solution to this optimization problem is that,

$$\begin{aligned}\alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\ \alpha_2^* &= 0, \\ b_M^* &= v_A, \\ \pi_M(\alpha_1^*, \alpha_2^*, b_M^*) &= d_1[\theta_1(m_1 + r_1) - v_A] + d_2(\theta_0 m_0 - v_A).\end{aligned}$$

The equilibrium holds when $\theta_0 m_0 \geq v_A$.

Notice that when configuration is the equilibrium, the manufacturer gives a positive participation rate while bidding by itself. The manufacturer submits the lowest possible bid v_A that can keep the second position for itself and minimize the cost per click for R_1 at the same time.

[2] The channel profit is not affected by b_1 given that it is higher than b_M, b_A, b_2.

By exchanging R_1 and R_2 in the solution above, we can get the manufacturer's optimal profit under (R_2, M), $\pi_M(\alpha_1^*, \alpha_2^*, b_M^*) = d_1[\theta_2(m_2 + r_2) - v_A] + d_2(\theta_0 m_0 - v_A)$, which is smaller than the manufacturer's optimal profit under (R_1, M) given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$. Therefore, (R_2, M) cannot be the position configuration in equilibrium.

- Consider the position configuration (R_1, A).

For this position configuration to be the equilibrium, we need to have

$$b_1 > b_A > \max\{b_M, b_2\}. \tag{OA-15}$$

The SNE condition and the LB selection rule that guard against R_2's deviation from position 3 to position 2 imply that,

$$b_2 = \frac{\theta_2 r_2}{1 - \alpha_2} \tag{OA-16}$$

Given (OA-15) and (OA-16), we can show that the SNE condition that guards against R_2's deviation to position 1 is has been satisfied.

Given b_M, the SNE condition and the LB selection rule that guard against the outside advertiser A's deviation from position 2 to position 1 imply that,

$$b_A = \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \max\{b_M, b_2\}. \tag{OA-17}$$

Given this equation and $b_A > \max\{b_M, b_2\}$, the SNE condition that guards against the outside advertiser A's deviation to position 3 has been satisfied.

The NE condition that guard against R_1's deviation from position 1 to position 2 imply that,

$$d_1[\theta_1 r_1 - (1 - \alpha_1) b_A] > d_2[\theta_1 r_1 - (1 - \alpha_1) \max\{b_M, b_2\}]. \tag{OA-18}$$

Given (OA-17) and (OA-18), the NE condition that guards against R_1's deviation to position 3 has been satisfied.

The manufacturer chooses α_1, α_2 and b_M to maximize its profit $\pi_M(\alpha_1, \alpha_2, b_M)$ subject to (OA-15)-(OA-18), where

$$\begin{aligned} \pi_M(\alpha_1, \alpha_2, b_M) &= d_1(\theta_1 m_1 - \alpha_1 b_A). \\ &= d_1 \left[\theta_1 m_1 - \alpha_1 \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} \max\left\{ b_M, \frac{\theta_2 r_2}{1 - \alpha_2} \right\} \right) \right], \end{aligned} \tag{OA-19}$$

which decreases in α_1, α_2 and b_M. Thus the manufacturer will choose $\alpha_2^* = 0, b_M^* = 0$. Substituting

(OA-17) to (OA-18), we get $\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A}$.

However, in order to prevent the manufacturer from deviating to the second position, we need to have $d_1(\theta_1 m_1 - \alpha_1 b_A) > d_1(\theta_1 m_1 - \alpha_1 b_A) + d_2(\theta_0 m_0 - b_A)$, which is equivalent to $b_A > \theta_0 m_0$. Given $b_M = 0, \alpha_2 = 0$, $b_A = \frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1}\theta_2 r_2$. If $\frac{d_1-d_2}{d_1} v_A + \frac{d_2}{d_1}\theta_2 r_2 < \theta_0 m_0$, advertiser A needs to bid at least $\theta_0 m_0$ to secure its position at the second position, and this will lower the manufacturer's profit as it increases the price per click for R_1. To avoid this, the manufacturer should commit not to participate in the bidding in the first stage if it finds that to stay in the third position (i.e., not to get displayed) is optimal . In such case, the equilibrium is the same as (R_1, A) in the basic model.

Therefore, the solution to this optimization problem is

$$\begin{aligned}
\alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\
\alpha_2^* &= 0, \\
b_M^* &= 0, \\
\pi_M(\alpha_1^*, \alpha_2^*, b_M^*) &= d_1[\theta_1(m_1 + r_1) - v_A] + d_2 \frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A}.
\end{aligned}$$

The equilibrium conditions are satisfied given $v_A > \theta_1 r_1, \theta_2 r_2$.

If we exchange the position of R_1 and R_2, we can get the manufacturer's optimal profit under (R_2, A), i.e., $\pi_M(\alpha_1^*, \alpha_2^*, b_M^*) = d_1[\theta_2(m_2 + r_2) - v_A] + d_2 \frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A}$. Again, it's lower than the manufacturer's optimal profit under (R_1, A) given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$, so (R_2, A) cannot be the equilibrium position configuration.

- Consider the position configuration (R_1, R_2).

Following similar reasoning as above, the manufacturer should commit not to participate in the bidding under this position configuration, so the optimal participation rates, equilibrium bids, and the manufacturer's optimal profit are the same as the position configuration (R_1, R_2) in the basic model, i.e.,

$$\begin{aligned}
\alpha_1^* &= 1 - \frac{\theta_1 r_1}{v_A}, \\
\alpha_2^* &= 1 - \frac{\theta_2 r_2}{v_A}, \\
b_M^* &= 0, \\
\pi_M(\alpha_1^*, \alpha_2^*, b_M^*) &= d_1[\theta_1(m_1 + r_1) - v_A] + d_2[\theta_2(m_2 + r_2) - v_A].
\end{aligned}$$

Similarly, the manufacturer's profit cannot be improved if we exchange the positions of R_1, R_2, so the position configuration (R_2, R_1) can be the equilibrium configuration.

• Consider the position configuration (A, R_1).

Following similar reasoning as above, the manufacturer quits the bidding under this position configuration, so the optimal participation rates, equilibrium bids, and the manufacturer's optimal profit are the same as the position configuration (A, R_1) in the basic model, i.e.,

$$\begin{aligned}\alpha_1^* &= \max\left\{1 - \frac{\theta_1 r_1}{\theta_2 r_2}, 0\right\},\\ \alpha_2^* &= 0,\\ b_M^* &= 0,\\ \pi_M(\alpha_1^*, \alpha_2^*, b_M^*) &= d_2[\theta_1(m_1 + r_1) - \max\{\theta_1 r_1, \theta_2 r_2\}].\end{aligned}$$

Similarly, the manufacturer's profit gets lower if we exchange the positions of R_1, R_2, so the position configuration (A, R_2) cannot be the position configuration in equilibrium.

OA.3. Identity-Dependent CTR (Proof of Theorem 3):

As a counterpart to the result in Section 2.1, Varian (2007) has shown that in equilibrium, $e_1 v_1 \geq e_2 v_2 \geq e_3 v_3$, where v_i denotes the profit per click for the advertiser at position i. The equilibrium bids are,

$$\begin{aligned}b_3 &= v_3,\\ b_2 &= \frac{x_1 - x_2}{x_1} v_2 + \frac{x_2}{x_1}\frac{e_3}{e_2} v_3.\end{aligned}$$

Similarly with the basic model, we assume that $e_A v_A > e_1 \theta_1 r_1, e_2 \theta_2 r_2$. We analyze the retailers and outside advertiser's bid as well as the manufacturer's choice of participation rate given the positions of all advertisers. Then we compare the manufacturer's profits among six all position configurations to identify the equilibrium.

We first consider the three position configurations where retailer 1 gets a higher position than retailer 2.

• Consider the position configuration (R_1, R_2).

This happens when,

$$\frac{e_1 \theta_1 r_1}{1 - \alpha_1} \geq \frac{e_2 \theta_2 r_2}{1 - \alpha_2} \geq e_A v_A. \tag{OA-20}$$

The manufacturer's profit is,

$$\pi_M(\alpha_1, \alpha_2) = x_1 e_1 \left[\theta_1 m_1 - \alpha_1 \frac{e_2}{e_1}\left(\frac{x_1 - x_2}{x_1}\frac{\theta_2 r_2}{1 - \alpha_2} + \frac{x_2}{x_1}\frac{e_A v_A}{e_2}\right)\right] + x_2 e_2 \left(\theta_2 m_2 - \alpha_2 \frac{e_A}{e_2} v_A\right),$$

which decreases in α_1, α_2. Therefore the manufacturer will choose the smallest α_1, α_2 that satisfy (OA-20). The optimal participation rates are,

$$\alpha_i^* = 1 - \frac{e_i \theta_i r_i}{e_A v_A}, \ i = 1, 2.$$

Correspondingly, the manufacturer's maximum profit is,

$$\pi_M(\alpha_1^*, \alpha_2^*) = x_1[e_1\theta_1(m_1 + r_1) - e_A v_A] + x_2[e_2\theta_2(m_2 + r_2) - e_A v_A].$$

- Consider the position configuration (R_1, A).

  This happens when,

$$\frac{e_1\theta_1 r_1}{1 - \alpha_1} \geq e_A v_A > \frac{e_2\theta_2 r_2}{1 - \alpha_2}. \tag{OA-21}$$

The manufacturer's profit is,

$$\pi_M(\alpha_1, \alpha_2) = x_1 e_1 \left[\theta_1 m_1 - \alpha_1 \frac{e_A}{e_1} \left(\frac{x_1 - x_2}{x_1} v_A + \frac{x_2}{x_1} \frac{e_2}{e_A} \frac{\theta_2 r_2}{1 - \alpha_2}\right)\right],$$

which decreases in α_1, α_2. Therefore the manufacturer will choose the smallest α_1, α_2 that satisfy (OA-21). The optimal participation rates are,

$$\begin{aligned} \alpha_1^* &= 1 - \frac{e_1\theta_1 r_1}{e_A v_A}, \\ \alpha_2^* &= 0. \end{aligned}$$

Correspondingly, the manufacturer's profit is

$$\pi_M(\alpha_1^*, \alpha_2^*) = x_1 \left[e_1\theta_1(m_1 + r_1) - e_A v_A\right] + x_2 \left[e_A v_A - e_1\theta_1 r_1 - e_2\theta_2 r_2 + \frac{e_1\theta_1 r_1 \cdot e_2\theta_2 r_2}{e_A v_A}\right].$$

- Consider the position configuration (A, R_1).

  This happens when,

$$e_A v_A > \frac{e_1\theta_1 r_1}{1 - \alpha_1} \geq \frac{e_2\theta_2 r_2}{1 - \alpha_2}. \tag{OA-22}$$

The manufacturer's profit is,

$$\pi_M(\alpha_1, \alpha_2) = x_2 e_1 \left[\theta_1 m_1 - \alpha_1 \frac{e_2}{e_1} \frac{\theta_2 r_2}{1 - \alpha_2}\right],$$

which decreases in α_1, α_2. Therefore the manufacturer will choose the smallest α_1, α_2 that satisfy

(OA-22). The optimal participation rates are,

$$\alpha_1^* = \frac{\max\{e_1\theta_1 r_1, e_2\theta_2 r_2\} - e_1\theta_1 r_1}{e_2\theta_2 r_2},$$
$$\alpha_2^* = 0.$$

Correspondingly, the manufacturer's profit is

$$\pi_M(\alpha_1^*, \alpha_2^*) = x_2\left[e_1\theta_1(m_1 + r_1) - \max\{e_1\theta_1 r_1, e_2\theta_2 r_2\}\right].$$

The other three configurations can be solved by symmetry. The remaining analysis is straightforward and the same with basic model, thus omitted.

OA.4. Price Competition (Proof of Theorem 4):

Step 1:

We have analyzed the case where the manufacturer sponsors both retailers to get displayed. The manufacturer's profit under the optimal participation rates is $\pi^*_{M(2)} = \bar{\theta}(w + t) - 2v_A$, where the subscript (2) denotes "under the case that the manufacturer sponsors two retailers".

Step 2:

Now we consider the case in which the manufacturer sponsors only one retailer to get displayed.

Since the two retailers are *ex ante* symmetric, we assume that retailer 1 gets displayed without loss of generalizability. One outside advertiser will take the second position. x_1 of the consumers on the Hotelling line will purchase from retailer 1, where x_1 solves $v - tx_1 - p_1 = 0$, or $x_1 = 1$ if $v - t - p_1 \geq 0$. The price per click for retailer 1 is v_A.

Retailer 1's profit is $\pi_1 = \bar{\theta}\min\{\frac{v-p_1}{t}, 1\}(p_1 - w) - (1-\alpha_1)v_A$. The optimal retail price for retailer 1 is $p_1^* = \max\{\frac{v+w}{2}, v-t\}$, and it will cover $x^* = \min\{\frac{v-w}{2t}, 1\}$ of consumers on the Hotelling line. Retailer 1's profit is

$$\pi_1^* = \begin{cases} \frac{\bar{\theta}}{4t}(v-w)^2 - (1-\alpha_1)v_A & \text{if } t \geq \frac{v-w}{2}, \\ \bar{\theta}(v-w-t) - (1-\alpha_1)v_A & \text{if } t \leq \frac{v-w}{2}. \end{cases}$$

The manufacturer's profit given his participation rates α_1 is

$$\pi_{M(1)} = \begin{cases} \bar{\theta}\frac{v-w}{2t}w - \alpha_1 v_A & \text{if } t \geq \frac{v-w}{2}, \\ \bar{\theta}w - \alpha_1 v_A & \text{if } t \leq \frac{v-w}{2}. \end{cases}$$

which decreases in α_1. Similarly, we use the subscript (1) denotes the case where the manufacturer sponsors one retailer. Therefore, the manufacturer will choose the lowest participation rate α_1 that can help retailer 1 get displayed, i.e., $\pi_1^* \geq 0$. This leads to

$$\alpha_1^* = \begin{cases} 1 - \frac{\bar{\theta}}{4tv_A}(v-w)^2 & \text{if } t \geq \frac{v-w}{2}, \\ 1 - \frac{\bar{\theta}(v-w-t)}{v_A} & \text{if } t \leq \frac{v-w}{2}. \end{cases}$$

The manufacturer's profit under the optimal participation rate α_1^* is

$$\pi_{M(1)}^* = \begin{cases} \frac{\bar{\theta}}{4t}(v^2 - w^2) - v_A & \text{if } t \geq \frac{v-w}{2}, \\ \bar{\theta}(v-t) - v_A & \text{if } t \leq \frac{v-w}{2}. \end{cases}$$

Step 3:

Suppose the manufacturer does not sponsor either retailer. The two outside advertisers will take both positions, and the manufacturer's profit will be zero.

Step 4:

Before we compare the manufacturer's profit under the cases of sponsoring two, one, zero retailers, we need to pin down the restrictions on parameter space based on our assumptions: (1) neither retailer can get displayed without support from the manufacturer, and (2) the Hotelling line is covered with two retailers.

Assumption (1) implies that $\alpha_i^* > 0$, because otherwise the manufacturer provide zero participation rate and the retailer(s) can still get displayed. For the case of manufacturer sponsoring two retailers, $\alpha_i^* > 0, i = 1, 2$ is equivalent to $t < 2v_A/\bar{\theta}$. For the case of manufacturer sponsoring only one retailer, we need to have $\alpha_1^* > 0$. Notice that α_1^* increases in t, and thus $\alpha_1^* > 0$ is equivalent to $t > v - w - \frac{v_A}{\bar{\theta}}$ when $t \leq \frac{v-w}{2}$, and equivalent to $t > \frac{\bar{\theta}}{4v_A}(v-w)^2$ when $t > \frac{v-w}{2}$. By combining the two cases, we have that $2v_A/\bar{\theta} \geq t > \max\left\{v - w - \frac{v_A}{\bar{\theta}}, \frac{\bar{\theta}}{4v_A}(v-w)^2\right\}$.

It is straightforward to show that assumptions (1) and (2) together imply the restriction: $t \in (B_0, \frac{2}{3}(v-w)]$, where

$$B_0 \equiv \begin{cases} \frac{\bar{\theta}}{4v_A}(v-w)^2, & \text{if } \frac{3}{8}(v-w) < \frac{v_A}{\bar{\theta}} < \frac{v-w}{2} \\ v - w - \frac{v_A}{\bar{\theta}}, & \text{if } \frac{v_A}{\bar{\theta}} \geq \frac{v-w}{2}. \end{cases}$$

Notice that assumption (1) also guarantees that when the manufacturer only sponsors retailer 1 to get displayed, retailer 2 has no incentive to deviate. Suppose retailer 2 deviates to take a position. For any price p_2, if the market is covered, retailer 2's profit will be less than the profit when retailer 2 is the monopoly, charging the same price. If the market is not covered, retailer 2 can be regarded as a monopoly. In both cases, retailer 2's profit cannot be higher than π_1^* since this is the highest possible profit that a monopoly can get on the Hotelling line. Our parameter restrictions ensure that π_1^* is less than v_A, which is the cost per click to get displayed. Therefore, retailer 2 has not incentive to get displayed without support from the manufacturer.

Step 5:

We first compare the manufacturer's optimal profits under the cases of sponsoring two versus one retailer.

For $t \geq \max\{\frac{v-w}{2}, \frac{\bar{\theta}}{4v_A}(v-w)^2\}$, it is more profitable for the manufacturer to sponsor one retailer if and only if $\frac{\bar{\theta}}{4t}(v^2-w^2)-v_A > \bar{\theta}(w+t)-2v_A$, which is equivalent to $t < \frac{v_A-\bar{\theta}w}{2\bar{\theta}}+\frac{\sqrt{(v_A-\bar{\theta}w)^2+\bar{\theta}^2(v^2-w^2)}}{2\bar{\theta}}$. Notice that $\frac{v_A-\bar{\theta}w}{2\bar{\theta}} + \frac{\sqrt{(v_A-\bar{\theta}w)^2+\bar{\theta}^2(v^2-w^2)}}{2\bar{\theta}} > \frac{v-w}{2}$ is equivalent to $v-w>0$, which always holds by assumption. Also $\frac{v_A-\bar{\theta}w}{2\bar{\theta}} + \frac{\sqrt{(v_A-\bar{\theta}w)^2+\bar{\theta}^2(v^2-w^2)}}{2\bar{\theta}} > 2\frac{v_A}{\bar{\theta}}$ is equivalent to $\frac{v+w}{2}\frac{v-w}{2} > wv_A$, which holds given $v_A < \frac{v-w}{2}$ and $w < \frac{v+w}{2}$. We have proven that when $\frac{3}{8}(v-w) < \frac{v_A}{\bar{\theta}} < \frac{v-w}{2}$, $2\frac{v_A}{\bar{\theta}} > \frac{2}{3}(v-w) > \frac{\bar{\theta}}{4v_A}(v-w)^2$.

Therefore, when $\frac{3}{8}(v-w) < \frac{v_A}{\bar{\theta}} < \frac{v-w}{2}$, for any $t \in (\frac{\bar{\theta}}{4v_A}(v-w)^2, \frac{2}{3}(v-w)]$, sponsoring one retailer is more profitable than sponsoring two retailers and this set is non-empty. When $\frac{v_A}{\bar{\theta}} \geq \frac{v-w}{2}$, for $t \in \left[\frac{v-w}{2}, \frac{v_A-\bar{\theta}w}{2\bar{\theta}} + \frac{\sqrt{(v_A-\bar{\theta}w)^2+\bar{\theta}(v^2-w^2)}}{2\bar{\theta}}\right)$, sponsoring one retailer is more profitable than sponsoring two retailers and this set is non-empty.

For $t < \frac{v-w}{2}$, it is more profitable for the manufacturer to sponsor one retailer if and only if $t < \frac{1}{2}(\frac{v_A}{\bar{\theta}} + v - w)$, which is naturally satisfied given that $t < \frac{v-w}{2}$. Therefore, when $t < \frac{v-w}{2}$, it is always more profitable for the manufacturer to sponsor only one retailer than to sponsor two retailers. This result is intuitive, since when $t < \frac{v-w}{2}$, one retailer can already covers the market. The manufacturer cannot get more demand by sponsoring two retailers, but will incur higher sponsoring cost. Thus when $t < \frac{v-w}{2}$, it is always more profitable to sponsor one retailer than to sponsor both retailers.

Summarizing the two cases and combining with the parameter restrictions we get in Step 4: denote

$$B_1 \equiv \frac{v_A - \bar{\theta}w}{2\bar{\theta}} + \frac{\sqrt{(v_A - \bar{\theta}w)^2 + \bar{\theta}(v^2 - w^2)}}{2\bar{\theta}},$$
$$B_2 \equiv \frac{2}{3}(v-w).$$

- For $t \in (B_0, \min\{B_1, B_2\}]$, it is more profitable for the manufacture to sponsor one retailer rather than to sponsor two retailers. Notice that this interval is always non-empty given our assumptions.

- For $t \in (\min\{B_1, B_2\}, B_2]$, it is more profitable for the manufacturer to sponsor two retailers rather than to sponsor one retailer. (If $B_1 \leq B_2$, then this interval is empty and there does not exist t such that it is more profitable to sponsor two retailers.)

Step 6:

Then we need to figure out whether the manufacturer would rather sponsor neither retailer for some t, i.e., to check whether $\max\{\pi^*_{M(1)}, \pi^*_{M(2)}\}$ is negative. We can classify t into three intervals, as listed below.

(1) When $\frac{v_A}{\bar{\theta}} \geq \frac{v-w}{2}$, there exists $t \in \left(v - w - \frac{v_A}{\bar{\theta}}, \frac{v-w}{2}\right]$, and for those t, $\pi^*_{M(1)} > \pi^*_{M(2)}$. Comparing $\pi^*_{M(1)} = \bar{\theta}(v-t) - v_A$ with 0, $\pi^*_{M(1)} \geq 0$ is equivalent to $t \leq v - \frac{v_A}{\bar{\theta}}$.

- If $v - \frac{v_A}{\bar{\theta}} \leq \frac{v-w}{2}$ (which is equivalent to $\frac{v_A}{\bar{\theta}} \geq \frac{1}{2}(v+w)$), then the manufacturer should sponsor one retailer when $t \in \left(v - w - \frac{v_A}{\bar{\theta}}, v - \frac{v_A}{\bar{\theta}}\right]$, and should sponsor zero retailer when $t \in \left(v - \frac{v_A}{\bar{\theta}}, \frac{v-w}{2}\right]$. Notice that the former interval is non-empty if and only if $v > \frac{v_A}{\bar{\theta}}$.
- If $v - \frac{v_A}{\bar{\theta}} > \frac{v-w}{2}$ (which is equivalent to $\frac{v_A}{\bar{\theta}} < \frac{1}{2}(v+w)$), then the manufacturer should sponsor one retailer for any $t \in \left(v - w - \frac{v_A}{\bar{\theta}}, \frac{v-w}{2}\right]$.

(2) For $t \in \left(\max\{\frac{v-w}{2}, \frac{\bar{\theta}}{4v_A}(v-w)^2\}, \min\{B_1, B_2\}\right]$, $\pi^*_{M(1)} > \pi^*_{M(2)}$ .Comparing $\pi^*_{M(1)} = \frac{\bar{\theta}}{4t}(v^2 - w^2) - v_A$ with 0.

- If $\frac{v_A}{\bar{\theta}} \geq \frac{1}{2}(v+w)$, then for any $t > \frac{v-w}{2}$, $\frac{\bar{\theta}}{4t}(v^2 - w^2) - v_A = \frac{v-w}{2t}\frac{v+w}{2}\bar{\theta} - v_A < \frac{v+w}{2}\bar{\theta} - v_A \leq 0$, which means that for any $t \in \left(\max\{\frac{v-w}{2}, \frac{\bar{\theta}}{4v_A}(v-w)^2\}, \min\{B_1, B_2\}\right]$, the manufacturer should sponsor zero retailer.
- If $\frac{v_A}{\bar{\theta}} < \frac{1}{2}(v+w)$, then $\frac{\bar{\theta}}{4t}(v^2 - w^2) - v_A \geq 0$ when $t \leq \frac{\bar{\theta}(v^2-w^2)}{4v_A}$. Thus the manufacturer should sponsor one retailer for $t \in \left(\max\{\frac{v-w}{2}, \frac{\bar{\theta}}{4v_A}(v-w)^2\}, \min\{\frac{\bar{\theta}(v^2-w^2)}{4v_A}, B_1, B_2\}\right]$. Notice that $\frac{\bar{\theta}(v^2-w^2)}{4v_A} > \frac{v-w}{2}$, $\frac{\bar{\theta}(v^2-w^2)}{4v_A} > \frac{\bar{\theta}}{4v_A}(v-w)^2$. We have also shown that $B_1, B_2 > \frac{v-w}{2}$, and $B_1, B_2 > \frac{\bar{\theta}}{4v_A}(v-w)^2$ when $\frac{3}{8}(v-w) < \frac{v_A}{\bar{\theta}} < \frac{v-w}{2}$. Thus the interval $\left(\max\{\frac{v-w}{2}, \frac{\bar{\theta}}{4v_A}(v-w)^2\}, \min\{\frac{\bar{\theta}(v^2-w^2)}{4v_A}, B_1, B_2\}\right]$ is non-empty. For $t \in \left(\min\{\frac{\bar{\theta}(v^2-w^2)}{4v_A}, B_1, B_2\}, \min\{B_1, B_2\}\right]$, the manufacturer should sponsor zero retailer.

(3) When $B_1 < B_2$, there exists $t \in (B_1, B_2]$ such that $\pi^*_{M(1)} < \pi^*_{M(2)}$. Comparing $\pi^*_{M(2)} = \bar{\theta}(w+t) - 2v_A$ to 0. $\pi^*_{M(2)} > 0$ is equivalent to $t > 2\frac{v_A}{\bar{\theta}} - w$. Thus the manufacturer should sponsor two retailers when $t \in (\max\{B_1, 2\frac{v_A}{\bar{\theta}} - w\}, B_2]$.

Notice that $2\frac{v_A}{\bar{\theta}} - w < B_2 \equiv \frac{2}{3}(v-w)$ is equivalent to $\frac{v_A}{\bar{\theta}} < \frac{1}{3}v + \frac{1}{6}w$. $B_1 < B_2$ is equivalent to $\frac{v_A}{\bar{\theta}} > \frac{7}{24}v - \frac{1}{24}w$.

Thus when $\frac{7}{24}v - \frac{1}{24}w < \frac{v_A}{\bar{\theta}} < \frac{1}{3}v + \frac{1}{6}w$, the manufacturer should sponsor zero retailer for $t \in \left(B_1, \max\{B_1, 2\frac{v_A}{\bar{\theta}} - w\}\right]$ and should sponsor two retailers for $t \in \left(\max\{B_1, 2\frac{v_A}{\bar{\theta}} - w\}, B_2\right]$. When $\frac{v_A}{\bar{\theta}} \geq \frac{1}{3}v + \frac{1}{6}w$, the manufacturer should sponsor zero retailer for any $t \in (B_1, B_2]$.

Summarizing the three intervals, we get the following result:

- If $v_A \geq \bar{\theta}v$, the manufacturer should sponsor neither retailer for any t under our parameter restrictions.

- If $\frac{\bar{\theta}}{2}(v+w) \leq v_A < \bar{\theta}v$, the manufacturer should sponsor one retailer for $t \in \left(v - w - \frac{v_A}{\bar{\theta}}, v - \frac{v_A}{\bar{\theta}}\right]$, and should sponsor neither retailer for $t \in (v - \frac{v_A}{\bar{\theta}}, B_2]$.

- If $\frac{3\bar{\theta}}{8}(v-w) < v_A < \frac{\bar{\theta}}{2}(v+w)$, the manufacturer should sponsor one retailer for

$$t \in \left(B_0, \min\left\{\frac{\bar{\theta}(v^2-w^2)}{4v_A}, B_1, B_2\right\}\right),$$

  and this interval is non-empty given the parameter assumptions. The manufacturer should sponsor two retailers for any

$$t \in \left[\max\left\{2\frac{v_A}{\bar{\theta}} - w, B_1\right\}, B_2\right],$$

  and this interval is non-empty only when $\frac{7}{24}v - \frac{1}{24}w < \frac{v_A}{\bar{\theta}} < \frac{1}{3}v + \frac{1}{6}w$. The manufacturer should sponsor neither retailer for any

$$t \in \left[\min\left\{\frac{\bar{\theta}(v^2-w^2)}{4v_A}, B_1, B_2\right\}, \min\left\{\max\left\{2\frac{v_A}{\bar{\theta}} - w, B_1\right\}, B_2\right\}\right).$$

OA.5. Endogenous Wholesale Contracts and Retail Prices:

OA.5.1. Proof of Lemma 2:

Proof. Suppose $v_1^* \geq v_2^*$, we first prove that retailer 1 will have a higher position than retailer 2 in equilibrium. We prove by contradiction. Suppose in equilibrium retailer 1 has a lower position than retailer 2, then we must have $v_1 < v_2$. If retailer 1 is in position 3, it can deviate by setting v_1 equal to v_1^*, and earn positive profit at position 2 or 1. Thus, it must be that retailer 1 takes position 2, and retailer 2 takes position 1. We have $v_1^* \geq v_2^* \geq v_A$. In this case, retailer 1's profit $\pi_{R_1} = d_2\left[v_1 - (1-\alpha_1)v_A\right]$, which increases with v_1, so retailer 1 will choose the largest possible v_1

given its position. We have $v_1 = v_2^*$, and $\pi_{R_1} = d_2 [v_2^* - (1-\alpha_1)v_A]$. Now, if retailer 1 deviates by setting $v_1 = v_1^*$, it will take the first position, and retailer 2 will take the second position. Similarly, given its position, retailer 2 will set $v_2 = v_2^*$ to maximize its profit. As a result, retailer 1 will earn,

$$\pi'_{R_1} = d_1 \left[v_1^* - (1-\alpha_1) \left(\frac{d_1 - d_2}{d_1} v_2^* + \frac{d_2}{d_1} v_A \right) \right] > d_1 \left[v_1 - (1-\alpha_1) \left(\frac{d_1 - d_2}{d_1} v_A + \frac{d_2}{d_1} v_A \right) \right] = \pi_{R_1}.$$

This implies that it is not an equilibrium for retailer 1 to take the second position while retailer 2 is taking the first position. To summarize, we have proved that in equilibrium, when $v_1^* \geq v_2^*$, retailer 1 has a higher position than retailer 2.

Next, suppose $v_1^* \geq v_2^*$, and we need to prove that (i) when $v_A > v_1^*$, the outside advertiser will take the first position; (ii) when $v_1^* \geq v_A > v_2^*$, the outside advertiser will take the second position; and (iii) when $v_2^* \geq v_A$, the outside advertiser will take the third position.

(i) is obvious: given $v_A > v_1^* \geq v_1$, the outsider advertiser must take a higher position than retailer 1.

(ii): When $v_1^* \geq v_A > v_2^*$, we have $v_A > v_2^* \geq v_2$, so the outside advertiser will always take a higher position than retailer 2. Suppose it takes position 1, then we must have $v_A > v_1$. In this case, retailer 1's profit is $\pi_{R_1} = d_2 [v_1 - (1-\alpha_1)v_2]$. Similarly as above, we can show that retailer 1 can earn a higher profit by setting v_1 as v_1^* and take the first position instead. Then (ii) is proved.

(iii) is straightforward to prove by contradiction. Suppose $v_2^* \geq v_A$ and the outside advertiser takes the second position. In this case, retailer 2 will take position 3 and earn zero profit, and it can deviate by setting v_2 at v_2^* and taking the second position instead, which will give its positive profit.

Given that the positions are determined by the rank of v_1^*, v_2^*, and v_A, we know that each retailer i' profit function increases with v_i, so in equilibrium, retailer i will set retail price $p_i^* = (1 + w_i)/2$, under which v_i takes the maximum value v_i^*. ∎

OA.5.2. Equilibrium Analysis of the Model with Linear Contracts (Proof of Theorem 5)

Without loss of generalizability, we assume that retailer 1 gets a higher position than retailer 2 in equilibrium. There are three possible position configurations then.

- Consider the position configuration (R_1, R_2).

The manufacturer's optimization problem is,

$$\begin{aligned}
&\max_{\alpha_i, w_i} d_1 \left[\bar{\theta}\frac{1-w_1}{2}(w_1-c) - \alpha_1\left(\frac{d_1-d_2}{d_1}\frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)} + \frac{d_2}{d_1}v_A\right)\right] + d_2\left[\bar{\theta}\frac{1-w_2}{2}(w_2-c) - \alpha_2 v_A\right] \\
\text{s.t.}\quad & \frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \geq \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)} \geq v_A, \\
& 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2.
\end{aligned}$$

The optimal solution is,

$$\begin{cases}
w_1^* = w_2^* = c,\ \alpha_1^* = \alpha_2^* = 1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\
w_1^* = w_2^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}},\ \alpha_1^* = \alpha_2^* = 0, & \text{when } \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \\
w_1^* = w_2^* = \frac{1+c}{2},\ \alpha_1^* = \alpha_2^* = 0, & \text{when } v_A \leq \frac{\bar{\theta}(1-c)^2}{16}
\end{cases}.$$

Correspondingly, the manufacturer's profit is,

$$\pi_M^* = \begin{cases}
(d_1+d_2)\left[\frac{\bar{\theta}(1-c)^2}{4} - v_A\right] & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\
(d_1+d_2)\left[(1-c)\sqrt{v_A\bar{\theta}} - 2v_A\right] & \text{when } \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \\
(d_1+d_2)\frac{\bar{\theta}(1-c)^2}{8} & \text{when } v_A \leq \frac{\bar{\theta}(1-c)^2}{16}
\end{cases}.$$

To summarize, in this case, the manufacturer sells to both retailers. The wholesale prices and participation rates for the two retailers are "symmetric", i.e., the manufacturer will only provide a marginally lower wholesale price or a marginally higher participation rate to retailer 1 in order to let it get a higher position than retailer 2. When the outside advertiser's profit per click is low, the manufacturer sets the monopolistic wholesale prices and provides zero participation rates to both retailers. When the outside advertiser's profit per click is medium, the manufacturer lowers wholesale prices but still provides zero participation rates. When the outside advertiser's profit per click is relatively high, the manufacturer only charges marginal production cost as wholesale prices, and provides positive participation rates to both retailers, so as to keep them in the first two positions.

- Consider the position configuration (R_1, A).

The manufacturer's optimization problem is,

$$\begin{aligned}
&\max_{\alpha_i, w_i} d_1 \left[\bar{\theta}\frac{1-w_1}{2}(w_1-c) - \alpha_1\left(\frac{d_1-d_2}{d_1}v_A + \frac{d_2}{d_1}\frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}\right)\right] \\
\text{s.t.} \quad & \frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \geq v_A > \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}, \\
& 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2.
\end{aligned}$$

The optimal solution is,

$$\begin{cases}
w_1^* = \frac{d_1 c + d_2}{d_1 + d_2}, w_2^* = 1, \alpha_1^* = 1 - \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1+d_2)^2 v_A}, \alpha_2^* \in [0,1), & v_A \geq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1+d_2)^2} \\
w_1^* = 1 - \sqrt{\frac{4v_A}{\bar{\theta}}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0,1), & \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1+d_2)^2} \\
w_1^* = \frac{1+c}{2}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0,1), & v_A \leq \frac{\bar{\theta}(1-c)^2}{16}
\end{cases}.$$

Correspondingly, the manufacturer's profit is,

$$\pi_M^* = \begin{cases}
\frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1+d_2)} - (d_1-d_2)v_A & \text{when } v_A \geq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1+d_2)^2} \\
d_1\left[(1-c)\sqrt{v_A\bar{\theta}} - 2v_A\right] & \text{when } \frac{\bar{\theta}(1-c)^2}{16} \leq v_A \leq \frac{\bar{\theta}(1-c)^2 d_1^2}{4(d_1+d_2)^2} \\
d_1\frac{\bar{\theta}(1-c)^2}{8} & \text{when } v_A \leq \frac{\bar{\theta}(1-c)^2}{16}
\end{cases}.$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1. When the outside advertiser's profit per click is relatively low, the manufacturer sets the monopolistic wholesale price and provides zero participation rate to the retailer at the same time. When the outside advertiser's profit per click is medium, the manufacturer lowers the wholesale price but still provides zero participation rate. A lower wholesale price leaves more profit margin to the retailer thus incentivizes its to bid higher so as to keep the first position. When the outside advertiser's profit per click is relatively high, the manufacturer will set a low wholesale price and provide positive participation rate at the same time so as to keep the retailer in the first position.

- Consider the position configuration (A, R_1).

The manufacturer's optimization problem is,

$$\begin{aligned}
&\max_{\alpha_i, w_i} d_2 \left[\bar{\theta}\frac{1-w_1}{2}(w_1-c) - \alpha_1\frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}\right] \\
\text{s.t.} \quad & v_A > \frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \geq \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}, \\
& 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2.
\end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = \frac{c+1}{2}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0,1), & v_A \geq \frac{\bar{\theta}(1-c)^2}{16} \\ w_1^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0,1), & v_A \leq \frac{\bar{\theta}(1-c)^2}{16} \end{cases}.$$

Correspondingly, the manufacturer's profit is,

$$\pi_M^* = \begin{cases} d_2 \frac{\bar{\theta}(1-c)^2}{8}, & v_A \geq \frac{\bar{\theta}(1-c)^2}{16} \\ d_2 \left[(1-c)\sqrt{v_A \bar{\theta}} - 2v_A \right], & v_A \leq \frac{\bar{\theta}(1-c)^2}{16} \end{cases}.$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1, and provides a zero participation rate. The manufacturer sets the monopolistic wholesale price when the outside advertiser's profit per click is relatively high; otherwise, it increases the wholesale price thus decreases the retailer's profit margin and its incentive to bid when the outside advertiser's profit per click is relatively low.

OA.5.3. Equilibrium Analysis of the Model with Two-Part Tariffs (Proof of Theorem 6)

The equilibrium analysis here parallels with that for the linear contracts above. For each of the three possible position configurations, we will formulate and then solve the manufacturer's channel profit maximization problem.

- Consider the position configuration (R_1, R_2).

  The manufacturer's optimization problem is,

$$\begin{aligned} \max_{\alpha_i, w_i} \; & d_1 \left[\bar{\theta} \frac{1-w_1}{2} \left(\frac{1+w_1}{2} - c \right) - \left(\frac{d_1 - d_2}{d_1} \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)} + \frac{d_2}{d_1} v_A \right) \right] \\ & + d_2 \left[\bar{\theta} \frac{1-w_2}{2} \left(\frac{1+w_2}{2} - c \right) - v_A \right] \\ \text{s.t.} \quad & \frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \geq \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)} \geq v_A, \\ & 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = c, w_2^* = 1 - \frac{d_2}{d_1}(1-c), \alpha_1^* \in [0,1), \alpha_2^* = 0, & v_A \leq \frac{d_2^2}{d_1^2} \frac{\bar{\theta}(1-c)^2}{4} \\ w_1^* = c, w_2^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, \alpha_1^* \in [0,1), \alpha_2^* = 0, & \frac{d_2^2}{d_1^2} \frac{\bar{\theta}(1-c)^2}{4} \leq v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \\ w_1^* = c, w_2^* = c, \alpha_1^* \in [1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, 1), \alpha_2^* = 1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, & v_A \geq \frac{\bar{\theta}(1-c)^2}{4}. \end{cases}$$

Correspondingly, the channel profit is,

$$\pi_C^* = \begin{cases} \frac{d_1^2+d_2^2}{d_1}\frac{\bar{\theta}(1-c)^2}{4} - 2d_2 v_A, & v_A \le \frac{d_2^2}{d_1^2}\frac{\bar{\theta}(1-c)^2}{4} \\ d_1\frac{\bar{\theta}(1-c)^2}{4} + d_2(1-c)\sqrt{v_A\bar{\theta}} - (d_1+2d_2)v_A, & \frac{d_2^2}{d_1^2}\frac{\bar{\theta}(1-c)^2}{4} \le v_A \le \frac{\bar{\theta}(1-c)^2}{4} \\ (d_1+d_2)\frac{\bar{\theta}(1-c)^2}{4} - (d_1+d_2)v_A, & v_A \ge \frac{\bar{\theta}(1-c)^2}{4}. \end{cases}.$$

To summarize, in this case, the manufacturer sells to both retailers. It sets the wholesale price as the production cost and provides high enough participation rate for retailer 1 to ensure it gets the first position. When v_A is relatively low, it sets the wholesale price higher than marginal production cost and provides zero participation rate for retailer 2; when v_A is relatively high, it sets the wholesale price at the marginal production cost and provides positive participation rate for retailer 2.

- Consider the position configuration (R_1, A).

  The manufacturer's optimization problem is,

$$\begin{aligned} &\max_{\alpha_i, w_i} d_1 \left[\bar{\theta}\frac{1-w_1}{2}\left(\frac{1+w_1}{2} - c\right) - \left(\frac{d_1-d_2}{d_1}v_A + \frac{d_2}{d_1}\frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}\right)\right] \\ \text{s.t.} \quad &\frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \ge v_A > \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}, \\ &0 \le \alpha_i < 1, 0 \le w_i \le 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = c, w_2^* = 1, \alpha_1^* \in [1 - \frac{\bar{\theta}(1-c)^2}{4v_A}, 1), \alpha_2^* \in [0,1), & v_A \ge \frac{\bar{\theta}(1-c)^2}{4} \\ w_1^* = c, w_2^* = 1, \alpha_1^* \in [0,1), \alpha_2^* \in [0,1), & v_A \le \frac{\bar{\theta}(1-c)^2}{4} \end{cases}.$$

Under both cases, the channel profit is,

$$\pi_C^* = d_1\frac{\bar{\theta}(1-c)^2}{4} - (d_1 - d_2)v_A.$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1. It sets the wholesale price as the marginal production cost, and provides the participation rate high enough to help retailer 1 outbid the outside advertiser. The participate rate does not influence the channel profit.

- Consider the position configuration (A, R_1).

The manufacturer's optimization problem is,

$$\begin{aligned} &\max_{\alpha_i, w_i} d_2 \left[\bar{\theta}\frac{1-w_1}{2}\left(\frac{1+w_1}{2}-c\right)-\frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}\right] \\ \text{s.t.} \quad & v_A > \frac{\bar{\theta}(1-w_1)^2}{4(1-\alpha_1)} \geq \frac{\bar{\theta}(1-w_2)^2}{4(1-\alpha_2)}, \\ & 0 \leq \alpha_i < 1, 0 \leq w_i \leq 1, \text{ for } i = 1, 2. \end{aligned}$$

The optimal solution is,

$$\begin{cases} w_1^* = c, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\ w_1^* = 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, w_2^* = 1, \alpha_1^* = 0, \alpha_2^* \in [0, 1), & v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \end{cases}.$$

Correspondingly, the channel profit is,

$$\pi_C^* = \begin{cases} d_2\frac{\bar{\theta}(1-c)^2}{4}, & v_A \geq \frac{\bar{\theta}(1-c)^2}{4} \\ d_2\left[(1-c)\sqrt{v_A\bar{\theta}} - v_A\right], & v_A \leq \frac{\bar{\theta}(1-c)^2}{4} \end{cases}.$$

To summarize, in this case, the manufacturer essentially only sells to retailer 1, and provides a zero participation rate. The manufacturer sets the wholesale price as the marginal production cost when the outside advertiser's profit per click is relatively high; otherwise, it increases the wholesale price thus decreases the retailer's profit margin and its incentive to bid when the outside advertiser's profit per click is relatively low.