Online Appendix to

Managing Consumer Deliberations in a Decentralized Distribution Channel

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1. Proof of Propositions

1.1. Proof of Proposition 6.

The proof is similar to the proof of Proposition 2. If the manufacturer desires to induce deliberation, the optimal wholesale price should be $w \in \{\frac{s+1}{2}, 1-\sqrt{2c}\}$, leading the retailer to price at either $p = \frac{3+s}{4}$ or $p = 1-\sqrt{2c}$ ($\frac{s+1}{2}$ is the wholesale price under standard double marginalization, and $1-\sqrt{2c}$ is the upper bound price). With $w = \frac{s+1}{2}$, the retail price is $p = \frac{s+3}{4}$ and the demand is $d = \frac{1-s}{4}$, and the manufacturer's profit is $\pi_m = \frac{(1-s)^2}{8}$. With $w = 1-\sqrt{2c}$, the retail price is $p = 1-\sqrt{2c}$ and the demand is $\sqrt{2c}$, and the manufacturer's profit is $(1-\sqrt{2c})\sqrt{2c}$. By comparing the manufacturer's profits, we can compute c_1, as described above.

c_2^s is computed similarly. Notice that the requirement conditions for inhibiting deliberation do not depend on s, and the equilibrium prices, given that deliberation is inhibited, are unchanged. The manufacturer's profit, however, is changed to $\pi_m = \frac{2c}{1-\sqrt{2c}} - s$. c_2^s is then derived by comparing profits.

It remains to check the monotonicity of c_1^s and c_2^s. The monotonicity is straightforward for c_1^s. As for c_2^s, let $t = 1-\sqrt{2c_2^s}$, then we have $(t-s)(1-t) = \frac{(1-t)^2}{t} - s$, or, equivalently, $-t^2 + st = \frac{1}{t} - 2$, taking partial derivatives with respect to s, we have $\frac{\partial t}{\partial s}(2t - \frac{1}{t^2} - s) = t$. Simple algebra yields that $\frac{\partial t}{\partial s} < 0$ holds when $c_2 \geq .021$, which always holds. Therefore, c_2^s is increasing in t. □

1.2. Proof of Proposition 7.

We consider two scenarios: inducing deliberation or inhibiting deliberation. Note that here "inducing" or "inhibiting" applies only to U consumers; S consumers always know their valuations.

When the manufacturer desires to induce deliberation, similarly to the proof of Proposition 2, the optimal prices $w \in \{\frac{1}{2}, 1-\frac{2}{\lambda}(\sqrt{2c}-\sqrt{2c\bar{\lambda}})\}$. We shall explain later how $1-\frac{2}{\lambda}(\sqrt{2c}-\sqrt{2c\bar{\lambda}})$ is calculated. When $w=\frac{1}{2}$, $p=\frac{3}{4}$ and total sales $S=1-p=\frac{1}{4}$. The manufacturer's profit is $\pi_1 = wS = \frac{1}{8}$. On the other hand, if the manufacturer wants the retailer to price at $p = 1-\sqrt{2c}$, as in Proposition 2, the following incentive compatibility constraint should be imposed,

$$(1-\sqrt{2c}-w)\sqrt{2c} \geq \Big(\frac{w+1}{2}-w\Big)\lambda\Big(1-\frac{w+1}{2}\Big) \tag{OA1}$$

The left hand side of Inequality (OA1) is the retailer's profit from selling to both S and U consumers; and the right hand side corresponds to the retailer's profit from selling to S consumers only. Solving this, the manufacturer faces the following constraint $w \leq 1-\frac{2}{\lambda}(\sqrt{2c}-\sqrt{2c\bar{\lambda}})$. When the manufacturer optimizes its profit at $w = 1-\frac{2}{\lambda}(\sqrt{2c}-\sqrt{2c\bar{\lambda}})$, the profit is $\pi_2 = (1-\frac{2}{\lambda}(\sqrt{2c}-\sqrt{2c\bar{\lambda}}))\sqrt{2c}$. At c_1, the manufacturer is indifferent between π_1 and π_2, and hence we have

$$c_1 = \frac{1}{32}\Big(3+3\sqrt{\bar{\lambda}}-2\lambda-2\sqrt{4(1+\sqrt{\bar{\lambda}})+\lambda(-5-3\sqrt{\bar{\lambda}}+\lambda)}\Big) \tag{OA2}$$

Consider now the scenario where the manufacturer desires to inhibit deliberation. To achieve this, the retail price should be $p=\min(\sqrt{2c},\frac{1}{2})$. When $c \leq \frac{1}{8}$, $p=\sqrt{2c}$ applies. The retailer may deviate from $p=\sqrt{2c}$ in the following two ways:

- Price at $p = 1-\sqrt{2c}$ and sell to both S and U consumers with sufficiently high valuations.
- Price at $p=\frac{1+w}{2}$ and sell only to S consumers with sufficiently high valuations.

When c is relatively small, the first way is more profitable; otherwise, the opposite is true. Hence, the following incentive compatibility constraint should be imposed when c is small.

$$(\sqrt{2c}-w)(\lambda(1-\sqrt{2c})+\bar{\lambda}) \geq (1-\sqrt{2c}-w)\sqrt{2c} \tag{OA3}$$

and we have $w \leq \frac{2c\bar{\lambda}}{1-(1+\lambda)\sqrt{2c}}$. When the retailer prices at $w = \frac{2c\bar{\lambda}}{1-(1+\lambda)\sqrt{2c}}$, the profit is optimized to $\pi_3^a = \frac{2\bar{\lambda}c(1-\lambda\sqrt{2c})}{1-\sqrt{2c}(1+\lambda)}$. The cutoff point c_2 solves

$$\pi_2 = \pi_3^a \tag{OA4}$$

When c further increases (but still smaller than $\frac{1}{2}$), the incentive compatibility constraint becomes

$$(\sqrt{2c}-w)(\lambda(1-\sqrt{2c})+\bar{\lambda}) \geq \Big(\frac{w+1}{2}-w\Big)\lambda\Big(1-\frac{w+1}{2}\Big) \tag{OA5}$$

Therefore, the optimal wholesale price is $w = 1 + 2\sqrt{2c} - \frac{2-2\sqrt{\bar{\lambda}(1-\sqrt{2c}\lambda)}}{\lambda}$ and the manufacturer's profit is $\pi_3^b = \big(1 + 2\sqrt{2c} - \frac{2-2\sqrt{\bar{\lambda}(1-\sqrt{2c}\lambda)}}{\lambda}\big)(\lambda(1-\sqrt{2c}) + \bar{\lambda})$. The cutoff point c_3 is given by $w_2 = w_3$. □

1.3. Proof of Proposition 9.

We first prove the following Lemma.

Lemma OA1 *Suppose that in equilibrium $p = 1 - \sqrt{2c}$; deliberation is induced. Then $w_1 = w_2 = p$ and the manufacturer takes all channel profit.*

Proof. Notice that $p = 1 - \sqrt{2c}$ is already the upper bound on retail price. Assume for contradiction that $w_1 < 1 - \sqrt{2c}$. Then, the manufacturer is strictly better off pricing at $w_1 = 1 - \sqrt{2c}$; as a result, $w_1 = w_2 = p = 1 - \sqrt{2c}$ and retail demand is not affected. On the other hand, if $w_1 = 1 - \sqrt{2c}$, the only possible pricing scheme is $w_1 = w_2 = p$. This proves the lemma. □

With Lemma OA1, when the manufacturer induces deliberation, there are two possibilities (corresponding to Regions I and II): $w_1 < w_2 < p < 1 - \sqrt{2c}$ and $w_1 = w_2 = p = 1 - \sqrt{2c}$. For the first case, it is easy to see that $w_1 = \frac{1}{2}, w_2 = \frac{3}{4}, p = \frac{7}{8}$ [1] and $\pi_m = \frac{1}{16}$; for the second case we have $\pi_m = (1 - \sqrt{2c})\sqrt{2c}$.

Next we consider the scenario where the manufacturer inhibits deliberation. Similarly to the base model, the maximum retail price is $p = \sqrt{2c}$. To prevent the retailer from deviation (to price at $w_2 = 1 - \sqrt{2c}$), the following incentive compatible condition should be satisfied[2]

$$(1 - \sqrt{2c} - w_2)\sqrt{2c} \leq \sqrt{2c} - w_2$$

Also, to prevent the distributor from deviation (to price at $w_1 = 1 - \sqrt{2c}$), the following incentive compatible condition is imposed,

$$(1 - \sqrt{2c} - w_1)\sqrt{2c} \leq w_2 - w_1$$

The pricing strategy is optimal when both inequalities are tight, i.e.,

$$w_1 = \frac{\sqrt{c}(6\sqrt{c} - 2\sqrt{2}c - \sqrt{2})}{(1-\sqrt{2c})^2}, w_2 = \frac{2c}{1 - \sqrt{2c}}$$

and that the manufacturer's profit is $\pi_m = w_1$.

The proposition follows by comparing the profit functions under each alternative pricing strategies. Finally, we check the validity of each scheme and complete the proof. □

[1] It is verified later that in equilibrium, the distributor has no incentive to deviate from this pricing in Region I.

[2] In fact, the retailer prices at either $\sqrt{2c}$ or $\frac{1+w_2}{2}$. It is checked later that it is more profitable to price at $p = \sqrt{2c}$ rather than $p = \frac{1+w_2}{2}$.

2. Analysis of the case $c > \frac{1}{8}$

Before presenting our results regarding the case $c > \frac{1}{8}$, let us recall a result in Lemma 1 that "When $c > \frac{1}{8}$, consumers do not deliberate; they make purchases if and only if $p \leq \frac{1}{2}$".

Thus, following Lemma 1, consumers never deliberate when $c > \frac{1}{8}$. If $w \leq \frac{1}{2}$, then the retailer optimally charges the upper bound price $p = \frac{1}{2}$ and serves the entire market; otherwise, since any price $p \geq w > \frac{1}{2}$ will lead to zero sales, it will not sell the product. Given the retailer's optimal response, the manufacturer's optimal strategy is to price at $w = \frac{1}{2}$. In this case, the retailer prices at $p = \frac{1}{2}$ and all consumers buy. Clearly, the channel is coordinated and the manufacturer takes the entire channel profit, and thus it has no incentive to integrate vertically.

As for consumer empowerment, our result does not change when taking the case $c > \frac{1}{8}$ into consideration. Still, the manufacturer's profit is maximized when $c \geq \frac{1}{8}$ (with $\pi_m = \frac{1}{2}$) and the retailer's profit is maximized when $c = \frac{7-3\sqrt{5}}{4}$ (with $\pi_r = \frac{7-3\sqrt{5}}{2}$). Therefore, their incentives to empower consumers are not affected.

Next, let us consider the equilibrium channel strategies in the presence of consumer heterogeneity in deliberation cost when $c > \frac{1}{8}$. Still, suppose that λ of the consumers are experts and know their private valuations whereas the remaining $\bar{\lambda} = 1 - \lambda$ of the consumers have to incur a deliberation cost c to assess their true valuations. Note that, here $w = \frac{1}{2}$ is no longer the optimal strategy to the manufacturer: the retailer could price at $p > w$ and still sell to the expert consumers only. To ensure that the retailer does not deviate, the following incentive compatibility constraint is needed:

$$\left(\frac{1}{2} - w\right)\left(\bar{\lambda} + \frac{1}{2}\lambda\right) \geq \left(\frac{w+1}{2} - w\right)\lambda\left(1 - \frac{w+1}{2}\right) \tag{OA6}$$

In equilibrium, $w = 2 - \frac{2-\sqrt{2(2-\lambda)\bar{\lambda}}}{\lambda}$ and $p = \frac{1}{2}$.

Finally, consider the multi-level channel equilibrium when $c > \frac{1}{8}$. It is verifiable that the equilibrium strategy is $w_1 = w_2 = p = \frac{1}{2}$. If the retailer deviates and prices at $p > \frac{1}{2}$, the demand will be zero, thus it has no incentive to deviate. The distributor does not deviate for the same reason. Therefore, the manufacturer takes the entire channel profit and the channel is coordinated.

3. Additional Analysis of the Model with Heterogeneous Deliberation Costs

In Section 5.2, we examine the model with two types of consumers, expert and regular consumers, and show that the effects of deliberation cost on channel strategies are similar with what we have observed in the basic model. Next, we show that our results regarding consumer empowerment and optimal quality decisions obtained in the basic model are also robust in this setting. However, the extended model with decisions on consumer empowerment or product quality is not analytically tractable. Consequently, we resort to numerical analysis. Here we report some results to show the robustness.

Consider the case where $\lambda = 0.3$ of the consumers are experts. We plot the equilibrium profits in Figure 1. The results are consistent with those observed in the main model. The figures clearly show that the manufacturer's profit is always increasing in deliberation cost, but the retailer's profit is not monotone in deliberation cost. Thus, the manufacturer always prefers a higher deliberation cost, but the retailer prefers an intermediate deliberation cost.

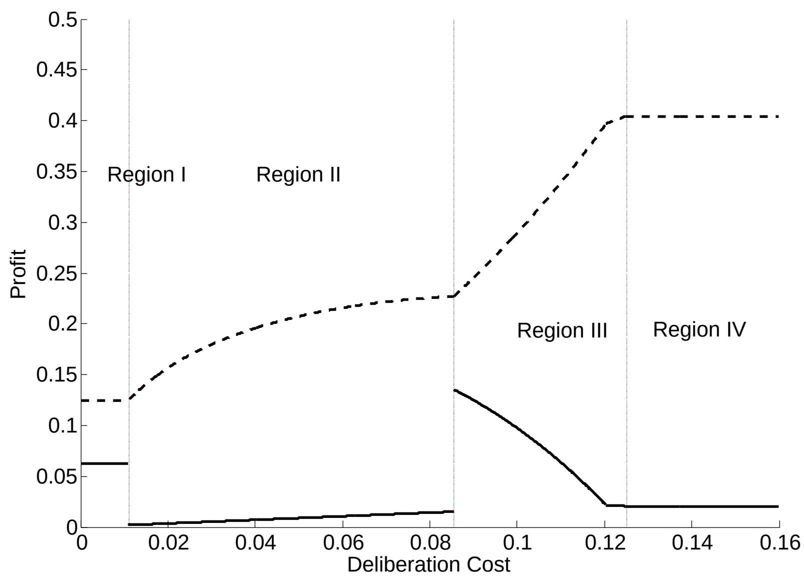

Figure 1 **The equilibrium profits when** $\lambda = 0.3$

Next, we investigate the manufacturer's quality decisions. Again, we assume that the manufacturer can choose any quality $q \in [0,1]$ and there is no cost of quality. We plot the optimal quality level for the manufacturer in Figure 2. Similar to the results of the basic model, the manufacturer may not always want to provide the highest-quality product even though the provision of quality is free. Under the intermediate range of deliberation cost, the manufacturer may prefer to offer an inferior product.

Our numerical analysis covers a wide range of λ values. As shown in Figures 3 to 5, our results on consumer empowerment and optimal product quality are consistent with the model of heterogenous deliberation costs.

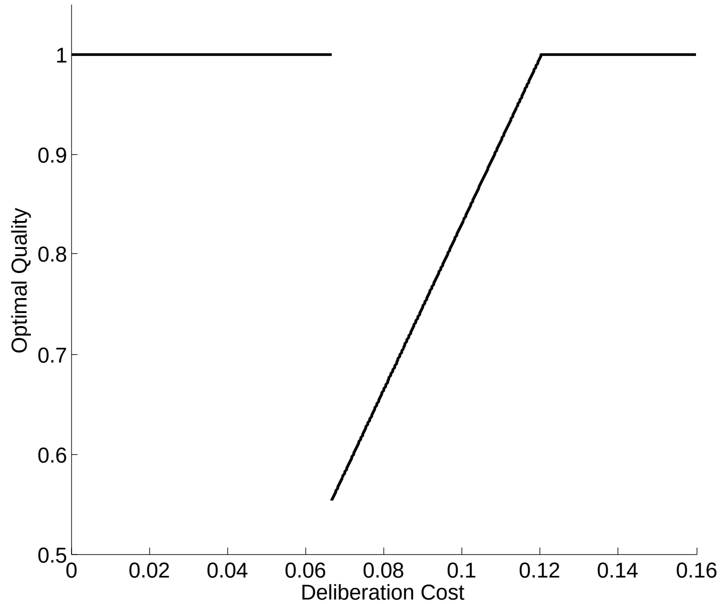

Figure 2 **The optimal quality when** $\lambda = 0.3$

4. Continuous Deliberation: A Simplified Model

Section 5.3 of the paper addresses continuous deliberation effort. However, under the model, ex-post, a consumer's valuation uncertainty is still either completely resolved (with probability ϕ) or completely unresolved (with probability $1-\phi$). In this section, we consider an alternative model to capture continuous deliberation outcome.

To simplify the analysis and obtain tractable results, we modify the model by assuming that that the prior distribution of consumer valuation follows the two-point distribution,

$$v_i = \begin{cases} h = \frac{1}{2} + k & \text{with probability } \frac{1}{2}, \\ l = \frac{1}{2} - k & \text{with probability } \frac{1}{2}. \end{cases}$$

As a technical assumption, we assume that $k \in [\frac{1}{14}, \frac{1}{2}]$ [3]. The expected valuation is still $E(v_i) = \frac{1}{2}$. Consumer i may invest in costly deliberation to acquire an unbiased signal s_i of her true valuation. Let $\tilde{h}$ and $\tilde{l}$ denote the signal indicating that the valuation will be h and l, respectively. Following Iyer, Narasimhan and Niraj (2007) and Jiang et al. (2016), let measure ρ denote the reliability of the signal, assuming that (i) when $\rho = 1$, the signal is perfectly reliable in the sense that $\Pr(\tilde{h}|h) = \Pr(\tilde{l}|l) = 1$, and when $\rho = 0$, the signal has no improvement over the prior. (ii) The signal is unbiased in the sense that $\Pr(h) = \Pr(\tilde{h})$. After imposing some consistency properties we can derive the that $\Pr(\tilde{h}|h) = \Pr(\tilde{l}|l) = \frac{1}{2} + \frac{\rho}{2}$ and hence $\Pr(h|\tilde{h}) = \Pr(l|\tilde{l}) = \frac{1}{2} + \frac{\rho}{2}$.

[3] If $k < \frac{1}{14}$, the manufacturer always induces the retailer to serve all consumers, and the model reverts to the conventional no deliberation setting.

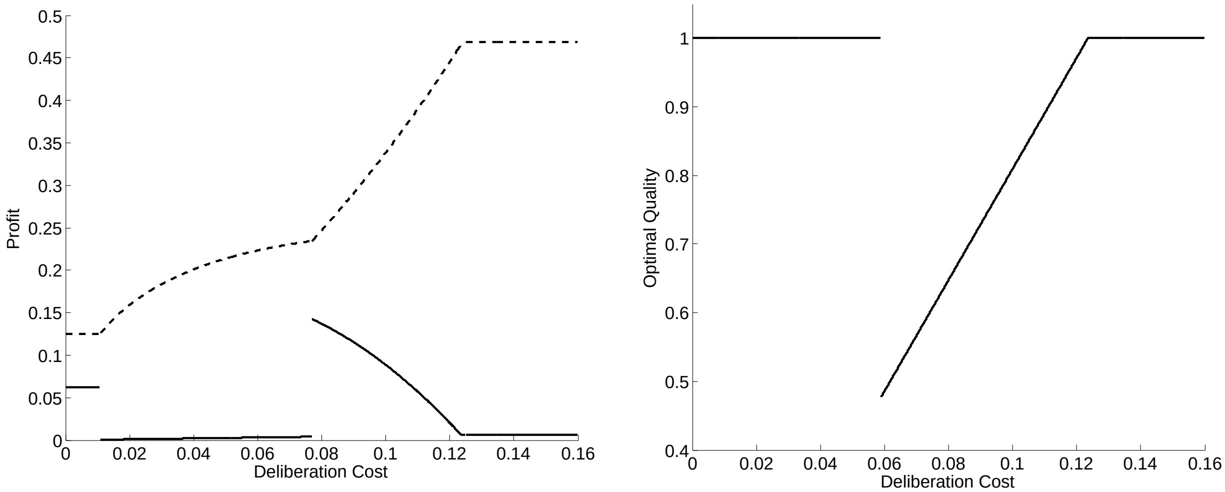

Figure 3 **The equilibrium profits and the optimal quality when** $\lambda = 0.1$

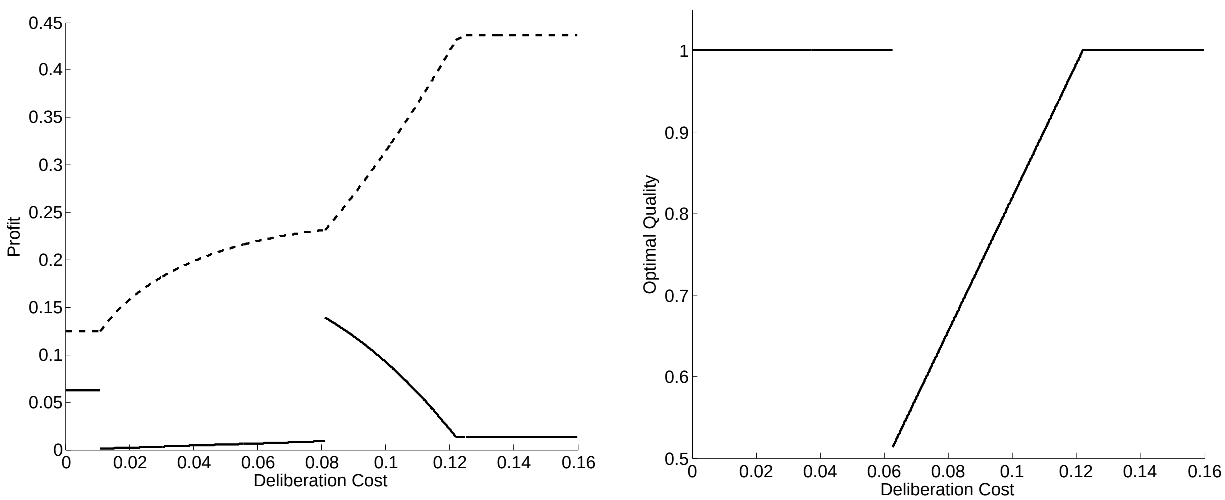

Figure 4 **The equilibrium profits and the optimal quality when** $\lambda = 0.2$

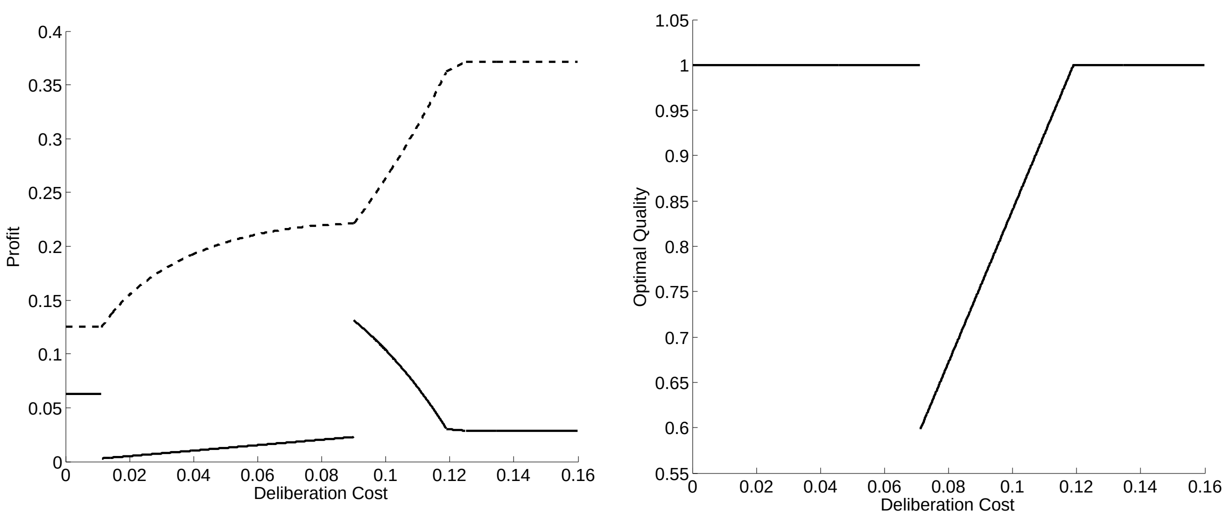

Figure 5 **The equilibrium profits and the optimal quality when** $\lambda = 0.4$

It follows that a consumer who receives signal $\tilde{h}$ updates her valuation to

$$E(v_i|\tilde{h}) = h\,\Pr(h|\tilde{h}) + l\,\Pr(l|\tilde{h}) = \frac{1}{2} + k\rho$$

and a consumer who receives signal $\tilde{l}$ updates her valuation to

$$E(v_i|\tilde{l}) = h\,\Pr(h|\tilde{l}) + l\,\Pr(l|\tilde{l}) = \frac{1}{2} - k\rho$$

The consumer must incur a "deliberation cost" to receive the signal, and, to obtain tractable results, we assume that the cost is $c_o \cdot \rho^2$, where c_o is the marginal cost of deliberation. In other words, the more reliable the signal, the more costly to acquire it, and a completely unreliable signal ($\rho = 0$) is free. The marginal cost of deliberation is increasing in the reliability of the signal. In the game, after observing the price, the consumer first decides the reliability of signal ρ and incurs the deliberation cost. Then she receives the signal s, and finally makes her purchase decision contingent on the signal she received.

Clearly, deliberation is beneficial to the consumer only if deliberation has the potential to alter her purchase decision. In other words, when deliberation takes place, the consumer should only make a purchase when the deliberation outcome is positive (i.e., $s = \tilde{h}$). The consumer's ex-ante expected surplus is therefore,

$$CS = \Pr(\tilde{h})\{E(v_i|\tilde{h}) - p\} - c_o \cdot \rho^2. \tag{OA7}$$

Solving for the consumer's optimal decision we have the following lemma:

Lemma OA2 *The consumers' deliberation decision is as follows.*

(i) *If $c_o \leq \frac{k}{4}$, there are three cases.*

- *If $p \leq \frac{1}{2} - k + 2c_o$, the consumer makes purchase without deliberation.*
- *If $\frac{1}{2} - k + 2c_o < p \leq \frac{1}{2} + k - 2c_o$, the consumer deliberates with $\rho = 1$, and buys when the signal is $\tilde{h}$.*
- *If $p > \frac{1}{2} + k - 2c_o$, the consumer neither deliberates nor buys.*

(ii) *If $c_o > \frac{k}{4}$, there are three cases.*

- *If $p \leq \frac{1}{2} - \frac{k^2}{8c_o}$, the consumer makes purchase without deliberation.*
- *If $\frac{1}{2} - \frac{k^2}{8c_o} < p \leq \frac{1}{2} + \frac{k^2}{8c_o}$, the consumer deliberates with $\rho = \frac{k}{4c_o}$, and buys when the signal is $\tilde{h}$.*
- *If $p > \frac{1}{2} + \frac{k^2}{8c_o}$, the consumer neither deliberates nor buys.*

Proof. If the consumer deliberate, she chooses ρ that maximizes CS in Equation OA7, and we can simplify CS as follows.

$$CS = \frac{1}{2}\left(\frac{1}{2} + k\rho - p\right) - c_o \cdot \rho^2$$

Optimizing the above consumer surplus we have

$$\rho^* = \begin{cases} \frac{k}{4c_0} & \text{if } k \leq c_0, \\ 1 & \text{otherwise.} \end{cases}$$

There are two alternative strategies: (A) If the consumer does not deliberate or buy (no-brianer no purchase), the expected consumer surplus is zero. (B) If the consumer buys without deliberation (no-brainer purchase), the expected consumer surplus is $E(v_i) - p$. By maximizing over the three options we prove the lemma. $\square$

Following Lemma OA2, we obtain the equilibrium channel strategies, which are summarized in the following proposition.

Proposition OA1 *The equilibrium channel strategies are as follows.*

(i) If $c_o \leq \frac{k}{4}$ and $c_o < \hat{c} = \frac{k}{2} - \frac{1}{28}$, the equilibrium prices are $w = p = \frac{1}{2} + k - 2c_o$, and the equilibrium profits are $\pi_m = \frac{w}{2}$, $\pi_r = 0$.

(ii) If $c_o \leq \frac{k}{4}$ and $c_o \geq \hat{c} = \frac{k}{2} - \frac{1}{28}$, the equilibrium prices are $w = \frac{1}{2} - 3k + 6c_o, p = \frac{1}{2} - k + 2c_o$, and the equilibrium profits are $\pi_m = w$, $\pi_r = p - w$.

(iii) If $c_o > \frac{k}{4}$ and $c_o < \hat{c} = \frac{7}{4}k^2$, the equilibrium prices are $w = p = \frac{1}{2} + \frac{k^2}{8c_o}$, and the equilibrium profits are $\pi_m = \frac{w}{2}$, $\pi_r = 0$.

(iv) If $c_o > \frac{k}{4}$ and $c_o \geq \hat{c} = \frac{7}{4}k^2$, the equilibrium prices are $w = \frac{1}{2} - \frac{3k^2}{8c_o}, p = \frac{1}{2} - \frac{k^2}{8c_o}$, and the equilibrium profits are $\pi_m = w$, $\pi_r = p - w$.

Moreover, the manufacturer's profit is decreasing in c_o when $c_o < \hat{c}$, and increasing in c_o otherwise. The manufacturer's profit is maximized at $c_o = +\infty$. The retailer's profit is zero when $c_o < \hat{c}$, and decreasing in c_o when $c_o > \hat{c}$. The retailer's profit is maximized at $c_o = \hat{c}$.

Proof. Let

$$\underline{p} = \begin{cases} \frac{1}{2} - k + 2c_o & \text{if } c_0 \leq k, \\ \frac{1}{2} - \frac{k^2}{8c_o} & \text{otherwise,} \end{cases}$$

and

$$\bar{p} = \begin{cases} \frac{1}{2} + k - 2c_o & \text{if } c_0 \leq k, \\ \frac{1}{2} + \frac{k^2}{8c_o} & \text{otherwise.} \end{cases}$$

Then, if $p \leq \underline{p}$, all consumers buy; if $p \in (\underline{p}, \bar{p}]$, half of the consumers buy; otherwise no consumers buy. In this sense, the retailer either charges $p = \underline{p}$, or $p = \bar{p}$, or does not sell.

As for the manufacturer, when it wants to induces the retailer to price at $p = \bar{p}$, it simply prices at $w = \bar{p}$, and the retailer makes no profit. In this case the demand is $\frac{1}{2}$, and $\pi_m = \frac{\bar{p}}{2}$. Otherwise, if the manufacturer wants to induce the retailer to price at $p = \underline{p}$, the following inequality must be satisfied:

$$\frac{1}{2}(\bar{p} - w) \leq \underline{p} - w$$

and the manufacturer's optimal price is $w = 2\underline{p} - \bar{p}$, and the corresponding profit is $\pi_m = 2\underline{p} - \bar{p}$. Hence, when $3\underline{p} \geq 4\bar{p}$, the manufacturer prices at $w = 2\underline{p} - \bar{p}$; otherwise it prices at $w = \bar{p}$. The proposition follows immediately. □

Figure 6 illustrates the equilibrium prices when $k = \frac{3}{20}$. Similar to the basic model, double marginalization is severe when c_o is intermediate, and becomes less severe (or vanishes) when c_o is small or very large. The equilibrium prices are slightly different from the basic model because we use two-point distribution instead of uniform distribution (and hence there is no standard double marginalization here).

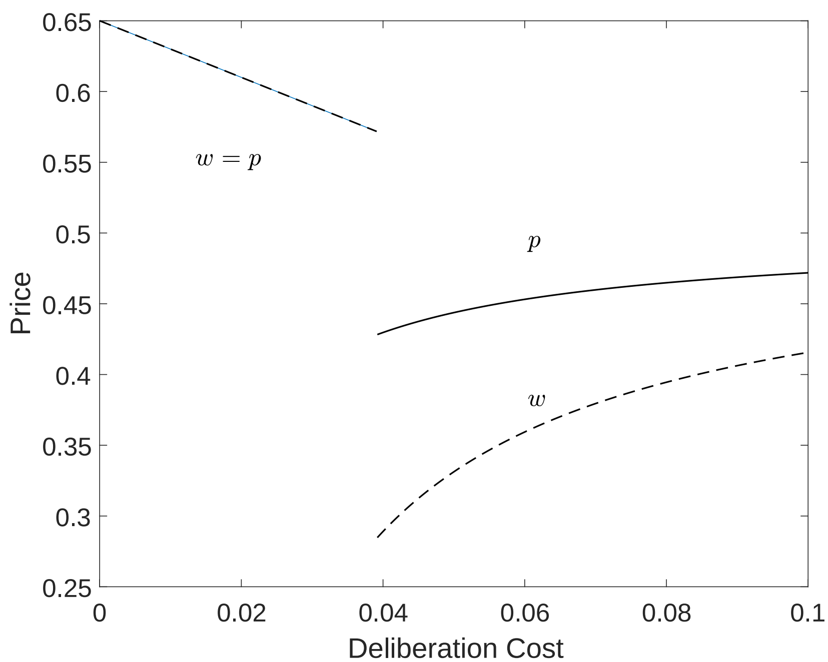

Figure 6 **Equilibrium channel prices,** $k = \frac{3}{20}$

Figure 7 illustrates the equilibrium profits when $k = \frac{3}{20}$. The manufacturer's profit is maximized when c_o approximates infinity. As for the retailer, its profit is maximized when $c_o = \frac{11}{280}$. The results that the manufacturer prefers a large deliberation cost whereas the retailer prefers a moderate deliberation cost is consistent with the basic model. However, it is noteworthy that the manufacturer's profit is no longer monotone in c_o. Again, this is driven by the nature of the two-point distribution.

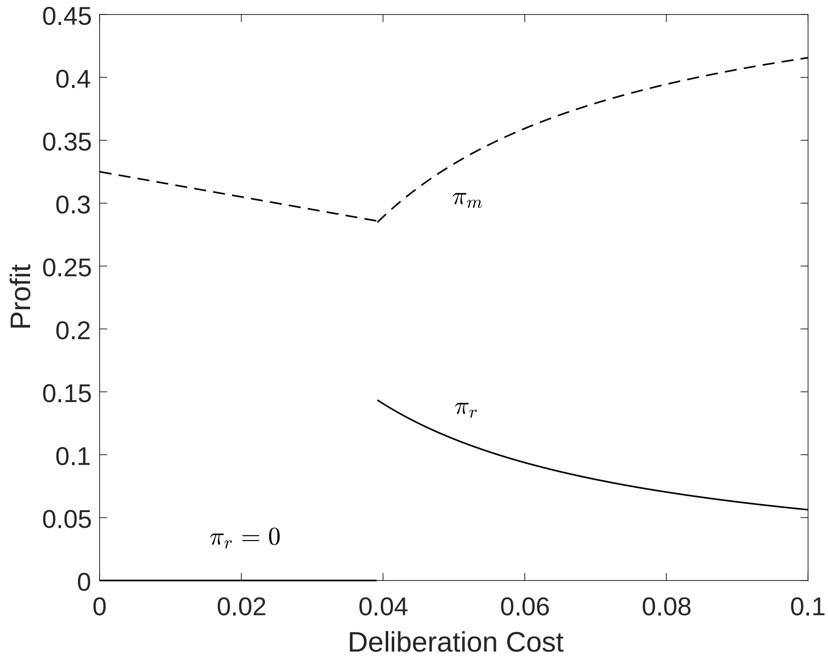

Figure 7 **Equilibrium channel profits,** $k = \frac{3}{20}$

Overall, the main insights from the basic model can be generalized to the current setup. First, double marginalization becomes less severe when c_o is small or very large. Second, the manufacturer prefers a very large deliberation cost whereas the retailer prefers an intermediate deliberation cost.

5. The Value of Vertical Integration

In this section we show that the benefit of vertical integration for the manufacturer is not monotone in deliberation cost. We extend the basic model by assuming that the manufacturer could integrate the retailer at a cost F.

Let Δ be the threshold cost under which the manufacturer is willing to integrate vertically, we have the following proposition, which follows immediately from the profit functions.

Proposition OA2 *The manufacturer is willing to integrate vertically iff $F \leq \Delta$, where Δ is the benefit of vertical integration.*

$$\Delta = \begin{cases} \frac{1}{8} & \text{if } c \leq \frac{3-2\sqrt{2}}{16}, \\ \frac{1}{4} - \sqrt{2c} + 2c & \text{if } \frac{3-2\sqrt{2}}{16} < c \leq \frac{1}{32}, \\ 2c & \text{if } \frac{1}{32} < c \leq \frac{7-3\sqrt{5}}{4}, \\ \frac{\sqrt{2c}-4c}{1-\sqrt{2c}} & \text{if } \frac{7-3\sqrt{5}}{4} < c \leq \frac{1}{8}, \\ 0 & \text{if } \frac{1}{8} < c, \end{cases}$$

The threshold Δ is plotted in Figure 8. It turns out that the manufacturer's benefit from vertical integration is not monotone in c, the deliberation cost. We can see that Δ is small when $c \approx \frac{1}{32}$ or when c is large. This is because double marginalization disappears when c is moderate or when c is large, and hence vertical integration is less profitable in these regions.

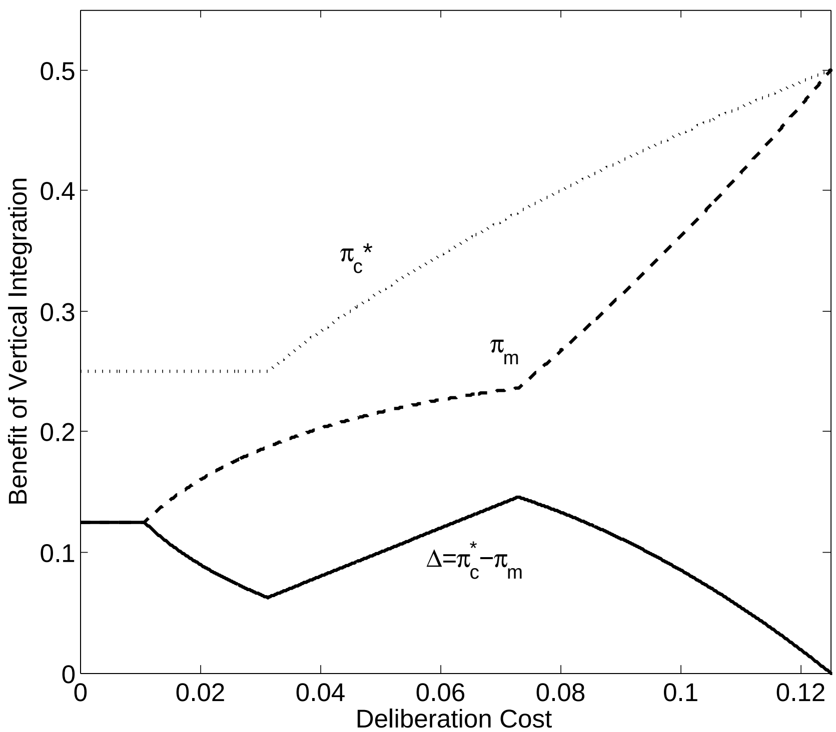

Figure 8 **The benefit of vertical integration. (π_c^*: the profit of integrated channel; π_m: the profit of the manufacturer in a decentralized channel)**

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