Online Appendix for
"The Performance Measurement Trap"
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Case When Salespeople Have Partial Information about Second Period Outside Option:

Assume now the following information structure that the salesperson has at the beginning of the game about his second period outside option. Assume at the beginning of the game that the salesperson has a signal (k, σ) about his second period outside option, where $k \in \{0, \ldots, K\}$ represents the reliability of σ and is uniformly distributed on $\{0, \ldots, K\}$, while σ is equal to $\mathbf{1}_{z>1/2}$ with probability k/K and is a random draw from $\{0, 1\}$ otherwise. Each of these $2K + 2$ possible (and equally likely) signals corresponds to a different value that the high type salesperson receiving such a signal places on the second period equilibrium offer.

Let $c_2^\star$ be the equilibrium second period c_2 when $\pi_1 = 1$ & $m(x_1) = 0$ and the manager expects efficient effort allocation in the first period (by the same argument as in Section 4.2, if the manager expects a distortion, c_2 will be lower). As before, by maximally distorting the x component of effort upwards, the high type salesperson can increase c_2 and, therefore, the value of the second period offer (since it can ensure $\pi_2 = 1$ by allocating effort efficiently in the second period), to $1/2$. Again, as before, the value of this action equals the probability of accepting the contract with $c_2 = 1/2$ times the expected increase in the payoff (which is $1/2 - \max\{z, c_2^\star\}$) conditional on accepting the second period offer. The difference lies in the evaluation of the probability of accepting the offer, which now is a function of the signal (k, σ).

Signal (k, σ) is uninformative with probability $1 - k/K$. In this case, z is uniformly distributed on $(0, 1)$ and therefore, the value of increasing c_2 from $c_2^\star$ to $1/2$ is $v^\star \equiv (1/2 - c_2^\star) \cdot c_2^\star + \int_{c_2^\star}^{1/2} (1/2 - z)\, dz$. With probability k/K, the signal is informative. In this case, if $\sigma = 1$, the salesperson is sure to leave even with $c_2 = 1/2$ and, therefore, has no value of increasing the second period offer. However, if $\sigma = 0$ (and the signal is true), the salesperson is sure to stay and values increasing the offer at $2v^\star$. Aggregating the above (taking into account the probability of the event $\{\pi_1 = 1$ & $m(x_1) = 0\}$), we find the values $v_{k,\sigma}$ the high type salesperson with signal (k, σ) places on increasing the second period offer. These values are ordered as:

$$v_{K,1} < v_{K-1,1} < \cdots < v_{0,1} < v_{0,0} < \ldots v_{K,0}. \tag{1}$$

These values represent the incentive the salesperson has to maximally distort his effort allocation toward $x = 2$ given signal (k, σ). Let dv be the smallest distance between any pair of the above values. Again, if the manager believes the effort allocation is distorted, dv will be greater. Thus, $dv > 0$ represents the lower bound on how different incentives are between the high type agents with different signals.

Similar to the argument leading to Proposition 2, one can now prove that if b is large enough, a potentially optimal first period contract (e.g., one where the weight on π_1 is bounded from above by a number independent of b) may only incentivize no–effort distortion by agents with one of the signal possibilities. The rest will be distorting x either maximally upward or maximally downward. Thus, for large K, almost all high type agents distort their first period effort allocation maximally, and we have the following proposition.

PROPOSITION 4: *If salespeople have some information about their second period outside option, it is impossible to profitably prevent maximal effort distortion by nearly all salespeople in the first period.*

PRECISE MEASUREMENT:

Allowing $a_\ell > 0$, which is important to achieve interesting results when $m(x) = x$, adds analytical complexity to the expressions of the benchmark model, but conceptually, the derivations are the same as in the main model. The expected profit given productivity α is $\frac{b+\alpha-(1-\alpha)B}{b+1}$. Therefore, given efficient allocation and negligible wage in the first period, the expected first-period profit is $\frac{2b+1+a_\ell+(1-a_\ell)B}{2(b+1)}$. The second-period contract given the probability P_h that the salesperson is of high-type is still of the form $c_2\mathbf{1}_{\pi_2=1}$, with

$$c_2 = c_2^{opt}(P_h) = \frac{P_h(1+b)^2 + (1-P_h)(b - B + a_\ell + a_\ell B)(b + a_\ell)}{2[(b + a_\ell)^2 + P_h(1 - a_\ell)(2b + 1 + a_h)]}, \tag{2}$$

and the second-period net profit given probability P_h of high type is

$$f(P_h) = \frac{1}{4}\frac{\left[(b + a_\ell)^2 + (1 - a_\ell)[(2b + 1 + a_\ell)P_h - (b + a_\ell)(1 - P_h)B]\right]^2}{(b + 1)^2[(b + a_\ell)^2 + (1 - a_\ell)(2b + 1 + a_l)P_h]}. \tag{3}$$

This still means $c_2 = 1/2$ if $P_h = 1$ but $c_2 = \frac{1}{2} - \frac{(1-a_\ell)B}{2(b+a_\ell)}$ if $P_h = 0$. Likewise, (given efficient effort allocation in the first period) if $\pi_1 = 0$ we still have $P_h = 0$, but if $\pi_1 = 1$, we have $P_h = \frac{b+1}{2b+1+a_\ell}$. Furthermore, if firm's belief about the type changes from low to high, c_2 increases by $\frac{(1-a_\ell)B}{2(b+a_\ell)}$.

The expected net profit in the second period with no measurement is

$$\pi_2^{nm} = \frac{1}{8}\frac{[(b+a_\ell)^2(b-B+a_\ell+a_\ell B)+(1+b)^3]^2}{(b+1)^3(2b+1+a_\ell)[(b+1)^2-(b+a_\ell)(1-a_\ell)]} + \frac{1}{8}\frac{(1-a_\ell)(b-B+a_\ell+a_\ell B)^2}{(b+1)^3}. \quad (4)$$

With measurement, if the low and high types choose different x_1 (i.e., if the equilibrium is separating), the firm effectively has complete information in the second period and is able to condition the second-period contract (c_2) on the salesperson's type. In this case, the second-period net profit is

$$\pi_2^{sep} = \frac{1}{8}\frac{(1+a_\ell)^2(b+1)^2+(1-a_\ell)(b-B)[(b+1)a_\ell+2a_\ell-(1-a_\ell)B]}{(b+1)^2}, \quad (5)$$

which is an improvement over the second-period net profit in the benchmark case by

$$\Delta_2^{sep} = \frac{1}{8}\frac{(1-a_\ell)^2(b+a_\ell)B^2}{(2b+1+a_\ell)[(b+1)^2-(b+a_\ell)(1-a_\ell)]}. \quad (6)$$

However, in a separating equilibrium, the low type salespeople will have an incentive to imitate the high type (if able) equal to $\frac{(1-a_\ell)B}{2(b+a_\ell)}\frac{b+a_\ell}{b+1}$ (the expected increase in c_2 times the probability of achieving $\pi_2 = 1$ under efficient allocation). Obviously, if $2\sqrt{a_\ell} < 1$ (i.e., $a_\ell < 1/4$), the low type salespeople are unable to imitate efficient x_1 of the high types, and the firm benefits from the introduction of the measurement. On the other hand, if $2\sqrt{a_\ell} > 1$ (i.e., $a_\ell > 1/4$), which we assume from now on, the firm has two potentially optimal strategies of ensuring type separation in the first period. The **first** strategy is to offer first-period pay c_l for $x = \sqrt{a_\ell}$ (i.e., the efficient allocation of low-type salespeople) just enough to make the low types indifferent between choosing $x_1 = \sqrt{a_\ell}$ (and receiving second-period offer corresponding to $P_h = 0$) and choosing $x_1 = 1$ (and receiving the second-period offer corresponding to $P_h = 1$). One can derive that this offer is

$$c_l = \frac{1}{8}\frac{(1-a_\ell)(2b-B+a_\ell B+2a_\ell)B(b+2\sqrt{a_\ell}-1)}{(b+1)^3}. \quad (7)$$

Note that since the value of being recognized as the high type is higher for the high type (since the probability of achieving $\pi_2 = 1$ is higher for them), the high type will still choose their efficient allocation in the first period. Therefore, following this strategy leaves the firm with the same first-period gross profit, but achieves separation in exchange for the cost $c_l/2$ (division by 2 due to 1/2 probability of the low type). Thus, this strategy achieves the net total profit higher than that of the benchmark case if and only if $c_l < 2\Delta_2^{sep}$.

The **second** potentially optimal strategy is to not offer any significant incentives in the first period, in which case the high type salespeople will choose x unavailable to the low-type salespeople (i.e., choose x just above $2\sqrt{a_\ell}$) to signal their high type. Although this causes the firm losses of some profits in the first period (due to the inefficient effort allocation by the high-type salespeople), the firm saves $c_l/2$ in the first-period wages relative to the first strategy above. The first-period profit loss due to the effort allocation distortion of the high-type salespeople is

$$c_{distsep} = \frac{1}{2}\frac{(1-2\sqrt{a_\ell})^2(B+1)}{b+1}. \quad (8)$$

Again, the benefit relative to the no measurement case is the full information about the salespeople types in the second period. If the first-period effort distortion is small (i.e., $2\sqrt{a_\ell}$ is not much higher than 1), the profit with this strategy is clearly higher than the profit under no measurement.

Finally, the **third** potentially optimal strategy when measurement is introduced is to incentivize the high-type salespeople not to separate in their choice of x_1 from the low-type ones by paying for a certain choice of x_1 enough for the high types not be willing to separate. In this case, $m(x)$ will not be informative of the type, and only the profit realization will be. Since the low-type salespeople achieve $\pi_1 = 1$ with a lower probability then the high type ones (keeping their choice of x_1 fixed and equal), the least costly way to induce the high type ones not to deviate is to make pay conditional on $\pi_1 = 1$ & $m(x_1) = x$. Although relative to the first strategy, more salespeople will get paid in the first period (the low type salespeople also get paid for choosing $x_1 = x$ when $\pi_1 = 1$), the cost per salesperson is lower because under this strategy, the benefit of deviation to a high x_1 is not changing the firm's belief of the high type from 0 to 1 but from a positive number (approximately $1/2$ when b is large) to 1.

Let us now consider which of the above three strategies may result in the firm's profit higher than it would have without measurement. If a_ℓ is only slightly above $1/4$, so that $2\sqrt{a_\ell}$ is only slightly above 1, the second strategy results in a small cost of distortion and has the benefit of information (which decreases in a_ℓ). On the other hand, the cost (of the first-period wages) in the first and the second strategies increases as a_ℓ decreases to $1/4$. Therefore, the second strategy is the best and results in the profit higher than without measurement. On the other hand, when a_ℓ approaches 1, the benefit of full information decreases to zero (see Equation (6)), but the cost of first-period distortion in the second strategy increases. Also, as $a_\ell \to 1$, the direct cost (of first-period wages) of the first strategy tends to zero (since the second-period wage conditional on $P_h = 0$ tends to the second-period wage conditional on $P_h = 1$), and its benefit of information is the same as that of the second strategy. Therefore, for a_ℓ close to 1, the second strategy results in a decrease in profits relative to both the first strategy and no measurement.

The first and the third strategies turn out to never improve profits relative to no measurement when $a_\ell \geq 1/4$ (if $a_\ell < 1/4$, the low type salespeople cannot imitate the high type ones, and therefore the second strategy achieves the first best). To prove that the first strategy never improves profits relative to no measurement, note that

$$c_l > \underline{c}_l \equiv \frac{1}{8}\frac{(1-a_\ell)(a_\ell B + 2a_\ell - B + 2b)Bb}{(b+1)^3}, \tag{9}$$

when $2\sqrt{a} > 1$ (the right hand side is obtained from the expression for c_l by replacing $2\sqrt{a_\ell}$ to 1). It is therefore sufficient to prove that $\underline{c}_l - 2\Delta_2^{sep} > 0$ for $b \geq B$ and $a_\ell > 1/4$. To do this, we rearrange $\underline{c}_l - 2\Delta_2^{sep} > 0$ as

$$\begin{aligned}\underline{c}_l - 2\Delta_2^{sep} = B(1-a_\ell)\big[&2a_\ell(ba_\ell^3 + Ba_\ell + 2b^4 B) + b(3ba_\ell + 7ba_\ell^3 - B + a_\ell^4 B + 6Ba_\ell^2) \\ &+ b^3(10ba_\ell - 3B + 4Ba_\ell + 5Ba_\ell^2) + 3b^2(2ba_\ell + 4ba_\ell^2 + Ba_\ell^2 - B + a_\ell^3 B + Ba_\ell)\big] \\ &\Big/\big[8(b+1)^3(2b+1+a_\ell)[(b+a_\ell)^2 + (1-a_\ell)(b+1)]\big]\,,\end{aligned} \tag{10}$$

and observe that the quantities within each pair of parentheses are positive for $b > B > 0$ and $a_\ell > 1/4$.

The third strategy is also always worse than the profit without measurement. The proof is similar to the above, albeit with longer expressions: we again construct an upper bound on the net total profit under the third strategy and show that it is below the net equilibrium profit without measurement. Specifically, to construct the upper bound, consider only the informational and direct-cost effects of the third strategy relative to the equilibrium without measurement (i.e., ignore the detriment of the effort distortion). Further, use the upper bound on the information the firm has in the second period by assuming that the induced distortion leads to the probabilities of the first-period profit outcomes as if $(2\sqrt{a_\ell} - x)x = 0$ for the low type (i.e., minimal value), and $(2-x)x = 1$ for the high type (i.e., maximal value), which also implies a lower bound on the direct cost of the first-period wage (because it both minimizes the payment to the low type salespeople and the high-type salespeople's incentive to establish that they are high type with probability one). Thus, we have the following upper bound on the net second-period profit under the third strategy:

$$\pi_2^{pool} \le \overline{\pi}_2^{pool} = f\left(\frac{b+1}{2b+1}\right)\frac{2b+1}{2b+2} + f(0)\frac{1}{2b+2}, \tag{11}$$

where $f(\cdot)$ is defined in Equation (3), and the high-type salespeople should receive at least

$$\underline{w}_1^{pool} = \frac{c_2^{opt}(1)^2}{2} - \frac{c_2^{opt}\left(\frac{b+1}{2b+1}\right)^2}{2} \tag{12}$$

expected compensation in the first period to prevent him from deviating to an unavailable for the low type salesperson level of x_1 in the first period (the function $c_2^{opt}(\cdot)$ is defined in Equation (2)). Given the above bounds and that half of the salespeople are of high type, in order to prove that the third strategy always results in a lower profit than the profit under no measurement, it suffices to show

$$\pi_2^{nm} - \left(\overline{\pi}_2^{pool} - \underline{w}_1^{pool}/2\right) > 0 \tag{13}$$

for $b \ge B \ge 0$. To show this, one can substitute $b = B + \beta$ into the left hand side and factor it to obtain a fraction with numerator being $(1 - a_\ell)$ times a polynomial in B, β and a_ℓ all coefficients of which are positive, and the denominator $((b+a_\ell)^2 b + (b+1)^3)^2((b+a_\ell)^2 + (1-a_\ell)(b+1))(2b+1+a_\ell)(b+1)$, which is also positive. We do not report the full expression as it is long. But to illustrate, if $a_\ell = 1/3$ and $b = B$, we have that the left hand side of Equation (13) is

$$\frac{3(18b^4 + 30b^3 + 19b^2 - 3)(27b^4 + 63b^3 + 72b^2 + 41b + 9)}{4(18b^3 + 33b^2 + 28b + 9)^2(9b^2 + 12b + 7)(3b + 2)} > 0, \quad \text{when } b > 1. \tag{14}$$

Thus, the only possibility for the total net profit to increase due to the introduction of the precise measurement of x is when the second strategy is optimal. Therefore, the total net profit decreases when a_ℓ is sufficiently high (for example, for $b = B = 1$, measurement decreases profits if $a_\ell > 0.32$ and for $b = B = 100$, measurement decreases profits when $a_\ell > 0.38$). This establishes the first claim of the proposition.

In fact, the third strategy is not only worse than not having the measurement, but is also always worse than the first strategy. To prove this, we need to use a slightly better upper bound on the profits under the third strategy, which we establish as follows. Let us still not count the detriment of the first-period's effort allocation distortion of the third strategy. But observe that under pooling on $x_1 = x \in [0, 2\sqrt{a_\ell}]$, conditional on $\pi_1 = 1$, we have that $P_h = \frac{(b+x(2-x)}{2b+x(2+x)+x(2\sqrt{a_\ell}-x)}$ increases in $x \in [0, 2\sqrt{a_l}]$, since $(P_h)'_x = \frac{1}{2}\frac{(1-\sqrt{a_\ell})(x^2+b)}{(b+x-x^2+x\sqrt{a_\ell})^2}$. Therefore, $x = 2\sqrt{a_\ell}$ leads to the best high-type identification in the event of $\pi_1 = 1$ and the least-costly incentive to prevent the high-type salespeople from separating if they do not take into account the possibility that their effort distortion may result in $\pi_0 = -B$. Thus, calculating the minimum payment required in the case $\pi_1 = 1$ to make the high-type salespeople indifferent between separating and not separating when they disregard the possibility of $\pi_1 = -B$ (a lower bound on the first-period wage cost), and calculating the second-period profits with the assumption that for $\pi_1 = 1$, the firm uses

$$P_h = P_{h1} \equiv \frac{(b + x(2-x)}{2b + x(2+x) + x(2\sqrt{a_\ell} - x)}, \quad \text{where } x = 2\sqrt{a_\ell}, \tag{15}$$

and for $\pi_1 = 0$, it uses $P_h = 0$, together with counting the probability of $\pi_1 = -B$ as being $b/2/(b+1)$ and the probability of $\pi_1 = 1$ as being $\frac{b+1}{2(b+1)}$ (i.e., as if "magically" all the high-type salespeople who should have obtained $\pi_1 = -B$ are switched back to $\pi_1 = 1$, and the firm gets the information about that only if $\pi_1 = -B$, thereby resulting in a higher second-period profit), we obtain an upper bound on the net second-period profit under the third strategy:

$$\pi_2^{pool} \leq \overline{\pi}_2^p \equiv f\left(P_{h1}\right)\frac{2b+1}{2b+2} + f(0)\frac{1}{2b+2}, \tag{16}$$

while the firm must offer the high-type salespeople first-period expected wage for choosing the pooling x_1 of at least

$$\underline{w}_1^p = \frac{c_2^{opt}(1)^2}{2} - \frac{c_2^{opt}\left(P_{h1}\right)^2}{2}. \tag{17}$$

Since the probability of $\pi_1 = 1$ is lower for the low type, the most efficient way to compensate the high-type salespeople for choosing the pooling x_1 is to condition the payment on $\pi_1 = 1$. Taking a lower bound $b/(b+1)$ on the probability that the low type achieves $\pi_1 = 1$, we obtain that the total spending on first-period wage is at least

$$\underline{c}_h \equiv \left(\frac{1}{2} + \frac{1}{2}\frac{b}{b+1}\right)\underline{w}_1^p. \tag{18}$$

We further use the lower bound on the profit under the first strategy by replacing its first-period wage cost c_l by

$$\overline{c}_l \equiv \frac{1}{8}\frac{(1-a_\ell)(a_\ell B + 2a_\ell - B + 2b)B}{(b+1)^2}, \tag{19}$$

which is obtained from c_l by replacing $2\sqrt{a_\ell}$ by its highest value 2. We then subtract the upper bound on the profit under the third strategy from the lower bound on the profit under the first strategy to obtain that the difference in total net profits is at least

$$(\pi_2^{sep} - \overline{c}_l) - (\overline{\pi}_2^{p} - \underline{c}_h), \tag{20}$$

which we now need to prove is positive when $b > B > 0$ and $a_\ell \in (1/4, 1)$. Do do this, it suffices to do the following: substitute $b = B + \beta$ and $a_\ell = a_L^2$ (now, we have $B > 0$, $\beta > 0$ and $a_L \in [1/2, 1]$), substitute $a_L = A + 1/2$ (now, $A \in [0, 1/2]$), and factor. By doing this, one obtains a fraction where the denominator is a complete square and the numerator is a product of $(1 - A^2)$ and a long polynomial. Thus, we need to prove that the polynomial is positive for $B > 0$, $\beta > 0$ and $A \in [0, 1/2]$. To do this, it suffices to replace all positive terms with A^i for $i \geq 1$ with 0 (decreasing the polynomial value for any positive values of B, β and A), and then substitute $A = 1/2$ (i.e., taking the maximal value of all negative terms) which turns out to be (after combining the terms with the same powers of β and B) a polynomial in B and β with only positive coefficients. Although these polynomials are too long to explicitly report here, the above procedure is technically straightforward. To illustrate Expression (20), consider again $B = 1$ and $a_\ell = 1/3$. In this case, Expression (20) becomes

$$\begin{aligned}
&\frac{43 - 20\sqrt{3}}{280368(b+1)^4(63b^2 + 24b\sqrt{3} + 9b + 16\sqrt{3} - 13 + 27b^3)^2} \times \\
&\times \Big[(1700744 - 538712\sqrt{3}) + (10546030 - 1621576\sqrt{3})b + (23349603 + 6280212\sqrt{3})b^2 \\
&\quad + (24599817 + 36043524\sqrt{3})b^3 + (63015408\sqrt{3} + 30127842)b^4 \\
&\quad + (62019756 + 47135520\sqrt{3})b^5 + (12992724\sqrt{3} + 70704495)b^6 \\
&\quad + (609444\sqrt{3} + 33671781)b^7 + 5677452b^8, \Big]
\end{aligned} \tag{21}$$

which is positive for all $b \geq 0$.

The calculation of the social welfare without measurement follows exactly the same procedure as in the main model, and therefore for the sake of brevity, we skip the details. The second-period social welfare without measurement turns out to be

$$W_{2B} = \frac{11}{16} + \frac{3}{16}\frac{(b + a_\ell - (1 - a_\ell)B)^2}{(b+1)^2} - \frac{3}{16}\frac{(a_\ell - 1)^2(b + a_\ell)B^2}{((b + a_\ell)^2 + (1 - a_\ell)(b+1))(2b + 1 + a_\ell)}. \tag{22}$$

The social welfare under the first strategy in the first period is exactly the same as without measurement (the only difference is the wage transfer from the firm to the high-type salespeople).

In the second period, the calculation of social welfare under the first strategy is similar to that without measurement, but simpler because types are fully identified. It turns out to be

$$W_{21} = \frac{11}{16} + \frac{3}{16}\frac{(b + a_\ell - (1 - a_\ell)B)^2}{(b+1)^2}. \tag{23}$$

Finally, under the second strategy, the total social welfare decreases relative to one under the first strategy by $c_{distsep}$ defined in Equation (8) due to the distortion of the first-period effort allocation. Clearly, the total welfare under the first strategy is higher than the total welfare without measurement. However, if the firm prefers the second strategy, total welfare ends up decreasing if

$$\frac{3(a_\ell - 1)^2(b + a_\ell)B^2}{((b + a_\ell)^2 + (1 - a_\ell)(b+1))(2b + 1 + a_\ell)} < \frac{8(1 - 2\sqrt{a_\ell})^2(B+1)}{b+1}. \tag{24}$$

For example, consider $b = B = 1$ and $a_\ell = 1/3$. In this case, with measurement, the second strategy provides the highest profit and results in the second-period welfare of $293/420 = 0.708(3)$, but at a cost of welfare loss of $7/6 - 2/\sqrt{3} \approx 0.011966$ in the first period (relative to no measurement), while with no measurement, the second-period welfare is $293/420 \approx 0.697619$. Therefore, measurement reduces the total welfare by $971/840 - 2/\sqrt{3} \approx 0.00125$. At higher values of b and B, social welfare may decrease even more. For example, when $b = B = 100$ and $a_\ell = 0.43$, social welfare reduces by 0.0185. This concludes the proof of the proposition.

Unexpected Measurement of the Second Period Effort After the First Period's Effort Allocation:

The optimal x_2 maximizes (14) in the paper. Using implicit differentiation of the first order condition $4 + 4w_2x_2 - 4x_2 - 3w_2x_2^2 + w_2b = 0$ on the optimal x_2, we obtain

$$\frac{\partial x_2}{\partial w_2} = \frac{4x_2 - 3x_2^2 + b}{4 - 4w_2 + 6w_2x_2}. \tag{25}$$

The firm's expected profit in the second period given $\pi_1 = 1\&m(x_1) = 0$ is

$$E(\pi_2|\pi_1 = 1\&m(x_1) = 0) = \phi_{10}\frac{a+b}{1+b}c_2(1+x_2w_2/2)[a - \frac{a+b}{1+b}c_2(1+x_2w_2/2)] + (1-\phi_{10})\frac{b}{1+b}c_2[0 - \frac{b}{1+b}c_2], \tag{26}$$

where ϕ_{10} is the probability of high type conditional on $\pi_1 = 1\&m(x_1) = 0$ which is $\phi_{10} = \frac{1+b}{1+3b}$

per (9) in the paper, $a = x_2(2 - x_2)$, and x_2 is a function of w_2, which is implicitly defined by the FOC on the salesperson's objective function (or explicitly by (15) in the paper).

The optimal c_2 and w_2 can then be found by taking the derivatives of (26) with respect to c_2 and w_2, using (25) for the derivative of x_2 with respect to w_2, and making those derivatives equal to zero.