Online Appendix for
Preference Learning and Demand Forecast

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OA.1 Field Experiment Implementation Details

OA.1.1 Procedure of Subject Selection and Assignment

We randomly select a fraction of the platform's Android servers. All users on these servers automatically entered the experiment once they accessed the game during the period of the experiment.

We have 20 treatment conditions (4 realization probabilities by 5 prices). The allocation algorithm directs each incoming user from condition 1 to condition 20, and then repeat the cycle. This pseudo-random procedure guarantees that the number of users is balanced across treatment conditions. As mentioned in the paper, the balance check of user characteristics confirms that there is no significant difference among participants across treatment conditions.

The random allocation happens at the first time a user enters the game during the experiment period. If the user enters the game again during the experiment period, she will see the same realization probability and price as the first time. Once a user has chosen "willing to buy" or "not willing to buy," she cannot come back to make a choice again.

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OA.1.2 Instructions to Participants

Step 1: Participants see the product, price, and realization probability

Once a participant is allocated to a treatment condition, she will be notified of the availability of the product as well as the realization probability and price. Specifically, suppose the participant is assigned to the condition with realization probability r and price p.

For $r = 0$, the instruction reads "Please choose whether you are willing to buy the lucky player package at the price below. Your choice is hypothetical and no transaction will happen."

For $r = 1/30$ or $1/2$, the instruction instead reads "If you choose 'willing to buy,' the system will run a lottery, and with probability r you will receive the product at the price below. If the lottery fails or you choose 'not willing to buy,' no transaction will happen."

For $r = 1$, the instruction reads "If you choose 'willing to buy,' you will receive the product at the price below. If you choose 'not willing to buy,' no transaction will happen."

The price p is displayed below the instruction. At the right-hand side of the screen, there are two buttons showing "willing to buy" and "not willing to buy" respectively, and the participant makes the purchase decision by clicking the corresponding button. Before making the choice, she can click on a product icon beside the price to view the content of the player package, which shows the six players in the package and the message that "This package contains [the six players below]. If you adopt this package, one of them will be randomly selected to be available to you."[OA1] The participant can also click on the picture of each player to see a more detailed description of the player's skill set.

Figure OA1 presents a screenshot of the choice task in the field experiment. Figure OA2 presents a screenshot of the content of the player package.

Step 2: Participants make the purchase decision

The participant decides whether to purchase the player package. After the participant makes the decision, if her choice is "not willing to buy" or she belongs to the conditions of $r = 0$, she will be shown a message that reads "Thank you for your support" with a "Confirm" button.

[OA1] The random selection feature is a product feature designed by the firm and should not be confused with random realization in our experiment design. The firm embeds this random selection feature in most of its products.

Figure OA1: Choice Task

Notes. The screenshot is taken for the condition in which realization probability equals 1/30 and price equals 2800 diamonds. Other conditions feature similar screens. Users can click on the product icon (the one with a question mark) to view the content of the player package.

Figure OA2: Content of the Player Package

Notes. Users can click on each player to obtain more information about the player's skills.

After the participant clicks the "Confirm" button, a message will pop up saying "Here is a short survey. You will receive two million golden coins if you complete the survey."[OA2] The

[OA2]Golden coins are a common form of reward on this platform. Users cannot purchase golden coins, and must earn them by completing various tasks in the game. Users can redeem golden coins in exchange for performance-enhancing tools in the game. By comparison, users must purchase diamonds in the game using real money, and can use diamonds to purchase players from the platform. Overall, golden coins and diamonds are obtained and used in separate ways. There is no direct conversion rate between them.

post-choice survey questions will then be shown to the participant (see Step 3).

If the participant chooses "willing to buy" and she belongs to the conditions of $r = 1$, she will receive the product and her account will be automatically charged price p.[OA3] She will be shown a message "Congratulations! You have received the product at price p." She will then be notified of the post-choice survey and shown the survey questions, as described above.

If the participant chooses "willing to buy" and she belongs to the conditions of $r = 1/30$ or $1/2$, the system will run a lottery with success probability r. If the lottery fails, the participant will be notified that "Unfortunately, you did not win the lottery" and then she will be directed to the post-choice survey. If the lottery succeeds, the participant will receive the product, her account will be automatically charged price p, and she will be notified that "Congratulations! The lottery succeeds and you have received the product at price p." Then the participant will be directed to the post-choice survey.

Step 3: Participants take the post-choice survey

In the survey, participants are asked to answer the following questions.

- Do you think the opportunity to buy this player package is rare?
  1. Yes
  2. Indifferent
  3. No
- How do your perceive the quality of the player package?
  1. Very poor
  2. Poor
  3. Moderate
  4. Good
  5. Very good

[OA3] If the participant chooses "willing to buy" but the balance of diamonds in her account is smaller than p, we will direct the participant to recharge her account so that she will be able to purchase the product.

- Which of the following soccer players is **not** included in the player package?

  (a) Kong Ka

  (b) Chen Tao

  (c) Sun Xiang

  (d) Peng Weiguo

  (e) Fan Zhiyi

  (f) I don't know

- How do you perceive the price of the player package?

  1. Very low
  2. Low
  3. Moderate
  4. High
  5. Very high

After the participant completes the survey, she is notified that she has received two million golden coins in her account. This concludes the participant's exposure to the field experiment.

OA.1.3 Summary Statistics by Experimental Condition

Table OA1 reports the number of users who entered the experiment, the number of users who completed the choice task, and the number of users who completed the survey in each of the 20 experimental conditions. The table also summarizes the mean values of Diamonds and VIP-Level of users who completed the choice task in each condition.

Table OA1: Summary Statistics for Each of the 20 Experimental Conditions

Realization Probability		Price (Diamonds) 1600	2000	2400	2800	3200
0	# Entered Experiment	271	271	271	271	271
	# Completed Choice	217	213	218	216	231
	# Completed Survey	176	171	170	179	186
	Diamonds	4003.53	3117.19	3600.67	3329.25	3037.35
	VIP-Level	2.95	3.18	3.19	2.93	2.90
1/30	# Entered Experiment	271	271	271	271	271
	# Completed Choice	194	191	180	166	189
	# Completed Survey	142	144	139	131	152
	Diamonds	3621.78	3702.71	2403.71	3074.34	3218.74
	VIP-Level	3.21	3.32	2.52	3.28	2.88
1/2	# Entered Experiment	271	271	271	271	271
	# Completed Choice	185	183	182	191	181
	# Completed Survey	145	144	144	148	142
	Diamonds	2757.05	3048.75	3150.38	2604.06	2739.93
	VIP-Level	2.79	2.92	3.32	2.96	2.81
1	# Entered Experiment	271	271	271	271	271
	# Completed Choice	178	170	184	188	175
	# Completed Survey	136	116	136	145	138
	Diamonds	3154.78	3210.11	3065.91	2603.24	2949.27
	VIP-Level	2.61	2.69	3.29	2.73	3.57

Notes. For Diamonds and VIP-Level, we report their mean values among all users who completed the choice task in each experimental condition.

OA.2 AIA Model Estimation and Prediction Details

We first draw three $N * K$ matrices of random numbers that are independent and identically distributed (i.i.d.), following the standard normal distribution, where N is the number of individual users who completed the choice task, and $K = 100$ is the number of iterations we will perform to simulate the average purchase probability of each individual. The three matrices are denoted as $\epsilon_1, \epsilon_2, \epsilon_3$. The draws are quasi-random. We first generate a two-dimensional Halton set with three columns, each column of which consists of an infinite sequence of evenly distributed numbers from $[0, 1]$. We then take the first $N * K$ elements of each column, and convert the numbers to draws from a standard normal distribution by taking the inverse of normal c.d.f. of them. Since the Halton set is more evenly distributed on $[0, 1]$ than direct random draws of the uniform distribution on $[0, 1]$, the random draws created in this way leads to better convergence performance compared to direct random draws and requires a smaller number of draws (Train 2009).

We also generate two $1 * T$ vectors, which are the Gauss-Hermite quadrature nodes and weights over $[-1, 1]$, where $T = 1,000$. These two vectors will be used to calculate the expectation of a function of a normally distributed random variable.

Given these draws, we can calculate the average purchase probability of each individual for any given set of parameters, and then calculate the log-likelihood of the observed data. The objective function is the sum of log-likelihood of the observed data.

Given a set of parameter values $(b_0, b_1, b_2, a_0, a_1, c_0, c_1, \sigma_v)$, we perform K iterations of calculation. Within each iteration, we take the following steps.

1. Simulate each individual's true valuation $v_i = b_0 + b_1 \text{Log-Diamonds}_i + b_2 \text{VIP-Level}_i + \sigma_v \epsilon_{1ik}$, for $i = 1, ..., N$, where ϵ_1 is an $N * K$ matrix of i.i.d. standard normal draws we have created, and ϵ_{1ik} is element (i, k) of the matrix.

2. Calculate each individual's prior uncertainty $\sigma_{0i} = \exp(a_0 + a_1 \text{VIP-Level}_i)$, for $i = 1, ..., N$.

3. Simulate each individual's prior belief $\mu_i = v_i + \sigma_{0i}\epsilon_{2ik}$, for $i = 1, ..., N$, where ϵ_2 is an $N * K$ matrix of i.i.d. standard normal draws we have created. Thus we have $\mu_i \sim N(v_i, \sigma_{0i}^2)$.

4. Calculate $\mathbb{E}[(v_i - p_i)^+]$, $i = 1, ..., N$, where p_i is the price assigned to individual i. The expectation is taken over individual i's belief about the distribution of v_i, which is $N(\mu_i, \sigma_{0i}^2)$. To achieve better convergence performance, we use the Gauss-Hermite quadrature method. That is, if a random variable $Y \sim N(\mu, \sigma^2)$, $\mathbb{E}[f(Y)] \approx \frac{1}{\sqrt{\pi}} \sum_{j=1}^{T} w_j f(\mu + \sqrt{2}\sigma x_j)$, where x_j, w_j are the Gauss-Hermite quadrature nodes and weights over $[-1, 1]$. In our case, $\mathbb{E}[(v_i - p_i)^+] \approx \frac{1}{\sqrt{\pi}} \sum_{j=1}^{T} w_j \cdot \left(\mu_i + \sqrt{2}\sigma_{0i} x_j - p_i\right)^+$.

5. Simulate each individual's effort cost $c_i = \exp(c_0 + c_1 \epsilon_{3ik})$, where ϵ_3 is the $N * K$ standard normal matrix we have drawn.

6. Calculate individual i's effort level $t_i^* = \min\left\{\frac{r_i}{c_i}\left(\mathbb{E}[(v_i - p_i)^+] - (\mu_i - p_i)^+\right), 1\right\}$, where r_i is the realization probability assigned to individual i. The terms c_i, $\mathbb{E}[(v_i - p_i)^+]$, and μ_i have been simulated in previous steps.

7. Calculate individual i's purchase probability for the k^{th} iteration: $\Pr_k(\text{Buy}_i = 1) = t_i^* \frac{\exp(v_i - p_i)}{1+\exp(v_i - p_i)} + (1 - t_i^*)\frac{\exp(\mu_i - p_i)}{1+\exp(\mu_i - p_i)}$.

After K iterations, for individual i, we average over the K draws to derive the individual's purchase probability $\Pr(\text{Buy}_i = 1) = \frac{1}{K}\sum_{k=1}^{K} \Pr_k(\text{Buy}_i = 1)$, and then calculate the sum of log-likelihood as $LL = \sum_{i=1}^{N}\left[\mathbf{1}(\text{Buy}_i = 1)\log \Pr(\text{Buy}_i = 1) + \mathbf{1}(\text{Buy}_i = 0)\log\left(1 - \Pr(\text{Buy}_i = 1)\right)\right]$.

Being able to calculate the simulated log-likelihood given a set of parameter values, we search over the parameter space and find the set of parameter values that maximizes the simulated log-likelihood. We restrict the value of σ_v and c_1 to be positive, since they represent the standard deviations of a normal distribution and a log-normal distribution. As discussed in the paper, the data of $r \in \{1/30, 1/2\}$ conditions are used to perform the estimation.

Given the parameter estimates, we follow the same steps as described above to calculate the purchase probability of each individual in the counterfactual case of $r = 1$: first draw random numbers, and then go over K iterations to simulate each individual's purchase probability given the estimated parameter values. Lastly, we aggregate the purchase probability to form the demand forecast of the AIA method.

References

Train, K. E. (2009). *Discrete Choice Methods with Simulation.* Cambridge University Press.