Online Appendix of "A Model of Brand Architecture Choice: A House of Brands vs. A Branded House"

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1 Each Firm Type's Preferred Branding under the HE

This section further explores each firm type's preferred branding regime when the HE exists for both branding regimes, i.e., $c < \min\{\overline{c}^{ind}, \overline{c}^{umb}\}$.

Proposition 1. *Suppose that $c < \min\{\overline{c}^{ind}, \overline{c}^{umb}\}$ such that the HE exists for both independent branding and umbrella branding. The average expected profit over all firm types is the same for the two branding regimes. The $(C,C)-$type firm obtains a greater profit under umbrella branding, whereas the $(I,I)-$type firm does so under independent branding. Finally, the $(C,I)-$ and $(I,C)-$type firms obtain a greater profit under umbrella branding if one of the following conditions holds: (i) $\mu < \frac{1}{2}$, (ii) $\frac{1}{2} \leq \mu < \frac{1}{2-\Delta}$ and α is sufficiently large, or (iii) $\frac{1}{2-\Delta} < \mu < \frac{1}{1+(1-\Delta)^2}$ and α and ρ are sufficiently large.*

Proof. Each type of the firm's per-period profit in the HE is defined in equations (12), (13) and (14) in the Appendix of the paper. The average expected profit over all firm types is the same for independent branding and umbrella branding because the firm extracts all surplus and, given the HE, the total surplus is the same in both branding regimes.

The profits of the $(C,C)-$type under independent branding and umbrella branding are compared. Using the properties in Lemma 1, $E\pi^{*HE}_{C,C}(umb) - E\pi^{*HE}_{C,C}(ind)$ boils down to

$$\frac{\alpha}{2}\big(\Delta(1-\Delta)(p^B(H,L;umb) - p^B(H,L;ind)) + (1-\Delta)^2(p^A(L,L;umb) - p^A(L,L;ind))\big)$$
$$+\frac{1-\alpha}{2}\big(\Delta(p^B(H,\varnothing;umb) - p^B(H,\varnothing;ind)) + (1-\Delta)(p^B(L,\varnothing;umb) - p^B(L,\varnothing;ind))\big)$$

If $\alpha = 1$, then the equation simplifies to $\frac{1}{2}\big(\Delta(1-\Delta)(p^B(H,L;umb) - p^B(H,L;ind)) + (1-\Delta)^2(p^B(L,L;umb) - p^B(L,L;ind))\big)$, which is positive because it is the same as $\frac{1-\Delta}{2}\frac{\Delta^2(1-\Delta)(1-\mu)^2(1-\mu\Delta+\mu\Delta\rho)\rho}{(1-\mu\Delta)\big((1-\mu\Delta)^2-\mu(1-\mu)\Delta^2\rho\big)(1-\Delta\rho-(1-\rho)\mu\Delta)}$. Additionally, if $\alpha = 0$, then the equation becomes $\frac{1}{2}\big(\Delta(p^B(H,\varnothing;umb) - p^B(H,\varnothing;ind)) + (1-\Delta)(p^B(L,\varnothing;umb) - p^B(L,\varnothing;ind))\big)$, which is also positive because it is the same as $\frac{(1-\mu)^2\Delta^2\rho}{1-\mu\Delta}$. Therefore, by continuity, for the entire parameter region, $E\pi^{*HE}_{C,C}(umb) - E\pi^{*HE}_{C,C}(ind) \geq 0$.

Next, comparing the profits of $(I,I)-$type, $E\pi^{*HE}_{I,I}(umb) - E\pi^{*HE}_{I,I}(ind) =$

$$\frac{\alpha}{2}\cdot\big(p^A(L,L;umb) - p^A(L,L;ind)\big) + \frac{1-\alpha}{2}\cdot\big(p^B(\varnothing,L;umb) - p^B(\varnothing,L;ind)\big),$$

which is negative because $p^A(L,L;umb)-p^A(L,L;ind) \leq 0$ and $p^B(\varnothing,L;umb)-p^B(\varnothing,L;ind) \leq 0$.

Finally, the profits for the mixed type (C,I) are compared. $E\pi^{*HE}_{C,I}(umb) - E\pi^{*HE}_{C,I}(ind) =$

$$\frac{\alpha}{2}\big(\frac{\Delta}{2}(p^B(H,L;umb) - p^B(H,L;ind)) + (1-\Delta)(p^A(L,L;umb) - p^A(L,L;ind))\big)$$
$$+\frac{1-\alpha}{2}\big(\frac{\Delta}{2}(p^B(H,\varnothing;umb) - p^B(H,\varnothing;ind)) + \frac{2-\Delta}{2}(p^B(L,\varnothing;umb) - p^B(L,\varnothing;ind))\big)$$

If $\alpha = 1$, then the equation becomes $\frac{1}{2}\big(\frac{\Delta}{2}(p^B(H,L;umb)-p^B(H,L;ind))+(1-\Delta)(p^A(L,L;umb)-p^A(L,L;ind))\big)$, which is positive if and only if either $\mu < \frac{1}{2-\Delta}$, or $\frac{1}{2-\Delta} \leq \mu < \frac{1}{1+(1-\Delta)^2}$ and $\rho > \frac{(1-\mu\Delta)(1-\mu(2-\Delta))}{\mu(1-\mu)\Delta(2-\Delta)}$. If $\alpha = 0$, then the equation becomes $\frac{1}{2}\big(\frac{\Delta}{2}(p^B(H,\varnothing;umb) - p^B(H,\varnothing;ind)) + \frac{2-\Delta}{2}(p^B(L,\varnothing;umb) - p^B(L,\varnothing;ind))\big) = \frac{(1-\mu)(1-2\mu)\Delta^2\rho}{2(1-\mu\Delta)}$, which is positive if and only if $\mu < \frac{1}{2}$. Given the continuity of $E\pi^{*HE}_{C,I}(umb) - E\pi^{*HE}_{C,I}(ind)$ in α, this proves the proposition. □

As long as the HE exists, the $(C,C)-$type firm prefers umbrella branding because it is confident that it can create a good track record in each product market. Then, a consumer will pool information across product markets, which will lead to her paying greater prices. On the other hand, the $(I,I)-$type firm prefers independent branding because it can only achieve a bad track record in each product market. Thus, it does not want the consumer to pool the bad track records across product markets.

The decision for the mixed types, (C,I) and (I,C), is less trivial. Recall that a good track record is more informative about the firm's capability type than a bad track record because only the competent firm can achieve a good track record with a positive probability Δ. This implies that the mixed type of firms may benefit from adopting umbrella branding, hoping that a probable good outcome in one product market will spillover to the other market in which the firm is doomed to always achieve a bad outcome. However, of course, this bad track record can also spillover in the opposite direction. Therefore, the net tradeoff will be determined by whether the positive spillover or the negative spillover is greater. If μ_0 is large, the positive spillover becomes weaker because consumers' prior beliefs are already optimistic. Therefore, as μ_0 becomes large, additional conditions are required for umbrella branding to be more profitable than independent branding. More specifically, the two product markets need to be sufficiently related in both dimensions, thus magnifying the positive spillover effect.

In summary, provided that the HE exists for both branding regimes so that the firm does not face commitment problems, higher market relatedness makes umbrella branding increasingly more attractive for all firm types, except for the $(I, I)-$type firm, which becomes worse off.

2 Proof of Claim 2 of Proposition 4

It is formally shown that all 12 potential equilibria cannot exist.

Case I. $\sigma^A_{C,C} = \sigma^B_{C,C} = 1$, and $\sigma^A_{C,I} = \sigma^B_{I,C} = 0$.

Since only the $(C, C)-$type firm can produce a high-quality product in each product market, the $(C, I)-$type firm will deviate by investing in a product market. Then, the consumer will misinterpret the firm's type as being the $(C, C)-$type, and thus be willing to pay more in both product markets. The statment is proven by showing that in this supposed equilibrium, the $(C, I)-$type firm has a greater incentive to invest in quality than the $(C, C)-$type firm does.

A formal proof is as follows. In this equilibrium, the $(C, C)-$type invests in product market A, but the $(C, I)-$type does not. Therefore, the equilibrium exists only if (a necessary condition) there are two thresholds, one for each type (C, C) and the other for the $(C, I)-$type firm, such that $c < \overline{c}^{br}_{C,C}$ and $c > \overline{c}^{br}_{C,I}$,[1] where

$$\begin{aligned}
\overline{c}^{br}_{C,C} &:= \delta\Delta \cdot \Bigg(\frac{\alpha}{2}\big(\Delta(V_{C,C}(H,H;br) - V_{C,C}(L,H;br)) + (1-\Delta)(V_{C,C}(H,L;br) - V_{C,C}(L,L;br))\big) \\
&\qquad\qquad + \frac{1-\alpha}{2}\big(V_{C,C}(H,\varnothing;br) - V_{C,C}(L,\varnothing;br)\big)\Bigg) \\
\overline{c}^{br}_{C,I} &:= \delta\Delta \cdot \Bigg(\frac{\alpha}{2}(V_{C,I}(H,L;br) - V_{C,I}(L,L;br)) + \frac{1-\alpha}{2}\big(V_{C,I}(H,\varnothing;br) - V_{C,I}(L,\varnothing;br)\big)\Bigg)
\end{aligned}$$

The necessary conditions for the equilibrium imply that $\overline{c}^{br}_{C,C} > \overline{c}^{br}_{C,I}$ must hold. Each expression is a weighted sum of differences in value functions of the form $V_\theta(H, h^B_t; br) - V_\theta(L, h^B_t; br)$. By equation (9) in the main text, $V_\theta(H, h^B_t; br) - V_\theta(L, h^B_t; br) = \frac{p^A(H,h^B_t;br) - p^A(L,h^B_t;br) + p^B(H,h^B_t;b}{2}$

Then, $\overline{c}^{br}_{C,C} > \overline{c}^{br}_{C,I}$ holds if and only if $V_\theta(H,H;br) - V_\theta(L,H;br) > V_\theta(H,L;br) - V_\theta(L,L;br)$ because $\overline{c}^{br}_{C,C}$ gives a greater weight to $V_\theta(H,H;br) - V_\theta(L,H;br)$ than $\overline{c}^{br}_{C,I}$ does. Equivalently, $p^A(H,H;br) - p^A(L,H;br) > p^A(H,L;br) - p^A(L,L;br)$ must hold. However, this can never hold. The left-hand side is zero because in this equilibrium, only the $(C,C)-$type firm invests in quality, so a track record of H reveals the firm to be of the $(C,C)-$type, which leads to $p^A(H,H;br) = p^A(L,H;br) = \Delta$. On the other hand, the right-hand side is positive because $p^A(H,L;br) = \Delta > p^A(L,L;br)$.

This proves that this supposed equilibrium cannot exist. In particular, as the intuitive logic indicates, the $(C,I)-$type firm has a greater incentive to invest in quality than the $(C,C)-$type firm does, so the former makes a profitable deviation.

Case II. $\sigma^A_{C,C} = \sigma^B_{C,C} = 0$, and $\sigma^A_{C,I} = \sigma^B_{I,C} = 1$.

The $(C,C)-$type firm will deviate and invest in product market A (for instance), thereby mimicking the $(C,I)-$type firm's strategy.

A formal proof is as follows. Following the same proof strategy as that for the first case, the equilibrium exists only if the two thresholds $\overline{c}^{br}_{C,C}$ and $\overline{c}^{br}_{C,I}$ satisfy $c > \overline{c}^{br}_{C,C}$ and $c < \overline{c}^{br,H*}_{C,I}$. Then, this equilibrium can exist if $\overline{c}^{br}_{C,C} < \overline{c}^{br}_{C,I}$. $\overline{c}^{br}_{C,C} := \delta\Delta \cdot \big(\frac{\alpha}{2}(V_{C,C}(H,L;br) -$

[1] Note that these thresholds are specific to this supposed equilibrium, and thus differ from the thresholds identified for the HE. For brevity, a notation for each equilibrium is omitted.

$V_{C,C}(L,L;br)) + \frac{1-\alpha}{2}(V_{C,C}(H,\varnothing;br) - V_{C,C}(L,\varnothing;br))\big)$, and $\overline{c}^{br}_{C,I} := \delta\Delta \cdot \big(\frac{\alpha}{2}(V_{C,I}(H,L;br) - V_{C,I}(L,L;br)) + \frac{1-\alpha}{2}(V_{C,I}(H,\varnothing;br) - V_{C,I}(L,\varnothing;br))\big)$. Since $V_\theta(\cdot;br)$ does not depend on θ, the two thresholds coincide exactly, i.e., $\overline{c}^{br}_{C,C} = \overline{c}^{br}_{C,I}$. Therefore, the necessary condition for the equilibrium cannot hold.

Case III. $\sigma^A_{C,C} = \sigma^B_{C,C} = 1$ and either $\sigma^A_{C,I} = 1$ and $\sigma^B_{I,C} = 0$, or $\sigma^A_{C,I} = 0$ and $\sigma^B_{I,C} = 1$ (two equilibria).

Without loss of generality, the former equilibrium is examined (this applies to all subsequent cases in this proof). The $(C,C)-$type is the only type of firm that can produce a high-quality product in product market B. Then, the $(I,C)-$type firm will find it profitable to deviate and optimal to invest in product market B. This deviation will allow the firm an opportunity to obtain a good outcome in product market B. Then, the consumer will believe the firm to be of the $(C,C)-$type that invests in both markets, and thus pay higher prices.

Case IV. $\sigma^A_{C,C} = \sigma^B_{C,C} = 0$ and either $\sigma^A_{C,I} = 1$ and $\sigma^B_{I,C} = 0$, or $\sigma^A_{C,I} = 0$ and $\sigma^B_{I,C} = 1$ (two equilibria).

If the $(C,I)-$type firm finds it optimal to invest in product market A, then the $(C,C)-$type firm will also choose to deviate from this equilibrium and mimic the $(C,I)-$type firm by investing in product market A. This will be a profitable deviation.

Case V. $\sigma^A_{C,I} = \sigma^B_{I,C} = 1$ and either $\sigma^A_{C,C} = 1$ and $\sigma^B_{C,C} = 0$, or $\sigma^A_{C,C} = 0$ and $\sigma^B_{C,C} = 1$ (two equilibria).

Since the $(I,C)-$type firm finds it optimal to invest in product market B, the $(C,C)-$type firm will also find it profitable to deviate and invest in product market B.

Case VI. $\sigma^A_{C,I} = \sigma^B_{I,C} = 0$ and either $\sigma^A_{C,C} = 1$ and $\sigma^B_{C,C} = 0$, or $\sigma^A_{C,C} = 0$ and $\sigma^B_{C,C} = 1$ (two equilibria).

In the former equilibrium, the $(C,I)-$type firm can deviate and mimic the $(C,C)-$type firm by investing in product market A. This will be a profitable deviation since the $(C,C)-$type firm finds it optimal to invest in product market A in each period.

Case VII. Either $\sigma^A_{C,C} = 1$ and $\sigma^B_{C,C} = 0$ and $\sigma^A_{C,I} = 0$ and $\sigma^B_{I,C} = 1$, or $\sigma^A_{C,C} = 0$ and $\sigma^B_{C,C} = 1$ and $\sigma^A_{C,I} = 1$ and $\sigma^B_{I,C} = 0$ (two equilibria).

As the $(C, C)-$type firm finds it optimal to invest in product market A, the $(C, I)-$type firm will deviate and mimic the strategy of the $(C, C)-$type firm and invest in product market A. This will be a profitable deviation. $\square$