APPENDIX: PRICING UNDER DYNAMIC COMPETITION WHEN LOYAL CONSUMERS STOCKPILE

Period 1 Mixed Strategies in Symmetric State 11

We can now use arguments similar to the ones to characterize the mixed strategy in state 00 (see section 3.2.1 and 3.2.2 in main paper) to obtain the mixed strategy in state 11. Corresponding to (2) we get

$$F_{3-i}^{11}(p) = \frac{\alpha+\beta}{\beta}\left(1-\frac{l^{11}}{p}\right), \quad p \in [l^{11}, t) \tag{6}$$

Define $\Delta_{3-i}^{11} \triangleq \alpha\lambda(-t+\delta_f r) - \delta_f r\left(1-F_{3-i}^{11}(t^-)\right)\bar{M}\beta \geq 0$. Depending on whether $\Delta_{3-i}^{11} < 0$ or $\Delta_{3-i}^{11} > 0$, we have

$$\alpha(h^{11}-t) + \Delta_{3-i}^{11} = 0 \text{ and } m_{3-i}^{11} = 0, \quad \text{if } \Delta_{3-i}^{11} < 0 \tag{7}$$

$$m_{3-i}^{11} = \frac{\Delta_{3-i}^{11}}{\beta\left(t-\frac{1}{2}\delta_f\bar{M}r\right)} > 0 \text{ and } h^{11} = t, \quad \text{if } \Delta_{3-i}^{11} \geq 0 \tag{8}$$

And, finally,

$$F_{3-i}^{11}(p) = \frac{(\alpha(1-\lambda)+\beta)}{\beta}\left(1-\frac{h^{11}}{p}\right) + \frac{h^{11}}{p}\left(F_{3-i}^{11}(t^-) + m_{3-i}^{11}\right) \; p \in [h^{11}, r) \tag{9}$$

The mixed strategy in state 11 is completely characterized by (6) and (9) after solving for l^{11}, m_{3-i}^{11} and h^{11} by using (7), (8) and the boundary condition that the mixing distribution $F_{3-i}^{11}(p)$ attain the value of 1 at *p* = *r* along with period 2 results. As in the case of state 00, a sufficient condition for *r* to be undominated in state 11 is also derived by comparing profits and is $t < r(1-\lambda(1-\delta_f))$.

Of the two upper bounds for *t*, based on *r* being undominated in states 00 and 11, the second one is tighter, and so for *r* to be undominated in both states a sufficient condition is $t < r\left(1-\lambda(1-\delta_f)\right)$. We now turn to the asymmetric states 01 and 10.

Period 1 Mixed Strategies in asymmetric States 01 and 10

Using arguments similar to the ones used to derive $F_{3-i}^{00}(p)$ and $F_{3-i}^{11}(p)$, we can derive the following:

$$F_{3-i}^{01}(p) = \frac{\alpha(1+\lambda)+\beta}{\beta}\left(1-\frac{l^{01}}{p}\right) > \frac{\alpha+\beta}{\beta}\left(1-\frac{l^{01}}{p}\right) = F_{3-i}^{10}(p), \quad p \in [l^{01}, t)$$

$$\Delta_{3-i}^{01} \triangleq \alpha\lambda(-t+\delta_f r) - \delta_f r\left(1-F_{3-i}^{01}(t^-)\right)\bar{M}\beta$$

$$> \alpha\lambda(-t+\delta_f r) - \delta_f r\left(1-F_{3-i}^{10}(t^-)\right)\bar{M}\beta = \Delta_{3-i}^{10} \tag{10}$$

We can then see that the mass points at *t* fall into one of three cases. In the first case if $\Delta_{3-i}^{10} > 0$, then both $F_{3-i}^{01}(p)$ and $F_{3-i}^{10}(p)$ have mass points m_{3-i}^{01} and m_{3-i}^{10} respectively at *t* and moreover, $m_{3-i}^{01} = \frac{\Delta_{3-i}^{01}}{\beta\left(t-\frac{1}{2}\delta_f\bar{M}r\right)} > \frac{\Delta_{3-i}^{10}}{\beta\left(t-\frac{1}{2}\delta_f\bar{M}r\right)} = m_{3-i}^{10} > 0$,

implying $\left(F_{3-i}^{01}(t) = F_{3-i}^{01}(t^-) + m_{3-i}^{01}\right) > \left(F_{3-i}^{10}(t^-) + m_{3-i}^{10} = F_{3-i}^{10}(t)\right)$. In the second case if $\Delta_{3-i}^{01} > 0 > \Delta_{3-i}^{10}$, then there will be a hole in the strategy $F_{3-i}^{10}(p)$, therefore there will also be a hole over the same interval for $F_{3-i}^{01}(p)$, between [*t*, *h*), and a mass point $m_{3-i}^{01} > \frac{\Delta_{3-i}^{01}}{\beta\left(t-\frac{1}{2}\delta_f\bar{M}r\right)} > 0$; so $\left(F_{3-i}^{01}(h) = F_{3-i}^{01}(t^-) + m_{3-i}^{01}\right) > \left(F_{3-i}^{10}(t^-) = F_{3-i}^{10}(h)\right)$. Finally, in the third case if $\Delta_{3-i}^{01} < 0$, there will be a hole in both strategies, $F_{3-i}^{01}(p)$ and $F_{3-i}^{10}(p)$, and moreover, since the interval of the hole should be equal for both, there will be a mass point $m_{3-i}^{01} > 0$; so again $\left(F_{3-i}^{01}(h) = F_{3-i}^{01}(t^-) + m_{3-i}^{01}\right) > \left(F_{3-i}^{10}(t^-) = F_{3-i}^{10}(h)\right)$.

Therefore, in all three cases, we see that

$$m_{3-i}^{01} > 0 \text{ and } F_{3-i}^{01}(h) > F_{3-i}^{10}(h), \;\; h \geq t. \tag{11}$$

We are now in a position to completely specify the mixed strategies in states 01 and 10.

$$F_i^{10}(p) \equiv F_{3-i}^{01}(p) = \frac{\alpha(1+\lambda)+\beta}{\beta}\left(1 - \frac{l}{p}\right), \;\; p \in [l, t) \tag{12}$$

$$F_i^{01}(p) \equiv F_{3-i}^{10}(p) = \frac{\alpha+\beta}{\beta}\left(1 - \frac{l}{p}\right), \qquad p \in [l, t) \tag{13}$$

$$m_{3-i}^{01} = \frac{\Delta_{3-i}^{01}}{\beta\left(t - \frac{1}{2}\delta_f \bar{M} r\right)}, m_{3-i}^{10} = \frac{\Delta_{3-i}^{10}}{\beta\left(t - \frac{1}{2}\delta_f \bar{M} r\right)}, \text{and } h = t \qquad if\ 0 \leq \Delta_{3-i}^{10} \tag{14}$$

$$m_{3-i}^{01} > m_{3-i}^{10} = 0, \text{and } h = \; t - \frac{\Delta_{3-i}^{10}}{\alpha} \qquad if\ \Delta_{3-i}^{10} < 0 \tag{15}$$

$$F_i^{10}(p) \equiv F_{3-i}^{01}(p) = F_{3-i}^{01}(h) + \frac{\alpha+\beta}{\beta}\left(1 - \frac{h}{p}\right), \qquad p \in [h, r) \tag{16}$$

$$F_i^{01}(p) \equiv F_{3-i}^{10}(p) = F_{3-i}^{10}(h) + \frac{\alpha(1-\lambda)+\beta}{\beta}\left(1 - \frac{h}{p}\right), \;\; p \in [h, r) \tag{17}$$

We can solve for m_{3-i}^{01}, h and l by using (14), (15) and the boundary condition that the mixing distribution $F_{3-i}^{01}(p)$ attain the value of 1 at $p = r$ along with period 2 results. Once l is known, we can use (10) to obtain m_{3-i}^{10}, if needed, and use (17) to obtain $F_{3-i}^{10}(r^-)$ and infer from that $M_i^{01} \equiv M_{3-i}^{10} = \left(1 - F_{3-i}^{10}(r^-)\right)$. To see if r is undominated in states 01 and 10, first note from (12) and (13) that $F_{3-i}^{01}(p) > F_{3-i}^{10}(p), p \in [l, t)$. And so, there are three possibilities; i) r is dominated in both states 01 and 10; ii) r is dominated in state 10, only for focal firm, in which case competing firm's mixing distribution, $F_{3-i}^{10}(p)$, contains a mass point m_d at p=r or; iii) r is undominated. It is easy to verify that if $t < r(1 - \lambda\left(1 - \delta_f\right))$, r is not dominated in both states 01 and 10. We can rule out the second case also, if $t(\alpha + \beta m_d) + \delta_f r(\alpha(1-\lambda) + \beta\bar{M}) < \alpha r(1-\lambda) + \delta_f \alpha r$, where $m_d = \left(1 - F_{3-i}^{10}(t)\right) = \frac{\alpha\lambda}{\alpha(1+\lambda)+\beta}$ because $F_i^{10}(t) = 1$. Thus, r is undominated in all states if

$$t < r\frac{1 - \lambda + \delta_f \lambda \eta_i - \delta_f \beta \overline{M/}\alpha}{1 - \lambda + \lambda\eta_i + \beta m_d/\alpha}$$

We have derived the mixed strategies assuming that stockpiling consumers adopt a mixed strategy by choosing to stockpile with probability $\eta_i = 0.5$ when they encounter a price of t. This has force in our model since the price of t in equilibrium is chosen with probability greater than zero, as evidenced by the mass point at t in the mixed strategies. The mass points also raise the possibility that both firms can have equal prices, of t, with non-zero probability. That too has force in our model and we have assumed that switching consumers adopt a mixed strategy by choosing brands with equal probability of $\phi_i = 0.5$ when encountering equal prices.

Lemma 1: *In the asymmetric state 01, in period 1, competing firm's pricing distribution,* $F_{3-i}^{01}(p)$ *does not a have mass point at r. Moreover, focal firm's pricing distribution,* $F_i^{01}(p)$ *has a mass point at r, denoted by* $M_i^{01} \geq 0$.

From (16) and (17), we know that

$$F_i^{10}(p) \equiv F_{3-i}^{01}(p) = F_{3-i}^{01}(h) + \frac{\alpha+\beta}{\beta}\left(1 - \frac{h}{p}\right), \qquad p \in [h, r)$$

$$F_i^{01}(p) \equiv F_{3-i}^{10}(p) = F_{3-i}^{10}(h) + \frac{\alpha(1-\lambda)+\beta}{\beta}\left(1 - \frac{h}{p}\right), \quad p \in [h, r)$$

Moreover, from (14) and (15) we know $F_{3-i}^{01}(h) > F_{3-i}^{10}(h), h \geq t$

This implies that

$$F_{3-i}^{01}(p) > F_{3-i}^{10}(p), \quad p \in [h, r)$$

Note that $f_{3-i}^{01}(p) > f_{3-i}^{10}(p), \ \ p \in [h, r)$. And since at most only one firm can have a mass point at *r*, the desired result follows.

Q.E.D.

This completes the characterization of mixed strategies in the first period of the 2-period model. The next section highlights the main results of an infinite period model; for detailed characterization of the infinite period model, please see web-appendix.

Mixed Strategy Equilibrium in an Infinite Period Model

Proposition 1: *The firm's value functions in each state in an infinite period model are*

$$V_i^{00} = V_i^{01} = \frac{r\alpha}{1-\delta_f}; \ V_i^{11} = \frac{r\alpha}{1-\delta_f} - r\alpha\lambda \ \ and \ \ \frac{r\alpha}{1-\delta_f} - r\alpha\lambda = V_i^{11} \leq V_i^{10} = \frac{r\alpha}{1-\delta_f} - r\alpha\lambda + r\beta M_i^{01}$$

Proof: Proof is by induction. Consider a finite horizon problem with *T* periods. Denote the value functions in the (*n*)th period, $1 < n < T$, by $V_{i,n}^{00}$, $V_{i,n}^{01}$, $V_{i,n}^{10}$, and $V_{i,n}^{11}$. Suppose also $V_{i,n}^{00} = V_{i,n}^{01}, V_{i,n}^{11} < V_{i,n}^{10}$ and the mixing distribution $F_{i,n}^{01}(r^-) < 1$, so $M_{i,n}^{01} > 0$.

Now consider the situation in the *n*-1th period. The value functions conditioned on state can now be evaluated.

$$\begin{aligned} V_{i,n-1}^{00} &= V_{i,n-1}^{00}(p_{i,n-1} = r) \\ &= \alpha r + \delta_f\left(\left(1 - F_{3-i,n-1}^{00}(t) + \frac{m_{3-i,n-1}^{00}}{2}\right)V_{i,n}^{00} + \left(F_{3-i,n-1}^{00}(t) - \frac{m_{3-i,n-1}^{00}}{2}\right)V_{i,n}^{01}\right) \\ &= \alpha r + \delta_f V_{i,n}^{00} \qquad (since\ V_{i,n}^{00} = V_{i,n}^{01}) \end{aligned}$$

The current period profit in period $n-1$, $\pi_{i,n-1}^{01}(p_{i,n-1} = r) = \alpha r + M_{3-i,n}^{01}\beta r$. Note that since $M_{i,n}^{01} > 0$, it msut be that $M_{i,n}^{10} = M_{3-i,n}^{01} = 0$. Therefore

$$\begin{aligned} V_{i,n-1}^{01} &= V_{i,n-1}^{01}(p_{i,n-1} = r) \\ &= \alpha r + \delta_f\left(\left(1 - F_{3-i,n-1}^{01}(t) + \frac{m_{3-i,n-1}^{01}}{2}\right)V_{i,n}^{00} + \left(F_{3-i,n-1}^{01}(t) - \frac{m_{3-i,n-1}^{01}}{2}\right)V_{i,n}^{01}\right) \\ &= \alpha r + \delta_f V_{i,n}^{00} \qquad (since\ V_{i,n}^{00} = V_{i,n}^{01}) \end{aligned}$$

The foregoing imply $V_{i,n-1}^{00}(p_{i,n-1}=r)=V_{i,n-1}^{01}(p_{i,n-1}=r)$. Next, we see that

$$\begin{aligned}
V_{i,n-1}^{11} = V_{i,n-1}^{11}(p_{i,n-1}=r) \\
&= \alpha(1-\lambda)r \\
&+ \delta_f\left(\left(1-F_{3-i,n-1}^{11}(t)+\frac{m_{3-i,n-1}^{11}}{2}\right)V_{i,n}^{00}+\left(F_{3-i,n-1}^{11}(t)-\frac{m_{3-i,n-1}^{11}}{2}\right)V_{i,n}^{01}\right) \\
&= \alpha(1-\lambda)r+\delta_f V_{i,n}^{00} \qquad (since\ V_{i,n}^{00}=V_{i,n}^{01})
\end{aligned}$$

Finally,

$$\begin{aligned}
V_{i,n-1}^{10} = V_{i,n-1}^{10}(p_{i,n-1}=r) \\
&= \alpha(1-\lambda)r+M_{3-i,n}^{10}\beta r \\
&+ \delta_f\left(\left(1-F_{3-i,n-1}^{10}(t)+\frac{m_{3-i,n-1}^{10}}{2}\right)V_{i,n}^{00}+\left(F_{3-i,n-1}^{10}(t)-\frac{m_{3-i,n-1}^{10}}{2}\right)V_{i,n}^{01}\right) \\
&= \alpha r+\delta_f V_{i,n}^{00} \qquad \left(since\ V_{i,n}^{00}=V_{i,n}^{01}\right) \\
&= V_{i,n-1}^{11}+M_{3-i,n}^{10}\beta r > V_{i,n-1}^{11}
\end{aligned}$$

To characterize $M_{i,n-1}^{01}$, we can proceed in a manner identical to the two period case and show that $M_{i,n-1}^{01}>M_{i,n-1}^{10}=0$. Thus, the value functions and the mass point in the *n*-1[th] period inherit the properties of the corresponding ones in the (*n*)[th] period. Indeed, by induction these properties are inherited for all periods $m<n$.

Invoking Table 1, we can see that in the last period, in period *T*, $V_{i,T}^{00}=V_{i,T}^{01}, V_{i,T}^{11}<V_{i,T}^{10}$ and $M_{i,T}^{01}>0$. The equilibrium for a finite horizon of arbitrary length, *T*, is then a sequence of strategies $\{\sigma_f(T),\sigma_1(T),\sigma_2(T),\sigma_3(T)\}$ that inherit the properties of the value functions. Moreover, since the equilibrium is Markov Perfect and all strategies are therefore conditioned only on payoff-relevant states, the finite horizon equilibrium converges to a steady state as $T\to\infty$, and the infinite horizon value functions inherit the desired properties.[1]

We can solve for the values in the infinite period case by evaluating the profits at *r*, $V_i^{01}=\alpha r+\delta_f\left(X_{3-i}^{01}V_i^{01}+(1-X_{3-i}^{01})V_i^{00}\right)$, where X_{3-i}^{01} is the probability that competing firm induces their loyal consumers to stockpile. Since $V_i^{00}=V_i^{01}$, we get

$$V_i^{01}=\frac{\alpha r}{1-\delta_f}=V_i^{00}.$$

Similarly, $V_i^{11}=\alpha(1-\lambda)r+\delta_f\left(X_{3-i}^{11}V_i^{01}+(1-X_{3-i}^{11})V_i^{00}\right)$ so $V_i^{11}=\frac{\alpha r}{1-\delta_f}-\lambda\alpha r$.

Finally, $V_i^{10}=\alpha(1-\lambda)r+M_i^{01}\beta r+\delta_f\left(X_{3-i}^{10}V_i^{01}+(1-X_{3-i}^{10})V_i^{00}\right)$ so $V_i^{10}=\frac{\alpha r}{1-\delta_f}-\lambda\alpha r+M_i^{01}\beta r$.

Q.E.D.

[1] It is well known in the case of repeated games finite horizon results do not extend to the infinite horizon case because standard dynamic programming convergence results cannot be invoked since strategies are conditioned on history that is not payoff relevant. We thank a reviewer for pointing this out.

Proposition 2: *Stockpiling loyal consumer's stockpile only if* $p \leq t$ *and do not stockpile if* $p > t$, *where the stockpiling threshold,* t *, is independent of the state and must satisfy:*

$$t = \delta_c \left(\frac{\bar{\pi}^0 \left(2\overline{pl}^0 - \overline{ph}^0 \right) + \overline{ph}^0 - \bar{\pi}^1 \overline{pl}^1}{1 + \delta_c \left(\bar{\pi}^0 - \bar{\pi}^1 \right)} \right)$$

Proof: Define $\bar{U}^0$ and $\bar{U}^1$ to be the utilities of the loyal consumer conditioned on their inventory being $I \in \{0,1\}$ respectively. The cost of stockpiling is the payment for the extra unit, p, incurred in the current period. The benefit of stockpiling occurs in the next period. The benefit is given by the difference in continuation utilities that are discounted, $\delta_c \left(\bar{U}^1 - \bar{U}^0 \right)$, where δ_c is the discount factor used by consumers. Therefore, for stockpiling to be optimal we require: $p < \delta_c \left(\bar{U}^1 - \bar{U}^0 \right)$.[2] We denote $t = \delta_c \left(\bar{U}^1 - \bar{U}^0 \right)$ as the stockpiling threshold. As we see the threshold is simply the difference in the appropriately discounted continuation utilities following a decision to either stockpile or not.

Recall that loyal consumers know the price distributions, $G^0(p)$ and $G^1(p)$, conditional on their own inventory state. Under steady state conditions, suppose loyal consumers have no inventory, $I = 0$. Let $\bar{\pi}^0$ denote the probability that they stockpile, buy 2 units and pay expected price $\overline{pl}^0$ when stockpile (Please refer to web-appendix A.2 for how these quantities can be computed). When they stockpile, they consume one unit in the current period and move to inventory state $I = 1$, in the next period. Conversely, with probability $\left(1 - \bar{\pi}^0\right)$ they buy only one unit at expected price of $\overline{ph}^0$ and remain in inventory state $I = 0$. Hence the steady state utility under inventory state $I = 0$ will be

$$\bar{U}^0 = \bar{\pi}^0 \left(r - 2\overline{pl}^0 + \delta_c \bar{U}^1 \right) + \left(1 - \bar{\pi}^0\right)\left(r - \overline{ph}^0 + \delta_c \bar{U}^0 \right)$$

Similarly, in inventory state $I = 1$ with probability $\bar{\pi}^1$ they will buy one unit and remain in inventory state $I = 1$ and with probability $\left(1 - \bar{\pi}^1\right)$ they will not buy and move to inventory state $I = 0$, which yields:

$$\bar{U}^1 = \bar{\pi}^1 \left(r - \overline{pl}^1 + \delta_c \bar{U}^1 \right) + \left(1 - \bar{\pi}^1\right)\left(r + \delta_c \bar{U}^0 \right)$$

From the above two equations we solve for $t = \delta_c \left(\bar{U}^1 - \bar{U}^0 \right)$ to get the desired result.

Q.E.D.

[2] Intuitively when current period price is lower than discounted expected price in the next period consumer should stockpile.

<u>Proposition 3</u>: *If* $\delta^* < \delta_c$ *stockpiling loyals stockpile with positive probability, where* $\delta^* = \dfrac{1-2\alpha}{(1-\alpha)\ln\left(\dfrac{1-\alpha}{\alpha}\right)}$.

<u>Proof</u>: Recall that no stockpiling implies that *s*=00 is an absorbing state. Suppose state *s*=00 is an absorbing state. If there is no stockpiling then every period is identical, and it is easy to characterize the mixed strategy pricing distribution following Narasimhan (1988). It is

$$F_{absorb}^{00}(p) = 1 - \frac{\alpha(r-p)}{(1-2\alpha)p}, \qquad \frac{r\alpha}{1-\alpha} \le p \le r$$

and the expected price and the lower bound of the support of prices respectively are:

$$E_{absorb}^{00} = \frac{r\alpha}{1-2\alpha}\ln\left(\frac{1-\alpha}{\alpha}\right) \text{ and } l^{00} = \frac{r\alpha}{1-\alpha}.$$

For *s*=00 to be an absorbing state, stockpiling loyal consumers' thresholds satisfy $t_1^*, t_2^* < l^{00}$. We will prove that a unilateral deviation by consumers of one of the firms, say firm 1, who use a different threshold to stockpile is unprofitable. Let such a threshold be $t_{deviate} > l^{00} > t^*$ that leads to stockpiling by them when price $p_1 = l^{00} < t_{deviate}$ is unprofitable for the consumers. Consider a period in which $p_1 = l^{00}$. The next period state, given firm 1 consumers deviate to $t_{deviate}$ is (10). Firm 2's loyal consumers stay on the equilibrium path and do not stockpile. Firms would now employ the Markov Perfect equilibrium mixed strategy conditional on state. It turns out that firm 1's strategy $F_{deviate}^{10}(p)$ in state (10) is identical to $F_{absorb}^{00}(p)$ since the equiprofit conditions for firm 2 after deviation are the same as on the equilibrium path. Therefore, in periods following the deviation, firm 1's stockpiling loyal consumers face prices that are identical to what they would have faced had they stayed on the equilibrium path. This implies that a necessary condition for 00 to be an absorbing state is that a deviation to stockpiling in any period by firm 1 consumers should be unprofitable to them even when firm 1 charges a price of l^{00} in that period. This leads to

$$l^{00} = \frac{r\alpha}{1-\alpha} > \delta_c E_{deviate}^{00} = \delta_c E_{absorb}^{00} = \delta_c \frac{r\alpha}{1-2\alpha}\ln\left(\frac{1-\alpha}{\alpha}\right)$$

Therefore, a sufficient condition for 00 to not be an absorbing state is

$$\frac{r\alpha}{1-\alpha} < \delta_c \frac{r\alpha}{1-2\alpha}\ln\left(\frac{1-\alpha}{\alpha}\right)$$

or equivalently

$$\delta_c > \delta^* = \frac{1-2\alpha}{(1-\alpha)\ln\left(\frac{1-\alpha}{\alpha}\right)}$$

Q.E.D.