Low-Price Guarantees in a Dual-Channel of Distribution—Online Appendix

Juncai Jiang[*] Chuan He[†]

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[*]Assitant Professor of Marketing, Pamplin College of Business, Virginia Tech, Blacksburg, VA 24061; email: jcjiang@vt.edu.

[†]Associate Professor of Marketing, Leeds School of Business, University of Colorado, Boulder, CO 80309; email: chuan.he@colorado.edu.

ONLINE APPENDIX

In this part of the Online Appendix, we derive the equilibrium prices and profits under each LPG policy. We first present the equilibria in a lemma and then provide proof.

No LPG

Lemma 1 *When the manufacturer does not offer LPG, the channel equilibrium is characterized as follows. Let* $b_1 \equiv \frac{3(2+\alpha)(1+\delta_D-\alpha)b_{LD}}{2(2-\alpha)(1+\delta_I+\alpha)}$ *and*

$$b_2 \equiv \frac{(2+\alpha)\begin{pmatrix}\sqrt{\alpha\left(1+\delta_D-\alpha\right)\Delta} \\ +\left(1+\delta_D-\alpha\right)\left(2\alpha^2+\alpha\left(11-\delta_I\right)+6\left(1+\delta_I\right)\right)\end{pmatrix}b_{LD}}{2(2-\alpha)^2\left(1+\delta_I+\alpha\right)^2},$$

($b_1 < b_2$) where $\Delta \equiv 13\alpha^4+\left(31+14\delta_I+5\delta_D\right)\alpha^3+2\left(5\delta_I^2-31\delta_D-15\delta_I+11\delta_I\delta_D+28\right)\alpha^2+4\left(16\delta_I-43\delta_D-10\delta_I^2-14\delta_I\delta_D+2\delta_I^2\delta_D-1\right)\alpha\ +\ 24\left(2\delta_I^2+\delta_I-5\delta_I\delta_D-\delta_I^2\delta_D-4\delta_D-1\right)$.
The equilibrium shopping strategy of LPG-sensitive comparison shoppers is given by

$$\beta^{N*} = \begin{cases} 1 & \text{if } 0 < b_{LI} \leq b_1 \\ 0 & \text{if } b_{LI} \geq b_2 \end{cases}. \tag{B-1}$$

The equilibrium wholesale price is given by

$$w^{N*} = \frac{2+\left(1-2\beta^{N*}\right)\alpha}{2\left(1+\delta_I+\left(1-2\beta^{N*}\right)\alpha\right)b_{LI}}. \tag{B-2}$$

The equilibrium retail price is given by

$$r^{N*} = \frac{3\left(2+\left(1-2\beta^{N*}\right)\alpha\right)}{4\left(1+\delta_I+\left(1-2\beta^{N*}\right)\alpha\right)b_{LI}}. \tag{B-3}$$

The equilibrium direct channel price is given by

$$p^{N*} = \frac{2-\left(1-2\beta^{N*}\right)\alpha}{2\left(1+\delta_D-\left(1-2\beta^{N*}\right)\alpha\right)b_{LD}}. \tag{B-4}$$

The equilibrium retail profit is given by

$$\pi^{N*} = \frac{\left(2 + \left(1 - 2\beta^{N*}\right)\alpha\right)^2}{64\left(1 + \delta_I + \left(1 - 2\beta^{N*}\right)\alpha\right) b_{LI}}. \tag{B-5}$$

The equilibrium manufacturer profit is given by

$$\Pi^{N*} = \frac{\left(2 + \left(1 - 2\beta^{N*}\right)\alpha\right)^2}{32\left(1 + \delta_I + \left(1 - 2\beta^{N*}\right)\alpha\right) b_{LI}} + \frac{\left(2 - \left(1 - 2\beta^{N*}\right)\alpha\right)^2}{16\left(1 + \delta_D - \left(1 - 2\beta^{N*}\right)\alpha\right) b_{LD}}. \tag{B-6}$$

The equilibrium channel profit is given by

$$\Pi_C^{N*} = \frac{3\left(2 + \left(1 - 2\beta^{N*}\right)\alpha\right)^2}{64\left(1 + \delta_I + \left(1 - 2\beta^{N*}\right)\alpha\right) b_{LI}} + \frac{\left(2 - \left(1 - 2\beta^{N*}\right)\alpha\right)^2}{16\left(1 + \delta_D - \left(1 - 2\beta^{N*}\right)\alpha\right) b_{LD}}. \tag{B-7}$$

Proof of Lemma 1

Proof. In the absence of LPG, LPG-sensitive consumers cannot take advantage of LPG and thus, the effective price they pay is the direct channel price $e^N = p^N$. Let β^N be the fraction of LPG-sensitive comparison shoppers who purchase from the direct channel in the no LPG case. The remaining $1 - \beta^N$ proportion of them will purchase from the indirect channel.

(I) When $e^N < r^N$

When $e^N < r^N$, all LPG-sensitive comparison shoppers will purchase from the direct channel ($\beta^{N*} = 1$). The profit functions of the manufacturer and retailer are given by

$$\begin{aligned}\Pi^N &= \frac{1+\alpha}{4} e^N \left(1 - b_{LD} e^N\right) + \frac{1}{4} p^N \left(1 - \delta_D b_{LD} p^N\right) \\ &\quad + \frac{1-\alpha}{4} w^N \left(1 - b_{LI} r^N\right) + \frac{1}{4} w^N \left(1 - \delta_I b_{LI} r^N\right),\end{aligned} \tag{B-8}$$

and

$$\pi^N = \frac{1-\alpha}{4}\left(r^N - w^N\right)\left(1 - b_{LI} r^N\right) + \frac{1}{4}\left(r^N - w^N\right)\left(1 - \delta_I b_{LI} r^N\right). \tag{B-9}$$

In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^N and r^N, respectively,

$$\begin{aligned}\max_{p^N>0} \Pi^N\left(p^N|w^N\right) &= \frac{1+\alpha}{4}e^N\left(1-b_{LD}e^N\right)+\frac{1}{4}p^N\left(1-\delta_D b_{LD}p^N\right) \\ &\quad +\frac{1-\alpha}{4}w^N\left(1-b_{LI}r^N\right)+\frac{1}{4}w^N\left(1-\delta_I b_{LI}r^N\right), \end{aligned} \tag{B-10}$$

and

$$\max_{r^N>0} \pi^N\left(r^N|w^N\right) = \frac{1-\alpha}{4}\left(r^N-w^N\right)\left(1-b_{LI}r^N\right)+\frac{1}{4}\left(r^N-w^N\right)\left(1-\delta_I b_{LI}r^N\right). \tag{B-11}$$

Solving the above maximization problems yields the optimal retail price response function $r^N\left(w^N\right) = \frac{w^N}{2}+\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$ and the equilibrium direct channel price $p^{N*} = \frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$. The equilibrium effective price is $e^{N*} = p^{N*} = \frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$.

In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^N while taking into account both players' stage 3 decisions:

$$\begin{aligned}\max_{w^N>0} \Pi^N\left(w^N\right) &= \frac{1+\alpha}{4}e^{N*}\left(1-b_{LD}e^{N*}\right)+\frac{1}{4}p^{N*}\left(1-\delta_D b_{LD}p^{N*}\right) \\ &\quad +\frac{1-\alpha}{4}w^N\left(1-b_{LI}r^N\left(w^N\right)\right)+\frac{1}{4}w^N\left(1-\delta_I b_{LI}r^N\left(w^N\right)\right). \end{aligned} \tag{B-12}$$

Solving the manufacturer's profit maximization problem, we obtain the equilibrium wholesale price $w^{N*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. It follows that the optimal retail price is $r^{N*} = r^N\left(w^{N*}\right) = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$. The equilibrium profits of the retailer and manufacturer are $\pi^{N*} = \pi^N\left(r^{N*}\right) = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$ and $\Pi^{N*} = \Pi^N\left(p^{N*}, w^{N*}\right) = \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}} + \frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}$, respectively. To ensure that $e^{N*} < r^{N*}$, we have $b_{LI} < \frac{3(2-\alpha)(1+\delta_D+\alpha)b_{LD}}{2(2+\alpha)(1+\delta_I-\alpha)} \equiv b_3$.

To ensure that there is no non-local deviation, we consider the case when the manufacturer unilaterally raises its direct channel price to slightly higher than the retail price, $p^{N*} = r^{N*}+\varepsilon$, where ε is an infinitesimally small positive number. In this way, the manufacturer directs LPG-sensitive comparison shoppers to the indirect channel. The manufacturer's equilibrium profit in this case is given by

$$\begin{aligned}\Pi_1^{N*} &= \frac{1-\alpha}{4}\left(r^{N*}+\varepsilon\right)\left(1-b_{LD}\left(r^{N*}+\varepsilon\right)\right)+\frac{1}{4}\left(r^{N*}+\varepsilon\right)\left(1-\delta_D b_{LD}\left(r^{N*}+\varepsilon\right)\right) \\ &\quad +\frac{1+\alpha}{4}w^{N*}\left(1-b_{LI}r^{N*}\right)+\frac{1}{4}w^{N*}\left(1-\delta_I b_{LI}r^{N*}\right), \end{aligned} \tag{B-13}$$

where $w^{N*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$ and $r^{N*} = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$. Thus, we have

$$\Pi_1^{N*} = \frac{(2-\alpha)\left(2b_{LI}\left(14+(14+\alpha)\delta_I-5\alpha(5-\alpha)\right)-9b_{LD}(2-\alpha)\left(1-\alpha+\delta_D\right)\right)}{64b_{LI}^2\left(1-\alpha+\delta_I\right)^2}. \quad \text{(B-14)}$$

Comparing Π^{N*} and Π_1^{N*}, we have

$$\Pi^{N*} - \Pi_1^{N*} > 0. \quad \text{(B-15)}$$

This indicates that the manufacturer does not have incentive to unilaterally raise its direct channel price.

We next consider the case when the retailer unilaterally lowers its retail price to slightly lower than the direct channel price, $r^{N*} = p^{N*} - \varepsilon$. In this way, the retailer undercuts the direct channel and therefore attracts LPG-sensitive comparison shoppers to the indirect channel. The retailer's equilibrium profit in this case is given by

$$\begin{aligned} \pi_1^{N*} &= \frac{1+\alpha}{4}\left(\left(p^{N*}-\varepsilon\right)-w^{N*}\right)\left(1-b_{LI}\left(p^{N*}-\varepsilon\right)\right) \\ &\quad +\frac{1}{4}\left(\left(p^{N*}-\varepsilon\right)-w^{N*}\right)\left(1-\delta_I b_{LI}\left(p^{N*}-\varepsilon\right)\right), \end{aligned} \quad \text{(B-16)}$$

where $w^{N*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$ and $p^{N*} = \frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$. Thus, we have

$$\pi_1^{N*} = \frac{(2+\alpha)\begin{pmatrix}(2+\alpha)b_{LI}\left(1-\alpha+\delta_I\right) \\ -(2-\alpha)b_{LD}\left(1+\delta_D+\alpha\right)\end{pmatrix}\left(2b_{LD}\left(1+\delta_D+\alpha\right)-b_{LI}\left(1+\alpha+\delta_I\right)\right)}{16b_{LD}^2 b_{LI}\left(1+\delta_D+\alpha\right)^2\left(1-\alpha+\delta_I\right)}. \quad \text{(B-17)}$$

Comparing π^{N*} and π_1^{N*}, we have

$$\pi^{N*} - \pi_1^{N*} > 0. \quad \text{(B-18)}$$

This indicates that the retailer does not have incentives to unilaterally lower its retail price in order to attract LPG-sensitive comparison shoppers to the indirect channel.

To sum up, when $b_{LI} < b_3$, the equilibrium outcome in (I) is stable and there is no non-local deviation.

(II) When $e^N > r^N$

Given that $e^N > r^N$, all LPG-sensitive comparison shoppers will purchase from the indirect channel ($\beta^{N*} = 0$). The profit functions of the manufacturer and retailer are given by

$$\begin{aligned}\Pi^N &= \frac{1-\alpha}{4}e^N\left(1-b_{LD}e^N\right)+\frac{1}{4}p^N\left(1-\delta_D b_{LD}p^N\right)\\&+\frac{1+\alpha}{4}w^N\left(1-b_{LI}r^N\right)+\frac{1}{4}w^N\left(1-\delta_I b_{LI}r^N\right),\end{aligned}\quad\text{(B-19)}$$

and

$$\pi^N = \frac{1+\alpha}{4}\left(r^N-w^N\right)\left(1-b_{LI}r^N\right)+\frac{1}{4}\left(r^N-w^N\right)\left(1-\delta_I b_{LI}r^N\right).\quad\text{(B-20)}$$

In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^N and r^N, respectively,

$$\begin{aligned}\max_{p^N>0}\Pi^N\left(p^N|w^N\right) &= \frac{1-\alpha}{4}e^N\left(1-b_{LD}e^N\right)+\frac{1}{4}p^N\left(1-\delta_D b_{LD}p^N\right)\\&+\frac{1+\alpha}{4}w^N\left(1-b_{LI}r^N\right)+\frac{1}{4}w^N\left(1-\delta_I b_{LI}r^N\right),\end{aligned}\quad\text{(B-21)}$$

and

$$\max_{r^N>0}\pi^N\left(r^N|w^N\right) = \frac{1+\alpha}{4}\left(r^N-w^N\right)\left(1-b_{LI}r^N\right)+\frac{1}{4}\left(r^N-w^N\right)\left(1-\delta_I b_{LI}r^N\right).\quad\text{(B-22)}$$

Solving the above maximization problems yields the optimal retail price response function $r^N\left(w^N\right) = \frac{w^N}{2}+\frac{2+\alpha}{2(1+\delta_I+\alpha)b_{LI}}$ and the equilibrium direct channel price $p^{N*} = \frac{2-\alpha}{2(1+\delta_D-\alpha)b_{LD}}$. The equilibrium effective price is $e^{N*} = p^{N*} = \frac{2-\alpha}{2(1+\delta_D-\alpha)b_{LD}}$.

In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^N while taking into account both players' Stage 3 decisions:

$$\begin{aligned}\max_{w^N>0}\Pi^N\left(w^N\right) &= \frac{1-\alpha}{4}e^{N*}\left(1-b_{LD}e^{N*}\right)+\frac{1}{4}p^{N*}\left(1-\delta_D b_{LD}p^{N*}\right)\\&+\frac{1+\alpha}{4}w^N\left(1-b_{LI}r^N\left(w^N\right)\right)+\frac{1}{4}w^N\left(1-\delta_I b_{LI}r^N\left(w^N\right)\right).\end{aligned}\quad\text{(B-23)}$$

Solving the manufacturer's profit maximization problem, we obtain the equilibrium wholesale price $w^{N*} = \frac{2+\alpha}{2(1+\delta_I+\alpha)b_{LI}}$. It follows that the optimal retail price is $r^{N*} = r^N\left(w^{N*}\right) = \frac{3(2+\alpha)}{4(1+\delta_I+\alpha)b_{LI}}$. The equilibrium profits of the retailer and manufacturer are $\pi^{N*} = \pi^N\left(r^{N*}\right) =$

$\frac{(2+\alpha)^2}{64(1+\delta_I+\alpha)b_{LI}}$ and $\Pi^{N*} = \Pi^N\left(p^{N*}, w^{N*}\right) = \frac{(2+\alpha)^2}{32(1+\delta_I+\alpha)b_{LI}} + \frac{(2-\alpha)^2}{16(1+\delta_D-\alpha)b_{LD}}$, respectively. To ensure that $e^{N*} > r^{N*}$, we have $b_{LI} > \frac{3(2+\alpha)(1+\delta_D-\alpha)b_{LD}}{2(2-\alpha)(1+\delta_I+\alpha)} \equiv b_1$.

To ensure that there is no non-local deviation, we consider the case when the manufacturer unilaterally lowers its direct channel price to slightly lower than the retail price, $p^{N*} = r^{N*} - \varepsilon$. In this way, the manufacturer is able to attract LPG-sensitive comparison shoppers to the direct channel. The manufacturer's equilibrium profit in this case is

$$\begin{aligned}\Pi_2^{N*} &= \frac{1+\alpha}{4}\left(r^{N*}-\varepsilon\right)\left(1-b_{LD}\left(r^{N*}-\varepsilon\right)\right)+\frac{1}{4}\left(r^{N*}-\varepsilon\right)\left(1-\delta_D b_{LD}\left(r^{N*}-\varepsilon\right)\right)\\ &\quad+\frac{1-\alpha}{4}w^{N*}\left(1-b_{LI}r^{N*}\right)+\frac{1}{4}w^{N*}\left(1-\delta_I b_{LI}r^{N*}\right), \qquad \text{(B-24)}\end{aligned}$$

where $w^{N*} = \frac{2+\alpha}{2(1+\delta_I+\alpha)b_{LI}}$ and $r^{N*} = \frac{3(2+\alpha)}{4(1+\delta_I+\alpha)b_{LI}}$. Thus, we have

$$\Pi_2^{N*} = \frac{(2+\alpha)\left(2b_{LI}\left(5\alpha(5+\alpha)+(14-\alpha)\delta_I+14\right)-9b_{LD}(2+\alpha)\left(1+\delta_D+\alpha\right)\right)}{64b_{LI}^2\left(1+\delta_I+\alpha\right)^2}. \qquad \text{(B-25)}$$

Comparing Π^{N*} and Π_2^{N*}, we have

$$\Pi^{N*} - \Pi_2^{N*}|_{b_{LI}=b_2} < 0 \qquad \text{(B-26)}$$

and

$$\Pi^{N*} - \Pi_2^{N*}|_{b_{LI}\to\infty} > 0. \qquad \text{(B-27)}$$

Since Π^{N*} and Π_2^{N*} are both decreasing functions of b_{LI}, there exists a unique solution to $\Pi^{N*}(b_{LI}) = \Pi_2^{N*}(b_{LI})$, which is given by

$$\begin{aligned}b_2 &= \arg\left\{\Pi^{N*}(b_{LI}) = \Pi_2^{N*}(b_{LI})\right\}\\ &\equiv \frac{(2+\alpha)\begin{pmatrix}\sqrt{\alpha\left(1+\delta_D-\alpha\right)\Delta}\\ +\left(1+\delta_D-\alpha\right)\left(2\alpha^2+\alpha\left(11-\delta_I\right)+6\left(1+\delta_I\right)\right)\end{pmatrix}b_{LD}}{2(2-\alpha)^2\left(1+\delta_I+\alpha\right)^2}, \qquad \text{(B-28)}\end{aligned}$$

$(b_2 > b_1)$, where $\Delta \equiv 13\alpha^4 + (31+14\delta_I+5\delta_D)\alpha^3 + 2\left(5\delta_I^2 - 31\delta_D - 15\delta_I + 11\delta_I\delta_D + 28\right)\alpha^2 + 4\left(16\delta_I - 43\delta_D - 10\delta_I^2 - 14\delta_I\delta_D + 2\delta_I^2\delta_D - 1\right)\alpha + 24\left(2\delta_I^2 + \delta_I - 5\delta_I\delta_D - \delta_I^2\delta_D - 4\delta_D - 1\right)$. When $b_1 < b_{LI} < b_2$, the manufacturer is better off by unilaterally lowering its direct channel price in order to uncut the retailer. Hence, the above equilibrium cannot be sustained.

Otherwise, when $b_{LI} \geq b_2$, the manufacturer has no incentives to deviate from the current equilibrium.

We next consider the case when the retailer unilaterally raises its retail price to slightly higher than the direct channel price, $r^{N*} = p^{N*} + \varepsilon$. In this way, the retailer drives LPG-sensitive comparison shoppers to the direct channel. The retailer's equilibrium profit in this case is

$$\begin{aligned} \pi_2^{N*} &= \frac{1-\alpha}{4}\left(\left(p^{N*}+\varepsilon\right)-w^{N*}\right)\left(1-b_{LI}\left(p^{N*}+\varepsilon\right)\right) \\ &\quad +\frac{1}{4}\left(\left(p^{N*}+\varepsilon\right)-w^{N*}\right)\left(1-\delta_I b_{LI}\left(p^{N*}+\varepsilon\right)\right), \end{aligned} \tag{B-29}$$

where $w^{N*} = \frac{2+\alpha}{2(1+\delta_I+\alpha)b_{LI}}$ and $p^{N*} = \frac{2-\alpha}{2(1+\delta_D-\alpha)b_{LD}}$, as above. Thus, we have

$$\pi_2^{N*} = \frac{(2-\alpha)\begin{pmatrix} 2b_{LD}\left(1+\delta_D-\alpha\right) \\ -b_{LI}\left(1+\delta_I-\alpha\right) \end{pmatrix}\left(b_{LI}(2-\alpha)\left(1+\delta_I+\alpha\right)-b_{LD}(2+\alpha)\left(1+\delta_D-\alpha\right)\right)}{16b_{LD}^2 b_{LI}\left(1+\delta_D-\alpha\right)^2\left(1+\delta_I+\alpha\right)}. \tag{B-30}$$

Comparing π^{N*} and π_2^{N*}, we have

$$\pi^{N*} - \pi_2^{N*} > 0. \tag{B-31}$$

This indicates that the retailer does not have incentives to unilaterally raises its retail price in order to push LPG-sensitive comparison shoppers to the direct channel. To sum up, the above equilibrium outcome in (II) is stable when $b_{LI} \geq b_2$.

Comparing three threshold values, we find the following relationship holds that $b_1 < b_3 < b_2$ ($\delta_I < \frac{4-(2-\delta_D)\alpha^2}{4\delta_D-\alpha^2}$ implies $b_1 < b_3$). After excluding the region with unstable equilibrium $b_1 < b_{LI} < b_2$, the above equilibrium outcomes in both (I) and (II) are stable when $b_{LI} \leq b_1$ and $b_{LI} \geq b_2$. Specifically, when $b_{LI} \leq b_1$, the equilibrium in (I) prevails. When $b_{LI} \geq b_2$, the equilibrium in (II) prevails.

Thus far, we have focused on the parameter range where $\delta_I < \frac{4-(2-\delta_D)\alpha^2}{4\delta_D-\alpha^2}$. This allows us to restrict our attention on the case of $b_1 < b_3$ only. Next, we briefly discuss the market outcome when $\delta_I > \frac{4-(2-\delta_D)\alpha^2}{4\delta_D-\alpha^2}$, or equivalently, $b_3 < b_1$. In this case, we can follow the same procedure to solve the model by backward induction. When $b_{LI} < b_3$, we have a stable equilibrium where $e^{N*} < r^{N*}$ and LPG-sensitive comparison shoppers purchase from the direct channel. When $b_{LI} \geq b_2$, we have a stable equilibrium where $e^{N*} > r^{N*}$ and LPG-sensitive comparison shoppers purchase from the indirect channel. Otherwise, when $b_3 < b_{LI} \leq b_1$, we have a stable equilibrium where $e^{N*} = r^{N*}$ and LPG-sensitive comparison shoppers follow a mixed shopping strategy,

$$\beta^{N*} = \frac{\begin{array}{c}(3(1-\delta_D+2\alpha)b_{LD}+2(1-\delta_I-2\alpha)b_{LI})+\\ \sqrt{(3(1-\delta_D)b_{LD}+2(1-\delta_I)b_{LI})^2+8(3b_{LD}-2b_{LI})(3(1+\delta_D)b_{LD}-2(1+\delta_I)b_{LI})}\end{array}}{4\alpha(3b_{LD}-2b_{LI})}. \quad \text{(B-32)}$$

Solving for the market equilibrium, we find that all the qualitative results in all lemmas and propositions still hold within this parameter range. ■

Low-Price Guarantee

Lemma 2 *When the manufacturer offers LPG, it is invoked when*

$$b_{LI} < \frac{\delta_D b_{LD}(7-2\alpha+2(1+\alpha)\mu b_{LD})-(1+\alpha)b_{LD}}{2(1+\delta_I-\alpha)}. \quad \text{(B-33)}$$

The wholesale price as a function of refund depth μ is given by

$$w^G(\mu) = \frac{7-2\alpha+2(1+\alpha)\mu b_{LD}}{2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}} \quad \text{(B-34)}$$

The retail price as a function of refund depth μ is given by

$$r^G(\mu) = \frac{7-2\alpha+2(1+\alpha)\mu b_{LD}}{4(1+\delta_I-\alpha)b_{LI}+2(1+\alpha)b_{LD}} \quad \text{(B-35)}$$

The direct channel price as a function of refund depth μ is given by

$$p^G(\mu) = \frac{1}{2\delta_D b_{LD}}. \quad \text{(B-36)}$$

The retail profit as a function of refund depth μ is given by

$$\pi^G(\mu) = \frac{((1+\delta_I-\alpha)b_{LI}(2b_{LD}(1+\alpha)\mu+2\alpha-1)-2(1+\alpha)(2-\alpha)b_{LD})^2}{16(1+\delta_I-\alpha)b_{LI}(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)b_{LD})^2}. \quad \text{(B-37)}$$

The manufacturer profit as a function of refund depth μ is given by

$$\Pi^G(\mu) = \frac{\begin{array}{c} (1+\delta_I-\alpha)\,b_{LI}\left((1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD}\right) \\ +8\,(1+\alpha)\,\delta_D\delta_I^2 b_{LD}b_{LI}^2\mu\,(1+\mu b_{LD}) \\ +4\delta_D\,(1+\alpha)\,b_{LD}^2\left(2-\alpha-(1-\alpha)\,b_{LI}\mu\right) \\ \times\left(2-\alpha-2\,(1-\alpha)\,b_{LI}\mu\right) \\ -\delta_D\,(1-\alpha)\,b_{LI}b_{LD}\left(\begin{array}{c} 17+4\alpha\,(1-\alpha)-8\delta_D\,(1-\alpha)^2\,b_{LI}b_{LD}\mu \\ -\delta_I b_{LI}\left(\begin{array}{c} 17+4\alpha\,(1-\alpha) \\ +4\,(1+\alpha)\left(\begin{array}{c} 3\,(2-\alpha)\,b_{LD} \\ -4b_{LI}\,(1-\alpha)\,(1+\mu b_{LD}) \end{array}\right) \end{array}\right) \end{array}\right) \end{array}}{16\delta_D b_{LD}\,(1+\delta_I-\alpha)\,b_{LI}\left(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD}\right)}. \quad \text{(B-38)}$$

When $0 < b_{LI} < \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, LPG is not invoked. Otherwise, when LPG is invoked, i.e., $b_{LI} > \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, the optimal refund depth is given by

$$\mu^* = \begin{cases} \frac{3(2-\alpha)b_{LD}-2(1+\delta_I-\alpha)b_{LI}}{4b_{LD}(1+\delta_I-\alpha)b_{LI}} & \text{if } \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \le b_1 \\ 0 & \text{if } b_{LI} \ge b_2 \end{cases}. \quad \text{(B-39)}$$

When the refund depth is chosen at the optimal level $\mu = \mu^*$, the equilibrium wholesale price is given by

$$w^{G*} = \begin{cases} \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}} & \text{if } \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \le b_1 \\ \frac{7}{2(1+\delta_I)b_{LI}+b_{LD}} - \frac{1}{(1+\delta_I)b_{LI}} & \text{if } b_{LI} \ge b_2 \end{cases}. \quad \text{(B-40)}$$

When the refund depth is chosen at the optimal level $\mu = \mu^*$, the equilibrium retail price is given by

$$r^{G*} = \begin{cases} \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}} & \text{if } \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \le b_1 \\ \frac{7}{4(1+\delta_I)b_{LI}+2b_{LD}} & \text{if } b_{LI} \ge b_2 \end{cases}. \quad \text{(B-41)}$$

When the refund depth is chosen at the optimal level $\mu = \mu^*$, the equilibrium direct channel price is given by

$$p^{G*} = \frac{1}{2\delta_D b_{LD}}. \quad \text{(B-42)}$$

When the refund depth is chosen at the optimal level $\mu = \mu^*$, the equilibrium retail profit is given by

$$\pi^{G*} = \begin{cases} \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}} & \textit{if } \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_1 \\ \frac{((1+\delta_I)b_{LI}+4b_{LD})^2}{16(1+\delta_I)b_{LI}(2(1+\delta_I)b_{LI}+b_{LD})^2} & \textit{if } b_{LI} \geq b_2 \end{cases}. \tag{B-43}$$

When the refund depth is chosen at the optimal level $\mu = \mu^*$, the equilibrium manufacturer profit is given by

$$\Pi^{G*} = \begin{cases} \frac{1+\delta_D+\delta_D\alpha}{16\delta_D b_{LD}} + \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}} & \textit{if } \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_1 \\ \frac{((1+\delta_I)b_{LI}-16\delta_D b_{LD})(2(1+\delta_I)b_{LI}+b_{LD})+49\delta_D(1+\delta_I)b_{LD}b_{LI}}{16\delta_D(1+\delta_I)b_{LD}b_{LI}(2(1+\delta_I)b_{LI}+b_{LD})} & \textit{if } b_{LI} \geq b_2 \end{cases}. \tag{B-44}$$

When the refund depth is chosen at the optimal level $\mu = \mu^*$, the equilibrium channel profit is given by

$$\Pi_C^{G*} = \begin{cases} \frac{1+\delta_D+\delta_D\alpha}{16\delta_D b_{LD}} + \frac{3(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}} & \textit{if } \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_1 \\ \frac{1}{16\delta_D b_{LD}} + \frac{49}{16(2(1+\delta_I)b_{LI}+b_{LD})} - \frac{1}{(1+\delta_I)b_{LI}} & \\ +\frac{((1+\delta_I)b_{LI}+4b_{LD})^2}{4(1+\delta_I)b_{LI}(2(1+\delta_I)b_{LI}+b_{LD})^2} & \textit{if } b_{LI} \geq b_2 \end{cases}. \tag{B-45}$$

Proof of Lemma 2

Proof. In this proof, we establish the rational expectations equilibrium, i.e., the manufacturer and retailer simultaneously set the direct channel price and retail price, with the expectations that LPG will be invoked in equilibrium. In other words, in a rational expectations equilibrium, the firms' expectations about the consumers' invoking LPG are fulfilled.

(I) When The Manufacturer Adopts PBG ($\mu > 0$)

Suppose that the manufacturer adopts PBG ($\mu > 0$). If firms' expectations are rational, the LPG-sensitive direct channel consumers will invoke LPG in Stage 4, $e^G = r^G - \mu$. This directly implies that $e^G < r^G$. As a result, LPG-sensitive comparison shoppers all purchase from the direct channel, i.e., $\beta^{G*} = 1$. The profit functions of the manufacturer and retailer can be rewritten as:

$$\begin{aligned} \Pi^G &= \frac{1+\alpha}{4} e^G \left(1 - b_{LD} e^G\right) + \frac{1}{4} p^G \left(1 - \delta_D b_{LD} p^G\right) \\ &\quad + \frac{1-\alpha}{4} w^G \left(1 - b_{LI} r^G\right) + \frac{1}{4} w^G \left(1 - \delta_I b_{LI} r^G\right), \end{aligned} \tag{B-46}$$

and

$$\pi^G = \frac{1-\alpha}{4}\left(r^G - w^G\right)\left(1 - b_{LI} r^G\right) + \frac{1}{4}\left(r^G - w^G\right)\left(1 - \delta_I b_{LI} r^G\right), \tag{B-47}$$

respectively. In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^G and r^G, respectively:

$$\begin{aligned}\max_{p^G>0} \Pi^G\left(p^G|w^G,\mu\right) &= \frac{1+\alpha}{4}e^G\left(1 - b_{LD}e^G\right) + \frac{1}{4}p^G\left(1 - \delta_D b_{LD} p^G\right) \\ &\quad + \frac{1-\alpha}{4}w^G\left(1 - b_{LI} r^G\right) + \frac{1}{4}w^G\left(1 - \delta_I b_{LI} r^G\right),\end{aligned} \tag{B-48}$$

and

$$\max_{r^G>0} \pi^G\left(r^G|w^G,\mu\right) = \frac{1-\alpha}{4}\left(r^G - w^G\right)\left(1 - b_{LI} r^G\right) + \frac{1}{4}\left(r^G - w^G\right)\left(1 - \delta_I b_{LI} r^G\right). \tag{B-49}$$

The above profit maximization problems yield the equilibrium direct channel price $p^{G*} = \frac{1}{2\delta_D b_{LD}}$ and the optimal retail price response function $r^G\left(w^G\right) = \frac{w^G}{2} + \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. As a result, the effective price e^G is given by $e^G\left(w^G\right) = \frac{w^G}{2} + \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}} - \mu$. To ensure that firms' belief on consumer behavior is consistent, we have the following condition:

$$r^G\left(w^G\right) < p^{G*}. \tag{B-50}$$

It follows that $\frac{w^G}{2} + \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}} < \frac{1}{2\delta_D b_{LD}}$. Rearranging the terms, we have $w^G < \frac{1}{\delta_D b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}}$. For the rational expectations equilibrium to exist, the following condition must hold: $w^G < \frac{1}{\delta_D b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}}$. Given that $w^G < \frac{1}{\delta_D b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}}$, the manufacturer maximizes its profit by choosing the wholesale price w^G in Stage 2, while taking into account both players' stage 3 decisions:

$$\begin{aligned}\max_{w^G>0} \Pi^G(w^G|\mu) &= \frac{1+\alpha}{4}e^G\left(w^G\right)\left(1 - b_{LD}e^G\left(w^G\right)\right) + \frac{1}{4}p^G\left(1 - \delta_D b_{LD} p^G\right) \\ &\quad + \frac{1-\alpha}{4}w^G\left(1 - b_{LI} r^G\left(w^G\right)\right) + \frac{1}{4}w^G\left(1 - \delta_I b_{LI} r^G\left(w^G\right)\right).\end{aligned} \tag{B-51}$$

The wholesale price as a function of refund depth μ is given by $w^G(\mu) = \frac{7-2\alpha+2(1+\alpha)\mu b_{LD}}{2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}}$. As a result, the retail price as a function of refund depth μ is given by $r^G(\mu) = r^G(w^G(\mu), \mu) = \frac{7-2\alpha+2(1+\alpha)\mu b_{LD}}{4(1+\delta_I-\alpha)b_{LI}+2(1+\alpha)b_{LD}}$. The manufacturer profit as a function of refund depth μ is given by

$$\Pi^G(\mu) = \frac{\begin{array}{c}(1+\delta_I-\alpha)\, b_{LI}\,(2\,(1+\delta_I-\alpha)\, b_{LI}+(1+\alpha)\, b_{LD}) \\ +\delta_D\,(1+\delta_I-\alpha)\, b_{LD} b_{LI}\,(17+4\alpha\,(1-\alpha)-8b_{LI}\,(1+\alpha)\,(1+\delta_I-\alpha)\,\mu) \\ -2\,(1+\alpha)\, b_{LD}\,(2-\alpha-(1+\delta_I-\alpha)\, b_{LI}\mu)\,(2-\alpha-2\,(1+\delta_I-\alpha)\, b_{LI}\mu)\end{array}}{16\delta_D\,(1+\delta_I-\alpha)\, b_{LI} b_{LD}\,(2\,(1+\delta_I-\alpha)\, b_{LI}+(1+\alpha)\, b_{LD})}. \tag{B-52}$$

The retail profit as a function of refund depth μ is given by

$$\pi^G(\mu) = \frac{((1-2\alpha)\,(1+\delta_I-\alpha)\, b_{LI}+2\,(1+\alpha)\,(2-\alpha)\, b_{LD}-2\,(1+\alpha)\, b_{LD}\mu)^2}{16\,(1+\delta_I-\alpha)\, b_{LI}\,(2\,(1+\delta_I-\alpha)\, b_{LI}+(1+\alpha)\, b_{LD})^2}. \tag{B-53}$$

In Stage 1, the manufacturer chooses the refund depth μ to maximize its profit:

$$\max_{\mu\geq 0} \Pi^G(\mu) = \frac{\begin{array}{c}(1+\delta_I-\alpha)\, b_{LI}\,(2\,(1+\delta_I-\alpha)\, b_{LI}+(1+\alpha)\, b_{LD}) \\ +\delta_D\,(1+\delta_I-\alpha)\, b_{LD} b_{LI}\,(17+4\alpha\,(1-\alpha)-8b_{LI}\,(1+\alpha)\,(1+\delta_I-\alpha)\,\mu) \\ -2\,(1+\alpha)\, b_{LD}\,(2-\alpha-(1+\delta_I-\alpha)\, b_{LI}\mu)\,(2-\alpha-2\,(1+\delta_I-\alpha)\, b_{LI}\mu)\end{array}}{16\delta_D\,(1+\delta_I-\alpha)\, b_{LI} b_{LD}\,(2\,(1+\delta_I-\alpha)\, b_{LI}+(1+\alpha)\, b_{LD})}. \tag{B-54}$$

The first-order condition of Π^G with respect to μ is given by

$$\frac{\partial \Pi^G}{\partial \mu} = \frac{(1+\alpha)\,(3\,(2-\alpha)\, b_{LD}-2\,(1+\delta_I-\alpha)\, b_{LI}\,(1+2b_{LD}\mu))}{8\delta_D\,(1+\delta_I-\alpha)\, b_{LI} b_{LD}\,(2\,(1+\delta_I-\alpha)\, b_{LI}+(1+\alpha)\, b_{LD})}. \tag{B-55}$$

It is easy to see that the unique solution that satisfy $\frac{\partial \Pi^G}{\partial \mu} = 0$ is $\overline{\mu} = \frac{3(2-\alpha)b_{LD}-2(1+\delta_I-\alpha)b_{LI}}{4(1+\delta_I-\alpha)b_{LD}b_{LI}}$. On evaluating the second-order condition we have

$$\frac{\partial^2 \Pi^G}{\partial \mu^2}|_{\mu=\overline{\mu}} = -\frac{1+\alpha}{2\delta\,(2\,(1+\delta_I-\alpha)\, b_{LI}+(1+\alpha)\, b_{LD})} < 0 \tag{B-56}$$

when $\overline{\mu} > 0$. Thus, there exists a unique optimal refund depth μ^* that maximizes Π^G:

$$\mu^* = \frac{3\,(2-\alpha)\, b_{LD}-2\,(1+\delta_I-\alpha)\, b_{LI}}{4\,(1+\delta_I-\alpha)\, b_{LD} b_{LI}}. \tag{B-57}$$

To ensure that $\mu^* > 0$, we need to have $b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. In this case, the market equilibrium is given by $w^{G*} = w^G(\mu^* = \mu_1) = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$, $r^{G*} = r^G(\mu^* = \mu_1) = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$, $p^{G*} = \frac{1}{2\delta_D b_{LD}}$,

$$\Pi^{G*} = \Pi^G(\mu^* = \mu_1) = \frac{1+\delta_D+\delta_D\alpha}{16\delta_D b_{LD}} + \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}}, \tag{B-58}$$

and $\pi^{G*} = \pi^G(\mu^* = \mu_1) = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$. In this case, LPG is effectively a price-beating guarantee (PBG). This equilibrium exists only if $w^{G*} < \frac{1}{\delta_D b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}}$, which holds when $b_{LI} > \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. To summarize, the PBG equilibrium exists when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$.

Otherwise if $w^G \geq \frac{1}{\delta_D b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}}$, firms form the belief that LPG will not be invoked ($r^{G*} \geq p^{G*}$) when making stage 3 decisions. In this case, the profit functions of the manufacturer and retailer are given by

$$\begin{aligned}\Pi^G &= \frac{1+\alpha}{4}e^G\left(1-b_{LD}e^G\right) + \frac{1}{4}p^G\left(1-\delta_D b_{LD}p^G\right) \\ &\quad + \frac{1-\alpha}{4}w^G\left(1-b_{LI}r^G\right) + \frac{1}{4}w^G\left(1-\delta_I b_{LI}r^G\right),\end{aligned} \tag{B-59}$$

and

$$\pi^G = \frac{1-\alpha}{4}\left(r^G - w^G\right)\left(1-b_{LI}r^G\right) + \frac{1}{4}\left(r^G - w^G\right)\left(1-\delta_I b_{LI}r^G\right). \tag{B-60}$$

One can see that this is the same as the no LPG case. Following the Proof of Lemma 1, we have the market equilibrium as follows: $p^{G*} = \frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$, $w^{G*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$, $r^{G*} = r^G\left(w^{G*}\right) = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$, $\pi^{G*} = \pi^G\left(r^{G*}\right) = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$ and $\Pi^{G*} = \Pi^G\left(p^{G*}, w^{G*}\right) = \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}} + \frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}$, respectively. This belief is rational when $w^{G*} \geq \frac{1}{\delta_D b_{LD}} - \frac{2-\alpha}{(1+\delta_I-\alpha)b_{LI}}$ and $r^{G*} > p^{G*}$. These two conditions can be rewritten as $b_{LI} \leq \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$ and $b_{LI} \leq \frac{3(2-\alpha)(1+\delta_D+\alpha)b_{LD}}{2(2+\alpha)(1+\delta_I-\alpha)}$. Since $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < \frac{3(2-\alpha)(1+\delta_D+\alpha)b_{LD}}{2(2+\alpha)(1+\delta_I-\alpha)}$, these two conditions can be combined into $b_{LI} \leq \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. To sum up,

i) When $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, the equilibrium refund depth is given by $\mu^* = \frac{3(2-\alpha)b_{LD}-2(1+\delta_I-\alpha)b_{LI}}{4(1+\delta_I-\alpha)b_{LD}b_{LI}}$ and the equilibrium wholesale price is given by $w^{G*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. With this wholesale price, both firms form the rational expectations that LPG (specifically, PBG) is invoked in equilibrium. The equilibrium direct channel price and retail price are given by $p^{G*} = \frac{1}{2\delta_D b_{LD}}$ and $r^{G*} = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$. Both firms' equilibrium profits are given by $\pi^{G*} = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$ and $\Pi^{G*} = \frac{1}{32}\left(\frac{2}{\delta_D b_{LD}} + \frac{2(1+\alpha)}{b_{LD}} + \frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}\right)$. The above rational expectations equilibrium holds when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$.

ii) When $b_{LI} \leq \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, the equilibrium wholesale price is given by $w^{G*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. With this wholesale price, both firms form the rational expectations that LPG is not invoked in equilibrium. The equilibrium direct channel price and retail price are given by $p^{G*} = \frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$ and $r^{G*} = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$. Both firms' equilibrium profits are given by $\pi^{G*} = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$ and $\Pi^{G*} = \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}} + \frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}$. The above rational expectations equilibrium holds when $b_{LI} \leq \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. If LPG in not invoked in equilibrium in this case, there is no incentive for the manufacturer to offer LPG in the first place because LPG brings in no benefit.

(II) When The Manufacturer Adopts PMG ($\mu = 0$)

Suppose that the manufacturer adopts PMG ($\mu = 0$). If firms' expectations are rational, the LPG-sensitive direct channel consumers will invoke LPG in Stage 4, $e^G = r^G - \mu$. This directly implies that $e^G = r^G$. As a result, both channels evenly split the LPG-sensitive comparison shopper segment, i.e., $\beta^{G*} = \frac{1}{2}$. The profit functions of the manufacturer and retailer can be rewritten as:

$$\begin{aligned}\Pi^G &= \frac{1}{4}e^G\left(1 - b_{LD}e^G\right) + \frac{1}{4}p^G\left(1 - \delta_D b_{LD}p^G\right) \\ &\quad + \frac{1}{4}w^G\left(1 - b_{LI}r^G\right) + \frac{1}{4}w^G\left(1 - \delta_I b_{LI}r^G\right), \end{aligned} \tag{B-61}$$

and

$$\pi^G = \frac{1}{4}\left(r^G - w^G\right)\left(1 - b_{LI}r^G\right) + \frac{1}{4}\left(r^G - w^G\right)\left(1 - \delta_I b_{LI}r^G\right). \tag{B-62}$$

In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^G and r^G, respectively:

$$\begin{aligned}\max_{p^G>0} \Pi^G\left(p^G|w^G,\mu\right) &= \frac{1}{4}e^G\left(1 - b_{LD}e^G\right) + \frac{1}{4}p^G\left(1 - \delta_D b_{LD}p^G\right) \\ &\quad + \frac{1}{4}w^G\left(1 - b_{LI}r^G\right) + \frac{1}{4}w^G\left(1 - \delta_I b_{LI}r^G\right), \end{aligned} \tag{B-63}$$

and

$$\max_{r^G>0} \pi^G\left(r^G|w^G,\mu\right) = \frac{1}{4}\left(r^G - w^G\right)\left(1 - b_{LI}r^G\right) + \frac{1}{4}\left(r^G - w^G\right)\left(1 - \delta_I b_{LI} r^G\right). \tag{B-64}$$

The above profit maximization problems yield the equilibrium direct channel price $p^{G*} = \frac{1}{2\delta_D b_{LD}}$ and the optimal retail price response function $r^G\left(w^G\right) = \frac{1}{(1+\delta_I)b_{LI}} + \frac{w^G}{2}$. It follows that the effective price e^G is given by $e^G\left(w^G\right) = r^G\left(w^G\right) - \mu = \frac{1}{(1+\delta_I)b_{LI}} + \frac{w^G}{2}$. To ensure that firms' belief on consumer behavior is consistent, we have the following condition:

$$r^G\left(w^G\right) < p^{G*}. \tag{B-65}$$

It follows that $\frac{1}{(1+\delta_I)b_{LI}} + \frac{w^G}{2} < \frac{1}{2\delta_D b_{LD}}$. Rearranging the terms, we have $w^G < \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$. For the rational expectations equilibrium to exist, the following condition must hold: $w^G < \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$. Given that $w^G < \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$, the manufacturer maximizes its profit by choosing the wholesale price w^G while taking into account both players' stage 3 decisions:

$$\begin{aligned}\max_{w^G>0} \Pi^G(w^G|\mu \quad = \quad 0) &= \frac{1}{4}e^G\left(w^G\right)\left(1 - b_{LD}e^G\left(w^G\right)\right) + \frac{1}{4}p^G\left(1 - \delta_D b_{LD} p^G\right) \\ &+ \frac{1}{4}w^G\left(1 - b_{LI}r^G\left(w^G\right)\right) + \frac{1}{4}w^G\left(1 - \delta_I b_{LI} r^G\left(w^G\right)\right). \end{aligned} \tag{B-66}$$

In this case, the market equilibrium is given by $w^{G*} = w^G\left(\mu^* = 0\right) = \frac{7}{2(1+\delta_I)b_{LI}+b_{LD}} - \frac{1}{(1+\delta_I)b_{LI}}$, $r^{G*} = r^G\left(\mu^* = 0\right) = \frac{7}{4(1+\delta_I)b_{LI}+2b_{LD}}$, $p^{G*} = \frac{1}{2\delta_D b_{LD}}$, $\Pi^{G*} = \Pi^G\left(\mu^* = 0\right) = \frac{1}{16\delta_D b_{LD}} + \frac{49}{16(2(1+\delta_I)b_{LI}+b_{LD})} - \frac{1}{(1+\delta_I)b_{LI}} = \frac{((1+\delta_I)b_{LI}-16\delta_D b_{LD})(2(1+\delta_I)b_{LI}+b_{LD})+49\delta_D(1+\delta_I)b_{LD}b_{LI}}{16\delta_D(1+\delta_I)b_{LD}b_{LI}((1+\delta_I)b_{LI}+b_{LD})}$, and

$$\pi^{G*} = \pi^G\left(\mu^* = 0\right) = \frac{\left(\left(1+\delta_I\right)b_{LI} + 4b_{LD}\right)^2}{16\left(1+\delta_I\right)b_{LI}\left(2\left(1+\delta_I\right)b_{LI} + b_{LD}\right)^2}.$$

This equilibrium exists only if $w^{G*} < \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$, which can be rewritten as

$$\frac{7}{2\left(1+\delta_I\right)b_{LI} + b_{LD}} + \frac{1}{\left(1+\delta_I\right)b_{LI}} - \frac{1}{\delta_D b_{LD}} < 0. \tag{B-67}$$

This inequality holds when $b_{LI} > \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. To summarize, the PMG equilibrium exists when $b_{LI} > \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$.

Otherwise if $w^G > \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$, firms form the belief that LPG will not be invoked ($r^{G*} \geq p^{G*}$) when making stage-three decisions. In this case, the profit functions of the manufacturer and retailer are given by

$$\begin{aligned}\Pi^G &= \frac{1+\alpha}{4}e^G\left(1-b_{LD}e^G\right)+\frac{1}{4}p^G\left(1-\delta_D b_{LD}p^G\right)\\ &\quad+\frac{1-\alpha}{4}w^G\left(1-b_{LI}r^G\right)+\frac{1}{4}w^G\left(1-\delta_I b_{LI}r^G\right),\end{aligned} \tag{B-68}$$

and

$$\pi^G = \frac{1-\alpha}{4}\left(r^G - w^G\right)\left(1-b_{LI}r^G\right)+\frac{1}{4}\left(r^G-w^G\right)\left(1-\delta_I b_{LI}r^G\right). \tag{B-69}$$

One can see that this is the same as the no LPG case. Following the Proof of Lemma 1, we have the market equilibrium as follows: $p^{G*} = \frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$, $w^{G*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$, $r^{G*} = r^G\left(w^{G*}\right) = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$, $\pi^{G*} = \pi^G\left(r^{G*}\right) = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$ and $\Pi^{G*} = \Pi^G\left(p^{G*}, w^{G*}\right) = \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}} + \frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}$, respectively.

For this belief to be rational, the following two conditions must hold that 1) the wholesale price is greater than the threshold, $w^{G*} \geq \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$, and 2) LPG is not invoked, i.e., $r^{G*} > p^{G*}$. These two conditions can be rewritten as $b_{LI} \leq \frac{(6-5\alpha+6\delta_I-\alpha\delta_I)\delta_D b_{LD}}{2\left(1-\alpha+2\delta_I-\alpha\delta_I+\delta_I^2\right)}$ and $b_{LI} \leq b_2$. These two conditions can be combined into

$$b_{LI} \leq \min\left\{\frac{\delta_D\left(6-5\alpha+6\delta_I-\alpha\delta_I\right)b_{LD}}{2\left(1-\alpha+2\delta_I-\alpha\delta_I+\delta_I^2\right)}, b_2\right\}. \tag{B-70}$$

Next, we compare the manufacturer's profit in the region $b_{LI} \leq \min\left\{\frac{\delta_D(6-5\alpha+6\delta_I-\alpha\delta_I)b_{LD}}{2\left(1-\alpha+2\delta_I-\alpha\delta_I+\delta_I^2\right)}, b_2\right\}$. As in the proof of Proposition **??**, the manufacturer's profit with the belief that LPG will be invoked ($w^{G*} < \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$) is greater than that with the belief that LPG will not be invoked ($w^{G*} \geq \frac{1}{\delta_D b_{LD}} - \frac{2}{(1+\delta_I)b_{LI}}$). To sum up,

i) When $b_{LI} > \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, the equilibrium refund depth is given by $\mu^* = 0$ and the equilibrium wholesale price is given by $w^{G*} = \frac{7}{2(1+\delta_I)b_{LI}+b_{LD}} - \frac{1}{(1+\delta_I)b_{LI}}$. With this wholesale price, both firms form the rational expectations that LPG (specifically, PMG) is invoked in equilibrium. The equilibrium direct channel price and retail price are given by $p^{G*} = \frac{1}{2\delta_D b_{LD}}$ and $r^{G*} = \frac{7}{4(1+\delta_I)b_{LI}+2b_{LD}}$. Both firms' equilibrium profits are given by $\pi^{G*} = \frac{((1+\delta_I)b_{LI}+4b_{LD})^2}{16(1+\delta_I)b_{LI}(2(1+\delta_I)b_{LI}+b_{LD})^2}$ and $\Pi^{G*} = \frac{1}{16\delta_D b_{LD}} + \frac{49}{16((1+\delta_I)b_{LI}+b_{LD})} - \frac{1}{(1+\delta_I)b_{LI}}$. The above rational expectations equilibrium holds when $b_{LI} > \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$.

ii) When $b_{LI} \leq \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, the equilibrium wholesale price is given by $w^{G*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. With this wholesale price, both firms form the rational expectations that LPG is not invoked in equilibrium. The equilibrium direct channel price and retail price are given by $p^{G*} = \frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$ and $r^{G*} = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$. Both firms' equilibrium profits are given by $\pi^{G*} = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$ and $\Pi^{G*} = \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}} + \frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}$. The above rational expectations equilibrium holds when $b_{LI} \leq \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$.

When PBG is feasible, it is easy to see that $\Pi^G(\mu^* > 0) > \Pi^G(\mu^* = 0)$, where $\Pi^G(\mu^* > 0)$ is the manufacturer profit when using PBG and $\Pi^G(\mu^* = 0)$ is the manufacturer profit when using PMG. This is because Π^G is increasing in μ^* in this region as in Proposition 2. This implies that the manufacturer always prefers PBG to PMG, as long as PBG is feasible. Thus, the manufacturer chooses PBG when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$ and PMG when $b_{LI} \geq \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. Given that we restrict our parameter to the regions where $b_{LI} \leq b_1$ and $b_{LI} \geq b_2$ and that the following relationships hold that $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_1 < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_2$, the market equilibrium can be characterized as:

(i) When $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_1$, the manufacturer chooses PBG; and

(ii) When $b_{LI} \geq b_2$, the manufacturer chooses PMG. ■

Two-Part Tariff and No LPG

Lemma 3 *When the manufacturer does not offer LPG, the channel equilibrium is characterized as follows. Let* $b_{T1} \equiv \frac{(2+\alpha)(1+\delta_D-\alpha)b_{LD}}{(2-\alpha)(1+\delta_I+\alpha)}$ *and*

$$b_{T2} \equiv \frac{(\alpha+2)\,b_{LD}\begin{pmatrix} b_{LD}\,(1+\delta_D-\alpha)\,(2+3\alpha+\alpha^2(1-\lambda)+(2+\alpha-2\alpha\lambda)\delta_I) \\ +\sqrt{\alpha\,(1+\delta_D-\alpha)\,\Delta_T} \end{pmatrix}}{(2-\alpha)^2\,(1+\delta_I+\alpha)^2},$$

($b_{T1} < b_{T2}$), where Δ_T is defined in the proof. The equilibrium shopping strategy of LPG-sensitive comparison shoppers is given by

$$\beta_T^{N*} = \begin{cases} 1 & \text{if } 0 < b_{LI} \leq b_{T1} \\ 0 & \text{if } b_{LI} \geq b_{T2} \end{cases}. \qquad \text{(B-71)}$$

The equilibrium wholesale price is given by

$$w_T^{N*} = 0. \tag{B-72}$$

The equilibrium retail price is given by

$$r_T^{N*} = \frac{2 + \left(1 - 2\beta_T^{N*}\right)\alpha}{2\left(1 + \delta_I + \left(1 - 2\beta_T^{N*}\right)\alpha\right) b_{LI}}. \tag{B-73}$$

The equilibrium direct channel price is given by

$$p_T^{N*} = \frac{2 - \left(1 - 2\beta_T^{N*}\right)\alpha}{2\left(1 + \delta_D - \left(1 - 2\beta_T^{N*}\right)\alpha\right) b_{LD}}. \tag{B-74}$$

The equilibrium retail profit is given by

$$\pi_T^{N*} = \frac{(1 - \lambda)\left(2 + \left(1 - 2\beta_T^{N*}\right)\alpha\right)^2}{16\left(1 + \delta_I + \left(1 - 2\beta_T^{N*}\right)\alpha\right) b_{LI}}. \tag{B-75}$$

The equilibrium manufacturer profit is given by

$$\Pi_T^{N*} = \frac{\lambda\left(2 + \left(1 - 2\beta_T^{N*}\right)\alpha\right)^2}{16\left(1 + \delta_I + \left(1 - 2\beta_T^{N*}\right)\alpha\right) b_{LI}} + \frac{\left(2 - \left(1 - 2\beta_T^{N*}\right)\alpha\right)^2}{16\left(1 + \delta_D - \left(1 - 2\beta_T^{N*}\right)\alpha\right) b_{LD}}. \tag{B-76}$$

The equilibrium channel profit is given by

$$\Pi_{TC}^{N*} = \frac{\left(2 + \left(1 - 2\beta_T^{N*}\right)\alpha\right)^2}{16\left(1 + \delta_I + \left(1 - 2\beta_T^{N*}\right)\alpha\right) b_{LI}} + \frac{\left(2 - \left(1 - 2\beta_T^{N*}\right)\alpha\right)^2}{16\left(1 + \delta_D - \left(1 - 2\beta_T^{N*}\right)\alpha\right) b_{LD}}. \tag{B-77}$$

Proof of Lemma 3

Proof. In the absence of LPG, LPG-sensitive consumers cannot take advantage of LPG and thus, the effective price they pay is the direct channel price $e_T^N = p_T^N$. Let β_T^N be the fraction of LPG-sensitive comparison shoppers who purchase from the direct channel in the no LPG case. The remaining $1 - \beta_T^N$ proportion of them will purchase from the indirect channel.

<u>(I) When $e_T^N < r_T^N$</u>

When $e_T^N < r_T^N$, all LPG-sensitive comparison shoppers will purchase from the direct channel ($\beta_T^{N*} = 1$). The profit functions of the manufacturer and retailer are given by

$$\begin{aligned} \Pi_T^N &= \frac{1+\alpha}{4} e_T^N \left(1 - b_{LD} e_T^N\right) + \frac{1}{4} p_T^N \left(1 - \delta_D b_{LD} p_T^N\right) \\ &+ \frac{1-\alpha}{4} w_T^N \left(1 - b_{LI} r_T^N\right) + \frac{1}{4} w_T^N \left(1 - \delta_I b_{LI} r_T^N\right) + \lambda F^N, \end{aligned} \tag{B-78}$$

and

$$\pi_T^N = \frac{1-\alpha}{4} \left(r_T^N - w_T^N\right) \left(1 - b_{LI} r_T^N\right) + \frac{1}{4} \left(r_T^N - w_T^N\right) \left(1 - \delta_I b_{LI} r_T^N\right) - \lambda F^N. \tag{B-79}$$

In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^N and r^N, respectively,

$$\begin{aligned} \max_{p_T^N > 0} \Pi_T^N \left(p_T^N | w_T^N\right) &= \frac{1+\alpha}{4} e_T^N \left(1 - b_{LD} e_T^N\right) + \frac{1}{4} p_T^N \left(1 - \delta_D b_{LD} p_T^N\right) \\ &+ \frac{1-\alpha}{4} w_T^N \left(1 - b_{LI} r_T^N\right) + \frac{1}{4} w_T^N \left(1 - \delta_I b_{LI} r_T^N\right) + \lambda F^N, \end{aligned} \tag{B-80}$$

and

$$\max_{r_T^N > 0} \pi_T^N \left(r_T^N | w_T^N\right) = \frac{1-\alpha}{4} \left(r_T^N - w_T^N\right) \left(1 - b_{LI} r_T^N\right) + \frac{1}{2} \left(r_T^N - w_T^N\right) \left(1 - \delta_I b_{LI} r_T^N\right) - \lambda F^N. \tag{B-81}$$

Solving the above maximization problems yields the optimal retail price response function $r_T^N \left(w_T^N\right) = \frac{w_T^N}{2} + \frac{2-\alpha}{2(1+\delta_I - \alpha) b_{LI}}$ and the equilibrium direct channel price $p_T^{N*} = \frac{2+\alpha}{2(1+\delta_D+\alpha) b_{LD}}$. The equilibrium effective price is $e_T^{N*} = p_T^{N*} = \frac{2+\alpha}{2(1+\delta_D+\alpha) b_{LD}}$. Replace r_T^N with $r_T^N \left(w_T^N\right)$ in $\pi_T^N \left(r_T^N | w_T^N\right)$ and we find that $\frac{\partial \pi_T^N}{\partial w_T^N} < 0$. It implies that w_T^N should be set as low as possible to maximize π_T^N.

In Stage 2, the manufacturer maximizes its profit by choosing $\left\{w_T^N, F^N\right\}$ while taking into account both players' stage 3 decisions:

$$\begin{aligned}\max_{F^N>0, w_T^N\geq 0} \Pi_T^N\left(w_T^N, F^N\right) &= \frac{1+\alpha}{4} e_T^{N*}\left(1-b_{LD} e_T^{N*}\right)+\frac{1}{4} p_T^{N*}\left(1-\delta_D b_{LD} p_T^{N*}\right) \\ &+\frac{1-\alpha}{4} w_T^N\left(1-b_{LI} r_T^N\left(w_T^N\right)\right) \\ &+\frac{1}{4} w_T^N\left(1-\delta_I b_{LI} r_T^N\left(w_T^N\right)\right)+\lambda F^N .\end{aligned} \tag{B-82}$$

Solving the manufacturer's profit maximization problem, we obtain the equilibrium wholesale price $w_T^{N*}=0$ and $F^{N*}=\frac{(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}}$. It follows that the optimal retail price is $r_T^{N*}=r_T^N\left(w_T^{N*}\right)=\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. The equilibrium profits of the retailer and manufacturer are $\pi_T^{N*}=\pi_T^N\left(r_T^{N*}, F^{N*}\right)=\frac{(1-\lambda)(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}}$ and $\Pi_T^{N*}=\Pi_T^N\left(p_T^{N*}, w_T^{N*}, F^{N*}\right)=\frac{\lambda(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}}+\frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}$, respectively. To ensure that $e^{N*}<r^{N*}$, we have $b_{LI}<\frac{(2-\alpha)(1+\delta_D+\alpha)b_{LD}}{(2+\alpha)(1+\delta_I-\alpha)}\equiv b_{T3}$.

To ensure that there is no non-local deviation, we consider the case when the manufacturer unilaterally raises its direct channel price to slightly higher than the retail price, $p_T^{N*}=e_T^{N*}=r_T^{N*}+\varepsilon$, where ε is an infinitesimally small positive number. In this way, the manufacturer directs LPG-sensitive comparison shoppers to the indirect channel. The manufacturer's equilibrium profit in this case is given by

$$\begin{aligned}\Pi_{T1}^{N*} &= \frac{1-\alpha}{4}\left(r_T^{N*}+\varepsilon\right)\left(1-b_{LD}\left(r_T^{N*}+\varepsilon\right)\right)+\frac{1}{4}\left(r_T^{N*}+\varepsilon\right)\left(1-\delta_D b_{LD}\left(r_T^{N*}+\varepsilon\right)\right) \\ &+\frac{1+\alpha}{4} w_T^{N*}\left(1-b_{LI} r_T^{N*}\right)+\frac{1}{4} w_T^{N*}\left(1-\delta_I b_{LI} r_T^{N*}\right)+\lambda F^N,\end{aligned} \tag{B-83}$$

where $w_T^{N*}=0$ and $r_T^{N*}=\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. Thus, we have $\Pi_{T1}^{N*}=\frac{(2-\alpha)^2(2(1+\delta_I-\alpha)b_{LI}-(1+\delta_D-\alpha)b_{LD})}{16(1+\delta_I-\alpha)^2 b_{LI}^2}+\frac{\lambda(2-\alpha)\left((2+3\alpha)\delta_I+2-3\alpha-\alpha^2\right)}{16(1+\delta_I-\alpha)^2 b_{LI}}$. Comparing Π_T^{N*} and Π_{T1}^{N*}, we have

$$\Pi_T^{N*}-\Pi_{T1}^{N*}>0. \tag{B-84}$$

This indicates that the manufacturer does not have incentive to unilaterally raise its direct channel price.

We next consider the case when the retailer unilaterally lowers its retail price to slightly lower than the direct channel price, $r^{N*}=p^{N*}-\varepsilon$. In this way, the retailer undercuts the direct channel and therefore attracts LPG-sensitive comparison shoppers to the indirect channel. The retailer's equilibrium profit in this case is given by

$$\begin{aligned}\pi_{T1}^{N*} &= \frac{1+\alpha}{4}\left(\left(p_T^{N*}-\varepsilon\right)-w_T^{N*}\right)\left(1-b_{LI}\left(p_T^{N*}-\varepsilon\right)\right) \\ &+\frac{1}{4}\left(\left(p_T^{N*}-\varepsilon\right)-w_T^{N*}\right)\left(1-\delta_I b_{LI}\left(p_T^{N*}-\varepsilon\right)\right)-\lambda F^N, \end{aligned} \tag{B-85}$$

where $w^{N*}=0$ and $p^{N*}=\frac{2+\alpha}{2(1+\delta_D+\alpha)b_{LD}}$. Thus, we have

$$\pi_{T1}^{N*}=\frac{(1-\lambda)(2+\alpha)^2\left(2\left(1+\delta_D+\alpha\right)b_{LD}-\left(1+\delta_I+\alpha\right)b_{LI}\right)}{16b_{LD}^2\left(1+\delta_D+\alpha\right)^2}. \tag{B-86}$$

Comparing π^{N*} and π_1^{N*}, we have

$$\pi_T^{N*}-\pi_{T1}^{N*}>0. \tag{B-87}$$

This indicates that the retailer does not have incentives to unilaterally lower its retail price in order to attract LPG-sensitive comparison shoppers to the indirect channel.

To sum up, the equilibrium when $b_{LI}<b_{T3}$ is stable and does not suffer from non-local deviation.

(II) When $e_T^N>r_T^N$

Given that $e_T^N>r_T^N$, all LPG-sensitive comparison shoppers will purchase from the indirect channel ($\beta_T^{N*}=0$). The profit functions of the manufacturer and retailer are given by

$$\begin{aligned}\Pi_T^N &= \frac{1-\alpha}{4}e_T^N\left(1-b_{LD}e_T^N\right)+\frac{1}{4}p_T^N\left(1-\delta_D b_{LD}p_T^N\right) \\ &+\frac{1+\alpha}{4}w_T^N\left(1-b_{LI}r_T^N\right)+\frac{1}{4}w_T^N\left(1-\delta_I b_{LI}r_T^N\right)+\lambda F^N, \end{aligned} \tag{B-88}$$

and

$$\pi_T^N=\frac{1+\alpha}{4}\left(r_T^N-w_T^N\right)\left(1-b_{LI}r_T^N\right)+\frac{1}{4}\left(r_T^N-w_T^N\right)\left(1-\delta_I b_{LI}r_T^N\right)-\lambda F^N. \tag{B-89}$$

In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p_T^N and r_T^N, respectively,

$$\max_{p_T^N>0} \Pi_T^N \left(p_T^N|w_T^N\right) = \frac{1-\alpha}{4} e_T^N \left(1-b_{LD}e_T^N\right) + \frac{1}{4} p_T^N \left(1-\delta_D b_{LD} p_T^N\right) + \frac{1+\alpha}{4} w_T^N \left(1-b_{LI} r_T^N\right) + \frac{1}{4} w_T^N \left(1-\delta_I b_{LI} r_T^N\right) + \lambda F^N, \quad \text{(B-90)}$$

and

$$\max_{r_T^N>0} \pi_T^N \left(r_T^N|w_T^N\right) = \frac{1+\alpha}{4}\left(r_T^N - w_T^N\right)\left(1-b_{LI}r_T^N\right) + \frac{1}{2}\left(r_T^N - w_T^N\right)\left(1-\delta_I b_{LI} r_T^N\right) - \lambda F^N. \quad \text{(B-91)}$$

Solving the above maximization problems yields the optimal retail price response function $r_T^N\left(w_T^N\right) = \frac{w_T^N}{2} + \frac{2+\alpha}{2(1+\delta_I+\alpha)b_{LI}}$ and the equilibrium direct channel price $p_T^{N*} = \frac{2-\alpha}{2(1+\delta_D-\alpha)b_{LD}}$. The equilibrium effective price is $e_T^{N*} = p_T^{N*} = \frac{2-\alpha}{2(1+\delta_D-\alpha)b_{LD}}$. Replace r_T^N with $r_T^N\left(w_T^N\right)$ in $\pi_T^N\left(r_T^N|w_T^N\right)$ and we find that $\frac{\partial \pi_T^N}{\partial w_T^N} < 0$. It implies that w_T^N should be set as low as possible to maximize π_T^N.

In Stage 2, the manufacturer maximizes its profit by choosing $\left\{w_T^N, F^N\right\}$ while taking into account both players' Stage 3 decisions:

$$\max_{F^N>0, w_T^N\geq 0} \Pi_T^N\left(w_T^N, F^N\right) = \frac{1-\alpha}{4} e_T^{N*}\left(1-b_{LD}e_T^{N*}\right) + \frac{1}{4}p_T^{N*}\left(1-\delta_D b_{LD} p_T^{N*}\right) + \frac{1+\alpha}{4} w_T^N\left(1-b_{LI}r_T^N\left(w_T^N\right)\right) + \frac{1}{4} w_T^N\left(1-\delta_I b_{LI} r_T^N\left(w_T^N\right)\right) + \lambda F^N. \quad \text{(B-92)}$$

Solving the manufacturer's profit maximization problem, we obtain the equilibrium wholesale price $w_T^{N*} = 0$ and $F^{N*} = \frac{(2+\alpha)^2}{16(1+\delta_I+\alpha)b_{LI}}$. It follows that the optimal retail price is $r_T^{N*} = r_T^N\left(w_T^{N*}\right) = \frac{2+\alpha}{2(1+\delta_I+\alpha)b_{LI}}$. The equilibrium profits of the retailer and manufacturer are $\pi_T^{N*} = \pi_T^N\left(r_T^{N*}, F^{N*}\right) = \frac{(1-\lambda)(2+\alpha)^2}{16(1+\delta_I+\alpha)b_{LI}}$ and $\Pi_T^{N*} = \Pi_T^N\left(p_T^{N*}, w_T^{N*}, F^{N*}\right) = \frac{\lambda(2+\alpha)^2}{16(1+\delta_I+\alpha)b_{LI}} + \frac{(2-\alpha)^2}{16(1+\delta_D-\alpha)b_{LD}}$, respectively. To ensure that $e^{N*} < r^{N*}$, we have $b_{LI} < \frac{(2+\alpha)(1+\delta_D-\alpha)b_{LD}}{(2-\alpha)(1+\delta_I+\alpha)} \equiv b_{T1}$.

To ensure that there is no non-local deviation, we consider the case when the manufacturer unilaterally lowers its direct channel price to slightly lower than the retail price, $p_T^{N*} = r_T^{N*} - \varepsilon$. In this way, the manufacturer is able to attract LPG-sensitive comparison shoppers to the direct channel. The manufacturer's equilibrium profit in this case is

$$\begin{aligned}\Pi_{T2}^{N*} &= \frac{1+\alpha}{4}\left(r_T^{N*}-\varepsilon\right)\left(1-b_{LD}\left(r_T^{N*}-\varepsilon\right)\right)+\frac{1}{4}\left(r_T^{N*}-\varepsilon\right)\left(1-\delta_D b_{LD}\left(r_T^{N*}-\varepsilon\right)\right)\\ &+\frac{1-\alpha}{4}w_T^{N*}\left(1-b_{LI}r_T^{N*}\right)+\frac{1}{4}w_T^{N*}\left(1-\delta_I b_{LI}r_T^{N*}\right)+\lambda F^N, \end{aligned} \quad \text{(B-93)}$$

where $w_T^{N*}=0$ and $r_T^{N*}=\frac{2+\alpha}{2(1+\delta_I+\alpha)b_{LI}}$. Thus, we have $\Pi_{T2}^{N*}=\frac{(2+\alpha)^2(2(1+\delta_I+\alpha)b_{LI}-(1+\delta_D+\alpha)b_{LD})}{16b_{LI}^2(1+\delta_I+\alpha)^2}+\frac{\lambda(2+\alpha)\left(2+3\alpha-\alpha^2+(2-3\alpha)\delta_I\right)}{16b_{LI}(1+\delta_I+\alpha)^2}$. Comparing Π_T^{N*} and Π_{T2}^{N*}, we have

$$\Pi_T^{N*}-\Pi_{T2}^{N*}|_{b_{LI}=b_{T3}}<0. \quad \text{(B-94)}$$

and

$$\Pi_T^{N*}-\Pi_{T2}^{N*}|_{b_{LI}\to\infty}>0. \quad \text{(B-95)}$$

Since both Π_T^{N*} and Π_{T2}^{N*} are both decreasing functions of b_{LI}, there exists a unique solution to $\Pi_T^{N*}(b_{LI})=\Pi_2^{N*}(b_{LI})$, which is given by

$$\begin{aligned} b_{T2} &= \arg\left\{\Pi^{N*}(b_{LI})=\Pi_2^{N*}(b_{LI})\right\}\\ &\equiv \frac{(\alpha+2)\,b_{LD}\begin{pmatrix} b_{LD}(1+\delta_D-\alpha)\begin{pmatrix}2+3\alpha+\alpha^2(1-\lambda)\\ +(2+\alpha-2\alpha\lambda)\delta_I\end{pmatrix}\\ +\sqrt{\alpha(1+\delta_D-\alpha)\Delta_T}\end{pmatrix}}{(2-\alpha)^2(1+\delta_I+\alpha)^2}, \end{aligned} \quad \text{(B-96)}$$

$(b_{T2}>b_{T1})$, where $\Delta_T\equiv\left(2\lambda-\lambda^2-2\right)\alpha^4+\left(4\lambda-4\delta_I+\lambda^2+6\lambda\delta_I-2\lambda\delta_D-4\lambda^2\delta_I+\lambda^2\delta_D-4\right)\alpha^3+\left(8\delta_D-4\delta_I-2\lambda-2\delta_I^2-4\lambda^2\delta_I^2+10\lambda\delta_I-6\lambda\delta_D+4\lambda\delta_I^2+4\lambda^2\delta_I+4\lambda^2\delta_I\delta_D-6\lambda\delta_I\delta_D-2\right)\alpha^2+\left(16\delta_D-4\lambda+4\lambda^2\delta_I^2-8\lambda\delta_I-4\lambda\delta_D+16\delta_I\delta_D+4\lambda\delta_I^2-4\lambda\delta_I^2\delta_D+4\lambda^2\delta_I^2\delta_D-16\lambda\delta_I\delta_D\right)\alpha+\left(8\delta_D-8\lambda\delta_I+16\delta_I\delta_D-8\lambda\delta_I^2+8\delta_I^2\delta_D-8\lambda\delta_I^2\delta_D-8\lambda\delta_I\delta_D\right)$.

When $b_{T3}<b_{LI}\leq b_{T2}$, the manufacturer is better off by unilaterally lowering its direct channel price in order to uncut the retailer. Hence, the above equilibrium cannot be sustained. Otherwise, when $b_{LI}\geq b_{T2}$, the manufacturer has no incentive to deviate from the current equilibrium.

We next consider the case when the retailer unilaterally raises its retail price to slightly higher than the direct channel price, $r_T^{N*}=p_T^{N*}+\varepsilon$. In this way, the retailer drives LPG-sensitive comparison shoppers to the direct channel. The retailer's equilibrium profit in this case will be

$$\pi_{T2}^{N*} = \frac{1-\alpha}{4}\left(\left(p_T^{N*}+\varepsilon\right)-w_T^{N*}\right)\left(1-b_{LI}\left(p_T^{N*}+\varepsilon\right)\right) + \frac{1}{4}\left(\left(p_T^{N*}+\varepsilon\right)-w_T^{N*}\right)\left(1-\delta_I b_{LI}\left(p_T^{N*}+\varepsilon\right)\right)-\lambda F^N, \tag{B-97}$$

where $w_T^{N*}=0$ and $p_T^{N*}=\frac{2-\alpha}{2(1+\delta_D-\alpha)b_{LD}}$, as above. Thus, we have

$$\pi_2^{N*} = \frac{(1-\lambda)(2-\alpha)^2\left(2(1+\delta_D-\alpha)b_{LD}-(1+\delta_I-\alpha)b_{LI}b_{LI}\right)}{16(1+\delta_D-\alpha)^2 b_{LD}^2}. \tag{B-98}$$

Comparing π^{N*} and π_2^{N*}, we have

$$\pi_T^{N*}-\pi_{T2}^{N*}>0. \tag{B-99}$$

This indicates that the retailer does not have incentive to unilaterally raises its retail price in order to push LPG-sensitive comparison shoppers to the direct channel.

To sum up, the above equilibrium outcome is stable when $b_{LI}\le b_{T1}$ and $b_{LI}\ge b_{T2}$. ■

Two-Part Tariff and Low-Price Guarantee

Lemma 4 *When* $0<b_{LI}\le\frac{\delta_D(2-\alpha)b_{LD}}{1+\delta_I-\alpha}$*, LPG is not invoked. Otherwise, when LPG is invoked, i.e.,* $b_{LI}>\frac{\delta_D(2-\alpha)b_{LD}}{1+\delta_I-\alpha}$*, the optimal refund depth is given by*

$$\mu_T^* = \max\left\{0,\frac{(2-\alpha)b_{LD}-(1+\delta_I-\alpha)b_{LI}}{2(1+\delta_I-\alpha)b_{LD}b_{LI}}\right\}. \tag{B-100}$$

When the refund depth is chosen at the optimal level $\mu_T=\mu_T^*$*, the equilibrium wholesale price is given by*

$$w_T^{G*}=0 \tag{B-101}$$

When the refund depth is chosen at the optimal level $\mu_T=\mu_T^*$*, the equilibrium retail price is given by*

$$r_T^{G*} = \begin{cases} \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}} & \text{if } \frac{\delta_D(2-\alpha)b_{LD}}{1+\delta_I-\alpha}<b_{LI}\le b_{T1} \\ \frac{1}{(1+\delta_I)b_{LI}} & \text{if } b_{LI}\ge b_{T2} \end{cases}. \tag{B-102}$$

When the refund depth is chosen at the optimal level $\mu_T = \mu_T^*$, *the equilibrium direct channel price is given by*

$$p_T^{G*} = \frac{1}{2\delta_D b_{LD}}. \tag{B-103}$$

When the refund depth is chosen at the optimal level $\mu_T = \mu_T^*$, *the equilibrium retail profit is given by*

$$\pi_T^{G*} = \begin{cases} \frac{(1-\lambda)(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}} & \text{if } \frac{\delta_D(2-\alpha)b_{LD}}{1+\delta_I-\alpha} < b_{LI} \leq b_{T1} \\ \frac{1-\lambda}{4(1+\delta_I)b_{LI}} & \text{if } b_{LI} \geq b_{T2} \end{cases}. \tag{B-104}$$

When the refund depth is chosen at the optimal level $\mu_T = \mu_T^*$, *the equilibrium manufacturer profit is given by*

$$\Pi_T^{G*} = \begin{cases} \frac{1}{16\delta_D b_{LD}} + \frac{1+\alpha}{16 b_{LD}} + \frac{\lambda(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}} & \text{if } \frac{\delta_D(2-\alpha)b_{LD}}{1+\delta_I-\alpha} < b_{LI} \leq b_{T1} \\ \frac{1}{16\delta_D b_{LD}} + \frac{(1+\delta_I)b_{LI}-b_{LD}}{4(1+\delta_I)^2 b_{LI}^2} + \frac{\lambda}{4(1+\delta_I)b_{LI}} & \text{if } b_{LI} \geq b_{T2} \end{cases}. \tag{B-105}$$

When the refund depth is chosen at the optimal level $\mu_T = \mu_T^*$, *the equilibrium channel profit is given by*

$$\Pi_{TC}^{G*} = \begin{cases} \frac{1}{16\delta_D b_{LD}} + \frac{1+\alpha}{16 b_{LD}} + \frac{(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}} & \text{if } \frac{\delta_D(2-\alpha)b_{LD}}{1+\delta_I-\alpha} < b_{LI} \leq b_{T1} \\ \frac{1}{16\delta_D b_{LD}} + \frac{(1+\delta_I)b_{LI}-b_{LD}}{4(1+\delta_I)^2 b_{LI}^2} + \frac{1}{4(1+\delta_I)b_{LI}} & \text{if } b_{LI} \geq b_{T2} \end{cases}. \tag{B-106}$$

Proof of Lemma 4

Proof. Suppose that the manufacturer adopts PBG ($\mu > 0$). This implies $e_T^G = r_T^G - \mu_T < r_T^G$. In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p_T^G and r_T^G, respectively:

$$\begin{aligned} \max_{p_T^G>0} \Pi_T^G\left(p_T^G | w_T^G, \mu_T\right) &= \frac{1+\alpha}{4} e_T^G\left(1 - b_{LD} e_T^G\right) + \frac{1}{4} p_T^G\left(1 - \delta_D b_{LD} p_T^G\right) \\ &+ \frac{1-\alpha}{4} w_T^G\left(1 - b_{LI} r_T^G\right) + \frac{1}{4} w_T^G\left(1 - \delta_I b_{LI} r_T^G\right) + \lambda F^G, \end{aligned} \tag{B-107}$$

and

$$\max_{r_T^G>0} \pi^G\left(r_T^G | w_T^G, \mu\right) = \frac{1-\alpha}{4}\left(r_T^G - w_T^G\right)\left(1 - b_{LI} r_T^G\right) + \frac{1}{4}\left(r_T^G - w_T^G\right)\left(1 - \delta_I b_{LI} r_T^G\right) - \lambda F^G. \tag{B-108}$$

The above profit maximization problems yield the equilibrium direct channel price $p_T^{G*} = \frac{1}{2\delta_D b_{LD}}$ and the optimal retail price response function $r_T^G\left(w_T^G\right) = \frac{w_T^G}{2} + \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$.

In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^G while taking into account both players' stage-three decisions:

$$\begin{aligned}\max_{F^G>0, w_T^G\geq 0} \Pi_T^G(w_T^G|\mu) &= \frac{1+\alpha}{4}e_T^G\left(1-b_{LD}e_T^G\right)+\frac{1}{4}p_T^G\left(1-\delta_D b_{LD}p_T^G\right) \\ &+\frac{1-\alpha}{4}w_T^G\left(1-b_{LI}r_T^G\left(w_T^G\right)\right)+\frac{1}{4}w_T^G\left(1-\delta_I b_{LI}r_T^G\left(w_T^G\right)\right)+\lambda F^G.\end{aligned} \tag{B-109}$$

The wholesale price as a function of refund depth μ_T is given by $w_T^{G*}=0$. As a result, the retail price as a function of refund depth μ_T is given by $r_T^{G*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. The manufacturer profit as a function of refund depth μ is given by

$$\begin{aligned}\Pi_T^G\left(\mu_T\right) &= \frac{\begin{array}{c}\left(1+\alpha\right)\left(2-\alpha-2\left(1+\delta_I-\alpha\right)b_{LI}\mu_T\right)\\ \left(2\left(1+\delta_I-\alpha\right)b_{LI}-\left(2-\alpha\right)b_{LD}+2b_{LD}\left(1+\delta_I-\alpha\right)b_{LI}\mu_T\right)\end{array}}{16\left(1+\delta_I-\alpha\right)^2 b_{LI}^2} \\ &+\frac{1}{16\delta_D b_{LD}}+\frac{\lambda\left(2-\alpha\right)^2}{16\left(1+\delta_I-\alpha\right)b_{LI}}.\end{aligned}$$

The retail profit as a function of refund depth μ_T is given by $\pi_T^G\left(\mu_T\right)=0$. Note that PBG is invoked in equilibrium only when $p_T^{G*} > r_T^{G*}$. This inequality holds when $b_{LI} > \frac{\delta_D(2-\alpha)b_{LD}}{1+\delta_I-\alpha}$.

In Stage 1, the manufacturer chooses the refund depth μ_T to maximize its profit:

$$\begin{aligned}\max_{\mu_T\geq 0}\Pi_T^G(\mu_T) &= \frac{\begin{array}{c}\left(1+\alpha\right)\left(2-\alpha-2\left(1+\delta_I-\alpha\right)b_{LI}\mu_T\right)\\ \times\left(2\left(1+\delta_I-\alpha\right)b_{LI}-\left(2-\alpha\right)b_{LD}+2b_{LD}\left(1+\delta_I-\alpha\right)b_{LI}\mu_T\right)\end{array}}{16\left(1+\delta_I-\alpha\right)^2 b_{LI}^2} \\ &+\frac{1}{16\delta_D b_{LD}}+\frac{\lambda\left(2-\alpha\right)^2}{16\left(1+\delta_I-\alpha\right)b_{LI}}.\end{aligned} \tag{B-110}$$

The first-order condition of Π_T^G with respect to μ_T is given by

$$\frac{\partial\Pi_T^G}{\partial\mu_T} = -\frac{\left(1+\alpha\right)\left(\alpha b_{LD}+\left(1+\delta_I-\alpha\right)b_{LI}-2b_{LD}+2\left(1+\delta_I-\alpha\right)b_{LI}b_{LD}\mu_T\right)}{4\left(1+\delta_I-\alpha\right)b_{LI}}. \tag{B-111}$$

It is easy to see that the unique solution that satisfy $\frac{\partial \Pi_T^G}{\partial \mu_T}=0$ is $\mu_{T1}=\frac{(2-\alpha)b_{LD}-(1+\delta_I-\alpha)b_{LI}}{2(1+\delta_I-\alpha)b_{LD}b_{LI}}$. On evaluating the second-order condition we have

$$\frac{\partial^2 \Pi_T^G}{\partial \mu_T^2}|_{\mu_T=\mu_{T1}}=-\frac{(1+\alpha)\,b_{LD}}{2}<0 \tag{B-112}$$

Thus, there exists a unique optimal refund depth μ_T^* that maximizes Π_T^G:

$$\mu_T^*=\max\{0,\mu_{T1}\}=\max\left\{0,\frac{(2-\alpha)\,b_{LD}-(1+\delta_I-\alpha)\,b_{LI}}{2\,(1+\delta_I-\alpha)\,b_{LD}b_{LI}}\right\}. \tag{B-113}$$

When $\mu_T^*=\frac{(2-\alpha)b_{LD}-(1+\delta_I-\alpha)b_{LI}}{2(1+\delta_I-\alpha)b_{LD}b_{LI}}$, or equivalently

$$\frac{\delta_D\,(2-\alpha)\,b_{LD}}{1+\delta_I-\alpha}<b_{LI}<\frac{(2-\alpha)\,b_{LD}}{1+\delta_I-\alpha},$$

the profit of the manufacturer is given by $\pi_T^{G*}=\frac{\lambda(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}}+\frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}$. The retailer's equilibrium profit is given by $\pi_T^{G*}=\frac{(1-\lambda)(2-\alpha)^2}{16(1+\delta_I-\alpha)b_{LI}}$. In this case, LPG is effectively a price-beating guarantee (PBG).

Suppose that the manufacturer adopts PMG ($\mu=0$). This implies that $e^G=r^G$. In this case, both channels evenly split the LPG-sensitive consumers. In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p_T^G and r_T^G, respectively:

$$\begin{aligned}\max_{p_T^G>0}\Pi_T^G\left(p_T^G|w_T^G,\mu\right) &= \frac{1}{4}e_T^G\left(1-b_{LD}e_T^G\right)+\frac{1}{4}p_T^G\left(1-\delta_D b_{LD}p_T^G\right)\\ &\quad+\frac{1}{4}w_T^G\left(1-b_{LI}r_T^G\right)+\frac{1}{4}w_T^G\left(1-\delta_I b_{LI}r_T^G\right)+\lambda F^G,\end{aligned} \tag{B-114}$$

and

$$\max_{r_T^G>0}\pi_T^G\left(r_T^G|w_T^G,\mu\right)=\frac{1}{4}\left(r_T^G-w_T^G\right)\left(1-b_{LI}r_T^G\right)+\frac{1}{4}\left(r_T^G-w_T^G\right)\left(1-\delta_I b_{LI}r_T^G\right)-\lambda F^G. \tag{B-115}$$

The above profit maximization problems yield the equilibrium direct channel price $p_T^{G*}=\frac{1}{2\delta_D b_{LD}}$ and the optimal retail price response function $r_T^G\left(w_T^G\right)=\frac{2+(1+\delta_I)b_{LI}w_T^G}{2(1+\delta_I)b_{LI}}$.

In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w_T^G while taking into account both players' stage-three decisions:

$$\max_{w^G>0} \Pi_T^G(w_T^G|\mu \;=\; 0) = \frac{1}{4}e_T^G\left(w_T^G\right)\left(1 - b_{LD}e_T^G\left(w_T^G\right)\right) + \frac{1}{4}p_T^G\left(1-\delta_D b_{LD} p_T^G\right)$$
$$+\frac{1}{4}w_T^G\left(1 - b_{LI}r_T^G\left(w_T^G\right)\right) + \frac{1}{4}w^G\left(1-\delta_I b_{LI} r_T^G\left(w_T^G\right)\right) + \lambda F^G \quad \text{(B-116)}$$

In this case, the market equilibrium is given by $w_T^{G*} = 0$, $r_T^{G*} = \frac{1}{(1+\delta_I)b_{LI}}$, $p_T^{G*} = \frac{1}{2\delta_D b_{LD}}$, $F_T^G = \frac{1}{4(1+\delta_I)b_{LI}}$, $\Pi_T^{G*} = \frac{1}{16\delta_D b_{LD}} + \frac{(1+\delta_I)b_{LI}-b_{LD}}{4(1+\delta_I)^2 b_{LI}^2} + \frac{\lambda}{4(1+\delta_I)b_{LI}}$, and $\pi_T^{G*} = \pi_T^G(\mu^* = 0) = \frac{1-\lambda}{4(1+\delta_I)b_{LI}}$.
■

Model Extensions

In this part of the Online Appendix, we extend our main model in several different directions. First, our analyses in Section 4 is predicated on the manufacturer's product quality being exogenously given. We endogenize the manufacturer's product quality decision and show that our main insights still hold. Second, based on the empirical observation that the manufacturer implements PBG in different forms, we examine the implications of alternative forms of PBGs. Third, we consider alternative game sequences with regard to when the manufacturer determines the refund depth and show that our results are invariant. In sum, this appendix is devoted to various sensitivity analysis for establishing the robustness of our key findings.

Endogenous Product Quality

In Section 4, we assume that the manufacturer's product quality is exogenously determined. In practice, manufacturers are able to adjust the product quality based on their LPG policies. For instance, in many travel markets such as car rental and cruise, manufacturers are able to enhance quality by adding more cars/cruises. In the airlines and hotel industry, travel suppliers can change their quality offerings over time. In this subsection, we relax the exogenous quality assumption and allow the manufacturer to choose its quality level t.

The sequence of moves in the model is as follows:

- Stage 1: The manufacturer chooses the quality t.

- Stage 2: The manufacturer determines the refund depth μ when it offers LPG.
- Stage 3: The manufacturer sets the wholesale price w.
- Stage 4: The manufacturer and the retailer simultaneously set the direct channel price p and the retail price r, respectively.
- Stage 5: Consumers make purchase decisions.

In solving the model, we adopt backward induction as the standard solution concept. Formally, consumers' demand is the same as the main model, except that the demand potential is given by $\sqrt{t}$. This assumption captures the idea that the demand potential increases with product quality but at a decreasing rate. The cost of quality is assumed to be quadratic in quality, $\frac{1}{2}t^2$. In a special case when $t = 1$ and quality is costless, this setup is reduced to our main model in Section 4.

The derivation of equilibria is provided toward the end of the Online Appendix. Following similar analyses in Section 4, we find that when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_1$, the manufacturer optimally chooses PBG and the equilibrium product quality under PBG is higher than that under no LPG, i.e., $t^{G*} > t^{N*}$. Furthermore, when $b_2 \leq b_{LI} < \bar{b}$, PMG induces higher product quality than no LPG, i.e., $t^{G*} > t^{N*}$; however, no LPG is chosen and $t^{N*} > t^{G*}$ when $b_{LI} < \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$ or $b_{LI} > \bar{b}$.

Result 1 *The retailer prefers PBG when* $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_1$*, PMG when* $b_2 \leq b_{LI} < \overrightarrow{b}$*, and no LPG when* $b_{LI} > \overrightarrow{b}$*, where* $b_2 < \overrightarrow{b} < \bar{b}$*.*

Result 1 establishes two important findings. First, when product quality is endogenous, the retailer could benefit from LPG when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_1$ and $b_2 \leq b_{LI} < \overrightarrow{b}$. This is in contrast with Proposition **??**, where we show that the retailer is weakly worse off with LPG in a market where product quality is fixed. This indicates that LPG could be a win-win strategy for both channel members. Second, the benefit of LPG is not always guaranteed for the retailer. Higher product quality is not enough to offset the negative impact of LPG on the retailer when $\overrightarrow{b} < b_{LI} < \bar{b}$. Therefore, the misalignment between the manufacturer and the retailer may still exists, and this demonstrates the robustness of Proposition **??**.

Price-Beating Guarantees with Alternative Refund Formats

In this subsection, we explore LPGs with alternative refund formats that are also prevalent in the online travel industry. PBG with percentage refund is popular with hotels and car rental companies. For instance, Hilton's LPG policy states that "If you find a lower qualified price for the same accommodation and terms at any point before your reservation is made or up to 24 hours after making your reservation through one of the official Hilton booking channels, we will honor the lower price and take an additional 25% off the room rate for each night of your stay."[1] This type of PBG (denoted by P) allows the manufacturer to beat the retail price by offering a percentage-off discount γ $(0 < \gamma < 1)$. γ can be seen as the refund depth. Thus, the effective price that LPG-sensitive consumers pay from the direct channel is given by

$$e^P = \begin{cases} (1-\gamma)\, r^P & \text{if } p^P < r^P \\ p^P & \text{otherwise} \end{cases}. \tag{B-117}$$

The retail profit is given by $\pi^P = \frac{1-\alpha}{4}\left(r^P - w^P\right) q_{LI}\left(r^P\right) + \frac{1}{4}\left(r^P - w^P\right) q_{HI}\left(r^P\right)$ and the manufacturer profit is given by $\Pi^P = \frac{1+\alpha}{4} e^P q_{LD}\left(e^P\right) + \frac{1}{4} p^P q_{HD}\left(p^P\right) + \frac{1-\alpha}{4} w^P q_{LI}\left(r^P\right) + \frac{1}{4} w^P q_{HI}\left(r^P\right)$. A summary of the equilibrium prices and profits can be found toward the end of this Online Appendix.

Cruise lines such as Carnival, Royal Caribbean, Norwegian, Princess, and Celebrity promise to beat the retail price by a percentage of the price difference. For instance, on its website Royal Caribbean states that "When you reserve with Royal Caribbean and subsequently find a lower rate advertised by Royal Caribbean within 48 hours from the time the reservation is made, we will honor that lower eligible rate by applying an onboard credit to the reservation equal to 110% of the price difference."[2] We denote this type of PBG by V. Let ρ $(\rho > 0)$ be the percentage of the price difference between the direct channel price and retail price. Here, ρ can be interpreted as the refund depth. Thus, the effective price that LPG-sensitive

[1] Source: https://hiltonworldwide3.hilton.com/en/best-price-guarantee/overview.html [accessed in August 2020].

[2] Source: https://www.royalcaribbean.com/faq/questions/best-price-guarantee-policy [accessed in August 2020].

consumers pay from the direct channel is given by

$$e^V = \begin{cases} r^V - \rho\left(p^V - r^V\right) & \text{if } p^V < r^V \\ p^V & \text{otherwise} \end{cases}. \tag{B-118}$$

The retail profit is given by $\pi^V = \frac{1-\alpha}{4}\left(r^V - w^V\right) q_{LI}\left(r^V\right) + \frac{1}{4}\left(r^V - w^V\right) q_{HI}\left(r^V\right)$ and the manufacturer profit is given by $\Pi^V = \frac{1+\alpha}{4} e^V q_{LD}\left(e^V\right) + \frac{1}{4} p^V q_{HD}\left(p^V\right) + \frac{1-\alpha}{4} w^V q_{LI}\left(r^V\right) + \frac{1}{4} w^V q_{HI}\left(r^V\right)$. A summary of the equilibrium prices and profits can be found toward the end of this Online Appendix.

Result 2 *Different forms of PBGs are equivalent in optimality.*

The intuition for Result 2 is as follows. Regardless of which form of PBGs is used, the optimal refund depth is always determined such that the effective price that LPG-sensitive direct channel consumers pay maximizes the manufacturer's profit. As such, our choice of using fixed refund to model PBG is a robust approach without loss of generality.

Alternative Game Sequences

In our main model, the manufacturer decides whether to offer LPG and, if so, the refund depth, followed by other events. The rationale for such game sequence is that the LPG strategy is a price commitment and it needs to be consistent over time. Put differently, firms have greater flexibility in changing their prices than LPG policy. That said, we observe that some manufacturers change the refund depth over time. For instance, when the authors first began this project, the extra refund that InterContinental Hotels Group offered was "first night free" while the current form of the extra refund is "40,000 IHG Rewards Club points." Similarly, Hilton Worldwide and Choice International changed the refund depth from "\$50 credit" and "first night free" to "25% off the retail price" and "\$50 reward card", respectively. The implications are that the manufacturers tend to calibrate the refund depth over time. This suggests alternative game sequence compared to what we adopt in the main model. In this subsection, we explore whether our results are sensitive to alternative game sequences.

We first examine a sequence of events where refund depth is chosen after pricing decisions are made:

- Stage 1: The manufacturer decides whether to offer LPG.
- Stage 2: The manufacturer sets the wholesale price w.
- Stage 3: The manufacturer and the retailer simultaneously set the direct channel price p and the retail price r, respectively.
- Stage 4: The manufacturer determines the refund depth μ if PBG is adopted.
- Stage 5: Consumers make purchase decisions.

Next, we examine a sequence of events where refund depth is chosen simultaneously with the direct channel price and retail price:

- Stage 1: The manufacturer decides whether to offer LPG.
- Stage 2: The manufacturer sets the wholesale price w.
- Stage 3: The manufacturer and the retailer simultaneously set the direct channel price p and refund depth μ and the retail price r, respectively.
- Stage 4: Consumers make purchase decisions.

We solve the model by backward induction. In the following result, we establish the robustness of our findings to alternative game sequences.

Result 3 *The equilibrium outcome is invariant whether the manufacturer chooses the refund depth before, simultaneously, or after firms' pricing decisions.*

PROOFS OF RESULTS IN THE MODEL EXTENSIONS

Proof of Result 1

Proof. (1) Consider the case in which the manufacturer does not offer LPG.

(I) When $e^N < r^N$

When $e^N < r^N$, all LPG-sensitive comparison shoppers will purchase from the direct channel ($\beta^{N*} = 1$). The profit functions of the manufacturer and retailer are given by

$$\begin{aligned}\Pi^N\left(p^N, w^N, t^N\right) &= \frac{1+\alpha}{4} e^N\left(\sqrt{t^N} - b_{LD} e^N\right) + \frac{1}{4} p^N\left(\sqrt{t^N} - \delta_D b_{LD} p^N\right) \\ &+ \frac{1-\alpha}{4} w^N\left(\sqrt{t^N} - b_{LI} r^N\right) + \frac{1}{4} w^N\left(\sqrt{t^N} - \delta_I b_{LI} r^N\right) - \frac{1}{2} t^{N2},\end{aligned} \tag{C-1}$$

and

$$\pi^N\left(r^N\right) = \frac{1-\alpha}{4}\left(r^N - w^N\right)\left(\sqrt{t^N} - b_{LI} r^N\right) + \frac{1}{4}\left(r^N - w^N\right)\left(\sqrt{t^N} - \delta_I b_{LI} r^N\right). \tag{C-2}$$

Here, $e^N = p^N$. We solve the model by backward induction. In Stage 4, the manufacturer and the retailer simultaneously maximize their profits by choosing p^N and r^N, respectively,

$$\begin{aligned}\max_{p^N > 0} \Pi^N\left(p^N | w^N\right) &= \frac{1+\alpha}{4} e^N\left(\sqrt{t^N} - b_{LD} e^N\right) + \frac{1}{4} p^N\left(\sqrt{t^N} - \delta_D b_{LD} p^N\right) \\ &+ \frac{1-\alpha}{4} w^N\left(\sqrt{t^N} - b_{LI} r^N\right) + \frac{1}{4} w^N\left(\sqrt{t^N} - \delta_I b_{LI} r^N\right) - \frac{1}{2} t^{N2}\end{aligned} \tag{C-3}$$

and

$$\max_{r^N > 0} \pi^N\left(r^N | w^N\right) = \frac{1-\alpha}{4}\left(r^N - w^N\right)\left(\sqrt{t^N} - b_{LI} r^N\right) + \frac{1}{4}\left(r^N - w^N\right)\left(\sqrt{t^N} - \delta_I b_{LI} r^N\right). \tag{C-4}$$

Solving the above maximization problems yields the optimal retail price response function $r^N\left(w^N, t^N\right) = \frac{w^N}{2} + \frac{(2-\alpha)\sqrt{t^N}}{2(1+\delta_I-\alpha) b_{LI}}$ and the equilibrium direct channel price $e^N\left(t^N\right) = p^N\left(t^N\right) = \frac{(2+\alpha)\sqrt{t^N}}{2(1+\delta_D+\alpha) b_{LD}}$.

In Stage 3, the manufacturer maximizes its profit by choosing the wholesale price w^N

while taking into account both players' stage-three decisions:

$$\max_{w^N>0} \Pi^N\left(w^N\right) = \frac{1+\alpha}{4}e^N\left(t^N\right)\left(\sqrt{t^N}-b_{LD}e^N\left(t^N\right)\right)+\frac{1}{4}p^N\left(t^N\right)\left(\sqrt{t^N}-\delta_D b_{LD}p^N\left(t^N\right)\right) \quad \text{(C-5)}$$
$$+\frac{1-\alpha}{4}w^N\left(t^N\right)\left(\sqrt{t^N}-b_{LI}r^N\left(t^N\right)\right)+\frac{1}{4}w^N\left(t^N\right)\left(\sqrt{t^N}-\delta_I b_{LI}r^N\left(t^N\right)\right)-\frac{1}{2}t^{N2}.$$

The wholesale price charged by the manufacturer as a function of product quality t^N is given by $w^N\left(t^N\right) = \frac{(2-\alpha)\sqrt{t^N}}{2(1+\delta_I-\alpha)b_{LI}}$. It follows that the retail price as a function of product quality t^N is $r^N\left(t^N\right) = \frac{3(2-\alpha)\sqrt{t^N}}{4(1+\delta_I-\alpha)b_{LI}}$. The profits of the manufacturer and the retailer as functions of product quality t^N are $\Pi^N\left(t^N\right) = \left(\frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}}+\frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}\right)t^N-\frac{1}{2}t^{N2}$ and $\pi^N\left(t^N\right) = \frac{(2-\alpha)^2 t^N}{64(1+\delta_I-\alpha)b_{LI}}$, respectively. In Stage 1, the manufacturer maximizes its profit by choosing t^N,

$$\max_{t^N>0}\Pi^N\left(t^N\right) = \left(\frac{(2-\alpha)^2}{32\left(1+\delta_I-\alpha\right)b_{LI}}+\frac{(2+\alpha)^2}{16\left(1+\delta_D+\alpha\right)b_{LD}}\right)t^N-\frac{1}{2}t^{N2}. \quad \text{(C-6)}$$

Solving the maximization problem yields the equilibrium product quality

$$t^{N*} = \frac{(2-\alpha)^2}{32\left(1+\delta_I-\alpha\right)b_{LI}}+\frac{(2+\alpha)^2}{16\left(1+\delta_D+\alpha\right)b_{LD}},$$

the equilibrium retail profit $\pi^{N*} = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}\left(\frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}}+\frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}\right)$, and the equilibrium manufacturer profit $\Pi^{N*} = \frac{1}{2}\left(\frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}}+\frac{(2+\alpha)^2}{16(1+\delta_D+\alpha)b_{LD}}\right)^2$. To ensure that $e^{N*} < r^{N*}$, we have $b_{LI} < \frac{3(2-\alpha)(1+\delta_D+\alpha)b_{LD}}{2(1+\delta_I-\alpha)(2+\alpha)} \equiv b_2$. By checking non-local deviations, we find that the above equilibrium outcomes are stable.

(II) When $e^N > r^N$

In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^N and r^N, respectively,

$$\max_{p^N>0}\Pi^N\left(p^N|w^N\right) = \frac{1-\alpha}{4}e^N\left(\sqrt{t^N}-b_{LD}e^N\right)+\frac{1}{4}p^N\left(\sqrt{t^N}-\delta_D b_{LD}p^N\right) \quad \text{(C-7)}$$
$$+\frac{1+\alpha}{4}w^N\left(\sqrt{t^N}-b_{LI}r^N\right)+\frac{1}{4}w^N\left(\sqrt{t^N}-\delta_I b_{LI}r^N\right)-\frac{1}{2}t^{N2}$$

and

$$\max_{r^N>0} \pi^N\left(r^N|w^N\right) = \frac{1+\alpha}{4}\left(r^N - w^N\right)\left(\sqrt{t^N} - b_{LI}r^N\right) + \frac{1}{4}\left(r^N - w^N\right)\left(\sqrt{t^N} - \delta_I b_{LI} r^N\right). \tag{C-8}$$

Solving the above maximization problems yields the optimal retail price response function $r^N\left(w^N, t^N\right) = \frac{w^N}{2} + \frac{(2+\alpha)\sqrt{t^N}}{2(1+\delta_I+\alpha)b_{LI}}$ and the equilibrium direct channel price $e^N\left(t^N\right) = p^N\left(t^N\right) = \frac{(2-\alpha)\sqrt{t^N}}{2(1+\delta_D-\alpha)b_{LD}}$.

In Stage 3, the manufacturer maximizes its profit by choosing the wholesale price w^N while taking into account both players' stage-three decisions:

$$\begin{aligned}\max_{w^N>0} \Pi^N\left(w^N\right) &= \frac{1-\alpha}{4}e^N\left(t^N\right)\left(\sqrt{t^N} - b_{LD}e^N\left(t^N\right)\right) + \frac{1}{4}p^N\left(t^N\right)\left(\sqrt{t^N} - \delta_D b_{LD} p^N\left(t^N\right)\right) \\ &\quad + \frac{1+\alpha}{4}w^N\left(t^N\right)\left(\sqrt{t^N} - b_{LI}r^N\left(t^N\right)\right) + \frac{1}{4}w^N\left(t^N\right)\left(\sqrt{t^N} - \delta_I b_{LI} r^N\left(t^N\right)\right) - \frac{1}{2}t^{N2}.\end{aligned} \tag{C-9}$$

The wholesale price charged by the manufacturer as a function of product quality t^N is given by $w^N\left(t^N\right) = \frac{(2+\alpha)\sqrt{t^N}}{2(1+\delta_I+\alpha)b_{LI}}$. It follows that the retail price as a function of product quality t^N is $r^N\left(t^N\right) = \frac{3(2+\alpha)\sqrt{t^N}}{4(1+\delta_I+\alpha)b_{LI}}$. The profits of the manufacturer and the retailer as functions of product quality t^N are $\Pi^N\left(t^N\right) = \left(\frac{(2+\alpha)^2}{32(1+\delta_I+\alpha)b_{LI}} + \frac{(2-\alpha)^2}{16(1+\delta_D-\alpha)b_{LD}}\right)t^N - \frac{1}{2}t^{N2}$ and $\pi^N\left(t^N\right) = \frac{(2+\alpha)^2 t^N}{64(1+\delta_I+\alpha)b_{LI}}$, respectively. In Stage 1, the manufacturer maximizes its profit by choosing t^N,

$$\max_{t^N>0} \Pi^N\left(t^N\right) = \left(\frac{(2+\alpha)^2}{32\left(1+\delta_I+\alpha\right)b_{LI}} + \frac{(2-\alpha)^2}{16\left(1+\delta_D-\alpha\right)b_{LD}}\right)t^N - \frac{1}{2}t^{N2}. \tag{C-10}$$

Solving the maximization problem yields the equilibrium product quality

$$t^{N*} = \frac{(2+\alpha)^2}{32\left(1+\delta_I+\alpha\right)b_{LI}} + \frac{(2-\alpha)^2}{16\left(1+\delta_D-\alpha\right)b_{LD}}, \tag{C-11}$$

the equilibrium retail profit $\pi^{N*} = \frac{(2-\alpha)^2}{64(1+\delta_I+\alpha)b_{LI}}\left(\frac{(2+\alpha)^2}{32(1+\delta_I+\alpha)b_{LI}} + \frac{(2-\alpha)^2}{16(1+\delta_D-\alpha)b_{LD}}\right)$, and the equilibrium manufacturer profit $\Pi^{N*} = \frac{1}{2}\left(\frac{(2+\alpha)^2}{32(1+\delta_I+\alpha)b_{LI}} + \frac{(2-\alpha)^2}{16(1+\delta_D-\alpha)b_{LD}}\right)^2$. To ensure that $e^{N*} > r^{N*}$, we have $b_{LI} < \frac{3(2+\alpha)(1+\delta_D-\alpha)b_{LD}}{2(1+\delta_I+\alpha)(2-\alpha)} \equiv b_1$. By checking non-local deviations, we find that the above equilibrium outcomes are stable when $b_{LI} \geq b_2$.

To summarize, the above equilibrium is stable when $b_{LI} \leq b_1$ and $b_{LI} \geq b_2$.

(2) When the manufacturer offers LPG, we have $e^G = r^G - \mu \leq r^G$. Next, we discuss the cases when the manufacturer uses PMG ($\mu = 0$) or PBG ($\mu > 0$) separately.

<u>(I) The Manufacturer Adopts PBG ($\mu > 0$)</u>

This implies $e^G = r^G - \mu < r^G$. In Stage 4, the manufacturer and the retailer simultaneously maximize their profits by choosing p^G and r^G, respectively:

$$\begin{aligned}\max_{p^G>0} \Pi^G\left(p^G|w^G,\mu\right) &= \frac{1+\alpha}{4}e^G\left(\sqrt{t^G}-b_{LD}e^G\right)+\frac{1}{4}p^G\left(\sqrt{t^G}-\delta_D b_{LD}p^G\right)\\ &+\frac{1-\alpha}{4}w^G\left(\sqrt{t^G}-b_{LI}r^G\right)+\frac{1}{4}w^G\left(\sqrt{t^G}-\delta_I b_{LI}r^G\right),\end{aligned} \quad \text{(C-12)}$$

and

$$\max_{r^G>0} \pi^G\left(r^G|w^G,\mu\right) = \frac{1-\alpha}{4}\left(r^G-w^G\right)\left(\sqrt{t^G}-b_{LI}r^G\right)+\frac{1}{4}\left(r^G-w^G\right)\left(\sqrt{t^G}-\delta_I b_{LI}r^G\right). \quad \text{(C-13)}$$

The above profit maximization problems yield the equilibrium direct channel price $p^{G*} = \frac{\sqrt{t^G}}{2\delta_D b_{LD}}$ and the optimal retail price response function $r^G\left(w^G\right) = \frac{w^G}{2} + \frac{(2-\alpha)\sqrt{t^G}}{2(1+\delta_I-\alpha)b_{LI}}$.

In Stage 3, the manufacturer maximizes its profit by choosing the wholesale price w^G while taking into account both players' stage-three decisions:

$$\begin{aligned}\max_{w^G>0} \Pi^G(w^G|\mu) &= \frac{1+\alpha}{4}e^G\left(w^G\right)\left(\sqrt{t^G}-b_{LD}p_e^G\left(w^G\right)\right)+\frac{1}{4}p^G\left(\sqrt{t^G}-\delta_D b_{LD}p^G\right) \quad \text{(C-14)}\\ &+\frac{1-\alpha}{4}w^G\left(\sqrt{t^G}-b_{LI}r^G\left(w^G\right)\right)+\frac{1}{4}w^G\left(\sqrt{t^G}-\delta_I b_{LI}r^G\left(w^G\right)\right).\end{aligned}$$

The wholesale price as a function of refund depth μ is given by

$$w^G\left(\mu\right) = \left(\frac{7-2\alpha+2\left(1+\alpha\right)\mu b_{LD}}{2\left(1+\delta_I-\alpha\right)b_{LI}+\left(1+\alpha\right)b_{LD}} - \frac{2-\alpha}{\left(1+\delta_I-\alpha\right)b_{LI}}\right)\sqrt{t^G}. \quad \text{(C-15)}$$

As a result, the retail price as a function of refund depth μ is given by $r^G\left(\mu\right) = r^G(w^G\left(\mu\right),\mu) = \frac{(7-2\alpha+2(1+\alpha)\mu b_{LD})\sqrt{t^G}}{4(1+\delta_I-\alpha)b_{LI}+2(1+\alpha)b_{LD}}$. LPG is invoked only when $r^G\left(\mu\right) > p^{G*}$. It holds when $b_{LI} < \frac{\delta_D b_{LD}(7-2\alpha+2(1+\alpha)\mu b_{LD})-(1+\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. The manufacturer profit as a function of refund depth μ

is given by

$$\Pi^G(\mu) = t^G \frac{\begin{array}{c}(1+\delta_I-\alpha)\,b_{LI}\,(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD})+\\ \delta_D b_{LD}\,(1+\delta_I-\alpha)\,b_{LI}\begin{pmatrix}17+4\alpha\,(1-\alpha)\\ -8\,(1+\alpha)\,(1+\delta_I-\alpha)\,b_{LI}\mu\end{pmatrix}\\ -2b_{LD}\,(1+\alpha)\,(2-\alpha-(1+\delta_I-\alpha)\,b_{LI}\mu)\\ \times(2-\alpha-2\,(1+\delta_I-\alpha)\,b_{LI}\mu)\end{array}}{16\delta_D\,(1+\delta_I-\alpha)\,b_{LI}b_{LD}\,(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD})} - \frac{t^{G2}}{2}. \tag{C-16}$$

The retail profit as a function of refund depth μ is given by

$$\pi^G(\mu) = \frac{((1-2\alpha)\,(1+\delta_I-\alpha)\,b_{LI}+2\,(1+\alpha)\,(2-\alpha)\,b_{LD}-2\,(1+\alpha)\,b_{LD}\mu)^2}{16\,(1+\delta_I-\alpha)\,b_{LI}\,(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD})^2}t^G. \tag{C-17}$$

In Stage 2, the manufacturer chooses the refund depth μ to maximize its profit:

$$\max_{\mu\geq 0}\Pi^G(\mu) = t^G \frac{\begin{array}{c}(1+\delta_I-\alpha)\,b_{LI}\,(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD})+\\ \delta_D b_{LD}\,(1+\delta_I-\alpha)\,b_{LI}\begin{pmatrix}17+4\alpha\,(1-\alpha)\\ -8\,(1+\alpha)\,(1+\delta_I-\alpha)\,b_{LI}\mu\end{pmatrix}\\ -2b_{LD}\,(1+\alpha)\,(2-\alpha-(1+\delta_I-\alpha)\,b_{LI}\mu)\\ \times(2-\alpha-2\,(1+\delta_I-\alpha)\,b_{LI}\mu)\end{array}}{16\delta_D\,(1+\delta_I-\alpha)\,b_{LI}b_{LD}\,(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD})} - \frac{t^{G2}}{2}. \tag{C-18}$$

The first-order condition of Π^G with respect to μ is given by

$$\frac{\partial\Pi^G}{\partial\mu} = t^G \frac{(1+\alpha)\,(3\,(2-\alpha)\,b_{LD}-2\,(1+\delta_I-\alpha)\,b_{LI}\,(1+2b_{LD}\mu))}{8\delta_D\,(1+\delta_I-\alpha)\,b_{LI}b_{LD}\,(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD})}. \tag{C-19}$$

It is easy to see that the unique solution that satisfy $\frac{\partial\Pi^G}{\partial\mu}=0$ is $\overline{\mu}=\frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}-\frac{1}{2b_{LD}}$. On evaluating the second-order condition we have

$$\frac{\partial^2\Pi^G}{\partial\mu^2}|_{\mu=\overline{\mu}} = -\frac{(1+\alpha)\,b_{LD}t^G}{2\delta_D b_{LD}\,(2\,(1+\delta_I-\alpha)\,b_{LI}+(1+\alpha)\,b_{LD})} < 0 \tag{C-20}$$

when $\overline{\mu} > 0$. Thus, there exists a unique optimal refund depth μ^* that maximizes Π^G:

$$\mu^* = \max\{0, \overline{\mu}\} = \max\left\{0, \frac{3(2-\alpha)b_{LD} - 2(1+\delta_I-\alpha)b_{LI}}{4b_{LD}(1+\delta_I-\alpha)b_{LI}}\right\}. \quad \text{(C-21)}$$

This gives us the market equilibrium as a function of t^G. To guarantee that $\mu^* > 0$, we need $b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. Then in Stage 1 we maximize the manufacturer profit with respect to t^G and we obtain

$$\max_{\mu \geq 0} \Pi^G(\mu) = \frac{t^G}{32}\left(\frac{2}{\delta_D b_{LD}} + \frac{2(1+\alpha)}{b_{LD}} + \frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}\right) - \frac{t^{G2}}{2}. \quad \text{(C-22)}$$

Solving the maximization problem yields the equilibrium product quality $t^{G*} = \frac{2}{\delta_D b_{LD}} + \frac{2(1+\alpha)}{b_{LD}} + \frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}$, the equilibrium retail profit

$$\pi^{G*} = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}\left(\frac{2(1+\delta_D+\delta_D\alpha)}{b_{LD}} + \frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}\right), \quad \text{(C-23)}$$

and the equilibrium manufacturer profit

$$\Pi^{G*} = \frac{1}{2}\left(\frac{2(1+\delta_D+\delta_D\alpha)}{b_{LD}} + \frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}\right)^2. \quad \text{(C-24)}$$

In this case, LPG is effectively a price-beating guarantee (PBG). This equilibrium exists only when LPG is invoked in equilibrium, or equivalently, $p^{G*} > r^{G*}$. This inequality holds when $b_{LI} > \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$. To summarize, the PBG equilibrium exists when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$.

Suppose that the manufacturer adopts PMG ($\mu = 0$). This implies $e^G = r^G$. In this case, both channels evenly split the LPG-sensitive consumers. In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^G and r^G, respectively:

$$\begin{aligned}\max_{p^G > 0} \Pi^G\left(p^G | w^G, \mu\right) &= \frac{1}{4}e^G\left(\sqrt{t^G} - b_{LD}e^G\right) + \frac{1}{4}p^G\left(\sqrt{t^G} - \delta_D b_{LD} p^G\right) \\ &+ \frac{1}{4}w^G\left(\sqrt{t^G} - b_{LI}r^G\right) + \frac{1}{4}w^G\left(\sqrt{t^G} - \delta_I b_{LI} r^G\right), \quad \text{(C-25)}\end{aligned}$$

and

$$\max_{r^G>0} \pi^G \left(r^G|w^G,\mu\right) = \frac{1}{4}\left(r^G - w^G\right)\left(\sqrt{t^G} - b_{LI}r^G\right) + \frac{1}{4}\left(r^G - w^G\right)\left(\sqrt{t^G} - \delta_I b_{LI} r^G\right). \tag{C-26}$$

The above profit maximization problems yield the equilibrium direct channel price $p^{G*} = \frac{\sqrt{t^G}}{2\delta_D b_{LD}}$ and the optimal retail price response function $r^G\left(w^G\right) = \frac{2+(1+\delta_I)b_{LI}w^G}{2(1+\delta_I)b_{LI}}$.

In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^G while taking into account both players' stage-three decisions:

$$\begin{aligned}\max_{w^G>0} \Pi^G(w^G|\mu &= 0) = \frac{1}{4}e^G\left(w^G\right)\left(\sqrt{t^G} - b_{LD}e^G\left(w^G\right)\right) + \frac{1}{4}p^G\left(\sqrt{t^G} - \delta_D b_{LD} p^G\right) \\ &+\frac{1}{4}w^G\left(\sqrt{t^G} - b_{LI}r^G\left(w^G\right)\right) + \frac{1}{4}w^G\left(\sqrt{t^G} - \delta_I b_{LI} r^G\left(w^G\right)\right).\end{aligned} \tag{C-27}$$

In this case, the market equilibrium is given by $t^{G*} = \frac{1}{16\delta_D b_{LD}} + \frac{49}{16(2(1+\delta_I)b_{LI}+b_{LD})} - \frac{1}{(1+\delta_I)b_{LI}}$, $\Pi^{G*} = \Pi^G\left(\mu^* = 0\right) = \frac{1}{2}\left(\frac{1}{16\delta_D b_{LD}} + \frac{49}{16(2(1+\delta_I)b_{LI}+b_{LD})} - \frac{1}{(1+\delta_I)b_{LI}}\right)^2$, and $\pi^{G*} = \pi^G\left(\mu^* = 0\right) = \frac{((1+\delta_I)b_{LI}+4b_{LD})^2}{16(1+\delta_I)b_{LI}(2(1+\delta_I)b_{LI}+b_{LD})^2}\left(\frac{1}{16\delta_D b_{LD}} + \frac{49}{16(2(1+\delta_I)b_{LI}+b_{LD})} - \frac{1}{(1+\delta_I)b_{LI}}\right)$. ■

Low-Price Guarantee with Alternative Refund Format

Lemma 5 *When the manufacturer offers PBG with refund in the format of a percentage-off discount from the retail price, it is invoked when* $b_{LI} > \frac{\left((7-2\alpha-(1+\alpha)\gamma)\delta_D-(1+\alpha)(1-\gamma)^2\right)b_{LD}}{2(1+\delta_I-\alpha)}$. *The wholesale price as a function of refund depth* γ *is given by*

$$w^P\left(\gamma\right) = \frac{\left(3-(1+\alpha)\gamma\right)\left(1+\delta_I-\alpha\right)b_{LI} - \left(2-\alpha\right)\left(1+\alpha\right)\left(1-\gamma\right)^2 b_{LD}}{\left(1+\delta_I-\alpha\right)b_{LI}\left(2\left(1+\delta_I-\alpha\right)b_{LI} + \left(1+\alpha\right)\left(1-\gamma\right)^2 b_{LD}\right)} \tag{C-28}$$

The retail price as a function of refund depth γ *is given by*

$$r^P\left(\gamma\right) = \frac{7-\left(1+\alpha\right)\gamma - 2\alpha}{2\left(2\left(1+\delta_I-\alpha\right)b_{LI} + \left(1+\alpha\right)\left(1-\gamma\right)^2 b_{LD}\right)}. \tag{C-29}$$

The direct channel price as a function of refund depth γ *is given by*

$$p^P\left(\gamma\right) = \frac{1}{2\delta_D b_{LD}}. \tag{C-30}$$

The retail profit as a function of refund depth γ is given by

$$\pi^P(\gamma) = \frac{\left((1-(1+\alpha)\gamma-2\alpha)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}{16(1+\delta_I-\alpha)b_{LI}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}. \quad \text{(C-31)}$$

The manufacturer profit as a function of refund depth γ is given by

$$\begin{aligned}\Pi^P(\gamma) \;=\; & \frac{1}{16\delta_D b_{LD}} + \frac{(1+\alpha)(1-\gamma)(7-(1+\alpha)\gamma-2\alpha) \times (4(1+\delta_I-\alpha)b_{LI}-(1-\gamma)(5+\gamma-\alpha(4-\gamma))b_{LD})}{16\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2} \quad \text{(C-32)}\\ & + \frac{\left((1-(1+\alpha)\gamma-2\alpha)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right) \times \left((3-(1+\alpha)\gamma)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right)}{8(1+\delta_I-\alpha)b_{LI}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}.\end{aligned}$$

When $0 < b_{LI} < \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, PBG with percentage discount from the retail price is not invoked. When $b_{LI} > \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, PBG with percentage discount from the retail price is not feasible. Otherwise, when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \le b_2$, PBG with percentage discount from the retail price is feasible and invoked in equilibrium. In this case, the optimal refund depth is given by

$$\gamma^* = \frac{3(2-\alpha)b_{LD}-2(1+\delta_I-\alpha)b_{LI}}{3(2-\alpha)b_{LD}}. \quad \text{(C-33)}$$

When the refund depth is chosen at the optimal level $\gamma = \gamma^*$, the equilibrium wholesale price is given by

$$w^{P*} = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}. \quad \text{(C-34)}$$

When the refund depth is chosen at the optimal level $\gamma = \gamma^*$, the equilibrium retail price is given by

$$r^{P*} = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}. \quad \text{(C-35)}$$

When the refund depth is chosen at the optimal level $\gamma = \gamma^*$, the equilibrium direct channel price is given by

$$p^{P*} = \frac{1}{2\delta_D b_{LD}}. \quad \text{(C-36)}$$

When the refund depth is chosen at the optimal level $\gamma = \gamma^*$, the equilibrium retail profit is given by

$$\pi^{P*} = \frac{(2-\alpha)^2}{64\,(1+\delta_I-\alpha)\,b_{LI}}. \tag{C-37}$$

When the refund depth is chosen at the optimal level $\gamma = \gamma^*$, the equilibrium manufacturer profit is given by

$$\Pi^{P*} = \frac{1}{16\delta_D b_{LD}} + \frac{1+\alpha}{16 b_{LD}} + \frac{(2-\alpha)^2}{32\,(1+\delta_I-\alpha)\,b_{LI}}. \tag{C-38}$$

When the refund depth is chosen at the optimal level $\gamma = \gamma^*$, the equilibrium channel profit is given by

$$\Pi_C^{P*} = \frac{1}{16\delta_D b_{LD}} + \frac{1+\alpha}{16 b_{LD}} + \frac{3\,(2-\alpha)^2}{64\,(1+\delta_I-\alpha)\,b_{LI}}. \tag{C-39}$$

Proof of Lemma 5

Proof. Consider LPG with refund in the form of a percentage-off discount from the retail price. Since the model is reduced to the no LPG case when $p^P \le r^P$, we consider the case when $r^P < p^P$. In this case, we have $e^P = (1-\gamma)\,r^P$. In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^P and r^P, respectively:

$$\begin{aligned}\max_{p^P>0} \Pi^P\left(p^P|w^P,\gamma\right) &= \frac{1+\alpha}{4} e^P\left(1-b_{LD}e^P\right) + \frac{1}{4}p^P\left(1-\delta_D b_{LD} p^P\right) \\ &\quad + \frac{1-\alpha}{4} w^P\left(1-b_{LI}r^P\right) + \frac{1}{4} w^P\left(1-\delta_I b_{LI} r^P\right),\end{aligned} \tag{C-40}$$

and

$$\max_{r^P>0} \pi^P\left(r^P|w^P,\gamma\right) = \frac{1-\alpha}{4}\left(r^P-w^P\right)\left(1-b_{LI}r^P\right) + \frac{1}{4}\left(r^P-w^P\right)\left(1-\delta_I b_{LI} r^P\right). \tag{C-41}$$

The above profit maximization problems yield the equilibrium direct channel price $p^{P*} = \frac{1}{2\delta_D b_{LD}}$ and the optimal retail price response function $r^P\left(w^P\right) = \frac{w^P}{2} + \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. The effective price $e^P\left(w^P\right)$ can be written as $e^P\left(w^P\right) = \frac{w^P}{2} + \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$.

In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^P while taking into account both players' stage 3 decisions:

$$\max_{w^P>0} \Pi^P(w^P|\gamma) = \frac{1+\alpha}{4} e^P\left(w^P\right)\left(1-b_{LD}e^P\left(w^P\right)\right)+\frac{1}{4}p^P\left(1-\delta_D b_{LD}p^{P*}\right) + \frac{1-\alpha}{4}w^P\left(1-b_{LI}r^P\left(w^P\right)\right)+\frac{1}{4}w^P\left(1-\delta_I b_{LI}r^P\left(w^P\right)\right). \quad \text{(C-42)}$$

Solving the profit maximization problem yields the wholesale price as a function of refund depth γ, $w^P(\gamma) = \frac{(3-(1+\alpha)\gamma)(1+\delta_I-\alpha)-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}}{(1+\delta_I-\alpha)b_{LI}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)}$, the retail price as a function of refund depth γ, $r^P(\gamma) = r^P(w^P(\gamma),\gamma) = \frac{7-(1+\alpha)\gamma-2\alpha}{2\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)}$, the retail profit as a function of refund depth γ, $\pi^P(\gamma) = \frac{\left((1-(1+\alpha)\gamma-2\alpha)(1+\delta_I-\alpha)-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}{16(1+\delta_I-\alpha)b_{LI}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}$, and the manufacturer profit as a function of refund depth γ,

$$\Pi^P(\gamma) = \frac{1}{16\delta_D b_{LD}} + \frac{(1+\alpha)(1-\gamma)(7-(1+\alpha)\gamma-2\alpha) \times (4(1+\delta_I-\alpha)b_{LI}-(1-\gamma)(5+\gamma-\alpha(4-\gamma))b_{LD})}{16\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2} + \frac{\left((1-(1+\alpha)\gamma-2\alpha)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right) \times \left((3-(1+\alpha)\gamma)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right)}{8(1+\delta_I-\alpha)b_{LI}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}. \quad \text{(C-43)}$$

Note that PBG is invoked in equilibrium only when $r^P(\gamma) < p^{P*}$. This inequality holds when $b_{LI} > \frac{\left((7-2\alpha-(1+\alpha)\gamma)\delta_D-(1+\alpha)(1-\gamma)^2\right)b_{LD}}{2(1+\delta_I-\alpha)}$.

In Stage 1, the manufacturer chooses the refund depth γ:

$$\max_{1\geq\gamma\geq 0} \Pi^P(\gamma) = \frac{1}{16\delta_D b_{LD}} + \frac{(1+\alpha)(1-\gamma)(7-(1+\alpha)\gamma-2\alpha) \times (4(1+\delta_I-\alpha)b_{LI}-(1-\gamma)(5+\gamma-\alpha(4-\gamma))b_{LD})}{16\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2} + \frac{\left((1-(1+\alpha)\gamma-2\alpha)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right) \times \left((3-(1+\alpha)\gamma)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1-\gamma)^2 b_{LD}\right)}{8(1+\delta_I-\alpha)b_{LI}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}. \quad \text{(C-44)}$$

The first-order condition of Π^P with respect to γ is given by

$$\frac{\partial \Pi^P}{\partial \gamma} = \frac{(1+\alpha)(7-(1+\alpha)\gamma - 2\alpha)(2(1+\delta_I-\alpha)b_{LI} - 3(2-\alpha)(1-\gamma)b_{LD})}{8\left(2(1+\delta_I-\alpha)b_{LI} + (1+\alpha)(1-\gamma)^2 b_{LD}\right)^2}. \quad \text{(C-45)}$$

The unique solution that satisfies $\frac{\partial \Pi^P}{\partial \gamma} = 0$ and $1 > \gamma > 0$ is $\overline{\gamma} = \frac{3(2-\alpha)b_{LD} - 2(1+\delta_I-\alpha)b_{LI}}{3(2-\alpha)b_{LD}}$. On evaluating the second-order condition, we have

$$\begin{aligned} \frac{\partial^2 \Pi^P}{\partial \gamma^2}|_{\gamma=\overline{\gamma}} &= -\frac{81(2-\alpha)^4(1+\alpha)b_{LD}^2}{32(1+\delta_I-\alpha)^2 b_{LI}^2\left((1+\alpha)(1+\delta_I-\alpha)b_{LI} + 9(2-\alpha)^2 b_{LD}\right)} \\ &< 0 \end{aligned} \quad \text{(C-46)}$$

Thus, there exists a unique optimal refund depth γ^* that maximizes Π^P:

$$\gamma^* = \overline{\gamma} = \frac{3(2-\alpha)b_{LD} - 2(1+\delta_I-\alpha)b_{LI}}{3(2-\alpha)b_{LD}}. \quad \text{(C-47)}$$

When $\gamma^* = \overline{\gamma}$, or equivalently $b_{LI} < \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, the market equilibrium is given by $w^{P*} = w^P(\gamma^* = \overline{\gamma}) = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$, the equilibrium retail price is given by $r^{P*} = r^P(\gamma^* = \overline{\gamma}) = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$, the equilibrium direct channel price is given by $p^{P*} = \frac{1}{2\delta_D b_{LD}}$, the equilibrium retail profit is given by $\pi^{P*} = \pi^P(\gamma^* = \overline{\gamma}) = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$, and the equilibrium manufacturer profit is given by $\Pi^{P*} = \Pi^P(\gamma^* = \overline{\gamma}) = \frac{1}{32}\left(\frac{2}{\delta_D b_{LD}} + \frac{2(1+\alpha)}{b_{LD}} + \frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}\right)$. It is easy to see that the equilibrium outcome is the same as that under LPG with fixed amount of refund in Section 4.2. ■

Lemma 6 *When the manufacturer offers PBG with refund in the format of a percentage-off discount from the price difference between the two channels, it is invoked when*

$$(1-(1+\alpha)\rho)(1+\delta_I-\alpha)b_{LI} > \begin{pmatrix} (7-2\alpha+(1+\alpha)\rho)\delta_D \\ -(1+\alpha)(1+(7-3\alpha)\rho) \end{pmatrix} b_{LD}.$$

The channel equilibrium can be characterized as follows. The wholesale price as a function *of refund depth* ρ *is given by*

$$w^V(\rho) = \frac{\begin{array}{c}\delta_D\left((3+\rho+\alpha\rho)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1+\rho)^2 b_{LD}\right)\\ -\rho(1-\alpha)(1+\delta_I-\alpha)(1+(3-\alpha)\rho)b_{LI}\end{array}}{(1+\delta_I-\alpha)b_{LI}\left(\begin{array}{c}\delta_D\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD}\right)\\ +2\rho^2(1+\alpha)(1+\delta_I-\alpha)b_{LI}\end{array}\right)} \tag{C-48}$$

The retail price as a function of refund depth ρ *is given by*

$$r^V(\rho) = \frac{(\alpha(\rho-2)+\rho+7)\delta_D+(1+\alpha)\rho(1+(7-3\alpha)\rho)}{\begin{array}{c}2\delta_D(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD})\\ +4(1+\alpha)\rho^2(1+\delta_I-\alpha)b_{LI}\end{array}}. \tag{C-49}$$

The direct channel price as a function of refund depth ρ *is given by*

$$p^V(\rho) = \frac{\begin{array}{c}2(1-\rho-\alpha\rho)(1+\delta_I-\alpha)b_{LI}\\ +(1+\alpha)(1+\rho)(1+7\rho-3\alpha\rho)b_{LD}\end{array}}{\begin{array}{c}2\delta_D b_{LD}(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD})\\ +4(1+\alpha)\rho^2(1+\delta_I-\alpha)b_{LD}b_{LI}\end{array}}. \tag{C-50}$$

The retail profit as a function of refund depth ρ *is given by*

$$\pi^V(\rho) = \frac{\left(\begin{array}{c}\delta_D b_{LD}((1-\rho-2\alpha-\alpha\rho)(1+\delta_I-\alpha)b_{LI}+2(2-\alpha)(1+\alpha)(1+\rho)^2 b_{LD})\\ +(1+\alpha)\rho(1-\rho+\alpha\rho)(1+\delta_I-\alpha)b_{LD}b_{LI}\end{array}\right)^2}{16(1+\delta_I-\alpha)b_{LI}\left(\begin{array}{c}\delta_D b_{LD}(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD})\\ +2(1+\alpha)\rho^2(1+\delta_I-\alpha)b_{LD}b_{LI}\end{array}\right)^2}. \tag{C-51}$$

The manufacturer profit as a function of refund depth ρ is given by

$$\Pi^V(\rho) = \frac{\begin{array}{c} \delta_D b_{LD} \left(\begin{array}{c} (1+\delta_I-\alpha) b_{LI} \left(\rho^2+14\rho+17+2\alpha\left(\rho^2+5\rho+2\right)-\alpha^2\left(4+4\rho-\rho^2\right)\right) \\ -4(2-\alpha)^2(1+\alpha)(1+\rho)^2 b_{LD} \end{array} \right) \\ +(1+\delta_I-\alpha) b_{LI} \left(\begin{array}{c} 2(1-\rho-\alpha\rho)^2(1+\delta_I-\alpha) b_{LI} \\ +(1+\alpha)\left(1+14\rho+17\rho^2-\alpha\rho(6+10\rho-\alpha\rho)\right) b_{LD} \end{array} \right) \end{array}}{16(1+\delta_I-\alpha) b_{LI} \left(\begin{array}{c} \delta_D b_{LD}\left(2(1+\delta_I-\alpha) b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD}\right) \\ +2(1+\alpha)\rho^2(1+\delta_I-\alpha) b_{LD} b_{LI} \end{array} \right)}. \tag{C-52}$$

When $0 < b_{LI} < \frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, PBG with percentage discount from the price difference is not invoked. When $b_{LI} > \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, PBG with percentage discount from the price difference is not feasible. Otherwise, when $\frac{3\delta_D(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)} < b_{LI} \leq b_2$, PBG with percentage discount from the price difference is feasible and invoked in equilibrium. In this case, the optimal refund depth is given by

$$\rho^* = \frac{\delta_D b_{LD}\left(3(2-\alpha)b_{LD} - 2(1+\delta_I-\alpha) b_{LI}\right)}{b_{LD}\left(2(1+\delta_I-\alpha) b_{LI} - 3\delta_D(2-\alpha)b_{LD}\right)}. \tag{C-53}$$

When the refund depth is chosen at the optimal level $\rho = \rho^*$, the equilibrium wholesale price is given by

$$w^{V*} = \frac{2-\alpha}{2(1+\delta_I-\alpha) b_{LI}}. \tag{C-54}$$

When the refund depth is chosen at the optimal level $\rho = \rho^*$, the equilibrium retail price is given by

$$r^{V*} = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha) b_{LI}}. \tag{C-55}$$

When the refund depth is chosen at the optimal level $\rho = \rho^*$, the equilibrium direct channel price is given by

$$p^{V*} = \frac{1}{2\delta_D b_{LD}}. \tag{C-56}$$

When the refund depth is chosen at the optimal level $\rho = \rho^*$, the equilibrium retail profit is given by

$$\pi^{V*} = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha) b_{LI}}. \tag{C-57}$$

When the refund depth is chosen at the optimal level $\rho = \rho^*$, the equilibrium manufacturer *profit is given by*

$$\Pi^{V*} = \frac{1}{16\delta_D b_{LD}} + \frac{1+\alpha}{16 b_{LD}} + \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha) b_{LI}}. \tag{C-58}$$

When the refund depth is chosen at the optimal level $\rho = \rho^*$, the equilibrium channel profit is given by

$$\Pi_C^{V*} = \frac{1}{16\delta_D b_{LD}} + \frac{1+\alpha}{16 b_{LD}} + \frac{3(2-\alpha)^2}{64(1+\delta_I-\alpha) b_{LI}}. \tag{C-59}$$

Proof of Lemma 6

Proof. Consider LPG with refund in the form of a percentage-off discount from the price difference between both channels. Since the model is reduced to the no LPG case when $p^V \leq r^V$, we consider the case when $p^V > r^V$. In this case, we have $e^V = r^V - \rho\left(p^V - r^V\right)$. In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^V and r^V, respectively:

$$\begin{aligned} \max_{p^V>0} \Pi^V\left(p^V | w^V, \rho\right) &= \frac{1+\alpha}{4} e^V\left(1 - b_{LD} e^V\right) + \frac{1}{4} p^V\left(1 - \delta_D b_{LD} p^V\right) \\ &\quad + \frac{1-\alpha}{4} w^V\left(1 - b_{LI} r^V\right) + \frac{1}{4} w^V\left(1 - \delta_I b_{LI} r^V\right), \end{aligned} \tag{C-60}$$

and

$$\max_{r^V>0} \pi^V\left(r^V | w^V\right) = \frac{1-\alpha}{4}\left(r^V - w^V\right)\left(1 - b_{LI} r^V\right) + \frac{1}{4}\left(r^V - w^V\right)\left(1 - \delta_I b_{LI} r^V\right). \tag{C-61}$$

The above profit maximization problems yield the optimal direct channel price response function $p^V\left(w^V, \rho\right) = \frac{\left(1-\rho-\alpha\rho+\rho(1+\alpha)(1+\rho) w^V b_{LD}\right)(1+\delta_I-\alpha) b_{LI} + \rho(1+\alpha)(2-\alpha)(1+\rho) b_{LD}}{2(1+\delta_I-\alpha)((1+\alpha)\rho^2+\delta_D) b_{LI} b_{LD}}$ and the optimal retail price response function $r^V\left(w^V\right) = \frac{w^V}{2} + \frac{2-\alpha}{2(1+\delta_I-\alpha) b_{LI}}$.

In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^V while taking into account both players' stage-three decisions:

$$\begin{aligned} \max_{w^V>0} \Pi^V(w^V | \rho) &= \frac{1+\alpha}{4} e^V\left(w^V, \rho\right)\left(1 - b_{LD} e^V\left(w^V, \rho\right)\right) + \frac{1}{4} p^V\left(w^V, \rho\right)\left(1 - \delta_D b_{LD} p^V\left(w^V, \rho\right)\right) \\ &\quad + \frac{1-\alpha}{4} w^V\left(1 - b_{LI} r^V\left(w^V\right)\right) + \frac{1}{4} w^V\left(1 - \delta_I b_{LI} r^V\left(w^V\right)\right). \end{aligned} \tag{C-62}$$

Solving the maximization problem yields the wholesale price as a function of refund depth ρ,

$w^V(\rho) = \frac{\delta_D\left((3+\rho+\alpha\rho)(1+\delta_I-\alpha)b_{LI}-(2-\alpha)(1+\alpha)(1+\rho)^2 b_{LD}\right)-\rho(1-\alpha)(1+\delta_I-\alpha)(1+(3-\alpha)\rho)b_{LI}}{(1+\delta_I-\alpha)b_{LI}\left(\delta_D\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD}\right)+2\rho^2(1+\alpha)(1+\delta_I-\alpha)b_{LI}\right)}$, the retail price as a function of refund depth ρ,

$$\begin{aligned} r^V(\rho) &= r^V(w^V(\rho)) \\ &= \frac{(\alpha(\rho-2)+\rho+7)\,\delta_D + (1+\alpha)\,\rho\,(1+(7-3\alpha)\,\rho)}{\begin{array}{c} 2\delta_D\,(2\,(1+\delta_I-\alpha)\,b_{LI} + (1+\alpha)(1+\rho)^2 b_{LD}) \\ +4(1+\alpha)\rho^2\,(1+\delta_I-\alpha)\,b_{LI} \end{array}}, \end{aligned} \tag{C-63}$$

the direct channel price as a function of refund depth ρ,

$$\begin{aligned} p^V(\rho) &= p^V(w^V(\rho),\rho) \\ &= \frac{\begin{array}{c} 2(1-\rho-\alpha\rho)\,(1+\delta_I-\alpha)\,b_{LI} \\ +(1+\alpha)(1+\rho)(1+7\rho-3\alpha\rho)b_{LD} \end{array}}{\begin{array}{c} 2\delta_D b_{LD}\,(2\,(1+\delta_I-\alpha)\,b_{LI} + (1+\alpha)(1+\rho)^2 b_{LD}) \\ +4(1+\alpha)\rho^2\,(1+\delta_I-\alpha)\,b_{LD} b_{LI} \end{array}}, \end{aligned} \tag{C-64}$$

the retail profit as a function of refund depth ρ,

$$\pi^V(\rho) = \frac{\left(\begin{array}{c} \delta_D b_{LD}\,((1-\rho-2\alpha-\alpha\rho)\,(1+\delta_I-\alpha)\,b_{LI} + 2(2-\alpha)(1+\alpha)(1+\rho)^2 b_{LD}) \\ +(1+\alpha)\rho(1-\rho+\alpha\rho)\,(1+\delta_I-\alpha)\,b_{LD} b_{LI} \end{array}\right)^2}{16\,(1+\delta_I-\alpha)\,b_{LI}\left(\begin{array}{c} \delta_D b_{LD}\,(2\,(1+\delta_I-\alpha)\,b_{LI} + (1+\alpha)(1+\rho)^2 b_{LD}) \\ +2(1+\alpha)\rho^2\,(1+\delta_I-\alpha)\,b_{LD} b_{LI} \end{array}\right)^2}, \tag{C-65}$$

and the manufacturer profit as a function of refund depth ρ,

$$\Pi^V(\rho) = \frac{\begin{array}{c} \delta_D b_{LD}\left(\begin{array}{c} (1+\delta_I-\alpha)\,b_{LI}\,(\rho^2+14\rho+17+2\alpha\,(\rho^2+5\rho+2)-\alpha^2\,(4+4\rho-\rho^2)) \\ -4\,(2-\alpha)^2\,(1+\alpha)\,(1+\rho)^2\,b_{LD} \end{array}\right) \\ +\,(1+\delta_I-\alpha)\,b_{LI}\left(\begin{array}{c} 2\,(1-\rho-\alpha\rho)^2\,(1+\delta_I-\alpha)\,b_{LI} \\ +(1+\alpha)\,(1+14\rho+17\rho^2-\alpha\rho\,(6+10\rho-\alpha\rho))\,b_{LD} \end{array}\right) \end{array}}{16\,(1+\delta_I-\alpha)\,b_{LI}\left(\begin{array}{c} \delta_D b_{LD}\,(2\,(1+\delta_I-\alpha)\,b_{LI} + (1+\alpha)(1+\rho)^2 b_{LD}) \\ +2(1+\alpha)\rho^2\,(1+\delta_I-\alpha)\,b_{LD} b_{LI} \end{array}\right)}. \tag{C-66}$$

To ensure that PBG is invoked in equilibrium, we need $p^V(\rho) > r^V(\rho)$. The inequality holds when $(1-(1+\alpha)\rho)(1+\delta_I-\alpha)b_{LI} > ((7-2\alpha+(1+\alpha)\rho)\delta_D-(1+\alpha)(1+(7-3\alpha)\rho))b_{LD}$.

In Stage 1, the manufacturer chooses the refund depth ρ:

$$\max_{\rho\geq 0}\Pi^V(\rho) = \frac{\begin{array}{c}\delta_D b_{LD}\left(\begin{array}{c}(1+\delta_I-\alpha)b_{LI}\left(\rho^2+14\rho+17+2\alpha\left(\rho^2+5\rho+2\right)-\alpha^2\left(4+4\rho-\rho^2\right)\right)\\ -4(2-\alpha)^2(1+\alpha)(1+\rho)^2 b_{LD}\end{array}\right)\\ +(1+\delta_I-\alpha)b_{LI}\left(\begin{array}{c}2(1-\rho-\alpha\rho)^2(1+\delta_I-\alpha)b_{LI}\\ +(1+\alpha)\left(1+14\rho+17\rho^2-\alpha\rho(6+10\rho-\alpha\rho)\right)b_{LD}\end{array}\right)\end{array}}{16(1+\delta_I-\alpha)b_{LI}\left(\begin{array}{c}\delta_D b_{LD}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD}\right)\\ +2(1+\alpha)\rho^2(1+\delta_I-\alpha)b_{LD}b_{LI}\end{array}\right)} \tag{C-67}$$

The first-order condition of Π^V with respect to ρ is given by

$$\frac{\partial\Pi^P}{\partial\rho} = \frac{\begin{array}{c}\delta_D b_{LD}\left(\begin{array}{c}(1+\delta_I-\alpha)b_{LI}\left(\rho^2+14\rho+17+2\alpha\left(\rho^2+5\rho+2\right)-\alpha^2\left(4+4\rho-\rho^2\right)\right)\\ -4(2-\alpha)^2(1+\alpha)(1+\rho)^2 b_{LD}\end{array}\right)\\ +(1+\delta_I-\alpha)b_{LI}\left(\begin{array}{c}2(1-\rho-\alpha\rho)^2(1+\delta_I-\alpha)b_{LI}\\ +(1+\alpha)\left(1+14\rho+17\rho^2-\alpha\rho(6+10\rho-\alpha\rho)\right)b_{LD}\end{array}\right)\end{array}}{16(1+\delta_I-\alpha)b_{LI}\left(\begin{array}{c}\delta_D b_{LD}\left(2(1+\delta_I-\alpha)b_{LI}+(1+\alpha)(1+\rho)^2 b_{LD}\right)\\ +2(1+\alpha)\rho^2(1+\delta_I-\alpha)b_{LD}b_{LI}\end{array}\right)}. \tag{C-68}$$

Two solutions that satisfy $\frac{\partial\Pi^V}{\partial\rho}=0$ are $\overline{\rho}=\frac{\delta_D(3(2-\alpha)b_{LD}-2(1+\delta_I-\alpha)b_{LI})}{2(1+\delta_I-\alpha)b_{LI}-3\delta_D(2-\alpha)b_{LD}}$ and

$$\overline{\overline{\rho}} = \frac{2(1+\delta_I-\alpha)b_{LI}+(1+\alpha-\delta_D(7-2\alpha))b_{LD}}{(\alpha+1)\left(2(1+\delta_I-\alpha)b_{LI}+(\delta_D-7+3\alpha)b_{LD}\right)}. \tag{C-69}$$

On evaluating the second-order condition, we have

$$\frac{\partial^2\Pi^V}{\partial\rho^2}|_{\rho=\overline{\rho}} < 0 \tag{C-70}$$

and

$$\frac{\partial^2\Pi^V}{\partial\rho^2}|_{\rho=\overline{\overline{\rho}}} > 0. \tag{C-71}$$

Thus, there exists a unique optimal refund depth ρ^* that maximizes Π^V:

$$\rho^* = \overline{\rho} = \frac{\delta_D b_{LD}\left(3(2-\alpha)b_{LD} - 2\left(1+\delta_I-\alpha\right)b_{LI}\right)}{b_{LD}\left(2\left(1+\delta_I-\alpha\right)b_{LI} - 3\delta_D(2-\alpha)b_{LD}\right)}. \tag{C-72}$$

Otherwise, when $\rho^* = \overline{\rho}$, or equivalently $b_{LI} < \frac{3(2-\alpha)b_{LD}}{2(1+\delta_I-\alpha)}$, the market equilibrium is given by $w^{V*} = w^V\left(\rho^* = \overline{\rho}\right) = \frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$, the equilibrium retail price is given by $r^{V*} = r^V\left(\rho^* = \overline{\rho}\right) = \frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$, the equilibrium direct channel price is given by $p^{V*} = \frac{1}{2\delta_D b_{LD}}$, the equilibrium retail profit is given by $\pi^{V*} = \pi^V\left(\rho^* = \overline{\rho}\right) = \frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$, and the equilibrium manufacturer profit is given by $\Pi^{V*} = \Pi^V\left(\rho^* = \overline{\rho}\right) = \frac{1}{16\delta_D b_{LD}} + \frac{1+\alpha}{16 b_{LD}} + \frac{(2-\alpha)^2}{32(1+\delta_I-\alpha)b_{LI}}$. It is easy to see that the equilibrium outcome is the same as that under LPG with fixed amount of refund in Section 4.2. ■

Proof of Result 2

Proof. The proof of Result 2 directly follows from the proofs of Lemma 5 and 6. ■

Proof of Result 3

Proof. Note that under PMG, the extra refund depth is fixed at $\mu = 0$. Thus, the manufacturer does not need to choose the refund depth. Put differently, the equilibrium outcome is the same no matter when the manufacturer sets the refund depth. Hence, in this proof, we restrict our attention to the case of PBG where $\mu > 0$. This implies that $e^G = r^G - \mu < r^G$, as it is the case under PBG. Since the refund depth is announced before consumers make purchase decision, alternative game sequences do not influence consumers' purchase behavior. Thus, the profit functions of the manufacturer and retailer are

$$\begin{aligned} \Pi^G &= \frac{1+\alpha}{4} e^G\left(1 - b_{LD}e^G\right) + \frac{1}{4}p^G\left(1 - \delta_D b_{LD} p^G\right) \\ &\quad + \frac{1-\alpha}{4} w^G\left(1 - b_{LI}r^G\right) + \frac{1}{4}w^G\left(1 - \delta_I b_{LI} r^G\right), \end{aligned} \tag{C-73}$$

and

$$\pi^G = \frac{1-\alpha}{4}\left(r^G - w^G\right)\left(1 - b_{LI}r^G\right) + \frac{1}{4}\left(r^G - w^G\right)\left(1 - \delta_I b_{LI} r^G\right), \tag{C-74}$$

respectively. We first evaluate the game sequence in which the refund depth is chosen after firms simultaneously decide the direct channel price p and the retail price r.

In Stage 4, the manufacturer maximizes its profit by choosing the refund depth μ:

$$\begin{aligned}\max_{\mu>0}\Pi^G\left(\mu|w^G,p^G,r^G\right) &= \frac{1+\alpha}{4}e^G\left(1-b_{LD}e^G\right)+\frac{1}{4}p^G\left(1-\delta_D b_{LD}p^G\right)\\ &+\frac{1-\alpha}{4}w^G\left(1-b_{LI}r^G\right)+\frac{1}{4}w^G\left(1-\delta_I b_{LI}r^G\right), \quad \text{(C-75)}\end{aligned}$$

Solving the profit maximization problem, we find that the refund depth as a function of retail price is given by $\mu\left(r^G\right)=r^G-\frac{1}{2b_{LD}}$. It follows that the equilibrium effective price is given by $e^{G*}=r^G-\mu\left(r^G\right)=\frac{1}{2b_{LD}}$. Going back to Stage 3, the manufacturer and retailer simultaneously maximize their profits by choosing p^G and r^G, respectively:

$$\begin{aligned}\max_{p^G>0}\Pi^G\left(p^G|w^G\right) &= \frac{1+\alpha}{4}e^{G*}\left(1-b_{LD}e^{G*}\right)+\frac{1}{4}p^G\left(1-\delta_D b_{LD}p^G\right)\\ &+\frac{1-\alpha}{4}w^G\left(1-b_{LI}r^G\right)+\frac{1}{4}w^G\left(1-\delta_I b_{LI}r^G\right), \quad \text{(C-76)}\end{aligned}$$

and

$$\max_{r^G>0}\pi^G\left(r^G|w^G\right)=\frac{1-\alpha}{4}\left(r^G-w^G\right)\left(1-b_{LI}r^G\right)+\frac{1}{4}\left(r^G-w^G\right)\left(1-\delta_I b_{LI}r^G\right). \quad \text{(C-77)}$$

The above profit maximization problems yield the equilibrium direct channel price $p^{G*}=\frac{1}{2\delta_D b_{LD}}$ and the optimal retail price response function $r^G\left(w^G\right)=\frac{w^G}{2}+\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^G while taking into account both players' stage-three decisions:

$$\begin{aligned}\max_{w^G>0}\Pi^G(w^G) &= \frac{1+\alpha}{4}e^{G*}\left(1-b_{LD}e^{G*}\right)+\frac{1}{4}p^{G*}\left(1-\delta_D b_{LD}p^{G*}\right)\\ &+\frac{1-\alpha}{4}w^G\left(1-b_{LI}r^G\left(w^G\right)\right)+\frac{1}{4}w^G\left(1-\delta_I b_{LI}r^G\left(w^G\right)\right). \quad \text{(C-78)}\end{aligned}$$

Solving this profit maximization problem, we find that the equilibrium wholesale price is given by $w^{G*}=\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. Hence, the market equilibrium is given by $w^{G*}=\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$, $r^{G*}=r^G\left(w^G=w^{G*}\right)=\frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$, $p^{G*}=\frac{1}{2\delta_D b_{LD}}$, $\pi^{G*}=\pi^G\left(w^G=w^{G*}\right)=\frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$, and $\Pi^{G*}=\Pi^G\left(w^G=w^{G*}\right)=\frac{1}{32}\left(\frac{2}{\delta_D b_{LD}}+\frac{2(1+\alpha)}{b_{LD}}+\frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}\right)$. One can easily verify that the equilibrium outcome is the same as in Lemma 2.

Next, we evaluate the game sequence in which the refund depth is chosen simultaneously with the direct channel price and retail price.

In Stage 3, the manufacturer and the retailer simultaneously maximize their profits by choosing p^G and μ and r^G, respectively:

$$\max_{p^G>0,\mu>0} \Pi^G\left(p^G,\mu|w^G\right) = \frac{1+\alpha}{4}e^{G*}\left(1-b_{LD}e^{G*}\right)+\frac{1}{4}p^G\left(1-\delta_D b_{LD}p^G\right) + \frac{1-\alpha}{4}w^G\left(1-b_{LI}r^G\right)+\frac{1}{4}w^G\left(1-\delta_I b_{LI}r^G\right), \quad \text{(C-79)}$$

and

$$\max_{r^G>0}\pi^G\left(r^G|w^G\right)=\frac{1-\alpha}{4}\left(r^G-w^G\right)\left(1-b_{LI}r^G\right)+\frac{1}{4}\left(r^G-w^G\right)\left(1-\delta_I b_{LI}r^G\right). \quad \text{(C-80)}$$

The above profit maximization problems yield the optimal refund depth response function $\mu\left(w^G\right)=\frac{w^G}{2}+\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}-\frac{1}{2b_{LD}}$, the equilibrium direct channel price $p^{G*}=\frac{1}{2\delta_D b_{LD}}$, and the optimal retail price response function $r^G\left(w^G\right)=\frac{w^G}{2}+\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. It follows that the equilibrium effective price is given by $e^{G*}=r^G\left(w^G\right)-\mu\left(w^G\right)=\frac{1}{2b_{LD}}$. In Stage 2, the manufacturer maximizes its profit by choosing the wholesale price w^G while taking into account both players' stage 3 decisions:

$$\max_{w^G>0}\Pi^G(w^G) = \frac{1+\alpha}{4}e^{G*}\left(1-b_{LD}e^{G*}\right)+\frac{1}{4}p^{G*}\left(1-\delta_D b_{LD}p^{G*}\right) + \frac{1-\alpha}{4}w^G\left(1-b_{LI}r^G\left(w^G\right)\right)+\frac{1}{4}w^G\left(1-\delta_I b_{LI}r^G\left(w^G\right)\right). \quad \text{(C-81)}$$

Solving this profit maximization problem, we find that the equilibrium wholesale price is given by $w^{G*}=\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$. Hence, the market equilibrium is given by $w^{G*}=\frac{2-\alpha}{2(1+\delta_I-\alpha)b_{LI}}$, $r^{G*}=r^G\left(w^G=w^{G*}\right)=\frac{3(2-\alpha)}{4(1+\delta_I-\alpha)b_{LI}}$, $p^{G*}=\frac{1}{2\delta_D b_{LD}}$, $\pi^{G*}=\pi^G\left(w^G=w^{G*}\right)=\frac{(2-\alpha)^2}{64(1+\delta_I-\alpha)b_{LI}}$, and $\Pi^{G*}=\Pi^G\left(w^G=w^{G*}\right)=\frac{1}{32}\left(\frac{2}{\delta_D b_{LD}}+\frac{2(1+\alpha)}{b_{LD}}+\frac{(2-\alpha)^2}{(1+\delta_I-\alpha)b_{LI}}\right)$. One can easily verify that the equilibrium outcome is the same as in Lemma 2. ■