Online Appendix
Designing an Online Retail Marketplace: Leveraging Information from Sponsored Advertising

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Online Appendix

OA1 Detailed Calculations for the Main Model

This online appendix provides more details in calculating consumer choice, sellers' bids and the platform's ad revenue in the main model. Section OA1.1 and Section OA1.2 complete the calculation for consumer choice and the seller's pay-per-impression bid, respectively. Section OA1.3 and Section OA1.4 derive the seller's bid and the platform's ad revenue under pay-per-click bid, respectively.

OA1.1 Solution for Consumer Choice

Below we provide a step-by-step calculation of the three operators defined in Appendix A1.

$$
\begin{aligned}
\widehat{E}^{AO}_{\lambda_{k1},q_2,\lambda_{k2}}(\cdot) =& \mathbb{E}_{\lambda_{k1},q_2,\lambda_{k2}}[\ (\cdot)\ \mathbb{1}_{\{q_1>q_2, Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}],\\
=& \int_0^1\int_0^1\int_0^1 (\cdot)\ \cdot \mathbb{1}_{\{q_2<q_1, q_2<q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})\}}\ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
=& \int_0^1\int_0^{\lambda_{k1}}\int_0^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1} + \int_0^1\int_{\lambda_{k1}}^1\int_0^{\max\{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1}),0\}} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
=& \int_0^1\int_0^{\lambda_{k1}}\int_0^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1} + \int_0^1\int_{\lambda_{k1}}^{\min\{\lambda_{k1}+\frac{T}{1-T}q_1,1\}}\int_0^{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
=& \int_0^1\int_0^{\lambda_{k1}}\int_0^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1} + \int_0^{\max\{1-\frac{T}{1-T}q_1,0\}}\int_{\lambda_{k1}}^{\lambda_{k1}+\frac{T}{1-T}q_1}\int_0^{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1}\\
&+ \int_{\max\{1-\frac{T}{1-T}q_1,0\}}^1\int_{\lambda_{k1}}^1\int_0^{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
=& \begin{cases}
\begin{aligned}&\int_0^1\int_0^{\lambda_{k1}}\int_0^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1}\\
&+\int_0^{1-\frac{T}{1-T}q_1}\int_{\lambda_{k1}}^{\lambda_{k1}+\frac{T}{1-T}q_1}\int_0^{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1}\\
&+\int_{1-\frac{T}{1-T}q_1}^1\int_{\lambda_{k1}}^1\int_0^{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\end{aligned} & 0<T\le\frac{1}{1+q_1},\\
\int_0^1\int_0^{\lambda_{k1}}\int_0^{q_1} dq_2\ d\lambda_{k2}\ d\lambda_{k1} + \int_0^1\int_{\lambda_{k1}}^1\int_0^{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1}, & T>\frac{1}{1+q_1}.
\end{cases}
\end{aligned}
$$

In arriving at the third equality, note that $q_1 \le q_1 - \frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})$ if and only if $\lambda_{k2}\le\lambda_{k1}$. Furthermore, when $\lambda_{k2}>\lambda_{k1}$, $\max\{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1}),0\}$ is non-zero if and only if $\lambda_{k2}<\lambda_{k1}+\frac{T}{1-T}q_1$. This leads to the fourth equality. The next step stems from the fact that $\min\{\lambda_{k1}+\frac{T}{1-T}q_1,1\}$ is equal to $\lambda_{k1}+\frac{T}{1-T}q_1$ if and only if $\lambda_{k1}\le 1-\frac{T}{1-T}q_1$, and is equal to 1 otherwise. To further simplify the expression, the result depends on the value of q_1. When $T\le\frac{1}{1+q_1}$, we can replace $\max\{1-\frac{T}{1-T}q_1,0\}$ by $1-\frac{T}{1-T}q_1$, and when $T>\frac{1}{1+q_1}$, it is 0. This brings us to equation (A1), which is a well-defined integral in the multiple-dimensional uniformly-distributed parameter space.

$$
\begin{aligned}
&\widehat{E}^{A}_{\lambda_{k1},q_2,\lambda_{k2}}(\cdot)=\mathbb{E}_{\lambda_{k1},q_2,\lambda_{k2}}[\,(\cdot)\ \mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}<Tq_2+(1-T)\lambda_{k2}\}}],\\
&=\int_0^1\int_0^1\int_0^1 (\cdot)\ \mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}<Tq_2+(1-T)\lambda_{k2}\}}\ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&=\int_0^1\int_{\lambda_{k1}}^1\int_{\max\{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1}),0\}}^{q_1} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&=\int_0^1\int_{\lambda_{k1}}^{\min\{\lambda_{k1}+\frac{T}{1-T}q_1,1\}}\int_{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1}+\int_0^1\int_{\min\{\lambda_{k1}+\frac{T}{1-T}q_1,1\}}^1\int_0^{q_1} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&=\int_0^{\max\{1-\frac{T}{1-T}q_1,0\}}\int_{\lambda_{k1}}^{\lambda_{k1}+\frac{T}{1-T}q_1}\int_{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1}\\
&\quad+\int_{\max\{1-\frac{T}{1-T}q_1,0\}}^1\int_{\lambda_{k1}}^1\int_{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1}+\int_0^{\max\{1-\frac{T}{1-T}q_1,0\}}\int_{\lambda_{k1}+\frac{T}{1-T}q_1}^1\int_0^{q_1} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&=\begin{cases}
\int_0^{1-\frac{T}{1-T}q_1}\int_{\lambda_{k1}}^{\lambda_{k1}+\frac{T}{1-T}q_1}\int_{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1} & \\
\quad+\int_{1-\frac{T}{1-T}q_1}^1\int_{\lambda_{k1}}^1\int_{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1} & \\
\quad+\int_0^{1-\frac{T}{1-T}q_1}\int_{\lambda_{k1}+\frac{T}{1-T}q_1}^1\int_0^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1}, & 0<T\le\frac{1}{1+q_1},\\
\int_0^1\int_{\lambda_{k1}}^1\int_{q_1-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1} (\cdot)\ dq_2\ d\lambda_{k2}\ d\lambda_{k1}, & T>\frac{1}{1+q_1}.
\end{cases}
\end{aligned}
$$

$$
\begin{aligned}
&\widehat{E}^{O}_{\lambda_{k1},q_2,\lambda_{k2}}(\cdot)=\mathbb{E}_{\lambda_{k1},q_2,\lambda_{k2}}[\,(\cdot)\ \mathbb{1}_{\{q_1<q_2,Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}],\\
&=\int_0^1\int_0^1\int_0^1 (\cdot)\ \mathbb{1}_{\{q_2>q_1,q_2<q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})\}}\ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&=\int_0^1\int_0^{\lambda_{k1}}\int_{q_1}^{\min\{q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2}),1\}} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&=\int_0^1\int_0^{\max\{\lambda_{k1}-\frac{T}{1-T}(1-q_1),0\}}\int_{q_1}^1 (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1}+\int_0^1\int_{\max\{\lambda_{k1}-\frac{T}{1-T}(1-q_1),0\}}^{\lambda_{k1}}\int_{q_1}^{q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&=\int_{\min\{\frac{T}{1-T}(1-q_1),1\}}^1\int_0^{\lambda_{k1}-\frac{T}{1-T}(1-q_1)}\int_{q_1}^1 (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1}+\int_0^{\min\{\frac{T}{1-T}(1-q_1),1\}}\int_0^{\lambda_{k1}}\int_{q_1}^{q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1},\\
&\quad+\int_{\min\{\frac{T}{1-T}(1-q_1),1\}}^1\int_{\lambda_{k1}-\frac{T}{1-T}(1-q_1)}^{\lambda_{k1}}\int_{q_1}^{q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1}\\
&=\begin{cases}
\int_{\frac{T}{1-T}(1-q_1)}^1\int_0^{\lambda_{k1}-\frac{T}{1-T}(1-q_1)}\int_{q_1}^1 (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1} & \\
\quad+\int_{\frac{T}{1-T}(1-q_1)}^1\int_{\lambda_{k1}-\frac{T}{1-T}(1-q_1)}^{\lambda_{k1}}\int_{q_1}^{q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1} & \\
\quad+\int_0^{\frac{T}{1-T}(1-q_1)}\int_0^{\lambda_{k1}}\int_{q_1}^{q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1}, & 0<T\le\frac{1}{2-q_1},\\
\int_0^1\int_0^{\lambda_{k1}}\int_{q_1}^{q_1+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})} (\cdot)\ \ dq_2\ d\lambda_{k2}\ d\lambda_{k1}, & T>\frac{1}{2-q_1}.
\end{cases}
\end{aligned}
$$

Next, we provide more details in calculating the consumer's expected match probability with seller 2 in scenario (a). A consumer's conditional expected match probability with seller 2 can be obtained by applying the same operator $\widehat{E^{AO}}$ to $\theta_k q_2 + (1-\theta_k)\lambda_{k2}$. This leads to,

$$
\begin{aligned}
&\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k2}\ \mathbb{1}_{\{q_1>q_2, Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}],\\
=&\mathbb{E}_{q_1}\left[\widehat{E^{AO}_{\lambda_{k1},q_2,\lambda_{k2}}}\left(\theta_k q_2 + (1-\theta_k)\lambda_{k2}\right)\right],\\
=&\begin{cases} \frac{3\theta_k T^3+T^3-9\theta_k T^2-10T^2+5\theta_k T+20T-10}{120(T-1)^3}, & 0<T\le\frac{1}{2},\\ \frac{-\theta_k-13\theta_k T^3+49T^3-23T^2+4\theta_k T+4T}{120T^3}, & \frac{1}{2}<T\le 1.\end{cases}
\end{aligned}
$$

Dividing the above expression by the probability of scenario (a) happening, a consumer's expected match utility with seller 2 in this scenario is,

$$
\begin{aligned}
&\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k2}\mid q_1>q_2, Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}],\\
=&\begin{cases} \frac{-10+5T(4+\theta_k)+T^3(1+3\theta_k)-T^2(10+9\theta_k)}{5(-1+T)(6-8T+T^2)}, & 0<T\le\frac{1}{2},\\ \frac{-23T^2+T^3(49-13\theta_k)-\theta_k+4T(1+\theta_k)}{5T-30T^2+85T^3}, & \frac{1}{2}<T\le 1.\end{cases}
\end{aligned}
$$

Now consider scenario (b). In this scenario, again, consumers infer that $q_1 > q_2$ under the belief that sellers' bids increase with their quality. However, the other seller 2 shows up in the first organic results, implying that $Tq_1 + (1-T)\lambda_{k1} < Tq_2 + (1-T)\lambda_{k2}$. We derive $\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\mid q_1 > q_2, Tq_1+(1-T)\lambda_{k1} < Tq_2+(1-T)\lambda_{k2}]$ in a similar way to Scenario (a). The only difference is that we apply operator $\widehat{E}^A$ instead of $\widehat{E}^{AO}$ to $m_{k1} = \theta_k q_1 + (1-\theta_k)\lambda_{k1}$, when seller 1 only wins the ad slot in this case.

$$
\begin{aligned}
&\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\ \mathbb{1}_{\{q_1>q_2, Tq_1+(1-T)\lambda_{k1}<Tq_2+(1-T)\lambda_{k2}\}}],\\
=&\mathbb{E}_{q_1}\left[\widehat{E}^A_{\lambda_{k1},q_2,\lambda_{k2}}\left(\theta_k q_1 + (1-\theta_k)\lambda_{k1}\right)\right],\\
=&\mathbb{E}_{q_1}\begin{cases} f_2(q_1), & 0<T\le\frac{1}{1+q_1},\\ \frac{(T-1)(\theta_k-4\theta_k q_1-1)}{24T}, & T>\frac{1}{1+q_1},\end{cases}\\
=&\begin{cases} \int_0^1 f_2(q_1)\ dq_1, & 0<T\le\frac{1}{2},\\ \int_0^{\frac{1-T}{T}} f_2(q_1)\ dq_1 + \int_{\frac{1-T}{T}}^1 \frac{(T-1)(\theta_k-4\theta_k q_1-1)}{24T}\ dq_1, & T>\frac{1}{2},\end{cases}\\
=&\begin{cases} \frac{-10(\theta_k+1)+13(\theta_k+2)T^3-(39\theta_k+55)T^2+5(7\theta_k+8)T}{120(T-1)^3}, & 0<T\le\frac{1}{2},\\ -\frac{\theta_k+3(\theta_k+2)T^3-7T^2-4\theta_k T+T}{120T^3}, & T>\frac{1}{2},\end{cases}
\end{aligned}
$$

where $f_2(q_1) = \frac{q_1}{24(T-1)^3}\Big(6T\left(-2\theta_k + 2\theta_k q_1^2 + 5\theta_k q_1 + q_1 + 2\right) + T^3\big(-4\theta_k + (3\theta_k+1)q_1^3 + (8\theta_k+4)q_1^2 + 6(\theta_k+1)q_1 + 4\big) - 4T^2\left(-3\theta_k + \theta_k q_1^3 + (5\theta_k+1)q_1^2 + (6\theta_k+3)q_1 + 3\right) + 4(\theta_k - 3\theta_k q_1 - 1)\Big)$ upon evaluating equation (A2).

Next, the probability of Scenario (b) happening $P(q_1 > q_2, Tq_1 + (1-T)\lambda_{k1} < Tq_2 + (1-T)\lambda_{k2})$ is given by,

$$
\begin{aligned}
&\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\mathbb{1}_{\{q_1>q_2, Tq_1+(1-T)\lambda_{k1}<Tq_2+(1-T)\lambda_{k2}\}}],\\
=&\mathbb{E}_{q_1}\left[\widehat{E}^A_{\lambda_{k1},q_2,\lambda_{k2}}(\mathbb{1})\right],\\
=&\begin{cases} \frac{11T^2-16T+6}{24(T-1)^2}, & 0<T\le\frac{1}{2},\\ \frac{-5T^2+6T-1}{24T^2}, & \frac{1}{2}<T\le 1.\end{cases}
\end{aligned}
$$

Given these, a consumer's expected match probability with seller 1 is,

$$\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\ |\ q_1 > q_2, Tq_1 + (1-T)\lambda_{k1} < Tq_2 + (1-T)\lambda_{k2}],$$
$$= \begin{cases} \frac{-10(\theta_k+1)+13(\theta_k+2)T^3-(39\theta_k+55)T^2+5(7\theta_k+8)T}{5(T-1)(11T^2-16T+6)}, & 0 < T \le \frac{1}{2}, \\ \frac{\theta_k-3(\theta_k+2)T^2-3\theta_k T+T}{5(1-5T)T}, & \frac{1}{2} < T \le 1. \end{cases}$$

A consumer's expected utility with seller 2, $\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_2\ \mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k2}<Tq_1+(1-T)\lambda_{k2}\}}]$, is equivalent to $\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\ \mathbb{1}_{\{q_1<q_2,Tq_1+(1-T)\lambda_{k2}>Tq_1+(1-T)\lambda_{k2}\}}]$. The latter can be solved by applying $\hat{E}^O$ (given by equation (A3)) to m_{k1}. This results in,

$$\begin{aligned}
&\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_2\ \mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k2}<Tq_1+(1-T)\lambda_{k2}\}}],\\
=&\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\ \mathbb{1}_{\{q_1<q_2,Tq_1+(1-T)\lambda_{k2}>Tq_1+(1-T)\lambda_{k2}\}}],\\
=&\mathbb{E}_{q_1}\left[\widehat{E}^O_{\lambda_{k1},q_2,\lambda_{k2}}\big(\theta_k q_1 + (1-\theta_k)\,\lambda_{k1}\big)\right],\\
=&\mathbb{E}_{q_1}\begin{cases} f_3(q_1), & 0 < T \le \frac{1}{2-q_1}, \\ -\frac{(T-1)(\theta_k(4q_1-3)+3)}{24T}, & T > \frac{1}{1+q_1}, \end{cases}\\
=&\begin{cases} \int_0^1 f_3(q_1)\ dq_1, & 0 < T \le \frac{1}{2}, \\ \int_0^{\frac{2T-1}{T}} -\frac{(T-1)(\theta_k(4q_1-3)+3)}{24T}\ dq_1 + \int_{\frac{2T-1}{T}}^1 f_3(q_1)\ dq_1, & T > \frac{1}{2}, \end{cases}\\
=&\begin{cases} \frac{10(\theta_k-2)+(29-13\theta_k)T^3+(39\theta_k-80)T^2-35(\theta_k-2)T}{120(T-1)^3}, & 0 < T \le \frac{1}{2}, \\ \frac{(T-1)\left(-\theta_k+(3\theta_k-19)T^2+(3\theta_k+4)T\right)}{120T^3}, & T > \frac{1}{2}, \end{cases}
\end{aligned}$$

where $f_3(q_1) = \frac{(1-q_1)}{24(T-1)^3}\Big(-6T\left(5\theta_k + 2\theta_k q_1^2 - 9\theta_k q_1 + q_1 - 5\right) + T^3\big(-13(\theta_k-1) + (3\theta_k+1)q_1^3 - (17\theta_k + 3)q_1^2 + (31\theta_k-3)q_1\big) - 4T^2\left(-9\theta_k + \theta_k q_1^3 - 8\theta_k q_1^2 + (19\theta_k-3)q_1 + 9\right) + \theta_k(8-12q_1) - 8\Big)$ in the above expression.

Dividing the above expression by the probability of Scenario (b) happening, we find that if clicking on the seller in the first organic slot, consumers expect a utility of,

$$\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_2\ |\ q_1 > q_2, Tq_1 + (1-T)\lambda_{k1} < Tq_2 + (1-T)\lambda_{k2}],$$
$$= \begin{cases} \frac{T^3(29-13\theta_k)+10(-2+\theta_k)-35T(-2+\theta_k)+T^2(-80+39\theta_k)}{5(-1+T)(6-16T+11T^2)}, & 0 < T \le \frac{1}{2}, \\ \frac{T^2(19-3\theta_k)+\theta_k-T(4+3\theta_k)}{5(1-5T)T}, & \frac{1}{2} < T \le 1. \end{cases}$$

OA1.2 Solution for Sellers' Bids under Pay-per-impression

In this section, we provide further details for deriving the seller's bid in Appendix A2. For the second term in Appendix A2, borrowing the results from equation (A2), we have

$$\begin{aligned}
&\frac{\partial}{\partial q_1'}\int_0^{q_1'} \mathbb{E}_{\lambda_{k1},\lambda_{k2},\theta_k>\theta^T}\left[\big(1-\phi\big)\ m_{k1}\ \mathbb{1}_{\{Tq_1'+(1-T)\lambda_{k1}<Tq_2+(1-T)\lambda_{k2}\}}\right]dq_2,\\
=&\frac{\partial}{\partial q_1'}\mathbb{E}_{\theta_k>\theta^T}\left[\mathbb{E}_{q_2,\lambda_{k1},\lambda_{k2}}[\big(1-\phi\big)\ m_{k1}\ \mathbb{1}_{\{Tq_1'+(1-T)\lambda_{k1}<Tq_2+(1-T)\lambda_{k2},q_1'>q_2\}}]\right],\\
=&(1-\phi)\frac{\partial}{\partial q_1'}\mathbb{E}_{\theta_k>\theta^T}\left[\hat{E}^A_{\lambda_{k1},q_2,\lambda_{k2}}(m_{k1})\right],
\end{aligned}$$

$$
=(1-\phi)\frac{\partial}{\partial q_1'}\mathbb{E}_{\theta_k>\theta^T}\begin{cases}
\int_0^{1-\frac{T}{1-T}q_1'}\int_{\lambda_{k1}}^{\lambda_{k1}+\frac{T}{1-T}q_1'}\int_{q_1'-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1'}\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)\,dq_2\,d\lambda_{k2}\,d\lambda_{k1} \\
\quad+\int_{1-\frac{T}{1-T}q_1'}^{1}\int_{\lambda_{k1}}^{1}\int_{q_1'-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1'}\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)dq_2\,d\lambda_{k2}\,d\lambda_{k1} \\
\quad+\int_0^{1-\frac{T}{1-T}q_1'}\int_{\lambda_{k1}+\frac{T}{1-T}q_1'}^{1}\int_0^{q_1'}\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)\,dq_2\,d\lambda_{k2}\,d\lambda_{k1}, & 0<T\le\frac{1}{1+q_1'}, \\
\int_0^1\int_{\lambda_{k1}}^1\int_{q_1'-\frac{1-T}{T}(\lambda_{k2}-\lambda_{k1})}^{q_1'}\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)dq_2\,d\lambda_{k2}\,d\lambda_{k1}, & T>\frac{1}{1+q_1'},
\end{cases}
$$

$$
=(1-\phi)\frac{\partial}{\partial q_1'}\begin{cases}
\frac{q_1'\left(23T^2-30T+10\right)}{192(T-1)(-13T^3+39T^2-35T+10)^2}\Big(4q_1\left(29T^3-103T^2+100T-30\right) \\
\quad\left(\left(q_1'^2+3q_1'+3\right)T^2-3(q_1'+2)T+3\right)+\left(23T^2-30T+10\right) \\
\quad\left(-4\left(q_1'^2+3q_1'+3\right)T^2+\left(q_1'^3+4q_1'^2+6q_1'+4\right)T^3+6(q_1'+2)T-4\right)\Big), & T\le\frac{1}{2}, \\
\frac{q_1'}{192(T-1)(1-3T(T+1))^2}\Big((2-7T)^2((q_1'+2)T-2)\big((q_1'(q_1'+2)+2)T^2 \\
\quad-2(q_1'+2)T+2\big)-4q_1(7T-2)(T(19T+3)-2) \\
\quad\left((q_1'(q_1'+3)+3)T^2-3(q_1'+2)T+3\right)\Big), & \frac{1}{2}<T\le\frac{1}{1+q_1'}, \\
\frac{(T-1)^2(7T-2)(4q_1(T(19T+3)-2)+(9-7T)T-2)}{192T(1-3T(T+1))^2}, & T>\frac{1}{1+q_1'},
\end{cases}
$$

$$
=(1-\phi)\begin{cases}
\frac{\left(23T^2-30T+10\right)(q_1'T+T-1)^2\left(q_1\left(87T^3-309T^2+300T-90\right)+\left(23T^2-30T+10\right)(q_1'T+T-1)\right)}{48(T-1)(-13T^3+39T^2-35T+10)^2}, & T\le\frac{1}{2}, \\
\frac{(7T-2)(q_1'T+T-1)^2\left(q_1\left(-57T^2-9T+6\right)+(7T-2)(q_1'T+T-1)\right)}{48(T-1)(3T^2+3T-1)^2}, & \frac{1}{2}<T\le\frac{1}{1+q_1'}, \\
0, & T>\frac{1}{1+q_1'}.
\end{cases}
$$

For the third term, equation (A3) leads to

$$
\frac{\partial}{\partial q_1'}\int_{q_1'}^1\mathbb{E}_{\lambda_{k1},\lambda_{k2},\theta_k<\theta^T}\left[(1-\phi)\;m_{k1}\;\mathbb{1}_{\{Tq_1'+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}\right]dq_2,
$$

$$
=\frac{\partial}{\partial q_1'}\mathbb{E}_{\theta_k<\theta^T}\left[\mathbb{E}_{q_2,\lambda_{k1},\lambda_{k2}}[(1-\phi)\;m_{k1}\;\mathbb{1}_{\{Tq_1'+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2},q_1'<q_2\}}]\right],
$$

$$
=(1-\phi)\frac{\partial}{\partial q_1'}\mathbb{E}_{\theta_k<\theta^T}\left[\hat{E}^O_{\lambda_{k1},q_2,\lambda_{k2}}(m_{k1})\right],
$$

$$
=\frac{\partial}{\partial q_1'}\mathbb{E}_{\theta_k<\theta^T}\begin{cases}
\int_{\frac{T}{1-T}(1-q_1')}^1\int_0^{\lambda_{k1}-\frac{T}{1-T}(1-q_1')}\int_{q_1'}^1\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)\,dq_2\,d\lambda_{k2}\,d\lambda_{k1} \\
\quad+\int_{\frac{T}{1-T}(1-q_1')}^1\int_{\lambda_{k1}-\frac{T}{1-T}(1-q_1')}^{\lambda_{k1}}\int_{q_1'}^{q_1'+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})}\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)\,dq_2\,d\lambda_{k2}\,d\lambda_{k1} \\
\quad+\int_0^{\frac{T}{1-T}(1-q_1')}\int_0^{\lambda_{k1}}\int_{q_1'}^{q_1'+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})}\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)dq_2\,d\lambda_{k2}\,d\lambda_{k1}, & 0<T\le\frac{1}{2-q_1'}, \\
\int_0^1\int_0^{\lambda_{k1}}\int_{q_1'}^{q_1'+\frac{1-T}{T}(\lambda_{k1}-\lambda_{k2})}\left(\theta_k q_1+(1-\theta_k)\lambda_{k1}\right)dq_2\,d\lambda_{k2}\,d\lambda_{k1}, & T>\frac{1}{2-q_1'},
\end{cases}
$$

$$
=(1-\phi)\frac{\partial}{\partial q_1'}\begin{cases}
\frac{\left(1-q_1'\right)\left(3T^3-25T^2+30T-10\right)}{192(T-1)^3(-13T^3+39T^2-35T+10)^2} \\
\quad\Big(4q_1\left(3T^4-28T^3+55T^2-40T+10\right)\left(\left(q_1'^2-5q_1'+7\right)T^2+3(q_1'-3)T+3\right) \\
\quad+\left(49T^3-131T^2+110T-30\right)\left(\left(q_1'^3-3q_1'^2-3q_1'+13\right)T^3\right. \\
\quad\left.+12(q_1'-3)T^2-6(q_1'-5)T-8\right)\Big), & T\le\frac{1}{2}, \\
\frac{\left(1-q_1'\right)T(13T-3)}{192(T-1)^3(3T^2+3T-1)^2}\Big(T^5\left(52q_1\left(q_1'^2-5q_1'+7\right)-q_1'^3+3q_1'^2+3q_1'-13\right) \\
\quad+T^4\left(q_1\left(-64q_1'^2+476q_1'-916\right)+3\left(5q_1'^3-15q_1'^2-19q_1'+77\right)\right) \\
\quad+2T^3\left(6q_1\left(q_1'^2-21q_1'+68\right)-2q_1'^3+6q_1'^2+99q_1'-311\right) \\
\quad+2T^2(6q_1(3q_1'-25)-69q_1'+301)+12T(3q_1+2q_1'-20)+32\Big), & \frac{1}{2}<T\le\frac{1}{1+q_1'}, \\
-\frac{(T-1)(13T-3)(T(4q_1(13T-3)-3T+45)-12)}{192(1-3T(T+1))^2}, & T>\frac{1}{1+q_1'},
\end{cases}
$$

$$
=(1-\phi)\begin{cases}
\frac{\left(3T^3-25T^2+30T-10\right)((q_1'-2)T+1)^2\left(3q_1(3T^4-28T^3+55T^2-40T+10)+(49T^3-131T^2+110T-30)(q_1'T+T-2)\right)}{48(1-T)^3(-13T^3+39T^2-35T+10)^2}, & T\le\frac{1}{2}, \\
\frac{T(13T-3)((q_1'-2)T+1)^2\left(T^3(-39q_1+q_1'+1)+T^2(48q_1-15q_1'-17)+T(-9q_1+4q_1'+34)-8\right)}{48(T-1)^3(3T^2+3T-1)^2}, & \frac{1}{2}<T\le\frac{1}{1+q_1'}, \\
0, & T>\frac{1}{1+q_1'}.
\end{cases}
$$

Combined together, we have for $\frac{\partial U_{q_1}(q_1')}{\partial q_1'}$:

$$\frac{\partial U_{q_1}(q_1')}{\partial q_1'} = \begin{cases} \frac{1}{4}(2q_1+1), & T > \frac{1}{q_1'+1}, \\ -\frac{3q_1(T-1)\left(\left(q_1'^2+2q_1'-1\right)T^2-2(q_1'-1)T-1\right)-3\left(q_1'^2+2q_1'-2\right)T^2+\left(q_1'^3+3q_1'^2+3q_1'-2\right)T^3+3(q_1'-2)T+2}{12(T-1)^3}, & T \leq \frac{1}{q_1'+1}, \end{cases}$$
$$+ \begin{cases} \frac{(7T-2)(q_1'T+T-1)^2\left(q_1\left(-57T^2-9T+6\right)+(7T-2)(q_1'T+T-1)\right)}{48(T-1)(3T^2+3T-1)^2}, & \frac{1}{2} < T \leq \frac{1}{q_1'+1}, \\ \frac{\left(23T^2-30T+10\right)(q_1'T+T-1)^2\left(q_1\left(87T^3-309T^2+300T-90\right)+\left(23T^2-30T+10\right)(q_1'T+T-1)\right)}{48(T-1)(-13T^3+39T^2-35T+10)^2}, & 0 < T < \frac{1}{2}, \end{cases}$$
$$+ \begin{cases} \frac{T(13T-3)((q_1'-2)T+1)^2\left(T^3(-39q_1+q_1'+1)+T^2(48q_1-15q_1'-17)+T(-9q_1+4q_1'+34)-8\right)}{48(T-1)^3(3T^2+3T-1)^2}, & \frac{1}{2} < T < \frac{1}{2-q_1'}, \\ f_4(q_1), & 0 < T < \frac{1}{2}, \end{cases}$$
$$- b(q_1').$$

where $f_4(q_1) \equiv -\frac{\left(3q_1\left(3T^4-28T^3+55T^2-40T+10\right)+\left(49T^3-131T^2+110T-30\right)(q_1'T+T-2)\right)}{48(T-1)^3(-13T^3+39T^2-35T+10)^2}\left(3T^3-25T^2+30T-10\right)((q_1'-2)T+1)^2$.

OA1.3 Solution for Sellers' Bids under Pay-per-click

In this section, we characterize the solution for the pay-per-click equilibrium bid $b^{CPC}(q)$. From $B^{CPC}(q) = \int_0^q N(q)\ b^{CPC}(q_j)dq_j = \int_0^q b(q_j)dq_j$, we have $\int_0^q b^{CPC}(q_j)dq_j = \frac{\int_0^q b(q_j)dq_j}{N(q)}$. Taking the derivative with respect to q on both sides, we obtain

$$b^{CPC}(q) = \frac{b(q)N(q) - N'(q)\int_0^q\ b(q_i)\ d\ q_i}{N'(q)},$$

where $b(q)$ is a seller's equilibrium pay-per-impression bid, and $N(q)$ is the expected number of clicks a seller obtains from the ad slot. Now we need to find the expression for $N(q)$, which depends on the degree of cannibalization. That is, conditioning on the same seller winning both the ad slot and the organic slot, we assume that $0 \leq \alpha \leq 1$ proportion of consumers click on the ad slot. Then,

$$N(q_i) = \alpha \int_0^{q_i} \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\mathbb{1}_{\{Tq_i+(1-T)\lambda_{ki}>Tq_j+(1-T)\lambda_{kj}\}}\right]dq_j + \int_0^{q_i} \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\mathbb{1}_{\{Tq_i+(1-T)\lambda_{ki}<Tq_j+(1-T)\lambda_{kj},\theta_k>\theta^T\}}\right]dq_j.$$

The analytical solution for $b^{CPC}(q_i)$ can be obtained by substituting the expression for $N(q_i)$ as specified above, and $b(q_i)$ as in the main analysis. Based on contour plot from Mathematica (see Figure OA1), the pay-per-click bid increases with q as long as the degree of cannibalization α is insignificant (and T is not too large as in our main analysis). If α is sufficiently large, sellers with large q may bid 0 to avoid cannibalization, and this contradicts our assumption that the bid function increases with q. Therefore, we restrain our analysis to the case with a limited degree of cannibalization. Then, the bid is qualitatively the same as the pay-per-impression bid in the sense that the bid function increases with q_1 given $T \leq T^{Sales}$.

OA1.4 Platform's Advertising Revenue under Pay-per-click

In this section, we prove that the platform's expected ad revenues under pay-per-click and pay-per-impression are the same. Since our main insights depend on how the platform's two streams of revenues change with T, and both streams of revenues are the same under pay-per-click and pay-per-impression, our key takeaways stay valid.

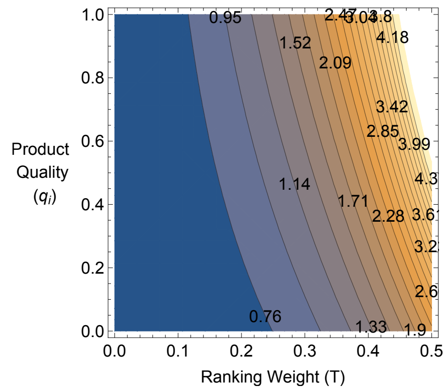

Figure OA1: Sellers' bidding strategy $b(q_i)$ under a pay-per-click auction. In the figure, $\phi = 0.3$, $\alpha = 0$.

Denote by $b^{CPC}(q_i)$ the equilibrium pay-per-click bid of a seller with q_i, and denote by $N(q_i)$ the expected number of clicks he obtains from the ad slot. If a seller with product quality q_i deviates to bid $b^{CPC}(q_i')$, then the expected number of clicks he obtains from the ad slot also changes to $N(q_i')$. Under a pay-per-click auction, we can write his expected profit by deviating to q_i' as follows:

$$\begin{aligned} U_{q_i}^{CPC}(q_i') = & \int_0^{q_i'} \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\left(1-\phi\right)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}>Tq_j+(1-T)\lambda_{kj}\}}\right]dq_j \\ & + \int_0^{q_i'} \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\left(1-\phi\right)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}<Tq_j+(1-T)\lambda_{kj},\theta_k>\theta^T\}}\right]dq_j \\ & + \int_{q_i'}^{1} \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\left(1-\phi\right)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}>Tq_j+(1-T)\lambda_{kj},\theta_k<\theta^T\}}\right]dq_j \\ & - \int_0^{q_i'} N(q_i')\ b^{CPC}(q_j)dq_j. \end{aligned}$$

To facilitate the calculation, we define,

$$B^{CPC}(q_i') \equiv \int_0^{q_i'} N(q_i')\ b^{CPC}(q_j)dq_j.$$

Similar to the analysis for the pay-per-impression case, we take the derivative of $U_{q_i}^{CPC}(q_i')$ with respect to q_i' and obtain,

$$\begin{aligned} \frac{\partial U_{q_i}^{CPC}(q_i')}{\partial q_i'} = & \frac{\partial\left(\int_0^{q_i'} \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\left(1-\phi\right)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}>Tq_j+(1-T)\lambda_{kj}\}}\right]dq_j\right)}{\partial q_i'} \\ & + \frac{\partial\left(\int_0^{q_i'} \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\left(1-\phi\right)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}<Tq_j+(1-T)\lambda_{kj},\theta_k>\theta^T\}}\right]dq_j\right)}{\partial q_i'} \\ & + \frac{\partial\left(\int_{q_i'}^1 \mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\left[\left(1-\phi\right)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}>Tq_j+(1-T)\lambda_{kj},\theta_k<\theta^T\}}\right]dq_j\right)}{\partial q_i'} \\ & - \frac{\partial\left(B^{CPC}(q_i')\right)}{\partial q_i'}. \end{aligned}$$

Notice that the first three items are the same as a seller's sales upon deviation under the pay-per-impression case. They add up to $b(q_i)$ at $q_i' = q_i$, where $b(q_i)$ is the equilibrium pay-per-impression bid we have solved in Section A2 in the Appendix. Since $U_{q_i}^{CPC}(q_i')$ is maximized at $q_i' = q_i$, we have,

$$\frac{\partial U_{q_i}^{CPC}(q_i')}{\partial q_i'}\Big|_{q_i'=q_i} \;=\; b(q_i) - \frac{\partial\Big(B^{CPC}(q_i')\Big)}{\partial q_i'}\Big|_{q_i'=q_i} \;=\; 0.$$

This leads to,

$$\frac{\partial\Big(B^{CPC}(q_i')\Big)}{\partial q_i'}\Big|_{q_i'=q_i} = \frac{\partial\Big(B^{CPC}(q_i)\Big)}{\partial q_i} = b(q_i), \forall q_i \in [0,1].$$

The first equal sign is valid since $B^{CPC}(q_i')$ is not a function of q_i. Since the above expression holds for any q_i, we integrate the two sides by the second equal sign and obtain,

$$B^{CPC}(q) - B^{CPC}(0) = \int_0^q \; b(q_i) \; d \; q_i.$$

Since $B^{CPC}(0) = 0$ by its definition, we have,

$$B^{CPC}(q) = \int_0^q \; b(q_i) \; d \; q_i.$$

The platform's expected advertising revenue under pay-per-click is,

$$2\int_0^1 \int_0^{q_2} N(q_2)\; b^{CPC}(q_1)\; dq_1 \; dq_2 = 2\int_0^1 B^{CPC}(q_2) dq_2 = 2\int_0^1 \int_0^{q_2} \; b(q_1)\; dq_1 \; dq_2.$$

The above expression is the same as the platform's expected ad revenue under pay-per-impression that we characterized in the previous section. Intuitively, when the bid is per-pay-click, each seller bids higher compared with the pay-per-impression bid but pays for clicks, which are fewer. Overall, the platform's ad revenue remains the same. This is because conditional on the winner of the ad auction, consumers' click behaviors remain the same under pay-per-impression and pay-per-click auctions. Then regardless of pay-per-impression or pay-per-click, the ad auctions generate the same value for a seller, and therefore they also lead to the same advertising revenue. Since pay-per-click generates the same expected demand and the same expected advertising revenue for the platform as per-per-impression, all our insights will hold qualitatively when the advertising fee is pay-per-click. For completeness, the analytical solution for the pay-per-click bid $b^{CPC}(q_i)$ is provided in Section OA1.3 in the Online Appendix.

OA1.5 Analysis for Full Range of T

The case for $T < 0$ and $T > 1$ can be solved in a similar way as the case for $0 < T < 1$. Due to the analytical complexity, we produce the contour plot of the platform's profit as a function of T and u_0 under $-3 < T < 3$ in Figure OA2. In this figure, the commission rate is optimally chosen to extract all surplus from a seller with an outside option of u_0, and the aim is to show how the platform's profit changes with T. We can observe that for any u_0, the platform's maximal profit under $T < 0$ or under $T > 1$ is always lower than its maximal profit under $0 \le T \le 1$. As a result, it suffices to focus on the parameter space with $0 \le T \le 1$, which leads to the maximal profit for the platform.

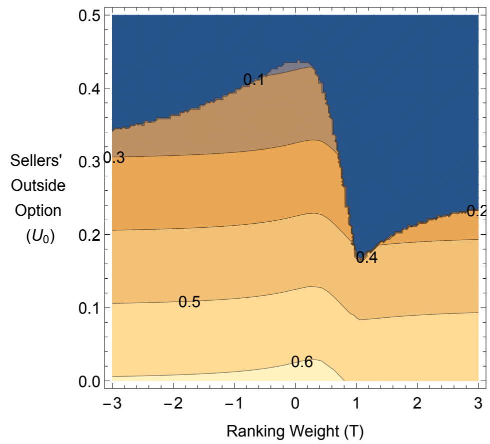

Figure OA2: Contour plots of platform revenue with a full range of T.

OA2 Analysis for Correlation between Platform's Information and Sellers' Information

We first update the consumer's expected match probabilities with the two sellers under Scenario (a) and Scenario (b). Under Scenario (a), the consumer's expected match probability with seller 1 is,

$$\frac{(1-\rho)\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\ \mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}]+\rho\mathbb{E}_{q_1,q_2}[q_1\mathbb{1}_{\{q_1>q_2\}}]}{(1-\rho)\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}]+\rho\mathbb{E}_{q_1,q_2}[\mathbb{1}_{\{q_1>q_2\}}]},$$

$$=\frac{(1-\rho)\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\ \mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}]+\rho\frac{1}{3}}{(1-\rho)\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}]+\rho\frac{1}{2}}.$$

Similarly, under Scenario (a), the consumer's expected match probability with seller 2 is,

$$\frac{(1-\rho)\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\ m_{k1}\ \mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}]+\rho\frac{1}{6}}{(1-\rho)\mathbb{E}_{q_1,\lambda_{k1},q_2,\lambda_{k2}}[\mathbb{1}_{\{q_1>q_2,Tq_1+(1-T)\lambda_{k1}>Tq_2+(1-T)\lambda_{k2}\}}]+\rho\frac{1}{2}}.$$

It can be readily seen that the consumer's match probability with seller 1 is higher than that with seller 2 in Scenario (a), and all consumers will click on seller 1.

In Scenario (b), as we have discussed in the main text, the consumer's conditional match probabilities with the two sellers stay the same as in Section 4.1. Therefore, the cutoff value on θ_k in Scenario (b) also stays the same as in the main analysis.

In terms of the equilibrium bid, we update a seller's expected profits when deviating to bid $b^{Cor}(q_i')$, where "Cor" stands for "with correlation", as follows:

$$\begin{aligned}U_{q_i}^{Cor}(q_i') =&\rho\Big(\int_0^{q_i'}\mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k}\Big[\big(1-\phi\big)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}>Tq_j+(1-T)\lambda_{kj}\}}\Big]dq_j\\ &+\int_0^{q_i'}\mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k>\theta^T}\Big[\big(1-\phi\big)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}<Tq_j+(1-T)\lambda_{kj}\}}\Big]dq_j\\ &+\int_{q_i'}^{1}\mathbb{E}_{\lambda_{ki},\lambda_{kj},\theta_k<\theta^T}\Big[\big(1-\phi\big)\ m_{ki}\ \mathbb{1}_{\{Tq_i'+(1-T)\lambda_{ki}>Tq_j+(1-T)\lambda_{kj}\}}\Big]dq_j\Big)\\ &+(1-\rho)\ \int_0^{q_i'}\big(1-\phi\big)\ m_{ki}\ dq_j-\int_0^{q_i'}b^{Cor}(q_j)dq_j.\end{aligned}$$

From $\frac{\partial U^{Cor}_{q_i}(q_i')}{\partial q_i'}\Big| q_i = q_i' = 0$, we can obtain,

$$b^{Cor}(q_i) = (1-\rho)\ b(q_i) + \rho\ \left(1-\phi\right)\ q_i,$$

where $b(q_i)$ is the seller's bid with $\rho = 0$ solved in Section A2 in the Appendix.

Following a similar logic, the consumer demand generated is equal to $D(T)$ (as given in Section 4.3.1) with a probability of $1-\rho$, and it is equal to $\frac{2}{3}$ with a probability of ρ, i.e., $(1-\rho)D(T)+\rho\frac{2}{3}$. s Then the platform's total revenue is given by $(1-\rho)\Pi(T,\phi)+\rho\left(\phi\frac{2}{3}+(1-\phi)\frac{1}{3}\right) = (1-\rho)\Pi(T,\phi)+\rho\frac{1}{3}(1+\phi)$, and each seller's expected net utility is given by $(1-\rho)U(T,\phi)+\rho\left((1-\phi)\frac{1}{3}-(1-\phi)\frac{1}{6}\right) = (1-\rho)U(T,\phi)+\rho\frac{1}{6}(1-\phi)$.

OA3 Using Platform Information to Rank Sponsored Results

In this section, we present the solution for an alternative scenario where the sponsored results are ranked based on both sellers' bids and the marketplace's information about personal fit. This is similar to the "quality score" adopted by popular search engines in ranking sponsored slots. To show that the platform can still benefit from strategic listing, we solve the case where the sponsored results are ranked based on both sellers' advertising bids (i.e., $b(q_i)$) and the platform's person fit information (i.e., λ_{ki}), but the organic results are ranked solely based on the platform's information about personal fit (i.e., λ_{ki}).

We find that the incremental value of strategic listing, relative to this alternative scenario, increases as consumers favor product quality more on average. To give a comprehensive view, we vary the mean value of θ_k by modifying the distribution of θ_k as $U[\mu, \mu+\frac{1}{2}]$. As Figure OA3 presents, compared with this alternative scenario, strategic listing can still benefit the marketplace. Intuitively, as product quality plays a more important role in consumer purchase, the information effect of strategic listing increases. We acknowledge that in the full scenario, the platform would benefit from using both the sellers' bids to rank organic results and its own personal fit information to rank sponsored results. Due to technical complexities, we do not provide the full solution.

OA4 No Commitment on T

In this section, we discuss the scenario where it may not be feasible to make the platform commit to a predetermined T. Without commitment to its ranking algorithm, the platform will choose an optimal T at $t = 4$, after the sellers make their participation and bidding decisions. We again solve the game using backward induction.

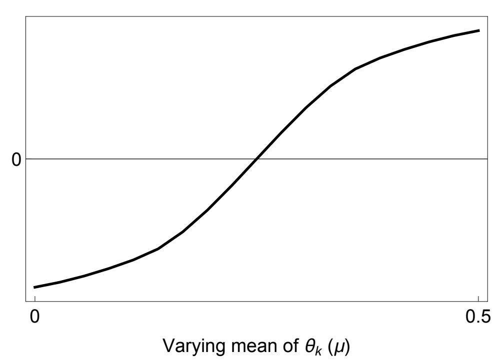

Figure OA3: Differences in platform revenue between the main case, and the alternative case where platform information is used to rank sponsored results. $\theta_k \sim U[\mu, \mu+\frac{1}{2}]$. $u_0 = 0.2$.

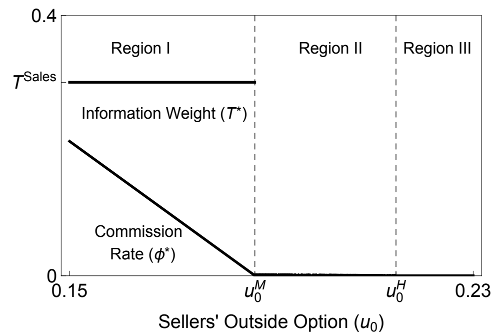

Figure OA4: Optimal information weight (T) and commission rate (ϕ) design when the platform does not commit to T. The regions are defined the same as in Figure 6. $\theta_k \sim U[0, 1]$.

The behavior of the consumers (at $t = 5$) is as derived in the main model. Next, we derive the equilibrium T chosen by the platform at $t = 4$. Notice that at this moment the platform's advertising revenue is fixed as sellers have already submitted their bids. Therefore, the platform will choose T to maximize product sales, which is equal to T^{Sales} according to Section 4.2. Recall that in our main analysis, when the sellers' outside option u_0 is small (i.e., in Region I in Figure 6, where $u_0 \leq u_0^M$) the platform also commits to information weight $T^* = T^{Sales}$ at the beginning of the game. Therefore, in this parameter space, our results from the main model hold unchanged.

However, unlike in the main analysis, sellers do not participate when their outside option is intermediate (i.e., in Region II in Figure 6, where $u_0^M \leq u_0 \leq u_0^H$). In this region, under the main analysis, the platform commits to $T^* < T^{Sales}$ at $t = 0$ to ensure sellers' participation. In the current scenario, however, listing strategies are decided at $t = 4$, by which time sellers' participation has already been decided. Once sellers decide to join, the platform will set $T^* = T^{Sales}$ to maximize product sales. This will leave sellers a smaller surplus than their outside option. Anticipating this, sellers will not join at $t = 1$. Finally, in Region III where sellers' outside option is large (i.e., $u_0 > u_0^H$), sellers do not join under any chosen commission rate expecting that T will be set at T^{Sales}. Compared with Proposition 3 and Figure 4.3, our results are the same in Regions I and III; the only difference is that sellers fail to participate in Region II. We illustrate this result in Figure OA4.

The above result implies that our insights that the platform can benefit from strategic listing still holds, but sellers will participate in a smaller parameter space than the case in which the platform can commit to how it will use information from the sponsored side to influence the organic side. Furthermore, if it is feasible for the platform to publish and commit to its ranking algorithm at the beginning of the game, then it is weakly optimal for it to do so; specifically, when sellers' participation is more difficult to induce, the platform can strictly benefit from committing to T early to enable a marketplace. Counterintuitively, while today's walled-garden tech giants like Amazon and Google rarely detail their algorithms to the public, making them transparent may be a win-win strategy for all stakeholders.