Supplemental Appendices:

Influencers: The Power of Comments

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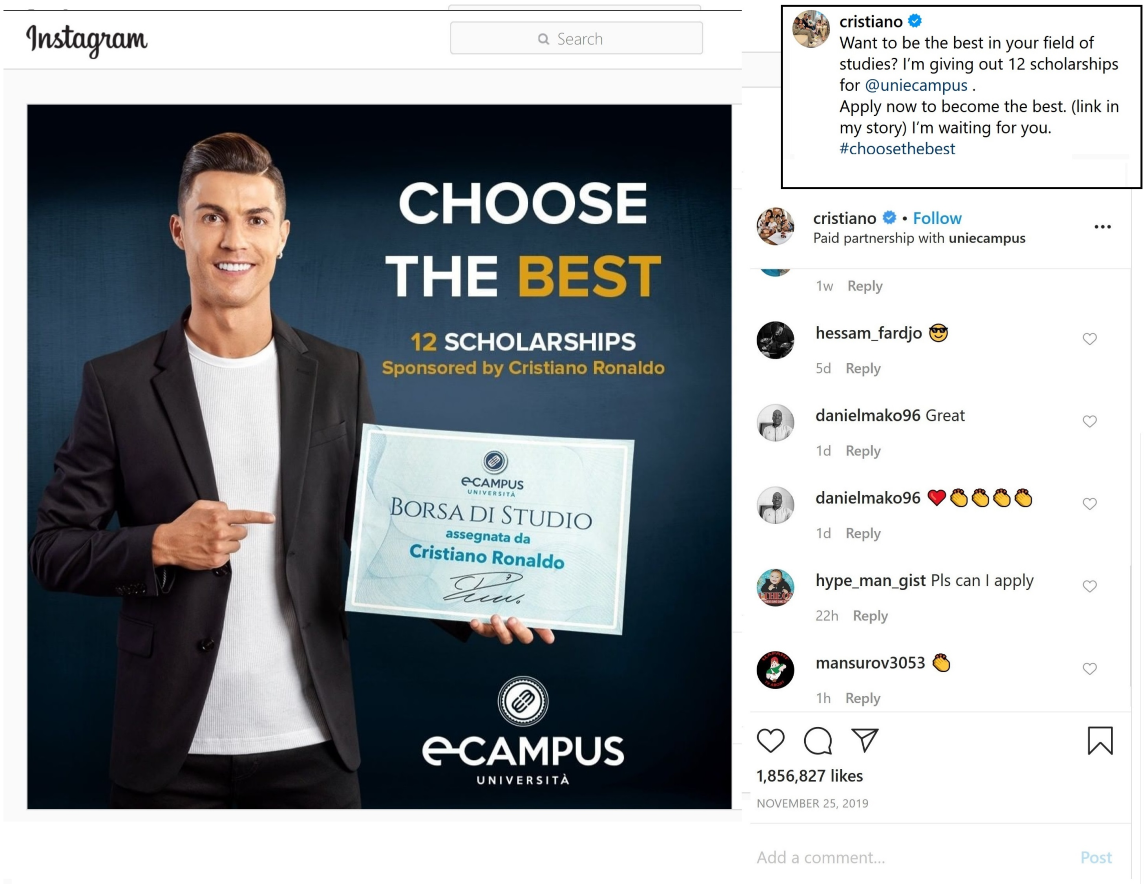
Abstract

Appendix A presents real world examples of endorsements by a celebrity influencer and a micro-influencer. Appendix B contains two model extensions. Appendix C presents formal proofs of all results.

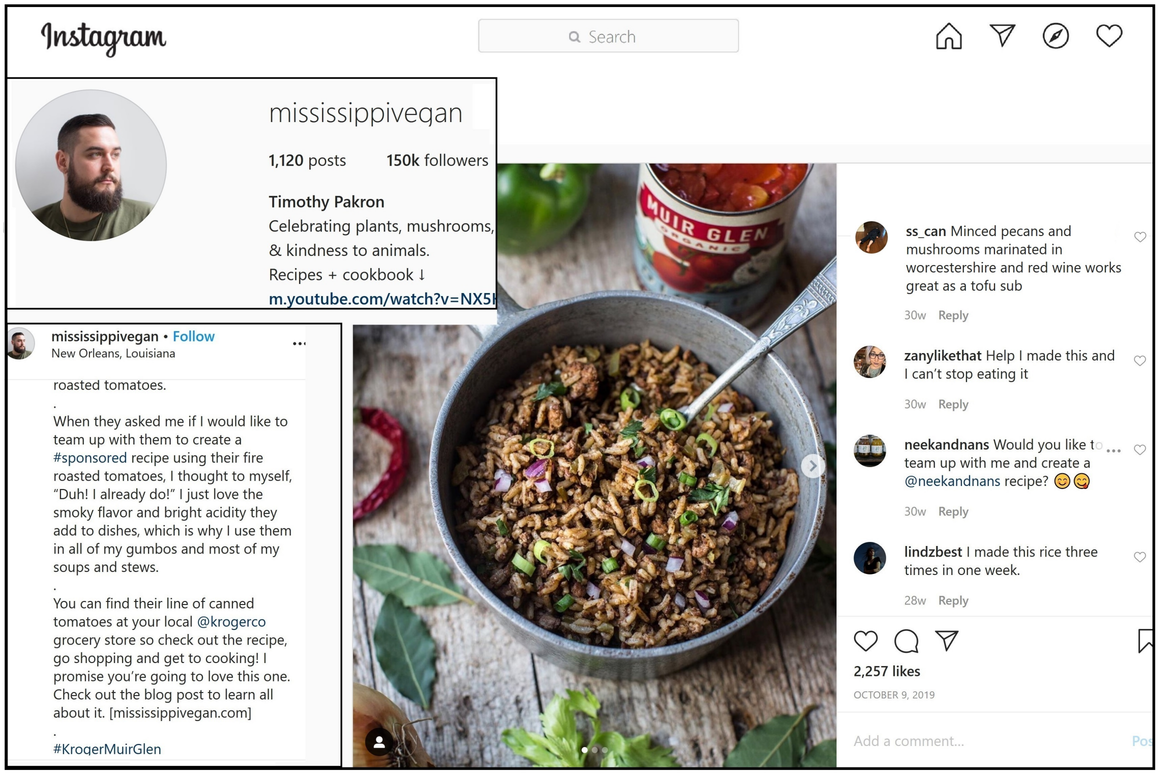
Appendix A: Influencer Examples

Cristiano Ronaldo, Sponsored Post on Instagram



Retrieved May 15, 2020, https://www.instagram.com/p/B5SFl7YK0Px/

Vegan Chef, Sponsored Post on Instagram



Retrieved May 15, 2020, https://www.instagram.com/p/B3ar1oKngcJ/

Appendix B: Model extensions

This appendix contains two model extensions. These extensions allow for influencer reputation and the influencer to delete uninformative comments.

Model extension: Influencer reputation

This model extension relaxes the assumption that the influencer's endorsement policy is binding, and we derive conditions, for an infinitely repeated version of the game, in which reputation effects compel the influencer not to deviate from his chosen endorsement policy.

The influencer first announces a non-binding endorsement policy that specifies whether he endorses all products or only high quality products. There is then a new product the influencer can endorse in each period $t \in \{1, 2, \ldots\}$. Similar to previous reputation models (e.g., Kreps and Wilson 1982; Bar-Isaac 2003; Selove 2019), all players learn at the end of period t about any deviation from the influencer's endorsement policy, for example, through news stories or online word-of-mouth. We show that, under some conditions, this threat of loss of reputation provides an incentive for the influencer to maintain a policy of endorsing only high quality products.

The game timing in each period t is the following:

1. Nature chooses the quality level for the period t product, which the influencer observes.

2. The influencer decides whether to endorse the product. If he does not endorse the product, the subgame for period t ends with no payoffs for the period. Otherwise, the firm and the influencer agree to a fixed payment the firm makes

to the influencer in period t such that they proportionally split the expected profits.

3. The firm sets the period t product price for the first group of customers.

4. The first group of customers read the influencer's post, learn the product's current price, have the option to buy the product, and leave comments.

5. The firm sets the period t product price for the second group of customers.

6. The second group of customers read the influencer's post and learn the product's current price. They can then read comments and have the option to buy the product.

In a one-shot version of this game, with a nonbinding endorsement policy, the influencer would endorse all products. The influencer could not credibly promise to endorse only high quality products, because he would then have an incentive to endorse the product and earn positive profits even if it has low quality. Therefore, the only possible equilibrium of a one-shot version of this game involves the influencer endorsing all products and customer beliefs that the endorsement does not convey any quality information. We focus on an equilibrium of our repeated game in which, if the influencer ever deviates from his announced endorsement policy, players revert to this one-shot equilibrium in all subsequent periods. This punishment strategy provides the strongest possible incentive for the influencer not to deviate because it implies, if the influencer ever violates an announced policy of endorsing only high quality products, then followers will always believe he endorses all products and never again believe his claim that the products he endorse all have high quality.

We focus our analysis on a micro-influencer, although we could also perform similar derivations of reputation effects for a celebrity influencer. Consider a proposed equilibrium in which the influencer endorses only high quality products, and suppose the fraction of informative comments ($\overline{\lambda}$) is high enough that the conditions of Lemma 3 hold. If the influencer deviates and endorses a low quality product in period t, Lemma 3 implies he will generate the following profits for the period:

$$\left[\beta\left(\overline{q}-C\right)+(1-\beta)\underline{q}\left(1-C-\frac{R}{\overline{q}\overline{\lambda}}\right)\right]M_V \tag{7}$$

Profits for this equilibrium deviation involve customers beliefs that quality is high ($\overline{q}$), but in the second period only a low fraction of customers ($\underline{q}$) find the product is a good fit because quality is actually low.

This deviation from the proposed equilibrium causes all future periods to revert to the equilibrium in which the influencer endorses all products instead of only high quality products. The resulting change in expected profits in each future period is given by expression (6). If Proposition 2 implies the influencer should endorse only high quality products, this expression is negative, that is, profits are lower when the influencer endorses all products. Letting δ denote the influencer's discount factor, this threat of lost future profits is sufficient to prevent the influencer from endorsing a low quality if the following condition holds:

Condition 1.

$$\begin{aligned}\left[\beta\left(\overline{q}-C\right)+(1-\beta)\underline{q}\left(1-C-\frac{R}{\overline{q}\overline{\lambda}}\right)\right] &\leq \\ \frac{\delta(1-\alpha)}{1-\delta}\left[-\underline{q}+\beta C+(1-\beta)\underline{q}C+\frac{R(1-\beta)}{\overline{\lambda}}\right]&\end{aligned} \tag{8}$$

Intuitively, when this condition holds, the short-term profits from endorsing a low quality product in any period t are less than the discounted value of the lost future profits that result from imposing additional reading costs on followers by moving to an equilibrium in which the influencer endorses all products. The following proposition states this result formally.

Proposition 5. *If Condition 1 holds, a micro-influencer can sustain a policy of endorsing only high quality products, even if its endorsement policy is not binding.*

Model extension: Deleting uninformative comments

We now allow the influencer to delete some of the uninformative comments on his post.

Previous research has developed models in which a seller can write biased reviews of their product which, in equilibrium, results in less informative comments (Mayzlin 2006), or can make informative comments easier to find by allowing customer reviews on the seller's website (Chen and Xie 2008). Similarly, we allow the influencer to choose a policy that affects the informativeness of comments. At the same time he sets his endorsement policy, the influencer can decide either to keep all of the comments on his post or to delete some uninformative comments. If a micro-influencer has a policy of deleting uninformative comments, the fraction of informative comments on his post increases to $\overline{\lambda}^*$, where $\overline{\lambda}^* \in (\overline{\lambda}, 1]$. If a celebrity influencer deletes uninformative comments, the fraction of informative comments increases to $\underline{\lambda}^*$, where $\underline{\lambda}^* \in (\underline{\lambda}, 1]$. Note we allow for the possibility that the influencer is not able to delete all uninformative comments, so the fraction of informative comments could remain less than one.

Equations (4) and (5) show that a micro-influencer's equilibrium profits are strictly increasing in $\overline{\lambda}$. Therefore, by reducing reading costs for second period customers, a policy of deleting uninformative comments increases profits for a micro-influencer.

For a celebrity influencer, we let $\underline{\lambda}^*$ be high enough that the conditions of Lemma 3 hold for this fraction of informative comments. Therefore, if a celebrity influencer has a policy of deleting uninformative comments, customers in the second period read comments on his post, and profits can then be derived using the same equations as for a micro-influencer in the original version of the model.

By comparing equation (2) with equation (4), and comparing (3) with (5), we see that deleting uninformative comments affects profits for the celebrity influencer in two ways. First, by allowing second period customers to learn whether the product is a good fit, this policy allows the seller to avoid the cost of producing the product for customers for whom it is a bad fit. Second, it causes customers in the second period to incur costs of reading comments about the product. The net effect can go in either direction, that is, a policy of deleting uninformative comments could increase profits if production costs are relatively large, but could also reduce profits if the reading costs are relatively large.

These results are formalized in the following proposition.

Proposition 6. *For a micro-influencer, deleting uninformative comments increases profits. For a celebrity influencer, deleting uninformative comments increases the profits from a given endorsement policy if and only if* $(1 - \widetilde{q})C > \frac{R}{\underline{\lambda}^*}$*, where* $\widetilde{q}$ *is expected quality given the influencer's endorsement policy.*

Thus, a celebrity influencer may or may not want to delete uninformative comments. Intuitively, if production costs are low relative to the cost of reading comments, it is more profitable to have all customers purchase the product without reading comments, so a celebrity influencer prefers to keep uninformative comments on his post.

Appendix C: Proofs

Proof of Lemma 1

We consider all feasible search strategies for followers and derive conditions in which each strategy is optimal. One possible strategy is to continue reading comments until finding an informative comment and then purchase the product if it is a good fit. Given that a fraction λ of comments are informative, in expectation one has to read $\frac{1}{\lambda}$ comments to find an informative comment. Furthermore, given that there is probability $\widetilde{q}$ the product has good fit for a given follower, the expected utility of this strategy is $\widetilde{q}(1-P_2)-\frac{R}{\lambda}$. Another possible strategy is to purchase immediately without reading any comments, which generates expected utility $\widetilde{q}-P_2$. A third possible strategy is not reading any comments and not purchasing, which generates utility 0.

Among these three strategies, reading comments before deciding whether to purchase generates the highest expected utility if $(1-\widetilde{q})P_2 > \frac{R}{\lambda}$ and $\widetilde{q}(1-P_2)-\frac{R}{\lambda} \geq 0$, purchasing immediately generates the highest expected utility if $(1-\widetilde{q})P_2 \leq \frac{R}{\lambda}$ and $\widetilde{q}-P_2 \geq 0$, and not reading comments or purchasing generates the highest expected utility otherwise.

Another feasible strategy would be to read a finite number of comments and stop searching if an informative comment has not yet been found. However, reading an uninformative comment has no effect on the probability λ that the next comment will be informative or on the probability $\widetilde{q}$ that the product has good fit. Furthermore, reading an uninformative comment has no effect on the expected utility of stopping search and making a purchase decision immediately, which generates utility $\max\{0, \widetilde{q}-P_2\}$. Therefore, the optimal threshold level of utility needed to stop search does not change over time (Kohn and Shavell 1974). After reading an uninformative comment, the follower still faces the same dynamic optimization problem they faced before reading the comment. Thus, if it is optimal to read the first comment, it must also be optimal to continue reading comments after finding an uninformative comment, and reading a finite number of uninformative comments and then stopping search cannot be strictly better than continuing to search until an informative comment is found. QED

Proof of Lemma 2

We first compute the equilibrium in the second period. Based on Lemma 1, followers read comments only if $\frac{R}{\lambda} < (1 - \widetilde{q})P_2$ and $\frac{R}{\lambda} < \widetilde{q}(1 - P_2)$. The second inequality implies $P_2 < 1$, so together these inequalities imply $\frac{R}{\lambda} < 1 - \widetilde{q}$. Therefore, if the fraction of informative comments λ is small enough that $\frac{R}{\lambda} > 1 - \underline{q}$, there cannot be an equilibrium in which followers read comments. Under this condition, followers may purchase the product immediately without reading comments if the price is low enough given their quality beliefs. Furthermore, a high price cannot serve as a high quality signal because a low quality firm would have an incentive to mimic this high price. Thus, in equilibrium, followers base their quality beliefs only on the influencer's endorsement policy, and the firm sets price $\overline{q}$ if the influencer endorses only high quality products and price $\alpha\overline{q} + (1 - \alpha)\underline{q}$ if the influencer endorses all products. Similar logic implies the firm also follows this same price strategy in the first period. QED

Proof of Lemma 3

We first consider the case in which the influencer endorses only high quality products, which implies $\widetilde{q} = \overline{q}$. In the second period, based on Lemma 1, the highest possible price for which followers would purchase immediately is $\frac{R}{(1-\overline{q})\lambda}$. Setting this price and selling to all followers generates second period profits $(1-\beta)(\frac{R}{(1-\overline{q})\lambda}-C)M_V$. Alternatively, for small values of $\frac{R}{\lambda}$, the firm can set a higher price, $1-\frac{R}{\overline{q}\lambda}$, in which case followers read comments before deciding whether to purchase. Setting this price and selling to a fraction $\overline{q}$ of followers generates second period profits $(1-\beta)\overline{q}(1-\frac{R}{\overline{q}\lambda}-C)M_V$. If $\frac{R}{\lambda}$ is sufficiently small, profits from the latter strategy are higher, and the firm prefers to set a higher price so followers read comments.

In the first period, the firm can set price $\overline{q}$, which generates first period profits $\beta(\overline{q}-C)M_V$ and results in followers writing informative comments. Alternatively, the firm can set a higher price, which results in zero first period profits and no informative comments, in which case second period profits will be $(1-\beta)(\overline{q}-C)M_V$. If $\frac{R}{\lambda}$ is sufficiently small, the strategy of selling to first period followers so there are informative comments that are read by followers in the second period is more profitable.

We now consider the case in which the influencer endorses all products. We will show that, under some conditions, the intuitive criterion implies a strategy of setting a low price so customers do not read comments is perceived as a low quality signal. For this strategy, the maximum second period price is $\frac{R}{(1-\underline{q})\lambda}$, which results in second period profits $(1-\beta)(\frac{R}{(1-\underline{q})\lambda}-C)M_V$. Alternatively, the firm can set a higher price $1-\frac{R}{[\alpha\overline{q}+(1-\alpha)\underline{q}]\lambda}$, which results in followers reading comments, so a fraction $\overline{q}$ buy the product if quality is a high and a fraction $\underline{q}$ buy if quality is low. If $\frac{R}{\lambda}$ is low enough that $\underline{q}\Big[1-\frac{R}{[\alpha\overline{q}+(1-\alpha)\underline{q}]\lambda}-C\Big]>\frac{R}{(1-\underline{q})\lambda}-C$, a low quality firm prefers to pool with the high quality firm and set a high price so that followers read comments.

Furthermore, we now derive conditions for which a potential pooling equilibrium in which both firm types set price $\frac{R}{(1-[\alpha\overline{q}+(1-\alpha)\underline{q}])\lambda}$ and customers purchase immediately cannot satisfy the intuitive criterion. If $\frac{R}{\lambda}$ is sufficiently small, a high quality firm would prefer to deviate to price $1-\frac{R}{\overline{q}\lambda}$ if customers believe this higher price signals high quality. Furthermore, because a higher fraction of customers purchase after search if the firm has high quality than if it has low quality, there must exist a price such that the high quality firm prefers to deviate and the low quality firm does not if such a price deviation signals high quality. Thus, if a high quality firm prefers to deviate from the pooling equilibrium to signal high quality, a pooling equilibrium on the low price cannot satisfy the intuitive criterion.

In addition, if $\frac{R}{\lambda}$ is sufficiently small, it is more profitable to set first period price $\alpha\overline{q} + (1-\alpha)\underline{q}$ so that followers buy the product and write informative comments, rather than setting a higher price so they do not buy or write informative comments. QED

Proof of Proposition 1

The profit functions for a celebrity influencer given by (2) and (3) follow from Lemma 2. If $\underline{q} > C$, the profits in (2) are greater than the profits in (3), which implies profits are maximized by endorsing all products. If $\underline{q} < C$, the profits in (3) are greater than in (2), which implies profits are maximized by endorsing only a high quality product. Because the seller and influencer split the profits proportionally, the influencer chooses the endorsement policy that maximizes expected profits, which is a policy of endorsing all products. QED

Proof of Proposition 2

The profit functions given by (4) and (5) follow from Lemma 3. If $(1-\beta)\left(\frac{R}{\lambda}\right) > \underline{q} - \beta C - (1-\beta)\underline{q}C$, the profits in (5) are greater than the profits in (4), which implies for a micro-influencer profits are maximized by endorsing only high quality products. If this inequality is reversed, the profits in (4) are greater, and the influencer maximizes profits by endorsing all products. Because the seller and influencer split the profits proportionally, the influencer chooses whichever endorsement policy maximizes expected profits. QED

Proof of Proposition 3 For a celebrity influencer, the proof of Lemma 2 shows that, even if first period followers who buy the product write informative comments, followers in the second period do not read the comments. Therefore, $N = 0$, which implies all followers prefer to write uninformative comments, including those who purchase the product. The rest of the analysis for a celebrity influencer is the same as in Lemma 2 and Proposition 1.

For a micro-influencer, the proof of Lemma 3 shows that followers in the second period read comments if $\frac{R}{\lambda}$ is sufficiently small. As shown in the body of the paper, each commenter in the first period then expects $\frac{1-\beta}{\beta}$ second period followers to read her comment. Given this value of N, followers who purchase the product prefer to write an informative comment if $(S_i - S_u)\frac{1-\beta}{\beta} > k_i - k_u$. Furthermore, first period followers derive utility $e - k_u + S_u\frac{1-\beta}{\beta}$ if they do not buy and the product and they write an uninformative comment, and those who are interested in the product are willing to pay $\widetilde{q} + (S_i - S_u)\frac{1-\beta}{\beta} - (k_i - k_u)$ to reflect the utility from consuming the product, the incremental status utility from writing an informative comment, and the incremental cost of writing an informative comment. The equilibrium utility for first period followers, regardless of whether they buy the product, is then $e - k_u + S_u\frac{1-\beta}{\beta}$. Second period followers pay the same price as in the main model and derive equilibrium utility $e - k_u$. Therefore, second period followers wait to read in the second period if their cost of reading the post earlier exceeds the incremental utility of first period followers. The equilibrium endorsement policy then follows from similar analysis to the proof of Proposition 2.

Proof of Proposition 4 A policy of endorsing only high quality products for which the influencer has expertise leads to probability $\gamma\alpha$ of an endorsement, and conditional on an endorsement, expected quality is $\overline{q}$. A policy of endorsing high quality products for which the influencer has expertise and also all products for which he lacks expertise leads to probability $\gamma\alpha+(1-\gamma)$ of an endorsement, and conditional on an endorsement, expected quality is $\frac{\gamma\alpha\overline{q}+(1-\gamma)\widehat{q}}{\gamma\alpha+(1-\gamma)}$. For a celebrity influencer, similar derivations to those in the original model show that the latter policy generates higher profits if $\widehat{q} > C$. For a micro-influencer, similar derivations to those in the original model show that the latter policy generates higher profits if $\widehat{q} - \beta C - (1-\beta)\widehat{q}C > (1-\beta)\left(\frac{R}{\overline{\lambda}}\right)$. Finally, a policy of endorsing all products leads to probability one of an endorsement, and conditional on an endorsement, expected quality is $\widehat{q}$. Similar derivations to those in the original model yield conditions in which this policy generates the greatest profits. QED

Proof of Proposition 5 Lemma 3 implies the profits from deviating and endorsing a low quality product are given by expression (7). Lemma 3 also implies the reduction in each period's future profits from moving to an equilibrium in which the influencer endorses all products is given by expression (6). Under Condition 1, the discounted value of this loss in future profits is greater than the one time benefit from deviating and endorsing a low quality product. Therefore, the influencer can sustain the equilibrium in which he endorses only high quality products, and the threat of moving to the bad equilibrium in which he endorses all products prevents him from deviating. QED

Proof of Proposition 6 For a micro-influencer, equations (4) and (5) show that profits are strictly increasing in $\overline{\lambda}$. For a celebrity influencer, comparing equation (2) with equation (4) and substituting $\overline{\lambda} = \underline{\lambda}^*$ shows that deleting comments increases the profits from endorsing all products if $(1 - [\alpha\overline{q} + (1-\alpha)\underline{q}])C > \frac{R}{\underline{\lambda}^*}$, and comparing (3) with (5) and substituting $\overline{\lambda} = \underline{\lambda}^*$ shows that deleting comments increases the profits from endorsing only high quality products if $(1-\overline{q})C > \frac{R}{\underline{\lambda}^*}$. QED