The Interactions of Customer Reviews and Price and Their Dual Roles in Conveying Quality Information

Online Appendix

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In this appendix, we provide the analysis and the proofs of results which are not presented in the main paper.

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B1 Proof of Lemma 1.

Part a). When a review system is absent, we first consider a pooling equilibrium under which both types of sellers choose the same first-period price p_{1LH}. As the first-period price does not provide any information on the seller's quality, we have $b_1(p_{1LH}) = \frac{1}{2}$. The same first-period price by both types of sellers further implies that the equilibrium second-period price has to be the same for both types of sellers. To see this, suppose that one type of seller charges a higher second-period price. Given the same first-period profits, the other type of seller can deviate to the high second-period price to improve profits, contradicting the equilibrium requirement. Therefore, the second-period price p_{2LH} also does not contain any seller's quality information, which implies $b_2(p_{1LH}, p_{2LH}) = \frac{1}{2}$.

With $b_1(p_{1LH}) = \frac{1}{2}$ and $b_2(p_{1LH}, p_{2LH}) = \frac{1}{2}$, each seller makes sales in both periods, we have $p^*_{1LH} \leq \overline{\alpha}$ and $p^*_{2LH} \leq \overline{\alpha}$. We next show that $p^*_{1LH} = \overline{\alpha}$ and $p^*_{2LH} = \overline{\alpha}$ are the equilibrium prices. This is because in period $i \in \{1, 2\}$, if under certain belief $b_i(p'_i)$, one type of seller can deviate to a price $p'_i \neq \overline{\alpha}$ to improve total profits, the other type of seller can do the same. As a result, an off-equilibrium belief $b_i(p'_i) = \frac{1}{2}$ is valid according to the Intuitive Criterion. With this off-equilibrium belief, no seller would deviate to a price $p'_i < \overline{\alpha}$ as it only lowers the seller's profits of period i without affecting its profits of the other period. No seller would deviate to a price $p'_i > \overline{\alpha}$ because it will result in zero sales in period i without affecting its profits of the other period.

Furthermore, both types of sellers earn the same profits

$$\Pi_H^{pooling*} = \Pi_L^{pooling*} = \frac{1+N}{2} \cdot \overline{\alpha} \tag{B1}$$

Given a pooling equilibrium, it is immediately seen that the one with $p^*_{1LH} = \overline{\alpha}$ and $p^*_{2LH} = \overline{\alpha}$ leads to the highest profits to both types of sellers. As discussed in the paper, we focus attention on this focal pooling equilibrium even when other pooling equilibria may exist.

We next consider the possibility of a separating equilibrium under which both types of sellers differ in their first period prices. Because the first-period price is observable in both periods, all consumers can tell the seller's quality. This means the second-period equilibrium prices will be $p^*_{2H} = \alpha_H$ and $p^*_{2L} = \alpha_L$. In the first period, the L-seller will also set $p^*_{1L} = \alpha_L$, because knowing the L-seller's type a consumer will not pay any price higher than α_L. We therefore have the L-seller's expected profits $\Pi_L^{separating*} = \frac{1+N}{2} \cdot \alpha_L$. The H-seller's profits can be written as $\Pi_H^{separating} = \frac{1}{2} \cdot p^*_{1H} + \frac{N}{2} \cdot \alpha_H$.

Note that by deviating to $p^*_{1L} = \alpha_L$, the H-seller will be misrecognized as an L-seller and earn the profits $\Pi_L^{separating*}$. The equilibrium condition requires $\Pi_H^{separating} \geq \Pi_L^{separating*}$. Similarly, by deviating to p^*_{1H}, the L-seller will be misrecognized as an H-seller and earn the profits $\Pi_H^{separating}$. The equilibrium condition then requires $\Pi_L^{separating*} \geq \Pi_H^{separating}$. The two inequalities imply $\Pi_H^{separating} = \Pi_L^{separating*}$, which yields $p^*_{1H} = (1+N) \cdot \alpha_L - N \cdot \alpha_H$. The expected profits of each type of sellers under the separating equilibrium are

$$\Pi_H^{separating*} = \Pi_L^{separating*} = \frac{1+N}{2} \cdot \alpha_L \tag{B2}$$

Lastly, from (B1) and (B2), we have $\Pi_H^{pooling*} > \Pi_H^{separating*}$ and $\Pi_L^{pooling*} > \Pi_L^{separating*}$.

Therefore, we focus attention on the pooling equilibrium when a review system is absent.

Part b). In the presence of a review system and with $\lambda = 0$, all consumers will leave reviews as long as they make purchase. As discussed in the paper, unbiased reviews will reveal the seller's quality to second-period consumers as the H-seller's ratio $\frac{r_1}{r_0} = \frac{\alpha_H}{1-\alpha_H}$ is different from the L-seller's ratio $\frac{r_1}{r_0} = \frac{\alpha_L}{1-\alpha_L}$.

Note that a seller may also charge a high price which leads to no sales in the first period, and as a result does not receive any reviews. However, this cannot happen in equilibrium. To see the reason, first consider an H-seller. The H-seller can always set $p_{1H} \leq \alpha_L$, which will make first-period consumers to purchase and leave reviews. The reviews will convey the H-seller's quality to the second-period consumers and enable it to charge the highest possible second-period price $p_{2H} = \alpha_H$. Therefore, an H-seller never considers making no sales in the first period regardless of consumes' belief. According to Step 1 of the Intuitive Criterion, if the second-period consumers ever observe no reviews in the first period, it could only possibly be from an L-seller, and as a result the second-period consumers' willingness-to-pay becomes α_L. Given this, an L-seller will also not consider making no sales in the first-period, as it can be better off by charging $p_{1L} = \alpha_L$ and still being able to charge $p_{2L} = \alpha_L$ in the second period.

Given both types of sellers make sales in the first-period and therefore the reviews will reveal the seller's quality, we have $p^*_{2H} = \alpha_H$ and $p^*_{2L} = \alpha_L$ in the second period. Then, only the pooling equilibrium that both types of sellers choose the same first-period price p_{1LH} is possible, because if they differ in first-period price, the seller that sets a lower first-period price can be better off by mimicking the other type's first-period price. Consider the highest possible first-period price $p^*_{1LH} = \overline{\alpha}$ and examine all possible deviation.

- Deviation to $p'_1 < p^*_{1LH}$.

  No seller would deviate to such a p'_1 as the deviation only lowers the first-period profits without improving the second-period profits.

- Deviation to $p'_1 \in (p^*_{1LH}, \alpha_H]$.

  Note that under the belief $b_1(p'_1) = 1$, the deviation would be profitable for both types of sellers. This means both types of sellers are identified from Step 1 of the Intuitive Criterion. Therefore, the lowest deviation profits are when $b_1(p'_1) = 0$, in which case the seller makes no sales in the first period and charges at most $p_2 = \alpha_L$ in the second period. Both types of sellers earn higher equilibrium profits than the lowest deviation profits. The equilibrium survives the Intuitive Criterion according to its Step 2.

- Deviation to $p'_1 > \alpha_H$.

  Under this price, the first-period consumers will not make purchase and no reviews are generated. The H-seller will not consider the deviation regardless of consumer belief as the deviation hurts its first-period profits and cannot improve its second-period profits. The seller that can be identified from Intuitive Criterion's Step 1, if any, can only be the L-seller. With $b_1(p'_1) = 0$, the L-seller also will not deviate to p'_1. Therefore, the equilibrium survives the Intuitive Criterion.

Lastly, the H-seller's profits are $\Pi^*_H = \frac{1}{2} \cdot \overline{\alpha} + \frac{N}{2} \cdot \alpha_H$, higher than the L-seller's profits $\Pi^*_L = \frac{1}{2} \cdot \overline{\alpha} + \frac{N}{2} \cdot \alpha_L$.

B2 Proof of Lemma 2.

We examine the subgame in the second period. Suppose that the seller's first-period price is p_1 and the realization of the reviews are $\{r_0, r_1\}$.

First, consider the situation that the first-period price and reviews already perfectly reveal a seller's type (i.e., $b_2(p_1, r_0, r_1) = 1$ or 0). Then, the second-period price will not further facilitate consumers' belief updating, which implies $b_2(p_1, r_0, r_1, p_2^{**}) = b_2(p_1, r_0, r_1)$.

Second, consider the situation that the first-period price and reviews cannot perfectly reveal a seller's type (i.e., $0 < b_2(p_1, r_0, r_1) < 1$). Let $S_H(p_1, r_0, r_1)$ and $S_L(p_1, r_0, r_1)$ respectively denote the support of the second-period price points of the H-seller and the L-seller.

- We first show that both $S_H(p_1, r_0, r_1)$ and $S_L(p_1, r_0, r_1)$ must contain only one single price point in equilibrium. As the second-period is the last period, the seller will not consider a second-period price that leads to no sales. Given that the second-period demand is positive and does not vary with the second-period price, there can only be one optimal second-period price for the seller. Therefore, both $S_H(p_1, r_0, r_1)$ and $S_L(p_1, r_0, r_1)$ contain only one price point in equilibrium. We use p_{2H}^{**} and p_{2L}^{**} to denote the H-seller and L-seller's equilibrium prices in the subgame of the second-period.

- Next, it is easy to see that $p_{2H}^{**} = p_{2L}^{**}$: If $p_{2H}^{**} \neq p_{2L}^{**}$, the seller who sets a lower second-period price can always deviate to the higher second-period price chosen by the other type of seller to strictly improve the second-period profits. Given $p_{2H}^{**} = p_{2L}^{**}$, the second-period price still cannot cause consumers' belief updating (i.e, $b_2(p_1, r_0, r_1, p_2^{**}) = b_2(p_1, r_0, r_1)$).

This completes the proof.

B3 Proof of Claim 1.

It is trivial that neither type of sellers will set a price that does not lead to any sales in the second period. Therefore, each type of seller will only consider p_2 which leads to $D_2(m_2, p_1, r_0, r_1, p_2) = m_2$.

We then consider the first period. Given that consumers are homogeneous, only $D_1(m_1, p_1) = m_1$ or $D_1(m_1, p_1) = 0$ can happen. The case that $D_1(m_1, p_1) = 0$ happens if the seller sets p_1 higher than the first-period consumers' willingness-to-pay and will lead to no customer reviews generated. However, neither type of seller will consider such a price for the following reason. Note that an H-seller can set $p_{1H} = \lambda$, a price under which all first-period consumers make purchase (given $\lambda < \alpha_L < \alpha_H$) and leave reviews. The reviews will then reveal the H-seller's quality to the second-period consumers and enable the H-seller to charge $p_{2H} = \alpha_H$. This means an H-seller will never consider making zero sales in the first-period as it only hurts the first-period profits without improving the second-period profits. The Intuitive Criterion then implies that zero reviews, if ever happen, could only be caused by an L-seller's high price. Therefore, the second-period consumers will pay at most $p_2 = \alpha_L$ if not observing any reviews from the first-period. Knowing this, an L-seller will also never consider a first-period price that leads to no first-period sales as it can be better off by setting $p_{1L} = \lambda$, inducing first-period consumers to purchase, and charging $p_{2L} = \alpha_L$ in the second period. Taken together, both types of seller will only consider the first price such that $D_1(m_1, p_1) = 0$.

B4 The Derivation of Consumer Belief in Section 4.2.1

We present the details of the derivations of the second-period consumer belief shown in equation (4), (5), and (6) in the main paper. Consider three cases as follows.

- *Case 1:* $p_1 \in [\lambda, 1-\lambda]$

  In this case, first-period consumers with good experiences will leave positive reviews, and those with bad experiences will leave negative reviews. As a result, the ratio of positive and negative reviews $\frac{r_1}{r_0}$ is equal to $\frac{m\cdot\alpha_H}{m\cdot(1-\alpha_H)} = \frac{\alpha_H}{1-\alpha_H}$ for the H-seller but is equal to $\frac{m\cdot\alpha_L}{m\cdot(1-\alpha_L)} = \frac{\alpha_L}{1-\alpha_L}$ for the L-seller. This means the ratio $\frac{r_1}{r_0}$ is fully indicative of the seller's type. Therefore, we have

$$b_2(p_1 \in [\lambda, 1-\lambda], r_0, r_1) = \begin{cases} 1 & \text{if } \frac{r_1}{r_0} = \frac{\alpha_H}{1-\alpha_H} \\ 0 & \text{if } \frac{r_1}{r_0} = \frac{\alpha_L}{1-\alpha_L} \end{cases} \tag{B3}$$

  as shown in equation (4) in the main paper.

- *Case 2:* $p_1 \in (1-\lambda, 1]$

  In this case, first-period consumers with bad experiences will leave negative reviews, but those with good experiences will not leave reviews. Therefore, for both types of sellers, we will have $r_1 = 0$ and $r_0 > 0$.

  Without $\{r_0, r_1\}$, a second-period consumer has the information p_1 and therefore should hold the same belief as the first-period consumers (i.e., b_1). The reviews information $\{r_1, r_0\}$ can convey more information for consumers to infer the seller's quality. We use "#NR" as the abbreviation of "the number of negative reviews" and derive $b_2(p_1 > 1-\lambda, r_0, r_1) = \Pr(\{\text{H-seller}\}|\{\#\text{NR} = r_0\})$.

  Note that r_0 is a random variable and follows a continuous distribution since the first period market size, m_1, follows the distribution $U[0,1]$. Therefore, the conditional probability $\Pr(\{\text{H-seller}\}|\{\#\text{NR} = r_0\})$ cannot be defined as $\Pr(\{\text{H-seller}\}|\{\#\text{NR} = r_0\}) = \frac{\Pr(\{\text{H-seller}, \#\text{NR}=r_0\})}{\Pr(\{\text{NR}=r_0\})}$ because "$\text{NR} = r_0$" is an event of probability zero. Instead, based on the standard definition of probability conditional on an event of probability zero, it should be defined as the limit of $\Pr(\{\text{H-seller}\}|\{\#\text{NR} \in (r_0-\epsilon, r_0]\}) = \frac{\Pr(\{\text{H-seller}, \#\text{NR}\in(r_0-\epsilon, r_0]\})}{\Pr(\{\text{NR}\in(r_0-\epsilon, r_0]\})}$ as ϵ goes to zero (Ash, 2008).

$$\begin{aligned} & b_2(p_1 > 1-\lambda, r_0, r_1) \\ =& \Pr(\{\text{H-seller}\}|\{\#\text{NR} = r_0\}) \\ =& \lim_{\epsilon\to 0} \Pr(\{\text{H-seller}\}|\{\#\text{NR} \in (r_0-\epsilon, r_0]\}) \\ =& \lim_{\epsilon\to 0} \frac{\Pr(\{\text{H-seller}\}, \{\#\text{NR} \in (r_0-\epsilon, r_0]\})}{\Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\})} \\ =& \lim_{\epsilon\to 0} \frac{b_1 \cdot \Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\}|\text{H-seller})}{b_1 \cdot \Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\}|\text{H-seller}) + (1-b_1)\cdot \Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\}|\text{L-seller})} \end{aligned} \tag{B4}$$

  Now consider the following two situations

1. If $r_0 \in (0, 1-\alpha_H]$, then (B4) becomes

$$\begin{aligned}
& b_2(p_1 > 1-\lambda, r_0, r_1) \\
= & \lim_{\epsilon \to 0} \frac{b_1 \cdot \Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\}|\text{H-seller})}{b_1 \cdot \Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\}|\text{H-seller}) + (1-b_1) \cdot \Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\}|\text{L-seller})} \\
= & \lim_{\epsilon \to 0} \frac{b_1 \cdot \Pr(\{m \in (\frac{r_0-\epsilon}{1-\alpha_H}, \frac{r_0}{1-\alpha_H})\}]}{b_1 \cdot \Pr(\{m \in (\frac{r_0-\epsilon}{1-\alpha_H}, \frac{r_0}{1-\alpha_H})\}) + (1-b_1) \cdot \Pr(\{m \in (\frac{r_0-\epsilon}{1-\alpha_L}, \frac{r_0}{1-\alpha_L})\})} \\
= & \lim_{\epsilon \to 0} \frac{b_1 \cdot \frac{\epsilon}{1-\alpha_H}}{b_1 \cdot \frac{\epsilon}{1-\alpha_H} + (1-b_1) \cdot \frac{\epsilon}{1-\alpha_L}} \\
= & \frac{b_1}{b_1 + (1-b_1) \cdot \frac{1-\alpha_H}{1-\alpha_L}}
\end{aligned} \tag{B5}$$

2. If $r_0 \in (1-\alpha_H, 1-\alpha_L]$, then $\Pr(\{\#\text{NR} \in (r_0-\epsilon, r_0]\}|\text{H-seller}) = 0$. As a result, (B4) becomes

$$b_2(p_1 > 1-\lambda, r_0, r_1) = 0 \tag{B6}$$

Combining (B5) and (B6), we have

$$b_2(p_1 > 1-\lambda, r_0, r_1) = \begin{cases} \frac{b_1}{b_1+(1-b_1)\cdot\frac{1-\alpha_H}{1-\alpha_L}} & \text{if } r_0 \in (0, 1-\alpha_H] \\ 0 & \text{if } r_0 \in (1-\alpha_H, 1-\alpha_L] \end{cases}$$

which is equation (5) in the main paper.

- *Case 3:* $p_1 \in [0, \lambda)$

  Following the same approach as in Case 2, we can derive

$$b_2(p_1 < \lambda, r_0, r_1) = \begin{cases} \frac{b_1}{b_1+(1-b_1)\cdot\frac{\alpha_H}{\alpha_L}} & \text{if } r_1 \in (0, \alpha_L] \\ 1 & \text{if } r_1 \in (\alpha_L, \alpha_H] \end{cases}$$

  as shown in equation (6) in the main paper. We omit the details to avoid repetition.

B5 Proof of Proposition 3.

We first calculate the total consumer surplus without or with a review system, and then make comparison to prove the proposition. Note that in the second period, the seller always sets the price that is equal to consumers' willingness to pay in both the settings we analyzed. This means that the consumer surplus in the second period is zero, and therefore the total consumer surplus is the same as the first-period consumer surplus.

1. Consumer Surplus When a Review System is Absent.

From part a) of Lemma 1, a seller sets each period's price equal to the consumer's willingness to pay. Therefore, consumer surplus is 0.

$$CS_0 = 0 \tag{B7}$$

The subscript "0" denotes the no review system situation.

2. Consumer Surplus When a Review System Exists.

We derive the consumer surplus under the separating equilibrium and under the pooling equilibrium in sequence.

- Separating equilibrium.

  From Proposition 1, if $\alpha_L > 1-\lambda$ and $N \geq \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L} \cdot \frac{1-\alpha_L}{1-\alpha_H}$, we can calculate the consumer surplus as

  $$CS^{s1} = (\frac{1}{2} \cdot (\alpha_H - (1-\lambda)) + \frac{1}{2} \cdot (\alpha_L - \alpha_L)) \cdot \frac{1}{2} = \frac{\alpha_H - (1-\lambda)}{4} \tag{B8}$$

  That is, with probability $\frac{1}{2}$, a first-period consumer faces an H-seller and has surplus $(\alpha_H - (1-\lambda))$; and with probability $\frac{1}{2}$, a first-period consumer faces an H-seller and has surplus $(\alpha_L - \alpha_L)$. In addition, the expected market size of the first period is $\frac{1}{2}$.

  If $\alpha_L > 1-\lambda$ and $N < \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L} \cdot \frac{1-\alpha_L}{1-\alpha_H}$, the consumer surplus can be similarly derived as

  $$\begin{aligned} CS^{s2} &= (\frac{1}{2} \cdot (\alpha_H - (\alpha_L - N \cdot \frac{1-\alpha_H}{1-\alpha_L} \cdot (\alpha_H - \alpha_L))) + \frac{1}{2} \cdot (\alpha_L - \alpha_L)) \cdot \frac{1}{2} \\ &= \frac{1}{4} \cdot (\alpha_H - \alpha_L)(1 + N \cdot \frac{1-\alpha_H}{1-\alpha_L}) \end{aligned} \tag{B9}$$

- Pooling equilibrium.

  From Proposition 2, if $\alpha_L \leq 1-\lambda$, we observe the pooling equilibrium and the consumer surplus can be similarly calculated as

  $$\begin{aligned} CS^{p} &= (\frac{1}{2} \cdot (\alpha_H - \min\{1-\lambda, \overline{\alpha}\}) + \frac{1}{2} \cdot (\alpha_L - \min\{1-\lambda, \overline{\alpha}\}) \cdot \frac{1}{2} \\ &= \frac{\max\{0, \overline{\alpha} - (1-\lambda)\}}{2} \end{aligned} \tag{B10}$$

With the consumer surplus in both settings, we can see that consumer surplus is zero when a review system is absent, but can be positive in the presence of a review system. Furthermore, under a separating equilibrium when a review system exists, consumer surplus is always positive across, and therefore is higher than when a review system is absent.

Lastly, when a review system exists and with the consumer surplus expression under each equilibrium region, we can write the consumer welfare as a function of λ in the entire parameter space. It can be easily verified that the consumer surplus weakly increases with λ in all settings.

B6 The Analysis in Section 5.1.1: A Review System Without Historical Price Information.

We first establish a proposition to characterize the equilibrium. We then discuss the possibility of multiple equilibria.

B6.1 Equilibrium Characterization.

Proposition B1. *If the historical price is unobservable, the equilibrium is as follows.*

(i.) *Separating Equilibrium in the First Period.*

Define $N'' \equiv \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L}$. *The two types of sellers vary in first-period prices if* $\alpha_L > 1-\lambda$ *and* $N \geq N''$. *In this case, the H-seller charges a lower first-period price than the L-seller,* $p^*_{1H} = 1-\lambda < \alpha_L = p^*_{1L}$. *The H-seller receives both positive and negative reviews, while the L-seller receives only negative reviews. In the second period, the H-seller charges* $p^*_{2H} = \alpha_H$ *and the L-seller charges* $p^*_{2L} = \alpha_L$.

(ii.) *Pooling Equilibrium in the First Period.*

The two types of sellers charge the same first-period price if (1) $\alpha_L \leq 1-\lambda$, *or (2)* $\overline{\alpha} > 1-\lambda$ *and* $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\overline{\alpha}'}$, *where* $\overline{\alpha}' \equiv \frac{\alpha_H+\alpha_L-2\alpha_H\alpha_L}{2-\alpha_H-\alpha_L}$, *which is the expected seller quality after observing an* $r_0 \leq 1-\alpha_H$ *of negative reviews.*

- *In Case (1), both types of sellers charge the first-period price* $p^*_{1LH} = \min\{\overline{\alpha}, 1-\lambda\}$, *and receive both positive and negative reviews. In the second period, the H-seller charges* $p^*_{2H} = \alpha_H$ *and the L-seller charges* $p^*_{2L} = \alpha_L$.
- *In Case (2), both types of sellers charge the first-period price* $p^*_{1LH} = \overline{\alpha}$, *and receive only negative reviews. In the second period, the H-seller sets* $p^*_{2H} = \overline{\alpha}'$. *If* $r_0 > 1-\alpha_H$, *which is only feasible if the seller is L, the L-seller sets* $p^*_{2L} = \alpha_L$. *Otherwise, the L-seller sets* $p^*_{2L} = \overline{\alpha}'$.

Furthermore, the H-seller earns higher profits than the L-seller in both the separating equilibrium and the pooling equilibrium.

Proof of Proposition B1.

Note that even without historical price, the H-seller can always charge $p_{1H} = \lambda$ in the first period, rely on unbiased reviews convey its quality, and charge $p_{2H} = \alpha_H$ in the second period. Then, following the similar reasoning in the proof of Claim 1, in equilibrium we have $D_1 = m_1$ and $D_2 = m_2$ for both types of sellers. We then prove the proposition in two parts by characterizing the separating equilibrium and the pooling equilibrium in sequence.

Part (i): Separating Equilibrium.

As the two types of sellers charge different first-period prices, the first-period consumers can identify the seller's quality. This means $p^*_{1L} \leq \alpha_L$. We consider the three cases as follows.

- *Case 1:* $p^*_{1L} \in (1-\lambda, 1]$

  In this case, given $p^*_{1L} \leq \alpha_L$, it requires that

$$\alpha_L > 1 - \lambda \tag{B11}$$

Now consider the H-seller's first-period price p_{1H}. First, the H-seller will not charge $p_{1H} < 1-\lambda$, because the H-seller is better off charging $p_{1H} = 1-\lambda$, a price that will lead to sales (because $1-\lambda < \alpha_L$) and generate unbiased reviews (so that the H-seller can charge $p_{2H} = \alpha_H$ in the second period). Second, any price $p_{1H} > 1-\lambda$ cannot be an equilibrium. To see this, note that if $p_{1H} > 1-\lambda$, both types of sellers receive only negative reviews. Because the first-period price is not observable by the second-period consumers, the type of seller which charges a lower first-period price can always deviate to the other type of seller's higher first-period price: Doing so will improve the first-period profits and will not affect the second-period profits (as it will not affect the reviews generated). As a result the only possible equilibrium p_{1H} is

$$p^*_{1H} = 1 - \lambda \tag{B12}$$

Since $p^*_{1H} = 1-\lambda$ can generate reviews that fully reveal the H-seller's quality, while $p^*_{1L} \in (1-\lambda, 1]$ generates only negative reviews, the second-period consumers will be able to identify both the H-seller and the L-seller. This implies $p^*_{2H} = \alpha_H$ and $p^*_{2L} = \alpha_L$. Furthermore, because the L-seller's second-period price is already the lowest possible (i.e., $p^*_{2L} = \alpha_L$), the L-seller in the first-period will not lower its price from $p_{1L} = \alpha_L$ because it won't help with the second-period profits. Therefore, we have $p^*_{1L} = \alpha_L$. Then, the profits of each type of seller are

$$\Pi^*_H = \frac{1}{2} \cdot (1-\lambda) + \frac{N}{2} \cdot \alpha_H \tag{B13}$$

$$\Pi^*_L = \frac{1}{2} \cdot \alpha_L + \frac{N}{2} \cdot \alpha_L \tag{B14}$$

Next we consider each seller's incentive of deviation to the other seller's price. Because $\alpha_L > 1-\lambda$, apparently the L-seller has no incentive to deviate from $p^*_{1L} = \alpha_L$ to $p'_{1L} = 1-\lambda$, because the price $1-\lambda$ lowers the L-seller's first-period profits and still generate unbiased reviews which reveal the L-seller's quality to the second-period consumers. For the H-seller, if deviating from $p^*_{1H} = 1-\lambda$ to $p'_{1H} = \alpha_L$, it will be recognized as the L-seller in the second period. The deviation profits are $\Pi_H(p'_{1H} = \alpha_L) = \frac{1}{2} \cdot \alpha_L + \frac{N}{2} \cdot \alpha_L$. The no deviation requires $\Pi^*_H \geq \Pi_H(p'_{1H} = \alpha_L)$, which yields

$$N \geq \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L} \tag{B15}$$

Last we consider each seller's incentive of deviation to an off-equilibrium price $p'_1 \notin \{1-\lambda, \alpha_L\}$.

  - <u>Deviation to $p'_1 < 1-\lambda$.</u>

The H-seller will not consider such a deviation regardless of consumer belief because it only lowers the first-period profits but cannot improve its second-period profits. According to Intuitive Criterion's Step 1, if $p_1' < 1-\lambda$ could ever be observed, it could only come from the L-seller (i.e., $b_1(p_1') = 0$). Since $p_1' < 1-\lambda$ will generate positive reviews, $b_1(p_1' < 1-\lambda) = 0$ further means $b_2(r_0, r_1 > 0, p_2) = 0$. As a result, the L-seller will not deviate to $p_1' < 1-\lambda$ because the deviation also lowers the first-period profits without helping with the second-period profits. Thus the equilibrium survives the Intuitive Criterion according to its Step 2.

– Deviation to $p_1' \in (1-\lambda, \alpha_L)$.

  Such a deviation will generate only negative reviews. As the second-period consumers cannot observe the first-period price, the H-seller will be recognized as the L-seller by the second-period consumers if making such a deviation. Note that (B15) ensures that the H-seller will not deviate to $p_1' = \alpha_L$ and to be recognized as the L-seller. Then, the H-seller will also not deviate to $p_1' < \alpha_L$ and to be recognized as the L-seller. For the L-seller, such a deviation is also unprofitable, because it only lowers the first-period profits but not changing the customer reviews and thus the second-period profits.

– Deviation to $p_1' \in (\alpha_L, \alpha_H]$.

  Note that with $b_1(p_1') = 0$, the deviation to p_1' is profitable for the L-seller. Therefore, the L-seller will be identified from the Intuitive Criterion's Step 1. As a result, the lowest deviation profits are when $b_1(p_1') = 0$, in which case a seller cannot make sales in the first period, does not receive any reviews, and charges at most $p_2 = \alpha_L$ in the second period. It's immediately seen that both types of seller's equilibrium profits are higher than this lowest deviation profits. The equilibrium survives the Intuitive Criterion according to its Step 2.

– Deviation to $p_1' > \alpha_H$.

  Under such a price, the first-period consumers will not make purchase and no reviews will be generated. For the similar reason as discussed in the case "$p_1' < 1-\lambda$", the deviation to $p_1' > \alpha_H$ is not profitable to either seller and the equilibrium survives the Intuitive Criterion.

- *Case 2:* $p_{1L}^* \in [\lambda, 1-\lambda]$

  This case cannot hold in equilibrium. To see the reason, note that $p_{1L}^* \in [\lambda, 1-\lambda]$ generates unbiased reviews can reveal a seller's quality. This implies $p_{2L}^* = \alpha_L$. Furthermore, the H-seller will not undercut the L-seller's first-period price because at least choosing $p_{1H} = p_{1L}^*$ can convey its high quality to the second-period consumers. Then given a separating equilibrium, we must have $p_{1H}^* > p_{1L}^*$. However, the L-seller will deviate to the H-seller's price, because it can still charge $p_{2L}^* = \alpha_L$ and improve its first-period profits. This suggests that $p_{1L}^* \in [\lambda, 1-\lambda]$ cannot be an equilibrium.

- *Case 3:* $p_{1L}^* \in [0, \lambda)$

  This case cannot hold in equilibrium. To see the reason, first consider the situation that $p_{1H} \in [0, \lambda)$. In this case, both types of sellers' first-period prices generate only positive reviews. Then, the seller that charges a lower first-period price can improve its profits by deviating to the higher first-period price, without affecting the customer reviews generated. Second consider the situation $p_{1H} \in [\lambda, 1]$. In this case, the H-seller will generate negative reviews while the L-seller only generate positive reviews. As a result, the second-period consumers can identify a seller's quality, which implies that $p^*_{2L} = \alpha_L$ and $p^*_{2H} = \alpha_H$. It is then immediately seen that the L-seller has incentive to deviate to the H-seller's first-period price, because it can improve the first-period profits and still charge $p^*_{2L} = \alpha_L$ in the second period. This suggests that $p^*_{1L} \in [0, 1-\lambda)$ cannot be an equilibrium.

To summarize the three cases, defining $N'' = \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L}$, the equilibrium characterization stated in the proposition follows. Furthermore, (B15) implies that the H-seller earns (weakly) higher profits than the L-seller.

Lastly, we provide one example of the belief that is valid and sustains the equilibrium as follows: $b_1(p_1) = 1$ if $p_1 = p^*_{1H}$ and $b_1(p_1) = 0$ otherwise. It should be noted that this is not the only belief that sustains the equilibrium.

Part (ii): Pooling Equilibrium.

Under a pooling equilibrium, we have

$$b_1(p^*_{1LH}) = \frac{1}{2} \tag{B16}$$

We consider the three cases as follows.

- *Case 1:* $p^*_{1LH} \in [\lambda, 1-\lambda]$.

  The analysis of this case is the same as Case 2 in the proof of Proposition 2. We therefore omit the details to avoid repetition, but only summarize the equilibrium outcome. That is, the condition for $p^*_{1LH} \in [\lambda, 1-\lambda]$ is

  $$\alpha_L \leq 1 - \lambda \tag{B17}$$

  The two types of sellers charge the same first-period price but differ in their second-period prices

  $$p^*_{1LH} = \min\{1-\lambda, \overline{\alpha}\} \tag{B18}$$
  $$p^*_{2H} = \alpha_H \tag{B19}$$
  $$p^*_{2L} = \alpha_L \tag{B20}$$

  The profits of each seller are

  $$\Pi^*_H = \frac{1}{2} \cdot \min\{1-\lambda, \overline{\alpha}\} + \frac{N}{2} \cdot \alpha_H \tag{B21}$$
  $$\Pi^*_L = \frac{1}{2} \cdot \min\{1-\lambda, \overline{\alpha}\} + \frac{N}{2} \cdot \alpha_L \tag{B22}$$

  and the H-seller earns higher profits than the L-seller.

- *Case 2:* $p^*_{1LH} \in (1-\lambda, 1]$.

In this case, both types of sellers receive only negative reviews. Given $p^*_{1LH} \in (1-\lambda, 1]$ and $b_1(p^*_{1LH}) = \frac{1}{2}$, we must have

$$\overline{\alpha} > 1-\lambda \tag{B23}$$

so that p^*_{1LH} can lead to consumer purchase. Furthermore, we also need to have $p^*_{1LH} \geq \alpha_L$: if $p^*_{1LH} < \alpha_L$, the L-seller can deviate to $p'_1 = \alpha_L$, which can improve its first-period profits without changing the customer reviews and thus the second-period profits.

Based on the reviews generated, we have the second-period consumers' belief as

$$b_2(r_0, r_1) = \begin{cases} \frac{1-\alpha_L}{2-\alpha_L-\alpha_H} & \text{if } r_0 \in (0, 1-\alpha_H] \\ 0 & \text{if } r_0 \in (1-\alpha_H, 1-\alpha_L] \end{cases} \tag{B24}$$

Therefore, if $r_0 \in (0, 1-\alpha_H]$, both types of sellers will charge the same second-period price denoted as p^*_{2LH}; If $r_0 \in (1-\alpha_H, 1-\alpha_L]$, the second-period consumers identify the L-seller, and as a result the L-seller charges $p^*_{2L} = \alpha_L$. We can write each seller's profits as

$$\Pi^*_H = \frac{1}{2} \cdot p^*_{1LH} + \frac{N}{2} \cdot p^*_{2LH} \tag{B25}$$

$$\Pi^*_L = \frac{1}{2} \cdot p^*_{1LH} + \frac{N}{2} \cdot \left(\frac{1-\alpha_H}{1-\alpha_L} \cdot p^*_{2LH} + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L\right) \tag{B26}$$

We then consider each seller's incentive of deviation.

- Deviation to a price $p'_1 = 1-\lambda$.
  The price $p'_1 = 1-\lambda$ is lower than p^*_{1LH} and will generate unbiased reviews to reveal a seller's quality. Therefore, an L-seller will never consider the deviation regardless of consumers' belief. Therefore, if any seller can be identified from Step 1 of Intuitive Criterion, it could only be the H-seller (i.e., $b_1(p'_1 = 1-\lambda) = 1$). With $b_1(p'_1 = 1-\lambda) = 1$, the H-seller's deviation profits are $\Pi_H(p'_1 = 1-\lambda) = \frac{1}{2} \cdot (1-\lambda) + \frac{N}{2} \cdot \alpha_H$. According to Intuitive Criterion's Step 2, we need $\Pi^*_H \geq \Pi_H(p'_1 = 1-\lambda)$, which yields

$$N \leq \frac{p^*_{1LH} - (1-\lambda)}{\alpha_H - p^*_{2LH}} \tag{B27}$$

- Deviation to a price $p'_1 < 1-\lambda$.
  Note that given the condition (B27), the H-seller will not deviate to $p'_1 = 1-\lambda$, which enables the H-seller to earn the highest possible second-period profits. As a result, the H-seller will also not deviate to $p'_1 < 1-\lambda$. Thus if any seller can be identified from Step 1 of the Intuitive Criterion, it could only be the L-seller (i.e., $b_1(p'_1 < 1-\lambda) = 0$). As $p'_1 < 1-\lambda$ generates positive reviews, $b_1(p'_1 < 1-\lambda) = 0$ further implies that $b_2(r_0, r_1 > 0) = 0$. Therefore, the L-seller will not deviate to $p'_1 < 1-\lambda$ and the equilibrium survives the Intuitive Criterion.
- Deviation to a price $p'_1 \in (1-\lambda, p^*_{1LH})$.

For each seller, charging $p_1' \in (1-\lambda, p_{1LH}^*)$ will generate the same customer reviews as charging p_{1LH}^*, and therefore will not change the second-period profits. Therefore, no seller will consider the deviation $p_1' \in (1-\lambda, p_{1LH}^*)$.

– Deviation to a price $p_1' \in (p_{1LH}^*, \alpha_H]$.

  Note that with $b_1(p_1') = 1$, both types of sellers will deviate to p_1', as the deviation does not affect the reviews but only improves each seller's first-period profits. Then both types of sellers are identified from Step 1 of the Intuitive Criterion. Therefore the lowest profits of deviating to $p_1'(p_{1LH}^*, \alpha_H]$ are when $b_1(p_1') = 0$, under which case the seller cannot make purchase in the first period, does not receive any reviews, and charges at most $p_2 = \alpha_L$ in the second period. It is immediately seen that both types of sellers' equilibrium profits are higher than the lowest deviation profits. The equilibrium survives the Intuitive Criterion according its Step 2.

– Deviation to a price $p_1' > \alpha_H$

  Note under this price, consumers in the first-period will not make purchase. Further note that the H-seller's profits from deviating to $p_1' > \alpha_H$ are strictly lower than that deviating to $p_1' = 1-\lambda$. Since condition (B27) ensures that the H-seller will never deviate to $p_1' = 1-\lambda$, it further implies that the H-seller will never deviate to $p_1' > \alpha_H$ regardless of consumers' belief. Then for the similar reason as discussed in the case "Deviation to a price $p_1' < 1-\lambda$", the deviation to $p_1' > \alpha_H$ is not profitable for the L-seller either. The equilibrium survives the Intuitive Criterion.

The analysis shows that $p_{1LH}^* \in (1-\lambda, 1]$ can be equilibrium which survives the Intuitive Criterion. Note that $p_{1LH}^* \leq \overline{\alpha}$, and $p_{2LH}^* \leq \frac{1-\alpha_L}{2-\alpha_L-\alpha_H} \cdot \alpha_H + \frac{1-\alpha_H}{2-\alpha_L-\alpha_H} \cdot \alpha_L = \frac{\alpha_H+\alpha_L-2\alpha_H\alpha_L}{2-\alpha_L-\alpha_H}$, we define $\overline{\alpha}' \equiv \frac{\alpha_H+\alpha_L-2\alpha_H\alpha_L}{2-\alpha_L-\alpha_H}$ and have

$$p_{1LH}^* = \overline{\alpha} \tag{B28}$$

$$p_{2LH}^* = \overline{\alpha}' \tag{B29}$$

which are the prices that benefit both types of sellers. From (B23), (B27), and the expressions of p_{1LH}^* and p_{2LH}^* given in (B28) and (B29), the equilbirium condition is

$$\overline{\alpha} > 1-\lambda \text{ and } N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H - \overline{\alpha}'} \tag{B30}$$

Lastly, from (B25) and (B26) it is immediately seen that the H-seller earns higher profits than the L-seller.

- *Case 3:* $p_{1LH}^* \in [0, \lambda)$.

  This case cannot hold in equilibrium. This is because given $\alpha_L > \lambda$, the H-seller can be better off by deviating from p_{1LH}^* to $p_1' = \lambda$, so that it earns higher first-period profits and also uses reviews to reveal its quality to the second-period consumers.

Cases 1 and 2 in the analysis above corresponds to the Case (1) and (2) in the proposition statement. This completes the proof of the pooling equilibrium.

Lastly, we provide one example of the belief that is valid and sustains the equilibrium as follows: $b_1(p_1) = \frac{1}{2}$ if $p_1 = p^*_{1LH}$ and $b_1(p_1) = 0$. It should be noted that this is not the only belief that sustains the equilibrium.

B6.2 The Multiple Equilibria Region.

In Proposition B1, the pooling equilibrium characterized in Case (2) can emerge together with either the separating equilibrium, or the pooling equilibrium characterized in Case (1). When such multiple equilibria happen, we explained in section 5.1.1 of the main paper that both types of sellers earn higher profits under the pooling equilibrium characterized in Case (2). Now we present the comparison details.

First, the parameter region of the separating equilibrium is $Rn_s = \{\alpha_L > 1-\lambda$ and $N \geq \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L}\}$ (see (B11) and (B15)), while the condition for the Case (2) pooling equilibrium is $Rn_{p2} = \{\overline{\alpha} > 1-\lambda$ and $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\overline{\alpha}'}\}$. It is easy to see that $Rn_s \cap Rn_{p2} \neq \emptyset$, implying that the separating equilibrium and the Case (2) pooling equilibrium can simultaneously exist. Under the separating equilibrium, as shown in (B13) and (B14) we have

$$\Pi^{s*}_H = \frac{1}{2}\cdot(1-\lambda) + \frac{N}{2}\cdot\alpha_H \tag{B31}$$

$$\Pi^{s*}_L = \frac{1}{2}\cdot\alpha_L + \frac{N}{2}\cdot\alpha_L \tag{B32}$$

Under the Case (2) pooling equilibrium, as shown in (B25) and (B26) and given $p^*_{1LH} = \overline{\alpha}$ and $p^*_{2LH} = \overline{\alpha}'$ we have

$$\Pi^{p2*}_H = \frac{1}{2}\cdot\overline{\alpha} + \frac{N}{2}\cdot\overline{\alpha}' \tag{B33}$$

$$\Pi^{p2*}_L = \frac{1}{2}\cdot\overline{\alpha} + \frac{N}{2}\cdot(\frac{1-\alpha_H}{1-\alpha_L}\cdot\overline{\alpha}' + \frac{\alpha_H-\alpha_L}{1-\alpha_L}\cdot\alpha_L) \tag{B34}$$

Note that in the region $Rn_s \cap Rn_{p2}$ we have $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\overline{\alpha}'}$, which implies $\Pi^{p2*}_H \geq \Pi^{s*}_H$. It is also immediately seen that $\Pi^{p2*}_L > \Pi^{s*}_L$. Therefore, when both the separating equilibrium and the Case (2) pooling equilibrium exist, both types of sellers earn higher profits under the Case (2) pooling equilibrium.

Second, the parameter region of the Case (1) pooling equilibrium is $Rn_{p1} = \{\alpha_L \leq 1-\lambda\}$ (see (B17)). It is easy to see that $Rn_{p1} \cap Rn_{p2} \neq \emptyset$, implying that the Case (1) pooling equilibrium and the Case (2) pooling equilibrium can simultaneously exist. Under Case (1) pooling equilibrium, as shown in (B53) and (B54), and also given $\overline{\alpha} > 1-\lambda$, we have

$$\Pi^{p1*}_H = \frac{1}{2}\cdot(1-\lambda) + \frac{N}{2}\cdot\alpha_H \tag{B35}$$

$$\Pi^{p1*}_L = \frac{1}{2}\cdot(1-\lambda) + \frac{N}{2}\cdot\alpha_L \tag{B36}$$

Once again, the region $Rn_{p1} \cap Rn_{p2}$ requires $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\overline{\alpha}'}$, which implies $\Pi^{p2*}_L \geq \Pi^{p1*}_L$. Furthermore, the region $Rn_{p1} \cap Rn_{p2}$ requires $\overline{\alpha} > \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\overline{\alpha}'}$, which implies $\Pi^{p2*}_H > \Pi^{p1*}_H$. Therefore, when both the Case (1) pooling equilibrium and the Case (2) pooling equilibrium exist, both types of sellers earn higher profits under the Case (2) pooling equilibrium.

B7 The Analysis in Section 5.1.2: A Percentage-based Review System.

Similar to the last section, we first establish a proposition to characterize the equilibrium, and then discuss the possibility of multiple equlibria. We also discuss the equilibrium outcome of a percentage-based system without historical price information.

B7.1 Equilibrium Characterization.

Proposition B2. *Under a percentage-based review system, the equilibrium is as follows.*

(i.) *Separating Equilibrium in the First Period.*

If $\alpha_L > 1-\lambda$ and $N \geq N'' \equiv \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L}$, the H-seller charges a lower first-period price than the L-seller with $p^*_{1H} = 1-\lambda < \alpha_L = p^*_{1L}$. The H-seller receives both positive and negative reviews, while the L-seller receives only negative reviews. In the second period, the H-seller charges $p^*_{2H} = \alpha_H$ and the L-seller charges $p^*_{2L} = \alpha_L$.

(ii.) *Pooling Equilibrium in the First Period.*

If $\alpha_L \leq 1-\lambda$, the pooling equilibrium characterized in the number-based system (i.e., both sellers choose $p^*_{1LH} = \min\{\overline{\alpha}, 1-\lambda\}$, receive unbiased reviews, and choose the second-period price equal to α_i) continues to hold.

In addition, if $\overline{\alpha} > 1-\lambda$ and $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\overline{\alpha}}$, a new pooling equilibrium emerges where both types of sellers charge the same first-period price $p^*_{1LH} = \overline{\alpha}$ and the same second-period price $p^*_{2LH} = \overline{\alpha}$. Both types of sellers only receive negative reviews.

Furthermore, the H-seller earns higher profits than the L-seller except for the new pooling equilibrium, under which both types of sellers earn the same profits.

Proof of Proposition B2.

Given a percentage-based review system, the H-seller can always charge $p_{1H} = \lambda$ in the first period, rely on ratio $R_p = \frac{r_1}{r_0+r_1} = \alpha_H$ to convey its quality, and charge $p_{2H} = \alpha_H$ in the second period. Following the similar reasoning in the proof of Claim 1, in equilibrium we have $D_1 = m_1$ and $D_2 = m_2$ for both types of sellers. We then prove the proposition in two parts by characterizing the separating equilibrium and the pooling equilibrium in sequence.

Part (i): Separating Equilibrium.

Because the two types of seller differ in their first-period prices which are observable to consumers in both periods, all consumers can tell a seller's quality. As a result, we have $p^*_{2H} = \alpha_H$, $p^*_{2L} = \alpha_L$, and $p^*_{1L} = \alpha_L$.

Following the same reasoning as in the analysis of the separating equilibrium of the main model (see the proof of Proposition 1), we have $p^*_{1H} < p^*_{1L}(= \alpha_L)$ and $(p^*_{1H}, p^*_{1L}) \cap [\lambda, 1-\lambda] = \emptyset$.

Given $\alpha_L \geq \lambda$, we must have $p^*_{1H} \geq 1 - \lambda$ which implies

$$\alpha_L > 1 - \lambda \tag{B37}$$

Consider the following two cases.

- *Case 1: $p^*_{1H} = 1 - \lambda$.*

  In this case, the H-seller's positive review ratio is $R_p = \alpha_H$. The expected profits of each seller are

  $$\Pi^*_H = \frac{1}{2} \cdot (1 - \lambda) + \frac{N}{2} \cdot \alpha_H \tag{B38}$$

  $$\Pi^*_L = \frac{1 + N}{2} \cdot \alpha_L \tag{B39}$$

  Consider each seller's incentive of deviation to the other seller's strategy. It is easily seen that the L-seller will not deviate from $p^*_{1L} = \alpha_L$ to $p'_{1L} = 1 - \lambda$, because the price $1 - \lambda$ is lower than $p^*_{1L} = \alpha_L$ and still reveals the L-seller's quality through reviews. If the H-seller deviates from $p^*_{1H} = 1 - \lambda$ to $p'_{1H} = \alpha_L$, it will be recognized as the L-seller. The deviation profits are $\Pi_H(p'_{1H} = \alpha_L) = \frac{1+N}{2} \cdot \alpha_L$. No deviation condition requires $\Pi^*_H \geq \Pi_H(p'_{1H} = \alpha_L)$, which yields

  $$N \geq \frac{\alpha_L - (1 - \lambda)}{\alpha_H - \alpha_L} \tag{B40}$$

  The condition (B40) also implies that the H-seller earns strictly higher profits than the L-seller.

  We next consider each seller's incentive of deviation to an off-equilibrium price $p'_1 \notin \{1 - \lambda, \alpha_L\}$.

  - <u>Deviation to $p'_1 < 1 - \lambda$.</u>

    Regardless of consumers' belief $b_1(p'_1)$, the H-seller will not consider such a deviation as it only lowers its first-period profits but cannot improve its second-period profits. Thus if any seller can be identified from the Intuitive Criterion, it could be only be the L-seller. With $b_1(p'_1) = 0$, the deviation profits are lower than the L-seller's equilibrium profits. Thus the equilibrium satisfies the Intuitive Criterion according to its Step 2.

  - <u>Deviation to $p'_1 \in (1 - \lambda, \alpha_L)$.</u>

    Note that with $b_1(p'_1) = 1$, the H-seller finds the deviation profitable. Thus the H-seller is identified from Intuitive Criterion's Step 1. However, the H-seller cannot be the only seller that is identified from Intuitive Criterion's Step 1, as otherwise consumers will hold $b_1(p'_1) = 1$ and the H-seller would indeed deviate. Therefore, we need the L-seller also to be identified from Intuitive Criterion's Step 1. The L-seller achieves the highest deviation profits when $b_1(p'_1) = 1$ (and accordingly $b_2(p'_1, R_p(p'_1)) = 1$), which are

$$\Pi_L(p_1') = \frac{1}{2} \cdot p_1' + \frac{N}{2} \cdot \alpha_H \tag{B41}$$

We need $\Pi_L(p_1') \geq \Pi_L^*$, $\forall p_1' \in (1-\lambda, \alpha_L)$, which yields the condition

$$N \geq \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L} \tag{B42}$$

which is the same as (B40). With this condition, the L-seller will also be identified from Intuitive Criterion's Step 1. Therefore, for both types of sellers, the lowest deviation profits are when $b_1(p_1') = 0$. It's clear that the L-seller will not deviate to p_1' given $b_1(p_1') = 0$. Furthermore, condition (B40) ensures that the H-seller will not deviate to $p_1' = p_{1L}^* = \alpha_L$, and therefore will not deviate to $p_1' < \alpha_L$ to pretend to be the L-seller. The equilibrium satisfies the Intuitive Criterion according to its Step 2.

– Deviation to $p_1' \in (\alpha_L, \alpha_H]$.

  With $b_1(p_1') = 1$, such a deviation is profitable to both types of sellers. Therefore, both types of sellers are identified from Intuitive Criterion's Step 1. Then the deviation profits are the lowest when $b_1(p_1') = 0$, in which case the first-period consumers will not purchase and the second-period consumers would like to pay $p_2 = \alpha_L$ at most. It is immediately seen that both types of sellers will not deviate to p_1'. The equilibrium survives the Intuitive Criterion according to its Step 2.

– Deviation to $p_1' > \alpha_H$.

  This is a price under which the seller cannot make sales. For the similar reason discussed in the case "$p_1' < 1-\lambda$", the deviation to $p_1' > \alpha_H$ is not profitable and the equilibrium survives the Intuitive Criterion.

- *Case 2:* $p_{1H}^* > 1-\lambda$.

  In this case, both the H-seller and the L-seller have the positive review ratio $R_p = 0$. The expected profits of each seller are

$$\Pi_H^* = \frac{1}{2} \cdot p_{1H}^* + \frac{N}{2} \cdot \alpha_H \tag{B43}$$

$$\Pi_L^* = \frac{1+N}{2} \cdot \alpha_L \tag{B44}$$

  Note that if deviating to the H-seller's price p_{1H}^*, the L-seller can pretend to be an H-seller and earn Π_H^*. The equilibrium condition requires $\Pi_L^* \geq \Pi_H^*$. Similarly, if deviating to the L-seller's price $p_{1L}^* = \alpha_L$, the H-seller can pretend to be an L-seller and earn Π_L^*. The equilibrium condition requires $\Pi_H^* \geq \Pi_L^*$. As a result, we must have $\Pi_H^* = \Pi_L^*$, which yields

$$p_{1H}^* = (1+N) \cdot \alpha_L - N \cdot \alpha_H \tag{B45}$$

The condition $p_{1H}^* > 1 - \lambda$ requires

$$N < \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L} \tag{B46}$$

And the profits of each seller are

$$\Pi_H^* = \Pi_L^* = \frac{1+N}{2} \cdot \alpha_L \tag{B47}$$

Lastly, for a deviation price $p_1' < p_{1H}^*$, neither seller will consider it because the deviation will result in profit loss regardless of consumers' belief. For a deviation price $p_1' > p_{1H}^*$, the lowest deviation profits happen under the belief $b_1(p_1') = 0$ which makes the deviation not profitable for either seller. This also ensures that the equilibrium to satisfy the Intuitive Criterion.

In summary, the separating equilibrium is characterized as follows.

- If $\alpha_L > 1 - \lambda$ and $N \geq \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L}$, the equilibrium is $p_{1H}^* = 1 - \lambda$ and $p_{1L}^* = \alpha_L$. The H-seller earns higher profits.
- If $\alpha_L > 1 - \lambda$ and $N < \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L}$, the equilibrium is $p_{1H}^* = (1+N)\alpha_L - N\alpha_H$ and $p_{1L}^* = \alpha_L$. The two sellers earn the same profits.

The separating equilibrium when $N < \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L}$, however, will always coexist with a pooling equilibrium, while both types of sellers earn higher profits under the pooling equilibrium. We present the details in Part (ii) of the proof.

Lastly, we provide an example of the belief that is valid and sustains the equilibrium as follows: $b_1(p_1) = 1$ if $p_1 = p_{1H}^*$ and $b_1(p_1) = 0$ otherwise. It should be noted that this is not only belief that sustains the equilibrium.

Part (ii): Pooling Equilibrium.

Under a pooling equilibrium, we have

$$b_1(p_{1LH}^*) = \frac{1}{2} \tag{B48}$$

Consider the three cases as follows.

- *Case 1:* $p_{1LH}^* \in [\lambda, 1-\lambda]$.

  The analysis of this case is the same as Case 2 in the proof of Proposition 2. Given its repetitive nature, we omit the details but only summarize the equilibrium outcome. That is, the condition for $p_{1LH}^* \in [\lambda, 1-\lambda]$ is

  $$\alpha_L \leq 1 - \lambda \tag{B49}$$

  The two types of sellers charge the same first-period price but differ in their second-period prices

$$p_{1LH}^{*} = \min\{1-\lambda, \overline{\alpha}\} \tag{B50}$$

$$p_{2H}^{*} = \alpha_H \tag{B51}$$

$$p_{2L}^{*} = \alpha_L \tag{B52}$$

The profits of each seller are

$$\Pi_H^{*} = \frac{1}{2} \cdot \min\{1-\lambda, \overline{\alpha}\} + \frac{N}{2} \cdot \alpha_H \tag{B53}$$

$$\Pi_L^{*} = \frac{1}{2} \cdot \min\{1-\lambda, \overline{\alpha}\} + \frac{N}{2} \cdot \alpha_L \tag{B54}$$

and the H-seller earns higher profits than the L-seller.

- *Case 2:* $p_{1LH}^{*} \in (1-\lambda, 1]$.

  In this case, both types of sellers receive only negative reviews and therefore the positive review ratio $R_p = 0$. Given $p_{1LH}^{*} \in (1-\lambda, 1]$ and $b_1(p_{1LH}^{*}) = \frac{1}{2}$ (so that $p_{1LH}^{*} \leq \overline{\alpha}$), we must have

  $$\overline{\alpha} > 1-\lambda \tag{B55}$$

  so that p_{1LH}^{*} can lead to consumer purchase. We further have the second-period consumers' belief as $b_2(p_{1LH}^{*}, R_p(p_{1LH}^{*}) = 0) = \frac{1}{2}$. This means both types of sellers will charge the same second-period price, denoted as p_{2LH}^{*}. We can write each seller's profits as

  $$\Pi_H^{*} = \frac{1}{2} \cdot p_{1LH}^{*} + \frac{N}{2} \cdot p_{2LH}^{*} \tag{B56}$$

  $$\Pi_L^{*} = \frac{1}{2} \cdot p_{1LH}^{*} + \frac{N}{2} \cdot p_{2LH}^{*} \tag{B57}$$

  Given $p_{1LH}^{*} \leq \overline{\alpha}$, $p_{2LH}^{*} \leq \overline{\alpha}$, and we focus on the equilibrium that each type of seller earn higher profits, we have

  $$p_{1LH}^{*} = p_{2LH}^{*} = \overline{\alpha} \tag{B58}$$

  We then show that this is indeed an equilibrium by considering each seller's incentive of deviation to an off-equilibrium price.

  - <u>Deviation to a price $p_1' = 1-\lambda$.</u>

    The price $p_1' = 1-\lambda$ is lower than p_{1LH}^{*} and will generate the ratio R_p that can reveal a seller's quality. Therefore, an L-seller will never consider the deviation regardless of consumers' belief. If any seller can be identified from Intuitive Criterion's Step 1, it could only be the H-seller. The equilibrium holds only if the H-seller finds the deviation not profitable under $b_1(p_1') = 1$. Note that the deviation profits are

$\Pi_H(p_1' = 1-\lambda) = \frac{1}{2}\cdot(1-\lambda) + \frac{N}{2}\cdot\alpha_H$. On solving $\Pi_H^* \geq \Pi_H(p_1' = 1-\lambda)$ we have

$$N \leq \frac{\overline{\alpha} - (1-\lambda)}{\alpha_H - \overline{\alpha}} \tag{B59}$$

With condition (B59), no sellers will be identified from Intuitive Criterion's Step 1, suggesting that no seller finds the deviation to $p_1' = 1-\lambda$ profitable regardless of consumers' belief.

– Deviation to a price $p_1' < 1-\lambda$.
The condition (B59) implies that the H-seller will not deviate to $p_1' = 1-\lambda$, which enables the H-seller to earn the highest possible second-period profits. As a result, it also ensures that the H-seller will not deviate to $p_1' < 1-\lambda$. Then only the L-seller could be possibly identified from the Intuitive Criterion's Step 1, i.e., $b_1(p_1') = 0$. However, given $b_1(p_1') = 0$, the L-seller will not deviate to $p_1' < 1-\lambda$ either. Thus the equilibrium holds and survives the Intuitive Criterion.

– Deviation to a price $p_1' > 1-\lambda$.
In this situation, since both types of sellers earn the same profits without deviation (see (B56) and (B57)) and $R_p(p_1') = 0$ for both types of sellers, if the deviation benefits the H-seller, it also benefits the L-seller. Therefore, when $b_1(p_1') = 0$ the deviation profits are the lowest and neither seller will find the deviation profitable. As a result, the equilibrium survives the Intuitive Criterion.

In summary, with the condition

$$\overline{\alpha} > 1-\lambda \text{ and } N \leq \frac{\overline{\alpha} - (1-\lambda)}{\alpha_H - \overline{\alpha}} \tag{B60}$$

we have the pooling equilibrium $p_{1LH}^* = p_{2LH}^* = \overline{\alpha}$. In addition, both sellers earn the same profits

$$\Pi_H^* = \Pi_L^* = \frac{1+N}{2}\cdot\overline{\alpha} \tag{B61}$$

- *Case 3:* $p_{1LH}^* \in [0, \lambda)$.

  Given $\alpha_L > \lambda$, it is easy to see that $p_{1LH}^* \in [0, \lambda)$ cannot hold in equilibrium. This is because the H-seller can improve profits by deviating from p_{1LH}^* to $p_1' = \lambda$, so that it earns higher first-period profits and also uses the positive review ratio $R_p = \alpha_H$ to reveal its quality to the second-period consumers.

Cases 1 and 2 in the pooling equilibrium analysis above correspond to the characterization of the pooling equilibrium in the proposition.

Furthermore, it is seen that the condition of Case 2 of the separating equilibrium (i.e., $\alpha_L > 1-\lambda$ and $N < \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L}$) is a subset of the Case 2 of the pooling equilibrium (i.e., $\overline{\alpha} > 1-\lambda$ and $N < \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\overline{\alpha}}$), but both types of sellers earn higher profits under the Case 2 pooling equilibrium (i.e., $\frac{1+N}{2}\cdot\overline{\alpha}$) than that under the Case 2 separating equilibrium (i.e., $\frac{1+N}{2}\cdot\alpha_L$). Therefore, we will not focus attention on Case 2 of the separating equilibrium.

Lastly, we provide one example of the belief that is valid and sustains the equilibrium as follows: $b_1(p_1) = \frac{1}{2}$ if $p_1 = p^*_{1LH}$ and $b_1(p_1) = 0$ otherwise. It should be noted that this is not the only belief that sustains the equilibrium.

B7.2 The Multiple Equilibria Region.

From the proof of Proposition B2, it is seen that the condition of the second pooling equilibrium has overlap with the condition of the separating equilibrium and the condition of the first pooling equilibrium , which suggests the existence of multiple equilibria. Following the same analysis as in Section B4, we can show that when multiple equilibria happen, both types of sellers earn higher profits under the second pooling equilibrium. We omit the details to avoid repetition.

B7.3 A Percentage-based System without Historical Price Information.

In the last part of Section 5.1.2 of the main paper, we also state that the removal of historical price information given a percentage-based system will not affect the equilibrium characterization as shown in Proposition B2. The derivation of the equilibrium when historical price is removed is similar to the proof of Proposition B2. We omit the details to avoid repetition.

B8 The Analysis in Section 5.1.3.

To show that Proposition 3 is robust under either a system without historical price information or a system that is percentage-based, we calculate the total consumer surplus in each setting and then make comparisons to that when a review system is absent (see section B5). Same as in the proof of Proposition 3, in the second period the seller always sets the price that is equal to consumers' willingness to pay. Therefore the consumer surplus in the second period is zero. The total consumer surplus is the same as the first-period consumer surplus.

1. Consumer Surplus under a Review System Without Historical Price Information.

We derive the consumer surplus under the separating equilibrium and under the pooling equilibrium in sequence.

- Separating equilibrium.

  From Proposition B1, a separating equilibrium exists if $\alpha_L > 1 - \lambda$ and $N \geq \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L}$.the consumer surplus is

$$CS^s_{NHP} = (\frac{1}{2} \cdot (\alpha_H - (1-\lambda)) + \frac{1}{2} \cdot (\alpha_L - \alpha_L)) \cdot \frac{1}{2} = \frac{\alpha_H - (1-\lambda)}{4} \tag{B62}$$

  Here the subscript"NHP" is the abbreviation for no-historical-price.

- Pooling equilibrium.

  From Proposition 3, if $\alpha_L \leq 1 - \lambda$, a pooling equilibrium where reviews are unbiased can exist. The consumer surplus is

$$CS^{p1}_{NHP} = (\frac{1}{2}\cdot(\alpha_H - \min\{1-\lambda, \overline{\alpha}\}) + \frac{1}{2}\cdot(\alpha_L - \min\{1-\lambda, \overline{\alpha}\})\cdot\frac{1}{2} \tag{B63}$$

$$= \frac{\max\{0, \overline{\alpha} - (1-\lambda)\}}{2} \tag{B64}$$

If $\overline{\alpha} > 1-\lambda$ and $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H - \overline{\alpha}'}$ a pooling equilibrium with only negative reviews can exist. Since in this equilibrium, a seller will charge a price that is equal to consumers' willingness to pay in both periods, we have

$$CS^{p2}_{NHP} = 0 \tag{B65}$$

2. Consumer Surplus under a Percentage-based Review System.

We derive the consumer surplus under the separating equilibrium and under the pooling equilibrium in sequence.

- Separating equilibrium.

  From Proposition B2, a separating equilibrium exists if $\alpha_L > 1-\lambda$ and $N \geq \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L}$.the consumer surplus is

$$CS^{s}_{PB} = (\frac{1}{2}\cdot(\alpha_H - (1-\lambda)) + \frac{1}{2}\cdot(\alpha_L - \alpha_L))\cdot\frac{1}{2} = \frac{\alpha_H - (1-\lambda)}{4} \tag{B66}$$

  Here the subscript"PB" denotes the percentage-based system.

- Pooling equilibrium.

  From Proposition 4, if $\alpha_L \leq 1-\lambda$, a pooling equilibrium where the positive review ratio can reveal a seller's quality can exist. The consumer surplus is

$$CS^{p1}_{PB} = (\frac{1}{2}\cdot(\alpha_H - \min\{1-\lambda, \overline{\alpha}\}) + \frac{1}{2}\cdot(\alpha_L - \min\{1-\lambda, \overline{\alpha}\})\cdot\frac{1}{2} \tag{B67}$$

$$= \frac{\max\{0, \overline{\alpha} - (1-\lambda)\}}{2} \tag{B68}$$

  If $\overline{\alpha} > 1-\lambda$ and $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H - \overline{\alpha}}$ a pooling equilibrium with only $R_p = 0$ can exist. In this equilibrium, a seller will charge a price that is equal to consumers' willingness to pay in both periods. Therefore, we have

$$CS^{p2}_{PB} = 0 \tag{B69}$$

Recall from Section B5 that consumer surplus is zero when a review system is absent. We see from the consumer surplus derived above that it can be positive in both settings. Furthermore, under a separating equilibrium, consumer surplus is always positive, and therefore is higher than when a review system is absent.

Lastly, under each different type of review systems, with the consumer surplus expression under each equilibrium region, we can write the consumer welfare as a function of λ in the entire parameter space (in case of multiple equilibria, as discussed in Section B6.2 and B7.2 we choose the one that both sellers earn higher profits). It can be easily verified that the consumer surplus weakly increases with λ in both settings.

B9 Proof of Lemma 3.

The approach of deriving the equilibrium given risk-averse aversion is the same as that given risk-neutral consumers. As a result, the proof of Lemma 3 is highly similar to that of Lemma 1, Proposition 1, and Proposition 2. Due to its repetitive nature, we omit the details.

B10 Proof of Proposition 4.

We prove two parts of the proposition in sequence.

Part (a): Total Welfare.

Consider a situation that a consumer's perception of the seller's probability of delivering a good experience being α and the seller's price being p. After the transaction, the consumer surplus is $\alpha - \alpha(1-\alpha) - p = \alpha^2 - p$, and the seller profits are p. Therefore, the total welfare from the transaction is $\alpha^2 - p + p = \alpha^2$, which is also the consumer's willingness to pay. This means the total welfare can be calculated as the total willingness to pay of all consumers.

In the absence of a review system, from Lemma 3 we know that each consumer's willingness to pay is $\overline{\alpha}^2$. Therefore, the total welfare is

$$TW_0 = \frac{1+N}{2} \cdot \overline{\alpha}^2 \tag{B70}$$

In the presence of a review system, from Lemma 3, if $\alpha_L^2 > 1-\lambda$, the separating equilibrium emerges, as a result consumers of both periods can tell the seller's quality. The expected total welfare is $\frac{1+N}{2} \cdot (\frac{1}{2} \cdot \alpha_H^2 + \frac{1}{2} \cdot \alpha_L^2) = \frac{1+N}{4} \cdot (\alpha_H^2 + \alpha_L^2)$. If $\alpha_L^2 \leq 1-\lambda$, the pooling equilibrium emerges. In this case the first-period consumers cannot identify the seller's quality but the second-period consumers can. The expected total welfare is $\frac{1}{2} \cdot \overline{\alpha}^2 + \frac{N}{2} \cdot (\frac{1}{2} \cdot \alpha_H^2 + \frac{1}{2} \cdot \alpha_L^2) = \frac{\overline{\alpha}^2}{2} + \frac{N}{4} \cdot (\alpha_H^2 + \alpha_L^2)$. In summary we have

$$TW_r(\lambda) = \begin{cases} \frac{\overline{\alpha}^2}{2} + \frac{N}{4} \cdot (\alpha_H^2 + \alpha_L^2) & \text{if } \lambda \leq 1 - \alpha_L^2 \\ \frac{1+N}{4} \cdot (\alpha_H^2 + \alpha_L^2) & \text{if } \lambda > 1 - \alpha_L^2 \end{cases} \tag{B71}$$

Note that

$$TW_r(\lambda) - TW_0 = \begin{cases} \frac{N}{2} \cdot (\frac{\alpha_H^2 + \alpha_L^2}{2} - \overline{\alpha}^2) & \text{if } \lambda \leq 1 - \alpha_L^2 \\ \frac{1+N}{2} \cdot (\frac{\alpha_H^2 + \alpha_L^2}{2} - \overline{\alpha}^2) & \text{if } \lambda > 1 - \alpha_L^2 \end{cases} \tag{B72}$$

Thus $TW_r(\lambda) - TW_0 > 0$. Furthermore,

$$TW_r(\lambda > 1 - \alpha_L^2) - TW_r(\lambda \leq 1 - \alpha_L^2) = \frac{1}{2} \cdot (\frac{\alpha_H^2 + \alpha_L^2}{2} - \overline{\alpha}^2) > 0 \tag{B73}$$

Therefore, $TW_r(\lambda)$ weakly increases with λ, and is strictly higher when $\lambda > 1 - \alpha_L^2$ than when $\lambda \leq 1 - \alpha_L^2$.

Part (b): Consumer Surplus and Seller's Average Profits.

Because the seller in the second period always sets the price equal to a consumer's willingness to pay, the second-period consumer surplus is 0. Therefore, the total consumer surplus is the same as the consumer surplus in the first-period.

When a review system is absent, the total consumer surplus is

$$CS_0 = 0 \tag{B74}$$

And the average profits of the two sellers are

$$\Pi_0^{ave} = \frac{1+N}{2} \cdot \overline{\alpha}^2 \tag{B75}$$

With a review system, consider the separating equilibrium and pooling equilibrium separately.

- Separating Equilibrium.

  From Lemma 3, the separating equilibrium emerges when $\alpha_L^2 > 1 - \lambda$. The consumer surplus is

$$\begin{aligned} CS_r &= (\frac{1}{2} \cdot (\alpha_H^2 - p_{1H}^*) + \frac{1}{2} \cdot (\alpha_L^2 - p_{1L}^*)) \cdot \frac{1}{2} \\ &= \begin{cases} \frac{\alpha_H^2 - (1-\lambda)}{4} & \text{if } N \geq \frac{\alpha_L^2 - (1-\lambda)}{\alpha_H^2 - \alpha_L^2} \cdot \frac{1-\alpha_L}{1-\alpha_H} \\ \frac{1}{4} \cdot (\alpha_H^2 - \alpha_L^2) \cdot (1 + N \cdot \frac{1-\alpha_H}{1-\alpha_L}) & \text{if } N < \frac{\alpha_L^2 - (1-\lambda)}{\alpha_H^2 - \alpha_L^2} \cdot \frac{1-\alpha_L}{1-\alpha_H} \end{cases} \end{aligned} \tag{B76}$$

  And the average profits of the two sellers are

$$\begin{aligned} \Pi_r^{ave} &= (\frac{p_{1H}^*}{2} + \frac{p_{1L}^*}{2}) \cdot \frac{1}{2} + (\frac{p_{2H}^*}{2} + \frac{p_{2L}^*}{2}) \cdot \frac{N}{2} \\ &= \begin{cases} \frac{1}{2} \cdot (\frac{1-\lambda}{2} + \frac{\alpha_L^2}{2}) + \frac{N}{2} \cdot (\frac{\alpha_H^2}{2} + \frac{\alpha_L^2}{2}) & \text{if } N \geq \frac{\alpha_L^2 - (1-\lambda)}{\alpha_H^2 - \alpha_L^2} \cdot \frac{1-\alpha_L}{1-\alpha_H} \\ \frac{1}{2} \cdot (\alpha_L^2 - \frac{1}{2} \cdot N \cdot \frac{1-\alpha_H}{1-\alpha_L} \cdot (\alpha_H^2 - \alpha_L^2)) + \frac{N}{2} \cdot (\frac{\alpha_H^2}{2} + \frac{\alpha_L^2}{2}) & \text{if } N < \frac{\alpha_L^2 - (1-\lambda)}{\alpha_H^2 - \alpha_L^2} \cdot \frac{1-\alpha_L}{1-\alpha_H} \end{cases} \end{aligned} \tag{B77}$$

  It is immediately seen that $CS_r > CS_0$. Furthermore, if $N > \max\{\frac{\alpha_L^2 - (1-\lambda)}{\alpha_H^2 - \alpha_L^2} \cdot \frac{1-\alpha_L}{1-\alpha_H}, \frac{2\overline{\alpha}^2 - (1-\lambda+\alpha_L^2)}{2(\alpha_H^2 + \alpha_L^2 - 2\overline{\alpha}^2)}\}$, we have $\Pi_r^{ave} > \Pi_0^{ave}$.

- Pooling Equilibrium.

  From Lemma 3, the pooling equilibrium emerges when $\alpha_L^2 \leq 1 - \lambda$. The consumer surplus is

$$CS_r = (\overline{\alpha}^2 - p_{1LH}^*) \cdot \frac{1}{2} = \frac{\max\{0, \overline{\alpha}^2 - (1-\lambda)\}}{2} \tag{B78}$$

And the average profits of the two sellers are

$$\Pi_r^{ave} = (\frac{p_{1LH}^*}{2} + \frac{p_{1LH}^*}{2}) \cdot \frac{1}{2} + (\frac{p_{2H}^*}{2} + \frac{p_{2L}^*}{2}) \cdot \frac{N}{2} = \frac{\min\{\overline{\alpha}^2, 1-\lambda\}}{2} + \frac{N}{4} \cdot (\alpha_H^2 + \alpha_L^2) \tag{B79}$$

Again, we can see that $CS_r \geq CS_0$. Furthermore, if $N > \frac{2\max\{0, \overline{\alpha}^2 - (1-\lambda)\}}{\alpha_H^2 + \alpha_L^2 - 2\overline{\alpha}^2}$, we have $\Pi_r^{ave} > \Pi_0^{ave}$.

In summary, a review system weakly improves consumers' surplus, and if N is large, strictly improves the average profits of the two types of sellers.

B11 Proof of Corollary 1.

With the equilibrium characterized in Lemma 3, we can write down the expected profits of the H-seller and the L-seller as

$$\Pi_H^* = \begin{cases} \frac{1}{2} \cdot (1-\lambda) + \frac{N}{2} \cdot \alpha_H^2 & \text{if } \alpha_L^2 > 1-\lambda, N \geq \frac{\alpha_L^2 - (1-\lambda)}{\alpha_H^2 - \alpha_L^2} \cdot \frac{1-\alpha_L}{1-\alpha_H} \\ \frac{1}{2} \cdot (\alpha_L^2 - N \cdot \frac{1-\alpha_H}{1-\alpha_L} \cdot (\alpha_H^2 - \alpha_L^2)) + \frac{N}{2} \cdot \alpha_H^2 & \text{if } \alpha_L^2 > 1-\lambda, N < \frac{\alpha_L^2 - (1-\lambda)}{\alpha_H^2 - \alpha_L^2} \cdot \frac{1-\alpha_L}{1-\alpha_H} \\ \frac{1}{2} \cdot \overline{\alpha}^2 + \frac{N}{2} \cdot \alpha_H^2 & \text{if } \alpha_L^2 \leq 1-\lambda, \overline{\alpha}^2 \leq 1-\lambda \\ \frac{1}{2} \cdot (1-\lambda) + \frac{N}{2} \cdot \alpha_H^2 & \text{if } \alpha_L^2 \leq 1-\lambda, \overline{\alpha}^2 > 1-\lambda \end{cases} \tag{B80}$$

$$\Pi_L^* = \begin{cases} \frac{1+N}{2} \cdot \alpha_L^2 & \text{if } \alpha_L^2 > 1-\lambda \\ \frac{1}{2} \cdot \overline{\alpha}^2 + \frac{N}{2} \cdot \alpha_L^2 & \text{if } \alpha_L^2 \leq 1-\lambda, \overline{\alpha}^2 \leq 1-\lambda \\ \frac{1}{2} \cdot (1-\lambda) + \frac{N}{2} \cdot \alpha_L^2 & \text{if } \alpha_L^2 \leq 1-\lambda, \overline{\alpha}^2 > 1-\lambda \end{cases} \tag{B81}$$

Both Π_H^* and Π_L^* are piecewise functions. It is first verified that the corollary holds in each of the function's sub regions. Further note that both Π_H^* and Π_L^* are continuous in the entire parameter space. Therefore, the corollary holds in the entire parameter space.

B12 Comparison Across Different Review Systems in Section 5.2.

In the last part of Section 5.2 we focus on the risk-averse consumers and describe the comparisons of total welfare, consumer surplus, and average seller profits across the main model system, the system with historical price information hidden from consumers, and the percentage-based system. We present the details here.

The equilibrium outcome, the total welfare, consumer surplus, and the average seller profits under the main model are shown in Lemma 3 and the proof of Proposition 4. For the system with historical price information hidden from consumers and the percentage-based system, the equilibrium derivation is very similar to that in proof of Proposition B1 and Proposition B2. To avoid repetition, we omit the derivation details, but only report the equilibrium characterization, the total welfare, the consumer surplus, and the average seller profits. We then based on the results to make comparisons.

1. The review system with historical price information hidden from consumers.

- Equilibrium Characterization.

  If $\alpha_L^2 > 1-\lambda$ and $N \geq \frac{\alpha_L^2-(1-\lambda)}{\alpha_H^2-\alpha_L^2}$, a separating equilibrium exists. In this case, the H-seller charges a lower first-period price than the L-seller, $p_{1H}^* = 1-\lambda < p_{1L}^* = \alpha_L^2$. The H-seller receives both positive and negative reviews , while the L-seller receives only negative reviews. In the second period, the H-seller charges $p_{2H}^* = \alpha_H^2$ and the L-seller charges $p_{2L}^* = \alpha_L^2$.

  If (1) $\alpha_L^2 \leq 1-\lambda$, or (2) $\overline{\alpha}^2 > 1-\lambda$ and $N \leq \frac{\overline{\alpha}^2-(1-\lambda)}{\alpha_H^2-\overline{\alpha'}^2}$ where $\overline{\alpha'} \equiv \frac{\alpha_H+\alpha_L-2\alpha_H\alpha_L}{2-\alpha_H-\alpha_L}$, a pooling equilibrium exists.

  - In Case (1), both types of sellers charge the first-period price $p_{1LH}^* = \min\{\overline{\alpha}^2, 1-\lambda\}$ and receive both positive and negative reviews. In the second period, the H-seller charges $p_{2H}^* = \alpha_H^2$ and the L-seller charges $p_{2L} = \alpha_L^2$.
  - In Case (2), both types of sellers charge the first-period price $p_{1LH}^* = \min\{\overline{\alpha}^2, 1-\lambda\}$, and receive only negative reviews. In the second period, if $r_0 > 1-\alpha_H$, the L-seller sets $p_{2L}^* = \alpha_L^2$. Otherwise, both sellers set $p_{2LH}^* = \overline{\alpha'}^2$.

  Furthermore, the Case (2) pooling equilibrium can simultaneously exist with either the separating equilibrium or the Case (1) pooling equilibrium. In this case, we focus on the Case (2) pooling equilibrium because both types of sellers earn higher profits compared to that under the alternative equilibrium.

- Total Welfare.

  Under the separating equilibrium, the total welfare is

$$TW_{NHP} = \frac{1+N}{4} \cdot (\alpha_H^2 + \alpha_L^2) \tag{B82}$$

  Under the Case (1) pooling equilibrium, the total welfare is

$$TW_{NHP} = \frac{\overline{\alpha}^2}{2} + \frac{N}{4} \cdot (\alpha_H^2 + \alpha_L^2) \tag{B83}$$

  Under the Case (2) pooling equilibrium, the total welfare is

$$TW_{NHP} = \frac{\overline{\alpha}^2}{2} + \frac{N}{4} \cdot \left(\frac{2-\alpha_L-\alpha_H}{1-\alpha_L} \cdot \overline{\alpha'}^2 + \frac{\alpha_H-\alpha_L}{1-\alpha_L} \cdot \alpha_L^2\right) \tag{B84}$$

- Consumer Surplus.

  Under the separating equilibrium, the consumer surplus is

$$CS_{NHP} = \frac{\alpha_H^2-(1-\lambda)}{4} \tag{B85}$$

Under the Case (1) pooling equilibrium, the total welfare is

$$CS_{NHP} = \frac{\max\{0, \overline{\alpha}^2 - (1-\lambda)\}}{2} \tag{B86}$$

Under the Case (2) pooling equilibrium, the total welfare is

$$CS_{NHP} = 0 \tag{B87}$$

- Average seller profits.

  Under the separating equilibrium, the average seller profits are

$$\Pi_{NHP}^{ave} = \frac{1}{2} \cdot (\frac{1-\lambda}{2} + \frac{\alpha_L^2}{2}) + \frac{N}{2} \cdot (\frac{\alpha_H^2}{2} + \frac{\alpha_L^2}{2}) \tag{B88}$$

  Under the Case (1) pooling equilibrium, the average seller profits are

$$\Pi_{NHP}^{ave} = \frac{\min\{\overline{\alpha}^2, 1-\lambda\}}{2} + \frac{N}{4} \cdot (\alpha_H^2 + \alpha_L^2) \tag{B89}$$

  Under the Case (2) pooling equilibrium, the average seller profits are

$$\Pi_{NHP}^{ave} = \frac{\overline{\alpha}^2}{2} + \frac{N}{4} \cdot (\frac{2-\alpha_L-\alpha_H}{1-\alpha_L} \cdot \overline{\alpha'}^2 + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L^2) \tag{B90}$$

2. The percentage-based review system.

- Equilibrium Characterization.

  If $\alpha_L^2 > 1-\lambda$ and $N \geq \frac{\alpha_L^2-(1-\lambda)}{\alpha_H^2-\alpha_L^2}$, a separating equilibrium exists. In this case, the H-seller charges a lower first-period price than the L-seller, $p_{1H}^* = 1-\lambda < p_{1L}^* = \alpha_L^2$. The H-seller receives a positive and negative reviews, while the L-seller receives only negative reviews. In the second period, the H-seller charges $p_{2H}^* = \alpha_H^2$ and the L-seller charges $p_{2L}^* = \alpha_L^2$.

  If (1) $\alpha_L^2 \leq 1-\lambda$, or (2) $\overline{\alpha}^2 > 1-\lambda$ and $N \leq \frac{\overline{\alpha}^2-(1-\lambda)}{\alpha_H^2-\overline{\alpha}^2}$, a pooling equilibrium exists.

  - In Case (1), both types of sellers charge the first-period price $p_{1LH}^* = \min\{\overline{\alpha}^2, 1-\lambda\}$ and receive both positive and negative reviews. In the second period, the H-seller charges $p_{2H}^* = \alpha_H^2$ and the L-seller charges $p_{2L} = \alpha_L^2$.
  - In Case (2), both types of sellers charge the first-period price $p_{1LH}^* = \min\{\overline{\alpha}^2, 1-\lambda\}$, and receive only negative reviews. In the second period, both sellers set $p_{2LH}^* = \overline{\alpha}^2$.

  Furthermore, the Case (2) pooling equilibrium can coexist with either the separating equilibrium or the Case (1) pooling equilibrium. In this case, we focus on the Case (2) pooling equilibrium because both types of sellers earn higher profits compared to that under the alternative equilibrium.

- Total Welfare.

  Under the separating equilibrium, the total welfare is

$$TW_{PB} = \frac{1+N}{4} \cdot (\alpha_H^2 + \alpha_L^2) \tag{B91}$$

  Under the Case (1) pooling equilibrium, the total welfare is

$$TW_{PB} = \frac{\overline{\alpha}^2}{2} + \frac{N}{4} \cdot (\alpha_H^2 + \alpha_L^2) \tag{B92}$$

  Under the Case (2) pooling equilibrium, the total welfare is

$$TW_{PB} = \frac{1+N}{2} \cdot \overline{\alpha}^2 \tag{B93}$$

- Consumer Surplus.

  Under the separating equilibrium, the consumer surplus is

$$CS_{PB} = \frac{\alpha_H^2 - (1-\lambda)}{4} \tag{B94}$$

  Under the Case (1) pooling equilibrium, the total welfare is

$$CS_{PB} = \frac{\max\{0, \overline{\alpha}^2 - (1-\lambda)\}}{2} \tag{B95}$$

  Under the Case (2) pooling equilibrium, the total welfare is

$$CS_{PB} = 0 \tag{B96}$$

- Average seller profits.

  Under the separating equilibrium, the average seller profits are

$$\Pi_{PB}^{ave} = \frac{1}{2} \cdot (\frac{1-\lambda}{2} + \frac{\alpha_L^2}{2}) + \frac{N}{2} \cdot (\frac{\alpha_H^2}{2} + \frac{\alpha_L^2}{2}) \tag{B97}$$

  Under the Case (1) pooling equilibrium, the average seller profits are

$$\Pi_{PB}^{ave} = \frac{\min\{\overline{\alpha}^2, 1-\lambda\}}{2} + \frac{N}{4} \cdot (\alpha_H^2 + \alpha_L^2) \tag{B98}$$

  Under the Case (2) pooling equilibrium, the average seller profits are

$$\Pi_{PB}^{ave} = \frac{1+N}{2} \cdot \overline{\alpha}^2 \tag{B99}$$

With the derivation of the total welfare, consumer surplus, and average seller profits, we can make comparisons across all different systems. Upon making comparisons, we find that the parameter space can be partitioned into three regions

$$\text{Region 1} = \{\overline{\alpha}^2 \leq 1-\lambda\} \cup \{N > \frac{\overline{\alpha}^2 - (1-\lambda)}{\alpha_H^2 - \overline{\alpha'}^2}\} \tag{B100}$$

$$\text{Region 2} = \{\overline{\alpha}^2 > 1-\lambda\} \cap \{N \in (\frac{\overline{\alpha}^2 - (1-\lambda)}{\alpha_H^2 - \overline{\alpha}^2}, \frac{\overline{\alpha}^2 - (1-\lambda)}{\alpha_H^2 - \overline{\alpha'}^2}]\} \tag{B101}$$

$$\text{Region 3} = \{\overline{\alpha}^2 > 1-\lambda\} \cap \{N \leq \frac{\overline{\alpha}^2 - (1-\lambda)}{\alpha_H^2 - \overline{\alpha}^2}\} \tag{B102}$$

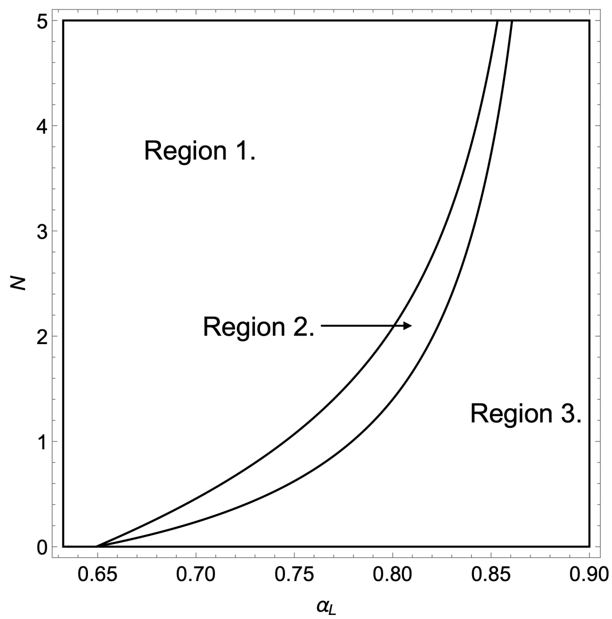

Figure 1: The three regions (in a setting where $\{\alpha_H = 0.9, \lambda = 0.4\}$)

In Region 1, the main model system, the system with historical price information hidden from consumers, and the percentage-based system lead to the same equilibrium outcomes, and therefore, the same total welfare, consumer surplus, and average seller profits.

In Region 2, both the main model system and the percentage-based system lead to the same total welfare, which is higher than that under the system with historical price information hidden from consumers. Next, the average seller profits under the system with historical price information hidden from consumers is higher than that under the main model system, which in turn is weakly higher than that under the percentage-based system. Last, the consumer welfare is weakly higher under a percentage-based system than that under the main model system, while that under a system with historical price information hidden from consumers is the lowest.

In Region 3, the main model system leads to the strictly highest total welfare while the percentage-based system leads to the strictly lowest total welfare. The main model system also has the highest consumer surplus, while the other two systems lead to the same but lower consumer surplus. Furthermore, the average seller profits under the system with historical price information hidden from consumers are the highest, while that under the main model system are the lowest.

Figure 1 visualizes the three regions in the parameter space. In summary, the main model system always leads to the highest total welfare. Even when an alternative system may lead to the same level total welfare (i.e., the two other systems in Region 1 and the percentage-based system in Region 2), it is dominated by the main model system in terms of the average seller profits.

B13 The Analysis in Section 6: Relaxing the Assumption of Homogeneous Consumers.

Due to the main model's simplifying assumption of homogeneous consumers, the seller covers the whole market in each period (see Claim 1). In reality, it is conceivable that the first-period reviews may affect the second-period demand. In this part, we adopt a more general second-period demand function and assess the robustness of our results. Specifically, we assume that in the second period, given belief b_2 and the seller's price p_2, a total proportion of $q(b_2, p_2) \in [0, 1]$ of consumers make purchases. As a result, the seller's second-period profits are $\Pi_2 = m_2 \cdot q(b_2, p_2) \cdot p_2$. We assume that $\frac{\partial q(b_2,p_2)}{\partial b_2} > 0$. The assumption captures that if the reviews lead to a higher perceived seller quality (i.e., b_2 increases) they increase the seller's future demand. This assumption leads to $\frac{\partial \Pi_2}{\partial b_2} > 0$, which has an immediate implication that if the seller's quality is revealed to consumers the H-seller earns higher profits than the L-seller.

We define $\pi_2 \equiv q(b_2, p_2) \cdot p_2$, then

$$\Pi_2 = m_2 \cdot \pi_2 \tag{B103}$$

We let $p_2^*(b_2)$ denote the price that maximized $\pi_2(b_2, p_2)$,[1] and use the notation

$$\pi_2^*(b_2) \equiv \pi_2(b_2, p_2 = p_2^*(b_2)) \tag{B104}$$

Consider two beliefs $b_2' > b_2''$. With the condition $\frac{\partial q}{\partial b_2} > 0$, we have

$$\pi_2^*(b_2') = \pi_2(b_2', p_2^*(b_2') \geq \pi_2(b_2', p_2^*(b_2'')) > \pi_2^*(b_2'') \tag{B105}$$

Therefore, we have $\frac{\partial \pi_2^*(b_2)}{\partial b_2} > 0$. An immediate result is that $\pi_2^*(1) > \pi_2^*(0)$. That is, if sellers' types are known to consumers, the H-seller earns higher profits than the L-seller.

The derivation of equilibrium is highly similar to that in the main model (see the proof of Proposition 1 and 2). We omit the details to avoid repetition. The equilibrium characterization is as follows:

Separating Equilibrium: *If $\alpha_L > 1 - \lambda$, the H-seller charges a lower first-period price than the L-seller. Specifically, define $\widetilde{N} \equiv \frac{\alpha_L - (1-\lambda)}{\pi_2^*(1) - \pi_2^*(0)} \cdot \frac{1-\alpha_L}{1-\alpha_H}$.*

- *If $N \geq \widetilde{N}$, the equilibrium prices are $p_{1H}^* = 1 - \lambda$ and $p_{1L}^* = \alpha_L$. Under these prices, the*

[1] This is simply the optimal price that the seller will choose when the second-period consumers hold the belief b_2. The existence of an optimal price can be ensured by regular conditions such as $\pi_2(b_2, p_2)$ being concave and having a solution that satisfies the first-order condition.

H-seller receives both positive and negative reviews, while the L-seller receives only negative reviews.

- *If $N < \widetilde{N}$, the equilibrium prices are $p_{1H}^* = \alpha_L - N \cdot \frac{1-\alpha_H}{1-\alpha_L} \cdot (\pi_2^*(1) - \pi_2^*(0))$ and $p_{1L}^* = \alpha_L$. Under these prices, both types of sellers receive only negative reviews.*

In the second period, the L-seller charges $p_2^*(0)$ while the H-seller charges $p_2^*(1)$. Moreover, the H-seller earns higher total profits in two periods than the L-seller.

Pooling Equilibrium: *If $\alpha_L \leq 1 - \lambda$, both types of sellers choose the same first-period price while the second-period consumers can identify the seller's quality through reviews. Specifically,*

- *In the first period, both types of sellers charge $p_{1LH}^* = \min\{\overline{\alpha}, 1 - \lambda\}$, and receive both positive and negative reviews.*

- *In the second period, the L-seller charges $p_2^*(0)$ while the H-seller charges $p_2^*(1)$.*

Moreover, the H-seller earns higher profits than the L-seller.

It is immediately seen that the characterizations of the separating equilibrium and the pooling equilibrium correspond to Proposition 1 and 2 in the main paper.

B14 The Analysis in Section 6: Relaxing the Assumption of A Continuum of Consumers.

In the main paper, we assume a continuum of consumers to facilitate the analysis. With a continuum (infinite number of) of consumers, unbiased reviews perfectly reveal a seller's type. The assumption captures the real world situation with large number of customers, such that the aggregate information from customer reviews that are free from selection bias can perfectly convey a seller's quality. As a result, two conditions which are essential for our main results are ensured: (1) The H-seller may have incentive to lower its first-period price to induce unbiased reviews (i.e., both positive and negative reviews). Despite the first-period profit loss, the H-seller benefits from better revealing its quality and accordingly earning higher second-period profits. (2) The L-seller may not deviate from a high first-period price that results in biased review (negative reviews only) to mimic the H-seller because, by doing so, it incurs a profit loss in the first period and its second-period profits are lower than the H-seller's second-period profits.

When the number of consumers is finite in the first period, reviews may fail to fully convey the seller's type even when all consumers write reviews. For example, the L-seller may be fortunate enough to receive all positive reviews and the H-seller may happen to receive all negative reviews so that consumers won't be able to update (falsify) their belief with the observed review in the second period if they believe that the seller is the H-seller or the L-seller. As a result, the requirement (2) above is no longer satisfied, as the L-seller can deviate to the H-seller's price to perfectly mimic the H-seller and earn the same profits as the H-seller in both periods. However, we can show that the two requirements and our key results can be restored if we extend/modify our model slightly with even more realistic assumptions.

Below we briefly outline the two model formulations, present the analysis, demonstrate the robustness of the equilibrium in which the H-seller would lower its price which generates full reviews to signal its type. In both formulations, we use M_1 and M_2 to respectively denote the number of consumers in the first and the second period.

B14.1 Model Formulation 1.

Following the same idea as in section 5.5 that the market demand can be affected by reviews, we can extend our model by assuming that

$$M_2 = (1 + k \cdot r_1) \cdot M_{20} \tag{B106}$$

Here M_{20} denote the base second-period market size when the seller has no positive reviews. The formulation (B106) captures the positive demand effect from positive reviews in influencing the second-period demand. It may reflect the network effect of positive reviews and/or the fact that platform may give sellers with more positive reviews a more prominent slot, which results in higher online traffic and demand for such sellers. In what follows, we demonstrate that the separating equilibrium with

$$p_{1H}^* = 1 - \lambda < \alpha_L = p_{1L}^*,$$
$$p_{2H}^* = \alpha_H, \text{ and } p_{2L}^* = \alpha_L$$

still exists and further characterize the equilibrium conditions.

To see this, note that with $p_{1H}^* = 1 - \lambda < \alpha_L = p_{1L}^*$, the H-seller and the L-seller's total profits are

$$\Pi_H^* = M_1(1 - \lambda) + (1 + kM_1\alpha_H) \cdot M_{20} \cdot \alpha_H \tag{B107}$$
$$\Pi_L^* = M_1\alpha_L + M_{20}\alpha_L \tag{B108}$$

If the H-seller mimics the L-seller by charging $p'_{1H} = \alpha_L$, it will be recognized as the L-seller and earn profits

$$\Pi'_H(p'_{1H} = \alpha_L) = M_1\alpha_L + M_{20}\alpha_L \tag{B109}$$

If the L-seller mimics the H-seller by charging $p'_{1H} = 1 - \lambda$, it will be recognized as the H-seller and earn profits

$$\Pi'_L(p'_{1L} = 1 - \lambda) = M_1(1 - \lambda) + (1 + kM_1\alpha_L) \cdot M_{20} \cdot \alpha_H \tag{B110}$$

No deviation requires

$$\Pi_H^* \geq \Pi'_H(p'_{1H} = \alpha_L) \tag{B111}$$
$$\Pi_L^* \geq \Pi'_L(p'_{1L} = 1 - \lambda) \tag{B112}$$

which yields

$$M_{20} \in [\frac{M_1(\alpha_L - (1 - \lambda))}{kM_1\alpha_H^2 + \alpha_H - \alpha_L}, \frac{M_1(\alpha_L - (1 - \lambda))}{kM_1\alpha_H\alpha_L + \alpha_H - \alpha_L}] \tag{B113}$$

Given $\alpha_L > 1-\lambda$, it is immediately seen that $[\frac{M_1(\alpha_L-(1-\lambda))}{kM_1\alpha_H^2+\alpha_H-\alpha_L}, \frac{M_1(\alpha_L-(1-\lambda))}{kM_1\alpha_H\alpha_L+\alpha_H-\alpha_L}]$ is not a null set.

For the possible deviation to off-equilibrium price $p_1' \notin \{1-\lambda, \alpha_L\}$, we use the same approach as in the main model to show that deviation is not profitable and the equilibrium satisfies Intuitive Criterion. We omit the tedious details.

Therefore, we have demonstrated the robustness of our result in model formulation 1.

B14.2 Model Formulation 2.

In this formulation, we assume that with certain probability the second-period consumers do not observe the first-period price. We believe that the assumption is quite realistic, as historical price is usually less observable than customer reviews.

Let $\mu \in [0,1]$ and $1-\mu$ denote the probability that second-period consumers observe and do not observe the first-period price respectively. The general analysis with a finite number of consumers would be intractable due to the excessive cases to be analyzed. Therefore, we demonstrate the robustness of the key result using the simplest case that there is only one first-period consumer ($M_1 = 1$).

We demonstrate the robustness of the separating equilibrium with

$$p_{1H}^* = 1-\lambda < \alpha_L = p_{1L}^* \tag{B114}$$

Note that under these prices, the H-seller can generate two review outcomes $\{r_0 = 1, r_1 = 0\}$ and $\{r_0 = 0, r_1 = 1\}$; the L-seller can generate two review outcomes $\{r_0 = 1, r_1 = 0\}$ and $\{r_0 = 0, r_1 = 0\}$. Therefore, the review outcome $\{r_0 = 1, r_1 = 0\}$ is the one that can be generated by both types of sellers. If consumers do not observe the first-period price, they will update their belief from the reviews in the following way.

$$b_2(\{r_0 = 0, r_1 = 1\}) = 1 \tag{B115}$$

$$b_2(\{r_0 = 0, r_1 = 0\}) = 0 \tag{B116}$$

$$b_2(\{r_0 = 1, r_1 = 0\}) = \frac{\frac{1}{2}\cdot(1-\alpha_H)}{\frac{1}{2}\cdot(1-\alpha_H)+\frac{1}{2}\cdot(1-\alpha_L)} = \frac{1-\alpha_H}{2-\alpha_H-\alpha_L} \tag{B117}$$

We let

$$p_{2H}^*(\{r_0 = 0, r_1 = 1\}) = \alpha_H \tag{B118}$$

$$p_{2L}^*(\{r_0 = 0, r_1 = 0\}) = \alpha_L \tag{B119}$$

$$\begin{aligned} p_{2H}^*(\{r_0 = 1, r_1 = 0\}) = p_{2L}^*(\{r_0 = 1, r_1 = 0\}) &= \frac{1-\alpha_H}{2-\alpha_H-\alpha_L}\cdot\alpha_H + \frac{1-\alpha_L}{2-\alpha_H-\alpha_L}\cdot\alpha_L \\ &= \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L} \end{aligned} \tag{B120}$$

Next we show that the equilibrium characterized by first-period prices (B114) and second-period prices (B118),(B119), and (B120) exist, and characterize the equilibrium conditions.

First, note that (B118) and (B119) immediately hold, as when the review outcome $\{r_0 = 0, r_1 = 1\}$ or $\{r_0 = 0, r_1 = 0\}$ appears, the seller's type is conveyed to consumers regardless of whether they observe the first-period price or not. Next, consider the review outcome

$\{r_0 = 1, r_1 = 0\}$. If the H-seller prefers $p^*_{2H}(\{r_0 = 1, r_1 = 0\}) = \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}$ which always sells to second-period consumers, to deviating to $p'_{2H} = \alpha_H$ which sells to second-period consumers only if they observe the first-period price, we need

$$\frac{(1-\alpha_H)\alpha_H + (1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L} \geq \mu \cdot \alpha_H \tag{B121}$$

The condition yields

$$\mu \leq \frac{(1-\alpha_H)\alpha_H + (1-\alpha_L)\alpha_L}{\alpha_H(2-\alpha_H-\alpha_L)} \tag{B122}$$

Similarly, given the review outcome $\{r_0 = 1, r_1 = 0\}$, if the L-seller prefers $p^*_{2L}(\{r_0 = 1, r_1 = 0\}) = \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}$ which sells to second-period consumers only if they do not observe the first-period price, to deviating to $p'_{2L} = \alpha_L$ which always sells to second-period consumers, we need

$$(1-\mu)\frac{(1-\alpha_H)\alpha_H + (1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L} \geq \alpha_L \tag{B123}$$

The condition yields

$$\mu \leq \frac{(1-\alpha_H)(\alpha_H-\alpha_L)}{\alpha_H(1-\alpha_H)+(1-\alpha_L)\alpha_L} \tag{B124}$$

For the possible second-period deviation to off-equilibrium price $p'_2(\{r_0 = 1, r_1 = 0\}) \neq \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}$, one can show that with the belief (conditional on consumers do not observe the first-period price) $b_2(r_0 = 1, r_1 = 0, p'_2(\{r_0 = 1, r_1 = 0\}) \neq \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}) = 0$, neither type of seller will deviate from $p_2(\{r_0 = 1, r_1 = 0\}) = \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}$. The belief is valid according to the Intuitive Criterion.

We can then write down each type of seller's equilibrium profits as follows.

$$\Pi^*_H = 1-\lambda + M_2 \cdot \left(\alpha_H \cdot \alpha_H + (1-\alpha_H) \cdot \frac{(1-\alpha_H)\alpha_H + (1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}\right) \tag{B125}$$

$$\Pi^*_L = \alpha_L + M_2 \cdot \left(\alpha_L \cdot \alpha_L + (1-\alpha_L) \cdot (1-\mu)\frac{(1-\alpha_H)\alpha_H + (1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}\right) \tag{B126}$$

We then examine each type of seller's incentive of deviating to the other seller's first-period price. Consider the H-seller and the L-seller in sequence.

- For the H-seller, by deviating from $p^*_{1H} = 1-\lambda$ to $p'_{1H} = \alpha_L$, it earns profits

$$\Pi'_H(p'_{1H} = \alpha_L) = \alpha_L + M_2 \cdot \left(\alpha_H \cdot \alpha_L + (1-\alpha_H) \cdot (1-\mu)\frac{(1-\alpha_H)\alpha_H + (1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}\right) \tag{B127}$$

We need to have $\Pi_H^* \geq \Pi_H'(p_{1H}')$, which gives us

$$M_2 \geq \frac{(2-\alpha_H-\alpha_L)(\alpha_L-(1-\lambda))}{\alpha_H(\alpha_H-\alpha_L)(2-\alpha_H-\alpha_L)+(1-\alpha_H)((1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L)\mu} \tag{B128}$$

- For the L-seller, by deviating from $p_{1L}^* = \alpha_L$ to $p_{1L}' = 1-\lambda$, it earns profits

$$\Pi_L'(p_{1L}' = 1-\lambda) = 1-\lambda + M_2 \cdot \left(\alpha_L \cdot \alpha_H + (1-\alpha_L) \cdot \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}\right) \tag{B129}$$

We need to have $\Pi_L^* \geq \Pi_L'(p_{1L}')$, which gives us

$$M_2 \leq \frac{(2-\alpha_H-\alpha_L)(\alpha_L-(1-\lambda))}{-\alpha_H^2(\alpha_L(1-\mu)+\mu)+\alpha_H(\alpha_L(2-\mu)+\mu)+\alpha_L(\mu-2\alpha_L(1+\mu)+\alpha_L^2(1+\mu))} \tag{B130}$$

Lastly, we consider the deviation to an off-equilibrium first-period price $p_1' \notin \{1-\lambda, \alpha_L\}$.

For deviation $p_1' \in (1-\lambda, \alpha_L)$, first consider the H-seller. If the H-seller ever finds the deviation profitable, then the H-seller must also finds the deviation profitable under $b_1(p_1') = 1$. Also note that the price $p_1' \in (1-\lambda, \alpha_L)$ will result in two possible review outcome: $\{r_0 = 1, r_1 = 0\}$ and $\{r_0 = 0, r_1 = 0\}$.

- If $\{r_0 = 1, r_1 = 0\}$ is generated, given (B122), the H-seller will choose $p_{2H} = \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}$ and sell to all second-period consumers.
- If $\{r_0 = 0, r_1 = 0\}$ is generated, given (B124), the H-seller will also choose $p_{2H} = \alpha_L$ to sell to all second-period consumers.

Therefore, given $b_1(p_1') = 1$, the deviation profits are

$$\Pi_H'(p_1') = p_1' + M_2 \cdot \left(\alpha_H \cdot \alpha_L + (1-\alpha_H) \cdot \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}\right) \tag{B131}$$

Similarly, under the belief $b_1(p_1') = 1$, the L-seller's deviation profits to $p_1' \in (1-\lambda, \alpha_L)$ are

$$\Pi_L'(p_1') = p_1' + M_2 \cdot \left(\alpha_L \cdot \alpha_L + (1-\alpha_L) \cdot \frac{(1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L}{2-\alpha_H-\alpha_L}\right) \tag{B132}$$

If deviation to $p_1' \in (1-\lambda, \alpha_L)$ is not profitable, we need either (i) both H-seller and L-seller have profitable deviation under $b_1(p_1') = 1$, in which case the belief $b_1(p_1') = 0$ is valid to prevent deviation, or (ii) H-seller does not have profitable deviation under $b_1(p_1') = 1$, in which case the Intuitive Criterion will rule out the H-seller from its step 1, and pin down the belief $b_1(p_1') = 0$ which prevents the L-seller from deviation. Upon solving these conditions, we obtain

$$M_2 \geq \frac{(2-\alpha_H-\alpha_L)(\alpha_L-(1-\lambda))}{\alpha_H(\alpha_H-\alpha_L)(2-\alpha_H-\alpha_L)+(1-\alpha_L)((1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L)\mu} \tag{B133}$$

For deviation $p_1' < 1-\lambda$ or $p_1' > \alpha_L$, using the same approach can show that deviation is not profitable for either type of seller, with no further conditions required. We omit the details to avoid repetition.

Combining and simplifying the conditions (B114),(B122),(B124), (B128),(B130), and(B133), we obtain the equilibrium condition as

$$\begin{aligned}
\alpha_L >& 1-\lambda,\\
\mu \le& \frac{(1-\alpha_H)(\alpha_H-\alpha_L)}{\alpha_H(1-\alpha_H)+(1-\alpha_L)\alpha_L},\\
M_2 \in& [\frac{(2-\alpha_H-\alpha_L)(\alpha_L-(1-\lambda))}{\alpha_H(\alpha_H-\alpha_L)(2-\alpha_H-\alpha_L)+(1-\alpha_H)((1-\alpha_H)\alpha_H+(1-\alpha_L)\alpha_L)\mu},\\
& \frac{(2-\alpha_H-\alpha_L)(\alpha_L-(1-\lambda))}{-\alpha_H^2(\alpha_L(1-\mu)+\mu)+\alpha_H(\alpha_L(2-\mu)+\mu)+\alpha_L(\mu-2\alpha_L(1+\mu)+\alpha_L^2(1+\mu))}].
\end{aligned}$$

Note that $\{M_2=1,\alpha_H=0.8,\alpha_L=0.65,\lambda=0.47,\mu=0.07\}$ satisfy the equilibrium condition. This means the equilibrium region is not a null set.

Therefore, we have demonstrated the robustness of our result in model formulation 2.

B14.3 Other Formulations.

It is worth mentioning that the two formulations presented above are the simple ones but by no means are the only possible formulations that may restore our main results with finite number of consumers. For example, we may assume that the seller does not fully know its own type, i.e., α_H and α_L are random variables with the H-seller having higher expected prior of α than the L-seller. Then the realized reviews may be used by consumers in the second period to update the prior and form the posterior belief of α. Although such a model is mathematically challenging, intuitively we can expect that the requirement (2) will also be met in such a setting with the finite number of consumers. This is because the realized reviews will be more negative for the L-seller so that the posterior expectation of α on the L-seller is lower than that on the H-seller even if the L-seller mimics the H-seller in the first period (i.e., when L-seller mimics the H-seller by giving the same signal on the prior of α). In fact, our current setting can be viewed as the limiting case of that general setup when the distribution of $\alpha_H(\alpha_L)$ degenerated to a single point (e.g., without randomness) and the number of consumers goes to infinite.

B15 Proof of the Nonexistence of a Semi-Separating Equilibrium.

In this section, we first characterize the structure of a semi-separating equilibrium if it can exist. We then prove the nonexistence of a semi-separating equilibrium by demonstrating that given the equilibrium's structure, at least one type of seller has profitable deviation.

B15.1 The Structure of A Semi-Separating Equilibrium.

A semi-separating equilibrium is where some type(s) randomize between pooling and separating signals. In the most general case, the L and H seller randomly select from two sets of prices,

$\mathcal{P}_{1L}$ and $\mathcal{P}_{1H}$, respectively, such that $\mathcal{P}_{1L} \neq \mathcal{P}_{1H}$ and $\mathcal{P}_{1L} \cap \mathcal{P}_{1H} \neq \emptyset$. We now characterize the conditions that a semi-separating equilibrium needs to satisfy.

1. *Either there is no separating price for the L-seller (i.e., $\mathcal{P}_{1L} \subset \mathcal{P}_{1H}$), or the separating price for the L-seller is unique and equals to α_L (i.e., $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1H}) = \{\alpha_L\}$).*

   Suppose multiple prices belong to the set $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1H}$. Then for any price $p'_{1L} \in (\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1H}$ that separates the L-seller from the H-seller in the semi-separating equilibrium, consumers' belief is $b_1 = 0$ when seeing this price p'_{1L}, and accordingly can only charge $p_{2L} = \alpha_L$ in the second period. If $p'_{1L} \neq \alpha_L$, then the L-seller has strict incentive to deviate to $p_{1L} = \alpha_L$ to increase its first-period profit without hurting the second-period profit. Therefore, either $\mathcal{P}_{1L} \subset \mathcal{P}_{1H}$ or $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1H}) = \{\alpha_L\}$ must hold.

2. *Either there is no separating price for the H-seller (i.e. $\mathcal{P}_{1H} \subset \mathcal{P}_{1L}$), or the separating price for the H-seller in the semi-separating equilibrium is unique(i.e., $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1L} = \{p_{1H}\}$).*

   Suppose multiple prices belong to the set $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1L}$. Let p'_{1H} and p''_{1H} be two of them. Without loss of generality, we let $p'_{1H} < p''_{1H}$. Consumers' belief is $b_1 = 1$ when seeing either of these two prices. Then the H-seller has strict incentive to shift all the probability on p'_{1H} to p''_{1H}. This process can be repeated until there is only one price left in the set $\{p_{1H}\} = (\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1L}$.

3. *If $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1L} = \{p_{1H}\}$, the separating price for the H-seller in the semi-separating equilibrium is lower than all the pooling prices in the semi-separating equilibrium.*

   Suppose not, for any pooling price $p'_1 \in \mathcal{P}_{1L} \cap \mathcal{P}_{1H}$ such that $p_{1H} > p'_1$, the H seller would have strict incentive to shift the probability on p'_1 to p_{1H} to earn higher profits in both periods. Repeat this process, then all the pooling prices lower than p_{1H} will have zero probability.

4. *A pooing price in the semi-separating equilibrium is strictly larger than $1 - \lambda$.*

   Suppose a pooling price $p'_1 \leq 1 - \lambda$ exists. Note that p'_1 cannot be lower than λ because otherwise the H-seller would be strictly better off by deviating to $p_{1H} = \lambda$, so that it improves its first-period profits and also conveys its high quality through unbiased reviews. Next, suppose $p'_1 \in [\lambda, 1 - \lambda]$. In a semi-separating equilibrium, at least one of L and H seller mixes between a separating price and a pooing price. If the H-seller mixes between p'_1 and $p_{1H} < p'_1$, she would be strictly better off by shifting all the probability on p_{1H} to p'_1, so that improves its first-period profits and still conveys its high quality through unbiased reviews. If the L-seller mixes between p'_1 and $p_{1L} = \alpha_L > p'_1$, she would be strictly better off by shifting all the probability on p'_1 to $p_{1L} = \alpha_L$, so that it improves its first-period profits without earning less second-period profits. Therefore, $p'_1 \in [\lambda, 1 - \lambda]$ cannot be a pooling price in a semi-separating equilibrium.

   In summary, denoting a pooling price as $p_{1LH} \in \mathcal{P}_{1L} \cap \mathcal{P}_{1H}$, we must have $p_{1LH} > 1 - \lambda$.

5. *If $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1H} = \{\alpha_L\}$, then we must have $p_{1LH} < p_{1L} = \alpha_L, \forall p_{1LH} \in \mathcal{P}_{1L} \cap \mathcal{P}_{1H}$.*

   Suppose that $\exists p_{1LH}$ such that $p_{1LH} > p_{1L} = \alpha_L$. In this case, comparing with choosing $p_{1L} = \alpha_L$, the L-seller is strictly better off by charging p_{1LH} which improves its first-period profits without hurting its second-period profits. violating the requirement that the L-seller is indifferent between p_{1LH} and $p_{1L} = \alpha_L$.

6. *<u>If $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1L} = \{p_{1H}\}$, then we must have $p_{1H} \geq 1 - \lambda$.</u>*

   Suppose $p_{1H} < 1 - \lambda$. From point 4 we know that $p_{1LH} > 1 - \lambda$. This means regardless of consumer belief, the L-seller strictly prefers p_{1LH} to the price $p_1' = 1 - \lambda$, as the price p_1' price generates lower first-period profits and reveals the L-seller's quality. Then according to Intuitive Criterion, we have $b_1(p_1' = 1 - \lambda) = 1$. As a result, the H-seller will deviate form $p_{1H} < 1 - \lambda$ to $p_1' = 1 - \lambda$ to improve profits.

The analysis above suggest three possibilities of a semi-separating equilibrium. The superscript * represent the equilibrium of the whole game.

- **Case 1:** $\mathcal{P}_{1H} \subset \mathcal{P}_{1L}$ **and** $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1H} = \{\alpha_L\}$.

  In this case, $\forall p^*_{1LH} \in \mathcal{P}_{1L} \cap \mathcal{P}_{1H}$, we have

$$1 - \lambda < p^*_{1LH} < p^*_{1L} = \alpha_L \tag{B134}$$

- **Case 2:** $\mathcal{P}_{1L} \subset \mathcal{P}_{1H}$ **and** $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1L} = \{p^*_{1H}\}$.

  In this case, $\forall p^*_{1LH} \in \mathcal{P}_{1L} \cap \mathcal{P}_{1H}$, we have

$$1 - \lambda \leq p^*_{1H} < p^*_{1LH} \tag{B135}$$

- **Case 3:**$(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1H} = \{\alpha_L\}$ **and** $(\mathcal{P}_{1L} \cup \mathcal{P}_{1H}) - \mathcal{P}_{1L} = \{p^*_{1H}\}$.

  In this case, $\forall p^*_{1LH} \in \mathcal{P}_{1L} \cap \mathcal{P}_{1H}$, we have

$$1 - \lambda \leq p^*_{1H} < p^*_{1LH} < p^*_{1L} = \alpha_L \tag{B136}$$

We then prove none of these three cases can exist.

B15.2 The Nonexistence of a Semi-Separating Equilibrium Characterized in Case 1.

Consider one of the equilibrium price $p^*_{1LH} \in \mathcal{P}_{1L} \cap \mathcal{P}_{1H}$. This price generates only negative reviews. If the number of negative reviews $r_0 \in (0, 1 - \alpha_H]$, the second-period consumers cannot fully identify the seller's type, and as a result both types of sellers will charge the same equilibrium price p^*_{2LH}; [2] If the number of negative reviews $r_0 \in (1 - \alpha_H, 1 - \alpha_L]$, the L-seller will be identified, and as a result it charges the second-period price $p^*_{2L} = \alpha_L$. If the L-seller is indifferent between p^*_{1LH} and p^*_{1L}, we have

$$\frac{1}{2} \cdot p^*_{1LH} + \frac{N}{2} \cdot (\frac{1 - \alpha_H}{1 - \alpha_L} \cdot p^*_{2LH} + \frac{\alpha_H - \alpha_L}{1 - \alpha_L} \cdot \alpha_L) = \frac{1 + N}{2} \cdot \alpha_L \tag{B137}$$

[2] As there may be multiple p^*_{1LH}, there can also be multiple p^*_{2LH}. With a slight abuse of notation, in the entire section B15, the p^*_{2LH} refers to the price that follows the focal p^*_{1LH} we are considering and the resulting reviews $r_0 \in (0, 1 - \alpha_H]$.

which gives us

$$p_{1LH}^* = \alpha_L - N \cdot \frac{1-\alpha_H}{1-\alpha_L} \cdot (p_{2LH}^* - \alpha_L) \tag{B138}$$

With the expression of p_{1LH}^*, we can also write down the H-seller's profit function and relate it to the L-seller's profit function

$$\begin{aligned}
\Pi_H^* &= \frac{1}{2} \cdot p_{1LH}^* + \frac{N}{2} \cdot p_{2LH}^* \\
&= \frac{1}{2} \cdot \alpha_L + \frac{N}{2} \cdot (\frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot p_{2LH}^* + \frac{1-\alpha_H}{1-\alpha_L} \cdot \alpha_L) \\
&= \frac{1+N}{2} \cdot \alpha_L + \frac{N}{2} \cdot \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot (p_{2LH}^* - \alpha_L) \\
&= \Pi_L^* + \frac{N}{2} \cdot \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot (p_{2LH}^* - \alpha_L)
\end{aligned} \tag{B139}$$

Next consider the deviation $p_1' = 1-\lambda$. This is a price that can generate reviews to fully reveal a seller's quality. As $1-\lambda < p_{1LH}^* < p_{1L}^*$, the L-seller will never consider the deviation $p_1' = 1-\lambda$ regardless of consumers' belief. Therefore, if any seller can be identified from Step 1 of the Intuitive Criterion, it could only be the H-seller. If the equilibrium holds, then the H-seller must have no incentive to deviate to $p_1' = 1-\lambda$ given $b_1'(p_1' = 1-\lambda) = 1$. This means $\Pi_H^* \geq \Pi_H(p_1' = 1-\lambda) = \frac{1}{2}(1-\lambda) + \frac{N}{2} \cdot \alpha_H$, which further implies $\exists p_1'' \geq 1-\lambda$, such that

$$\Pi_H^* = \frac{1}{2} \cdot p_1'' + \frac{N}{2} \cdot \alpha_H \tag{B140}$$

Then given $\Pi_H^* = \Pi_L^* + \frac{N}{2} \cdot \frac{\alpha_H-\alpha_L}{1-\alpha_L} \cdot (p_{2LH}^* - \alpha_L)$ (see (B139)), we have

$$\Pi_L^* + \frac{N}{2} \cdot \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot (p_{2LH}^* - \alpha_L) = \frac{1}{2} \cdot p_1'' + \frac{N}{2} \cdot \alpha_H \tag{B141}$$

Therefore,

$$\begin{aligned}
\Pi_L^* &= \frac{1}{2} \cdot p_1'' + \frac{N}{2} \cdot (\alpha_H - \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot p_{2LH}^* + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L) \\
&> \frac{1}{2} \cdot p_1'' + \frac{N}{2} \cdot (\alpha_H - \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_H + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L) \\
&= \frac{1}{2} \cdot p_1'' + \frac{N}{2} \cdot (\frac{1-\alpha_H}{1-\alpha_L} \cdot \alpha_H + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L) \\
&\geq \Pi_L(p_1 = p_1'')
\end{aligned} \tag{B142}$$

That is, the L-seller will not deviate to p_1'' regardless of the consumers' belief. By continuity, the L-seller will not deviate to $p_1'' + \epsilon$ regardless of consumers' belief. Therefore, only the H-seller can be identified from Step 1 of the Intuitive Criterion. Since with $b_1(p_1'') = 1$ the H-seller is indifferent between charging p_{1LH}^* and deviating to p_1'', then with $b_1(p_1'' + \epsilon) = 1$, the H-seller has strict incentive of deviating to $p_1'' + \epsilon$. This means the equilibrium cannot hold. In other words, a semi-separating equilibrium characterized in case 1 does not exist.

B15.3 The Nonexistence of a Semi-Separating Equilibrium Characterized in Case 2.

In this case, the H-seller's indifference condition requires

$$\frac{1}{2} \cdot p_{1H}^* + \frac{N}{2} \cdot \alpha_H = \frac{1}{2} \cdot p_{1LH}^* + \frac{N}{2} \cdot p_{2LH}^* \tag{B143}$$

which gives us

$$p_{1LH}^* = p_{1H}^* + N \cdot (\alpha_H - p_{2LH}^*) \tag{B144}$$

We can write down the L-seller's profit function and substitute in the expression of p_{1LH}^*

$$\begin{aligned}
\Pi_L^* &= \frac{1}{2} \cdot p_{1LH}^* + \frac{N}{2} \cdot (\frac{1-\alpha_H}{1-\alpha_L} \cdot p_{2LH}^* + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L) \\
&= \frac{1}{2} \cdot p_{1H}^* + \frac{N}{2} \cdot (\alpha_H - \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot p_{2LH}^* + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L) \\
&> \frac{1}{2} \cdot p_{1H}^* + \frac{N}{2} \cdot (\alpha_H - \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_H + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L) \\
&= \frac{1}{2} \cdot p_{1H}^* + \frac{N}{2} \cdot (\frac{1-\alpha_H}{1-\alpha_L} \cdot \alpha_H + \frac{\alpha_H - \alpha_L}{1-\alpha_L} \cdot \alpha_L) \\
&\geq \Pi_L(p_1 = p_{1H}^*)
\end{aligned}$$

Thus the L-seller strictly prefers p_{1LH}^* to p_{1H}^*. By continuity, the L-seller strictly prefers p_{1LH}^* to $p_{1H}^* + \epsilon$. Therefore, for the off-equilibrium deviation $p_{1H}^* + \epsilon$, only the H-seller can possibly be identified from Intuitive Criterion's Step 1. With $b_1(p_{1H}^* + \epsilon) = 1$ and $b_2(p_{1H}^* + \epsilon, r_0, r_1) = 1$, it is immediately seen that the H-seller has strict incentive of deviating to $p_{1H}^* + \epsilon$. Therefore, a semi-separating equilibrium characterized in case 2 does not exist.

B15.4 The Nonexistence of a Semi-Separating Equilibrium Characterized in Case 3.

The proof here is the same as the proof in case 2. Consequently, the H-seller has strict incentive to deviate to $p_{1H}^* + \epsilon$. Therefore, a semi-separating equilibrium characterized in case 3 does not exist.

In summary, we have proved that a semi-separating equilibrium does not exist.

B16 Heterogeneity In Consumers' Behavior of Leaving Reviews.

In this section, we allow consumers to be heterogeneous in their behavior of leaving reviews. Specifically, we let β proportion of consumers have the threshold $\lambda = 0$ so that they always leave reviews, and the remaining $1 - \beta$ proportion of consumers have the threshold $\lambda > 0$ so that the reviews from these consumers may be subject to selection bias. We further allow randomness in β by assuming $\beta \sim U(0,1)$ and let the distribution to be common knowledge to the seller and all consumers. With a little abuse of notation but for ease of exposition, we let $\lambda > 0$ denote the threshold of the consumers whose reviews may be subject to selection bias. The remaining setting of the model is the same as that in the main model.

It is worth noting the need of randomness in β. If β is fixed, then the seller's type will be perfectly revealed even when the reviews are subject to selection bias. To see this, consider the situation that $p_1 > 1-\lambda$ so that the $1-\beta$ proportion of consumers with $\lambda > 0$ leave negative reviews when they receive bad experiences, but do not leave reviews when they receive positive experiences. In this situation, the ratio $\frac{r_1}{r_0} = \frac{m_2 \cdot \beta \cdot \alpha_i}{m_2 \cdot \beta(1-\alpha_i) + m_2 \cdot (1-\beta) \cdot (1-\alpha_i)} = \frac{\beta \alpha_i}{1-\alpha_i}, i \in \{L, H\}$ differs across the two types of sellers and fully conveys the seller's type. Therefore, the uncertainty in the proportion of customers who always leave reviews is essential. In this way, when reviews are subject to selection bias, consumers cannot tell whether an observed $\{r_0, r_1\}$ is from an H-seller in a market with small proportion of consumers who always write reviews, or is from an L-seller in a market with large proportion of consumers who always write reviews. Furthermore, the assumption on the randomness of β seems to be reasonable: In reality, although not everyone writes reviews, it is unlikely that consumers and the firm can predict the realized proportion of customers who write reviews and those who do not.

First note that the price $p_1 = \lambda$ makes all first-period consumers make purchase and leave reviews. The ratio $\frac{r_1}{r_0} = \frac{\alpha_i}{1-\alpha_i}$, $i \in \{L, H\}$ will reveal the seller's type. For the same reason discussed in Claim 1, in equilibrium we must have $D_1 = m_1$ and $D_2 = m_2$. Then, for the same reason discussed in Lemma 2, we have $b_2(p_1, r_0, r_1, p_2) = b_2(p_1, r_0, r_1)$. As in Section 4.2.1 in the main paper, we next derive the second-period consumer belief. Noting that the seller's price in each period falls in the range $p \in [0, 1]$, we consider the three cases.

- *Case 1:* $p_1 \in [\lambda, 1-\lambda]$

  This is the case that all first-period consumers leave positive reviews if receiving good experiences, and leave bad reviews if receiving bad experiences. Thus the ratio $\frac{r_1}{r_0}$ fully conveys the seller's quality. Therefore, we have

$$b_2(p_1 \in [\lambda, 1-\lambda], r_0, r_1) = \begin{cases} 1 & \text{if } \frac{r_1}{r_0} = \frac{\alpha_H}{1-\alpha_H} \\ 0 & \text{if } \frac{r_1}{r_0} = \frac{\alpha_L}{1-\alpha_L} \end{cases} \tag{B145}$$

- *Case 2:* $p_1 \in (1-\lambda, 1]$

  In this case, the first-period consumers with $\lambda > 0$ do not leave reviews when having good experiences. Therefore, the number of positive and negative reviews generated by a type-i seller are

$$\#\text{NR} = m_1 \cdot (1-\alpha_i) \tag{B146}$$

$$\#\text{PR} = m_1 \cdot \alpha_i \cdot \beta \tag{B147}$$

where $i \in \{H, L\}$. Then, we have

$$
\begin{aligned}
& b_2(p_1 > 1-\lambda, r_0, r_1) \\
= & \Pr(\{\text{H-seller}\}|\{\#\text{NR} = r_0, \#\text{PR} = r_1\}) \\
= & \lim_{\epsilon_1 \to 0, \epsilon_2 \to 0} \Pr(\{\text{H-seller}\}|\{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\}) \\
= & \lim_{\epsilon_1 \to 0, \epsilon_2 \to 0} \frac{\Pr(\{\text{H-seller}\}, \{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\})}{\Pr(\{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\})} \\
= & \lim_{\epsilon_1 \to 0, \epsilon_2 \to 0} \frac{b_1 \cdot \Pr(\{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\}|\{\text{H-seller}\})}{\sum_{i=L,H} b_i \cdot \Pr(\{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\})|\{\text{i-seller}\})}
\end{aligned}
\tag{B148}
$$

where $b_H \equiv b_1$ and $b_L \equiv (1 - b_1)$.

Consider the four possible situations.

- Situation 1: $r_0 \in (1 - \alpha_H, 1 - \alpha_L]$.
  In this situation, $\Pr(\{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\})|\{\text{H-seller}\}) = 0$. Therefore

  $$b_2(p_1 > 1 - \lambda, r_0, r_1) = 0 \tag{B149}$$

  This is the situation that there are too many negative reviews which cannot be generated by an H-seller. As a result, the L-seller's type is revealed.

- Situation 2: $r_0 \leq 1 - \alpha_H$ and $r_1 \in (\alpha_L, \alpha_H]$.
  In this situation, $\Pr(\{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\})|\{\text{H-seller}\}) = 1$, Therefore

  $$b_2(p_1 > 1 - \lambda, r_0, r_1) = 1 \tag{B150}$$

  This is the situation that there are too many positive reviews which cannot be generated by an L-seller. As a result, the H-seller's type is revealed.

- Situation 3: $r_0 \leq 1 - \alpha_H$, $r_1 \leq \alpha_L$, and $r_1 \in (r_0 \cdot \frac{\alpha_L}{1-\alpha_L}, r_0 \cdot \frac{\alpha_H}{1-\alpha_H}]$.
  Note that

  $$\frac{\#\text{PR}}{\#\text{NR}} = \beta \cdot \frac{\alpha_i}{1 - \alpha_i} \leq \frac{\alpha_i}{1 - \alpha_i}$$

  Therefore, if $r_1 \in (r_0 \cdot \frac{\alpha_L}{1-\alpha_L}, r_0 \cdot \frac{\alpha_H}{1-\alpha_H}]$, the reviews can only be generated by an H-seller. As a result, $\Pr(\{\#\text{NR} \in (r_0 - \epsilon_1, r_0], \#\text{PR} \in (r_1 - \epsilon_2, r_1]\})|\{\text{H-seller}\}) = 1$, and therefore

  $$b_2(p_1 > 1 - \lambda, r_0, r_1) = 1 \tag{B151}$$

  This is the situation that the ratio of positive reviews is very high which cannot be generated by an L-seller. As a result, the H-seller's type is revealed.

- Situation 4: $r_0 \leq 1 - \alpha_H$, $r_1 \leq \alpha_L$, and $r_1 \in (0, r_0 \cdot \frac{\alpha_L}{1-\alpha_L}]$.

In this case, for small enough ϵ_1 and ϵ_2

$$
\begin{aligned}
&\Pr(\{\#\text{NR} \in (r_0-\epsilon_1, r_0], \#\text{PR} \in (r_1-\epsilon_2, r_1]\} | \{\text{H-seller}\}) \\
=&\iint_{\{m_1(1-\alpha_H)\in(r_0-\epsilon_1,r_0], m_1\cdot\alpha_H\cdot\beta\in(r_1-\epsilon_2,r_1]\}} d\{m_1,\beta\} \\
=&\int_{\frac{r_0-\epsilon_1}{1-\alpha_H}}^{\frac{r_0}{1-\alpha_H}} \int_{\frac{r_1-\epsilon_2}{\alpha_H\cdot m_1}}^{\frac{r_1}{\alpha_H\cdot m_1}} d\beta dm_1 \\
=&\int_{\frac{r_0-\epsilon_1}{1-\alpha_H}}^{\frac{r_0}{1-\alpha_H}} \frac{\epsilon_2}{\alpha_H\cdot m_1} dm_1 \\
=&\frac{\epsilon_2}{\alpha_H}\cdot(\log r_0 - \log(r_0-\epsilon_1))
\end{aligned}
\tag{B152}
$$

Similarly, we have

$$
\begin{aligned}
&\Pr(\{\#\text{NR} \in (r_0-\epsilon_1, r_0], \#\text{PR} \in (r_1-\epsilon_2, r_1]\} | \{\text{L-seller}\}) \\
=&\frac{\epsilon_2}{\alpha_L}\cdot(\log r_0 - \log(r_0-\epsilon_1))
\end{aligned}
\tag{B153}
$$

Then, (B148) becomes

$$
\begin{aligned}
&b_2(p_1 > 1-c, r_0, r_1) = \\
=&\lim_{\epsilon_1\to 0,\epsilon_2\to 0} \frac{b_1\cdot\frac{\epsilon_2}{\alpha_H}\cdot(\log r_0-\log(r_0-\epsilon_1))}{b_1\cdot\frac{\epsilon_2}{\alpha_H}\cdot(\log r_0-\log(r_0-\epsilon_1)) + (1-b_1)\cdot\frac{\epsilon_2}{\alpha_L}\cdot(\log r_0-\log(r_0-\epsilon_1))} \\
=&\frac{b_1}{b_1+(1-b_1)\cdot\frac{\alpha_H}{\alpha_L}}
\end{aligned}
\tag{B154}
$$

Putting together the four situations, we have the following consumers' belief

$$
b_2(p_1 \in (1-\lambda, 1], r_0, r_1) = \begin{cases} 0 & \text{if } r_0 \in (1-\alpha_H, 1-\alpha_L] \\ 1 & \text{if } r_0 \in (0, 1-\alpha_H], r_1 \in [r_0\cdot\frac{\alpha_H}{1-\alpha_H}, \alpha_H] \\ \frac{b_1}{b_1+(1-b_1)\cdot\frac{\alpha_H}{\alpha_L}} & \text{if } r_0 \in (0, 1-\alpha_H], r_1 \in (0, r_0\cdot\frac{\alpha_H}{1-\alpha_H}] \end{cases}
\tag{B155}
$$

- *Case 3:* $p_1 \in [0, \lambda)$

  In this case, the first-period consumers with $\lambda > 0$ do not leave reviews when having bad experiences. Following the same approach as in Case 2, we have the following consumers' belief

$$
b_2(p_1 \in [0, \lambda), r_0, r_1) = \begin{cases} 1 & \text{if } r_1 \in (\alpha_L, \alpha_H] \\ 0 & \text{if } r_1 \in (0, \alpha_L], r_0 \in (r_1\cdot\frac{1-\alpha_H}{\alpha_H}, 1-\alpha_L] \\ \frac{b_1}{b_1+(1-b_1)\cdot\frac{1-\alpha_H}{1-\alpha_L}} & \text{if } r_1 \in (0, \alpha_L], r_0 \in (0, r_1\cdot\frac{1-\alpha_H}{\alpha_H}] \end{cases}
\tag{B156}
$$

With the derivation of consumers' belief characterized in (B145), (B155), and (B156), we follow the steps as those in deriving the equilibrium in the main model. To avoid repetition, we omit the details and present the equilibrium characterization here.

Separating Equilibrium: *If $\alpha_L > 1-\lambda$ and $N \geq N' \equiv \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L} \cdot \frac{1-\alpha_L}{1-\alpha_H}$, the H-seller charges a lower first-period price than the L-seller. Specifically, the equilibrium prices are $p_{1H}^* = 1-\lambda$ and $p_{1L}^* = \alpha_L$. In the second period, the H-seller charges $p_{2H}^* = \alpha_H$ and the L-seller charges $p_{2L}^* = \alpha_L$. Moreover, the H-seller earns higher total profits in two periods than the L-seller.*

Pooling Equilibrium: *The two types of sellers charge the same first-period price if (1) $\alpha_L \leq 1-\lambda$, or (2) $\overline{\alpha} > 1-\lambda$ and $N \leq \frac{\overline{\alpha}-(1-\lambda)}{\alpha_H-\alpha_L} \cdot \frac{\alpha_H+\alpha_L}{\alpha_L} \cdot \frac{1-\alpha_L}{1-\alpha_H}$*

- *In Case (1), both types of sellers charge the first-period price $p_{1LH}^* = \min\{\overline{\alpha}, 1-\lambda\}$. In the second period, consumers can fully identify the seller's quality through reviews. The H-seller charges $p_{2H}^* = \alpha_H$ and the L-seller charges $p_{2L}^* = \alpha_L$.*

- *In Case (2), both types of sellers charge the first-period price $p_{1LH}^* = \min\{\overline{\alpha}, 1-\lambda\}$.In the second period, consumers may not fully identify the seller's quality, in which case both types of sellers charge $p_{2LH}^* = \frac{2\alpha_H\alpha_L}{\alpha_L+\alpha_H}$; If consumers can fully identify the seller's quality, The H-seller charges $p_{2H}^* = \alpha_H$ and the L-seller charges $p_{2L}^* = \alpha_L$.*

Moreover, the H-seller earns higher profits than the L-seller.

The equilibrium characterization covers the entire parameter space. It is seen that the dual roles of customer reviews and price in conveying quality information are robust.

B17 The D1-Criterion.

In the paper we study the equilibrium that survives the Intuitive Criterion (Cho and Kreps, 1987). In this section we apply the D1-equilibrium (Banks and Sobel, 1987) to examine the robustness of the equilibrium. In what follows we show that the applying the D1-Criterion yields the same equilibrium outcome as applying the Intuitive Criterion.

The application of the D1-Criterion also has two steps: Step 1, for a price p' that is off the equilibrium path, find the types of sellers that is more likely to deviate to p'. Here the likelihood of deviation depends on the set of consumer belief $b_1(p_1')$ that could make the deviation profitable (this will be clearer in the following analysis). Step 2: After restricting consumers' belief on which type deviates to p' to those identified from Step 1, the equilibrium survives the D1-Criterion if both types of seller's equilibrium profits are weakly higher than the lowest profits from deviation to p'.

Note that the set of equilibria surviving the D1-Criterion are a subset of those surviving the Intuitive Criterion. As no semi-separating equilibrium satisfies the Intuitive Criterion, no semi-separating equilibrium satisfies the D1-Criterion. Therefore, to show that applying the D1-Criterion result in the same equilibrium outcome as applying the Intuitive Criterion, we only need to show that the separating equilibrium characterized in Proposition 1 and the pooling equilibrium characterized in Proposition 2 also survive the D1-Criterion.

B17.1 The Separating Equilibrium in Proposition 1.

The equilibrium emerges when $\alpha_L > 1-\lambda$. According to Proposition 1, we have the two cases as follows.

- *Case 1:* $N \geq N' \equiv \frac{\alpha_L-(1-\lambda)}{\alpha_H-\alpha_L} \cdot \frac{1-\alpha_L}{1-\alpha_H}$.
  Recall from Proposition 1 that in this situation, the H-seller charges $p_{1H}^* = 1-\lambda$ and the L-seller charges $p_{1L}^* = \alpha_L$. The H-seller receives unbiased reviews while the L-seller receives only negative reviews.Consider the deviation to a off-equilibrium price p_1'. Let B_H and B_L denote the set of first-period beliefs $b_1(p_1')$ under which the H-seller and the L-seller find the deviation strictly profitable. Let B_H^0 and B_L^0 denote the set of first-period beliefs $b_1(p_1')$ under which the H-seller and the L-seller are indifferent between deviation and sticking with the equilibrium.

  - Deviation to a price $p_1' < 1-\lambda$.
    It is immediately seen that such a deviation lowers the H-seller's profits regardless of consumers' belief. Therefore,
    $$B_H \cup B_H^0 = \emptyset \tag{B157}$$
    Next, if $B_L \neq \emptyset$, then $B_H \cup B_H^0 \subset B_L$. According to D1-Criterion's Step 1, we have $b_1(p_1' < 1-\lambda) = 0$, under which the L-seller will not deviate to p_1'. If $B_L = \emptyset$, then the equilibrium immediately holds. Therefore, the equilibrium survives the D1-Criterion.
  - Deviation to a price $p_1' \in (1-\lambda, \alpha_L]$.
    Let b_1' denote the belief associated with p_1'. The H-seller's profits of deviating to p_1' are
    $$\Pi_H'(p_1') = \frac{p_1'}{2} + \frac{N}{2} \cdot (\frac{b_1'(1-\alpha_L)}{b_1'(1-\alpha_L)+(1-b_1')(1-\alpha_H)} \cdot \alpha_H + \frac{(1-b_1')(1-\alpha_H)}{b_1'(1-\alpha_L)+(1-b_1')(1-\alpha_H)} \cdot \alpha_L) \tag{B158}$$
    Comparing it with the H-seller's equilibrium profits, we have
    $$B_H = \{b_1 | b_1 > \frac{1-\alpha_H}{\alpha_H-\alpha_L} \cdot \frac{M(\alpha_H-\alpha_L)-(p_1-(1-\lambda))}{N(1-\alpha_H)+p_1-(1-\lambda)}\} \tag{B159}$$
    $$B_H^0 = \{b_1 | b_1 = \frac{1-\alpha_H}{\alpha_H-\alpha_L} \cdot \frac{M(\alpha_H-\alpha_L)-(p_1-(1-\lambda))}{N(1-\alpha_H)+p_1-(1-\lambda)}\} \tag{B160}$$
    With $N \geq N'$, we have $\frac{1-\alpha_H}{\alpha_H-\alpha_L} \cdot \frac{N(\alpha_H-\alpha_L)-(p_1-(1-\lambda))}{N(1-\alpha_H)+p_1-(1-\lambda)} \in (0,1)$. Therefore, $B_H \neq \emptyset$ and $B_H^0 \neq \emptyset$.
    Similarly, the L-seller's profits of deviating to p_1' are
    $$\Pi_L(p_1')' = \frac{p_1'}{2} + \frac{N}{2} \cdot \Big(\frac{1-\alpha_H}{1-\alpha_L} \cdot (\frac{b_1'(1-\alpha_L)}{b_1'(1-\alpha_L)+(1-b_1')(1-\alpha_H)} \cdot \alpha_H + \frac{(1-b_1')(1-\alpha_H)}{b_1'(1-\alpha_L)+(1-b_1')(1-\alpha_H)} \cdot \alpha_L) + \frac{\alpha_H-\alpha_L}{1-\alpha_L} \cdot \alpha_L\Big) \tag{B161}$$

Comparing it with the L-seller's equilibrium profits, we have

$$B_L = \{b_1 | b_1 > \frac{1-\alpha_H}{\alpha_H - \alpha_L} \cdot \frac{\alpha_L - p_1'}{N(1-\alpha_H) - (p_1' - \alpha_L)}\} \tag{B162}$$

$$B_L^0 = \{b_1 | b_1 = \frac{1-\alpha_H}{\alpha_H - \alpha_L} \cdot \frac{\alpha_L - p_1'}{N(1-\alpha_H) - (p_1' - \alpha_L)}\} \tag{B163}$$

It can be easily verified that

$$B_H \cup B_H^0 \subset B_L \tag{B164}$$

According to D1-Criterion's Step 1, we have $b_1(p_1' \in (1-\lambda, \alpha_L)) = 0$, under which neither seller would deviate to p_1'. Therefore, the equilibrium survives the D1-Criterion.

– Deviation to a price $p_1' \in (\alpha_L, \alpha_H]$.

Two immediate observations can be made from this range of deviation. First, as long as consumers will make purchase at $p_1' \in (\alpha_L, \alpha_H]$, the L-seller has strict incentive to deviate as the deviation will improve its first-period profits without hurting its second-period profits. Second, as long as consumers do not make purchase at p_1', the H-seller will not deviate to p_1' as the deviation will hurt its first-period profits without improving its second-period profits. The two observations imply that

$$B_H \cup B_H^0 \subseteq B_L \tag{B165}$$

If $B_H \cup B_H^0 \subset B_L$, then according to Step 1 of the D1-Criterion, we have $b_1(p_1') = 0$; If $B_H \cup B_H^0 = B_L$, then the deviation profits are the lowest when $b_1(p_1') = 0$. With $b_1(p_1') = 0$, neither type of seller will deviate to p_1'. Therefore, the equilibrium survives the D1-Criterion.

– Deviation to a price $p_1' > \alpha_H$.

This is a price that consumers will not make purchases. Therefore, we have

$$B_H \cup B_H^0 = \emptyset \tag{B166}$$

Then for the similar reason discussed in the "$p_1' > \alpha_H$" case, neither type of seller will deviate to $p_1' > \alpha_H$ and the equilibrium survives the D1-Criterion.

- *Case 2:* $N < N' \equiv \frac{\alpha_L - (1-\lambda)}{\alpha_H - \alpha_L} \cdot \frac{1-\alpha_L}{1-\alpha_H}$

Recall from Proposition 1 that in this situation, the H-seller charges $p_{1H}^* = \alpha_L - N \cdot \frac{1-\alpha_H}{1-\alpha_L} \cdot (\alpha_H - \alpha_L)$ and the L-seller charges $p_{1L}^* = \alpha_L$. Both types of sellers receive only negative reviews. Consider the deviation to a off-equilibrium price p_1'.

– Deviation to a price $p_1' < p_{1H}^*$.

Such a deviation lowers the H-seller's profits regardless of consumers' belief, so we have

$$B_H \cup B_H^0 = \emptyset \tag{B167}$$

If $B_L \neq \emptyset$, then $B_H \cup B_H^0 \subset B_L$. According to D1-Criterion's Step 1, we have $b_1(p_1' < p_{1H}^*) = 0$, under which the L-seller will not deviate to p_1'. If $B_L = \emptyset$, then the equilibrium immediately holds. Therefore, the equilibrium survives the D1-Criterion.

– Deviation to a price $p_1' \in (p_{1H}^*, \alpha_L]$.

Following the same approach as in the $N \geq N'$ case, we derive

$$B_H = \{b_1 | b_1 > \frac{1-\alpha_H}{\alpha_H - \alpha_L} \cdot \frac{N(\alpha_H - \alpha_L)^2 + (\alpha_L - p_1')(1-\alpha_L)}{N(1-\alpha_H)(1+\alpha_H - 2\alpha_L) - (\alpha_L - p_1')(1-\alpha_L)}\} \quad \text{(B168)}$$

$$B_H^0 = \{b_1 | b_1 = \frac{1-\alpha_H}{\alpha_H - \alpha_L} \cdot \frac{N(\alpha_H - \alpha_L)^2 + (\alpha_L - p_1')(1-\alpha_L)}{N(1-\alpha_H)(1+\alpha_H - 2\alpha_L) - (\alpha_L - p_1')(1-\alpha_L)}\} \quad \text{(B169)}$$

$$B_L = \{b_1 | b_1 > \frac{1-\alpha_H}{\alpha_H - \alpha_L} \cdot \frac{\alpha_L - p_1'}{N(1-\alpha_H) - (p_1' - \alpha_L)}\} \quad \text{(B170)}$$

$$B_L^0 = \{b_1 | b_1 = \frac{1-\alpha_H}{\alpha_H - \alpha_L} \cdot \frac{\alpha_L - p_1'}{N(1-\alpha_H) - (p_1' - \alpha_L)}\} \quad \text{(B171)}$$

and we can verify that

$$B_H \cup B_H^0 \subset B_L \quad \text{(B172)}$$

We have $b_1(p_1' \in (p_{1H}^*, \alpha_L)) = 0$. According to D1-Criterion's Step 1. As a result, neither seller would deviate to p_1' and the equilibrium survives the D1-Criterion.

– Deviation to a price $p_1' \in (\alpha_L, \alpha_H]$.

For the same reason discussed in the $N \geq N'$ case, such a deviation is not profitable for both types of sellers and the equilibrium survives the D1-Criterion.

– Deviation to a price $p_1' > \alpha_H$.

For the same reason discussed in the $N \geq N'$ case, such a deviation is not profitable for both types of sellers and the equilibrium survives the D1-Criterion.

The analysis shows that the equilibrium characterized in Proposition 1 also survives the D1-Criterion. Therefore, applying both the Intuitive Criterion and the D1-Criterion yield the same separating equilibrium characterized in Proposition 1.

B17.2 The Pooling Equilibrium in Proposition 2.

Recall from the proof of Proposition 2 (see Section A2 in the main appendix) that when $\alpha_L \leq 1-\lambda$, all equilibria with $p_{1LH}^* \in [\alpha_L, \min\{1-\lambda, \overline{\alpha}\}]$ survive the Intuitive Criterion. We will show that all equilibria with $p_{1LH}^* \in [\alpha_L, \min\{1-\lambda, \overline{\alpha}\}]$ also survive the D1-Criterion.

Consider the deviation to an off-equilibrium price p_1'.

- Deviation to $p_1' < p_{1LH}^*$

  Regardless of consumers' belief, such a deviation lowers the H-seller's firs-period profits without improving its second-period profits, so we have

$$B_H \cup B_H^0 = \emptyset \quad \text{(B173)}$$

According to D1-Criterion's Step 1, we have $b_1(p_1' < p_{1LH}^*) = 0$, under which the L-seller will not deviate to p_1'. If $B_L = \emptyset$, then the equilibrium immediately holds. Therefore, the equilibrium survives the D1-Criterion.

- Deviation to $p_1' > p_{1LH}^*$

  It is easy to see that as long as consumers will make purchase at $p_1' > p_{1LH}^*$, the L-seller has strict incentive to deviate as the deviation will improve its first-period profits without hurting its second-period profits; As long as consumers do not make purchase at p_1', the H-seller will not deviate to p_1' as the deviation will hurt its first-period profits without improving its second-period profits. Therefore, we have

$$B_H \cup B_H^0 \subseteq B_L \tag{B174}$$

  If $B_H \cup B_H^0 \subset B_L$, then according to Step 1 of the D1-Criterion, we have $b_1(p_1') = 0$; If $B_H \cup B_H^0 = B_L$, then the deviation profits are the lowest when $b_1(p_1') = 0$. With $b_1(p_1') = 0$, neither type of seller will deviate to p_1'. Therefore, the equilibrium survives the D1-Criterion.

Therefore, the pooling equilibria that satisfy the Intuitive Criterion also satisfy the D1-Criterion. In other words, the D1-Criterion cannot further refine the equilibrium. Therefore, we focus on the equilibrium that both types of sellers earn the highest profits, i.e., $p_{1LH}^* = \min\{1-\lambda, \alpha_L\}$ as characterized in Proposition 2.

B18 The Pricing Pattern in the Children's Literature Category.

In the introduction section, we state that in the children's literature category, we observe a similar pricing pattern as that in the science fiction novels category. We present the details here.

In the children's literature, the Newbery Medal is considered one of the highest awards, and is given to the author of "the most distinguished contribution to American literature for children".[3] Each year the committee awards one book the Newbery Medal and several leading contenders the Newbery Honors. We collect the introductory prices of books which won the Newbery Medal and books which won the Newbery Honors and present them in the following table. We find a similar pattern as in the science fiction category that the Newbery Medal winners tend to have a lower introductory price than the Newbery Honors winners.

[3] See `https://www.ala.org/alsc/awardsgrants/bookmedia/newbery`. Accessed October 2023

	The introductory price of the book that wins Newbery Medal (in USD)	The average introductory prices of Newbery Honor books (in USD)
2023	14.49	17.32
2022	15.49	15.87
2021	12.00	16.96
2020	14.90	14.95
2019	11.50	11.50
2018	10.00	12.17
2017	9.99	12.07

Table 1: Prices in the Children's Literature Category

We further collect the number of pages of each book and calculate the per-page price of each book. The result is shown in the next table.

	The introductory per-page price of the book that wins Newbery Medal (in USD)	The average introductory per-page prices of Newbery Honor books (in USD)
2023	0.035	0.057
2022	0.046	0.192
2021	0.039	0.095
2020	0.058	0.110
2019	0.031	0.041
2018	0.031	0.171
2017	0.025	0.115

Table 2: Per-page Prices in the Children's Literature Category

It is seen that the Newbery Medal winners tend to have a lower introductory per-page price than the Newbery Honors winners.

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