Online Appendix to "Restricting Speculative Reselling: When 'How Much' Is the Question"

Proof of Lemma 1

In Period 1, consumers strictly prefer to purchase immediately if $u_1 > u_2$, where the expression of u_1 is given in Equation (3) and that of u_2 is given in Equation (4). By solving the inequality, we obtain that first-period consumers strictly prefer to purchase immediately if

$$\begin{aligned} p_f < \widetilde{w}_1(D_s, D_{1c}) &= \begin{cases} \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})(2-K+D_s)}, & \text{if } D_s \leq 2-K \\ \frac{3(2-K+D_s)}{8(2-D_{1c})}, & \text{if } D_s > 2-K \end{cases} \\ &= \max\{\frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})(2-K+D_s)}, \frac{3(2-K+D_s)}{8(2-D_{1c})}\}, \end{aligned} \tag{A1}$$

where D_s and D_{1c} are the equilibrium first-period sales to speculators and consumers, respectively (i.e., what an individual first-period consumer anticipates would happen in the equilibrium). They strictly prefer not to purchase immediately if $p_f > \widetilde{w}_1(D_s, D_{1c})$ and they are indifferent if $p_f = \widetilde{w}_1(D_s, D_{1c})$. We have:

$$\frac{\partial \widetilde{w}_1(D_s, D_{1c})}{\partial D_{1c}} = \begin{cases} \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})^2(2-K+D_s)}, & \text{if } D_s \leq 2-K \\ \frac{3(2-K+D_s)}{8(2-D_{1c})^2}, & \text{if } D_s > 2-K \end{cases} \geq 0. \tag{A2}$$

Thus, if $p_f < \widetilde{w}_1(D_s, 0)$, then early consumers always strictly prefer to purchase immediately and there is only one possible outcome: $D_{1c} = \min\{1, K - D_s\}$. If $p_f > \widetilde{w}_1(D_s, \min\{1, K - D_s\})$, then early consumers always strictly prefer not to purchase immediately and there is also only one possible outcome: $D_{1c} = 0$. However, if $\widetilde{w}_1(D_s, 0) \leq p_f \leq \widetilde{w}_1(D_s, \min\{1, K - D_s\})$, then there are three possible equilibrium outcomes: $D_{1c} = 0$, $D_{1c} = \min\{1, K - D_s\}$, and $D_{1c} = \widehat{D}_{1c}$, where $\widehat{D}_{1c}$ is the solution of the equation $p_f = \widetilde{w}_1(D_s, D_{1c})$.

Recall that speculators' profit is given in Equation (5). By checking the sign of the first-order derivative of Π_s with respect to D_{1c}, we obtain that speculators' profit strictly increases in D_{1c}. Thus, when multiple values of D_{1c} could be sustained in equilibrium, speculators prefer the highest D_{1c}.

Let us now move to the firm's preference. We have that $D_{1c} > 0$ requires that there is a shortage in the second period. This implies that the firm sells all the capacity (and makes a profit of $\Pi_f = p_f \times K$) at a price p_f that satisfies $p_f \leq \widetilde{w}_1(D_s, D_{1c})$. Further, recall that $\frac{\partial \widetilde{w}_1(D_s, D_{1c})}{\partial D_{1c}} \geq 0$. Thus, a higher D_{1c} relaxes the constraint on the firm's price (while sales are constant) and is therefore preferable by the firm.

The above proves that in the case of multiple equilibria, both the firm and speculators prefer the highest D_{1c} (i.e., $D_{1c} = \min\{1, K - D_s\}$). This completes the proof of Lemma 1. $\square$

Derivation of the Equilibrium of the Main Model

This analysis contains three steps. First, we derive the firm's optimal choice of price constrained by $D_{1c}=0$. Second, we derive the firm's optimal choice of price constrained by $D_{1c}>0$. Finally, we compare the firm's payoffs in the above two cases to derive the equilibrium.

Step 1: Derive the firm's optimal choice of price constrained by $D_{1c}=0$*.*

Note that when no early consumer purchases in the first period (i.e., when $D_{1c}=0$), speculators' total payoff as a function of D_s reduces to:

$$\Pi_s(D_s)=\begin{cases}\frac{2-K}{2}\times D_s-p_f\times D_s, & \text{if } D_s\le 2-K\\ \frac{(2-K+D_s)^2}{8}-p_f\times D_s, & \text{if } D_s>2-K\end{cases}. \tag{A3}$$

To maximize the total payoff, speculators choose $D_s=\beta$ if $\beta\le 2-K$ and $p_f<\frac{2-K}{2}$, or if $\beta>2-K$ and $p_f<\frac{(2-K+\beta)^2}{8\beta}$; $D_s=0$ if $\beta\le 2-K$ and $p_f>\frac{2-K}{2}$, or if $\beta>2-K$ and $p_f>\frac{(2-K+\beta)^2}{8\beta}$; D_s equals any value in $[0,\beta]$ if $\beta\le 2-K$ and $p_f=\frac{2-K}{2}$; $D_s=0$ or β if $\beta>2-K$ and $p_f=\frac{(2-K+\beta)^2}{8\beta}$.

Given the speculators' decision, if $\beta\le 2-K$, then the firm's payoff is

$$\begin{aligned}\Pi_f=&\begin{cases}p_f\times K, & \text{if } p_f<\frac{2-K}{2}, \text{ or if } p_f=\frac{2-K}{2} \text{ and } D_s>0\\ p_f\times\min\{K,2(1-p_f)\}, & \text{if } p_f>\frac{2-K}{2}, \text{ or if } p_f=\frac{2-K}{2} \text{ and } D_s=0\end{cases}\\ =&p_f\times\min\{K,2(1-p_f)\}\end{aligned}$$

and is maximized at $p_f=\max\{\frac{2-K}{2},\frac{1}{2}\}$.

If $2-K<\beta\le K$, then the firm's payoff is

$$\begin{aligned}\Pi_f=&\begin{cases}p_f\times K, & \text{if } p_f<\frac{(2-K+\beta)^2}{8\beta}, \text{ or if } p_f=\frac{(2-K+\beta)^2}{8\beta} \text{ and } D_s=\beta\\ p_f\times\min\{K,2(1-p_f)\}, & \text{if } p_f>\frac{(2-K+\beta)^2}{8\beta}, \text{ or if } p_f=\frac{(2-K+\beta)^2}{8\beta} \text{ and } D_s=0\end{cases}\\ =&\begin{cases}p_f\times K, & \text{if } p_f<\frac{(2-K+\beta)^2}{8\beta}, \text{ or if } p_f=\frac{(2-K+\beta)^2}{8\beta} \text{ and } D_s=\beta\\ p_f\times 2(1-p_f), & \text{if } p_f>\frac{(2-K+\beta)^2}{8\beta}, \text{ or if } p_f=\frac{(2-K+\beta)^2}{8\beta} \text{ and } D_s=0\end{cases}\end{aligned}$$

and is maximized at $p_f=\frac{1}{2}$, or $p_f=\frac{1}{2K}$ if $D_s=\beta=K$.

To sum up, if $K\le 1$, then in equilibrium, $p_f=\frac{2-K}{2}$, D_s equals any value in $[0,\beta]$, $D_{1c}=0$, and $p_s=\frac{2-K}{2}$. The firm's and speculators' profits are, respectively, $\Pi_f=\frac{(2-K)K}{2}$ and $\Pi_s=0$. The surpluses of consumers who entered in Period 1 (early) and in Period 2 (late) are $CS_{early}=CS_{late}=\frac{K^2}{8}$ and $CS_{total}=\frac{K^2}{4}$. The social welfare is $\frac{(4-K)K}{4}$. Note that in this case, technically, there are many equilibria (corresponding to every $D_s\in[0,\beta]$), but all of them are payoff-equivalent.

If $K>1$ and $\beta<K$, then in equilibrium, $p_f=\frac{1}{2}$, $D_s=0$, and $D_{1c}=0$. The firm's and speculators' profits are, respectively, $\Pi_f=\frac{1}{2}$ and $\Pi_s=0$. The consumer surpluses are $CS_{early}=CS_{late}=\frac{1}{8}$ and $CS_{total}=\frac{1}{4}$. The social welfare is $\frac{3}{4}$.

If $K>1$ and $\beta=K$, then in equilibrium, either $p_f=\frac{1}{2}$, $D_s=0$, and $D_{1c}=0$, or $p_f=\frac{1}{2K}$, $D_s=\beta=K$, $D_{1c}=0$, and $p_s=\frac{1}{2}$. Regardless of the equilibrium decisions, the firm's and speculators' profits are, respectively, $\Pi_f = \frac{1}{2}$ and $\Pi_s = 0$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{1}{8}$ and $CS_{total} = \frac{1}{4}$. The social welfare is $\frac{3}{4}$.

Step 2: Derive the firm's optimal choice of price constrained by $D_{1c} > 0$.

In the proof of Lemma 1, we have shown that when D_{1c} is positive, the firm's profit is $\Pi_f = p_f \times K$. Thus, the firm's problem is equivalent to maximizing p_f (subject to that $D_{1c} > 0$ is possible in equilibrium with this p_f). Therefore, the optimal p_f is obtained by finding the maximal p_f under which the inequality $u_1 \geq u_2$ is satisfied. Note that solving this inequality is not trivial because u_2 is a function of p_f, D_s, and D_{1c}, where D_{1c} is dependent on p_f and D_s, and D_s is dependent on p_f. Furthermore, as we derived above, D_s and D_{1c} are discontinuous in p_f.

To solve this inequality, we first substitute D_{1c} by the value most preferable by the firm and speculators (i.e., $D^*_{1c} = \min\{1, K - D_s\}$) as proven in Lemma 1. Next, we solve for the speculators' optimal choice of D_s given p_f (and constrained by $D_s \leq \beta$). Then, we obtain the upper bound on p_f to derive the equilibrium decisions and payoffs subject to the constraint $D_{1c} > 0$.

Note that given $D^*_{1c} = \min\{1, K - D_s\}$, we can rewrite the speculators' problem (Equation (5)) for the $K \leq \frac{3}{2}$ case as

$$\max_{D_s \leq \beta} \Pi_s(D_s) = \begin{cases} (2-K) \times D_s - p_f \times D_s, & \text{if } D_s \leq K-1 \\ \frac{2-K}{2-K+D_s} \times D_s - p_f \times D_s, & \text{if } K-1 < D_s \leq 2-K \\ \frac{2-K+D_s}{4} - p_f \times D_s, & \text{if } D_s > 2-K \end{cases}, \tag{A4}$$

and that for the $\frac{3}{2} < K < 2$ case as

$$\max_{D_s \leq \beta} \Pi_s(D_s) = \begin{cases} (2-K) \times D_s - p_f \times D_s, & \text{if } D_s \leq 2-K \\ \frac{(2-K+D_s)^2}{4} - p_f \times D_s, & \text{if } 2-K < D_s \leq K-1 \\ \frac{2-K+D_s}{4} - p_f \times D_s, & \text{if } D_s > K-1 \end{cases}. \tag{A5}$$

It is convenient to split the rest of the derivation in the following cases.

Case: $\beta \leq K-1$ *and* $\beta \leq 2-K$. Here, given $D_s \leq \beta \leq K-1$, we have $D^*_{1c} = 1$. Given $D_s \leq \beta \leq 2-K$, $u_1 \geq u_2$ corresponds to $p_f \leq \widetilde{w}_1(D_s, D^*_{1c}) = \frac{(2-K)(2-K+2D_s)}{2(2-D^*_{1c})(2-K+D_s)} \leq \frac{2-K}{2-D^*_{1c}} = 2-K$. Thus, speculators' profit $\Pi_s(D_s) = (2-K) \times D_s - p_f \times D_s$ is maximized at $D^*_s = \beta$. In turn, the upper bound of p_f can be written as $\widetilde{w}_1(D^*_s, D^*_{1c}) = \frac{(2-K)(2-K+2D^*_s)}{2(2-D^*_{1c})(2-K+D^*_s)} = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$. Therefore, the equilibrium decisions are $p^*_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$, $D^*_s = \beta$, $D^*_{1c} = 1$, and $p^*_s = 2-K$. The firm's and speculators' profits are, respectively, $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}$ and $\Pi_s = \frac{(2-K)^2\beta}{2(2-K+\beta)}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{1}{2} - \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$ and $CS_{total} = 1 - \frac{(2-K)(2-K+2\beta)}{2-K+\beta}$ and the social welfare is $\frac{-2+4K-K^2}{2}$.

Case: $K-1 < \beta \leq 2-K$. Here, to maximize the joint payoff (given in Equation (A5)), the speculators choose $D_s = K-1$ if $p_f > (2-K)^2$, $D_s = (\frac{1}{\sqrt{p_f}} - 1)(2-K)$ if $\begin{cases} \frac{1}{4} < p_f \leq (2-K)^2 \\ \beta \geq (\frac{1}{\sqrt{p_f}} - 1)(2-K) \end{cases}$, and $D_s = \beta$ if $p_f \le \frac{1}{4}$ or $\begin{cases} \frac{1}{4} < p_f \le (2-K)^2 \\ \beta < (\frac{1}{\sqrt{p_f}} - 1)(2-K) \end{cases}$. Note that the highest price p_f under which early consumers purchase immediately equals $\frac{(2-K)K}{2}$ when $D_s = K-1$, $\frac{4}{9}$ when $D_s = (\frac{1}{\sqrt{p_f}} - 1)(2-K) = \frac{2-K}{2}$, and $\frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$ when $D_s = \beta$. To sum up, if $\begin{cases} 1 < K \le \frac{4}{3} \\ K-1 < \beta \le \frac{2-K}{2} \end{cases}$, the equilibrium decisions are $p_f^* = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$, $D_s^* = \beta$, $D_{1c}^* = K - \beta$, and $p_s^* = 2-K$. The firm's and speculators' profits are $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}$ and $\Pi_s = \frac{(2-K)^2\beta}{2(2-K+\beta)^2}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{\beta^2}{2(2-K+\beta)^2}$ and $CS_{total} = \frac{\beta^2}{(2-K+\beta)^2}$ and the social welfare is $\frac{2K-K^2+2\beta}{4-2K+2\beta}$. If $\begin{cases} 1 < K \le \frac{4}{3} \\ \frac{2-K}{2} < \beta \le 2-K \end{cases}$, the equilibrium decisions are $p_f^* = \frac{4}{9}$, $D_s^* = \frac{2-K}{2}$, $D_{1c}^* = \frac{3K-2}{2}$, and $p_s^* = \frac{2}{3}$. The firm's and speculators' profits are $\Pi_f = \frac{4}{9}K$ and $\Pi_s = \frac{2-K}{9}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{1}{18}$ and $CS_{total} = \frac{1}{9}$ and the social welfare is $\frac{K+1}{3}$. Alternatively, if $\begin{cases} \frac{4}{3} < K \le \frac{3}{2} \\ K-1 < \beta \le 2-K \end{cases}$, the equilibrium decisions are $p_f^* = \frac{(2-K)K}{2}$, $D_s^* = K-1$, $D_{1c}^* = 1$, and $p_s^* = 2-K$. The firm's and speculators' profits are $\Pi_f = \frac{(2-K)K^2}{2}$ and $\Pi_s = \frac{(K-1)(2-K)^2}{2}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{(K-1)^2}{2}$ and $CS_{total} = (K-1)^2$ and the social welfare is $\frac{-2+4K-K^2}{2}$.

Case: $K-1 \le 2-K < \beta$. Here, to maximize the joint payoff, the speculators choose $D_s = K-1$ if $p_f > (2-K)^2$, $D_s = (\frac{1}{\sqrt{p_f}} - 1)(2-K)$ if $\frac{1}{4} < p_f \le (2-K)^2$, and $D_s = \beta$ if $p_f \le \frac{1}{4}$. Note that the highest price p_f under which early consumers purchase immediately is $p_f = \frac{K(2-K)}{2}$ when $D_s = K-1$, $p_f = \frac{4}{9}$ when $D_s = (\frac{1}{\sqrt{p_f}} - 1)(2-K) = \frac{2-K}{2}$, and $p_f = \frac{3}{8}$ when $D_s = \beta$. To sum up, if $\begin{cases} 1 < K \le \frac{4}{3} \\ K-1 \le 2-K < \beta \end{cases}$, the equilibrium decisions are $p_f^* = \frac{4}{9}$, $D_s^* = \frac{2-K}{2}$, $D_{1c}^* = \frac{3K-2}{2}$, and $p_s^* = \frac{2}{3}$. The firm's and speculators' profits are $\Pi_f = \frac{4}{9}K$ and $\Pi_s = \frac{2-K}{9}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{1}{18}$ and $CS_{total} = \frac{1}{9}$ and the social welfare is $\frac{K+1}{3}$. Alternatively, if $\begin{cases} \frac{4}{3} < K \le \frac{3}{2} \\ K-1 \le 2-K < \beta \end{cases}$, the equilibrium decisions are $p_f^* = \frac{(2-K)K}{2}$, $D_s^* = K-1$, $D_{1c}^* = 1$, and $p_s^* = 2-K$. The firm's and speculators' profits are: $\Pi_f = \frac{(2-K)K^2}{2}$ and $\Pi_s = \frac{(K-1)(2-K)^2}{2}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{(K-1)^2}{2}$ and $CS_{total} = (K-1)^2$ and the social welfare is $\frac{-2+4K-K^2}{2}$.

Case: $2-K < \beta \le K-1$. Here, the speculators' joint payoff is maximized at $D_s = \beta$ if $p_f \le \frac{(2-K+\beta)^2}{4\beta}$ or $D_s = 0$ if $p_f > \frac{(2-K+\beta)^2}{4\beta}$. To sell in the first period, the maximum price the firm can set is $p_f = \min\{\frac{(2-K+\beta)^2}{4\beta}, \frac{3(2-K+D_s)}{8(2-D_{1c})}\}$. To sum up, if $\begin{cases} \frac{3}{2} < K \le \frac{5}{3} \\ 2-K < \beta \le K-1 \end{cases}$ or $\begin{cases} \frac{5}{3} < K < 2 \\ 2-K < \beta \le 4-2K \end{cases}$, the equilibrium decisions are $p_f^* = \frac{3(2-K+\beta)}{8}$, $D_s^* = \beta$, $D_{1c}^* = 1$, and $p_s^* = \frac{2-K+\beta}{2}$. The firm's and speculators' profits are $\Pi_f = \frac{3(2-K+\beta)K}{8}$ and $\Pi_s = \frac{(2-K+\beta)^2}{4} - \frac{3(2-K+\beta)}{8}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{-2+3K-3\beta}{8}$ and $CS_{total} = \frac{-2+3K-3\beta}{4}$ and the social welfare is $\frac{4-K^2-4\beta-\beta^2+2K(2+\beta)}{8}$.

If $\begin{cases} \frac{5}{3} < K < 2 \\ 4-2K < \beta \leq K-1 \end{cases}$, on the other hand, the equilibrium decisions are $p_f^* = \frac{(2-K+\beta)^2}{4\beta}$, $D_s^* = \beta$, $D_{1c}^* = 1$, and $p_s^* = \frac{2-K+\beta}{2}$. The firm's and speculators' profits are $\Pi_f = \frac{(2-K+\beta)^2 K}{4\beta}$ and $\Pi_s = 0$. The consumer surpluses are $CS_{early} = \frac{2\beta-(2-K+\beta)^2}{4\beta}$, $CS_{late} = \frac{-2(K-1)(2-K)^2+(8-8K+3K^2)\beta-2\beta^2-\beta^3}{8\beta}$, and $CS_{total} = \frac{-2K^3+K^2(8+3\beta)-4K(2+\beta)+\beta(4-4\beta-\beta^2)}{8\beta}$ and the social welfare is $\frac{4-K^2-4\beta-\beta^2+2K(2+\beta)}{8}$.

Case: $2-K < K-1 < \beta$. Here, to maximize the joint payoff, the speculators choose $D_s = 0$ if $p_f > \frac{1}{4(K-1)}$, $D_s = K-1$ if $\frac{1}{4} < p_f \leq \frac{1}{4(K-1)}$, and $D_s = \beta$ if $p_f \leq \frac{1}{4}$. Note that the highest price p_f under which early consumers purchase immediately is $\frac{3}{8}$ when $D_s \geq K-1 > 2-K$. Thus, $p_f^* = \min\{\frac{3}{8}, \frac{1}{4(K-1)}\}$. To sum up, if $\begin{cases} \frac{3}{2} < K \leq \frac{5}{3} \\ 2-K < K-1 < \beta \end{cases}$, the equilibrium decisions are $p_f^* = \frac{3}{8}$, $D_s^* = K-1$, $D_{1c}^* = 1$, and $p_s^* = \frac{1}{2}$. The firm's and speculators' profits are $\Pi_f = \frac{3K}{8}$ and $\Pi_s = \frac{5-3K}{8}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{1}{8}$ and $CS_{total} = \frac{1}{4}$ and the social welfare is $\frac{7}{8}$. If $\begin{cases} \frac{5}{3} < K < 2 \\ 2-K < K-1 < \beta \end{cases}$, on the other hand, the equilibrium decisions are $p_f^* = \frac{1}{4(K-1)}$, $D_s^* = K-1$, $D_{1c}^* = 1$, and $p_s^* = \frac{1}{2}$. The firm's and speculators' profits are $\Pi_f = \frac{K}{4(K-1)}$ and $\Pi_s = 0$. The consumer surpluses are $CS_{early} = \frac{2K-3}{4(K-1)}$, $CS_{late} = \frac{1}{8}$, and $CS_{total} = \frac{5K-7}{8(K-1)}$ and the social welfare is $\frac{7}{8}$.

Step 3: Compare the firm's payoffs subject to the constraint of zero or positive D_{1c}, which are derived in Steps 1 and 2, and summarize the equilibrium.

Here, we compare the firm's expected payoffs derived in Step 1 and Step 2, respectively,

$$\Pi_f|_{D_{1c}=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$$

and

$$\Pi_f|_{D_{1c}>0} = \begin{cases} \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \beta \leq K-1 \text{ and } \beta \leq 2-K \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } 1 < K \leq \frac{4}{3} \text{ and } K-1 < \beta \leq \frac{2-K}{2} \\ \frac{4}{9}K, & \text{if } 1 < K \leq \frac{4}{3} \text{ and } \beta > \frac{2-K}{2} \\ \frac{(2-K)K^2}{2}, & \text{if } \frac{4}{3} < K \leq \frac{3}{2} \text{ and } \beta > K-1 \\ \frac{3(2-K+\beta)K}{8}, & \text{if } \frac{3}{2} < K \leq \frac{5}{3} \text{ and } 2-K < \beta \leq K-1, \\ & \text{or if } \frac{5}{3} < K < 2 \text{ and } 2-K < \beta \leq 4-2K \\ \frac{3K}{8}, & \text{if } \frac{3}{2} < K \leq \frac{5}{3} \text{ and } \beta > K-1 \\ \frac{(2-K+\beta)^2 K}{4\beta}, & \text{if } \frac{5}{3} < K < 2 \text{ and } 4-2K < \beta \leq K-1 \\ \frac{K}{4(K-1)}, & \text{if } \frac{5}{3} < K < 2 \text{ and } \beta > K-1 \end{cases}.$$

When the former is larger than the latter, the equilibrium decisions are as described in Step 1, whereas when the latter is larger than the former, the equilibrium decisions are as described in Step 2.

Specifically, if $K \leq 1$, then $\Pi_f^* = \frac{(2-K)K}{2}$. If $1 < K \leq \frac{9}{8}$, then:

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \\ \frac{1}{2}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)}) < \beta \leq K \end{cases}.$$

If $\frac{9}{8} < K \leq \frac{4}{3}$, then:

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq \frac{2-K}{2} \\ \frac{4K}{9}, & \text{if } \frac{2-K}{2} < \beta \leq K \end{cases}.$$

If $\frac{4}{3} < K \leq \frac{3}{2}$, then:

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)K^2}{2}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{3}{2} < K \leq \frac{3+\sqrt{3}}{3}$, then:

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq 2-K \\ \frac{3(2-K+\beta)K}{8}, & \text{if } 2-K < \beta \leq K-1 \\ \frac{3K}{8}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{3+\sqrt{3}}{3} < K \leq \frac{5}{3}$, then:

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{4-6K+3K^2}{3K} \\ \frac{3(2-K+\beta)K}{8}, & \text{if } \frac{4-6K+3K^2}{3K} < \beta \leq K-1 \\ \frac{3K}{8}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{5}{3} < K \leq \frac{3+\sqrt{5}}{3}$, then:

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{4-6K+3K^2}{3K} \\ \frac{3(2-K+\beta)K}{8}, & \text{if } \frac{4-6K+3K^2}{3K} < \beta \leq 4-2K \\ \frac{(2-K+\beta)^2K}{4\beta}, & \text{if } 4-2K < \beta < K-1 \\ \frac{K}{4(K-1)}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{3+\sqrt{5}}{3} < K < 2$, then:

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(K-1)^2+\sqrt{1-4K+2K^2}}{K} \\ \frac{(2-K+\beta)^2K}{4\beta}, & \text{if } \frac{(K-1)^2+\sqrt{1-4K+2K^2}}{K} < \beta < K-1 \\ \frac{K}{4(K-1)}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

Note that in the above analysis, we assume $K<2$ (i.e., the firm's capacity is not enough to serve all consumers) to focus on the case of interest to us. But following a similar analysis as above, it is easy to show that if $K\geq 2$, $\Pi_f|_{D_{1c}=0}=\frac{1}{2}$ and $\Pi_f|_{D_{1c}>0}=\begin{cases}\frac{(2-K+\beta)^2K}{4\beta}, & \text{if } K-2<\beta\leq K-1\\ \frac{(2-K+\beta)K}{4\beta}, & \text{if } \beta>K-1\end{cases}$ ($D_{1c}>0$ is only feasible when $\beta>K-2$). Therefore, if $K\geq 2$, $\Pi_f^*=\max\{\Pi_f|_{D_{1c}=0},\Pi_f|_{D_{1c}>0}\}=\frac{1}{2}=\Pi_f|_{D_{1c}=0}$.

This completes the analysis of Section 4.2. $\square$

Proof of Proposition 1

Proof of Part (1)

When $\beta=0$, the equilibrium price of the firm is $p_f=\frac{2-K}{2}$ if $K\leq 1$ and $p_f=\frac{1}{2}$ if $K>1$. In either case, consumers only purchase in Period 2. That is, there are no first-period sales to consumers.

When $\beta>0$, sales to both speculators and consumers are positive in Period 1 if and only if $\Pi_f|_{D_{1c}>0}>\Pi_f|_{D_{1c}=0}$, where $\Pi_f|_{D_{1c}>0}$ and $\Pi_f|_{D_{1c}=0}$ are derived in the analysis of Section 4.2. By calculation, we obtain that sales to both speculators and consumers are positive in Period 1 if and only if $\begin{cases}0<\beta\leq\frac{3-\sqrt{3}}{3}\\ K_A(\beta)<K<K_B(\beta)\end{cases}$ or $\begin{cases}\frac{3-\sqrt{3}}{3}<\beta\leq\frac{7}{16}\\ K_A(\beta)<K<\frac{6+3\beta+\sqrt{-12+36\beta+9\beta^2}}{6}\end{cases}$ or $\begin{cases}\frac{7}{16}<\beta\leq\frac{6-2\sqrt{5}}{3}\\ \frac{9}{8}<K<\frac{6+3\beta+\sqrt{-12+36\beta+9\beta^2}}{6}\end{cases}$ or $\begin{cases}\frac{6-2\sqrt{5}}{3}<\beta\leq 1\\ \frac{9}{8}<K<K_C(\beta)\end{cases}$ or $\begin{cases}1<\beta\leq\frac{9}{8}\\ \frac{9}{8}<K<2\end{cases}$ or $\begin{cases}\frac{9}{8}<\beta<2\\ \beta\leq K<2\end{cases}$, where $K_A(\beta)$ is the smallest root of the equation $K^3+K^2(-2\beta-5)+K(6\beta+8)-\beta^2-4\beta-4=0$, $K_B(\beta)$ is the second smallest root of the equation $K^3+K^2(-2\beta-4)+K(4\beta+5)-\beta-2=0$, and $K_C(\beta)$ is the second smallest root of the equation $K^3+K^2(-2\beta-4)+K(\beta^2+4\beta+4)-2\beta=0$.

This completes the proof of Part (1) of Proposition 1.

Proof of Part (2)

We first prove that the first statement holds for any $K\in(1,\frac{4}{3})$ but not for any other K.

For any $K\in(1,\frac{4}{3})$, choose $p_f\in(\frac{4}{9},\frac{(2-K)K}{2})$. Then for the given p_f, if $\beta<\frac{(2-K)(2p_f-2+K)}{2(2-K-p_f)}$, we have $D_s=0$, $D_{1c}=0$, and $D_s+D_{1c}=0$; if $\frac{(2-K)(2p_f-2+K)}{2(2-K-p_f)}<\beta\leq K-1$, we have $D_s=\beta$, $D_{1c}=1$, and $D_s+D_{1c}=1+\beta$; if $K-1<\beta\leq\frac{(2-K)(1-2p_f+\sqrt{1-2p_f})}{2p_f}$, we have $D_s=\beta$, $D_{1c}=K-\beta$, and $D_s+D_{1c}=K$; if $\beta>\frac{(2-K)(1-2p_f+\sqrt{1-2p_f})}{2p_f}$, we have $D_s=0$, $D_{1c}=0$, and $D_s+D_{1c}=0$. Clearly, for a given p_f in a range, first-period demand can first increase in β and then collapse.

For any $K\leq 1$, if $p_f>\frac{2-K}{2}$, then $D_s=0$, $D_{1c}=0$, and $D_s+D_{1c}=0$, which means that first-period demand never increases in β. If $p_f=\frac{2-K}{2}$, then D_s can equal any value in $[0,\beta]$, which means that first-period demand does not collapse as β increases. If $p_f<\frac{2-K}{2}$, then $D_s=\beta$ and again, first-period demand does not collapse as β increases.

For any $\frac{4}{3} \leq K < \frac{3}{2}$, if $p_f > \frac{(2-K)K}{2}$, then $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, which means that first-period demand never increases in β. If $p_f \leq \frac{(2-K)K}{2}$, then $D_s + D_{1c} = K$ for any $\beta \geq K - 1$, which means that first-period demand never collapse as β increases.

For any $\frac{3}{2} \leq K < \frac{5}{3}$, if $p_f > \frac{3}{8}$, then $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, which means that first-period demand never increases in β. If $\frac{1}{4} < p_f \leq \frac{3}{8}$, then $D_s = K - 1$, $D_{1c} = 1$, and $D_s + D_{1c} = K$ for any $\beta \geq K - 1$, which means that first-period demand does not collapse as β increases. If $p_f \leq \frac{1}{4}$, then $D_s = \beta$, $D_{1c} = K - \beta$, and $D_s + D_{1c} = K$ for any $\beta \geq K - 1$ and again, first-period demand does not collapse as β increases.

For any $\frac{5}{3} \leq K < 2$, if $p_f > \frac{1}{4(K-1)}$, then $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, which means that first-period demand never increases in β. If $\frac{1}{4} < p_f \leq \frac{1}{4(K-1)}$, then $D_s = K - 1$, $D_{1c} = 1$, and $D_s + D_{1c} = K$ for any $\beta \geq K - 1$, which means that first-period demand does not collapse as β increases. If $p_f \leq \frac{1}{4}$, then $D_s = \beta$, $D_{1c} = K - \beta$, and $D_s + D_{1c} = K$ for any $\beta \geq K - 1$ and again, first-period demand does not collapse as β increases.

For any $K \geq 2$, if $p_f > \frac{1}{2K}$, then $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, which means that first-period demand never increases in β. If $\frac{1}{4(K-1)} < p_f \leq \frac{1}{2K}$, then $D_s = \beta$, $D_{1c} = K - \beta$, and $D_s + D_{1c} = K$ for any $\beta \geq \frac{K-2}{1-4p_f}$, which means that first-period demand does not collapse as β increases. If $p_f \leq \frac{1}{4(K-1)}$, then $D_s = \beta$, $D_{1c} = K - \beta$, and $D_s + D_{1c} = K$ for any $\beta \geq K - 2 + 2p_f + 2\sqrt{p_f(K - 2 + p_f)}$, which means that first-period demand does not collapse as β increases.

As above, we prove the first statement in Part (2) of Proposition 1.

Next, we prove that the second statement holds for any $K \in (1, \frac{9}{8}]$ but not for any other K.

For any $K \in (1, \frac{9}{8})$, if $\beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2}$, we have $p_f = \frac{1}{2}$, $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$; if $\frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K - 1$, we have $p_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$, $D_s = \beta$, $D_{1c} = 1$, and $D_s + D_{1c} = 1 + \beta$; if $K - 1 < \beta \leq (2 - K)(K - 1 + \sqrt{K(K-1)})$, we have $p_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$, $D_s = \beta$, $D_{1c} = K - \beta$, and $D_s + D_{1c} = K$; and if $(2 - K)(K - 1 + \sqrt{K(K-1)}) < \beta < K$, we have $p_f = \frac{1}{2}$, $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$. If $\beta = K$, the firm is indifferent between setting $p_f = \frac{1}{2}$, which corresponds to $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, and setting $p_f = \frac{1}{2K}$, which corresponds to $D_s = K$, $D_{1c} = 0$, and $D_s + D_{1c} = K$. Clearly, given the optimal p_f, first-period demand can first increase in β and then collapse.

For $K = \frac{9}{8}$, if $\beta \leq \frac{7}{496}$, we have $p_f = \frac{1}{2}$, $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$; if $\frac{7}{496} < \beta \leq \frac{1}{8}$, we have $p_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)} = \frac{7(7+16\beta)}{16(7+8\beta)}$, $D_s = \beta$, $D_{1c} = 1$, and $D_s + D_{1c} = 1 + \beta$; if $\frac{1}{8} < \beta \leq \frac{7}{16}$, we have $p_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2} = \frac{7(7+16\beta)}{2(7+8\beta)^2}$, $D_s = \beta$, $D_{1c} = K - \beta$, and $D_s + D_{1c} = K$. If $\frac{7}{16} < \beta \leq \frac{9}{8}$, the firm is indifferent between setting $p_f = \frac{1}{2}$, which corresponds to $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, and setting $p_f = \frac{4}{9}$, which corresponds to $D_s = \frac{2-K}{2}$, $D_{1c} = \frac{3K-2}{2}$, and $D_s + D_{1c} = K$. Clearly, given the optimal p_f, first-period demand can first increase in β and then collapse.

For any $K \leq 1$, in equilibrium we have $p_f = \frac{2-K}{2}$, D_s equals any value in $[0, \beta]$, and $D_{1c} = 0$, which means that first-period demand does not collapse as β increases.

For any $\frac{9}{8} < K < 2$, $\Pi_f|_{D_{1c}>0} > \Pi_f|_{D_{1c}=0}$ if $\beta > K-1$, which means that in equilibrium $D_{1c} > 0$ if $\beta > K-1$. In other words, first-period demand does not collapse as β increases.

For $K = 2$, if $\beta < 1$, we have $p_f = \frac{1}{2}$, $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$; if $\beta \geq 1$, the firm is indifferent between setting $p_f = \frac{1}{2}$, which corresponds to $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, and setting $p_f = \frac{1}{4}$, which corresponds to $D_s + D_{1c} = 2$. Here, first-period demand does not first increase in β and then collapse.

For any $K > 2$, if $\beta < K$, we have $p_f = \frac{1}{2}$, $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$; if $\beta = K$, the firm is indifferent between setting $p_f = \frac{1}{2}$, which corresponds to $D_s = 0$, $D_{1c} = 0$, and $D_s + D_{1c} = 0$, and setting $p_f = \frac{1}{2K}$, which corresponds to $D_s = K$, $D_{1c} = 0$, and $D_s + D_{1c} = K$. Here, first-period demand does not first increase in β and then collapse.

This completes the proof of Part (2) of Proposition 1.

Proof of Part (3)

Without speculators' presence, the firm's equilibrium profit is $\Pi_f^* = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$.

With speculators' presence, the firm's equilibrium profit is $\Pi_f^* = \max\{\Pi_f|_{D_{1c}>0}, \Pi_f|_{D_{1c}=0}\} \geq \Pi_f|_{D_{1c}=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$. Thus, speculators' presence never hurts the firm.

Note that with speculators' presence, the sales to consumers in the first period can only be positive in equilibrium if $\Pi_f|_{D_{1c}>0} \geq \Pi_f|_{D_{1c}=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$. Here, the equality only holds for K and β in a set of measure zero (on the boundary between the equilibrium with $D_{1c} > 0$ and $D_{1c} = 0$). Therefore, compared with the case without speculators, firm's profit is almost always strictly higher when sales to consumers in the first period are positive.

This completes the proof of Part (3) of Proposition 1. $\square$

Proof of Proposition 2

In Step 3 of the equilibrium derivation above, we listed the firm's equilibrium profit as a function of K and β. It is easy to see that $\frac{d\Pi_f^*}{d\beta} \geq 0$ if $\beta < K-1$ and $\frac{d\Pi_f^*}{d\beta} \leq 0$ if $\beta > K-1$. This shows that the firm's equilibrium profit is first (weakly) increasing and then (weakly) decreasing in β.

Moreover, we can conclude that 1) when $K \leq 1$ or $K \geq 2$, Π_f^* is constant in β; 2) when $1 < K < \frac{4}{3}$, Π_f^* achieves its strict maximum at $\beta = K-1$; and 3) when $\frac{4}{3} \leq K < 2$, Π_f^* achieves its maximum at any $\beta \in [K-1, K]$. This shows that an intermediate value of β is strictly optimal for the firm when K is intermediate, and the proof is complete. $\square$

Proof of Proposition 3

Based on the analysis of Section 4.2, we list the expressions for the consumer surplus.

If $K \leq 1$, then $CS_{total} = \frac{K^2}{4}$. If $1 < K \leq \frac{9}{8}$, then:

$$CS_{total} = \begin{cases} \frac{1}{4}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ 1 - \frac{(2-K)(2-K+2\beta)}{2-K+\beta}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{\beta^2}{(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \\ \frac{1}{4}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)}) < \beta \leq K \end{cases}.$$

If $\frac{9}{8} < K \leq \frac{4}{3}$, then:

$$CS_{total} = \begin{cases} \frac{1}{4}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ 1 - \frac{(2-K)(2-K+2\beta)}{2-K+\beta}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{\beta^2}{(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq \frac{2-K}{2} \\ \frac{1}{9}, & \text{if } \frac{2-K}{2} < \beta \leq K \end{cases}.$$

If $\frac{4}{3} < K \leq \frac{3}{2}$, then:

$$CS_{total} = \begin{cases} \frac{1}{4}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ 1 - \frac{(2-K)(2-K+2\beta)}{2-K+\beta}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ (K-1)^2, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{3}{2} < K \leq \frac{3+\sqrt{3}}{3}$, then:

$$CS_{total} = \begin{cases} \frac{1}{4}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ 1 - \frac{(2-K)(2-K+2\beta)}{2-K+\beta}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq 2-K \\ \frac{3K-2-3\beta}{4}, & \text{if } 2-K < \beta \leq K-1 \\ \frac{1}{4}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{3+\sqrt{3}}{3} < K \leq \frac{5}{3}$, then:

$$CS_{total} = \begin{cases} \frac{1}{4}, & \text{if } \beta \leq \frac{4-6K+3K^2}{3K} \\ \frac{3K-2-3\beta}{4}, & \text{if } \frac{4-6K+3K^2}{3K} < \beta \leq K-1 \\ \frac{1}{4}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{5}{3} < K \leq \frac{3+\sqrt{5}}{3}$, then:

$$CS_{total} = \begin{cases} \frac{1}{4}, & \text{if } \beta \leq \frac{4-6K+3K^2}{3K} \\ \frac{3K-2-3\beta}{4}, & \text{if } \frac{4-6K+3K^2}{3K} < \beta \leq 4-2K \\ \frac{-2K^3+K^2(8+3\beta)-4K(2+\beta)+\beta(4-4\beta-\beta^2)}{8\beta}, & \text{if } 4-2K < \beta < K-1 \\ \frac{5K-7}{8(K-1)}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

If $\frac{3+\sqrt{5}}{3} < K \leq 2$, then:

$$CS_{total} = \begin{cases} \frac{1}{4}, & \text{if } \beta \leq \frac{(K-1)^2+\sqrt{1-4K+2K^2}}{K} \\ \frac{-2K^3+K^2(8+3\beta)-4K(2+\beta)+\beta(4-4\beta-\beta^2)}{8\beta}, & \text{if } \frac{(K-1)^2+\sqrt{1-4K+2K^2}}{K} < \beta < K-1 \\ \frac{5K-7}{8(K-1)}, & \text{if } K-1 < \beta \leq K \end{cases}.$$

Note that in the region where consumer demand is positive in the first period (i.e., when $K > 1$ and $\Pi_f^* > \frac{1}{2}$), consumer surplus is continuous in β. Moreover, in this region, whenever $\frac{d\Pi_f^*}{d\beta} \neq 0$, we have $\frac{d\Pi_f^*}{d\beta} \cdot \frac{dCS_{total}}{d\beta} < 0$, and whenever $\frac{d\Pi_f^*}{d\beta} = 0$, we have $\frac{dCS_{total}}{d\beta} = 0$. Thus, in the region where consumer demand is positive in the first period, β affects consumer surplus in the opposite direction from how it affects firm profit.

Note that the intermediate value of β that strictly maximizes the firm's profit is $\beta = K - 1$ for $1 < K < \frac{4}{3}$. By comparison, we can observe that such β also minimizes consumer surplus.

When $1 < K < \frac{4}{3}$, we can list expressions for social welfare as follows.

If $1 < K \leq \frac{9}{8}$, then:

$$SW = \begin{cases} \frac{3}{4}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{-2+4K-K^2}{2}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{2K-K^2+2\beta}{2(2-K+\beta)}, & \text{if } K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \\ \frac{3}{4}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)}) < \beta \leq K \end{cases}.$$

If $\frac{9}{8} < K < \frac{4}{3}$, then:

$$SW = \begin{cases} \frac{3}{4}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{-2+4K-K^2}{2}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{2K-K^2+2\beta}{2(2-K+\beta)}, & \text{if } K-1 < \beta \leq \frac{2-K}{2} \\ \frac{K+1}{3}, & \text{if } \frac{2-K}{2} < \beta \leq K \end{cases}.$$

Here, if $1 < K \leq \frac{4-\sqrt{2}}{2}$, $SW|_{\beta=K-1} = \frac{-2+4K-K^2}{2} \leq SW|_{\beta=\beta'}$ for any $\beta' \neq K-1$, meaning that the intermediate value of β that strictly maximizes the firm's profit may also minimize the social welfare. If $\frac{4-\sqrt{2}}{2} < K < \frac{4}{3}$, on the other hand, $SW|_{\beta=K-1} = \frac{-2+4K-K^2}{2} > \frac{3}{4} = SW|_{\beta=0}$, meaning that the intermediate value of β that strictly maximizes the firm's profit sometimes increases social welfare relative to $\beta = 0$.

This completes the proof of Proposition 3. □

Proof of Proposition 4

First, we analyze the case with exogenous β. For $\beta \in (0,1)$,

$$\Pi_f^*|_{K=\beta+1} = \begin{cases} \frac{(1-\beta)(1+\beta)^2}{2}, & \text{if } \beta \leq \frac{1}{2} \\ \frac{3(\beta+1)}{8}, & \text{if } \frac{1}{2} < \beta \leq \frac{2}{3} \\ \frac{\beta+1}{4\beta}, & \text{if } \frac{2}{3} < \beta < 1 \end{cases}.$$

Here, $\Pi_f^*|_{K=\beta+1} > \max\{0, \frac{1}{2}\} = \max\{\Pi_f^*|_{K=0}, \Pi_f^*|_{K=2}\}$, meaning that an intermediate K is strictly optimal for the firm given an exogenous $\beta \in (0,1)$.

For $\beta \in [1, \frac{5}{3}]$, $\Pi_f^*|_{K=\frac{5}{3}} = \frac{5}{8} > \max\{0, \frac{1}{2}\} = \max\{\Pi_f^*|_{K=0}, \Pi_f^*|_{K=2}\}$, meaning that an intermediate K is strictly optimal for the firm given an exogenous $\beta \in [1, \frac{5}{3}]$.

For $\beta \in (\frac{5}{3}, 2)$, $\Pi_f^*|_{K=\beta} = \frac{\beta}{4(\beta-1)} > \max\{0, \frac{1}{2}\} = \max\{\Pi_f^*|_{K=0}, \Pi_f^*|_{K=2}\}$, meaning that an intermediate K is strictly optimal for the firm given an exogenous $\beta \in (\frac{5}{3}, 2)$.

Then we move to the case with endogenous β. If speculation is allowed and the optimal β is chosen (i.e., $\beta = K - 1$ for $1 < K < 2$ and any β otherwise), then the firm's equilibrium profit is

$$\Pi_f^*|_{\beta=\beta^*(K)} \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{(2-K)K^2}{2}, & \text{if } 1 < K \leq \frac{3}{2} \\ \frac{3K}{8}, & \text{if } \frac{3}{2} < K \leq \frac{5}{3} \\ \frac{K}{4(K-1)}, & \text{if } \frac{5}{3} < K < 2 \end{cases}, \tag{A6}$$

which achieves the strict maximum at $K = \frac{5}{3}$. This shows that an intermediate K is strictly optimal for the firm even for endogenous β.

If there is no speculation, the firm's equilibrium profit $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$ achieves the maximum at $K = 1 < \frac{5}{3}$. This shows that the presence of speculators can induce the firm to choose a larger capacity.

This completes the proof of Proposition 4. □

Proof of Proposition 5

Let $\beta = \frac{1}{4}$, then in equilibrium, the firm's profit is

$$\Pi_f^* = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } 1 < K \leq \widehat{K} \\ \frac{4(2-K)K(5-2K)}{(9-4K)^2}, & \text{if } \widehat{K} < K \leq \frac{5}{4} \\ \frac{(2-K)K(5-2K)}{9-4K}, & \text{if } \frac{5}{4} < K \leq \frac{3}{2} \\ \frac{1}{2}, & \text{if } K > \frac{3}{2} \end{cases},$$

consumer surplus is

$$CS_{total} = \begin{cases} \frac{K^2}{4}, & \text{if } K \leq 1 \\ \frac{1}{4}, & \text{if } 1 < K \leq \widehat{K} \\ \frac{1}{(9-4K)^2}, & \text{if } \widehat{K} < K \leq \frac{5}{4} \\ \frac{-11+14K-4K^2}{9-4K}, & \text{if } \frac{5}{4} < K \leq \frac{3}{2} \\ \frac{1}{4}, & \text{if } K > \frac{3}{2} \end{cases},$$

and social welfare is

$$SW = \begin{cases} \frac{(4-K)K}{4}, & \text{if } K \leq 1 \\ \frac{3}{4}, & \text{if } 1 < K \leq \widehat{K} \\ \frac{1+4K-2K^2}{9-4K}, & \text{if } \widehat{K} < K \leq \frac{5}{4} \\ \frac{-2+4K-K^2}{2}, & \text{if } \frac{5}{4} < K \leq \frac{3}{2} \\ \frac{3}{4}, & \text{if } K > \frac{3}{2} \end{cases},$$

where $\widehat{K} \approx 1.04$ is the smallest root of the equation $-81 + 152K - 88K^2 + 16K^3 = 0$.

Here, if $K < 1$ or $\widehat{K} < K < \frac{5}{4}$, we have $\frac{d\Pi_f^*}{dK} > 0$ and $\frac{dCS_{total}}{dK} > 0$. This shows that a larger capacity may simultaneously increase the firm's profit and consumer surplus given an exogenous value of β.

Let $\beta = \beta^*(K)$ (which equals $K-1$ for $1<K<2$ and any value otherwise), then in equilibrium, the firm's profit is given in Equation (A6), and consumer welfare is

$$CS_{total} = \begin{cases} \frac{K^2}{4}, & \text{if } K \leq 1 \\ (K-1)^2, & \text{if } 1 < K \leq \frac{3}{2} \\ \frac{1}{4}, & \text{if } \frac{3}{2} < K \leq \frac{5}{3} \\ \frac{5K-7}{8(K-1)}, & \text{if } \frac{5}{3} < K < 2 \end{cases}.$$

Here, if $K<1$ or $1<K<\frac{4}{3}$, we have $\frac{d\Pi_f^*}{dK}>0$ and $\frac{dCS_{total}}{dK}>0$. This shows that a larger capacity may simultaneously increase the firm's profit and consumer surplus given the optimal value of β.

This proves the first part of Proposition 5.

Next, turn to the effect of K on social welfare. While deriving the equilibrium, we obtain the expressions defining social welfare as a piecewise-differentiable function of K. We observe that:

(i) The first-order derivative of social welfare with respect to K within each differentiable piece is non-negative, and

(ii) Social welfare is continuous as a function of K both within the region where $D_{1c}=0$ and within the region where $D_{1c}>0$.

Therefore, social welfare weakly increases in K inside each of these regions ($D_{1c}=0$ and $D_{1c}>0$).

However, social welfare may be discontinuously decreasing when a change in K leads to the equilibrium switching from the region with $D_{1c}=0$ to the region with $D_{1c}>0$ or vice versa.

For example, for $\beta=\frac{1}{4}$, as K increases from a little below $\widehat{K}$ to a little above $\widehat{K}$ (i.e., as the equilibrium switches from the one with zero D_{1c} to the one with positive D_{1c}), there is a drop in social welfare. As K increases from a little below $\frac{3}{2}$ to a little above $\frac{3}{2}$ (i.e., as the equilibrium switches from the one with positive D_{1c} to the one with zero D_{1c}), there is another drop in social welfare.

This completes the proof of Proposition 5. $\square$

Details of the Analysis in Section 5.1

The firm's expected profit subject to the constraint $D_{1c}=0$ is the same as in the main model. In the following, similar to the analysis of the main model, we first derive the firm's optimal choice of price and the corresponding profit under the constraint $D_{1c}>0$. Then, we compare the firm's payoffs subject to the constraint of zero and positive D_{1c}. Finally, we show that the main results remain to hold.

When speculators compete with each other, an early consumer's expected utility of purchasing immediately and that of deciding on purchase in Period 2 are, respectively, $u_1 = Ev - p_f = \frac{1}{2} - p_f$, and

$$\begin{aligned} u_2 &= \int_{1-\frac{K_2}{M}}^{1} (v-p_f)^+ dv + \int_{0}^{1-\frac{K_2}{M}} (v-p_s^*)^+ dv \\ &= \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1} (v-p_f) dv + \int_{\frac{2-K}{2-D_{1c}}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}} (v-\frac{2-K}{2-D_{1c}}) dv \\ &= \frac{1}{2} - p_f - \frac{(2-K)^2 - 2(2-D_{1c})(2-K)p_f + 2(2-K-2p_f+D_{1c}\cdot p_f)D_s}{2(2-D_{1c})^2}. \end{aligned}$$

The consumer may purchase immediately only if $u_1 \geq u_2$ (i.e., if $p_f \leq \widetilde{w}_1(D_s, D_{1c}) = \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})(2-K+D_s)}$). Similar to the analysis of the main model, conditional on positive D_{1c}, the firm's objective of maximizing the expected profit is equivalent to choosing the maximal p_f satisfying $p_f \leq \widetilde{w}_1(D_s, D_{1c})$.

In addition, given $p_f \leq \widetilde{w}_1 < p_s^*$, an individual speculator expects positive profit if he acquires one unit of the product, and thus speculators will purchase as long as the upper limit β is not reached. In other words, $D_s^* = \beta$.

Furthermore, $\frac{\partial \widetilde{w}_1(D_s, D_{1c})}{\partial D_{1c}} \geq 0$, meaning that the higher the value of D_{1c}, the higher the price early consumers are willing to pay immediately. In the case of multiple equilibria, we again focus on the equilibrium yielding the firm's highest profit (i.e., $D_{1c}^* = \min\{1, K - D_s\} = \min\{1, K-\beta\}$).

Based on the above observations, conditional on positive D_{1c}, if $\beta \leq K-1$, it is optimal for the firm to set $p_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$. In turn, $D_s = \beta$, $D_{1c} = 1$, and $p_s = 2-K$. The firm's and individual speculator's profits are, respectively, $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}$ and $\Pi_{s,individual} = \frac{(2-K)^2}{2(2-K+\beta)}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{1}{2} - \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}$ and $CS_{total} = 1 - \frac{(2-K)(2-K+2\beta)}{(2-K+\beta)}$. The social welfare is $\frac{-K^2+4K-2}{2}$. If $\beta > K-1$, on the other hand, $p_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$. In turn, $D_s = \beta$, $D_{1c} = K-\beta$, and $p_s = \frac{2-K}{2-K+\beta}$. The firm's and individual speculator's profits are $\Pi_f = \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}$ and $\Pi_{s,individual} = \frac{(2-K)^2}{2(2-K+\beta)^2}$. The consumer surpluses are $CS_{early} = CS_{late} = \frac{\beta^2}{2(2-K+\beta)^2}$ and $CS_{total} = \frac{\beta^2}{(2-K+\beta)^2}$. The social welfare is $\frac{2K-K^2+2\beta}{4-2K+2\beta}$.

By comparing the above expected firm's profit subject to the constraint of positive D_{1c} with that subject to the constraint of zero D_{1c}, we can obtain the firm's (unconditional) equilibrium profit as $\Pi_f^* = \frac{(2-K)K}{2}$ if $K \leq 1$, and

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2}{-1+4K-2K^2} \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)}, & \text{if } \frac{(2-K)(K-1)^2}{-1+4K-2K^2} < \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) \\ \frac{1}{2}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)}) < \beta \leq K \end{cases}$$

if $1 < K \leq \frac{1+\sqrt{5}}{2}$, and $\Pi_f^* = \frac{1}{2}$ if $K > \frac{1+\sqrt{5}}{2}$. Here, it is easy to see that $\frac{d\Pi_f^*}{d\beta} \geq 0$ if $\beta < K-1$, and $\frac{d\Pi_f^*}{d\beta} \leq 0$ if $\beta > K-1$. This shows that the firm's equilibrium profit is first increasing and then decreasing in β.

Note that when $1 < K < \frac{1+\sqrt{5}}{2}$, Π_f^* achieves its strict maximum at $\beta = K-1$. This shows that an intermediate value of β is strictly optimal for the firm when K is intermediate even if speculators compete with each other. By comparing the range of K here (i.e., $1 < K < \frac{1+\sqrt{5}}{2}$) with that in the main model (i.e., $1 < K < \frac{4}{3}$), we know that when speculators compete with each other, it is more likely that an intermediate value of β is strictly optimal for the firm. In other words, the firm has more incentive to restrict speculation.

As for the effect of β on consumer surplus, note that conditional on positive D_{1c}, $\frac{d\Pi_f}{d\beta} \times \frac{dCS_{total}}{d\beta} < 0$, meaning that β influences consumer surplus in the opposite direction from firm profit, and the value of β that strictly maximizes firm's profit also strictly minimizes consumer surplus.

For $K = \frac{8}{5}$, in equilibrium, consumer surplus equals $\frac{9}{25}$ at $\beta = \frac{3}{5}$ and $\frac{1}{4} < \frac{9}{25}$ at $\beta = 0$, meaning that an intermediate level of speculation may lead to higher consumer surplus compared with that in the region where the equilibrium first-period consumer purchase is zero.

Last, we move to the firm's decision on capacity. If speculation is allowed and the optimal β is chosen (i.e., $\beta = K-1$ for $1 < K \leq \frac{1+\sqrt{5}}{2}$ and β equals any positive value otherwise), then the firm's equilibrium profit is $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{(2-K)K^2}{2}, & \text{if } 1 < K \leq \frac{1+\sqrt{5}}{2} \\ \frac{1}{2}, & \text{if } K > \frac{1+\sqrt{5}}{2} \end{cases}$, which achieves the strict maximum at $K = \frac{4}{3}$. On the other hand, if there is no speculation, the firm's equilibrium profit is $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$, which achieves the maximum at $K = 1$. To sum up, an intermediate value of K is optimal for the firm even with zero unit cost production. Moreover, when speculation is allowed, the firm will choose a larger K (i.e., $\frac{4}{3}$) than when speculation is not allowed (i.e., 1).

This completes the analysis of Section 5.1 and proves that the main results hold under the assumption of competing speculators. $\square$

Details of the Analysis in Section 5.2

Similar to the main model, we start with the decisions in Period 2 and then analyze the decisions in Period 1. After we obtain the equilibrium decisions, we prove that the main results are robust when speculators price discriminate and prove the additional properties discussed in the main text.

Decisions in Period 2

In Period 2, a consumer with valuation v receives $u_{2,f} = v - p_f$ if she purchases from the firm, and

$$u_{2,s} = \begin{cases} v - p_{sh}, & \text{if } v \geq 1-\delta \\ v - p_{sl}, & \text{if } v < 1-\delta \end{cases} \tag{A7}$$

if she purchases from the speculators, and $u_{2,0}=0$ if she does not purchase at all.

As for the speculators, clearly, speculators are only relevant if $M(1-p_f)>K_2$. If $K_2>\delta M$, speculators only face the low-valuation consumers and sell $Q_l=\min\{D_s, M(1-p_{sl})-K_2\}$ units to them at the price p_{sl}. If $K_2\leq\delta M$, speculators face $\delta M-K_2$ high valuation consumers and all low-valuation consumers. In this case, speculators sell $Q_h=\min\{D_s, M(1-p_{sh})-K_2, \delta M-K_2\}$ to the high-valuation consumers at the price p_{sh} and $Q_l=\min\{D_s-Q_h, M(1-\delta-p_{sl})\}$ to the low-valuation consumers at the price p_{sl}. To sum up, we can write the speculators' second-period profit as:

$$\pi_{2s}=\begin{cases} p_{sh}\times\min\{D_s, M(1-p_{sh})-K_2, \delta M-K_2\} & \\ \qquad +p_{sl}\times\min\{D_s-\min\{D_s, M(1-p_{sh})-K_2, \delta M-K_2\}, M(1-\delta-p_{sl})\}, & \text{if } K_2\leq\delta M \\ p_{sl}\times\min\{D_s, M(1-p_{sl})-K_2\}, & \text{if } K_2>\delta M \end{cases}. \tag{A8}$$

Let:

$$\begin{aligned} A&=\frac{D_s}{M},\\ B&=\delta-\frac{K_2}{M},\\ C&=1-\frac{K_2}{M},\\ D&=1-\delta(=C-B),\\ \text{and } E&=\min\{A,B\}. \end{aligned}$$

Then the speculators' objective is equivalent to maximizing:

$$\begin{aligned} &\begin{cases} p_{sh}\times\min\{E, C-p_{sh}\}+p_{sl}\times\min\{A-\min\{E, C-p_{sh}\}, D-p_{sl}\}, & \text{if } B\geq 0\\ p_{sl}\times\min\{A, C-p_{sl}\}, & \text{if } B<0 \end{cases}\\ =&\begin{cases} p_{sh}\times E+p_{sl}\times\min\{A-E, D-p_{sl}\}, & \text{if } B\geq 0 \text{ and } p_{sh}\leq C-E, \text{ i.e., Case (i)a}\\ p_{sh}\times(C-p_{sh})+p_{sl}\times\min\{A-C+p_{sh}, D-p_{sl}\}, & \text{if } B\geq 0 \text{ and } p_{sh}>C-E, \text{ i.e., Case (i)b}\\ p_{sl}\times\min\{A, C-p_{sl}\}, & \text{if } B<0 \text{ i.e., Case (ii)} \end{cases}. \end{aligned} \tag{A9}$$

Note that $x\times\min\{a, b-x\}$ is maximized at $x=\max\{b-a, \frac{b}{2}\}$ for any $a\geq 0$. Therefore, for Case (i)a in Equation (A9), the (local) maxima points are $p_{sh}=C-E$ and $p_{sl}=\max\{D-A+E, \frac{D}{2}\}$. Here, further note that when $A\leq B$, speculators are indifferent in their choice of p_{sl}, as no unit is sold at that price. Thus, in this case, it is optimal to set $p_{sh}=C-A$ if $A\leq B$, and $\begin{cases} p_{sh}=C-B=D\\ p_{sl}=\max\{D-A+B, \frac{D}{2}\}=\max\{C-A, \frac{D}{2}\} \end{cases}$ if $A>B$.

In the Case (i)b, it is optimal to set $p_{sl} = \max\{D - A + C - p_{sh}, \frac{D}{2}\}$, which leads to $p_{sl} = D - A + C - p_{sh}$ if $p_{sh} \leq \frac{D}{2} - A + C$ and $p_{sl} = \frac{D}{2}$ if $p_{sh} > \frac{D}{2} - A + C$. That is, we can rewrite speculators' objective function in this case as:

$$
\begin{aligned}
&\begin{cases} p_{sh} \times (C - p_{sh}) + (D - A + C - p_{sh}) \times (A - C + p_{sh}), & \text{if } p_{sh} \leq \frac{D}{2} - A + C, \\ p_{sh} \times (C - p_{sh}) + \frac{D^2}{4}, & \text{if } p_{sh} > \frac{D}{2} - A + C, \end{cases} \\
= &\begin{cases} 2p_{sh}(\frac{-2A+3C+D}{2} - p_{sh}) - (A - C)(A - C - D), & \text{if } p_{sh} \leq \frac{D}{2} - A + C, \\ p_{sh} \times (C - p_{sh}) + \frac{D^2}{4}, & \text{if } p_{sh} > \frac{D}{2} - A + C. \end{cases}
\end{aligned}
$$

Note that $\frac{-2A+3C+D}{4} \leq \frac{D}{2} - A + C \iff A \leq \frac{C+D}{2} \iff \frac{C}{2} \leq \frac{-2A+3C+D}{4} (\leq \frac{D}{2} - A + C)$ and $\frac{-2A+3C+D}{4} > \frac{D}{2} - A + C \iff A > \frac{C+D}{2} \iff \frac{C}{2} > \frac{-2A+3C+D}{4} (> \frac{D}{2} - A + C)$. Therefore, the (unconstrained) maxima points of Case (i)b are $\begin{cases} p_{sh} = \frac{-2A+3C+D}{4} \\ p_{sl} = \frac{-2A+C+3D}{4} \end{cases}$ if $A \leq \frac{C+D}{2}$, and $\begin{cases} p_{sh} = \frac{C}{2} \\ p_{sl} = \frac{D}{2} \end{cases}$ if $A > \frac{C+D}{2}$.

In the Case (ii), the maxima point is $p_{sl} = \max\{C - A, \frac{C}{2}\}$.

To sum up, if $K_2 \leq \delta M$, then it is optimal to set $p_{sh} = \max\{\frac{2-K}{2-D_{1c}}, 1 - \delta, \frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4}, \frac{2-K+D_s}{2(2-D_{1c})}\}$, and set $p_{sl} = \max\{\frac{2-K}{2-D_{1c}}, \frac{1-\delta}{2}\}$ if $p_{sh} = 1 - \delta$, $p_{sl} = \frac{2-D_s-K}{4(2-D_{1c})} + \frac{3(1-\delta)}{4}$ if $p_{sh} = \frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4}$, and $p_{sl} = \frac{1-\delta}{2}$ if $p_{sh} = \frac{2-K+D_s}{2(2-D_{1c})}$. If $K_2 > \delta M$, on the other hand, it is optimal to set $p_{sl} = \max\{\frac{2-K}{2-D_{1c}}, \frac{2-K+D_s}{2(2-D_{1c})}\}$.

Decisions in Period 1

This analysis contains four steps. In the first step, we derive the firm's optimal choice of price constrained by $D_{1c} = 0$. In the second step, we derive the expressions of consumer willingness to pay in Period 1, $\widetilde{w}_1(D_s, D_{1c})$, which is obtained from the condition $u_1 \geq u_2$, and show two of its properties. In Step 3, we derive the firm's optimal choice of price constrained by $D_{1c} > 0$. Finally, we compare the firm's expected payoffs in the above two cases and obtain the equilibrium.

Step 1: Derive the firm's optimal choice of price constrained by $D_{1c} = 0$.

Note that when no early consumer purchases in the first period, then $D_{1c}=0$, $K_2=K-D_s$, and $M=2$. Thus:

$$
\begin{aligned}
\Pi_s(p_f,D_s) &= \begin{cases}
\frac{2-K}{2}\times D_s - p_f\times D_s, & \text{if } D_s \le \max\{2\delta-K, \min\{K-2\delta, 2-K\}\},\\
(1-\delta)(2\delta-K+D_s)+\frac{2-K}{2}(K-2\delta)-p_f\times D_s, & \\
& \text{if } K\le 1+\delta \text{ and } K-2\delta < D_s \le 3K-6\delta,\\
(1-\delta)(2\delta-K+D_s)+\frac{1-\delta}{2}\times 2(1-\delta-\frac{1-\delta}{2})-p_f\times D_s, & \\
& \text{if } K> 1+\delta \text{ and } K-2\delta < D_s \le 2+K-4\delta,\\
(\frac{6+D_s-3K}{8}+\frac{1-\delta}{4})(2-K+D_s-2(\frac{6+D_s-3K}{8}+\frac{1-\delta}{4})) & \\
\quad +(\frac{2-D_s-K}{8}+\frac{3(1-\delta)}{4})2(1-\delta-(\frac{2-D_s-K}{8}+\frac{3(1-\delta)}{4}))-p_f\times D_s, & \\
& \text{if } \max\{2\delta-K, 3K-6\delta\} < D_s \le 4-K-2\delta,\\
\frac{2-K+D_s}{4}(2-K+D_s-\frac{2-K+D_s}{2})+\frac{1-\delta}{2}\times 2(1-\delta-\frac{1-\delta}{2})-p_f\times D_s, & \\
& \text{if } \delta>\frac{1}{2} \text{ and } D_s > \max\{4-K-2\delta, 2+K-4\delta\},\\
\frac{2-K+D_s}{4}(2-K+D_s-\frac{2-K+D_s}{2})-p_f\times D_s, & \\
& \text{if } 2-K < D_s \le K-2\delta.
\end{cases}\\
&= \begin{cases}
(\frac{2-K}{2}-p_f)\times D_s, & \text{if } D_s \le \max\{2\delta-K, \min\{K-2\delta, 2-K\}\},\\
(1-\delta-p_f)D_s-\frac{(K-2\delta)^2}{2}, & \text{if } K\le 1+\delta \text{ and } K-2\delta < D_s \le 3K-6\delta,\\
(1-\delta-p_f)D_s+\frac{(1-\delta)^2}{2}-(1-\delta)(K-2\delta), & \\
& \text{if } K> 1+\delta \text{ and } K-2\delta < D_s \le 2+K-4\delta,\\
\frac{(D_s-2\delta+8-3K-8p_f)^2}{16}-\frac{1}{2}(2-K-2p_f)(4-K-2\delta-4p_f), & \\
& \text{if } \max\{2\delta-K, 3K-6\delta\} < D_s \le 4-K-2\delta,\\
\frac{(D_s+2-K-4p_f)^2}{8}+\frac{(1-\delta)^2}{2}+(2-K-2p_f)p_f, & \\
& \text{if } \delta>\frac{1}{2} \text{ and } D_s > \max\{4-K-2\delta, 2+K-4\delta\},\\
\frac{(D_s+2-K-4p_f)^2}{8}+(2-K-2p_f)p_f, & \text{if } 2-K < D_s \le K-2\delta.
\end{cases}
\end{aligned}
$$

Note that $\Pi_s(p_f,D_s)$ is convex in D_s. Thus:

$$
D_s^* = \begin{cases}
0, & \text{if } \Pi_s(p_f,0) > \Pi_s(p_f,\beta)\\
\beta, & \text{if } \Pi_s(p_f,\beta) \ge \Pi_s(p_f,0)
\end{cases}.
$$

(Note that if $\Pi_s(p_f,\beta)=\Pi_s(p_f,0)$, speculators are indifferent between $D_s=0$ and $D_s=\beta$. Similar to the main model, whichever choice speculators make in equilibrium when they are indifferent, the payoffs are equivalent. Thus, for ease of illustration, we assume that speculators choose $D_s=\beta$, which is weakly preferable by the firm.)

This corresponds to the firm's profit:

$$
\Pi_f = \begin{cases}
p_f\times \min\{K, 2(1-p_f)\}, & \text{if } \Pi_s(p_f,0) > \Pi_s(p_f,\beta),\\
p_f\times K, & \text{if } \Pi_s(p_f,\beta) \ge \Pi_s(p_f,0).
\end{cases}
$$

Here, when $\Pi_s(p_f,\beta)\ge \Pi_s(p_f,0)$, Π_f is maximized at $p_f=\frac{\pi_{2s}(\beta)}{\beta}$, where π_{2s} (Equation (A8)) is the speculators' second-period profit, corresponding to $\Pi_f|_{D_{1c}=0\ \&\ D_s>0}=\frac{\pi_{2s}(\beta)}{\beta}\times K$; whereas when $\Pi_s(p_f,\beta) < \Pi_s(p_f,0)$, Π_f is maximized at $p_f = \min\{\frac{\pi_{2s}(\beta)}{\beta} - \epsilon, \max\{\frac{2-K}{2}, \frac{1}{2}\}\}$, corresponding to $\Pi_f|_{D_{1c}=0 \& D_s=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \le 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$.

To sum up, conditional on zero first-period consumer demand, the firm's profit is $\Pi_f|_{D_{1c}=0} = \max\{\Pi_f|_{D_{1c}=0 \& D_s>0}, \Pi_f|_{D_{1c}=0 \ \& \ D_s=0}\}$, where $\Pi_f|_{D_{1c}=0 \& D_s>0} = \frac{\pi_{2s}(\beta)}{\beta}K$, $\Pi_f|_{D_{1c}=0 \ \& \ D_s=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \le 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$,

$$\frac{\pi_{2s}(\beta)}{\beta} = \begin{cases} \frac{2-K}{2}, & \text{if } \beta \le \max\{2\delta - K, \min\{K - 2\delta, 2 - K\}\}, \\ \frac{(1-\delta)(2\delta-K+\beta) + \frac{2-K}{2}(K-2\delta)}{\beta}, & \text{if } K \le 1+\delta \text{ and } K - 2\delta < \beta \le 3K - 6\delta, \\ \frac{(1-\delta)(2\delta-K+\beta) + \frac{(1-\delta)^2}{2}}{\beta}, & \text{if } K > 1+\delta \text{ and } K - 2\delta < \beta \le 2 + K - 4\delta, \\ (\frac{8-2\delta+\beta-3K}{8} \times \frac{2\delta+3\beta-K}{4} + \frac{8-6\delta-\beta-K}{8} \times \frac{-2\delta+\beta+K}{4})\frac{1}{\beta}, & \\ & \text{if } \max\{2\delta - K, 3K - 6\delta\} < \beta \le 4 - K - 2\delta, \\ \frac{(2-K+\beta)^2 + 4(1-\delta)^2}{8\beta}, & \text{if } \delta > \frac{1}{2} \text{ and } \beta > \max\{4 - K - 2\delta, 2 + K - 4\delta\}, \\ \frac{(2-K+\beta)^2}{8\beta}, & \text{if } 2 - K < \beta \le K - 2\delta, \end{cases} \tag{A10}$$

and

$$\frac{\pi_{2s}(\beta)}{\beta}K = \begin{cases} \frac{(2-K)K}{2}, & \text{if } \beta \le \max\{2\delta - K, \min\{K - 2\delta, 2 - K\}\}, \\ \frac{(1-\delta)(2\delta-K+\beta) + \frac{2-K}{2}(K-2\delta)}{\beta}K, & \text{if } K \le 1+\delta \text{ and } K - 2\delta < \beta \le 3K - 6\delta, \\ \frac{(1-\delta)(2\delta-K+\beta) + \frac{(1-\delta)^2}{2}}{\beta}K, & \text{if } K > 1+\delta \text{ and } K - 2\delta < \beta \le 2 + K - 4\delta, \\ (\frac{8-2\delta+\beta-3K}{8} \times \frac{2\delta+3\beta-K}{4} + \frac{8-6\delta-\beta-K}{8} \times \frac{-2\delta+\beta+K}{4})\frac{K}{\beta}, & \\ & \text{if } \max\{2\delta - K, 3K - 6\delta\} < \beta \le 4 - K - 2\delta, \\ \frac{(2-K+\beta)^2 + 4(1-\delta)^2}{8\beta}K, & \text{if } \delta > \frac{1}{2} \text{ and } \beta > \max\{4 - K - 2\delta, 2 + K - 4\delta\}, \\ \frac{(2-K+\beta)^2}{8\beta}K, & \text{if } 2 - K < \beta \le K - 2\delta. \end{cases}$$

The speculators' profit is $\Pi_s = 0$.

Step 2: Derive $\widetilde{w}_1(D_s, D_{1c})$ and show two of its properties

The early consumer compares $u_1 = Ev - p_f = \frac{1}{2} - p_f$ and

$$u_2 = \int_{\max\{1-\delta, 1-\frac{K_2}{M}\}}^{1} (v - p_f)^+ \, dv + \int_{1-\delta}^{\max\{1-\delta, 1-\frac{K_2}{M}\}} (v - p_{sh}^*)^+ \, dv$$
$$+ \int_{\min\{1-\delta, 1-\frac{K_2}{M}\}}^{1-\delta} (v - p_f)^+ \, dv + \int_{0}^{\min\{1-\delta, 1-\frac{K_2}{M}\}} (v - p_{sl}^*)^+ \, dv,$$

and may purchase immediately only if $u_1 \ge u_2$ (i.e., $p_f \le \widetilde{w}_1(D_s, D_{1c})$). Given the conditions that the parameters satisfy (i.e., the parameter regions), which correspond to different pairs of p_{sh} and p_{sl}, we derive $\widetilde{w}_1(D_s, D_{1c})$ and show the following two properties.

First, $\widetilde{w}_1(D_s, D_{1c}) \le p_{sh}$ if $K - D_s - D_{1c} \le M \cdot \delta$, and $\widetilde{w}_1(D_s, D_{1c}) \le p_{sl}$ if $K - D_s - D_{1c} > M \cdot \delta$. Thus, as long as they expect some early consumers to purchase immediately, speculators enter the market ($D_s \geq 0$). At the same time, it means that the firm can sell all its capacity and generate $\Pi_f = p_f \times K$ (i.e., maximizing the expected profit is equivalent to maximizing p_f).

Second, $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} \geq 0$. Thus, the larger the value of D_{1c}, the higher price an early consumer is willing to pay immediately.

We proceed by examining the following seven regions which together span all the parameter space.

Condition I: $\begin{cases} K_2 \leq \delta M \\ \max\{\frac{2-K}{2-D_{1c}}, 1-\delta, \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}, \frac{2-K+D_s}{2(2-D_{1c})}\} = \frac{2-K}{2-D_{1c}} \end{cases}$. In this case, $p^*_{sh} = \frac{2-K}{2-D_{1c}}$. Thus, $u_2 = \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1}(v-p_f)dv + \int_{\frac{2-K}{2-D_{1c}}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}(v-\frac{2-K}{2-D_{1c}})dv = \frac{1}{2} - p_f - \frac{(2-K)^2-2(2-D_{1c})(2-K)p_f+2(2-K-2p_f+D_{1c}\cdot p_f)D_s}{2(2-D_{1c})^2}$. Note that $u_1 = \frac{1}{2} - p_f \geq u_2$ leads to $p_f \leq \widetilde{w}_1(D_s,D_{1c}) = \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})(2-K+D_s)}$. Clearly, $\widetilde{w}_1(D_s,D_{1c}) = \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})(2-K+D_s)} \leq \frac{2-K}{2-D_{1c}}$, and $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} = \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})^2(2-K+D_s)} \geq 0$.

Condition II: $\begin{cases} K_2 \leq \delta M \\ \max\{\frac{2-K}{2-D_{1c}}, 1-\delta, \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}, \frac{2-K+D_s}{2(2-D_{1c})}\} = 1-\delta \\ \frac{2-K}{2-D_{1c}} \geq \frac{1-\delta}{2} \end{cases}$. In this case, $p_{sh} = 1-\delta$ and $p_{sl} = \frac{2-K}{2-D_{1c}}$. Thus, $u_2 = \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1}(v-p_f)dv + \int_{1-\delta}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}(v-(1-\delta))dv + \int_{\frac{2-K}{2-D_{1c}}}^{1-\delta}(v-\frac{2-K}{2-D_{1c}})dv$. Note that $u_1 = \frac{1}{2} - p_f \geq u_2$ leads to $p_f \leq \widetilde{w}_1(D_s,D_{1c}) = \frac{(2-K)(2-K+2D_s)-2(K-D_s-D_{1c}-(2-D_{1c})\delta)(K-D_{1c}-(2-D_{1c})\delta)}{2(2-D_{1c})(2-K+D_s)}$. Here, $\widetilde{w}_1(D_s,D_{1c}) \leq 1-\delta$ and $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} = \frac{2(2-D_{1c})^2(1-\delta)^2-(2-K)^2}{2(2-D_{1c})^2(2-K+D_s)} \geq 0$.

Condition III: $\begin{cases} K_2 \leq \delta M \\ \max\{\frac{2-K}{2-D_{1c}}, 1-\delta, \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}, \frac{2-K+D_s}{2(2-D_{1c})}\} = 1-\delta \\ \frac{2-K}{2-D_{1c}} < \frac{1-\delta}{2} \end{cases}$. In this case, $p_{sh} = 1-\delta$ and $p_{sl} = \frac{1-\delta}{2}$. Thus, $u_2 = \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1}(v-p_f)dv + \int_{1-\delta}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}(v-(1-\delta))dv + \int_{\frac{1-\delta}{2}}^{1-\delta}(v-\frac{1-\delta}{2})dv$. Note that $u_1 = \frac{1}{2} - p_f \geq u_2$ leads to $p_f \leq \widetilde{w}_1(D_s,D_{1c}) = \frac{(1-\delta)(8(2-K+D_s)-5(1-\delta)(2-D_{1c}))}{8(2-K+D_s)}$. Here, $\widetilde{w}_1(D_s,D_{1c}) \leq 1-\delta$ and $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} = \frac{5(1-\delta)^2}{8(2-K+D_s)} \geq 0$.

Condition IV: $\begin{cases} K_2 \leq \delta M \\ \max\{\frac{2-K}{2-D_{1c}}, 1-\delta, \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}, \frac{2-K+D_s}{2(2-D_{1c})}\} = \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4} \end{cases}$. In this case, $p_{sh} = \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}$ and $p_{sl} = \frac{2-D_s-K}{4(2-D_{1c})}+\frac{3(1-\delta)}{4}$. Thus, $u_2 = \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1}(v-p_f)dv + \int_{\frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}(v-(\frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}))dv + \int_{\frac{2-D_s-K}{4(2-D_{1c})}+\frac{3(1-\delta)}{4}}^{1-\delta}(v-(\frac{2-D_s-K}{4(2-D_{1c})}+\frac{3(1-\delta)}{4}))dv$. Note that $u_1 = \frac{1}{2} - p_f \geq u_2$ leads to $p_f \leq \widetilde{w}_1(D_s,D_{1c}) = \frac{4(4-2K+D_s)^2-8(2-K)^2-(K-D_s-D_{1c}-\delta(2-D_{1c}))^2}{16(2-D_{1c})(2-K+D_s)}$. Here, $\widetilde{w}_1(D_s,D_{1c}) \leq \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}$ and $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} = \frac{(1-\delta)^2(2-D_{1c})^2+7(2-K+D_s)^2-4D_s^2}{16(2-D_{1c})^2(2-K+D_s)} \geq 0$.

Condition V: $\begin{cases} K_2 \leq \delta M \\ \max\{\frac{2-K}{2-D_{1c}}, 1-\delta, \frac{6+D_s-3K}{4(2-D_{1c})}+\frac{1-\delta}{4}, \frac{2-K+D_s}{2(2-D_{1c})}\} = \frac{2-K+D_s}{2(2-D_{1c})} \end{cases}$. In this case, $p_{sh} = \frac{2-K+D_s}{2(2-D_{1c})}$ and $p_{sl} = \frac{1-\delta}{2}$. Thus, $u_2 = \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1}(v-p_f)dv + \int_{\frac{2-K+D_s}{2(2-D_{1c})}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}(v-\frac{2-K+D_s}{2(2-D_{1c})})dv + \int_{\frac{1-\delta}{2}}^{1-\delta}(v-

$\frac{1-\delta}{2})dv$. Note that $u_1 = \frac{1}{2} - p_f \geq u_2$ leads to $p_f \leq \widetilde{w}_1(D_s, D_{1c}) = \frac{3(2-K+D_s)^2-(1-\delta)^2(2-D_{1c})^2}{8(2-D_{1c})(2-K+D_s)}$. Here, $\widetilde{w}_1(D_s, D_{1c}) \leq \frac{2-K+D_s}{2(2-D_{1c})}$ and $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} = \frac{3(2-K+D_s)^2+(1-\delta)^2(2-D_{1c})^2}{8(2-D_{1c})^2(2-K+D_s)} \geq 0$.

Condition VI: $\begin{cases} K_2 > \delta M \\ \max\{\frac{2-K}{2-D_{1c}}, \frac{2-K+D_s}{2(2-D_{1c})}\} = \frac{2-K}{2-D_{1c}} \end{cases}$. In this case, $p_{sl} = \frac{2-K}{2-D_{1c}}$. Thus, $u_2 = \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1} (v-p_f)dv + \int_{\frac{2-K}{2-D_{1c}}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}} (v - \frac{2-K}{2-D_{1c}})dv$. Note that $u_1 = \frac{1}{2} - p_f \geq u_2$ leads to $p_f \leq \widetilde{w}_1(D_s, D_{1c}) = \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})(2-K+D_s)}$. Here, $\widetilde{w}_1(D_s, D_{1c}) \leq \frac{2-K}{2-D_{1c}}$ and $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} = \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})^2(2-K+D_s)} \geq 0$.

Condition VII: $\begin{cases} K_2 > \delta M \\ \max\{\frac{2-K}{2-D_{1c}}, \frac{2-K+D_s}{2(2-D_{1c})}\} = \frac{2-K+D_s}{2(2-D_{1c})} \end{cases}$. In this case, $p_{sl} = \frac{2-K+D_s}{2(2-D_{1c})}$. Thus, $u_2 = \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1} (v-p_f)dv + \int_{\frac{2-K+D_s}{2(2-D_{1c})}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}} (v - \frac{2-K+D_s}{2(2-D_{1c})})dv$. Note that $u_1 = \frac{1}{2} - p_f \geq u_2$ leads to $p_f \leq \widetilde{w}_1(D_s, D_{1c}) = \frac{3(2-K+D_s)}{8(2-D_{1c})}$. Here, $\widetilde{w}_1(D_s, D_{1c}) \leq \frac{2-K+D_s}{2(2-D_{1c})}$ and $\frac{\partial \widetilde{w}_1(D_s,D_{1c})}{\partial D_{1c}} = \frac{3(2-K+D_s)}{8(2-D_{1c})^2} \geq 0$.

To sum up the above derivations, we know that conditional on positive D_{1c}, the firm aims to maximize p_f. Moreover, in the equilibrium yielding the firm's highest profit, $D_{1c}^* = \max\{1, K - D_s\}$.

Step 3: Derive the equilibrium decisions subject to the constraint that $D_{1c} > 0$.

As discussed above, we focus on the equilibrium with $D_{1c}^* = \max\{1, K - D_s\}$.

If $\beta \leq K-1$, then $D_s \leq K-1$ and $D_{1c} = 1$. Speculators choose D_s to maximize

$$\Pi_s(p_f, D_s) = \begin{cases} (2-K)\times D_s - p_f \times D_s, & \text{if Condition I or VI is satisfied,} \\ (1-\delta)(\delta-(K-D_s-1)) + (2-K)(K-1-\delta) - p_f \times D_s, & \\ & \text{if Condition II is satisfied,} \\ (1-\delta)(\delta-(K-D_s-1)) + \frac{1-\delta}{2}(1-\delta-\frac{1-\delta}{2}) - p_f \times D_s, & \\ & \text{if Condition III is satisfied,} \\ (\frac{6+D_s-3K}{4}+\frac{1-\delta}{4})(2-K+D_s-(\frac{6+D_s-3K}{4}+\frac{1-\delta}{4})) & \\ \quad +(\frac{2-D_s-K}{4}+\frac{3(1-\delta)}{4})(1-\delta-(\frac{2-D_s-K}{4}+\frac{3(1-\delta)}{4})) - p_f \times D_s, & \\ & \text{if Condition IV is satisfied,} \\ \frac{2-K+D_s}{2}(2-K+D_s-\frac{2-K+D_s}{2}) + \frac{1-\delta}{2}(1-\delta-\frac{1-\delta}{2}) - p_f \times D_s, & \\ & \text{if Condition V is satisfied,} \\ \frac{2-K+D_s}{2}(2-K+D_s-\frac{2-K+D_s}{2}) - p_f \times D_s, & \text{if Condition VII is satisfied,} \end{cases}$$

$$= \begin{cases} (2-K-p_f)\times D_s, & \text{if } D_s \leq \max\{\min\{K-1, 1+\delta-K\}, \min\{K-1-\delta, 2-K\}\}, \\ (1-\delta-p_f)D_s - (K-1-\delta)^2, & \\ & \text{if } K \leq \frac{3+\delta}{2} \text{ and } K-1-\delta < D_s \leq \min\{K-1, 3(K-1-\delta)\}, \\ (1-\delta-p_f)D_s + \frac{(1-\delta)^2}{4} - (1-\delta)(K-1-\delta), & \\ & \text{if } K > \frac{3+\delta}{2} \text{ and } K-1-\delta < D_s \leq \min\{K-1, K-2\delta\}, \\ \frac{(D_s-\delta+7-3K-4p_f)^2}{8} - (2-K-p_f)(3-K-\delta-2p_f), & \\ & \text{if } \max\{1+\delta-K, 3(K-1-\delta)\} < D_s \leq \min\{K-1, 3-K-\delta\}, \\ \frac{(D_s+2-K-2p_f)^2}{4} + \frac{(1-\delta)^2}{4} + (2-K-p_f)p_f, & \\ & \text{if } \max\{K-2\delta, 3-K-\delta\} < D_s \leq K-1, \\ \frac{(D_s+2-K-2p_f)^2}{4} + (2-K-p_f)p_f, & \\ & \text{if } 2-K < D_s \leq K-1-\delta. \end{cases}$$

under the constraint of $D_s \le \beta$.

Note that Π_s is piece-wise differentiable and convex in D_s. Moreover, by checking its one-sided derivatives around the points where two subintervals touch (and note that Π_s is continuous in D_s at each of these points, and the right derivative is larger than the left derivative), we further conclude that $\Pi_s(p_f, D_s)$ is convex in D_s as long as $D_s \le K-1$. Therefore, the speculators' maximum profit is achieved at either $D_s = 0$ or $D_s = \beta$. To ensure positive first-period demand, the firm sets p_f such that $\Pi_s(p_f, \beta) \ge \Pi_s(p_f, 0)$, i.e.,

$$p_f \le \dot{w}_1 = \begin{cases} 2-K, & \text{if } \beta \le \max\{\min\{K-1, 1+\delta-K\}, \min\{K-1-\delta, 2-K\}\}, \\ 1-\delta-\frac{(K-1-\delta)^2}{\beta}, & \text{if } K \le \frac{3+\delta}{2} \text{ and } K-1-\delta < \beta \le \min\{K-1, 3(K-1-\delta)\}, \\ \frac{(1-\delta)(5+4\beta+3\delta-4K)}{4\beta}, & \text{if } K > \frac{3+\delta}{2} \text{ and } K-1-\delta < \beta \le \min\{K-1, K-2\delta\}, \\ \frac{(K-1-\delta)^2+\beta(14-2\delta-6K+\beta)}{8\beta}, & \text{if } \max\{1+\delta-K, 3(K-1-\delta)\} < \beta \le \min\{K-1, 3-K-\delta\}, \\ \frac{(2-K)(2-K+2\beta)+(1-\delta)^2+\beta^2}{4\beta}, & \text{if } \max\{K-2\delta, 3-K-\delta\} < \beta \le K-1, \\ \frac{(2-K+\beta)^2}{4\beta}, & \text{if } 2-K < \beta \le K-1-\delta, \end{cases}$$

where $\dot{w}_1$ is speculators' willingness to pay in Period 1, and $u_1 \ge u_2$, i.e.,

$$p_f \le \widetilde{w}_1(\beta, 1) = \begin{cases} \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}, & \text{if } \beta \le \max\{\min\{K-1, 1+\delta-K\}, \min\{K-1-\delta, 2-K\}\}, \\ \frac{2(1-\delta)(\beta+1-\delta)-(K-2\delta)^2}{2(2-K+\beta)}, & \text{if } K \le \frac{3+\delta}{2} \text{ and } K-1-\delta < \beta \le \min\{K-1, 3(K-1-\delta)\}, \\ \frac{(1-\delta)(11+8\beta+5\delta-8K)}{8(2-K+\beta)}, & \text{if } K > \frac{3+\delta}{2} \text{ and } K-1-\delta < \beta \le \min\{K-1, K-2\delta\}, \\ \frac{7(2-K+\beta)^2-2(1-\delta)(K-1-\beta)+1-\delta^2-4\beta^2}{16(2-K+\beta)}, & \\ & \text{if } \max\{1+\delta-K, 3(K-1-\delta)\} < \beta \le \min\{K-1, 3-K-\delta\}, \\ \frac{3(2-K+\beta)^2-(1-\delta)^2}{8(2-K+\beta)}, & \text{if } \max\{K-2\delta, 3-K-\delta\} < \beta \le K-1, \\ \frac{3(2-K+\beta)}{8}, & \text{if } 2-K < \beta \le K-1-\delta. \end{cases}$$

To sum up, if $\beta \le K-1$, in anticipation of speculators' and consumers' decisions, the firm sets

$$\begin{aligned} p_f = \widehat{p_f} &= \min\{\dot{w}_1, \widetilde{w}_1(\beta, 1)\} \\ &= \begin{cases} \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)}, & \text{if } \beta \le \max\{\min\{K-1, 1+\delta-K\}, \min\{K-1-\delta, 2-K\}\}, \\ \frac{2(1-\delta)(\beta+1-\delta)-(K-2\delta)^2}{2(2-K+\beta)}, & \text{if } K \le \frac{3+\delta}{2} \text{ and } K-1-\delta < \beta \le \min\{K-1, 3(K-1-\delta)\}, \\ \frac{(1-\delta)(5+4\beta+3\delta-4K)}{4\beta}, & \text{if } \frac{5+\delta}{3} < K \le \frac{15+\delta}{8} \text{ and } K-1-\delta < \beta \le \min\{\frac{2(2-K)(4K-5-3\delta)}{15-8K+\delta}, K-1, K-2\delta\}, \\ & \text{or if } K > \frac{15+\delta}{8} \text{ and } K-1-\delta < \beta \le \min\{K-1, K-2\delta\}, \\ \frac{(1-\delta)(11+8\beta+5\delta-8K)}{8(2-K+\beta)}, & \text{if } \frac{3+\delta}{2} < K \le \frac{15+\delta}{8} \text{ and} \\ & \qquad \max\{K-1-\delta, \frac{2(2-K)(4K-5-3\delta)}{15-8K+\delta}\} < \beta \le \min\{K-1, K-2\delta\}, \\ \frac{7(2-K+\beta)^2-2(1-\delta)(K-1-\beta)+1-\delta^2-4\beta^2}{16(2-K+\beta)}, & \\ & \text{if } \max\{1+\delta-K, 3(K-1-\delta)\} < \beta \le \min\{K-1, 3-K-\delta\}, \\ \frac{3(2-K+\beta)^2-(1-\delta)^2}{8(2-K+\beta)}, & \text{if } \max\{K-2\delta, 3-K-\delta\} < \beta \le \min\{\beta'(K,\delta), K-1\}, \\ \frac{(2-K)(2-K+2\beta)+(1-\delta)^2+\beta^2}{4\beta}, & \\ & \text{if } \max\{K-2\delta, \beta'(K,\delta)\} < \beta \le K-1, \\ \frac{3(2-K+\beta)}{8}, & \text{if } 2-K < \beta \le \min\{4-2K, K-1-\delta\}, \\ \frac{(2-K+\beta)^2}{4\beta}, & \text{if } 4-2K < \beta \le K-1-\delta, \end{cases} \end{aligned}$$

where $\beta'(K,\delta)$ is the third root of the equation $\beta^3+\beta(-3\delta^2+6\delta-3K^2+12K-15)+2\delta^2K-4\delta^2-4\delta K+8\delta+2K^3-12K^2+26K-20=0$. The firm's equilibrium profit is $\Pi_f=\widehat{p_f}\times K$, whereas the speculators' equilibrium profit is $\Pi_s(\widehat{p_f},\beta)$.

If $\beta>K-1$, then given the discussion above, either $D_s=0$, or $K-1\le D_s\le\beta$ and $D_{1c}=K-D_s$. In the former case, $\Pi_s(p_f,D_s)=0$, whereas in the latter case, speculators choose D_s to maximize

$$\Pi_s(p_f,D_s)=\begin{cases}\frac{2-K}{2-K+D_s}\times D_s-p_f\times D_s, & \text{if Condition I is satisfied,}\\ (1-\delta)(2-K+D_s)\delta+\frac{2-K}{2-K+D_s}(2-K+D_s)(1-\delta-\frac{2-K}{2-K+D_s})-p_f\times D_s, & \\ & \text{if Condition II is satisfied,}\\ (1-\delta)(2-K+D_s)\delta+\frac{1-\delta}{2}(2-K+D_s)(1-\delta-\frac{1-\delta}{2})-p_f\times D_s, & \\ & \text{if Condition III is satisfied,}\\ \frac{8-4K+2D_s-\delta(2-K+D_s)}{4(2-K+D_s)}(2-K+D_s)(1-\frac{8-4K+2D_s-\delta(2-K+D_s)}{4(2-K+D_s)}) & \\ \quad+(\frac{4-2K+D_s}{4-2K+2D_s}-\frac{3\delta}{4})(2-K+D_s)(1-\delta-(\frac{4-2K+D_s}{4-2K+2D_s}-\frac{3\delta}{4}))-p_f\times D_s, & \\ & \text{if Condition IV is satisfied,}\\ \frac{1}{2}(2-K+D_s)(1-\frac{1}{2})+\frac{1-\delta}{2}(2-K+D_s)(1-\delta-\frac{1-\delta}{2})-p_f\times D_s, & \\ & \text{if Condition V is satisfied,}\end{cases}$$

$$=\begin{cases}(\frac{2-K}{2-K+D_s}-p_f)\times D_s, & \text{if } K-1\le D_s<\frac{\delta(2-K)}{2-\delta},\\ \frac{(2-K)D_s}{2-K+D_s}+\delta(1-\delta)D_s-p_fD_s-\delta^2(2-K), & \\ & \text{if } \delta\le\frac{1}{2} \text{ and } \max\{K-1,\frac{3\delta(2-K)}{2-3\delta}\}\le D_s<\frac{(1+\delta)(2-K)}{1-\delta},\\ (\frac{(1-\delta)(1+3\delta)}{4}-p_f)D_s+\frac{(1-\delta)(1+3\delta)(2-K)}{4}, & \\ & \text{if } \delta\le\frac{1}{2} \text{ and } D_s\ge\max\{K-1,\frac{(1+\delta)(2-K)}{1-\delta}\},\\ \frac{((2-\delta)^2-8p_f)D_s^2+2(2-K)(4-2\delta+\delta^2-4p_f)D_s+(2-K)^2\delta}{8(2-K+D_s)}, & \\ & \text{if } \max\{K-1,\frac{\delta(2-K)}{2-\delta}\}\le D_s<\min\{\frac{3\delta(2-K)}{2-3\delta},\frac{(2-\delta)(2-K)}{\delta}\},\\ (\frac{2-2\delta+\delta^2}{4}-p_f)D_s+\frac{(2-2\delta+\delta^2)(2-K)}{4}, & \\ & \text{if } \delta>\frac{1}{2} \text{ and } D_s\ge\max\{K-1,\frac{(2-\delta)(2-K)}{\delta}\},\end{cases}$$

under the constraint of $D_s\le\beta$.

Here, it is easy to show that if $K-1\le D_s<\frac{\delta(2-K)}{2-\delta}$, then $\frac{\partial\Pi_s(p_f,D_s)}{\partial D_s}<0$ only if $\frac{(2-\delta)^2}{4}<p_f\le(2-K)^2$ and $(2-K)(\frac{1}{\sqrt{p_f}}-1)<D_s<\frac{\delta(2-K)}{2-\delta}$, or $p_f>(2-K)^2$. If $\delta\le\frac{1}{2}$ and $\max\{K-1,\frac{3\delta(2-K)}{2-3\delta}\}\le D_s<\frac{(1+\delta)(2-K)}{1-\delta}$, then $\frac{\partial\Pi_s(p_f,D_s)}{\partial D_s}<0$ only if $\frac{(1+3\delta)(1-\delta)}{4}<p_f\le\min\{\frac{4-8\delta+5\delta^2}{4},(2-K)^2+\delta(1-\delta)\}$ and $(2-K)(\frac{1}{\sqrt{p_f-\delta(1-\delta)}}-1)<D_s<\frac{(1+\delta)(2-K)}{1-\delta}$, or $p_f>\min\{\frac{4-8\delta+5\delta^2}{4},(2-K)^2+\delta(1-\delta)\}$. If $\delta\le\frac{1}{2}$ and $D_s\ge\max\{K-1,\frac{(1+\delta)(2-K)}{1-\delta}\}$, then $\frac{\partial\Pi_s(p_f,D_s)}{\partial D_s}<0$ only if $p_f>\frac{(1-\delta)(1+3\delta)}{4}$. If $\max\{K-1,\frac{\delta(2-K)}{2-\delta}\}\le D_s<\min\{\frac{3\delta(2-K)}{2-3\delta},\frac{(2-\delta)(2-K)}{\delta}\}$, then $\frac{\partial\Pi_s(p_f,D_s)}{\partial D_s}<0$ only if $\max\{\frac{4-8\delta+5\delta^2}{4},\frac{2-2\delta+\delta^2}{4}\}<p_f\le\min\{\frac{(2-\delta)^2}{4},\frac{(2-\delta)^2+4(2-K)^2}{8}\}$ and $(2-K)(\frac{2}{\sqrt{8p_f-(2-\delta)^2}}-1)\le D_s<\min\{\frac{3\delta(2-K)}{2-3\delta},\frac{(2-\delta)(2-K)}{\delta}\}$, or $p_f>\min\{\frac{(2-\delta)^2}{4},\frac{(2-\delta)^2+4(2-K)^2}{8}\}$. If $\delta>\frac{1}{2}$ and $D_s\ge\max\{K-1,\frac{(2-\delta)(2-K)}{\delta}\}$, then $\frac{\partial\Pi_s(p_f,D_s)}{\partial D_s}<0$ only if $p_f>\frac{2-2\delta+\delta^2}{4}$.

Similar to the $D_s\le K-1$ case, we can write the upper bound of p_f in the $D_s\ge K-1$ case as

$$\widehat{p_f} = \min\{\dot{w}_1, \widetilde{w}_1(D_s, K-D_s)\}$$
$$= \begin{cases} \frac{(2-K)(2-K+2D_s)}{2(2-K+D_s)^2}, & \text{if } K-1 \le D_s < \frac{\delta(2-K)}{2-\delta}, \\ \frac{(2-K)(2-K+2D_s)+2\delta[(1-\delta)D_s-\delta(2-K)](2-K+D_s)}{2(2-K+D_s)^2}, & \\ & \text{if } \delta \le \frac{1}{2} \text{ and } \max\{K-1, \frac{3\delta(2-K)}{2-3\delta}\} \le D_s < \frac{(1+\delta)(2-K)}{1-\delta}, \\ \frac{(1-\delta)(3+5\delta)}{8}, & \text{if } \delta \le \frac{1}{2} \text{ and } \max\{K-1, \frac{(1+\delta)(2-K)}{1-\delta}\} \le D_s < \frac{2(2-K)(1+3\delta)}{1-\delta}, \\ \frac{(1-\delta)(1+3\delta)(2-K+D_s)}{4D_s}, & \text{if } \delta \le \frac{1}{2} \text{ and } D_s \ge \max\{K-1, \frac{2(2-K)(1+3\delta)}{1-\delta}\}, \\ \frac{2(2-K)(2-K+2D_s)+D_s^2}{4(2-K+D_s)^2} - \frac{\delta^2}{16}, & \text{if } \max\{K-1, \frac{\delta(2-K)}{2-\delta}\} \le D_s < \min\{\frac{3\delta(2-K)}{2-3\delta}, \frac{(2-\delta)(2-K)}{\delta}\}, \\ \frac{2+2\delta-\delta^2}{8}, & \text{if } \delta > \frac{1}{2} \text{ and } \max\{K-1, \frac{(2-\delta)(2-K)}{\delta}\} \le D_s < \frac{2(2-K)(2-2\delta+\delta^2)}{-2+6\delta-3\delta^2}, \\ \frac{(2-2\delta+\delta^2)(2-K+D_s)}{4D_s}, & \text{if } \delta > \frac{1}{2} \text{ and } D_s \ge \max\{K-1, \frac{2(2-K)(2-2\delta+\delta^2)}{-2+6\delta-3\delta^2}\}. \end{cases}$$

We show the details of how we obtain the firm's optimal choice of price constrained by $D_{1c} > 0$ in the case of $0 \le \delta \le \frac{1}{2}$. If $0 \le \delta \le \frac{1}{2}$, we can rewrite speculators' expected profit in the $D_s \ge K-1$ case as

$$\Pi_s(p_f, D_s) = \begin{cases} (\frac{2-K}{2-K+D_s} - p_f) \times D_s, & \text{if } K-1 \le D_s < \frac{\delta(2-K)}{2-\delta}, \\ \frac{((2-\delta)^2-8p_f)D_s^2+2(2-K)(4-2\delta+\delta^2-4p_f)D_s+(2-K)^2\delta}{8(2-K+D_s)}, & \text{if } \max\{K-1, \frac{\delta(2-K)}{2-\delta}\} \le D_s < \frac{3\delta(2-K)}{2-3\delta}, \\ \frac{(2-K)D_s}{2-K+D_s} + \delta(1-\delta)D_s - p_f D_s - \delta^2(2-K), & \text{if } \max\{K-1, \frac{3\delta(2-K)}{2-3\delta}\} \le D_s < \frac{(1+\delta)(2-K)}{1-\delta}, \\ (\frac{(1-\delta)(1+3\delta)}{4} - p_f)D_s + \frac{(1-\delta)(1+3\delta)(2-K)}{4}, & \text{if } D_s \ge \max\{K-1, \frac{(1+\delta)(2-K)}{1-\delta}\}. \end{cases}$$

1. If $K-1 \le \beta < \frac{\delta(2-K)}{2-\delta}$, i.e., $\begin{cases} K < \frac{2+\delta}{2} \\ K-1 \le \beta < \frac{\delta(2-K)}{2-\delta} \end{cases}$, then $p_f \le \widetilde{w}_1 < \frac{(2-\delta)^2}{4}$ leads to $D_s = \beta$. In anticipation of speculators' and consumers' decisions, the firm sets $p_f = \widehat{p_f} = \widetilde{w}_1(\beta, K-\beta) = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$. Moreover, $p_{sh} = \frac{2-K}{2-K+\beta}$. The firm's equilibrium profit is $\Pi_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)^2\beta}{2(2-K+\beta)^2}$.

2. If $\max\{K-1, \frac{\delta(2-K)}{2-\delta}\} \le \beta < \frac{3\delta(2-K)}{2-3\delta}$, then $D_s = \min\{\beta, \max\{K-1, (2-K)(\frac{2}{\sqrt{8p_f-(2-\delta)^2}} - 1)\}\}$. In anticipation of speculators' and consumers' decisions, if $\begin{cases} 0 \le \delta \le \frac{2}{5} \\ K \le \frac{2+3\delta}{2} \\ \max\{K-1, \frac{\delta(2-K)}{2-\delta}\} \le \beta < \frac{3\delta(2-K)}{2-3\delta} \end{cases}$ or $\begin{cases} \frac{2}{5} < \delta \le \frac{1}{2} \\ K \le \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} \\ \max\{K-1, \frac{\delta(2-K)}{2-\delta}\} \le \beta < \frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)} \end{cases}$, the firm sets $p_f = \frac{2(2-K)(2-K+\beta)+\beta^2}{4(2-K+\beta)^2} - \frac{\delta^2}{16}$. In turn, $D_s = \beta$, $D_{1c} = K - \beta$, $p_{sh} = \frac{8-4K+2\beta-\delta(2-K+\beta)}{4(2-K+\beta)}$, and $p_{sl} = \frac{8-4K+2\beta-3\delta(2-K+\beta)}{4(2-K+\beta)}$. The firm's equilibrium profit is $\Pi_f = (\frac{2(2-K)(2-K+\beta)+\beta^2}{4(2-K+\beta)^2} - \frac{\delta^2}{16})K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{1}{16}(\delta^2(4-2K+3\beta) - 8\delta\beta + \frac{4\beta(8+2K^2-(2K-\beta)(4+\beta))}{(2-K+\beta)^2})$. If $\begin{cases} \frac{2}{5} < \delta \le \frac{1}{2} \\ K \le \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} \\ \frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)} \le \beta < \frac{3\delta(2-K)}{2-3\delta} \end{cases}$, the firm sets $p_f = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}$. In turn, $D_s =$

$\frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $D_{1c} = \frac{2[2K-8\delta+3\delta^2+2(2-K)\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $p_{sh} = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$, and $p_{sl} = \frac{8-9\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$. The firm's equilibrium profit is $\frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)[1+3\delta-\sqrt{-8+24\delta-9\delta^2}]}{9}$. If $\begin{cases} \frac{2}{5} < \delta \leq \frac{1}{2} \\ \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} < K \leq \frac{2+3\delta}{2} \\ K-1 \leq \beta < \frac{3\delta(2-K)}{2-3\delta}\} \end{cases}$, the firm sets $p_f = \frac{4-\delta^2+4K(2-K)}{16}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = \frac{6-\delta-2K}{4}$, $p_{sl} = \frac{6-3\delta-2K}{4}$. The firm's equilibrium profit is $\Pi_f = \frac{4-\delta^2+4K(2-K)}{16}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{4(1-K)(2\delta-5+4K-K^2)+\delta^2(1+K)}{16}$.

3. If $\max\{K-1, \frac{3\delta(2-K)}{2-3\delta}\} \leq \beta < \frac{(1+\delta)(2-K)}{1-\delta}$, then $D_s = \min\{\beta, \max\{K-1, (2-K)(\frac{2}{\sqrt{8p_f-(2-\delta)^2}} - 1), (2-K)(\frac{1}{\sqrt{p_f-\delta(1-\delta)}} - 1)\}\}$. In anticipation of speculators' and consumers' decisions, if $\begin{cases} 0 \leq \delta \leq \frac{2}{5} \\ K \leq \frac{2+3\delta}{2} \\ \frac{3\delta(2-K)}{2-3\delta} \leq \beta < \frac{(1+2\delta)(2-K)}{2(1-\delta)} \end{cases}$ or $\begin{cases} 0 \leq \delta \leq \frac{2}{5} \\ \frac{2+3\delta}{2} < K \leq \frac{4+2\delta}{3} \\ K-1 \leq \beta < \frac{(1+2\delta)(2-K)}{2(1-\delta)} \end{cases}$, the firm sets $p_f = \frac{(2-K)(2-K+2\beta)+2\delta[(1-\delta)\beta-\delta(2-K)](2-K+\beta)}{2(2-K+\beta)^2}$. In turn, $D_s = \beta$, $D_{1c} = K-\beta$, $p_{sh} = 1-\delta$, and $p_{sl} = \frac{2-K}{2-K+\beta}$. The firm's equilibrium profit is $\Pi_f = \frac{(2-K)(2-K+2\beta)+2\delta[(1-\delta)\beta-\delta(2-K)](2-K+\beta)}{2(2-K+\beta)^2}k$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)^2\beta+2\delta[(1-\delta)\beta-\delta(2-K)](2-K)(2-K+\beta)}{2(2-K+\beta)^2}$. If $\begin{cases} 0 \leq \delta \leq \frac{2}{5} \\ K \leq \frac{4+2\delta}{3} \\ \frac{(1+2\delta)(2-K)}{2(1-\delta)} \leq \beta < \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{(1-\delta)(4+5\delta)}{9}$. In turn, $D_s = \frac{(1+2\delta)(2-K)}{2(1-\delta)}$, $D_{1c} = \frac{3K-2-4\delta}{2(1-\delta)}$, $p_{sh} = 1-\delta$, and $p_{sl} = \frac{2(1-\delta)}{3}$. The firm's equilibrium profit is $\Pi_f = \frac{(1-\delta)(4+5\delta)}{9}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(1-\delta)(1+5\delta)(2-K)}{9}$. If $\begin{cases} \frac{2}{5} < \delta \leq \frac{1}{2} \\ K \leq \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} \\ \frac{3\delta(2-K)}{2-3\delta} \leq \beta < \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}$. In turn, $D_s = \frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $D_{1c} = \frac{2[2K-8\delta+3\delta^2+2(2-K)\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $p_{sh} = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$, and $p_{sl} = \frac{8-9\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$. The firm's equilibrium profit is $\frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)[1+3\delta-\sqrt{-8+24\delta-9\delta^2}]}{9}$. If $\begin{cases} 0 \leq \delta \leq \frac{2}{5} \\ \frac{4+2\delta}{3} < K \leq \frac{3+\delta}{2} \\ K-1 \leq \beta < \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$ or $\begin{cases} \frac{2}{5} < \delta \leq \frac{1}{2} \\ \frac{2+3\delta}{2} < K \leq \frac{3+\delta}{2} \\ K-1 \leq \beta < \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{(2-K)K}{2} + \delta(K-1-\delta)$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = 1-\delta$, and $p_{sl} = 2-K$. The firm's equilibrium profit is $\Pi_f = \frac{(2-K)K^2}{2} + \delta(K-1-\delta)K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)(-2-2\delta^2+2\delta(K-1)+3k-K^2)}{2}$. If $\begin{cases} \frac{2}{5} < \delta \leq \frac{1}{2} \\ \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} < K \leq \frac{2+3\delta}{2} \\ \frac{3\delta(2-K)}{2-3\delta} \leq \beta < \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{4-\delta^2+4K(2-K)}{16}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = \frac{6-\delta-2K}{4}$, $p_{sl} = \frac{6-3\delta-2K}{4}$. The firm's equilibrium profit is $\Pi_f = \frac{4-\delta^2+4K(2-K)}{16}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{4(1-K)(2\delta-5+4K-K^2)+\delta^2(1+K)}{16}$.

4. If $\beta \geq \max\{K-1, \frac{(1+\delta)(2-K)}{1-\delta}\}$, then similar to the analysis above, if $\begin{cases} 0 \leq \delta \leq \frac{2}{5} \\ K \leq \frac{4+2\delta}{3} \\ \beta \geq \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{(1-\delta)(4+5\delta)}{9}$. In turn, $D_s = \frac{(1+2\delta)(2-K)}{2(1-\delta)}$, $D_{1c} = \frac{3K-2-4\delta}{2(1-\delta)}$, $p_{sh} = 1-\delta$, and $p_{sl} = \frac{2(1-\delta)}{3}$. The firm's equilibrium profit is $\Pi_f = \frac{(1-\delta)(4+5\delta)}{9}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(1-\delta)(1+5\delta)(2-K)}{9}$. If $\begin{cases} \frac{2}{5} < \delta \leq \frac{1}{2} \\ K \leq \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} \\ \beta \geq \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}$. In turn, $D_s = \frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $D_{1c} = \frac{2[2K-8\delta+3\delta^2+2(2-K)\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $p_{sh} = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$, and $p_{sl} = \frac{8-9\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$. The firm's equilibrium profit is $\frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)[1+3\delta-\sqrt{-8+24\delta-9\delta^2}]}{9}$. If $\begin{cases} 0 \leq \delta \leq \frac{2}{5} \\ \frac{4+2\delta}{3} < K \leq \frac{3+\delta}{2} \\ \beta \geq \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$ or $\begin{cases} \frac{2}{5} < \delta \leq \frac{1}{2} \\ \frac{2+3\delta}{2} < K \leq \frac{3+\delta}{2} \\ \beta \geq \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{(2-K)K}{2} + \delta(K-1-\delta)$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = 1-\delta$, and $p_{sl} = 2-K$. The firm's equilibrium profit is $\Pi_f = \frac{(2-K)K^2}{2} + \delta(K-1-\delta)K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)(-2-2\delta^2+2\delta(K-1)+3k-K^2)}{2}$. If $\begin{cases} \frac{2}{5} < \delta \leq \frac{1}{2} \\ \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} < K \leq \frac{2+3\delta}{2} \\ \beta \geq \frac{(1+\delta)(2-K)}{1-\delta} \end{cases}$, the firm sets $p_f = \frac{4-\delta^2+4K(2-K)}{16}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = \frac{6-\delta-2K}{4}$, $p_{sl} = \frac{6-3\delta-2K}{4}$. The firm's equilibrium profit is $\Pi_f = \frac{4-\delta^2+4K(2-K)}{16}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{4(1-K)(2\delta-5+4K-K^2)+\delta^2(1+K)}{16}$. If $\begin{cases} \frac{3+\delta}{2} < K \leq \frac{5+11\delta}{3+5\delta} \\ \beta \geq K-1 \end{cases}$, the firm sets $p_f = \frac{(1-\delta)(3+5\delta)}{8}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = 1-\delta$, $p_{sl} = \frac{1-\delta}{2}$. The firm's equilibrium profit is $\Pi_f = \frac{(1-\delta)(3+5\delta)}{8}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(1-\delta)(5-3K+\delta(11-5K))}{8}$. If $\begin{cases} K > \frac{5+11\delta}{3+5\delta} \\ \beta \geq K-1 \end{cases}$, the firm sets $p_f = \frac{(1-\delta)(1+3\delta)}{4(K-1)}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = 1-\delta$, $p_{sl} = \frac{1-\delta}{2}$. The firm's equilibrium profit is $\Pi_f = \frac{(1-\delta)(1+3\delta)}{4(K-1)}K$, whereas the speculators' equilibrium profit is $\Pi_s = 0$.

Similarly, we can obtain the equilibrium decisions in the case of $\frac{1}{2} < \delta \leq 1$ and $\beta > K-1$ as follows:

1. If $\begin{cases} \frac{1}{2} < \delta \leq \frac{2}{3} \\ K \leq \frac{2+\delta}{2} \\ K-1 \leq \beta < \frac{\delta(2-K)}{2-\delta} \end{cases}$ or $\begin{cases} \frac{2}{3} < \delta \leq 1 \\ K \leq \frac{4}{3} \\ K-1 \leq \beta < \frac{2-K}{2} \end{cases}$, the firm sets $p_f = \widehat{p_f} = \widetilde{w}_1(\beta, K-\beta) = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}$. In turn, $D_s = \beta$, $D_{1c} = K-\beta$, $p_{sh} = \frac{2-K}{2-K+\beta}$. The firm's equilibrium profit is $\Pi_f = \frac{(2-K)(2-K+2\beta)}{2(2-K+\beta)^2}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)^2\beta}{2(2-K+\beta)^2}$.

2. If $\begin{cases} \frac{1}{2} < \delta \leq \frac{2}{3} \\ K \leq \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} \\ \max\{K-1, \frac{\delta(2-K)}{2-\delta}\} \leq \beta < \frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)} \end{cases}$, the firm sets $p_f = \frac{2(2-K)(2-K+\beta)+\beta^2}{4(2-K+\beta)^2} - \frac{\delta^2}{16}$. In turn, $D_s = \beta$, $D_{1c} = K-\beta$, $p_{sh} = \frac{8-4K+2\beta-\delta(2-K+\beta)}{4(2-K+\beta)}$, and $p_{sl} =$

$\frac{8-4K+2\beta-3\delta(2-K+\beta)}{4(2-K+\beta)}$. The firm's equilibrium profit is $\Pi_f = (\frac{2(2-K)(2-K+\beta)+\beta^2}{4(2-K+\beta)^2} - \frac{\delta^2}{16})K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{1}{16}(\delta^2(4-2K+3\beta) - 8\delta\beta + \frac{4\beta(8+2K^2-(2K-\beta)(4+\beta))}{(2-K+\beta)^2})$.

3. If $\begin{cases} \frac{1}{2} < \delta \leq \frac{2}{3} \\ K \leq \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} \\ \beta \geq \frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)} \end{cases}$, the firm sets $p_f = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}$. In turn, $D_s = \frac{(2-K)[\delta(8-3\delta)-2\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $D_{1c} = \frac{2[2K-8\delta+3\delta^2+2(2-K)\sqrt{-8+24\delta-9\delta^2}]}{(2-\delta)(2-3\delta)}$, $p_{sh} = \frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$, and $p_{sl} = \frac{8-9\delta+\sqrt{-8+24\delta-9\delta^2}}{12}$. The firm's equilibrium profit is $\frac{8-3\delta+\sqrt{-8+24\delta-9\delta^2}}{18}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(2-K)[1+3\delta-\sqrt{-8+24\delta-9\delta^2}]}{9}$.

4. If $\begin{cases} \frac{2}{3} < \delta \leq 1 \\ K \leq \frac{4}{3} \\ \beta \geq \frac{2-K}{2} \end{cases}$, the firm sets $p_f = \frac{4}{9}$. In turn, $D_s = \frac{2-K}{2}$, $D_{1c} = \frac{3K-2}{2}$, $p_{sh} = \frac{2}{3}$. The firm's equilibrium profit is $\Pi_f = \frac{4}{9}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{2-K}{9}$.

5. If $\begin{cases} \frac{2}{3} < \delta \leq 1 \\ \frac{4}{3} < K \leq \frac{2+\delta}{2} \\ \beta \geq K-1 \end{cases}$, the firm sets $p_f = \frac{(2-K)K}{2}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = 2-K$. The firm's equilibrium profit is $\Pi_f = \frac{(2-K)K^2}{2}$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{(K-1)(2-K)^2}{2}$.

6. If $\begin{cases} \frac{1}{2} < \delta \leq \frac{2}{3} \\ \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6} < K \leq \frac{4-\delta}{2} \\ \beta \geq K-1 \end{cases}$ or $\begin{cases} \frac{2}{3} < \delta \leq 1 \\ \frac{2+\delta}{2} < K \leq \frac{4-\delta}{2} \\ \beta \geq K-1 \end{cases}$, the firm sets $p_f = \frac{4-\delta^2+4K(2-K)}{16}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = \frac{6-\delta-2K}{4}$, $p_{sl} = \frac{6-3\delta-2K}{4}$. The firm's equilibrium profit is $\Pi_f = \frac{4-\delta^2+4K(2-K)}{16}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{4(1-K)(2\delta-5+4K-K^2)+\delta^2(1+K)}{16}$.

7. If $\begin{cases} \frac{1}{2} < \delta \leq 1 \\ \frac{4-\delta}{2} < K \leq \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2} \\ \beta \geq K-1 \end{cases}$, the firm sets $p_f = \frac{2+2\delta-\delta^2}{8}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = \frac{1}{2}$, and $p_{sl} = \frac{1-\delta}{2}$. The firm's equilibrium profit is $\Pi_f = \frac{2+2\delta-\delta^2}{8}K$, whereas the speculators' equilibrium profit is $\Pi_s = \frac{6-2K-\delta(2-\delta)(K+1)}{8}$.

8. If $\begin{cases} \frac{1}{2} < \delta \leq 1 \\ K > \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2} \\ \beta \geq K-1 \end{cases}$, the firm sets $p_f = \frac{2-2\delta+\delta^2}{4(K-1)}$. In turn, $D_s = K-1$, $D_{1c} = 1$, $p_{sh} = \frac{1}{2}$, and $p_{sl} = \frac{1-\delta}{2}$. The firm's equilibrium profit is $\Pi_f = \frac{2-2\delta+\delta^2}{4(K-1)}K$, whereas the speculators' equilibrium profit is $\Pi_s = 0$.

Step 4: Compare the firm's expected payoffs derived in Step 2 and 3, and obtain the equilibrium. This step is straightforward and it completes the detailed analysis of the decisions in Period 1. Figure 6 directly follows from this step.

Robustness of the Main Results and the Derivation of the Additional Findings Reported in Section 5.2

First, we prove that the region of β and K for positive first-period sales (weakly) increases when $\delta \in (0,1)$ compared to the case when $\delta = 0$.

Note that $\Pi_f|_{D_{1c}=0 \text{ \& } D_s=0}$ is constant in δ, whereas both $\Pi_f|_{D_{1c}>0}$ and $\Pi_f|_{D_{1c}=0 \text{ \& } D_s>0}$ are (weakly) larger when $\delta \in (0,1)$ than when $\delta = 0$. Thus, the region of positive sales in Period 1 is larger when $\delta \in (0,1)$ compared to the case when $\delta = 0$.

Second, we prove that an intermediate β can be strictly optimal for the firm and obtain the region of K for the strict optimality of an intermediate β to hold.

Denote the firm's expected profits subject to the constraint of $D_{1c} = 0$ and $D_{1c} > 0$ by, respectively, $\Pi_f|_{D_{1c}=0}$ and $\Pi_f|_{D_{1c}>0}$, of which the expressions are given in Step 1 and 3 in the detailed analysis of decisions in Period 1. Thus, the firm's equilibrium profit is $\Pi_f^* = \max\{\Pi_f|_{D_{1c}>0}, \Pi_f|_{D_{1c}=0}\} = \max\{\Pi_f|_{D_{1c}>0}, \Pi_f|_{D_{1c}=0\&D_s>0}, \Pi_f|_{D_{1c}=0\&D_s=0}\}$. Here, $\frac{\partial \Pi_f|_{D_{1c}>0}}{\partial \beta} \geq 0$ if $\beta < K-1$, $\frac{\partial \Pi_f|_{D_{1c}>0}}{\partial \beta} \leq 0$ if $\beta > K-1$, $\frac{\partial \Pi_f|_{D_{1c}=0\&D_s>0}}{\partial \beta} \geq 0$, and $\frac{\partial \Pi_f|_{D_{1c}=0\&D_s=0}}{\partial \beta} = 0$. Therefore, the firm's equilibrium profit is first (weakly) increasing, then (weakly) decreasing, and then (weakly) increasing in β.

To obtain the region of K where an intermediate value of β (i.e., $\beta = K-1$) is strictly optimal, it is equivalent to solve when:

$$\begin{cases} \frac{\partial \Pi_f|_{D_{1c}>0}}{\partial \beta}|_{\beta \to (K-1)^+} < 0 \\ \Pi_f|_{D_{1c}>0, \beta=K-1} > \Pi_f|_{D_{1c}=0\&D_s=0, \beta=K-1} = \frac{1}{2} \\ \Pi_f|_{D_{1c}>0, \beta=K-1} > \Pi_f|_{D_{1c}=0\&D_s>0, \beta=K} \end{cases}.$$

This leads to $\delta < \frac{1}{4}$ or $\delta > \frac{18}{37}$, and $\underline{K} < K < \overline{K}$. Here,

$$\overline{K} = \begin{cases} \frac{4+2\delta}{3}, & \text{if } 0 \leq \delta \leq \frac{1}{4}, \\ \frac{5}{3} - \frac{\sqrt{-8+24\delta-9\delta^2}}{6}, & \text{if } \frac{18}{37} < \delta \leq \frac{2}{3}, \\ \frac{4}{3}, & \text{if } \frac{2}{3} < \delta \leq 1, \end{cases}$$

whereas $\underline{K}$ is the second real root of $1 + 2\delta - 3\delta^2 + (2\delta + 2\delta^2)K - (2+2\delta)K^2 + K^3 = 0$ if $0 \leq \delta < \frac{1}{4}$, the second real root of $4\delta^2 + (12 - 8\delta + \delta^2)K - 12K^2 + 4K^3 = 0$ if $\frac{18}{37} < \delta \leq \frac{1}{2}(-3+\sqrt{17})$, and the second real root of $\delta^2 + (4-2\delta)K - 5K^2 + 2K^3 = 0$ if $\frac{1}{2}(-3+\sqrt{17}) < \delta \leq 1$.

This shows that an intermediate value of β is strictly optimal for the firm when K is intermediate.

Moreover, compared to the case with $\delta = 0$, when $\delta > 0$, we observe that both $\overline{K}$ and $\underline{K}$ increase, but $\overline{K} - \underline{K}$ decreases. Therefore, the speculators' ability to price discriminate shifts up, but shrinks, the range of the firm's capacity under which an intermediate value of β is strictly optimal.

Third, we prove that an intermediate K can be optimal for the firm.

If there is speculative reselling, note that for $\beta \in (0,1)$, we have $\beta + 1 \in (1,2)$, and

$$\Pi_f|_{D_{1c}>0, K=\beta+1} = \begin{cases} \frac{(1-\beta)(1+\beta)^2}{2}, & \text{if } \beta \leq \frac{\delta}{2} \\ \frac{(8-4\beta^2-\delta^2)(1+\beta)}{16}, & \text{if } 0 < \delta \leq \frac{1}{2} \text{ and } \frac{\delta}{2} < \beta \leq \frac{3\delta}{2}, \\ & \text{or if } \frac{1}{2} < \delta \leq 1 \text{ and } \frac{\delta}{2} < \beta \leq \frac{2-\delta}{2} \\ \frac{(1-\beta^2+2\beta\delta-2\delta^2)(1+\beta)}{2}, & \text{if } 0 \leq \delta \leq \frac{1}{2} \text{ and } \frac{3\delta}{2} < \beta \leq \frac{1+\delta}{2} \\ \frac{(1-\delta)(3+5\delta)(1+\beta)}{8}, & \text{if } 0 \leq \delta \leq \frac{1}{2} \text{ and } \frac{1+\delta}{2} < \beta \leq \frac{2+6\delta}{3+5\delta} \\ \frac{(1-\delta)(1+3\delta)(1+\beta)}{4\beta}, & \text{if } 0 \leq \delta \leq \frac{1}{2} \text{ and } \frac{2+6\delta}{3+5\delta} < \beta \leq 1 \\ \frac{(2+2\delta-\delta^2)(1+\beta)}{8}, & \text{if } \frac{1}{2} < \delta \leq 1 \text{ and } \frac{2-\delta}{2} < \beta \leq \frac{4-4\delta+2\delta^2}{2+2\delta-\delta^2} \\ \frac{(2-2\delta+\delta^2)(1+\beta)}{4\beta}, & \text{if } \frac{1}{2} < \delta \leq 1 \text{ and } \frac{4-4\delta+2\delta^2}{2+2\delta-\delta^2} < \beta \leq 1 \end{cases}.$$

Moreover,

$$\begin{aligned} \Pi_f^*|_{K=\beta+1} &\geq \Pi_f|_{D_{1c}>0, K=\beta+1} \\ &> \max\{0, \Pi_f|_{D_{1c}>0, K=2}, \frac{1}{2}\} \\ &= \max\{0, \Pi_f|_{D_{1c}>0, K=2}, \Pi_f|_{D_{1c}=0, K=2}\} \\ &= \max\{\Pi_f^*|_{K=0}, \Pi_f^*|_{K=2}\}. \end{aligned}$$

That is, an intermediate K is strictly optimal for an exogenous $\beta \in (0,1)$.

For $\beta \in [1,2)$, let $K = \widetilde{K} = \begin{cases} \max\{\beta, \frac{5+11\delta}{3+5\delta}\}, & \text{if } 0 \leq \delta \leq \frac{1}{2} \\ \max\{\beta, \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2}\}, & \text{if } \frac{1}{2} < \delta \leq 1 \end{cases}$. Then $\widetilde{K} \in [1,2)$,

$$\Pi_f|_{D_{1c}>0, K=\widetilde{K}} = \begin{cases} \frac{(1-\delta)(5+11\delta)}{8}, & \text{if } 0 \leq \delta \leq \frac{1}{2} \text{ and } 1 < \beta \leq \frac{5+11\delta}{3+5\delta} \\ \frac{(1-\delta)(1+3\delta)\beta}{4(\beta-1)}, & \text{if } 0 \leq \delta \leq \frac{1}{2} \text{ and } \beta > \frac{5+11\delta}{3+5\delta} \\ \frac{6-2\delta+\delta^2}{8}, & \text{if } \frac{1}{2} < \delta \leq 1 \text{ and } 1 < \beta \leq \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2} \\ \frac{(2-2\delta+\delta^2)\beta}{4(\beta-1)}, & \text{if } \frac{1}{2} < \delta \leq 1 \text{ and } \beta > \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2} \end{cases},$$

and

$$\begin{aligned} \Pi_f^*|_{K=\widetilde{K}} &\geq \Pi_f|_{D_{1c}>0, K=\widetilde{K}} \\ &> \max\{0, \Pi_f|_{D_{1c}>0, K=2}, \frac{1}{2}\} \\ &= \max\{0, \Pi_f|_{D_{1c}>0, K=2}, \Pi_f|_{D_{1c}=0, K=2}\} \\ &= \max\{\Pi_f^*|_{K=0}, \Pi_f^*|_{K=2}\}. \end{aligned}$$

That is, an intermediate K is strictly optimal for an exogenous $\beta \in [1,2)$.

Then we move to the case with endogenous β. Given that $\Pi_f^* = \max\{\Pi_f|_{D_{1c}>0}, \Pi_f|_{D_{1c}=0 \& D_s>0}, \Pi_f|_{D_{1c}=0 \& D_s=0}\}$, $\Pi_f|_{D_{1c}>0}$ is maximized at $\beta = K-1$, $\Pi_f|_{D_{1c}=0 \& D_s>0}$ is maximized at $\beta = K$, and $\Pi_f|_{D_{1c}=0 \& D_s=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$ is constant in β, the firm's optimal profit given K equals $\Pi_f^* = \max\{\Pi_f|_{D_{1c}>0,\beta=K-1}, \Pi_f|_{D_{1c}=0\&D_s>0,\beta=K}, \Pi_f|_{D_{1c}=0\&D_s=0}\} = \max\{\Pi_f|_{D_{1c}>0,\beta=K-1}, \Pi_f|_{D_{1c}=0\&D_s>0,\beta=K}\}$. Here,

$$\Pi_f|_{D_{1c}>0,\beta=K-1} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1, \\ \frac{(2-K)K^2}{2}, & \text{if } 0<\delta\leq 1 \text{ and } 1<K\leq\frac{2+\delta}{2}, \\ \frac{(4+8K-4K^2-\delta^2)K}{16}, & \text{if } 0<\delta\leq\frac{1}{2} \text{ and } \frac{2+\delta}{2}<K\leq\frac{2+3\delta}{2}, \\ & \text{or if } \frac{1}{2}<\delta\leq 1 \text{ and } \frac{2+\delta}{2}<K\leq\frac{4-\delta}{2}, \\ \frac{(2-K)K+2\delta(K-1)-2\delta^2}{2}K, & \text{if } 0\leq\delta\leq\frac{1}{2} \text{ and } \frac{2+3\delta}{2}<K\leq\frac{3+\delta}{2}, \\ \frac{(1-\delta)(3+5\delta)}{8}K, & \text{if } 0\leq\delta\leq\frac{1}{2} \text{ and } \frac{3+\delta}{2}<K\leq\frac{5+11\delta}{3+5\delta}, \\ \frac{(1-\delta)(1+3\delta)}{4(K-1)}K, & \text{if } 0<\delta\leq\frac{1}{2} \text{ and } K>\frac{5+11\delta}{3+5\delta}, \\ \frac{2+2\delta-\delta^2}{8}K, & \text{if } \frac{1}{2}<\delta\leq 1 \text{ and } \frac{4-\delta}{2}<K\leq\frac{6-2\delta+\delta^2}{2+2\delta-\delta^2}, \\ \frac{2-2\delta+\delta^2}{4(K-1)}K, & \text{if } \frac{1}{2}<\delta\leq 1 \text{ and } K>\frac{6-2\delta+\delta^2}{2+2\delta-\delta^2}, \end{cases}$$

and

$$\Pi_f|_{D_{1c}=0\&D_s>0,\beta=K} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } 0<\delta\leq 1 \text{ and } 0<K\leq\delta, \\ \frac{(4-K)K-\delta(2K-\delta)}{4}, & \text{if } 0<\delta<\frac{1}{2} \text{ and } \delta<K\leq 3\delta, \\ & \text{or if } \frac{1}{2}\leq\delta\leq 1 \text{ and } \delta<K\leq 2-\delta, \\ \frac{4(1-\delta)\delta+(2-K)(K-2\delta)}{2}, & \text{if } 0\leq\delta<\frac{1}{2} \text{ and } 3\delta<K\leq 1+\delta, \\ \frac{(1-\delta)(1+3\delta)}{2}, & \text{if } 0\leq\delta<\frac{1}{2} \text{ and } 1+\delta<K<2, \\ \frac{2-2\delta+\delta^2}{2}, & \text{if } \frac{1}{2}\leq\delta\leq 1 \text{ and } 2-\delta<K<2. \end{cases}$$

By simple comparison, we obtain that Π_f^* is strictly maximized at $K = \min\{\frac{5+11\delta}{3+5\delta}, \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2}\}$. That is, an intermediate K is strictly optimal for the firm even for endogenous β.

Forth, we prove that the presence of speculators can induce the firm to choose a larger capacity. If there is no speculative reselling, the firm's equilibrium profit $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K\leq 1 \\ \frac{1}{2}, & \text{if } K>1 \end{cases}$ achieves the maximum at $K=1$.

Note that $\min\{\frac{5+11\delta}{3+5\delta}, \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2}\} > 1$. Therefore, when speculation is allowed, the optimal value of K (i.e., $\min\{\frac{5+11\delta}{3+5\delta}, \frac{6-2\delta+\delta^2}{2+2\delta-\delta^2}\}$) is larger than that when speculation is not allowed (i.e., 1).

The above completes the analysis of Section 5.2 and proves that the main results hold with speculators' price discrimination ability. □

Details of the Analysis in Section 5.3

Similar to the main model, we start with the decisions in Period 2 and then analyze the decisions in Period 1. Although closed-form solutions for all parameter values are analytically complex, we show that an intermediate level of speculation may be optimal by presenting the result for specific parameter values.

Decisions in Period 2

If $K - D_{1c} \leq M\cdot\delta$, the firm's and speculators' capacity is not enough to serve any low-valuation consumers (i.e., those with $v < 1-\delta$). The firm and speculators compete for the high-valuation consumers (i.e., those with $v \geq 1-\delta$). The equilibrium prices will be the market-clearing price (i.e., $p^*_{sh} = p^*_{sl} = p^*_{f2} = \frac{2-K}{2-D_{1c}}$). The rest of the discussion is based on $K - D_{1c} > M \cdot \delta$. To simplify the analysis, we assume $\delta \leq \frac{1}{2}$.

As follows, we start by listing the speculators' potentially optimal pricing decisions given δ, K, D_s, D_{1c}, and p_{f2}. Given the speculators' best response to p_{f2}, we can then derive the firm's optimal pricing decision in Period 2.

Step 1: List speculators' potentially optimal decisions in Period 2 and the respective necessary conditions.

Clearly, $p_{sh} \geq p_{sl}$. We can write speculators' second-period profit as

$$\pi_{2s} = \begin{cases} p_{sh} \times \min\{D_s, M \cdot \delta - (K - D_s - D_{1c}), M(1 - \frac{K-D_s-D_{1c}}{M} - p_{sh})\} \\ \qquad + p_{sl} \times \min\{\max\{0, K - D_{1c} - M \cdot \delta, K - D_{1c} - M(1-p_{sh})\}, M(1-\delta-p_{sl})\}, \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } K - D_s - D_{1c} \leq M \cdot \delta \text{ and } p_{sh} > p_{f2} \\ p_{sl} \times \min\{D_s, M(1 - \frac{K-D_s-D_{1c}}{M} - p_{sl})\}, \qquad \text{if } K - D_s - D_{1c} > M \cdot \delta \text{ and } p_{sl} > p_{f2} \\ p_{sh} \times \min\{D_s, M \cdot \delta, M(1-p_{sh})\} \\ \qquad + p_{sl} \times \min\{\max\{0, D_s - M \cdot \delta, D_s - M(1-p_{sh})\}, M(1-\delta-p_{sl})\}, \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } p_{sh} \leq p_{f2} \end{cases}.$$

Note that it is never optimal to have $p_{sh} > p_{f2} \geq p_{sl}$ if $K - D_s - D_{1c} > M \cdot \delta$ because speculators can become better off by setting $p'_{sh} = p_{f2}$ and $p'_{sl} = p_{sl}$.

As follows, we list speculators' potential (global) optimal decisions in each of the three cases above and the necessary conditions for the potential optimality to hold. Then, it suffices to compare the speculators' expected profits across the three cases to obtain the speculators' global optimal pricing decisions in Period 2.

Case 1: $K - D_s - D_{1c} \leq M \cdot \delta$ and $p_{sh} > p_{f2}$.

(1.1) $p_{sh} = 1-\delta$ and $p_{sl} = \frac{2-K}{2-D_{1c}}$ if $1 - \delta \geq \max\{\frac{2-K+D_s}{2(2-D_{1c})}, \frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4}\}$, $\frac{2-K}{2-D_{1c}} \geq \frac{1-\delta}{2}$, and $p_{f2} < 1-\delta$. Correspondingly, $\pi_{2s} = (1-\delta) \times [(2-D_{1c})\delta - K + D_s + D_{1c}] + \frac{2-K}{2-D_{1c}} \times [K - D_{1c} - (2-D_{1c})\delta]$ if $\begin{cases} D_s \leq 2-K \\ \frac{K-D_s-2\delta}{1-\delta} \leq D_{1c} \leq \frac{3K-D_s-6\delta}{3(1-\delta)} \\ p_{f2} < 1-\delta \end{cases}$ or $\begin{cases} 2-K < D_s \leq 6-3K \\ \frac{2(K-1-\delta)}{1-\delta} \leq D_{1c} \leq \frac{3K-D_s-6\delta}{3(1-\delta)} \\ p_{f2} < 1-\delta \end{cases}$.

(1.2) $p_{sh} = 1-\delta$ and $p_{sl} = \frac{1-\delta}{2}$ if $1 - \delta \geq \max\{\frac{2-K+D_s}{2(2-D_{1c})}, \frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4}\}$, $\frac{2-K}{2-D_{1c}} < \frac{1-\delta}{2}$, and $p_{f2} < 1-\delta$. Correspondingly, $\pi_{2s} = (1-\delta) \times [(2-D_{1c})\delta - K + D_s + D_{1c}] + \frac{(2-D_{1c})(1-\delta)^2}{4}$ if $\begin{cases} 2-K < D_s \leq 6-3K \\ \frac{K-D_s-2\delta}{1-\delta} \leq D_{1c} < \frac{2(K-1-\delta)}{1-\delta} \\ p_{f2} < 1-\delta \end{cases}$ or $\begin{cases} D_s > 6-3K \\ \frac{K-D_s-2\delta}{1-\delta} \leq D_s \leq \frac{K-D_s+2-4\delta}{2(1-\delta)} \\ p_{f2} < 1-\delta \end{cases}$.

(1.3) $p_{sh} = \frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4}$ and $p_{sl} = \frac{2-D_s-K}{4(2-D_{1c})} + \frac{3(1-\delta)}{4}$ if $\frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4} \geq \max\{1-\delta, \frac{2-K+D_s}{2(2-D_{1c})}\}$. Correspondingly, $\pi_{2s} = \frac{(2-D_{1c})(1-\delta)^2}{8} - \frac{(1-\delta)(2-K-D_s)}{4} + \frac{(2-K+3D_s)^2-8D_s^2}{8(2-D_{1c})}$ if $\begin{cases} D_s \leq 4-2K \\ \frac{3K-D_s-6\delta}{3(1-\delta)} \leq D_{1c} < \frac{K-2\delta}{1-\delta} \\ p_{f2} < \frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4} \end{cases}$ or $\begin{cases} 4-2K < D_s \leq 6-3K \\ \frac{3K-D_s-6\delta}{3(1-\delta)} \leq D_{1c} < \frac{4-2\delta-D_s-K}{1-\delta} \\ p_{f2} < \frac{6+D_s-3K}{4(2-D_{1c})} + \frac{1-\delta}{4} \end{cases}$.

Case 2: $K - D_s - D_{1c} > M \cdot \delta$ *and* $p_{sl} > p_{f2}$.

(2.1) $p_{sl} = \frac{2-K}{2-D_{1c}}$ if $\frac{2-K}{2-D_{1c}} \geq \frac{2-K+D_s}{2(2-D_{1c})}$ and $p_{f2} < \frac{2-K}{2-D_{1c}}$. Correspondingly, $\pi_{2s} = \frac{2-K}{2-D_{1c}} D_s$ if $\begin{cases} D_s \leq 2-K \\ D_{1c} < \frac{K-D_s-2\delta}{1-\delta} \\ p_{f2} < \frac{2-K}{2-D_{1c}} \end{cases}$.

(2.2) $p_{sl} = \frac{2-K+D_s}{2(2-D_{1c})}$ if $\frac{2-K+D_s}{2(2-D_{1c})} > \frac{2-K}{2-D_{1c}}$ and $p_{f2} < \frac{2-K+D_s}{2(2-D_{1c})}$. Correspondingly, $\pi_{2s} = \frac{(2-K+D_s)^2}{4(2-D_{1c})}$ if $\begin{cases} D_s > 2-K \\ D_{1c} < \frac{K-D_s-2\delta}{1-\delta} \\ p_{f2} < \frac{2-K+D_s}{2(2-D_{1c})} \end{cases}$.

Case 3: $p_{sh} \leq p_{f2}$.

(3.1) $p_{sh} = p_{f2}$, $p_{sl} = p_{f2}$, and $\pi_{2s} = p_{f2} \times D_s$ if $p_{f2} \leq 1 - \frac{D_s}{2-D_{1c}}$.

(3.2) $p_{sh} = p_{f2}$ and $p_{sl} = 2 - \delta - \frac{D_s}{2-D_{1c}} - p_{f2}$ if $1 - \frac{D_s}{2-D_{1c}} < p_{f2} \leq \max\{1-\delta, 1-\frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}$, $p_{sh} \geq 1-\delta$ and $p_{f2} \geq 2 - \delta - \frac{D_s}{2-D_{1c}} - p_{f2} \geq \frac{1-\delta}{2}$. Correspondingly, $\pi_{2s} = (2-D_{1c})p_{f2}(1-p_{f2}) + (2 - \delta - \frac{D_s}{2-D_{1c}} - p_{f2})[D_s - (2-D_{1c})(1-p_{f2})]$ if $\max\{1 - \frac{D_s}{2-D_{1c}}, 1-\delta, 1-\frac{\delta}{2} - \frac{D_s}{2(2-D_{1c})}\} \leq p_{f2} \leq \min\{\max\{1-\delta, 1-\frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}, \frac{3-\delta}{2} - \frac{D_s}{2-D_{1c}}\}$.

(3.3) $p_{sh} = p_{f2}$ and $p_{sl} = 1 - \frac{D_s}{2-D_{1c}}$ if $1 - \frac{D_s}{2-D_{1c}} < p_{f2} \leq \max\{1-\delta, 1-\frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}$, $p_{sh} < 1-\delta$, and $p_{f2} \geq 1 - \frac{D_s}{2-D_{1c}} \geq \frac{1-\delta}{2}$. Correspondingly, $\pi_{2s} = p_{f2}(2-D_{1c})\delta + (1 - \frac{D_s}{2-D_{1c}})[D_s - (2-D_{1c})\delta]$ if $\begin{cases} \frac{(2-K)\delta}{1-\delta} < D_s \leq \frac{(2-K)(1+\delta)}{2(2-\delta)} \\ \frac{2\delta-D_s}{\delta} < D_{1c} < \frac{K-2\delta}{1-\delta} \\ 1 - \frac{D_s}{2-D_{1c}} < p_{f2} < 1-\delta \end{cases}$ or $\begin{cases} D_s > \frac{(2-K)(1+\delta)}{2(2-\delta)} \\ \frac{2\delta-D_s}{\delta} < D_{1c} < 2 - \frac{2D_s}{1+\delta} \\ 1 - \frac{D_s}{2-D_{1c}} < p_{f2} < 1-\delta \end{cases}$.

(3.4) $p_{sh} = p_{f2}$ and $p_{sl} = \frac{1-\delta}{2}$ if $1 - \frac{D_s}{2-D_{1c}} < p_{f2} \leq \max\{1-\delta, 1-\frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}$, $p_{sh} < 1-\delta$, and $p_{f2} \geq \frac{1-\delta}{2} > 1 - \frac{D_s}{2-D_{1c}}$. Correspondingly, $\pi_{2s} = p_{f2}(2-D_{1c})\delta + \frac{(2-D_{1c})(1-\delta)^2}{4}$ if $\begin{cases} D_s > \frac{(2-K)(1+\delta)}{2(2-\delta)} \\ 2 - \frac{2D_s}{1+\delta} < D_{1c} < \frac{K-2\delta}{1-\delta} \\ \frac{1-\delta}{2} \leq p_{f2} < 1-\delta \end{cases}$.

(3.5) $p_{sh} = p_{f2}$ and $p_{sl} = p_{f2}$ if $1 - \frac{D_s}{2-D_{1c}} < p_{f2} \leq \max\{1-\delta, 1-\frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}$, $p_{sh} < 1-\delta$, and $p_{f2} < \max\{\frac{1-\delta}{2}, 1 - \frac{D_s}{2-D_{1c}}\}$. Correspondingly, $\pi_{2s} = (2-D_{1c})p_{f2}(1-p_{f2})$ if $\begin{cases} D_s > \frac{(2-K)(1+\delta)}{2(2-\delta)} \\ 2 - \frac{2D_s}{1+\delta} < D_{1c} < \frac{K-2\delta}{1-\delta} \\ 1 - \frac{D_s}{2-D_{1c}} < p_{f2} < \frac{1-\delta}{2} \end{cases}$.

(3.6) $p_{sh} = 1 - \frac{D_s}{2-D_{1c}}$ if $p_{f2} \geq 1 - \frac{D_s}{2-D_{1c}} \geq \max\{1-\delta, 1-\frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}$. Correspondingly, $\pi_{2s} = (1 - \frac{D_s}{2-D_{1c}})D_s$ if $\begin{cases} D_s \leq \frac{(2-K)\delta}{2(1-\delta)} \\ D_{1c} < \frac{K-2\delta}{1-\delta} \\ p_{f2} \geq 1 - \frac{D_s}{2-D_{1c}} \end{cases}$ or $\begin{cases} D_s > \frac{(2-K)\delta}{2(1-\delta)} \\ D_{1c} \leq \frac{2\delta-2D_s}{\delta} \\ p_{f2} \geq 1 - \frac{D_s}{2-D_{1c}} \end{cases}$.

(3.7) $p_{sh} = 1 - \delta$ and $p_{sl} = 1 - \frac{D_s}{2-D_{1c}}$ if $p_{f2} \geq 1 - \delta \geq \max\{1 - \frac{D_s}{2-D_{1c}}, 1 - \frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}$ and $1 - \frac{D_s}{2-D_{1c}} \geq \frac{1-\delta}{2}$. Correspondingly, $\pi_{2s} = (2 - D_{1c})(1-\delta)\delta + (1 - \frac{D_s}{2-D_{1c}})[D_s - (2 - D_{1c})\delta]$ if $\begin{cases} \frac{3\delta(2-K)}{2(1-\delta)} < D_s \leq \frac{(2-K)(1+\delta)}{2(1-\delta)} \\ \frac{6\delta-2D_s}{3\delta} \leq D_{1c} < \frac{K-2\delta}{1-\delta} \\ p_{f2} \geq 1-\delta \end{cases}$ or $\begin{cases} D_s > \frac{(2-K)(1+\delta)}{2(1-\delta)} \\ \frac{6\delta-2D_s}{3\delta} \leq D_{1c} \leq 2 - \frac{2D_s}{1+\delta} \\ p_{f2} \geq 1-\delta \end{cases}$.

(3.8) $p_{sh} = 1 - \delta$ and $p_{sl} = \frac{1-\delta}{2}$ if $p_{f2} \geq 1 - \delta \geq \max\{1 - \frac{D_s}{2-D_{1c}}, 1 - \frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}, \frac{1}{2}\}$ and $1 - \frac{D_s}{2-D_{1c}} < \frac{1-\delta}{2}$. Correspondingly, $\pi_{2s} = (2 - D_{1c})(1-\delta)\delta + \frac{(2-D_{1c})(1-\delta)^2}{4}$ if $\begin{cases} D_s > \frac{(2-K)(1+\delta)}{2(1-\delta)} \\ 2 - \frac{2D_s}{1+\delta} < D_{1c} < \frac{K-2\delta}{1-\delta} \\ p_{f2} \geq 1-\delta \end{cases}$.

(3.9) $p_{sh} = 1 - \frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})}$ and $p_{sl} = 1 - \frac{3\delta}{4} - \frac{D_s}{2(2-D_{1c})}$ if $p_{f2} \geq 1 - \frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})} \geq \max\{1 - \frac{D_s}{2-D_{1c}}, 1-\delta, \frac{1}{2}\}$. Correspondingly, $\pi_{2s} = \frac{(2-D_{1c})\delta^2}{8} - \frac{D_s \cdot \delta}{2} + D_s - \frac{D_s^2}{2(2-D_{1c})}$ if $\begin{cases} \frac{(2-K)\delta}{2(1-\delta)} < D_s \leq \frac{3\delta(2-K)}{2(1-\delta)} \\ \frac{2\delta-2D_s}{\delta} \leq D_{1c} < \frac{K-2\delta}{1-\delta} \\ p_{f2} \geq 1 - \frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})} \end{cases}$ or $\begin{cases} D_s > \frac{3\delta(2-K)}{2(1-\delta)} \\ \frac{2\delta-2D_s}{\delta} \leq D_{1c} \leq \frac{6\delta-2D_s}{3\delta} \\ p_{f2} \geq 1 - \frac{\delta}{4} - \frac{D_s}{2(2-D_{1c})} \end{cases}$.

Step 2: Derive firm's pricing decision in Period 2.

In Step 1, we list the speculators' potentially optimal pricing decisions in Cases 1, 2, and 3, as well as the necessary conditions for the potential optimality to hold. Next, we need to compare the speculators' expected profits across the three cases discussed above to obtain the speculators' global optimal pricing decisions and in turn derive the firm's pricing decision. To reduce the number of case comparisons, we start by noting that

(i) Case 1 and Case 2 never co-exist.

(ii) Cases 1.1, 1.2, 2.1 and 2.2 never co-exist with Cases 3.6 to 3.9. Furthermore, Cases 3.6 to 3.9 cannot be globally optimal.

(iii) For Cases 1 and 2, $\pi_{2f} = p_{f2}(K - D_s - D_{1c})$. For Case 3.1, $\pi_{2f} = p_{f2} \times \min\{K - D_s - D_{1c}, (2 - D_{1c})(1 - p_{f2}) - D_s\}$. For Cases 3.2 to 3.9, $\pi_{2f} = 0$.

Based on the above observations, the optimal p_{f2} is such that one of the following three conditions holds: (I) speculators are indifferent between Case 3.1 and one of the sub-cases of Case 1 or 2, and $p_{f2} \geq \frac{1-\frac{D_s}{2-D_{1c}}}{2}$ or $p_{f2}(K - D_s - D_{1c}) \geq \frac{(2-D_s-D_{1c})^2}{4(2-D_{1c})}$; (II) $p_{f2} = \max\{\frac{2-K}{2-D_{1c}}, \frac{1-\frac{D_s}{2-D_{1c}}}{2}\}$ and $p_{f2} \times \min\{K - D_s - D_{1c}, (2 - D_{1c})(1 - p_{f2}) - D_s\} = p_{f2} \times [(2 - D_{1c})(1 - p_{f2}) - D_s] > p'_{f2}(K - D_s - D_{1c})$, where the value of p'_{f2} is such that speculators are indifferent between Case 3.1 and one of the sub-cases of Case 1 or 2; (III) speculators are indifferent between Cases 3.2 to 3.5 and one of the sub-cases of Case 1 or 2.

In the above Cases (I) and (III), speculators' prices are as in the Cases 1.1 to 2.2, whereas in the above Case (II), $p_{sh} = p_{sl} = p_{f2}$.

Decisions in Period 1

We start with the early consumers' and speculators' purchase decisions. Then, we analyze the firm's decisions on how many units to sell in Period 1 (i.e., the upper bound K_1 for $D_s + D_{1c}$) and the first-period price to charge (i.e., p_{f1}). Note that the closed-form solutions are complicated due to the numerous cases. Thus, we concentrate on presenting the algorithm of deriving the equilibrium decisions.

An early consumer compares the expected utility of purchasing immediately (u_1) and waiting (u_2), where $u_1 = \frac{1}{2} - p_{f1}$ and

$$u_2 = \begin{cases} \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1} (v - p_{f2})dv + \int_{p_{sh}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}} (v - p_{sh})dv + \int_{p_{sl}}^{1-\delta} (v - p_{sl})dv, & \\ & \text{if } K - D_s - D_{1c} \leq (2 - D_{1c}) \cdot \delta \text{ and } p_{sh} > p_{f2} \\ \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1} (v - p_{f2})dv + \int_{p_{sl}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}} (v - p_{sl})dv, & \\ & \text{if } K - D_s - D_{1c} > (2 - D_{1c}) \cdot \delta \text{ and } p_{sl} > p_{f2} \\ \int_{p_{f2}}^{1} (v - p_{f2})dv, & \text{if } p_{sh} = p_{sl} = p_{f2} \end{cases}.$$

An early consumer will purchase immediately only if $u_1 \geq u_2$.

Speculators choose $D_s \leq \beta$ to maximize their profit $\Pi_s(D_{1c}) = \pi_{2s} - p_{f1} \times D_s$ in anticipation of early consumers' purchase decisions.

In anticipation of the speculators' and early consumers' purchase decisions, the firm chooses p_{f1} and K_1 to maximize its total payoff over the two periods.

Note that the closed-form solutions are analytically complex due to numerous cases. Thus, we only provide here the process of deriving the equilibrium decisions given K and δ as follows:

1. Fix p_{f1}, K_1, D_s, and D_{1c}, then following the analysis of decisions in Period 2 above, obtain the pricing decisions in Period 2 and the corresponding π_{2s}, π_{2f}, and u_2.

2. Fix p_{f1}, K_1, and D_s, then obtain the speculators' expected total payoff constrained by $D_{1c} > 0$, $\Pi_s = \pi_{2s} - p_{f1} \times D_s$, given $D_{1c} = \min\{1, K_1 - D_s\}$.

3. Fix p_{f1}, K_1, and D_s, then obtain the speculators' expected total payoff constrained by $D_{1c} = 0$, $\Pi_s = \pi_{2s} - p_{f1} \times D_s$, given $D_{1c} = 0$.

4. Fix p_{f1} and K_1, then find the value of $D_s = D_s^*(p_{f1}, K_1)|_{D_{1c}>0}$ that maximizes Π_s obtained in Step 2. If given this D_s^* and $D_{1c} = \min\{1, K_1 - D_s^*\}$, $u_1 \geq u_2$ and $D_{1c} > 0$ are satisfied, then obtain the firm's expected total payoff constrained by $D_{1c} > 0$, $\Pi_f = p_{f1} \times (D_s + D_{1c}) + \pi_{2f}$.

5. Fix p_{f1} and K_1, then find the value of $D_s = D_s^*(p_{f1}, K_1)|_{D_{1c}=0}$ that maximizes Π_s obtained in Step 3. If given this D_s^* and $D_{1c} = 0$, $u_1 < u_2$ or $D_s^* = K_1$ is satisfied, then obtain the firm's expected total payoff constrained by $D_{1c} = 0$, $\Pi_f = p_{f1} \times (D_s + D_{1c}) + \pi_{2f} = p_{f1} \times D_s + \pi_{2f}$.

6. Find the value of $p_{f1} = p^*_{f1}$ and $K_1 = K^*_1$ that maximizes $\Pi_f = \max\{\Pi_f|_{D_{1c}>0}, \Pi_f|_{D_{1c}=0}\}$, where the value of $\Pi_f|_{D_{1c}>0}$ is obtained in Step 4 and the value of $\Pi_f|_{D_{1c}=0}$ is obtained in Step 5.

The equilibrium payoffs are obtained from the equilibrium decisions derived as above.

To show the strict optimality of an intermediate β, we follow the steps above for $K = \frac{3}{2}$ and $\delta = \frac{1}{4}$. The firm's profit is strictly maximized at an intermediate value of β (which is approximately 0.4642).

This completes the analysis of Section 5.3. $\square$

Details of the Analysis in Section 5.4

Similar to the analysis of the main model, we first derive the firm's optimal choice of price and the corresponding profit constrained by $D_{1c} > 0$. Then, we compare it to the firm's expected profit subject to the constraint of $D_{1c} = 0$ (i.e., $\Pi_f|_{D_{1c}=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$). Last, we show that the main results remain to hold under the proportional-rationing rule.

As in the main text, early consumers' expected utility of delaying the purchase decision until Period 2 is

$$\begin{aligned} u_2 &= \int_{p_f}^{1} \frac{K - D_s - D_{1c}}{M(1-p_f)}(v - p_f)dv + \int_{p^*_s}^{1} (1 - \frac{K - D_s - D_{1c}}{M(1-p_f)})(v - p^*_s)dv \\ &= \begin{cases} \frac{(1-p_f)(K-D_{1c}-D_s)}{2(2-D_{1c})} + \frac{(1-p_f)D_s^2}{2(2-D_{1c})(2-K+D_s-(2-D_{1c})p_f)}, & \text{if } D_s \leq \frac{2-K-(2-D_{1c})p_f}{1-2p_f} \\ \frac{1}{8} + \frac{(K-D_s-D_{1c})(4(1-p_f)^2-1)}{8(2-D_{1c})(1-p_f)}, & \text{if } D_s > \frac{2-K-(2-D_{1c})p_f}{1-2p_f} \end{cases}. \end{aligned}$$

Similar to the main model, conditional on positive first-period consumer demand, the firm will set a maximum price p_f such that both speculators and early consumers find it optimal to purchase immediately. The firm's expected payoff is $\Pi_f = p_f \times K$. Moreover, in the equilibrium yielding the firm's highest profit, $D^*_{1c} = \min\{1, K - D_s\}$.

Similar to the main model, we can show that the equilibrium firm's profit is

$$\Pi_f^* = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \le 1 \\ \frac{1}{2}, & \text{if } 1 < K \le \frac{9}{8} \text{ and } \beta > (2-K)(K-1+\sqrt{K(K-1)}), \\ & \text{or if } \frac{3}{2} < K \le \frac{7+\sqrt{7}}{6} \text{ and } \beta \le \frac{(K-1)(2K-3)(-1+4K-2K^2)}{1-12K+20K^2-8K^3}, \\ & \text{or if } \frac{7+\sqrt{7}}{6} < K \le \frac{7+\sqrt{13}}{6} \text{ and } \beta \le \frac{-6+21K-20K^2+6K^3}{2-8K+6K^2}, \\ & \text{or if } K > \frac{7+\sqrt{13}}{6} \text{ and } \beta \le \frac{K(1-4K+2K^2)}{2(K-1)^2} \\ \frac{8+6\beta-2K(3+\beta)+K^2-\sqrt{(8+6\beta-2K(3+\beta)+K^2)^2-4(2-K)(3-K+\beta)(2-K+2\beta)}}{6-2K+2\beta}K, & \\ & \text{if } 1 < K \le \frac{3}{2} \text{ and } \beta \le K-1, \\ & \text{or if } \frac{3}{2} < K \le \frac{7+\sqrt{7}}{6} \text{ and } \frac{(K-1)(2K-3)(-1+4K-2K^2)}{1-12K+20K^2-8K^3} < \beta \le \frac{8-8K+2K^2}{4K-5} \\ \frac{8(3-K+\beta)-5-\sqrt{16(3-K+\beta)^2-32(3-K+\beta)+25}}{8(3-K+\beta)}K, & \\ & \text{if } \frac{3}{2} < K \le \frac{7+\sqrt{7}}{6} \text{ and } \frac{8-8K+2K^2}{4K-5} < \beta \le K-1, \\ & \text{or if } \frac{7+\sqrt{7}}{6} < K \le \frac{5}{3} \text{ and } \frac{-6+21K-20K^2+6K^3}{2-8K+6K^2} < \beta \le K-1, \\ & \text{or if } \frac{5}{3} < K \le \frac{7+\sqrt{13}}{6} \text{ and } \frac{-6+21K-20K^2+6K^3}{2-8K+6K^2} < \beta \le \frac{(3-K)(2-K)}{4K-6} \\ \frac{1+4\beta-\sqrt{1-24\beta+16K\beta}}{8\beta}K, & \text{if } \frac{5}{3} < K \le \frac{7+\sqrt{13}}{6} \text{ and } \frac{(3-K)(2-K)}{4K-6} < \beta \le K-1, \\ & \text{or if } K > \frac{7+\sqrt{13}}{6} \text{ and } \frac{K(1-4K+2K^2)}{2(K-1)^2} < \beta \le K-1 \\ \frac{(2-K)(2-K+2\beta)K}{2(2-K+\beta)^2}, & \text{if } 1 < K \le \frac{9}{8} \text{ and } K-1 < \beta \le (2-K)(K-1+\sqrt{K(K-1)}), \\ & \text{or if } \frac{9}{8} < K \le \frac{4}{3} \text{ and } K-1 < \beta \le \frac{2-K}{2} \\ \frac{4K}{9}, & \text{if } \frac{9}{8} < K \le \frac{4}{3} \text{ and } \beta > \frac{2-K}{2} \\ \frac{(2-K)K^2}{2}, & \text{if } \frac{4}{3} < K \le \frac{3}{2} \text{ and } \beta > K-1 \\ \frac{3K}{8}, & \text{if } \frac{3}{2} < K \le \frac{5}{3} \text{ and } \beta > K-1 \\ \frac{K}{4(K-1)}, & \text{if } K > \frac{5}{3} \text{ and } \beta > K-1 \end{cases}.$$

Here, it is easy to see that $\frac{\partial \Pi_f^*}{\partial \beta} \ge 0$ if $\beta < K-1$, and $\frac{\partial \Pi_f^*}{\partial \beta} \le 0$ if $\beta > K-1$. This shows that when consumers have an equal probability of being served regardless of their valuations, the firm's equilibrium profit is first increasing and then decreasing in β.

Note that when $1 < K < \frac{4}{3}$, Π_f^* achieves its strict maximum at $\beta = K-1$. This shows that an intermediate value of β is strictly optimal for the firm when K is intermediate.

When $\beta \le 1$, $\Pi_f^*(K)|_{K=\beta+1} > \max\{\Pi_f^*(K)|_{K=0}, \Pi_f^*(K)|_{K=+\infty}\}$. This shows that an intermediate capacity is strictly optimal for the firm given an exogenous small level of speculative reselling.

Moreover, when β is chosen at the level optimal for the firm, the firm's equilibrium profit is

$$\Pi_f^*(\beta)|_{\beta=K-1} \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \le 1 \\ \frac{(2-K)K^2}{2}, & \text{if } 1 < K \le \frac{3}{2} \\ \frac{3K}{8}, & \text{if } \frac{3}{2} < K \le \frac{5}{3} \\ \frac{K}{4(K-1)}, & \text{if } \frac{5}{3} < K < 2 \end{cases},$$

which achieves the strict maximum at $K = \frac{5}{3}$. This shows that an intermediate capacity is strictly optimal for the firm given the endogenous optimal level of speculative reselling.

Last, when there is no speculation (i.e., $\beta = 0$), the firm's profit is maximized at $K = 3 - \sqrt{3}$. This value is smaller than $K = \frac{5}{3}$, the firm's optimal capacity given the optimal level of speculation. This shows that the presence of speculators can induce the firm to choose a higher capacity.

This completes the analysis of Section 5.4 and proves that the main results hold under the proportional-rationing rule. $\square$

Details of the Analysis in Section 5.5

In this analysis, we first derive the firm's equilibrium profit and then show that the main results remain to hold with the consumer resale possibility.

To derive the firm's equilibrium profit, we need to discuss two cases: (1) $p_f \le p_r^*$ and (2) $p_f > p_r^*$.

Case (1): $p_f \le p_r^*$

As in the main text, an early consumer's expected utility of delaying the purchase decision until Period 2 is

$$\begin{aligned} u_2 &= \int_{1-\frac{K_2}{M}}^{1} (v - p_f) dv + \int_{p_r^*}^{1-\frac{K_2}{M}} (v - p_r^*) dv \\ &= \begin{cases} \int_{1-\frac{K-D_{1c}-D_s}{2-D_{1c}}}^{1} (v-p_f)dv + \int_{\frac{2-K}{2}}^{1-\frac{K-D_{1c}-D_s}{2-D_{1c}}} (v - \frac{2-K}{2})dv, & \text{if } D_{1c} \le \alpha \\ \int_{1-\frac{K-D_{1c}-D_s}{2-D_{1c}}}^{1} (v-p_f)dv + \int_{\frac{2-K}{2-D_{1c}+\alpha}}^{1-\frac{K-D_{1c}-D_s}{2-D_{1c}}} (v - \frac{2-K}{2-D_{1c}+\alpha})dv, & \text{if } D_{1c} > \alpha \end{cases} \\ &= \begin{cases} \frac{2K(4-K)-(8-4K+K^2)D_{1c}-4(2-K)D_s}{8(2-D_{1c})} - \frac{K-D_{1c}-D_s}{2-D_{1c}} p_f, & \text{if } D_{1c} \le \alpha \\ \frac{(K-D_{1c}-D_s)(4-K-D_{1c}+D_s)}{2(2-D_{1c})^2} + \frac{((2-D_{1c}+\alpha)D_s+(2-K)\alpha)^2}{2(2-D_{1c})^2(2-D_{1c}+\alpha)^2} - \frac{K-D_{1c}-D_s}{2-D_{1c}} p_f, & \text{if } D_{1c} > \alpha \end{cases}, \end{aligned}$$

whereas her expected utility of purchasing immediately is $u_{1,NR} = \frac{1}{2} - p_f$ if she cannot resell the product, and $u_{1,R} = \frac{1}{2} - p_f + \frac{p_r^{*2}}{2} = \begin{cases} \frac{8-4K+K^2}{8} - p_f, & \text{if } D_{1c} \le \alpha \\ \frac{1}{2} + \frac{(2-K)^2}{2(2-D_{1c}+\alpha)^2} - p_f, & \text{if } D_{1c} > \alpha \end{cases}$ if she has the resale option.

Similar to Section 5.1, when expecting positive consumer demand in Period 1 (i.e., $u_{1,R} = \max\{u_{1,NR}, u_{1,R}\} \ge u_2$), speculators will purchase β units in Period 1 and the firm chooses p_f to maximize its expected payoff $\Pi_f = p_f \times K$.

Note that three cases may apply in the equilibrium with $p_f \le p_r^*$: (1a) even consumers without the resale option also purchase in the first period; (1b) only consumers with the resale option buy the products immediately; and (1c) no early consumers purchase in the first period.

If case (1a) applies and $\beta \le K - 1$, then $p_f = \frac{(2-K)[(2-K+2\beta)+2\alpha(2-K+\beta)]}{2(1+\alpha)^2(2-K+\beta)}$, $D_s = \beta$, $D_{1c} = 1$, and $\Pi_f = \frac{(2-K)[(2-K+2\beta)+2\alpha(2-K+\beta)]}{2(1+\alpha)^2(2-K+\beta)} K$.

If case (1a) applies and $K - 1 < \beta \le K - \alpha$, then $p_f = \frac{(2-K)(2-K+2\beta+2\alpha)}{2(2-K+\beta+\alpha)^2}$, $D_s = \beta$, $D_{1c} = K - \beta$, and $\Pi_f = \frac{(2-K)(2-K+2\beta+2\alpha)}{2(2-K+\beta+\alpha)^2} K$.

If case (1b) applies, then $p_f = \frac{2-K}{2}$, $D_s = \beta$, $D_{1c} = \min\{\alpha, K - \beta\}$, and $\Pi_f = \frac{(2-K)K}{2}$.

If case (1c) applies, we have already shown in the analysis of the main model that $\Pi_f = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$.

Case (2): $p_f > p_r^*$

As in the main text, no speculators enter the market in this case. Because the price in the resale market is lower than in the primary market and consumers prefer to buy from the resale market than from the primary market, in Period 2 the market-clearing condition is $\min\{D_{1c}, \alpha\} \times p_r = M(1-p_r)$, leading to $p_r^* = \begin{cases} \frac{2-D_{1c}}{2}, & \text{if } D_{1c} \leq \alpha \\ \frac{2-D_{1c}}{2-D_{1c}+\alpha}, & \text{if } D_{1c} > \alpha \end{cases}$. In Period 1, early consumers decide whether to purchase immediately or wait by comparing their expected utilities. The firm chooses p_f to maximize its expected profit.

Here, by substituting the expression of p_r^* derived above, an early consumer's expected utility of delaying the purchase decision until Period 2 is

$$\begin{aligned} u_2 &= \int_{p_r^*}^{1} (v - p_r^*) dv \\ &= \begin{cases} \int_{\frac{2-D_{1c}}{2}}^{1} (v - \frac{2-D_{1c}}{2}) dv, & \text{if } D_{1c} \leq \alpha \\ \int_{\frac{2-D_{1c}}{2-D_{1c}+\alpha}}^{1} (v - \frac{2-D_{1c}}{2-D_{1c}+\alpha}) dv, & \text{if } D_{1c} > \alpha \end{cases} \\ &= \begin{cases} \frac{D_{1c}^2}{8}, & \text{if } D_{1c} \leq \alpha \\ \frac{\alpha^2}{2(2-D_{1c}+\alpha)^2}, & \text{if } D_{1c} > \alpha \end{cases}, \end{aligned}$$

whereas her expected utility of purchasing immediately is $u_{1,NR} = \frac{1}{2} - p_f$ if she cannot resell the product, and $u_{1,R} = \int_0^1 \max\{v, p_r^*\} dv - p_f = \frac{1}{2} - p_f + \frac{p_r^{*2}}{2} = \begin{cases} \frac{8-4D_{1c}+D_{1c}^2}{8} - p_f, & \text{if } D_{1c} \leq \alpha \\ \frac{1}{2} + \frac{(2-D_{1c})^2}{2(2-D_{1c}+\alpha)^2} - p_f, & \text{if } D_{1c} > \alpha \end{cases}$ if she has the resale option. By simple calculation, we can show that $\max\{u_{1,NR}, u_{1,R}\} \geq u_2$ can never hold. Therefore, no early consumers purchase in the first period.

Summarize Cases (1) and (2)

By comparing the above cases, we know that the firm's equilibrium profit is $\Pi_f^* = \frac{(2-K)K}{2}$ if $K \leq 1$,

$$\Pi_f^* = \begin{cases} \frac{1}{2}, & \text{if } \beta \leq \frac{(2-K)(K-1)^2+2(2-K)(K-1)^2\alpha+(2-K)\alpha^2}{-1+4K-2K^2-2(K-1)^2\alpha-\alpha^2} \\ \frac{(2-K)[(2-K+2\beta)+2\alpha(2-K+\beta)]}{2(1+\alpha)^2(2-K+\beta)} K, & \text{if } \frac{(2-K)(K-1)^2+2(2-K)(K-1)^2\alpha+(2-K)\alpha^2}{-1+4K-2K^2-2(K-1)^2\alpha-\alpha^2} < \beta \leq K-1 \\ \frac{(2-K)(2-K+2\beta+2\alpha)}{2(2-K+\beta+\alpha)^2} K, & \text{if } K-1 < \beta \leq (2-K)(K-1+\sqrt{K(K-1)}) - \alpha \\ \frac{1}{2}, & \text{if } (2-K)(K-1+\sqrt{K(K-1)}) - \alpha < \beta \leq K \end{cases}$$

if $1 < K \leq \frac{1+\sqrt{5}}{2}$ and $\alpha \leq (2-K)\sqrt{K(K-1)} - (K-1)^2$, and $\Pi_f^* = \frac{1}{2}$ if $K > \frac{1+\sqrt{5}}{2}$ or $\alpha > (2-K)\sqrt{K(K-1)} - (K-1)^2$.

To obtain that an intermediate level of restriction on speculation may be optimal for the firm, note that when $1 < K \leq \frac{1+\sqrt{5}}{2}$ and $\alpha \leq (2-K)\sqrt{K(K-1)} - (K-1)^2$, Π_f^* achieves its strict maximum at $\beta = K-1$.

To obtain that an intermediate capacity may be optimal for the firm given an exogenous level of speculation, note that when $\beta \leq \frac{-1+\sqrt{5}}{2}$ and $\alpha \leq (1-\beta)\sqrt{\beta(1+\beta)}-\beta^2$, Π_f^* achieves its strict maximum at $K=\beta+1$.

To obtain that an intermediate capacity may be optimal for the firm even if speculation is at the optimal level (i.e., $\beta = K-1$), note that

$$\Pi_f^*|_{\beta=K-1} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{(2-K)K(K+2\alpha)}{2(1+\alpha)^2}, & \text{if } 1 < K \leq \frac{1+\sqrt{5}}{2} \text{ and } \alpha \leq (2-K)\sqrt{K(K-1)}-(K-1)^2 \\ \frac{1}{2}, & \text{if } K > \frac{1+\sqrt{5}}{2} \text{ or } \alpha > (2-K)\sqrt{K(K-1)}-(K-1)^2 \end{cases}$$

achieves its strict maximum at $K = \frac{2(1-\alpha+\sqrt{1+\alpha+\alpha^2})}{3}$ if $\alpha < \widehat{\alpha}$, where $\widehat{\alpha} \approx 0.357$ is the first root of the equation $-5+6\alpha+11\alpha^2+32\alpha^3=0$.

To obtain that speculative reselling can induce the firm to choose a larger capacity, note that if there is no speculation, similar to Section 5.1, the firm's profit achieves the maximum at $K=1$. On the other hand, we have shown above that if speculative reselling is chosen at the optimal level, the firm's profit achieves the maximum at $K = \frac{2(1-\alpha+\sqrt{1+\alpha+\alpha^2})}{3}$ if α is small enough.

This completes the analysis of Section 5.5 and proves that the main results hold with the consumer resale possibility. $\square$

Proof of Proposition 6

We have derived the firm's equilibrium profit Π_f^* given the consumer resale ability. Proposition 6 is proved by observing that $\frac{d\Pi_f^*}{d\alpha} \leq 0$. $\square$

Details of the Analysis in Section 5.6

Similar to the analysis of the main model, we first derive the firm's optimal choice of price and the corresponding profit constrained by $D_{1c} > 0$. Then, we compare it to the firm's expected profit subject to the constraint of $D_{1c} = 0$ (i.e., $\Pi_f|_{D_{1c}=0} = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } K > 1 \end{cases}$). Last, we show that the main results remain to hold with the marginal cost of speculative reselling.

Following logic similar as in the main model, the firm finds that positive first-period consumer purchases are profitable only if $K > 1$. Thus, when deriving the firm's optimal choice of price constrained by $D_{1c} > 0$, we focus on the $K > 1$ case.

In Period 2, as in the main text, speculators maximize the expected second-period profit

$$\begin{aligned} \pi_{2s} &= (p_s - c) \times \min\{D_s, M(1 - \frac{K_2}{M} - p_s)\} \\ &= (p_s - c) \times \min\{D_s, 2 - K + D_s - (2 - D_{1c})p_s\}. \end{aligned}$$

by setting $p_s^* = \max\{\frac{2-K}{2-D_{1c}}, \frac{\frac{2-K+D_s}{2-D_{1c}}+c}{2}\}$.

In Period 1, early consumers purchase only if their expected utility of purchasing immediately, $u_1 = \frac{1}{2} - p_f$, is higher than their expected utility of delaying the purchase decision until Period 2,

$$u_2 = \int_{1-\frac{K_2}{M}}^{1} (v - p_f)^+ dv + \int_0^{1-\frac{K_2}{M}} (v - p_s^*)^+ dv$$
$$= \begin{cases} \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1} (v-p_f)dv + \int_{\frac{2-K}{2-D_{1c}}}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}} (v - \frac{2-K}{2-D_{1c}})dv, & \text{if } D_s \leq (2-K)-(2-D_{1c})c \\ \int_{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}}^{1} (v-p_f)dv + \int_{\frac{1}{2}(\frac{2-K+D_s}{2-D_{1c}}+c)}^{1-\frac{K-D_s-D_{1c}}{2-D_{1c}}} (v - \frac{1}{2}(\frac{2-K+D_s}{2-D_{1c}}+c))dv, & \text{if } D_s > (2-K)-(2-D_{1c})c \end{cases}.$$

That is, $p_f \leq \widetilde{w}_1 = \begin{cases} \frac{(2-K)(2-K+2D_s)}{2(2-D_{1c})(2-K+D_s)}, & \text{if } D_s \leq (2-K)-(2-D_{1c})c \\ \frac{1}{8}(2c + \frac{3(2-K+D_s)}{2-D_{1c}} - \frac{(2-D_{1c})c^2}{2-K+D_s}), & \text{if } D_s > (2-K)-(2-D_{1c})c \end{cases}$. In the equilibrium yielding the firm's highest profit, $D_{1c}^* = \min\{1, K - D_s\}$.

Speculators choose how many units to purchase to maximize their total profit over the two periods, which equals

$$\Pi_s(D_s) = \begin{cases} (2-K-c-p_f)D_s, & \text{if } D_s \leq K-1 \\ (\frac{2-K}{2-K+D_s} - c - p_f)D_s, & \text{if } K-1 < D_s \leq \frac{(2-K)(1-c)}{1+c} \\ \frac{(2-K+D_s)(1-c)^2}{4} - p_f \times D_s, & \text{if } D_s > \frac{(2-K)(1-c)}{1+c} \end{cases}$$

if $c \leq 3-2K$, and

$$\Pi_s(D_s) = \begin{cases} (2-K-c-p_f)D_s, & \text{if } D_s \leq 2-K-c \\ \frac{(2-K+D_s-c)^2}{4} - p_f \times D_s, & \text{if } 2-K-c < D_s \leq K-1 \\ \frac{(2-K+D_s)(1-c)^2}{4} - p_f \times D_s, & \text{if } D_s > K-1 \end{cases}$$

if $c > 3-2K$. Specifically, when $c \leq 3-2K$, it is optimal to choose $D_s = 0$ if $p_f > 2-K-c$, $D_s = K-1$ if $(2-K)^2 - c < p_f \leq 2-K-c$, $D_s = (2-K)(\frac{1}{\sqrt{p_f+c}} - 1)$ if $\frac{(1-c)^2}{4} < p_f \leq (2-K)^2 - c$, and $D_s = K$ if $p_f \leq \frac{(1-c)^2}{4}$. When $c > 3-2K$, on the other hand, it is optimal to choose $D_s = 0$ if $p_f > \frac{(1-c)^2}{4(K-1)}$, $D_s = K-1$ if $\frac{(1-c)^2}{4} < p_f \leq \frac{(1-c)^2}{4(K-1)}$, and $D_s = K$ if $p_f \leq \frac{(1-c)^2}{4}$.

The firm chooses p_f in anticipation of speculators' and consumers' decisions to maximize its total profit.

Following the above steps, we obtain the equilibrium decisions subject to the constraint of $D_{1c} > 0$:

1. If $\begin{cases} 1 < K \leq \frac{4}{3} \\ c \leq \frac{8-10K+3K^2}{2} \end{cases}$, then $p_f = \frac{2-3c+2\sqrt{1+6c}}{9}$, $D_s = (2-K)(\frac{3}{1+\sqrt{1+6c}} - 1)$, $D_{1c} = K - (2-K)(\frac{3}{1+\sqrt{1+6c}} - 1)$, and $p_s = \frac{1+\sqrt{1+6c}}{3}$. The firm's and speculators' profits are, respectively, $\Pi_f = \frac{2-3c+2\sqrt{1+6c}}{9}K$ and $\Pi_s = \frac{(2-\sqrt{1+6c})(1-6c+\sqrt{1+6c})(2-K)}{9(1+\sqrt{1+6c})}$. Theconsumer surpluses are $CS_{early} = CS_{late} = \frac{5+6c-4\sqrt{1+6c}}{18}$ and $CS_{total} = \frac{5+6c-4\sqrt{1+6c}}{9}$. The social welfare is $\frac{1}{3}(K + 1 - \frac{3c(3-\sqrt{1+6c})(2-K)}{1+\sqrt{1+6c}})$.

2. If $\begin{cases} 1 < K \leq \frac{4}{3} \\ \frac{8-10K+3K^2}{2} < c \leq \frac{(2-K)^2}{2} \end{cases}$ or $\begin{cases} \frac{4}{3} < K \leq \sqrt{2} \\ c \leq \frac{(2-K)^2}{2} \end{cases}$ or $\begin{cases} \sqrt{2} < K \leq \frac{3}{2} \\ c \leq 3-2K \end{cases}$, then $p_f = \frac{(2-K)K}{2}$, $D_s = K-1$, $D_{1c} = 1$, and $p_s = 2-K$. The firm's and speculators' profits are, respectively, $\Pi_f = \frac{(2-K)K^2}{2}$

and $\Pi_s = \frac{(2-K)^2-2c}{2}(K-1)$. Consumer surpluses are $CS_{early} = CS_{late} = \frac{(K-1)^2}{2}$ and $CS_{total} = (K-1)^2$. Social welfare is $-1+c+2K-cK-\frac{K^2}{2}$.

3. If $\begin{cases} 1<K\le\sqrt{2} \\ \frac{(2-K)^2}{2} < c \le 3-2K \end{cases}$, then $p_f = 2-K-c$, $D_s = K-1$, $D_{1c} = 1$, and $p_s = 2-K$. The firm's and speculators' profits are, respectively, $\Pi_f = (2-K-c)K$ and $\Pi_s = 0$. Consumer surpluses are $CS_{early} = -\frac{3}{2}+c+K$, $CS_{late} = \frac{(K-1)^2}{2}$, and $CS_{total} = -1+c+\frac{K^2}{2}$. Social welfare is $-1+c+2K-cK-\frac{K^2}{2}$.

4. If $\begin{cases} 1<K\le\sqrt{2} \\ c>3-2K \end{cases}$ or $\begin{cases} \sqrt{2}<K\le\frac{5}{3} \\ c>1-2\sqrt{\frac{K-1}{K+1}} \end{cases}$ or $K>\frac{5}{3}$, then $p_f = \frac{(1-c)^2}{4(K-1)}$, $D_s = K-1$, $D_{1c} = 1$, and $p_s = \frac{1+c}{2}$. The firm's and speculators' profits are, respectively, $\Pi_f = \frac{(1-c)^2}{4(K-1)}K$ and $\Pi_s = 0$. Consumer surpluses are $CS_{early} = \frac{1}{2} - \frac{(1-c)^2}{4(K-1)}$, $CS_{late} = \frac{(1-c)^2}{8}$, and $CS_{total} = \frac{1}{2} - \frac{(1-c)^2}{4(K-1)} + \frac{(1-c)^2}{8}$. Social welfare is $\frac{7-6c+3c^2}{8}$.

5. If $\begin{cases} \sqrt{2}<K\le\frac{3}{2} \\ 3-2K<c\le 1-2\sqrt{\frac{K-1}{K+1}} \end{cases}$ or $\begin{cases} \frac{3}{2}<K\le\frac{5}{3} \\ c\le 1-2\sqrt{\frac{K-1}{K+1}} \end{cases}$, then $p_f = \frac{3+2c-c^2}{8}$, $D_s = K-1$, $D_{1c} = 1$, and $p_s = \frac{1+c}{2}$. The firm's and speculators' profits are, respectively, $\Pi_f = \frac{3+2c-c^2}{8}K$ and $\Pi_s = \frac{5-3K-2c(K+1)+c^2(K+1)}{8}$. Consumer surpluses are $CS_{early} = CS_{late} = \frac{(1-c)^2}{8}$ and $CS_{total} = \frac{(1-c)^2}{4}$. Social welfare is $\frac{7-6c+3c^2}{8}$.

By comparing the firm's expected profit subject to the constraint of $D_{1c} > 0$ with that subject to the constraint of $D_{1c} = 0$ (i.e., $\begin{cases} \frac{(2-K)K}{2}, & \text{if } K\le 1 \\ \frac{1}{2}, & \text{if } K>1 \end{cases}$), we can obtain the firm's (unconditional) equilibrium profit.

Next, we show that the main results remain to hold in this extension with the marginal cost of speculative reselling.

To obtain the result regarding the reselling cost c, note that for $1 < K \le \frac{4}{3}$, $\frac{d\Pi_f|_{D_{1c}>0}}{dc} > 0$ if $c < \frac{8-10K+3K^2}{2}$, $\frac{d\Pi_f|_{D_{1c}>0}}{dc} = 0$ if $\frac{8-10K+3K^2}{2} \le c \le \frac{(2-K)^2}{2}$, and $\frac{d\Pi_f|_{D_{1c}>0}}{dc} < 0$ if $c > \frac{(2-K)^2}{2}$; for $\frac{4}{3} < K \le \sqrt{2}$, $\frac{d\Pi_f|_{D_{1c}>0}}{dc} = 0$ if $c \le \frac{(2-K)^2}{2}$ and $\frac{d\Pi_f|_{D_{1c}>0}}{dc} < 0$ if $c > \frac{(2-K)^2}{2}$; for $\sqrt{2} < K \le \frac{3}{2}$, $\frac{d\Pi_f|_{D_{1c}>0}}{dc} = 0$ if $c \le 3-2K$, $\frac{d\Pi_f|_{D_{1c}>0}}{dc} > 0$ if $3-2K < c < 1-2\sqrt{\frac{K-1}{K+1}}$, and $\frac{d\Pi_f|_{D_{1c}>0}}{dc} < 0$ if $c > 1-2\sqrt{\frac{K-1}{K+1}}$; for $\frac{3}{2} < K \le \frac{5}{3}$, $\frac{d\Pi_f|_{D_{1c}>0}}{dc} > 0$ if $c < 1-2\sqrt{\frac{K-1}{K+1}}$, and $\frac{d\Pi_f|_{D_{1c}>0}}{dc} < 0$ if $c > 1-2\sqrt{\frac{K-1}{K+1}}$; for $\frac{5}{3} < K \le 2$, $\frac{d\Pi_f|_{D_{1c}>0}}{dc} < 0$. Moreover, $\frac{d\Pi_f|_{D_{1c}=0}}{dc} = 0$. Thus, $\Pi_f^* = \max\{\Pi_f|_{D_{1c}>0}, \Pi_f|_{D_{1c}=0}\}$ is first (weakly) increasing and then (weakly) decreasing in c.

When $1 < K < \frac{4}{3}$, Π_f^* achieves its strict maximum at any $c \in [\frac{8-10K+3K^2}{2}, \frac{(2-K)^2}{2}]$. When $\sqrt{2} < K < \frac{5}{3}$, Π_f^* achieves its strict maximum at $c = 1-2\sqrt{\frac{K-1}{K+1}}$. This shows that an intermediate value of c is strictly better than $c = 0$ and $c \to \infty$ if the firm's capacity K is intermediate.

To obtain the result regarding the firm's capacity K, note that when $c = \frac{2}{5}$, the firm's equilibrium profit

$$\Pi_f = \begin{cases} \frac{(2-K)K}{2}, & \text{if } K \leq 1 \\ \frac{1}{2}, & \text{if } 1 < K \leq \frac{5(\sqrt{85}-2)}{36} \\ \frac{2-3c+2\sqrt{1+6c}}{9}K, & \text{if } \frac{5(\sqrt{85}-2)}{36} < K \leq \frac{25-\sqrt{85}}{15} \\ \frac{(2-K)K^2}{2}, & \text{if } \frac{25-\sqrt{85}}{15} < K \leq \frac{2(5-\sqrt{5})}{5} \\ (2-K-c)K, & \text{if } \frac{2(5-\sqrt{5})}{5} < K \leq \frac{8+\sqrt{14}}{10} \\ \frac{1}{2}, & \text{if } K > \frac{8+\sqrt{14}}{10} \end{cases}$$

is strictly maximized at $K = \frac{2(5-\sqrt{5})}{5}$, meaning that an intermediate capacity may be optimal for the firm. Moreover, $K = \frac{2(5-\sqrt{5})}{5}$ is higher than $K = 1$, the firm's optimal capacity without speculative reselling, meaning that the presence of speculators can induce the firm to choose a higher capacity.

This completes the analysis of Section 5.6 and proves that the main results hold with the marginal cost of speculative reselling. $\square$