Online Appendix
Bricks Processing Returns for Clicks: Can Foes Become Friends?

Appendix A: Extensions

A.1. BORS Agreement with Showrooming

In our main model, customers who visit the offline store do not infer any information about the online products. However, in some cases, the online and offline products can share some common attributes (e.g., the prevalent style or fashion for apparel). In such cases, customers can infer some information about the online product by inspecting the offline product. Customers may also take advantage of the offline pre-sales service and store assistance to learn more about these common attributes (Janssen and Ke 2020). Hence, customers can reduce their uncertainty about their match with the online product by visiting the offline store. In this section, we model this uncertainty reduction and study its impact on the BORS agreement. Let μ be the extent of the reduction in online match uncertainty after a visit to the offline store. Since the online and offline products have differentiated brands in our model, showrooming customers may not be able to fully resolve their uncertainty upon visiting the offline store. Therefore, visiting the offline store reduces the probability of mismatch from an online purchase from λ to $\lambda - \mu$, where $\mu < \lambda$.[1] As a result, customers may decide to visit the offline store to reduce their online match uncertainty and subsequently purchase the product online. We label this behavior as "showrooming".

PROPOSITION 1. *In the setting where offline visits can reduce the online match uncertainty,*

a) consumers do not showroom under the BORS agreement,

b) consumers showroom under no agreement when the uncertainty reduction is high ($\mu > \frac{\delta}{h+\delta}$),

b) the BORS agreement benefits both retailers if and only if the online retailer relinquishes returners to the offline retailer ($\theta < \theta_2$) and $\theta > \tilde{\theta}_\pi$,

where
$$\tilde{\theta}_\pi = \begin{cases} \theta_\pi, & \mu \leq \dfrac{\delta}{h+\delta}, \\ \dfrac{\sqrt{1-\lambda}\left(\delta\left(\lambda^2 - (\lambda+3)\mu + \lambda + 3\right) + h(\lambda+3)(\lambda-\mu)\right)}{\lambda(\lambda+15)} + \dfrac{\delta(\lambda(-4\lambda+3\mu-3) - 3\mu + 3) - 3h(\lambda-1)(\lambda-\mu)}{\lambda(\lambda+15)}, & \text{otherwise}. \end{cases}$$

c) As the uncertainty reduction (μ) increases, the region where the agreement benefits both retailers expands, i.e., $\frac{d(\theta_2 - \tilde{\theta}_\pi)}{d\mu} \geq 0$.

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[1] As in our main model, we can show that the agreement cannot benefit both retailers unless some customers repurchase offline after returning offline (strategy ISS). Hence, we focus on the more interesting parameter space where at least some customers follow strategy ISS ($\lambda < 1/2$). Note that $\mu < \lambda < 1/2$ within our region of interest.

Proposition 5 highlights that customers are more likely to showroom under no agreement. A customer who is deciding between showrooming and purchasing from the online retailer directly faces the following trade-off – she can reduce the risk of return from an online purchase by visiting the offline store but has to incur the transportation cost in order to do so. Hence, it is only when this uncertainty reduction is high enough that the customer decides to engage in showrooming. Under the BORS agreement, since returns are not as costly, customers require a higher reduction in uncertainty to justify the transportation cost. In fact, we find that in our region of interest, no amount of uncertainty reduction is enough for customers to engage in showrooming under the agreement.

Additionally, we find that the BORS agreement can only benefit both retailers when the online retailer relinquishes returners to the offline retailer. When μ is low, and customers do not engage in showrooming in either case (agreement and no agreement), our analysis remains similar to the base model. When μ is high, and customers engage in showrooming in the no agreement case, the retailers can still benefit from the agreement. In this case, $\tilde{\theta}_\pi$ is a function of μ. Interestingly, we find that as μ increases (the value of showrooming increases) the region of the agreement expands. The intuition behind this result is that when the value of showrooming is high, removing showrooming behavior (under the agreement) is more beneficial for the offline retailer, making the agreement more likely. Therefore, our results show that, even in the presence of potential showrooming, the agreement can simultaneously benefit both retailers and dissuade customers from showrooming behavior.

A.2. BORS Agreement in the Presence of Another Competing Offline Retailer

In our main model, we abstract away from outside competition to focus on the interplay between the two retailers (one online and one offline) that enter into the BORS agreement. In this extension, we now consider a setting with three retailers $j \in \{I, S_1, S_2\}$: an online retailer (I) and two offline retailers (S_1 and S_2). The online retailer can partner with either one or both offline retailers. We start by assuming that both offline retailers are symmetric and then examine the more interesting case where the offline retailers are asymmetric.

In order to model this setting with three retailers, we use the Spokes model. There are three spokes, of length $\frac{1}{2}$ each, on a plane and each retailer is located at the origin of its corresponding spoke j. Consumers of unit mass are uniformly distributed on the spokes and each consumer, located on spoke j, considers at most two retailers – the local retailer j and one of the non-local retailers with a probability $\frac{1}{2}$. This assumption is standard in the literature grounded in the spokes model and ensures the existence of a pure strategy equilibrium (Chen and Riordan 2007, Amaldoss and He 2010, 2013, 2018).

LEMMA 1. *In a setting with one online and two symmetric offline retailers:*

- *the BORS agreement (with one or two offline retailers) is feasible (benefits all partnering retailers) only when the online retailer relinquishes returners to the partnering offline retailer(s),*
- *whenever the agreement is feasible with one offline retailer, it is also feasible with two offline retailers,*
- *the online retailer prefers partnering with two offline retailers over partnering with one offline retailer.*

Lemma 1 provides robustness to our key mechanism. Even in the presence of competition between offline retailers, the BORS agreement can benefit the online and the partnering offline retailer(s) thanks to the competition alleviation effect that results from relinquishing returners. In the symmetric setting, we find that whenever the agreement is feasible with one offline retailer, it is also feasible with two offline retailers. Furthermore, the online retailer prefers partnering with two retailers over one.

We now relax the symmetry assumption and consider the case where the two offline retailers may be asymmetric. Formally, we allow the two retailers S_1 and S_2 to be situated on asymmetric spokes of length $1/2-\alpha$ and $1/2+\alpha$, respectively, where $0 \leq \alpha < 1/2$. This decreases (increases) the number of customers who have S_1 (S_2) in their consideration set while keeping the number of customers who have I in their consideration set the same. We refer to the two retailers as "small" and "large" offline retailers, respectively, for the rest of this section.

PROPOSITION 2. *In a setting with one online and two offline retailers:*

- *the BORS agreement (with one or two offline retailers) is feasible (benefits all partnering retailers) only when the online retailer relinquishes returners to at least one of the partnering offline retailer(s),*
- *under certain conditions, the BORS agreement is feasible with one offline retailer but not with two offline retailers.*

Proposition 2 provides further robustness to our key mechanism, showing that our insights hold even under competition between asymmetric offline retailers. As in the symmetric case, the agreement may be feasible (benefit all involved parties) when the online retailer relinquishes returners to both offline retailers. In addition, we find that when the online retailer partners with both offline retailers, the agreement may benefit all three even if the online retailer relinquishes returners to only the small offline retailer. When the online retailer partners with only one offline retailer, again, as in the symmetric case, the agreement can be feasible when returners are relinquished. Interestingly, we find that, unlike the symmetric case, there exists some parameter space where the agreement with only one retailer is feasible, but the agreement with both retailers is not feasible. The intuition behind this result is that when the offline retailers are asymmetric, the smaller retailer may be worse off under the BORS agreement with both retailers, making it infeasible. Under BORS agreement with both, if the online retailer relinquishes returners only to the small retailer (S_1), the large retailer (S_2) becomes aggressive as it only faces the return flexibility effect. This, in turn, makes the small retailer worse off. Therefore, S_1 would prefer to be the sole partner of the online retailer. Even when the online retailer relinquishes to both retailers, the return flexibility effect may be too negative to be offset by the competition alleviation effect. In this case, the small retailer (S_1) would rather stay out of the BORS agreement to avoid the return flexibility effect and have the other two retailers enter into the BORS agreement.

A.3. Low Online Return Hassle Cost

In the main model, we considered the scenario where the online return hassle cost is higher than the transportation cost to the offline retailer ($h > \delta$). In this section, we extend the main model by considering a setting where the online retailer has alternative ways to reduce the return hassle of customers and investigate whether the BORS agreement can still benefit both retailers. For example, Amazon can provide customers with the option to return products at Amazon-owned locations such as Hub locker+ and Whole Foods Market stores (Amazon.com 2020), which may be more conveniently located than Kohl's stores. To explore the impact of these options on the BORS agreement, we study the setting where the hassle cost of returning to the online retailer via its provided solutions is lower than the cost of traveling to the offline retailer ($h \leq \delta$). Therefore, in this extended model, the offline retailer can no longer provide customers with a more convenient way (in terms of hassle costs) of returning products. We examine whether both retailers can still benefit from the BORS agreement despite this disadvantage.

PROPOSITION 3. *The BORS agreement can benefit both retailers even when the online retailer has alternate ways to reduce the return hassle cost of its customers ($h \leq \delta$). This happens when the brand differentiation is moderate: $\theta^l_\pi < \theta \leq \theta^l_2$, where $\theta^l_\pi = \theta_\pi$ and $\theta^l_2 = \frac{3h(1-\lambda)\left(\sqrt{1-\lambda}+1\right)-\delta\lambda\left(3\sqrt{1-\lambda}+1\right)}{2(3-\lambda)}$.*

Interestingly, this proposition shows that the BORS agreement can benefit both retailers even when the offline retailer no longer provides customers with a less costly way to return their online purchases. Intuitively, customers should not be interested in returning to the offline retailer in this case. However, our results show that not only do customers return to the offline store, but also the online retailer may decide to relinquish all returners to the offline retailer. The rationale behind customers' decision to return offline stems from the opportunity to make their repurchase at the offline retailer. By returning offline, they can kill two birds with one stone. They get to return the product, albeit at a higher cost, but then repurchase offline without having to incur additional purchasing costs (they would have incurred a waiting time cost upon ordering the replacement online). This can provide a higher utility to customers facing returns, increasing their expected utility of making the first purchase online. Therefore, returns become more flexible for online customers under the agreement, and the previously discussed return flexibility effect continues to exist even when $h \leq \delta$.

In addition, the online retailer may still choose to relinquish returners to the offline retailer. This leads to higher prices for both retailers owing to the competition alleviation effect. This happens when $\theta_p < \theta \leq \theta_2^l$ (see Figure A.1a). We find that, as in the main model, when the brand differentiation is moderate, the online retailer relinquishes customers to the offline retailer ($\theta \leq \theta_2^l$) and the benefit from the competition alleviation effect dominates any negative impact of the return flexibility effect ($\theta > \theta_\pi^l$). This makes the offline retailer better off under the agreement when $\theta_\pi^l < \theta \leq \theta_2^l$ (see Figure A.1b). As before, both the effects of competition alleviation and return flexibility are positive for the online retailer, ensuring it benefits from the agreement.

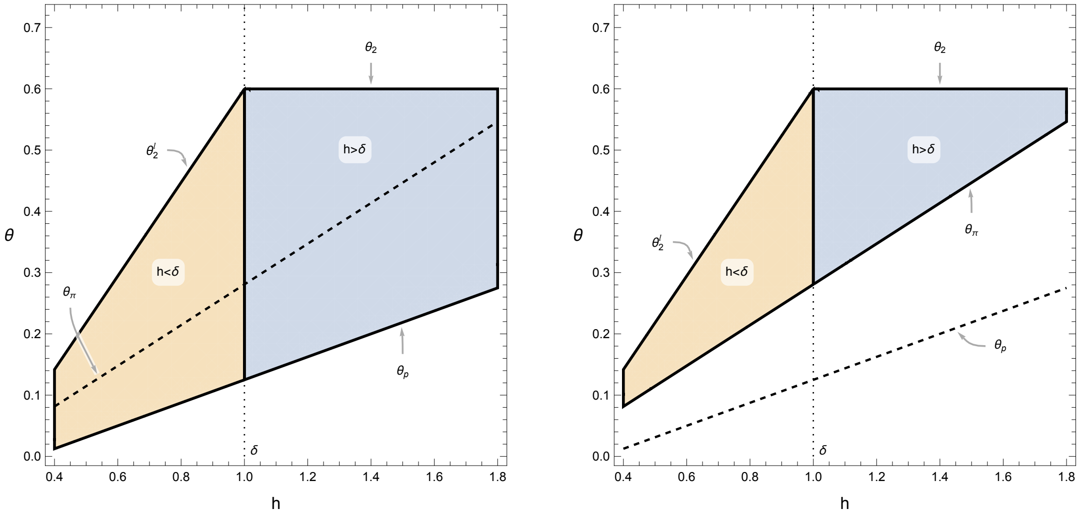
This extension provides robustness to our competition alleviation mechanism that drives the BORS agreement. Our findings show that the benefit of the BORS agreement goes above and beyond the simple explanation of reduced return hassle put forth by the business press. Moreover, this extension further reinforces the result that the BORS agreement benefits both retailers only when the online retailer decides to relinquish customers to the offline retailer. As such, our results remain the same qualitatively even when the online return hassle is lower than the offline transportation cost. However, the region where the agreement benefits both retailers changes.

COROLLARY 1. *When $h > \delta$, the range of brand differentiation values ($\theta_2 - \theta_\pi$) where the BORS agreement benefits both retailers reduces as the online return hassle cost (h) increases. When $h \leq \delta$, the range of brand differentiation values ($\theta_2^l - \theta_\pi$) where the BORS agreement benefits both retailers expands as the online return hassle cost (h) increases.*

Interestingly, when $h > \delta$, we find that the agreement is less likely as h increases. The range under which the agreement benefits both retailers is given by $\theta_2 - \theta_\pi$. We find that the lower threshold θ_π is increasing in h. As discussed earlier, θ_π is determined by the θ threshold above which the offline retailer makes a higher profit under the agreement as compared to no agreement. This threshold, in turn, is affected by the return flexibility effect, which imposes a negative force on the offline retailer's profit. Clearly, as h increases, the return flexibility effect becomes stronger. As a result, the level of brand differentiation between the two retailers has to be high enough for the competition alleviation effect to offset the return flexibility effect. We also find that the upper boundary θ_2 is independent of h when $h > \delta$. As θ_π increases with h and θ_2 does not depend on h, the range $\theta_2 - \theta_\pi$ decreases with h. In other words, when $h > \delta$, a BORS agreement between competing online and offline retailers is less likely to benefit them both when customers incur a high cost to return online.

However, as depicted in Figure A.1b, this range expands with h when the online hassle cost is low ($h \leq \delta$). This finding aligns with the intuition that the agreement becomes more useful to retailers as the online hassle cost increases.

Figure A.1 Region where the BORS agreement lifts prices / profits for both retailers ($\nu = 10$**,** $\lambda = 1/4$ **and** $\delta = 1$**)**



(a) Region with Higher Prices

(b) Region with Higher Profits

Unlike the main model, the online retailer's decision to relinquish returners (and hence, the threshold θ_2^l) now depends on h. As h increases, customers who decide to return and repurchase online incur a higher hassle cost. This makes attracting (not relinquishing) them costlier for the online retailer (it has to drop its price). This expands the relinquishing equilibrium range by increasing θ_2^l. Hence, the upper threshold of the range where the agreement benefits both retailers (θ_2^l) increases as h increases. As we already established, the lower threshold of the range (θ_π) that defines the region where the offline retailer is better off also increases as h increases. Since the online hassle cost clearly has a stronger impact on the online retailer as compared to the offline retailer, we find that the upper threshold (θ_2^l) increases at a faster rate than the lower threshold (θ_π). Therefore, the range of the brand differentiation values for which the agreement benefits both retailers expands as h increases.

Moreover, from Corollary 1, we can conclude that it is less likely for an online and an offline retailer to be better off under the agreement when the online hassle cost is either too high or too low (Figure A.1b). The range of θ where the agreement benefits both retailers is the widest when $h = \delta$ and shrinks as h moves away from δ in either direction.

A.4. BORS Agreement with Time Discounting

In our main model, we model customers' waiting time cost as additive in their utility in line with previous work (Chen et al. 2023, Hao and Kumar 2023). In this extension, we consider an alternate specification for customers' waiting time cost by modeling it as multiplicative due to time discounting. In this model, customers discount any future utility received after a waiting time. When the waiting time cost is additive, customers' purchase, return and repurchase decisions do not depend on the product valuation (v). Modeling the waiting time cost as multiplicative allows us to capture the effect of customers' product valuation on their decisions. If customers value a product highly, they experience a higher pain for waiting for the product and vice versa.

In the main model, to keep both retailers on an equal footing, we assumed both the offline transportation cost (d) and the online waiting time cost (δ) to be additive and equal. In this extension, the online waiting time cost is modeled as multiplicative. Hence, this naturally allows us to also explore the situation where the offline transportation cost is unequal (higher or lower) to the online waiting time cost.

In both settings (agreement and no agreement), a customer who purchases at the offline store does not have to wait for product delivery and hence gets the same utility as our base model, given by $v - \theta(1-x) - p_S - d$. A customer who purchases online has to wait for the product delivery after placing her order and making the payment p_I. With probability $(1-\lambda)$, she will find her match after waiting for one period and expects a utility $\delta(v - \theta x) - p_I$. With probability λ, she will find a mismatch. Under no agreement, she will immediately return after the delivery (expects a hassle cost δh), repurchase the product online, and hence wait for an additional time period (total of two time periods) for the product. Thus, with probability λ, she expects a utility $\delta^2(v - \theta x) - \delta h - p_I$.

Under the agreement, she will return the product after the delivery either online (expects a hassle cost δh) or offline (expects a hassle cost δd). If she plans to return online, she expects the same utility as under no agreement. If she plans to return offline, she can either repurchase the product online or offline. Repurchasing online entails waiting for an additional time period (total of two time periods) and an expected utility of $\delta^2(v - \theta x) - \delta d - p_I$. Repurchasing offline entails no additional waiting period. She will obtain a refund of the online price and pay the offline price, and expect a utility of $\delta(v - \theta(1-x) - p_S + p_I) - \delta d - p_I$.

As in our base model, we solve for the customer decisions and strategies using backward induction (while accounting for the timing of the decisions). There are two relevant cases under the agreement: i) the online retailer relinquishes returners, and ii) the online retailer does not relinquish returners. We solve for the various equilibria and summarize our findings in Proposition 4.

PROPOSITION 4. *When customers discount future utility, the BORS agreement can benefit both retailers only if the online retailer relinquishes returners to the offline retailer.*

Proposition 4 provides support to the results and mechanism in our main model. Relinquishing customers relaxes price competition between the two retailers, making both retailers better off under the agreement. Given the complexity of this model due to the multiplicative nature of the waiting time cost, we explore the conditions under which relinquishing happens, and both retailers are better off using a simulation. We run a simulation by generating $1M$ random values of the parameters and find that the agreement benefits both retailers when θ is moderate. We also investigate the impact of the product valuation (v) on the agreement. We find that the agreement benefits both retailers only when v is moderate. We provide some instances in Table A.1 in which we tabulate the v ranges where we find i) the relinquishing equilibrium and ii) both retailers to be better off under the agreement.

δ	θ	λ	v range for relinquishing equilibrium under the agreement	v range for both retailers to be better off under the agreement
0.5	1.25	0.25	$[1.71, 7.51]$	$[1.71, 6.64]$
0.25	1.5	0.1	$[0.870, 6.12]$	$[0.870, 4.12]$
0.75	1	0.2	$[2.20, 13.4]$	$[2.20, 12.4]$

Table A.1 **v range for the relinquishing equilibrium and the two retailers to be better off under the agreement for different parameter values ($h = 1.8$ and $d = 1.4$).**

We find that the agreement benefits both retailers when v is moderate. The intuition for this finding is as follows. When v is low, since the waiting time cost is multiplicative, the pain of waiting for the product is also low. Hence, the online retailer finds it easier to attract repurchases from customers who return offline, and therefore, it has a stronger incentive not to relinquish them to the offline retailer. Therefore, we find that the online retailer does not want to relinquish returners when v is sufficiently low. When v is high, the online retailer finds it optimal to relinquish returners, and the competition between the two relaxes. However, as v increases, the return flexibility effect also strengthens. In other words, the agreement makes it more likely that customers buy from the online retailer on the first purchase occasion. While customers who face a return have to wait for an additional period under no agreement (since they can only repurchase online), the agreement offers them an option to avoid this additional waiting period by repurchasing offline. Therefore, when v is sufficiently high, the positive competition alleviation effect cannot compensate for the loss from the return flexibility effect, and the offline retailer is worse off under the agreement.

A.5. Heterogeneous Transportation Cost

In our main model, we assume all customers have the same cost of transportation to the offline retailer. In this section, we relax this assumption by considering two equal-sized segments of customers – one with low transportation cost (δ) and one with high transportation cost ($\delta + \Delta$ where $\Delta > 0$). Both segments incur the same waiting time cost of shopping online. This extension allows us to investigate the robustness of our results in a setting where customers are homogeneous in their waiting time cost from purchasing online but are heterogeneous in their transportation cost when purchasing offline, depending on their geographical location.

Consider the case where both segments enjoy a lower transportation cost than the hassle of online return. In that case, all customers who face a return make their return offline. Since the transportation cost is sunk upon return, both segments behave identically for their repurchase decision. As a result, the online retailer can relinquish returners from either both segments together or none at all. Our key findings remain the same qualitatively under this scenario.

In this extension, we focus on the more interesting parameter space where the hassle of online return is higher than the transportation cost of the "low" segment but lower than the transportation cost of the "high" segment ($\delta < h < \delta + \Delta$). In this case, the low segment always returns offline, but the high segment may return online. Hence, they may differ in their repurchase behavior, and the online retailer can partially relinquish returners. The following proposition summarizes our results.

PROPOSITION 5. *When customers have heterogeneous transportation costs to the offline store, the BORS agreement can benefit both retailers only if the online retailer relinquishes at least the low transportation cost returners to the offline retailer.*

Proposition 5 illustrates that, as in our main model, both retailers benefit from the BORS agreement only when the online retailer relinquishes returners. However, the agreement can benefit both retailers even when only one of the two segments is relinquished. The two segments differ in their return and repurchase behavior. A "high" type customer incurs a higher return cost offline than online ($\delta + \Delta > h$). Hence, she returns online if she plans to repurchase online and returns offline when she plans to repurchase offline (as discussed in Section 7.3). A "low" type customer, on the other hand, incurs a lower return cost offline than online and hence, always returns offline. Since a "high" type customer can save some return hassle by returning and repurchasing online, all else equal, she is more likely to repurchase online (relative to a "low" type customer). Hence, if a "high" type customer chooses to repurchase offline, so would a "low" type customer. As a consequence, the online retailer cannot relinquish returners from the "high" segment without also relinquishing returners from the "low" segment (see Appendix B.15 for more details).

Therefore, we find three relevant cases under the agreement – Case 1 without relinquishing, Case 2 with relinquishing of the "low" type returners, and Case 3 with relinquishing of all returners. The competition alleviation effect, which stems from a reduction in price sensitivities, is present in both relinquishing cases but differs in strength. To illustrate this, we present the demands of the offline retailer under no agreement (Q_S^0) and the three cases under the agreement (Q_S^{1i} for Case i, where $i \in \{1,2,3\}$) in Equation A-1.

$$\begin{aligned}
Q_S^0 &= \frac{2\theta + 2\lambda(h+\delta) - \Delta}{4\theta} - \frac{p_S - p_I}{2\theta},\\
Q_S^{11} &= \frac{2\theta + \lambda(h+3\delta) - \Delta}{4\theta} - \frac{p_S - p_I}{2\theta},\\
Q_S^{12} &= \frac{2\theta + \lambda(h+\theta+2\delta) - \Delta}{4\theta} - \left(1 - \frac{\lambda}{2}\right)\frac{p_S - p_I}{2\theta},\\
Q_S^{13} &= \frac{2\theta + 2\lambda(\theta+\delta) - (1-\lambda)\Delta}{4\theta} - (1-\lambda)\frac{p_S - p_I}{2\theta}.
\end{aligned} \tag{A-1}$$

As is evident from Equation A-1, the magnitudes of the own and cross price sensitivities are equal to $\frac{1}{2\theta}$ in both Case 1 of the agreement and no agreement. Therefore, we do not find the competition alleviation effect in Case 1. However, the magnitudes of the price sensitivities drop to $\frac{1-\lambda/2}{2\theta}$ and $\frac{1-\lambda}{2\theta}$ in Case 2 and Case 3 of the agreement, respectively. Since $\frac{1-\lambda/2}{2\theta} > \frac{1-\lambda}{2\theta}$, the drop in price sensitivity is higher when all returners are relinquished (Case 3) as compared to the drop when only the "low" type returners are relinquished (Case 2). As a result, we expect the competition alleviation effect to be stronger when all returners are relinquished as compared to when only the "low" type returners are relinquished.

We find three equilibria under the agreement, corresponding to the three cases – $P1$ without relinquishing, $P2$ with relinquishing of "low" type returners and $P3$ with relinquishing of all returners. We present the θ ranges for each equilibrium and the range in which the agreement benefits both retailers in Table A.2 to

Δ	θ range for Eq P1 (no relinquishing)	Agreement benefits both retailers under Eq P1	θ range for Eq P2 (relinquish "low" returners)	Agreement benefits both retailers under Eq P2	θ range for Eq P3 (relinquish all returners)	Agreement benefits both retailers under Eq P3
0.55	$[1.189, +\infty)$	NO	$\emptyset$	$\emptyset$	$[0.402, 1.016]$	If $\theta > 0.538$
0.80	$[1.262, +\infty)$	NO	$[0.974, 1.176]$	YES	$[0.682, 0.894]$	YES
2.00	$[1.683, +\infty)$	NO	$[1.497, 1.574]$	YES	$\emptyset$	$\emptyset$

Table A.2 **θ range for the two retailers to be better off under the agreement for different values of Δ at $v = 20$, $\lambda = 0.25$, $h = 2$ and $\delta = 1.5$**

illustrate our findings. We observe that when Δ is low, only $P1$ and $P3$ exist, when Δ is medium, all $P1$, $P2$ and $P3$ exist, and when Δ is high, only $P1$ and $P2$ exist. In other words, the online retailer relinquishes all returners when Δ is low and relinquishes only the "low" type returners when Δ is high. The online retailer may relinquish all or only "low" type returners when Δ is moderate, depending on the level of differentiation. The intuition behind this finding is that as Δ increases, the "high" type returners are more inclined to (return directly to and) repurchase online as compared to (returning and) repurchasing offline. Hence, the online retailer is less inclined to relinquish the "high" type returners when Δ is high.

Further, we note that, in the no relinquishing equilibrium ($P1$), the BORS agreement cannot benefit both retailers (specifically, it cannot benefit the offline retailer). However, the agreement can potentially benefit both retailers in each of the two relinquishing equilibria ($P2$ and $P3$). The intuition for this result is similar to the base model. The return flexibility effect works against the offline retailer, rendering it worse off in $P1$. The competition alleviation effect, which only exists in $P2$ and $P3$, lifts the prices of the two retailers and can make both retailers better off.

A.6. In-store Shopping Cost

Our main model assumes that consumers only incur the transportation cost when buying from the offline retailer. However, in the real world, consumers may incur additional costs while shopping in the offline store. These costs may stem from the waiting time spent in a queue at checkout, the hassle of looking for products, and/or the general shopping experience in the store. We label these additional costs (on top of the transportation cost) incurred upon purchasing offline as "in-store shopping costs". Customers may have different sensitivities to such costs, and therefore, we consider two segments of customers: one with low in-store shopping cost (σ_l) and one with high in-store shopping cost (σ_h). For ease of exposition, we assume that $\sigma_l = 0$ and $\sigma_h = \sigma$, and that the segments are equal in size. Note that the in-store shopping cost is only incurred in case of an offline purchase, not upon a return made at the offline store. Hence, a customer who is returning offline and repurchasing online would incur the transportation cost δ but not the in-store shopping cost. However, a customer in segment $i \in \{l, h\}$ who purchases offline (either as a first purchase or after returning an online product) would incur both the transportation and in-store shopping costs $\delta + \sigma_i$. The following proposition summarizes our results.

PROPOSITION 6. *When buying offline involves additional in-store shopping costs over and above the transportation cost, the BORS agreement can benefit both retailers only if the online retailer relinquishes at least the low in-store shopping cost returners to the offline retailer.*

Proposition 6 illustrates that, as in our main model, both retailers can be better off under the agreement only when the online retailer relinquishes returners. However, the agreement can benefit both retailers even when only one of the two segments is relinquished. We find that the online retailer always relinquishes the low-type returners if it relinquishes the high-type returners. To understand the intuition behind this, consider a returner who, after making her return at the offline store, is deciding between repurchasing online vs repurchasing offline. The high-type returner incurs an additional in-store shopping cost upon repurchasing offline and, hence, is more inclined to repurchase online as compared to a low-type returner. Hence, if the high-type returner chooses to repurchase offline, so would the low-type returner. Therefore, we find three relevant cases under the agreement – Case 1 without relinquishing, Case 2 with relinquishing of the "low" type returners, and Case 3 with relinquishing of all returners. The competition alleviation effect, which stems from a reduction in price sensitivities, is present in both relinquishing cases but differs in strength. To illustrate this, we present the demands of the offline retailer under no agreement (Q_S^0) and the three cases under the agreement (Q_S^{1i} for Case i, where $i \in \{1, 2, 3\}$) in Equation A-2.

$$\begin{aligned}
Q_S^0 &= \frac{2\theta + 2(\delta + h)\lambda - \sigma}{4\theta} - \frac{p_S - p_I}{2\theta}, \\
Q_S^{11} &= \frac{2\theta + 4\delta\lambda - \sigma}{4\theta} - \frac{p_S - p_I}{2\theta}, \\
Q_S^{12} &= \frac{2\theta + (3\delta + \theta)\lambda - \sigma}{4\theta} - (1 - \frac{\lambda}{2})\frac{p_S - p_I}{2\theta}, \\
Q_S^{13} &= \frac{2\theta + 2(\delta + \theta)\lambda - \sigma(1 - \lambda)}{4\theta} - (1 - \lambda)\frac{p_S - p_I}{2\theta}.
\end{aligned} \tag{A-2}$$

As is evident from Equation A-2, the magnitudes of the own and cross price sensitivities are equal to $\frac{1}{2\theta}$ in both Case 1 of the agreement and no agreement. Therefore, we do not find the competition alleviation effect in Case 1. However, the magnitudes of the price sensitivities drop to $\frac{1-\lambda/2}{2\theta}$ and $\frac{1-\lambda}{2\theta}$ in Case 2 and Case 3 of the agreement, respectively. Since $\frac{1-\lambda/2}{2\theta} > \frac{1-\lambda}{2\theta}$, the drop in price sensitivity is higher when all returners are relinquished (Case 3) as compared to the drop when only the "low" type returners are relinquished (Case 2). Hence, the competition alleviation effect is stronger in Case 3 as compared to Case 2. Since the competition alleviation effect exists in both Cases 2 and 3 of the agreement, we find that the agreement can benefit both retailers in the equilibria arising from these two cases. Proposition 6 further establishes the robustness of our results by demonstrating that even relinquishing a portion of the returners can lead to the strategic competition alleviation effect and render both retailers better off under the agreement.

A.7. Return Processing Cost

In our main model, we assumed that there is no additional cost for the offline retailer when a product is returned offline. However, in practice, the offline retailer may incur a cost to process the returned products (e.g., packing, shipping back to the online retailer, temporary stocking). In this section, we extend the main model by examining a setting where the offline retailer incurs a processing cost $c > 0$ for each returned product to the offline store under the agreement. The retailers' profits under the agreement are then given by $\Pi_S = p_S Q_S - c Q_r$ and $\Pi_I = p_I Q_I$, where Q_r is the total number of returned products to the offline retailer. The following proposition summarizes the impact of c on the BORS agreement.

PROPOSITION 7. *When the offline retailer incurs a return processing cost (c), the BORS agreement benefits both retailers if and only if the online retailer relinquishes returners to the offline retailer and $c < \bar{c}$, where* $\bar{c} = \frac{\theta(6-\lambda)-\lambda\delta-\sqrt{9\theta^2(4-3\lambda)-6\theta\lambda(\delta(\lambda+2)-h(1-\lambda))+(1-\lambda)\lambda^2(\delta+h)^2}}{\lambda}$.

Proposition 7 shows that both retailers are better off under the agreement only when the online retailer relinquishes returners, and the processing cost is sufficiently low. Under the agreement, the offline retailer incurs a marginal cost c for λ proportion of customers who make their first purchase online (since with probability λ these customers would face a return and since $h > \delta$ all returners return offline). Hence, the BORS agreement has a direct negative impact on the offline retailer due to this additional cost. Additionally, as c increases, the offline retailer would reduce its price in order to attract more customers to make their first purchase online in order to reduce the number of returns it has to process. Hence, we find that as c increases, these two negative effects on total processing cost and price become stronger, making the offline retailer worse off.

A.8. Return Reconditioning Cost

In our main model, we assumed that there is no additional cost for the online retailer when a product is returned. However, in practice, the online retailer may incur a cost to recondition returned products (e.g., polishing, ironing, re-bagging/boxing) before it can sell them again. In this section, we extend the main model by examining a setting where the online retailer incurs a reconditioning cost $r > 0$ for each returned product before reselling it. This cost is incurred regardless of whether the return was made through the regular online channels or at the offline retailer's store. The cost r can also capture any loss due to the salvaging of these returned products in secondary channels. The retailers' profits are then given by $\Pi_S = p_S Q_S$ and $\Pi_I = p_I Q_I - r Q_r$, where Q_r is the total number of returned products.

In Section 5, we established that the return flexibility effect induces more customers to make their first purchase online under the agreement. Since λ proportion of these customers face a mismatch, the return flexibility effect also leads to more returns and hence, a higher total reconditioning cost for the online retailer. This imparts a negative force on the online retailer's profit, which increases with r. This negative force gives the online retailer an incentive to decrease the number of returns by increasing its price. A higher price would decrease the number of customers purchasing from the online retailer in the first place, leading to fewer returns. In other words, by increasing its price, the online retailer can dampen the return flexibility effect. As r increases, the online retailer's incentive to dampen the return flexibility effect increases.

Therefore, r impacts the retailers in two ways: i) by increasing the total return reconditioning cost for the online retailer and ii) by diminishing the return flexibility effect for both retailers. We summarize the overall impact of r on the BORS agreement in the following proposition.

PROPOSITION 8. *When the online retailer incurs a return reconditioning cost (r), the BORS agreement benefits both retailers if and only if the online retailer relinquishes returners to the offline retailer and* $\max\{0, \underline{r}\} < r < \bar{r}$, *where* $\bar{r} = \frac{h-\lambda(\delta+h)-\sqrt{1-\lambda}(\theta-h)+2\theta}{\lambda}$ *and* $\underline{r} = \bar{r} - \frac{6\theta}{\lambda}$.

Proposition 8 shows that both retailers are better off under the agreement only when the online retailer relinquishes returners, and the reconditioning cost is moderate. This is because the offline retailer is better off under the agreement only when the reconditioning cost is sufficiently high, and the online retailer is better off only when the reconditioning cost is not too high. The intuition behind this result is that the agreement increases the number of returns the online retailer receives; hence, as the reconditioning cost increases, the online retailer incurs a higher total cost from these returns. When r is sufficiently high, this cost dominates any potential benefits from the agreement (driven by both the competition alleviation and return flexibility effects), making the online retailer worse off. On the other hand, we know that the return flexibility effect hurts the offline retailer. Consequently, the offline retailer is better off under the agreement only when the positive competition alleviation effect dominates the negative return flexibility effect. As discussed, the negative return flexibility effect dampens as r increases. Therefore, the offline retailer is better off under the agreement only when r is above a threshold ($r > \max\{0, \underline{r}\}$).

A.9. BORS Agreement when the Online Retailer has a Market Advantage

Our main model assumes that both retailers have equal market strength. In this extension, we consider the case where the online retailer has a larger market advantage than its competitor. We model this advantage by placing the online retailer at $x = a$ and the offline retailer at $x = 1$, where $a \in (0, 1)$. Customers incur the same cost as in our main model $\theta(1 - x)$ on buying offline but now incur $\theta|x - a|$ on buying online. The customers located in $[0, a]$ represent the online retailer's "captive market." This captive market lends an advantage to the online retailer. We study the impact of this advantage on the BORS agreement.

PROPOSITION 9. *When the online retailer has a market advantage of size a, the BORS agreement benefits both retailers if and only if the online retailer relinquishes the returners and* $a > \max\{0, \underline{a}\}$*, where* $\underline{a} = \frac{(h-\theta)(1-\lambda)-\sqrt{(1-\lambda)(4\theta+(d+h)\lambda-h)^2}}{\theta(1-\lambda)}$.

Proposition 9 shows that the key results in our main model are robust even when the online retailer has a stronger market advantage than the offline retailer. The key driver of the benefit of the BORS agreement stems from the relinquishing of the returners, creating a positive competition alleviation effect that dominates the negative (on the offline retailer) return flexibility effect. Counter-intuitively, the online market advantage

(a) has to be sufficiently high for the offline retailer to benefit from the BORS agreement. As in our main model, the agreement always benefits the online retailer and it is the offline retailer that can be worse off under the agreement. The online retailer's advantage could make the agreement even less attractive for the offline retailer. However, we find that the online retailer's advantage may make the agreement more favorable (in comparison to no agreement) for the offline retailer. The intuition behind this result is that as the online retailer's captive market increases, the impact of the return flexibility is less negative (for the offline retailer), making the competition alleviation effect more likely to compensate for it. To see the intuition behind this result, we examine the offline retailer's demands under no agreement (Q_S^0) and under the agreement with relinquishing (Q_S^{12}) given in Equation A-3.

$$\begin{aligned} Q_S^0 &= \frac{\theta + \lambda(\delta + h)}{2\theta} - \frac{a}{2} - \frac{p_S - p_I}{2\theta}, \\ Q_S^{12} &= \frac{\theta + \lambda(\delta + \theta)}{2\theta} - \frac{a(1-\lambda)}{2} - (1-\lambda)\frac{p_S - p_I}{2\theta}. \end{aligned} \tag{A-3}$$

The captive market size does not impact the price sensitivities (competition alleviation effect) but impacts the base demands (return flexibility effect). Clearly, an increase in a always decreases the base demand of the offline retailer, but its impact is lower under the agreement. This is because λ proportion of the captive market (i.e., λa) would make the second purchase offline in the relinquishing equilibrium. As a increases, the number of customers who make their second purchase offline increases. This positive force dampens the negative return flexibility effect (for the offline retailer), making it easier for the positive competition alleviation effect to offset it.

A.10. BORS Agreement With Asymmetric Assortment

In our main setting, retailers have the same assortment, and each customer can find her match at each retailer. However, in practice, retailers can have asymmetric assortments. In this extension, we consider the situation where the online retailer has a deeper assortment than the offline retailer. Therefore, it is possible that a customer who visits the offline retailer does not find the product she seeks offline. As in our main model, a mismatch offers a lower value than a match. We denote this value by $\nu - \eta$. The customer is uncertain whether the offline retailer is carrying a variant that matches her before visiting the store. We denote the ex-ante probability of finding a preferred variant at the offline store by $\phi \in (0,1)$. Upon a visit to the offline store and discovering the offline assortment, a customer who faces a mismatch can choose to either purchase a non-matching product or purchase online. For ease of exposition, in this extension, we focus on the more interesting parameter space where, in equilibrium, each customer ends up with her match. Our results are robust even in the parameter space where, in equilibrium, customers may keep a non-matching product. As in our main setting, we consider the situation where the online retailer carries all variants. Hence, a customer who faces a mismatch after an online purchase returns the product and re-purchases another one.

A customer first decides whether to visit the offline retailer or not. A visit to the offline retailer informs her about the offline retailer's assortment. Second, she decides which retailer to purchase the product from. A purchase from the online retailer can happen even after she visits the offline retailer. Next, if she purchases online and the product does not match her, she decides where to return the product. In the case of the BORS agreement, she can also return the product at the competing offline retailer's store. Finally, she decides where to purchase the replacement from. Again, the repurchase can happen at the online retailer even if the product is returned offline in-store.

PROPOSITION 10. *When the retailers have an asymmetric assortment, the BORS agreement can benefit both retailers only if the online retailer relinquishes all returners who find their match offline.*

As in our main model, even when retailers have asymmetric assortments, and there is a likelihood that customers do not find their match offline, both retailers can still benefit from the BORS agreement. The key driver of this benefit, explained in our main model, i.e., competition alleviation effect due to relinquishing of returners, still applies and leads to an increase in profit. To illustrate this, we produce the demands of the offline retailer under no agreement (Q_S^0) and the two relevant cases under the agreement (Q_S^{11} for no relinquishing equilibrium) and (Q_S^{12} for the relinquishing equilibrium) in Equation A-4.

$$\begin{aligned}
Q_S^0 &= \frac{\phi(\delta+\lambda(\delta+h)+\theta)-\delta}{2\theta} - \phi\frac{p_S-p_I}{2\theta},\\
Q_S^{11} &= \frac{\delta(2\lambda\phi+\phi-1)+\theta\phi}{2\theta} - \phi\frac{p_S-p_I}{2\theta},\\
Q_S^{12} &= \frac{\phi(1+\lambda)(\delta+\theta)-\delta}{2\theta} - (1-\lambda)\phi\frac{p_S-p_I}{2\theta}.
\end{aligned} \tag{A-4}$$

As is evident from Equation A-4, the price sensitivities are the same under no agreement and non-relinquishing equilibrium under the agreement ($\frac{\phi}{2\theta}$). However, the price sensitivity drops in the relinquishing equilibrium under the agreement ($\frac{(1-\lambda)\phi}{2\theta}$), leading to the competition alleviation effect. In the relinquishing equilibrium, only the returners who find their match offline are relinquished, while those who do not find their match offline make their second purchase online.

A.11. BORS Agreement Between an Online Retailer and a Multichannel Retailer

In the main model, we considered the BORS agreement between an online retailer and an offline retailer. However, in practice, retailers such as Kohl's may also have an online channel. Therefore, in this extension, we investigate the BORS agreement between an online retailer and a multichannel retailer (with both offline and online channels). We assume that the multichannel retailer (M) is located at $x = 1$ and the online retailer (I) is located at $x = 0$. We denote the online channel of the multichannel retailer by M_I and the offline channel by M_S. Shopping online, either from the multichannel or the online retailer, involves a waiting time cost denoted by δ for all customers. However, customers are heterogeneous in terms of their transportation cost to the offline store such that a proportion $\alpha \in (0,1)$ of the customers have a low cost (δ), and the remaining customers $(1-\alpha)$ have a high cost ($\delta + \Delta$ where $\Delta > 0$). We consider the more interesting case where, in equilibrium, the customers who incur a high transportation cost would make their first purchase online from either M_I or I, while customers who incur a low transportation cost make their first purchase either offline from M_S or online from I.[2] Buying online from either retailer has a risk of mismatch and return. As in our main model, we assume that the online return hassle cost is higher than the offline transportation cost ($h > \delta + \Delta > \delta$). When a customer buys from the online channel of the multichannel retailer, she would return offline since it is less costly. Once she is at the offline store, she can repurchase the same brand from M_S. Customers who make the first purchase from the online retailer (I) can only return online under no agreement and either online or offline under the agreement. We solve both models to determine the conditions under which the BORS agreement benefits both retailers.

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[2] This case happens when $\Delta > \frac{\lambda\delta}{1-\lambda}$.

PROPOSITION 11. *When the BORS agreement is between a multichannel retailer and a pure online retailer, it benefits both retailers if and only if the pure online retailer relinquishes returners and* $\alpha < \min\{\bar{\alpha}, 1\}$*, where* α *represents the proportion of customers with a low shopping cost and* $\bar{\alpha} = \frac{(1-\lambda)(\delta+\Delta)(\lambda(\delta+h)+3\theta)-\lambda(1-\lambda)(\delta+\Delta)^2-\delta\theta(\lambda+3)-\sqrt{(1-\lambda)(\Delta\lambda(\delta+\theta)+3\Delta\theta+\delta\lambda(\theta-h))^2}}{\lambda\left(\delta^2-(1-\lambda)(\delta+\Delta)^2\right)}$.

As in our main model, the BORS agreement can make both retailers better off only when the online retailer relinquishes the returners. The agreement benefits the online retailer but may negatively impact the multichannel retailer due to the return flexibility effect. Relinquishing, however, creates a positive competition alleviation effect that can dominate this negative return flexibility effect (for the multichannel retailer). We find that the competition alleviation effect dominates the return flexibility effect only when the proportion of customers with a low transportation cost to the offline store is sufficiently low ($\alpha < \min\{\bar{\alpha}, 1\}$). The intuition behind this result is that the return flexibility effect increases with α. To understand this, note that α customers have a lower cost of traveling offline, which also means that the agreement would lower their return cost to a greater extent compared to the remaining $(1-\alpha)$ customers. Therefore, the number of customers who gain more from the agreement (in terms of reduction of their return cost) increases as α increases. In other words, the return flexibility effect is high when α is high, which can potentially dominate the competition alleviation effect. Thus, it is only when $\alpha < \min\{\bar{\alpha}, 1\}$, that we find that the competition alleviation effect dominates the return flexibility effect and both retailers are better off under the agreement.

Appendix B: Proofs

B.1. Technical Details of No Agreement Case

We use backward induction to solve for a forward-looking customer's decisions given retail prices p_j. On each decision occasion, the customer calculates her expected utility from each choice and picks the one that provides her with the highest expected utility. The customer's decision tree without the agreement is represented in Figure 1, and the decision variables are tabulated in Table 1.

A customer who makes her first purchase offline gets utility $U^0_{\{FP=S\}} = v - p_S - \theta(1-x) - \delta$. A customer who makes her first purchase online would find her match with probability $1-\lambda$, keep the product ($R = \phi$) and get utility $U^0_{\{R=\phi|FP=I,m=1\}} = v - p_I - \theta x - \delta$. With probability λ, she would find a mismatch and return online. She may either repurchase online or offline. The customer's expected utility of purchasing online is given by $U^0_{\{FP=I\}} = -\delta + (1-\lambda)U^0_{\{R=\phi|FP=I,m=1\}} + \lambda(-h + U^0_{\{SP\in\{I,S\}|FP=I,m=0,R=I\}})$.

We first show that repurchasing offline ($U^0_{\{SP=S|FP=I,m=0,R=I\}} = v - p_S - \theta(1-x) - \delta$) is dominated by repurchasing online ($U^0_{\{SP=I|FP=I,m=0,R=I\}} = v - p_I - \theta x - \delta$). Assuming the opposite, i.e., $U^0_{\{SP=S|FP=I,m=0,R=I\}} > U^0_{\{SP=I|FP=I,m=0,R=I\}}$. In this case, $U^0_{\{FP=I\}} = -\delta + (1-\lambda)U^0_{\{R=\phi|FP=I,m=1\}} + \lambda(-h + U^0_{\{SP=S|FP=I,m=0,R=I\}}) = -\delta + (1-\lambda)U^0_{\{SP=I|FP=I,m=0,R=I\}} + \lambda(-h + U^0_{\{SP=S|FP=I,m=0,R=I\}}) < -\delta - \lambda h + U^0_{\{SP=S|FP=I,m=0,R=I\}} < U^0_{\{FP=S\}}$. Hence, under this assumption since $U^0_{\{FP=I\}} < U^0_{\{FP=S\}}$, the customer would not have purchased online in the first place, which is a contradiction.

Hence, the customer always repurchases online and $U^0_{\{FP=I\}} = -\delta + (1-\lambda)U^0_{\{R=\phi|FP=I,m=1\}} + \lambda(-h + U^0_{\{SP=I|FP=I,m=0,R=I\}}) = v - \delta - p_I - \theta x - (\delta + h)\lambda$. We find that the market is covered when $v > \max\{\delta + h, \frac{3\theta+(2-\lambda)(\delta+h)}{2}\}$. The customer who is indifferent between purchasing online and offline has $U^0_{\{FP=I\}} = U^0_{\{FP=S\}}$, and is located at $x^0 = \frac{\theta-\lambda(\delta+h)-p_I+p_S}{2\theta}$. The resulting offline and online demands are given in Equation 1. We assume that

$$\theta > \frac{\lambda(\delta + h)}{3} \tag{B-1}$$

to ensure that in equilibrium, at least some customers buy from each retailer.

B.2. Proof of Lemma 1

We use backward induction to solve for a forward-looking customer's decisions given retail prices p_I and p_S. The customer's decision tree is represented in Figure 2, and the decision variables are tabulated in Table 1.

A customer who decides to make her first purchase offline gets utility $U^1_{\{FP=S\}} = v - p_S - \theta(1-x) - \delta$. A customer who makes her first purchase online would find her match with probability $1-\lambda$ and get utility $U^1_{\{R=\phi|FP=I,m=1\}} = v - p_I - \theta x - \delta$. With probability λ, she would find a mismatch. If she returns the product online, she may repurchase online and get utility $U^1_{\{SP=I|FP=I,m=0,R=I\}} = v - p_I - \theta x - \delta$ or repurchase offline and get utility $U^1_{\{SP=S|FP=I,m=0,R=I\}} = v - p_S - \theta(1-x) - \delta$. If she returns the product offline, she decides whether to make her second purchase online or offline. She gets utility $U^1_{\{SP=S|FP=I,m=0,R=S\}} = v - p_S - \theta(1-x)$ by repurchasing offline and $U^1_{\{SP=I|FP=I,m=0,R=S\}} = v - p_I - \theta(x) - \delta$ by repurchasing online. Upon returning online, the customer receives utility $U^1_{\{R=I|FP=I,m=0\}} = -h + \max\{U^1_{\{SP=I|FP=I,m=0,R=I\}}, U^1_{\{SP=S|FP=I,m=0,R=I\}}\}$ and upon returning offline, she receives utility $U^1_{\{R=S|FP=I,m=0\}} = -\delta + \max\{U^1_{\{SP=I|FP=I,m=0,R=S\}}, U^1_{\{SP=S|FP=I,m=0,R=S\}}\}$. It is easy to see that, since $h > \delta$, returning offline dominates returning online.

Comparing the two utilities upon returning offline, the customer chooses to repurchase online when $x < x^1_{SP}$, where $x^1_{SP} = \frac{\theta-\delta-p_I+p_S}{2\theta}$. We find that the market is covered when $v > \max\{\delta + h, \frac{3(\delta+2\theta)-2\lambda(\delta+\theta)}{3(1-\lambda)}\}$. Therefore, for a customer located at x,

$$U^1_{\{R=S|FP=I,m=0\}} = \begin{cases} v - p_I - \theta(x) - 2\delta & x < x^1_{SP}, \\ v - p_S - \theta(1-x) - \delta & \text{Otherwise.} \end{cases}$$

The expected utility of a customer from making her first purchase online is given by: $U^1_{\{FP=I\}} = -\delta + \lambda U^1_{\{R=S|FP=I,m=0\}} + (1-\lambda)U^1_{\{R=\phi|FP=I,m=1\}}$. For a customer located at x,

$$U^1_{\{FP=I\}} = \begin{cases} v - p_I - \theta x - (1+2\lambda)\delta & x < x^1_{SP}, \\ v - \delta - \theta x - (1-\lambda)p_I - \lambda p_S - \lambda((1-2x)\theta + \delta) & \text{Otherwise.} \end{cases}$$

The customer would compare her expected utility when $FP = I$ to when $FP = S$ to decide where to make her first purchase. Further, we can show that $U^1_{\{FP=S\}} < U^1_{\{FP=I\}}$ when $x < x^1_{SP}$. We compare expected utilities when $x \geq x^1_{SP}$, and find that $U^1_{\{FP=S\}} > U^1_{\{FP=I\}}$ when $x \geq x^1_{FP}$, where $x^1_{FP} = \frac{(1-\lambda)(\theta - p_I + p_S) - \lambda\delta}{2\theta(1-\lambda)}$. The expected utilities of a customer located at x are:

$$U^1 = \begin{cases} v - p_I - \theta x - (1+2\lambda)\delta & \text{(Strategy } ISI\text{) } x < x^1_{SP}, \\ v - \delta - \theta x - (1-\lambda)p_I - \lambda p_S - \lambda((1-2x)\theta + \delta) & \text{(Strategy } ISS\text{) } x^1_{SP} \leq x < x^1_{FP}, \\ v - p_S - \theta(1-x) - \delta & \text{(Strategy } S\text{)} \quad \text{Otherwise.} \end{cases}$$

B.3. Proof of Lemma 2

Since $x^1_{SP} < x^1_{FP}$, there are six cases (combinations) that lead to different demands for both retailers:

$$\begin{aligned} &\textbf{Case 1 } (\mathbf{0 < x^1_{SP} < x^1_{FP} < 1}): && Q^{11}_I = 1 - Q^{11}_S = \lambda x^1_{SP} + (1-\lambda)x^1_{FP} \\ &\textbf{Case 2 } (\mathbf{x^1_{SP} \leq 0 < x^1_{FP} < 1}): && Q^{12}_I = 1 - Q^{12}_S = (1-\lambda)x^1_{FP} \\ &\textbf{Case 3 } (\mathbf{0 < x^1_{SP} < 1 \leq x^1_{FP}}): && Q^{13}_I = 1 - Q^{13}_S = \lambda x^1_{SP} + (1-\lambda) \\ &\textbf{Case 4 } (\mathbf{x^1_{SP} \leq 0 < 1 \leq x^1_{FP}}): && Q^{14}_I = 1 - Q^{14}_S = (1-\lambda) \\ &\textbf{Case 5 } (\mathbf{x^1_{SP} < x^1_{FP} \leq 0}): && Q^{15}_I = 1 - Q^{15}_S = 0 \\ &\textbf{Case 6 } (\mathbf{1 \leq x^1_{SP} < x^1_{FP}}): && Q^{16}_I = 1 - Q^{16}_S = 1 \end{aligned} \tag{B-2}$$

Note that these cases are mutually exclusive and collectively exhaustive. All possible candidate pure strategy equilibria are accounted for in one of the enumerated cases. As we are searching for an equilibrium where each retailer has at least some customers making their first purchase from it, we focus on finding equilibria in the first two cases. We, however, have to ensure the consistency and stability of these equilibria and that there is no deviation to any of the different cases. We assume that

$$\theta > \max\{\frac{\lambda\delta\left(6 - 13\lambda + 3\sqrt{20\lambda^2 - 18\lambda + 3}\right)}{9 - 11\lambda^2 + 6\lambda}\mathbb{I}_{\mathbb{R}}, \frac{\lambda\delta}{3-\lambda}\}, \tag{B-3}$$

which is a necessary condition to ensure the stability of these equilibria, where $\mathbb{I}_{\mathbb{R}}$ indicates whether the first term is real ($\mathbb{I}_{\mathbb{R}} = 1$ if $20\lambda^2 - 18\lambda + 3 \geq 0$ and 0 otherwise). We provide further details regarding these deviations for Case 1 and Case 2 separately.

Case 1 (Not Relinquishing Returners): In Case 1 ($0 < x^1_{SP} < x^1_{FP} < 1$), we have $\Pi^{11}_S = \frac{p_S(2\delta\lambda+\theta+p_I-p_S)}{2\theta}$ and $\Pi^{12}_I = \frac{p_I(p_S-2\delta\lambda+\theta-p_I)}{2\theta}$. We solve the optimization problem for each retailer and obtain the following equilibrium candidate: $p^{11}_S = \frac{3\theta+2\delta\lambda}{3}$ and $p^{11}_I = \frac{3\theta-2\delta\lambda}{3}$. Substituting the equilibrium candidate in the condition for Case 1, we find that $\theta > \theta_{11}$ for this candidate equilibrium to be feasible, where $\theta_{11} = \frac{\delta(3-4\lambda)}{3}$. To derive the condition under which this equilibrium is stable, we examine all possible non-local deviations to the other cases. For such a deviation to exist to Case $i \neq 1$ for retailer $j \in \{I, S\}$, it must be the case that, conditional on the other retailer ($-j$) setting the candidate equilibrium price (p^{11}_{-j}), retailer j achieves a higher profit by deviating to a price that satisfies Case i. The condition to satisfy Case i is derived by substituting p_{-j} with p^{11}_{-j} in the condition of the case (see Equation B-2). The profit of retailer j upon deviation to Case i is given by $p_j Q^{1i}_j$ after substituting p_{-j} with p^{11}_{-j}. The condition for higher profit on deviation is derived by comparing this profit to the candidate equilibrium profit.

Retailer	Deviation to Case	Necessary Condition to Satisfy Non-local Case	Necessary Condition for Higher Profit on Deviation
S	2	$p_S \le \delta - \frac{2\delta\lambda}{3} \wedge p_S > \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)} \wedge p_S < 2\theta + \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}$	$\frac{6\theta+\delta\lambda(1+2\lambda)-\sqrt{\Delta_1}}{6(1-\lambda)} < p_S < \frac{6\theta+\delta\lambda(1+2\lambda)+\sqrt{\Delta_1}}{6(1-\lambda)} \wedge \Delta_1 > 0$
S	3	$p_S > \delta - \frac{2\delta\lambda}{3} \wedge p_S < 2\theta + \frac{1}{3}\delta(3-2\lambda) \wedge p_S \ge 2\theta + \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}$	$\frac{6\theta+\delta(3-2\lambda)-\sqrt{\Delta_2}}{6} < p_S < \frac{6\theta+\delta(3-2\lambda)+\sqrt{\Delta_2}}{6} \wedge \Delta_2 > 0$
S	4	$p_S \le \delta - \frac{2\delta\lambda}{3} \wedge p_S \ge 2\theta + \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}$	$p_S > \frac{(3\theta+2\delta\lambda)^2}{18\theta\lambda}$
S	5	$p_S \le \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}$	$p_S > \frac{(3\theta+2\delta\lambda)^2}{18\theta}$
S	6	$p_S \ge 2\theta + \frac{1}{3}\delta(3-2\lambda)$	Higher profit not possible
I	2	$p_I \ge 2\theta - \frac{1}{3}\delta(3-2\lambda) \wedge p_I < 2\theta - \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)} \wedge p_I > -\frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}$	$\frac{6\theta(1-\lambda)-\delta\lambda(1+2\lambda)-\sqrt{\Delta_3}}{6(1-\lambda)} < p_I < \frac{6\theta(1-\lambda)-\delta\lambda(1+2\lambda)+\sqrt{\Delta_3}}{6(1-\lambda)} \wedge \Delta_3 > 0$
I	3	$p_I < 2\theta - \frac{1}{3}\delta(3-2\lambda) \wedge p_I > -\frac{1}{3}\delta(3-2\lambda) \wedge p_I \le -\frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)} \wedge p_I > 0 \implies$ Not possible	$\frac{6\theta-\delta(3-2\lambda)\lambda-\sqrt{\Delta_4}}{6\lambda} < p_I < \frac{6\theta-\delta(3-2\lambda)\lambda+\sqrt{\Delta_4}}{6\lambda} \wedge \Delta_4 > 0$
I	4	$p_I \ge 2\theta - \frac{1}{3}\delta(3-2\lambda) \wedge p_I \le -\frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)} \wedge p_I > 0 \implies$ Not possible	$p_I > \frac{(3\theta-2\delta\lambda)^2}{18\theta(1-\lambda)}$
I	5	$p_I \ge 2\theta - \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}$	Higher profit not possible
I	6	$p_I \le -\frac{1}{3}\delta(3-2\lambda) \wedge p_I > 0 \implies$ Not possible	$p_I > \frac{(3\theta-2\delta\lambda)^2}{18\theta}$

Table B.1 Conditions on the deviating retailer's price to satisfy the non-local case and generate a higher profit for each potential non-local deviation from Case 1**,**
where $\Delta_1 = \lambda\left(36\theta(\theta-\delta(1-2\lambda)) - \delta^2\lambda(15-4\lambda(5+\lambda))\right)$, $\Delta_2 = \left(\delta^2\lambda(9-4(7-\lambda)\lambda) - 12\left(3\theta^2(1-\lambda)+\delta\lambda(\theta+2\theta\lambda)\right)\right)/\lambda$,
$\Delta_3 = 12\theta\lambda(1-\lambda)(\delta(3-2\lambda)-3\theta) - \delta^2\lambda^2(15-4\lambda(5+\lambda))$, $\Delta_4 = 36\theta(\theta(1-\lambda)-\delta\lambda(1-2\lambda)) + \delta^2\lambda^2(9-4(7-\lambda)\lambda)$.

We tabulate the necessary conditions on p_j for 1) satisfying each Case $i \neq 1$ and 2) for retailer j to achieve a higher profit by deviating to Case i in Table B.1. Each of the third and the fourth columns of Table B.1 imply an interval on the price of the deviating retailer. We find that these intervals do not overlap for all rows except for Retailer I deviating to Case 2. That is, the two necessary conditions on the price of the deviating retailer cannot hold together, and hence, there is no possible deviation for the offline retailer to any case and for the online retailer to a case other than Case 2.

We now focus on the deviation of Retailer I to Case 2. A profitable deviation happens when Case 1 is satisfied ($\theta > \theta_{11}$) and the necessary conditions in Column 3 and 4 are met. That is, there is a profitable deviation from Case 1 if and only if $\theta > \theta_{11}$, $\Delta_3 > 0$ and there exists $p_I \in \Omega_1$ such that Ω_1 is the deviation space defined by $\max\{2\theta - \frac{1}{3}\delta(3 - 2\lambda), -\frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}, \frac{6\theta(1-\lambda)-\delta\lambda(1+2\lambda)-\sqrt{\Delta_3}}{6(1-\lambda)}\} < p_I < \min\{2\theta - \frac{\delta\lambda(1+2\lambda)}{3(1-\lambda)}, \frac{6\theta(1-\lambda)-\delta\lambda(1+2\lambda)+\sqrt{\Delta_3}}{6(1-\lambda)}\}$.

- $\Delta_3 > 0 \iff \frac{\delta(3-2\lambda)}{6} - \frac{\delta(1-2\lambda)}{2\sqrt{1-\lambda}} < \theta < \frac{\delta(3-2\lambda)}{6} + \frac{\delta(1-2\lambda)}{2\sqrt{1-\lambda}}$. Since $\theta_{11} > \frac{\delta(3-2\lambda)}{6} - \frac{\delta(1-2\lambda)}{2\sqrt{1-\lambda}}$, it is easy to show that $\theta > \theta_{11} \wedge \Delta_3 > 0 \iff \theta_{11} < \theta < \theta_1$, where $\theta_1 = \frac{\delta(3-2\lambda)}{6} + \frac{\delta(1-2\lambda)}{2\sqrt{1-\lambda}}$.
- When $\theta_{11} < \theta < \theta_1$, $\Omega_1 \neq \emptyset$. For instance, $p_I = \theta - \frac{\delta\lambda(2\lambda+1)}{6(1-\lambda)} \in \Omega_1$.

Therefore, we find that when $\theta_{11} < \theta < \theta_1$, the online retailer has an incentive to deviate to Case 2. When $\theta \geq \theta_1$, neither retailer has an incentive to deviate from the candidate equilibrium in Case 1. Since $\theta_1 > \theta_{11}$, we find that Case 1 survives as an equilibrium if and only if $\theta \geq \theta_1$. For instance, for $v = 20$, $\lambda = 0.1$, $h = 1$, and $\delta = 0.6$, the parameter space that satisfies our assumptions is given by $\{\theta : \theta > 0.05\}$ and the non-relinquishing equilibrium exists when $\theta \geq 0.53$.

Case 2 (Relinquishing Returners): In Case 2, we have $x^1_{SP} \leq 0 < x^1_{FP} < 1$. In this case, all returners repurchase from the offline store. In other words, the online retailer relinquishes returners to the offline store. We have $\Pi^{12}_S = \frac{p_S(\lambda(\delta+\theta+p_S)+\theta-\lambda p_I+p_I-p_S)}{2\theta}$ and $\Pi^{12}_I = \frac{p_I(-\lambda(\delta+\theta+p_S)+\theta+(\lambda-1)p_I+p_S)}{2\theta}$. The condition for Case 2 entails the following constraint: $\delta + p_I - (\theta + p_S) \geq 0$. We solve the constrained optimization problem for each retailer using KKT. Given that there is one constraint, there are two possible responses by each retailer (one interior and one at the boundary). Hence, we have four potential equilibrium candidates. Out of these, only three candidates satisfy the original conditions for Case 2 (see Table B.2).

Table B.2

Candidate Name	p^{12}_S	p^{12}_I	Condition
21	$\frac{3\theta+\lambda(\delta+\theta)}{3(1-\lambda)}$	$\frac{3\theta-\lambda(\delta+\theta)}{3(1-\lambda)}$	$\theta \leq \frac{3\delta-5\delta\lambda}{3-\lambda}$
22	$\frac{2\delta\lambda+2\theta-\delta}{1-\lambda}$	$\frac{\delta(3\lambda-2)+\theta(3-\lambda)}{1-\lambda}$	$\theta > \frac{3\delta-5\delta\lambda}{3-\lambda}$
23	$\frac{3\delta\lambda-2\delta-\theta\lambda+\theta}{\lambda-1}$	$\frac{\delta(1-2\lambda)}{1-\lambda}$	$\theta > \frac{3\delta-5\delta\lambda}{3-\lambda}$

We can show that both candidates 22 and 23 do not survive as an equilibrium. If the offline retailer sets the price according to either candidate equilibrium, the online retailer always has an incentive to deviate to a price different from the one in the corresponding candidate equilibrium. For instance, the online retailer can deviate to a price $p_I = \delta(1-2\lambda)+\epsilon$ under Case 1, where $\epsilon > 0$ to obtain a higher profit. Therefore, only candidate 21 can survive as an equilibrium from Case 2.

Retailer	Deviation to Case	Necessary Condition to Satisfy Non-local Case	Necessary Condition for Higher Profit on Deviation
S	1	$p_S > \frac{3\delta-4\delta\lambda+2\theta\lambda}{3(1-\lambda)} \wedge p_S < \frac{6\theta+2\delta\lambda-4\theta\lambda}{3(1-\lambda)}$	$\frac{\theta(6-4\lambda)+\delta(5-6\lambda)\lambda-\sqrt{\Delta_5}}{6(1-\lambda)} < p_S < \frac{\theta(6-4\lambda)+\delta(5-6\lambda)\lambda+\sqrt{\Delta_5}}{6(1-\lambda)} \wedge \Delta_5 > 0$
S	3	$p_S > \frac{3\delta-4\delta\lambda+2\theta\lambda}{3(1-\lambda)} \wedge p_S < \frac{3(\delta+2\theta)-4(\delta+\theta)\lambda}{3(1-\lambda)} \wedge$ $p_S \geq \frac{6\theta+2\delta\lambda-4\theta\lambda}{3(1-\lambda)}$	$\frac{\delta(3-4\lambda)+\theta(6-4\lambda)-\sqrt{\Delta_6}}{6(1-\lambda)} < p_S < \frac{\delta(3-4\lambda)+\theta(6-4\lambda)+\sqrt{\Delta_6}}{6(1-\lambda)} \wedge \Delta_6 > 0$
S	4	$p_S \leq \frac{3\delta-4\delta\lambda+2\theta\lambda}{3(1-\lambda)} \wedge p_S \geq \frac{6\theta+2\delta\lambda-4\theta\lambda}{3(1-\lambda)}$	$p_S > \frac{(\delta\lambda+\theta(3+\lambda))^2}{18\theta\lambda(1-\lambda)}$
S	5	$p_S \leq \frac{2(\delta+\theta)\lambda}{3(1-\lambda)}$	$p_S > \frac{(\delta\lambda+\theta(3+\lambda))^2}{18\theta(1-\lambda)}$
S	6	$p_S \geq \frac{3(\delta+2\theta)-4(\delta+\theta)\lambda}{3(1-\lambda)}$	Higher profit not possible
I	1	$p_I < \frac{2\theta(3-\lambda)-\delta(3-4\lambda)}{3(1-\lambda)} \wedge p_I > \frac{2(2\theta-\delta)\lambda}{3(1-\lambda)}$	$\frac{2\theta(3-\lambda)-\delta(5-6\lambda)\lambda-\sqrt{\Delta_7}}{6(1-\lambda)} < p_I < \frac{2\theta(3-\lambda)-\delta(5-6\lambda)\lambda+\sqrt{\Delta_7}}{6(1-\lambda)} \wedge \Delta_7 > 0$
I	3	$p_I < \frac{2\theta(3-\lambda)-\delta(3-4\lambda)}{3(1-\lambda)} \wedge p_I > \frac{4(\delta+\theta)\lambda-3\delta}{3(1-\lambda)} \wedge$ $p_I \leq \frac{2(2\theta-\delta)\lambda}{3(1-\lambda)}$	$\frac{2\theta\left(3-3\lambda+2\lambda^2\right)-\delta(3-4\lambda)\lambda-\sqrt{\Delta_8}}{6(1-\lambda)\lambda} < p_I < \frac{2\theta\left(3-3\lambda+2\lambda^2\right)-\delta(3-4\lambda)\lambda+\sqrt{\Delta_8}}{6(1-\lambda)\lambda} \wedge \Delta_8 > 0$
I	4	$p_I \geq \frac{2\theta(3-\lambda)-\delta(3-4\lambda)}{3(1-\lambda)} \wedge p_I \leq \frac{2(2\theta-\delta)\lambda}{3(1-\lambda)}$	$p_I > \frac{(\theta(3-\lambda)-\delta\lambda)^2}{18\theta(1-\lambda)^2}$
I	5	$p_I \geq \frac{6\theta-2(\delta+\theta)\lambda}{3(1-\lambda)}$	Higher profit not possible
I	6	$p_I \leq \frac{4(\delta+\theta)\lambda-3\delta}{3(1-\lambda)}$	$p_I > \frac{(\theta(3-\lambda)-\delta\lambda)^2}{18\theta(1-\lambda)}$

Table B.3 Conditions on the deviating retailer's price to satisfy the non-local case and generate a higher profit for each potential non-local deviation from Case 2**,**

where $\Delta_5 = 4\delta\theta\lambda(9-2\lambda(12-7\lambda))+\delta^2\lambda^2(21-4\lambda(14-9\lambda))-4\theta^2\lambda(9-\lambda(9+\lambda))$, $\Delta_6 = 9\delta^2+12\theta(\delta+4\theta)-28(\delta+\theta)^2\lambda+20(\delta+\theta)^2\lambda^2-36\theta^2/\lambda$, $\Delta_7 = \lambda\left(4\theta^2(3-\lambda)^2-4\delta\theta(3-\lambda)(3-4\lambda)+\delta^2\lambda(21-4\lambda(14-9\lambda))\right)$, $\Delta_8 = \delta^2\lambda^2(9-4\lambda(7-5\lambda))-4\delta\theta\lambda(3-5\lambda)(3-2(2-\lambda)\lambda)+4\theta^2(9-\lambda(27-\lambda(36-\lambda(19-5\lambda))))$

.

We derive the conditions under which this happens by examining all possible non-local deviations to other cases. As in Case 1, for each retailer $j \in \{S, I\}$ and Case $i \neq 2$, we tabulate the necessary conditions on p_j for 1) satisfying deviation to Case i and 2) for retailer j, to achieve a higher profit by deviating to Case i in Table B.3. We find that these intervals do not overlap for all rows except for Retailer I deviating to Case 1. That is, the two necessary conditions on the price of the deviating retailer cannot hold together, and hence, there is no possible deviation for the offline retailer to any case and for the online retailer to a case other than Case 1.

We now focus on the deviation of Retailer I to Case 1. A profitable deviation happens when Case 2 is satisfied ($\theta \leq \frac{3\delta-5\delta\lambda}{3-\lambda}$) and the necessary conditions in Column 3 and 4 are met. That is, there is a profitable deviation from Case 2 if and only if $\theta \leq \frac{3\delta-5\delta\lambda}{3-\lambda}$, $\Delta_7 > 0$ and there exists $p_I \in \Omega_2$ such that Ω_2 is the deviation space defined by $\max\{\frac{2(2\theta-\delta)\lambda}{3(1-\lambda)}, \frac{2\theta(3-\lambda)-\delta(5-6\lambda)\lambda-\sqrt{\Delta_7}}{6(1-\lambda)}\} < p_I < \min\{\frac{2\theta(3-\lambda)-\delta(3-4\lambda)}{3(1-\lambda)}, \frac{2\theta(3-\lambda)-\delta(5-6\lambda)\lambda+\sqrt{\Delta_7}}{6(1-\lambda)}\}$. We have $\Delta_7 > 0 \iff \theta > \frac{\delta(3-4\lambda)+3\delta(1-2\lambda)\sqrt{1-\lambda}}{2(3-\lambda)} \vee \theta < \frac{\delta(3-4\lambda)-3\delta(1-2\lambda)\sqrt{1-\lambda}}{2(3-\lambda)}$.

- When $\theta < \frac{\delta(3-4\lambda)-3\delta(1-2\lambda)\sqrt{1-\lambda}}{2(3-\lambda)}$, It is easy to show that $\Omega_2 = \emptyset$.

- When $\theta > \frac{\delta(3-4\lambda)+3\delta(1-2\lambda)\sqrt{1-\lambda}}{2(3-\lambda)}$, $\Omega_2 \neq \emptyset$, i.e., there is always a $p_I \in \Omega_2$ that leads to a profitable deviation for I. For instance, a deviation to price $p_I = \frac{2\theta(3-\lambda)-\delta\lambda(5-6\lambda)}{6(1-\lambda)}$ leads to a higher profit for the online retailer when θ is above the mentioned threshold.

Therefore, we find that when $\theta_2 < \theta \leq \frac{3\delta-5\delta\lambda}{3-\lambda}$, the online retailer has an incentive to deviate from Case 2 (Candidate 21) to Case 1. When $\theta \leq \theta_2$, neither retailer has an incentive to deviate from Candidate 21, where $\theta_2 = \frac{\delta(3-4\lambda)+3\delta(1-2\lambda)\sqrt{1-\lambda}}{2(3-\lambda)}$. As $\theta_2 < \frac{3\delta-5\delta\lambda}{3-\lambda}$, Case 2 (Candidate 21) survives as an equilibrium if and only if $\theta \leq \theta_2$. For instance, for $v = 20$, $\lambda = 0.2$, $h = 1.5$, and $\delta = 1$, the parameter space that satisfies our assumptions is given by $\{\theta : \theta > 0.17\}$ and the relinquishing equilibrium exists when $\theta \leq 0.68$.

Since $\theta_2 < \theta_1$, there is no overlap in the conditions of the two equilibria, and hence, each equilibrium is unique. □

B.4. Proof of Proposition 1

We can derive the demands for the two retailers in equilibrium by substituting the relevant prices in Equations 1, 2 and 3. In no agreement, the equilibrium demands are given by $Q_S^0 = \frac{1}{2} + \frac{\lambda(\delta+h)}{6\theta}$ and $Q_I^0 = \frac{1}{2} - \frac{\lambda(\delta+h)}{6\theta}$. In the equilibrium in Case 1, the demands are given by $Q_S^{11} = \frac{1}{2} + \frac{\delta\lambda}{3\theta}$ and $Q_I^{11} = \frac{1}{2} - \frac{\delta\lambda}{3\theta}$. Clearly, $Q_S^0 > Q_S^{11}$ and $Q_I^0 < Q_I^{11}$ as $h > \delta$ in our main model.

In the equilibrium in Case 2, the demands are given by $Q_S^{12} = \frac{1}{2} + \frac{\lambda(\delta+\theta)}{6\theta}$ and $Q_I^{12} = \frac{1}{2} - \frac{\lambda(\delta+\theta)}{6\theta}$. Clearly, $Q_S^0 > Q_S^{12}$ and $Q_I^0 < Q_I^{12}$ when $\theta < h$. We know from Appendix B.3 that the equilibrium in Case 2 happens when $\theta \leq \theta_2$. We can show that $\theta_2 < \delta$. Since $\delta < h$ in our main model, this implies that the equilibrium in Case 2 happens only when $\theta < h$. Hence $Q_S^0 > Q_S^{12}$ and $Q_I^0 < Q_I^{12}$.

The equilibrium prices under no agreement are given by $p_S^0 = \frac{3\theta+\lambda(\delta+h)}{3}$ and $p_I^0 = \frac{3\theta-\lambda(\delta+h)}{3}$. The prices in equilibrium in Case 1 are given by $p_S^{11} = \frac{3\theta+2\delta\lambda}{3}$ and $p_I^{11} = \frac{3\theta-2\delta\lambda}{3}$. Clearly, $p_S^0 > p_S^{11}$ and $p_I^0 < p_I^{11}$ as $\delta < h$ in our main model.

The prices in equilibrium in Case 2 are given by $p_S^{12} = \frac{3\theta+\lambda(\delta+\theta)}{3(1-\lambda)}$ and $p_I^{12} = \frac{3\theta-\lambda(\delta+\theta)}{3(1-\lambda)}$. Clearly, $p_I^0 < p_I^{12}$ when $\theta < h$, which is the case under the equilibrium in Case 2 (as shown above). Comparing the expressions of the prices of the offline retailer, we can show that $p_S^0 < p_S^{12}$ only when $\theta > \theta_p$, where $\theta_p = \frac{h-\lambda(\delta+h)}{4}$. □

B.5. Proof of Lemma 3

We have the own and cross price sensitivities under no agreement equal to $1/2\theta$ (see Equation 1). Under the agreement, the own and cross price sensitivities under Case 1 (no relinquishing) are equal to $1/2\theta$ (see Equation 2) and under Case 2 (relinquishing) are equal to $(1-\lambda)/2\theta$ (see Equation 3). Clearly, the price sensitivities in Case 2 are (strictly) lower than that under no agreement.

B.6. Proof of Corollary 1

The proof of this corollary follows simply from Proposition 1. We know that the offline price is higher under the agreement if and only if $\theta_p < \theta \leq \theta_2$. We can easily show that $\theta > \theta_p$ can be expressed as $\lambda > \lambda_p$, $\delta > \delta_p$, and $h < h_p$ where $\lambda_p = \frac{h-4\theta}{\delta+h}$, $\delta_p = \frac{h(1-\lambda)-4\theta}{\lambda}$, and $h_p = \frac{4\theta+\lambda\delta}{1-\lambda}$.

We can easily show that $\theta \leq \theta_2 \iff \delta \geq \delta$ where $\delta_2 = \frac{2\theta(3-\lambda)\left((3-4\lambda)-3\sqrt{1-\lambda}(1-2\lambda)\right)}{\lambda\left(21-56\lambda+36\lambda^2\right)}$. We can also show that $\theta \leq \theta_2 \iff f(\lambda) = 3(1-2\lambda)\delta\sqrt{1-\lambda} + 2(2\delta-\theta)(\frac{\delta-5\theta}{2(2\delta-\theta)} + \frac{1}{2} - \lambda) \geq 0$. It is easy to see that $f(\lambda) \geq 0$ for all $\lambda \in (0, 1/2)$ when $\theta \leq \delta/5$. When $\theta > \delta/5$, we can show that there is a unique root of f that satisfies the condition of $\lambda \in (0, 1/2)$. Since $\theta < \delta$ (because $\theta_2 < \delta$), it is easy to show that $f(0) > 0$, $f(1/2) < 0$ and $f'(\lambda) < 0\ \forall \lambda \in (0, 1/2)$. Hence, $f(\lambda) \geq 0 \iff \lambda \leq \tilde{\lambda}_2$, where $\tilde{\lambda}_2$ is the unique solution of $f(\lambda) = 0$ such that $0 < \tilde{\lambda}_2 < \frac{1}{2}$. Combining these cases, we get $\theta \leq \theta_2 \iff \lambda \leq \lambda_2$, where $\lambda_2 = \begin{cases} \tilde{\lambda}_2, & \theta > \dfrac{\delta}{5}, \\ \dfrac{1}{2}, & \text{otherwise.} \end{cases}$

B.7. Proof of Proposition 2

The equilibrium profits under no agreement are given by $\Pi_I^0 = \frac{(3\theta-\lambda(\delta+h))^2}{18\theta}$ and $\Pi_S^0 = \frac{(3\theta+\lambda(\delta+h))^2}{18\theta}$. The profits in equilibrium in Case 1 are given by $\Pi_I^{11} = \frac{(3\theta-2\delta\lambda)^2}{18\theta}$ and $\Pi_S^{11} = \frac{(3\theta+2\delta\lambda)^2}{18\theta}$. We have $\Pi_S^{11} - \Pi_S^0 = \frac{\lambda(\delta-h)(p_S^{11}+p_S^0)}{6\theta}$ and $\Pi_I^{11} - \Pi_I^0 = \frac{\lambda(h-\delta)(p_I^{11}+p_I^0)}{6\theta}$. Clearly, $\Pi_S^0 > \Pi_S^{11}$ and $\Pi_I^0 < \Pi_I^{11}$ as $\delta < h$ in our main model.

The profits in equilibrium in Case 2 are given by $\Pi_I^{12} = \frac{(3\theta-\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$ and $\Pi_S^{12} = \frac{(3\theta+\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$. We have $(1-\lambda)\Pi_I^{12} - \Pi_I^0 = \frac{\lambda(h-\theta)((1-\lambda)p_I^{12}+p_I^0)}{6\theta}$. Clearly, $\Pi_I^0 < (1-\lambda)\Pi_I^{12} < \Pi_I^{12}$ when $\theta < h$, which is the case under the equilibrium in Case 2 (as shown in Appendix B.4). Comparing the expressions of the profits of the offline retailer, we can show that $\Pi_S^0 < \Pi_S^{12}$ only when $\theta > \theta_\pi$, where $\theta_\pi = \frac{\sqrt{1-\lambda}(\delta\lambda+h(\lambda+3))-4\delta\lambda+3h(1-\lambda)}{15+\lambda}$. Therefore, both retailers benefit from the agreement if and only if $\theta_\pi < \theta \leq \theta_2$. For instance, for $v = 20$, $\lambda = 0.3$, $h = 1.5$,

$\delta = 1$, the parameter space that satisfies our assumptions is given by $\{\theta : \theta > 0.25\}$ and both retailers benefit from the agreement when $0.41 < \theta \leq 0.52$.

Expressing the condition $\theta > \theta_\pi$ in terms of λ, δ and h, we get $\lambda > \lambda_\pi$, $\delta > \delta_\pi$ and $h < h_\pi$, where $\lambda_\pi = \frac{h^2+2\delta(h-4\theta)-6h\theta-\theta^2+(h-\theta)\sqrt{(h+2\delta)^2+2(7h+8\delta)\theta+\theta^2}}{2(h+\delta)^2}$, $\delta_\pi = \frac{h-4\theta-h\lambda+\sqrt{1-\lambda}(h-\theta)}{\lambda}$ and $h_\pi = \frac{\delta\lambda+\theta(3+\lambda)+\sqrt{1-\lambda}(-3\theta-\delta\lambda)}{\sqrt{1-\lambda}\lambda}$. $\square$

B.8. Proof of Proposition 3

Clearly, from the constraint in Equation 4, a feasible transfer payment exists only if the additional surplus created by the agreement ($\Sigma = \Pi_S^1 + \Pi_I^1 - \Pi_S^0 - \Pi_I^0$) is positive. The equilibrium profits under no agreement are given by $\Pi_I^0 = \frac{(3\theta-\lambda(\delta+h))^2}{18\theta}$ and $\Pi_S^0 = \frac{(3\theta+\lambda(\delta+h))^2}{18\theta}$. The profits in equilibrium in Case 1 are given by $\Pi_I^{11} = \frac{(3\theta-2\delta\lambda)^2}{18\theta}$ and $\Pi_S^{11} = \frac{(3\theta+2\delta\lambda)^2}{18\theta}$. The additional surplus created under the equilibrium in Case 1 can be written as $\Sigma_1 = \Pi_S^{11} + \Pi_I^{11} - \Pi_S^0 - \Pi_I^0 = \frac{(\delta-h)(3\delta+h)\lambda^2}{9\theta} < 0$.

The profits in equilibrium in Case 2 are given by $\Pi_I^{12} = \frac{(3\theta-\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$ and $\Pi_S^{12} = \frac{(3\theta+\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$. The additional surplus created under the equilibrium in Case 2 is given by $\Sigma_2 = \Pi_S^{12} + \Pi_I^{12} - \Pi_S^0 - \Pi_I^0 = \frac{\lambda\left(9\theta^2-\lambda(h-\theta)(2\delta+h+\theta)+\lambda^2(\delta+h)^2\right)}{9\theta(1-\lambda)}$. We can show that $\Sigma_2 > 0$ only when $\theta > \theta_{\text{barg}}$, where $\theta_{\text{barg}} = \frac{\sqrt{\lambda\left(9h(h+2\delta)-8(h+\delta)^2\lambda-(h+\delta)^2\lambda^2\right)}-\delta\lambda}{9+\lambda}$.

Hence, there exists a feasible transfer payment given by $\tau^* = \Pi_S^0 - \Pi_S^1 + \beta\Sigma = \frac{\lambda\left(\lambda\left(4(\beta-2)\delta\theta-(1-2\beta)\theta^2+(1-2\beta)h(2\delta+h)-6h\theta\right)-(1-2\beta)\lambda^2(\delta+h)^2+3\theta(2h-(5-6\beta)\theta)\right)}{18\theta(1-\lambda)}$ such that both retailers enter into the agreement when Equilibrium 2 exists ($\theta \leq \theta_2$) and the additional surplus is positive ($\theta > \theta_{\text{barg}}$). For instance, for $v = 20$, $\lambda = 0.3$, $h = 1.5$, $\delta = 1$, the parameter space that satisfies our assumptions is given by $\{\theta : \theta > 0.25\}$ and both retailers enter into the agreement when $0.30 < \theta \leq 0.52$.

We can also show that this region is (weakly) bigger than the region in Proposition 2. Since $\Pi_I^{12} > \Pi_I^0$ and $\Pi_S^{12} > \Pi_S^0 \implies \Sigma_2 > 0$, the BORS agreement is feasible with the transfer payment whenever it is feasible without it.

We have $\tau^* = \Pi_S^0 - \Pi_S^1 + \beta\Sigma$. Hence, $\tau^* > 0 \iff \beta > \frac{\Pi_S^0 - \Pi_S^{12}}{\Sigma}$. When $\theta_{barg} < \theta < \theta_\pi$, $\Pi_S^{12} - \Pi_S^0 < 0$. Hence, since $\beta > 0$, τ^* is positive $\forall\beta$. When $\theta_\pi < \theta \leq \theta_2$, τ^* is positive $\iff \beta > \frac{\Pi_S^0 - \Pi_S^1}{\Sigma}$. $\square$

B.9. Proof of Proposition 4 and Corollary 2

Under no agreement, the consumer surplus is given by $CS^0 = \int_0^{x^0} U^0_{\{FP=I\}} + \int_{x^0}^1 U^0_{\{FP=S\}}$, where $U^0_{\{FP=S\}} = v - p^0_S - \theta(1-x) - \delta$ and $U^0_{\{FP=I\}} = v - \delta - p^0_I - \theta x - (\delta + h)\lambda$, as derived in Appendix B.1. We substitute the equilibrium prices under no agreement (given in Section 3) and compute the consumer surplus. We find $CS^0 = \frac{-9\theta(4\delta+5\theta-4v)-18\theta\lambda(\delta+h)+\lambda^2(\delta+h)^2}{36\theta}$. The retailers' surplus is given by $RS^0 = \Pi^0_S + \Pi^0_I$ and the social surplus $TS^0 = CS^0 + RS^0$.

Under the agreement, the surplus depends on whether it is equilibrium $P1$ or equilibrium $P2$. Under $P1$ the consumer surplus is given by $CS^{11} = \int_0^{x^1_{SP}} U^{11}_{\{FP=I\}} + \int_{x^1_{SP}}^{x^1_{FP}} U^{11}_{\{FP=I\}} + \int_{x^1_{FP}}^1 U^{11}_{\{FP=S\}}$, where $U^{11}_{\{FP=S\}} = v - p^{11}_S - \theta(1-x) - \delta$ and

$$U^{11}_{\{FP=I\}} = \begin{cases} v - p^{11}_I - \theta x - (1+2\lambda)\delta & x < x^1_{SP}, \\ v - \delta - \theta x - (1-\lambda)p^{11}_I - \lambda p^{11}_S - \lambda((1-2x)\theta + \delta) & \text{Otherwise.} \end{cases},$$

Under $P2$ the consumer surplus is given by $CS^{12} = \int_0^{x^1_{FP}} U^{12}_{\{FP=I\}} + \int_{x^1_{FP}}^1 U^{12}_{\{FP=S\}}$, where $U^{12}_{\{FP=S\}} = v - p^{12}_S - \theta(1-x) - \delta$ and $U^{12}_{\{FP=I\}} = v - \delta - \theta x - (1-\lambda)p^{12}_I - \lambda p^{12}_S - \lambda((1-2x)\theta + \delta)$, as derived in Appendix B.2. We substitute the equilibrium prices under the corresponding agreement equilibrium (given in Table 2) and compute the consumer surplus. We find that $CS^{11} = \frac{\delta^2\lambda(-32(\lambda-1)\lambda-9)-36\delta\theta\left(\lambda^2-1\right)+9\theta(\lambda-1)(4v-5\theta)}{36\theta(\lambda-1)}$. and $CS^{12} = -\frac{\lambda^2(\delta+\theta)^2+18\theta\lambda(\delta-2v)-9\theta(4\delta+5\theta-4v)}{36\theta(\lambda-1)}$. The retailers' surplus is given by $RS^{11} = \Pi^{11}_S + \Pi^{11}_I$ and $RS^{12} = \Pi^{12}_S + \Pi^{12}_I$ and the social surplus $TS^{11} = CS^{11} + RS^{11}$ and $TS^{12} = CS^{12} + RS^{12}$.

Under P1 ($\theta \geq \theta_1$), we compare CS^{11} to CS^0 and find that CS^{11} is always higher than CS^0. Under P2 ($\theta \leq \theta_2$), we compare CS^{12} to CS^0 and find that when $\delta > \delta_{CS}$, we have $CS^{12} < CS^0$, for all $\theta \leq \theta_2$. When $\delta \leq \delta_{CS}$, we have $CS^{12} > CS^0 \iff \theta < \theta_{CS}$, where $\delta_{CS} = \frac{h\left(-((\lambda-118)\lambda)+6\sqrt{3}\sqrt{(\lambda-45)(\lambda-1)}-117\right)}{(\lambda-109)\lambda}$ and $\theta_{CS} = \frac{8\delta\lambda-\sqrt{81h^2+\lambda^3\left(-(\delta+h)^2\right)+\lambda^2(\delta+h)(109\delta+127h)-9h\lambda(26\delta+23h)}+9h(\lambda-1)}{\lambda-45}$. We now compare retailers' surplus. As we have shown in Appendix B.8, we have $\Sigma_1 = RS^{11} - RS^0 < 0 \;\; \forall \;\; \theta \geq \theta_1$ and $\Sigma_2 = RS^{12} - RS^0 > 0 \iff \theta_{barg} < \theta \leq \theta_2$. We can also show that under $P1$ ($\theta \geq \theta_1$), we have $TS^{11} > TS^0 \iff \theta > \theta_{TS1}$, where $\theta_{TS1} = \frac{-21\delta^2\lambda^2+21\delta^2\lambda-9\delta^2-5h^2\lambda^2+5h^2\lambda-10\delta h\lambda^2+10\delta h\lambda}{18(\lambda-1)(\delta-h)}$. Under $P2$ ($\theta \leq \theta_2$), we have $TS^{12} > TS^0 \iff \theta > \theta_{TS2}$ and $\delta \leq \delta_{TS}$, where $\delta_{TS} = \frac{h\left(\lambda(106-25\lambda)+6\sqrt{5}\sqrt{(\lambda-1)(5\lambda-9)}-81\right)}{\lambda(25\lambda-61)}$ and $\theta_{TS2} = \frac{4\delta\lambda+\sqrt{81h^2-25\lambda^3(\delta+h)^2+\lambda^2(\delta+h)(61\delta+151h)-9h\lambda(18\delta+23h)}+9h(\lambda-1)}{5\lambda-9}$.

The proof to Corollary 2 can be easily deduced by identifying the region where the customer's surplus is higher, and both the offline and online retailers are better off with transfer payment (the region where they can be better off without transfer payment is a subset of this region). The last happens when $\theta_{barg} < \theta < \theta_2$. The first, as derived above, happens when $\theta < \min\{\theta_2, \theta_{CS}\}$ and $\delta \leq \delta_{CS}$. Therefore, the win-win-win region is given by $\theta_{barg} < \theta < \min\{\theta_2, \theta_{CS}\}$ and $\delta \leq \delta_{CS}$. In addition, we can show that in this case, $\theta_{CS} < \theta_\pi$. Therefore, without transfer payment, we cannot have a win-win-win situation. $\square$

B.10. Proof of Proposition 5

We produce the decision trees and utilities of customers in Figures B.1 and B.2 under no agreement and agreement, respectively.

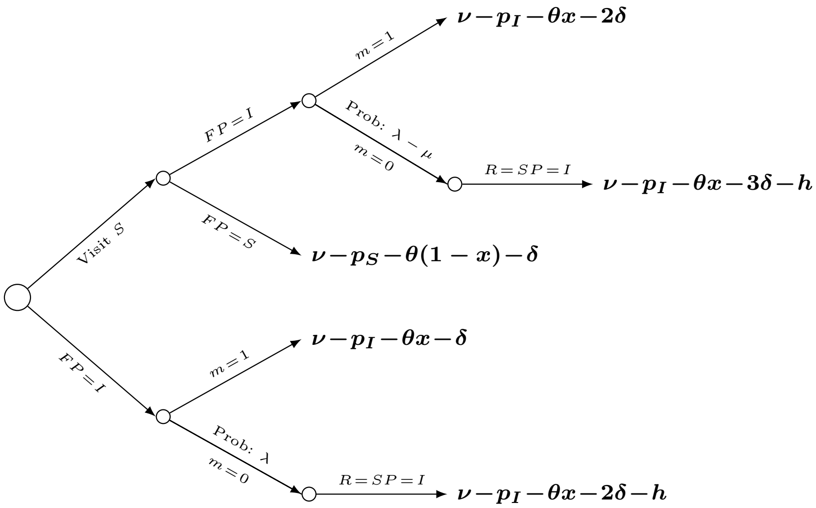

Figure B.1 Decision tree and utilities of a customer under no agreement

Under no agreement, a customer located at x can decide not to showroom and purchase online without an offline visit and get an expected utility $\nu - \delta - \lambda(h+\delta) - \theta x - p_I$ or showroom by visiting the offline store, reduce her uncertainty, and get an expected utility $\nu - 2\delta - (\lambda - \mu)(h+\delta) - \theta x - p_I$. Therefore, when $\mu \leq \frac{\delta}{\delta+h}$, the customer prefers no showrooming over showrooming.

Now we consider the agreement setting. We compare the utility of a customer from showrooming (visiting the offline store and then purchasing online) with her utility from directly purchasing online. First note that

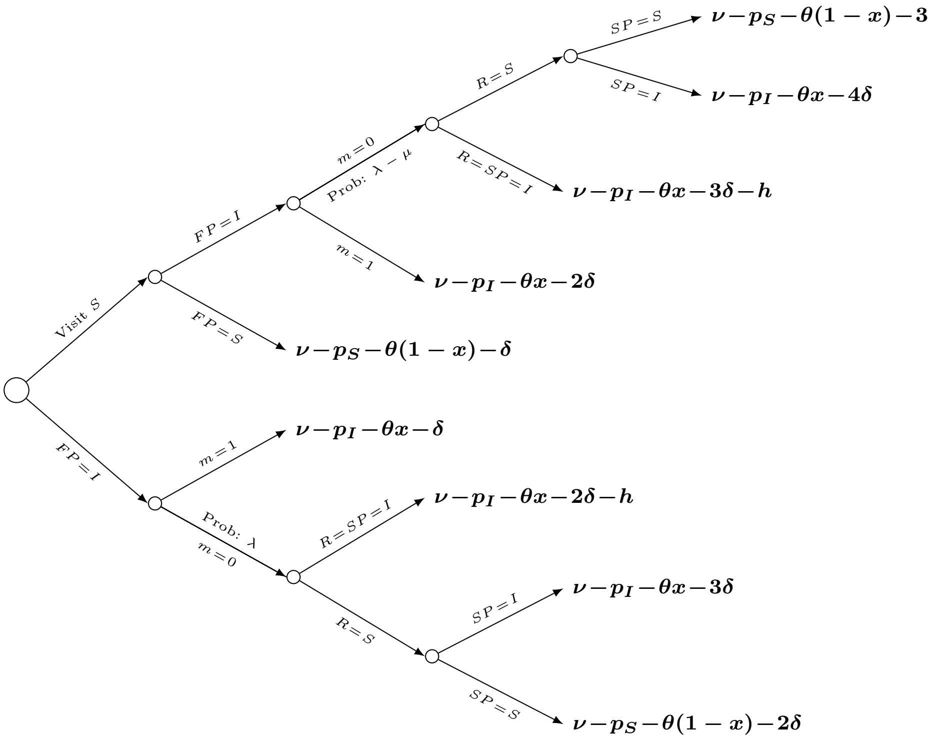

Figure B.2 Decision tree and utilities of a customer under the agreement

the second purchase decision ($SP = I$ or $SP = S$) of the customer is the same under both showrooming and no-showrooming cases. The customer would repurchase offline if ($\nu - \theta(1-x) - p_S > \nu - \theta x - \delta - p_I$) and vice versa. If the customer prefers repurchasing online, then her expected utility from showrooming is given by ($\nu - 2\delta - 2(\lambda - \mu)\delta - \theta x - p_I$) and not showrooming is given by ($\nu - \delta - 2\lambda\delta - \theta x - p_I$). If the customer prefers repurchasing offline, then her expected utility from showrooming is given by ($\nu - 2\delta + (1 - \lambda + \mu)(-\theta x - p_I) + (\lambda - \mu)(-\delta - \theta(1-x) - p_S)$) and not showrooming is given by ($\nu - \delta + (1-\lambda)(-\theta x - p_I) + \lambda(-\delta - \theta(1-x) - p_S)$). We can show that, in our region of interest ($\mu < \lambda < 1/2$), the customer's utility from not showrooming is higher in both cases. Hence, the customer does not showroom under the agreement.

Therefore, if $\mu \leq \frac{\delta}{\delta+h}$, customers do not engage in showrooming under both agreement and no agreement. Hence, we have the same equilibria and results as in our main model (see Section 3 and 4). However, if $\mu > \frac{\delta}{\delta+h}$, customers do not showroom under the agreement but showroom under no agreement. Therefore, we have the same equilibria under the agreement (see Section 4), but the results change under no agreement.

Consider $\mu > \frac{\delta}{\delta+h}$, following the proof in Appendix B.1, under no agreement, we obtain the following threshold: $x^0 = \frac{p_S - p_I - \delta(\lambda - \nu + 1) - h(\lambda - \nu) + \theta}{2\theta}$. Customers located at $x < x^0$ visit the offline store to reduce their uncertainty and then purchase online, and those located at $x \geq x^0$ purchase offline.

As in the main model, we solve for the different potential equilibria, checking for all possible non-local deviations. We are left with one equilibrium under no agreement when $\mu > \frac{\delta}{\delta+h}$. In Table B.4, we list all the equilibria for all possible values of μ.

Table B.4

μ	Eq Name	p_S	p_I	Π_S	Π_I
$\mu \leq \frac{\delta}{\delta+h}$	NPL	$\frac{3\theta+\lambda(\delta+h)}{3}$	$\frac{3\theta-\lambda(\delta+h)}{3}$	$\frac{(3\theta+\lambda(\delta+h))^2}{18\theta}$	$\frac{(3\theta-\lambda(\delta+h))^2}{18\theta}$
	PL1	$\frac{3\theta+2\delta\lambda}{3}$	$\frac{3\theta-2\delta\lambda}{3}$	$\frac{(3\theta+2\delta\lambda)^2}{18\theta}$	$\frac{(3\theta-2\delta\lambda)^2}{18\theta}$
	PL2	$\frac{3\theta+\lambda(\delta+\theta)}{3(1-\lambda)}$	$\frac{3\theta-\lambda(\delta+\theta)}{3(1-\lambda)}$	$\frac{(3\theta+\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$	$\frac{(3\theta-\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$
$\mu > \frac{\delta}{\delta+h}$	NPH	$\frac{3\theta+\delta+(\delta+h)(\lambda-\mu)}{3}$	$\frac{3\theta-\delta-(\delta+h)(\lambda-\mu)}{3}$	$\frac{(3\theta+\delta+(\delta+h)(\lambda-\mu))^2}{18\theta}$	$\frac{(3\theta-\delta-(\delta+h)(\lambda-\mu))^2}{18\theta}$
	PH1	$\frac{3\theta+2\delta\lambda}{3}$	$\frac{3\theta-2\delta\lambda}{3}$	$\frac{(3\theta+2\delta\lambda)^2}{18\theta}$	$\frac{(3\theta-2\delta\lambda)^2}{18\theta}$
	PH2	$\frac{3\theta+\lambda(\delta+\theta)}{3(1-\lambda)}$	$\frac{3\theta-\lambda(\delta+\theta)}{3(1-\lambda)}$	$\frac{(3\theta+\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$	$\frac{(3\theta-\lambda(\delta+\theta))^2}{18\theta(1-\lambda)}$

When $\mu \leq \frac{\delta}{\delta+h}$, we find the same results as in Proposition 2 and the agreement benefits both retailers only when the online retailer relinquishes returners. When $\mu > \frac{\delta}{\delta+h}$, clearly, the profit of the offline retailer is always lower under $PH1$ (no relinquishing) as compared to NPH as $h > \delta$ and $\lambda < 1/2$. Therefore, both retailers can be better off under the agreement only when returners are relinquished ($PH2$). It is easy to see that the profit of the online retailer under $PH2$ is always greater than its profit under NPH. Comparing the profit of the offline retailer under $PH2$ and NPH, the offline retailer is better off under $PH2$ if and only if $\theta > \frac{\sqrt{1-\lambda}\left(\delta\left(\lambda^2-(\lambda+3)\mu+\lambda+3\right)+h(\lambda+3)(\lambda-\mu)\right)}{\lambda(\lambda+15)} + \frac{\delta(\lambda(-4\lambda+3\mu-3)-3\mu+3)-3h(\lambda-1)(\lambda-\mu)}{\lambda(\lambda+15)}$. For instance, at parameter values $\nu = 20$, $\theta = 0.094$, $h = 0.948$, $\delta = 0.275$, $\mu = 0.408$, and $\lambda = 0.41$. Therefore, the offline retailer is better off under the BORS agreement when $\theta < \theta_2$ and $\theta > \tilde{\theta}_\pi$,

where
$$\tilde{\theta}_\pi = \begin{cases} \theta_\pi, & \mu \leq \frac{\delta}{h+\delta}, \\ \frac{\sqrt{1-\lambda}\left(\delta\left(\lambda^2-(\lambda+3)\mu+\lambda+3\right)+h(\lambda+3)(\lambda-\mu)\right)}{\lambda(\lambda+15)} + \frac{\delta(\lambda(-4\lambda+3\mu-3)-3\mu+3)-3h(\lambda-1)(\lambda-\mu)}{\lambda(\lambda+15)}, & \text{otherwise.} \end{cases}$$

When $\mu \leq \frac{\delta}{\delta+h}$ then $\frac{d(\theta_2-\tilde{\theta}_\pi)}{d\mu} = 0$. When $\mu > \frac{\delta}{\delta+h}$ then $\frac{d(\theta_2-\tilde{\theta}_\pi)}{d\mu} = \frac{\left(3(1-\lambda)+(\lambda+3)\sqrt{1-\lambda}\right)(\delta+h)}{\lambda(\lambda+15)} > 0$. □

B.11. Proof Of Lemma 1

We use the Spokes model for the setting with three retailers. There are three spokes on a plane and each retailer $j \in \{I, S_1, S_2\}$ is located at the origin of its corresponding spoke j. Consumers of unit mass are uniformly distributed on the spokes and each consumer, located on spoke j, considers at most two retailers – the local retailer j and one of the non-local retailers with a probability $1/2$. This assumption is standard in the literature grounded in the spokes model (Chen and Riordan 2007, Amaldoss and He 2010, 2013, 2018). Any consumer located on spoke j or on spoke j' considers retailers j and j' with a probability $1/2$, where $j, j' \in \{I, S_1, S_2\}$. Each retailer is situated on a spoke of length $1/2$ (Figure B.3). Since there is a unit mass of customers, their density is given by $2/3$.

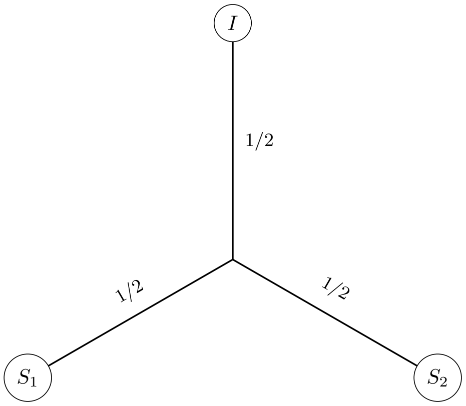

Figure B.3 Spokes Model

We solve three models: No agreement ($\Psi = 0$), Partnering with one offline retailer S_k ($\Psi = 1k$, $k \in \{1, 2\}$), and Partnering with both retailers ($\Psi = 2$).

No agreement $\Psi = 0$. Under no agreement, we look for the indifferent customer of each consideration set similarly to the no agreement in the base model. If a customer considers $\{I, S_i\}$, $i \in \{1, 2\}$, she buys from I when $x < x^{0IS_i}$ and buys from S_i otherwise, such that $x^{0IS_i} = \frac{-\delta\lambda - h\lambda + \theta - p_I + p_{S_i}}{2\theta}$. If she considers $\{S_2, S_1\}$, she buys from S_2 when $x < x^{0S_2S_1}$ and buys from S_1 otherwise, such that $x^{0S_2S_1} = \frac{\theta + p_{S_1} - p_{S_2}}{2\theta}$. Therefore, the demand of I is given by $Q_I^0 = \frac{2}{3} \times \frac{1}{2}\big(x^{0IS_1} + x^{0S_2I}\big)$. The demand of S_1 is given by $Q_{S_1}^0 = \frac{2}{3} \times \frac{1}{2}\big((1 - x^{0IS_1}) + (1 - x^{0S_2S_1})\big)$. The demand of S_2 is given by $Q_{S_2}^0 = \frac{2}{3} \times \frac{1}{2}\big((1 - x^{0IS_2}) + x^{0S_2S_1}\big)$.

Agreement with one offline retailer (S_k) $\Psi = 1k$. The online retailer can partner with either S_1 or S_2. Let k be the index of the offline retailer the online retailer partners with and $-k$ be the index of the offline retailer the online retailer does not partner with. Following the proof in Appendix B.2, under the agreement, whenever customers consider both partnering retailers ($\{I, S_k\}$), we obtain the following thresholds: $x_{SP}^{1S_k} = \frac{-\delta+\theta-p_I+p_{S_k}}{2\theta}$ and $x_{FP}^{1S_k} = x_{SP}^{1S_k} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$, where these customers follow Strategy ISI if located at $x < x_{SP}^{1S_k}$, follow Strategy ISS if located at $x_{SP}^{1S_k} \leq x < x_{FP}^{1S_k}$, and follow Strategy S otherwise. The other customers (those who consider $\{I, S_{-k}\}$ or $\{S_2, S_1\}$ follow the same strategies as in the no agreement case and have the same thresholds: $x^{1IS_{-k}} = x^{0IS_{-k}}$ and $x^{1S_2S_1} = x^{0S_2S_1}$. As in our base model, we derive the demand of each retailer (for each of the three consideration sets) depending on whether relinquishing happens or not. We then combine and normalize the demands from the consideration sets as in the no agreement case.

Agreement with both offline retailers $\Psi = 2$. Following the proof in Appendix B.2 and the agreement with one case, we solve for the thresholds for each consideration set. For the customers who consider $\{I, S_i\}$ we obtain these thresholds $x_{SP}^{2S_i} = \frac{-\delta+\theta-p_I+p_{S_i}}{2\theta}$ and $x_{FP}^{2S_i} = x_{SP}^{2S_i} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$. The customers who consider $\{S_2, S_1\}$ have the same thresholds $x^{2S_2S_1} = x^{0S_2S_1}$. As in our base model, we derive the demand of each retailer (for each of the three consideration sets) depending on whether relinquishing happens or not. We then combine and normalize the demands from the consideration sets as in the no agreement case.

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. We focus on the more interesting potential equilibria for each of the three models:

NP ($\Psi = 0$, No Agreement) : $0 < x^{0IS_1} < 1$, $0 < x^{0IS_2} < 1$, and $0 < x^{0S_2S_1} < 1$,

P1k1 ($\Psi = 1k$, No Relinquishing) : $0 < x_{SP}^{1S_k} < x_{FP}^{1S_k} < 1$, $0 < x^{1S_{-k}I} < 1$, and $0 < x^{1S_2S_1} < 1$,

P1k2 ($\Psi = 1k$, Relinquishing to S_k) : $x_{SP}^{1S_k} \leq 0 < x_{FP}^{1S_k} < 1$, $0 < x^{1S_{-k}I} < 1$, and $0 < x^{1S_2S_1} < 1$,

P21 ($\Psi = 2$, No Relinquishing) : $0 < x_{SP}^{2S_1} < x_{FP}^{2S_1} < 1$, $0 < x_{SP}^{2S_2} < x_{FP}^{2S_2} < 1$, and $0 < x^{2S_2S_1} < 1$,

P22 ($\Psi = 2$, Relinquishing to both): $x_{SP}^{2S_1} \leq 0 < x_{FP}^{2S_1} < 1$, $x_{SP}^{2S_2} \leq 0 < x_{FP}^{2S_2} < 1$, and $0 < x^{2S_2S_1} < 1$.

P23 ($\Psi = 2$, Relinquishing to S_1) : $x_{SP}^{2S_1} \leq 0 < x_{FP}^{2S_1} < 1 - \alpha$, $0 < x_{SP}^{2S_2} < x_{FP}^{2S_2} < 1 + \alpha$, and $0 < x^{2S_2S_1} < 1$,

P24 ($\Psi = 2$, Relinquishing to S_2): $0 < x_{SP}^{2S_1} < x_{FP}^{2S_1} < 1 - \alpha$, $x_{SP}^{2S_2} \leq 0 < x_{FP}^{2S_2} < 1 + \alpha$, and $0 < x^{2S_2S_1} < 1$, We find that when the online retailer partners with both retailers, there are only two equilibria: No relinquishing (P21) and relinquishing to both retailers (P22). This is because both offline retailers are symmetric and we obtain symmetric equilibria. When the online retailer partners with only one retailer, we also obtain two equilibria: no relinquishing (P111 or P121) and relinquishing to the partnering retailer (P112 or P122). We list the relevant equilibrium candidates below, the prices in Table B.5, and the profits in Table B.6.

Table B.5 Equilibria Prices

Eq	p_{S_1}	p_I	p_{S_2}
NP	$\frac{1}{5}\lambda(\delta+h)+\theta$	$\theta-\frac{2}{5}\lambda(\delta+h)$	$\frac{1}{5}\lambda(\delta+h)+\theta$
P111	$\frac{2\delta\lambda}{5}+\theta$	$\theta-\frac{1}{5}\lambda(3\delta+h)$	$\frac{1}{5}\lambda(\delta+h)+\theta$
P112	$\frac{\lambda(\delta(10-3\lambda)+h\lambda)+\theta(50-4\lambda(\lambda+5))}{2(2\lambda-5)(3\lambda-5)}$	$\frac{\delta(9\lambda-20)\lambda+\theta\left(4\lambda^2-40\lambda+50\right)+5h(\lambda-2)\lambda}{12\lambda^2-50\lambda+50}$	$\frac{(\lambda-2)\lambda(\delta+h)+2\theta(\lambda-5)}{4\lambda-10}$
P121	$\frac{1}{5}\lambda(\delta+h)+\theta$	$\theta-\frac{1}{5}\lambda(3\delta+h)$	$\frac{2\delta\lambda}{5}+\theta$
P122	$\frac{(\lambda-2)\lambda(\delta+h)+2\theta(\lambda-5)}{4\lambda-10}$	$\frac{\delta(9\lambda-20)\lambda+\theta\left(4\lambda^2-40\lambda+50\right)+5h(\lambda-2)\lambda}{12\lambda^2-50\lambda+50}$	$\frac{\lambda(\delta(10-3\lambda)+h\lambda)+\theta(50-4\lambda(\lambda+5))}{2(2\lambda-5)(3\lambda-5)}$
P21	$\frac{2\delta\lambda}{5}+\theta$	$\theta-\frac{4\delta\lambda}{5}$	$\frac{2\delta\lambda}{5}+\theta$
P22	$\frac{\delta\lambda+\theta(\lambda+5)}{5-3\lambda}$	$\frac{\delta(\lambda-2)\lambda+\theta(\lambda-5)(\lambda-1)}{(\lambda-1)(3\lambda-5)}$	$\frac{\delta\lambda+\theta(\lambda+5)}{5-3\lambda}$

Table B.6 Equilibria Profits

Eq	Π_{S_1}	Π_I	Π_{S_2}
NP	$\frac{(\lambda(\delta+h)+5\theta)^2}{75\theta}$	$\frac{(5\theta-2\lambda(\delta+h))^2}{75\theta}$	$\frac{(\lambda(\delta+h)+5\theta)^2}{75\theta}$
P111	$\frac{(2\delta\lambda+5\theta)^2}{75\theta}$	$\frac{(\lambda(3\delta+h)-5\theta)^2}{75\theta}$	$\frac{(\lambda(\delta+h)+5\theta)^2}{75\theta}$
P112	$-\frac{(\lambda-2)(\lambda(\delta(10-3\lambda)+h\lambda)+\theta(50-4\lambda(\lambda+5)))^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$-\frac{(\lambda-2)\left(\lambda^2(9\delta+5h+4\theta)-10\lambda(2\delta+h+4\theta)+50\theta\right)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$\frac{((\lambda-2)\lambda(\delta+h)+2\theta(\lambda-5))^2}{12\theta(5-2\lambda)^2}$
P121	$\frac{(\lambda(\delta+h)+5\theta)^2}{75\theta}$	$\frac{(\lambda(3\delta+h)-5\theta)^2}{75\theta}$	$\frac{(2\delta\lambda+5\theta)^2}{75\theta}$
P122	$\frac{((\lambda-2)\lambda(\delta+h)+2\theta(\lambda-5))^2}{12\theta(5-2\lambda)^2}$	$-\frac{(\lambda-2)\left(\lambda^2(9\delta+5h+4\theta)-10\lambda(2\delta+h+4\theta)+50\theta\right)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$-\frac{(\lambda-2)(\lambda(\delta(10-3\lambda)+h\lambda)+\theta(50-4\lambda(\lambda+5)))^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$
P21	$\frac{(2\delta\lambda+5\theta)^2}{75\theta}$	$\frac{(5\theta-4\delta\lambda)^2}{75\theta}$	$\frac{(2\delta\lambda+5\theta)^2}{75\theta}$
P22	$-\frac{(\lambda-2)(\delta\lambda+\theta(\lambda+5))^2}{6\theta(5-3\lambda)^2}$	$-\frac{(\delta(\lambda-2)\lambda+\theta(\lambda-5)(\lambda-1))^2}{3\theta(5-3\lambda)^2(\lambda-1)}$	$-\frac{(\lambda-2)(\delta\lambda+\theta(\lambda+5))^2}{6\theta(5-3\lambda)^2}$

Clearly, the profit of the offline retailer is always lower under P1k1 (no relinquishing) as compared to NP and is always lower under P21 (no relinquishing) as compared to NP as $h > \delta$. Therefore, the partnering retailers can be better off under the agreement only when returners are relinquished to at least one of the competing offline retailer(s) (in P1k2 and P22).

We can also show that whenever P1k2 exists, P22 also exists. That is, whenever relinquishing to one offline retailer is an equilibrium, relinquishing to both offline retailers is also an equilibrium. As in the main model, by checking all possible deviations, we can show that P22 exists if and only if $\theta \leq \theta_{22}$, where $\theta_{22} = \frac{\left(\left(-6\lambda+4\sqrt{1-\lambda}+13\right)\lambda-5\left(\sqrt{1-\lambda}+1\right)\right)\delta}{2\sqrt{1-\lambda}(\lambda-5)}$. Additionally, we can show that P1k2 is not an equilibrium for any $\theta > \theta_{22}$. This is because when $\theta > \theta_{22}$, the online retailer has a profitable deviation. Specifically, the online retailer can play the price $p_I = \frac{2\theta(\lambda(13\lambda-80)+100)-\lambda(\delta(\lambda(33\lambda-136)+130)+h(\lambda(9\lambda-40)+40))}{8(2\lambda-5)(3\lambda-5)}$ and deviate to the non relinquishing equilibrium. Furthermore, we can show that the profit of the partnering retailers (I and S_k) is higher in $P22$ as compared to $P1k2$. Hence, whenever the agreement is feasible in $P1k2$ it is also feasible in $P22$. Furthermore, the online retailer makes a higher profit in $P22$ as compared to $P1k2$. Therefore, we have: i)whenever the agreement is feasible with one offline retailer, it is also feasible with two offline retailers, and ii) the online retailer prefers partnering with two offline retailers over partnering with one offline retailer.

To show that the regions of interest are not empty, we provide a numerical instance: when $v = 20$, $h = 2.841$, $\lambda = 0.12$, $\delta = 2.453$, the region where $P22$ exists and the agreement benefits the three retailers as compared to no agreement is given by $\theta \in [0.939, 2.08]$; and the region where (P1k2) exists and the agreement benefits both retailers (I and S_k) as compared to no agreement is given by $\theta \in [1.47, 2.05]$. □

Proof of Proposition 2

We use the Spokes model for the setting with three retailers. There are three spokes on a plane and each retailer $j \in \{I, S_1, S_2\}$ is located at the origin of its corresponding spoke j. Consumers of unit mass are uniformly distributed on the spokes and each consumer, located on spoke j, considers at most two retailers – the local retailer j and one of the non-local retailers with a probability $1/2$. This assumption is standard in the literature grounded in the spokes model (Chen and Riordan 2007, Amaldoss and He 2010, 2013, 2018). Any consumer located on spoke j or on spoke j' considers retailers j and j' with a probability $1/2$, where $j, j' \in \{I, S_1, S_2\}$. We allow the two offline retailers to be asymmetric such that the retailer S_i, $i \in \{1, 2\}$ is situated on a spoke of length $1/2 + \alpha_i$, where $\alpha_1 = -\alpha$ and $\alpha_2 = \alpha$. Without loss of a generality we assume

$\alpha \in (0, 1/2)$ such that S_2 is the stronger retailer. Since there is a unit mass of customers, their density is given by $2/3$.

We solve three models: No agreement ($\Psi = 0$), Partnering with S_k ($\Psi = 1k,\ k \in \{1, 2\}$), and Partnering with Both retailers ($\Psi = 2$).

No agreement $\Psi = 0$. Under no agreement, we look for the indifferent customer of each consideration set similarly to the no agreement in the base model. If a customer considers $\{I, S_i\}$, she buys from I when $x < x^{0IS_i}$ and buys from S_i otherwise, such that $x^{0IS_i} = \frac{\alpha_i \theta - \delta\lambda - h\lambda + \theta - p_I + p_{S_i}}{2\theta}$. If she considers $\{S_2, S_1\}$, she buys from S_2 when $x < x^{0S_2S_1}$ and buys from S_1 otherwise, such that $x^{0S_2S_1} = \frac{\theta + p_{S_1} - p_{S_2}}{2\theta}$. Therefore, the demand of I is given by $Q_I^0 = \frac{2}{3} \times \frac{1}{2}\left(x^{0IS_1} + x^{0S_2I}\right)$. The demand of S_1 is given by $Q_{S_1}^0 = \frac{2}{3} \times \frac{1}{2}\left((1 - \alpha - x^{0IS_1}) + (1 - x^{0S_2S_1})\right)$. The demand of S_2 is given by $Q_{S_2}^0 = \frac{2}{3} \times \frac{1}{2}\left((1 + \alpha - x^{0IS_2}) + x^{0S_2S_1}\right)$.

Agreement with one offline retailer (S_k) $\Psi = 1k$. The online retailer can partner with either S_1 or S_2. Let k be the index of the offline retailer the online retailer partners with and $-k$ be the index of the offline retailer the online retailer does not partner with. Following the proof in Appendix B.2, under the agreement, whenever customers consider both partnering retailers ($\{I, S_k\}$), we obtain the following thresholds: $x_{SP}^{1S_k} = \frac{\alpha_k \theta - \delta + \theta - p_I + p_{S_k}}{2\theta}$ and $x_{FP}^{1S_k} = x_{SP}^{1S_k} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$, where these customers follow Strategy ISI if located at $x < x_{SP}^{1S_k}$, follow Strategy ISS if located at $x_{SP}^{1S_k} \le x < x_{FP}^{1S_k}$, and follow Strategy S otherwise. The other customers (those who consider $\{I, S_{-k}\}$ or $\{S_2, S_1\}$ follow the same strategies as in the no agreement case and have the same thresholds: $x^{1IS_{-k}} = x^{0IS_{-k}}$ and $x^{1S_2S_1} = x^{0S_2S_1}$. As in our base model, we derive the demand of each retailer (for each of the three consideration sets) depending on whether relinquishing happens or not. We then combine and normalize the demands from the consideration sets as in the no agreement case.

Agreement with both offline retailers $\Psi = 2$. Following the proof in Appendix B.2 and the agreement with one case, we solve for the thresholds for each consideration set. For the customers who consider $\{I, S_i\}$ we obtain these thresholds $x_{SP}^{2S_i} = \frac{\alpha_i \theta - \delta + \theta - p_I + p_{S_i}}{2\theta}$ and $x_{FP}^{2S_i} = x_{SP}^{2S_i} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$. The customers who consider $\{S_2, S_1\}$ have the same thresholds $x^{2S_2S_1} = x^{0S_2S_1}$. As in our base model, we derive the demand of each retailer (for each of the three consideration sets) depending on whether relinquishing happens or not. We then combine and normalize the demands from the consideration sets as in the no agreement case.

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. We focus on the more interesting potential equilibria for each of the three models:

NP ($\Psi = 0$, No Agreement) : $0 < x^{0IS_1} < 1-\alpha$, $0 < x^{0IS_2} < 1+\alpha$, and $0 < x^{0S_2S_1} < 1$,

P1k1 ($\Psi = 1k$, No Relinquishing) : $0 < x_{SP}^{1S_k} < x_{FP}^{1S_k} < 1+\alpha_k$, $0 < x^{1S_{-k}I} < 1+\alpha_{-k}$, and $0 < x^{1S_2S_1} < 1$,

P1k2 ($\Psi = 1k$, Relinquishing to S_k) : $x_{SP}^{1S_k} \leq 0 < x_{FP}^{1S_k} < 1+\alpha_k$, $0 < x^{1S_{-k}I} < 1-\alpha_{-k}$, and $0 < x^{1S_2S_1} < 1$,

P21 ($\Psi = 2$, No Relinquishing) : $0 < x_{SP}^{2S_1} < x_{FP}^{2S_1} < 1-\alpha$, $0 < x_{SP}^{2S_2} < x_{FP}^{2S_2} < 1+\alpha$, and $0 < x^{2S_2S_1} < 1$,

P22 ($\Psi = 2$, Relinquishing to both): $x_{SP}^{2S_1} \leq 0 < x_{FP}^{2S_1} < 1-\alpha$, $x_{SP}^{2S_2} \leq 0 < x_{FP}^{2S_2} < 1+\alpha$, and $0 < x^{2S_2S_1} < 1$.

P23 ($\Psi = 2$, Relinquishing to S_1) : $x_{SP}^{2S_1} \leq 0 < x_{FP}^{2S_1} < 1-\alpha$, $0 < x_{SP}^{2S_2} < x_{FP}^{2S_2} < 1+\alpha$, and $0 < x^{2S_2S_1} < 1$,

P24 ($\Psi = 2$, Relinquishing to S_2): $0 < x_{SP}^{2S_1} < x_{FP}^{2S_1} < 1-\alpha$, $x_{SP}^{2S_2} \leq 0 < x_{FP}^{2S_2} < 1+\alpha$, and $0 < x^{2S_2S_1} < 1$,

We list the relevant equilibrium candidates below, the prices in Table B.7, and the profits in Table B.8.

Table B.7 Equilibria Prices

Eq	p_{S_1}	p_I	p_{S_2}
NP	$\frac{1}{5}(\lambda(\delta+h)-(\alpha-5)\theta)$	$\theta-\frac{2}{5}\lambda(\delta+h)$	$\frac{1}{5}((\alpha+5)\theta+\lambda(\delta+h))$
P111	$-\frac{\alpha\theta}{5}+\frac{2\delta\lambda}{5}+\theta$	$\theta-\frac{1}{5}\lambda(3\delta+h)$	$\frac{1}{5}((\alpha+5)\theta+\lambda(\delta+h))$
P112	$\frac{-5(\alpha+4)\theta\lambda-10(\alpha-5)\theta+10\delta\lambda+\lambda^2(4(\alpha-1)\theta-3\delta+h)}{2(2\lambda-5)(3\lambda-5)}$	$\frac{\theta\left(-4(\alpha-1)\lambda^2+(11\alpha-40)\lambda+50\right)+\lambda(\delta(9\lambda-20)+5h(\lambda-2))}{12\lambda^2-50\lambda+50}$	$\frac{(\alpha+2)\theta\lambda-2(\alpha+5)\theta+(\lambda-2)\lambda(\delta+h)}{4\lambda-10}$
P121	$\frac{1}{5}(\lambda(\delta+h)-(\alpha-5)\theta)$	$\theta-\frac{1}{5}\lambda(3\delta+h)$	$\frac{\alpha\theta}{5}+\frac{2\delta\lambda}{5}+\theta$
P122	$\frac{-\alpha\theta(\lambda-2)+(\lambda-2)\lambda(\delta+h)+2\theta(\lambda-5)}{4\lambda-10}$	$\frac{\theta\lambda(4(\alpha+1)\lambda-11\alpha-40)+\delta\lambda(9\lambda-20)+5h(\lambda-2)\lambda+50\theta}{2(2\lambda-5)(3\lambda-5)}$	$\frac{5\lambda((\alpha-4)\theta+2\delta)+10(\alpha+5)\theta+\lambda^2(-4(\alpha+1)\theta-3\delta+h)}{2(2\lambda-5)(3\lambda-5)}$
P21	$-\frac{\alpha\theta}{5}+\frac{2\delta\lambda}{5}+\theta$	$\theta-\frac{4\delta\lambda}{5}$	$\frac{\alpha\theta}{5}+\frac{2\delta\lambda}{5}+\theta$
P22	$\frac{1}{6}\left(\theta\left(\frac{6\alpha(\lambda+1)}{2\lambda-5}-2\right)+\frac{6\delta\lambda+40\theta}{5-3\lambda}\right)$	$\frac{\delta(\lambda-2)\lambda+\theta(\lambda-5)(\lambda-1)}{(\lambda-1)(3\lambda-5)}$	$\frac{1}{6}\left(\theta\left(-\frac{6\alpha(\lambda+1)}{2\lambda-5}-2\right)+\frac{6\delta\lambda+40\theta}{5-3\lambda}\right)$
P23	$\frac{\theta(\alpha(\lambda(4\lambda-5)-10)-4\lambda(\lambda+5)+50)-2\delta(\lambda-5)\lambda}{2(2\lambda-5)(3\lambda-5)}$	$\frac{\theta\left(-4(\alpha-1)\lambda^2+(11\alpha-40)\lambda+50\right)+2\delta\lambda(7\lambda-15)}{12\lambda^2-50\lambda+50}$	$\frac{(\alpha+2)\theta\lambda-2(\alpha+5)\theta+2\delta(\lambda-2)\lambda}{4\lambda-10}$
P24	$\frac{-\alpha\theta(\lambda-2)+2\delta(\lambda-2)\lambda+2\theta(\lambda-5)}{4\lambda-10}$	$\frac{\theta(\lambda(4(\alpha+1)\lambda-11\alpha-40)+50)+2\delta\lambda(7\lambda-15)}{2(2\lambda-5)(3\lambda-5)}$	$\frac{\theta(\alpha(\lambda(5-4\lambda)+10)-4\lambda(\lambda+5)+50)-2\delta(\lambda-5)\lambda}{2(2\lambda-5)(3\lambda-5)}$

It is easy to see that the condition for $P24$ is not satisfied under the equilibrium candidate prices when $\alpha > 0$. Hence, relinquishing only to S_2 can not be an equilibrium. Furthermore, clearly, the profit of the offline retailer is always lower under P11 (no relinquishing) as compared to NP and is always lower under $P21$ (no relinquishing) as compared to NP as $h > \delta$. Therefore, the partnering retailers can be better off under the agreement only when returners are relinquished to at least one of the competing offline retailer(s) (in $P112$, $P122$, $P22$, and $P23$).

Table B.8 Equilibria Profits

Eq	Π_{S_1}	Π_I	Π_{S_2}
NP	$\frac{((\alpha-5)\theta-\lambda(\delta+h))^2}{75\theta}$	$\frac{(5\theta-2\lambda(\delta+h))^2}{75\theta}$	$\frac{((\alpha+5)\theta+\lambda(\delta+h))^2}{75\theta}$
P111	$\frac{((\alpha-5)\theta-2\delta\lambda)^2}{75\theta}$	$\frac{(\lambda(3\delta+h)-5\theta)^2}{75\theta}$	$\frac{((\alpha+5)\theta+\lambda(\delta+h))^2}{75\theta}$
P112	$-\frac{(\lambda-2)\left(-5(\alpha+4)\theta\lambda-10(\alpha-5)\theta+10\delta\lambda+\lambda^2(4(\alpha-1)\theta-3\delta+h)\right)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$-\frac{(\lambda-2)(\theta\lambda(-4\alpha\lambda+11\alpha+4\lambda-40)+\delta\lambda(9\lambda-20)+5h(\lambda-2)\lambda+50\theta)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$\frac{((\alpha+2)\theta\lambda-2(\alpha+5)\theta+(\lambda-2)\lambda(\delta+h))^2}{12\theta(5-2\lambda)^2}$
P121	$\frac{((\alpha-5)\theta-\lambda(\delta+h))^2}{75\theta}$	$\frac{(\lambda(3\delta+h)-5\theta)^2}{75\theta}$	$\frac{((\alpha+5)\theta+2\delta\lambda)^2}{75\theta}$
P122	$\frac{(-\alpha\theta(\lambda-2)+(\lambda-2)\lambda(\delta+h)+2\theta(\lambda-5))^2}{12\theta(5-2\lambda)^2}$	$-\frac{(\lambda-2)(\theta\lambda(4(\alpha+1)\lambda-11\alpha-40)+\delta\lambda(9\lambda-20)+5h(\lambda-2)\lambda+50\theta)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$-\frac{(\lambda-2)\left(5\lambda((\alpha-4)\theta+2\delta)+10(\alpha+5)\theta+\lambda^2(-4(\alpha+1)\theta-3\delta+h)\right)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$
P21	$\frac{((\alpha-5)\theta-2\delta\lambda)^2}{75\theta}$	$\frac{(5\theta-4\delta\lambda)^2}{75\theta}$	$\frac{((\alpha+5)\theta+2\delta\lambda)^2}{75\theta}$
P22	$-\frac{(\lambda-2)(\alpha\theta(\lambda(2-3\lambda)+5)+\delta\lambda(2\lambda-5)+\theta(\lambda+5)(2\lambda-5))^2}{6\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$-\frac{(\delta(\lambda-2)\lambda+\theta(\lambda-5)(\lambda-1))^2}{3\theta(5-3\lambda)^2(\lambda-1)}$	$-\frac{(\lambda-2)(\alpha\theta(\lambda+1)(3\lambda-5)+\delta\lambda(2\lambda-5)+\theta(\lambda+5)(2\lambda-5))^2}{6\theta\left(6\lambda^2-25\lambda+25\right)^2}$
P23	$-\frac{(\lambda-2)(\theta(\alpha(\lambda(5-4\lambda)+10)+4\lambda(\lambda+5)-50)+2\delta(\lambda-5)\lambda)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$-\frac{(\lambda-2)(\theta(\lambda(-4\alpha\lambda+11\alpha+4\lambda-40)+50)+2\delta\lambda(7\lambda-15))^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$\frac{((\alpha+2)\theta\lambda-2(\alpha+5)\theta+2\delta(\lambda-2)\lambda)^2}{12\theta(5-2\lambda)^2}$
P24	$\frac{(-\alpha\theta(\lambda-2)+2\delta(\lambda-2)\lambda+2\theta(\lambda-5))^2}{12\theta(5-2\lambda)^2}$	$-\frac{(\lambda-2)(\theta(\lambda(4(\alpha+1)\lambda-11\alpha-40)+50)+2\delta\lambda(7\lambda-15))^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$	$-\frac{(\lambda-2)(\theta(\alpha(\lambda(4\lambda-5)-10)+4\lambda(\lambda+5)-50)+2\delta(\lambda-5)\lambda)^2}{24\theta\left(6\lambda^2-25\lambda+25\right)^2}$

We show the existence of each of the relevant regions by providing a numerical example. For instance, when $v = 20$, $\alpha = 0.431$, $h = 3.315$, $\lambda = 0.118$, and $\delta = 2.62$, all relinquishing equilibria exist.

- $P112$ exists and makes S_1 and I better off when $2.5027 < \theta \leq 4.4913$,
- $P122$ exists and makes S_2 and I better off when $1.3153 < \theta \leq 1.4514$,
- $P22$ exists and makes S_1, S_2 and I better off when $1.3322 < \theta \leq 1.5087$,
- $P23$ exists and makes S_1, S_2 and I better off when $2.5099 < \theta \leq 4.5247$.

In addition, by examining the above regions and comparing the online retailer's profit whenever there is an overlap, we can show that the online retailer prefers:

- partnering with both and relinquishing to both ($P22$) when $1.3322 < \theta \leq 1.5087$,
- partnering with both and relinquishing to S_1 ($P23$) when $2.5099 < \theta \leq 4.5247$,
- partnering with only S_1 when $2.5027 < \theta \leq 2.5099$
- partnering with only S_2 when $1.3153 < \theta \leq 1.3322$

Therefore, there are conditions under which the online retailer might prefer to partner with only one offline retailer instead of both offline retailers. □

B.12. Proof of Proposition 3

The no agreement case is the same as in the main model even when $h \leq \delta$, and hence, we obtain the same equilibrium as described in Section 3. For the agreement case, we follow the same steps as in the main model and solve for the different potential equilibria that are consistent and stable such that there is no deviation. A necessary condition for these equilibria to exist is $\theta >$

$\max\{\frac{\lambda\left(\delta(4\lambda+3)-3\sqrt{\delta^2\lambda(3\lambda+2)-2\delta h(1-\lambda)(4\lambda+3)+9h^2(1-\lambda)^2}-9h(1-\lambda)\right)}{\lambda(11\lambda-6)-9}\mathbb{I}_{\mathbb{R}}, \frac{\lambda\delta}{3-\lambda}\}$, where $\mathbb{I}_{\mathbb{R}}$ indicates whether the first term is real ($\mathbb{I}_{\mathbb{R}} = 1$ if $\delta^2\lambda(3\lambda+2) - 2\delta h(1-\lambda)(4\lambda+3) + 9h^2(1-\lambda)^2 \geq 0$ and 0 otherwise).

When $\delta \geq \frac{1-\lambda}{\lambda}h$, the offline transportation cost is too high such that all customers would rather return online and repurchase online. Therefore, the agreement leads to exactly the same outcome as the no agreement equilibrium and hence, the agreement cannot make both retailers better off. However, when $\delta < \frac{1-\lambda}{\lambda}h$, customers may decide to return offline and repurchase offline under the agreement. Hereafter, we focus on the parameter space where $\delta < \frac{1-\lambda}{\lambda}h$. In this case, we find two equilibria under the agreement: one where the online retailer does not relinquish returners (P1) and one where the online retailer relinquishes returners (P2). We list these equilibria in Table B.9.

Table B.9

Eq Name	p_S	p_I	Π_S	Π_I	Condition
P1	$\frac{1}{3}\lambda(\delta+h)+\theta$	$\theta-\frac{1}{3}\lambda(\delta+h)$	$\frac{(\lambda(\delta+h)+3\theta)^2}{18\theta}$	$\frac{(\lambda(\delta+h)-3\theta)^2}{18\theta}$	$\theta > \frac{1}{6}\left(\frac{3h-3\lambda(\delta+h)}{\sqrt{1-\lambda}} - \delta\lambda + h(3-\lambda)\right)$
P2	$\frac{\theta(\lambda+3)+\delta\lambda}{3(1-\lambda)}$	$\frac{\theta(3-\lambda)-\delta\lambda}{3(1-\lambda)}$	$\frac{(\delta\lambda+\theta(\lambda+3))^2}{18\theta(1-\lambda)}$	$\frac{(\delta\lambda+\theta(\lambda-3))^2}{18\theta(1-\lambda)}$	$\theta < \frac{3h(1-\lambda)\left(\sqrt{1-\lambda}+1\right)-\delta\lambda\left(3\sqrt{1-\lambda}+1\right)}{2(3-\lambda)}$

As in the main model, the offline retailer does not earn a (strictly) higher profit under the agreement when there is no relinquishing (P1). However, when the online retailer relinquishes the returners (P2), both retailers can earn a higher profit under the agreement. When P2 exists ($\theta \leq \theta_2^l$), the offline retailer earns a higher profit under the agreement when $\theta > \theta_\pi^l$, where $\theta_2^l = \frac{3h(1-\lambda)\left(\sqrt{1-\lambda}+1\right)-\delta\lambda\left(3\sqrt{1-\lambda}+1\right)}{2(3-\lambda)}$ and $\theta_\pi^l = \theta_\pi = \frac{\sqrt{1-\lambda}(\delta\lambda+h(\lambda+3))-4\delta\lambda+3h(1-\lambda)}{15+\lambda}$. The online retailer is always better off under the BORS agreement. Therefore, both retailers are better off under the BORS agreement when $\theta_\pi^l < \theta \leq \theta_2^l$. For instance, for $v = 20$, $\lambda = 0.2$, $\delta = 1.5$, and $h = 1$, the parameter space that satisfies our assumptions is given by $\{\theta : \theta > 0.17\}$ and both retailers benefits from the agreement when $0.28 < \theta \leq 0.61$. □

B.13. Proof of Corollary 1

When $h > \delta$, we have $\theta_\pi = \frac{\sqrt{1-\lambda}(\delta\lambda+h(\lambda+3))-4\delta\lambda+3h(1-\lambda)}{15+\lambda}$ and $\theta_2 = \frac{\delta(3-4\lambda)+3\delta(1-2\lambda)\sqrt{1-\lambda}}{2(3-\lambda)}$. We find that $\frac{d(\theta_2-\theta_\pi)}{dh} = -\frac{3(1-\lambda)+(\lambda+3)\sqrt{1-\lambda}}{\lambda+15} < 0$.

When $h \leq \delta$, we have $\theta_\pi = \frac{\sqrt{1-\lambda}(\delta\lambda+h(\lambda+3))-4\delta\lambda+3h(1-\lambda)}{15+\lambda}$ and $\theta_2^l = \frac{3h(1-\lambda)(\sqrt{1-\lambda}+1)-\delta\lambda(3\sqrt{1-\lambda}+1)}{2(3-\lambda)}$. We find that $\frac{d(\theta_2^l-\theta_\pi)}{dh} = \frac{9(1-\lambda)(\lambda+3)+((1-\lambda)\lambda+11\lambda+27(1-2\lambda))\sqrt{1-\lambda}}{2(3-\lambda)(\lambda+15)} > 0$. □

B.14. Proof of Proposition 4

We use backward induction to solve for a forward-looking customer's decisions given retail prices p_j. On each decision occasion, the customer considers her expected utility from each choice and picks the one that provides her with the highest expected utility. The customer's decision trees with and without the agreement are represented in Figures B.4 and B.5, and the decision variables are tabulated in Table 1.

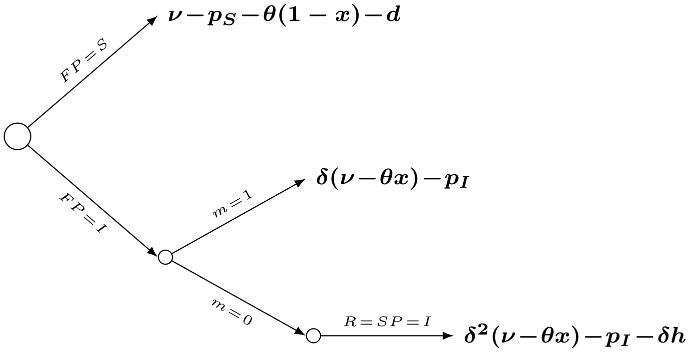

Figure B.4 Decision tree and anticipated utilities of a customer (at the time of first purchase) for each path under no agreement

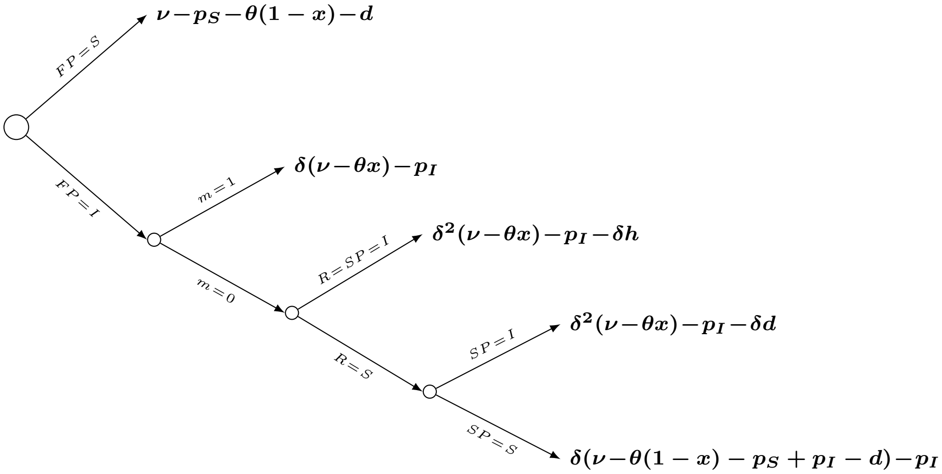

Figure B.5 Decision tree and anticipated utilities of a customer (at the time of first purchase) for each path under the agreement

Under no agreement, a customer who makes her first purchase offline gets utility $U^0_{\{FP=S\}} = v - p_S - \theta(1-x) - d$. A customer who makes her first purchase online would find her match with probability $1-\lambda$ and expect to get utility $U^0_{\{FP=I,m=1\}} = \delta(v-\theta x) - p_I$. With probability λ, she would find a mismatch and require a subsequent return and repurchase. She would expect a utility $U^0_{\{FP=I,m=0\}} = U^0_{\{FP=I,m=0,R=I,SP=I\}} = \delta^2(v-\theta x) - \delta(h) - p_I$. The customer's expected utility of purchasing online is given by $U^0_{\{FP=I\}} = \lambda U^0_{\{FP=I,m=0\}} + (1-\lambda)U^0_{\{FP=I,m=1\}}$. The customer who is indifferent between purchasing online and offline has $U^0_{\{FP=I\}} = U^0_{\{FP=S\}}$, and is located at $x^0 = \frac{\theta+d-\lambda(h\delta)-v(1-\delta)(1+\lambda\delta)+p_S-p_I}{\theta(1+\delta-\lambda(1-\delta)\delta)}$. Customers located at $x < x^0$ purchase online and those located at $x \geq x^0$ purchase offline.

Under the agreement, we first consider the second purchase decision of a customer who purchased online, found a mismatch and returned offline. Note that the second purchase decision is made after the customer has already waited for the first time period. We represent the expected utilities used to make this decision by $U^{1r}_{\{\text{decision path}\}}$. The customer expects a utility $U^{1r}_{\{R=S,SP=I\}} = \delta(v-\theta x)$ by repurchasing online and $U^{1r}_{\{R=S,SP=S\}} = v - \theta(1-x) - p_S + p_I$ by repurchasing offline. Comparing the two utilities, the customer chooses to repurchase online when $x < x^1_{SP}$, where $x^1_{SP} = \frac{\theta-v(1-\delta)-p_I+p_S}{\theta(1+\delta)}$. We now consider the return decision of a customer who purchased online and found a mismatch. The utility from returning online and repurchasing online is given by $\delta(v-\theta x) - h$. The utility from returning offline is given by $\max\{U^{1r}_{\{R=S,SP=I\}}, U^{1r}_{\{R=S,SP=S\}}\} - d$. Clearly, since $h > d$, returning online is dominated by returning offline.

We now consider the customer's first purchase decision. The customer's expected utility of purchasing online in case of a match is given by $U^1_{\{FP=I,m=1\}} = \delta(v-\theta x) - p_I$. The customer's expected utility in case of a mismatch ($U^1_{\{FP=I,m=0\}}$) depends on her repurchase decision: $U^1_{\{FP=I,m=0,R=S,SP=I\}} = \delta(\delta(v-\theta x) - d) - p_I$ (if $x < x^1_{SP}$) and $U^1_{\{FP=I,m=0,R=S,SP=S\}} = \delta(v - \theta(1-x) - p_S + p_I - d) - p_I$ (if $x \geq x^1_{SP}$). Therefore, for a customer located at x,

$$U^1_{\{FP=I,m=0\}} = \begin{cases} \delta(\delta(v-\theta x) - d) - p_I & x < x^1_{SP}, \\ \delta(v - \theta(1-x) - p_S + p_I - d) - p_I & \text{Otherwise.} \end{cases}$$

The customer's expected utility of making her first purchase online is given by: $U^1_{\{FP=I\}} = \lambda U^1_{\{FP=I,m=0\}} + (1-\lambda)U^1_{\{FP=I,m=1\}}$. The customer's expected utility of making her first purchase offline is given by $U^1_{\{FP=S\}} =$

$v - p_S - \theta(1-x) - d$. She would compare her expected utility when $FP = I$ to when $FP = S$ to decide where to make her first purchase. Solving the first purchase, we obtain two thresholds depending on whether we compare purchasing offline with purchasing online and repurchasing offline (x^1_{FP1}) or purchasing offline with purchasing and repurchasing online (x^1_{FP2}) such that $x^1_{FP1} = \frac{(1-\delta\lambda)(p_S-p_I)-\delta\lambda(d+\theta)-(1-\delta)\nu+d+\theta}{\theta(1+\delta(1-2\lambda))}$ and $x^1_{FP2} = \frac{p_S-p_I+d(1-\delta\lambda)-(1-\delta)\nu(\delta\lambda+1)+\theta}{\theta(1-(1-\delta)\delta\lambda+\delta)}$.

In order to identify all possible cases, we check all possible orderings of the different thresholds x^1_{SP}, x^1_{FP1}, and x^1_{FP2} and whether they are relevant (between 0 and 1). Out of the 120 permutations, we find that 9 different cases are possible. The expected profits of the two retailers are given by $\Pi_S = p_S Q_{SFP} + \delta p_S Q_{SSP}$ and $\Pi_I = p_I Q_{IFP} - \delta p_I Q_{SSP}$, where Q_{SFP} is the number of customers who make their first purchase offline, Q_{IFP} is the number of customers who make their first purchase online and Q_{SSP} is the number of customers who buy online, return, get a refund and repurchase offline.

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. Given that we are interested in the cases where, in equilibrium, each retailer has some customers who make their first purchase from it, and at least some customers follow Strategy ISS, we are left with one equilibrium under no agreement and two equilibria under the agreement (see Tables B.10 and B.11):

1. NP (No Agreement) such that $0 < x^0 < 1$,
2. P1 (No Relinquishing) such that $0 < x^1_{SP} < x^1_{FP1} < 1$,
3. P2 (Relinquishing Returners) such that $x^1_{SP} \le 0 < x^1_{FP1} < 1$.

Table B.10

Eq	p_S	p_I
NP	$\frac{2\delta\theta+(1-\delta)\nu-d+\delta\lambda(h-(1-\delta)(2\theta-\nu))+\theta}{3}$	$\frac{\theta(-(1-\delta)\delta\lambda+\delta+2)-(1-\delta)\nu(\delta\lambda+1)+d-\delta h\lambda}{3}$
P1	$\frac{-d(\delta+1)(1-\delta\lambda)^2+\delta\left(-(1-\delta)\delta\lambda^2(2\nu-\theta)-3(\delta+1)\theta\lambda+2\delta\theta-\delta\nu+3\theta\right)+\theta+\nu}{3\left(1+\delta-(1-\delta)\delta^2\lambda^2-\delta(\delta+1)\lambda\right)}$	$\frac{d(\delta+1)(1-\delta\lambda)^2+\delta\left(-(1-\delta)\delta\lambda^2(\theta-2\nu)-3(\delta+1)\theta\lambda+(\delta+3)\theta+\delta\nu\right)+2\theta-\nu}{3\left(1+\delta-(1-\delta)\delta^2\lambda^2-\delta(\delta+1)\lambda\right)}$
P2	$\frac{(-\delta\theta\lambda(\delta\lambda+2)+2\delta\theta+(\delta-1)\nu(\delta\lambda-1)+\theta)-d(1-\delta\lambda)^2}{3(1-\delta\lambda)^2}$	$\frac{1}{3}\left(\frac{2\theta-(1-\delta)\nu}{1-\delta\lambda}-\frac{(1-\delta)\theta}{(1-\delta\lambda)^2}+d+\theta\right)$

We can show that in P1, as $0 < x^1_{SP} < x^1_{FP1}$, the profit of the offline retailer is always lower under P1 (no relinquishing) as compared to NP. Therefore, both retailers can be better off under the agreement only when

Table B.11

Eq	Π_S	Π_I
NP	$\frac{(2\delta\theta+(1-\delta)\nu-d+\delta\lambda(h-(1-\delta)(2\theta-\nu))+\theta)^2}{9\theta(1+\delta-(1-\delta)\delta\lambda)}$	$\frac{(\theta(-(1-\delta)\delta\lambda+\delta+2)-(1-\delta)\nu(\delta\lambda+1)+d-\delta h\lambda)^2}{9\theta(1+\delta-(1-\delta)\delta\lambda)}$
P1	$\frac{\left((1-\delta)\delta^2\lambda^2(2\nu-\theta)+d(\delta+1)(1-\delta\lambda)^2+3\delta(\delta+1)\theta\lambda-(\delta+1)((2\delta+1)\theta+(1-\delta)\nu)\right)^2}{9(\delta+1)\theta\left(-(1-\delta)\delta^2\lambda^2-\delta(\delta+1)\lambda+\delta+1\right)(\delta(1-2\lambda)+1)}$	$\frac{\left(d(\delta+1)(1-\delta\lambda)^2+\delta\left(-(1-\delta)\delta\lambda^2(\theta-2\nu)-3(\delta+1)\theta\lambda+(\delta+3)\theta+\delta\nu\right)+2\theta-\nu\right)^2}{9(\delta+1)\theta\left(-(1-\delta)\delta^2\lambda^2-\delta(\delta+1)\lambda+\delta+1\right)(\delta(1-2\lambda)+1)}$
P2	$\frac{\left(d(1-\delta\lambda)^2-\theta(\delta(2-\lambda(\delta\lambda+2))+1)-(1-\delta)\nu(1-\delta\lambda)\right)^2}{9\theta(1-\delta\lambda)^2(\delta(1-2\lambda)+1)}$	$\frac{\left(d(1-\delta\lambda)^2+\theta(\delta(-\lambda)(4-\delta\lambda)+\delta+2)-(1-\delta)\nu(1-\delta\lambda)\right)^2}{9\theta(1-\delta\lambda)^2(\delta(1-2\lambda)+1)}$

the online retailer relinquishes the returners (P2). For instance, for parameter values $\nu = 5$, $h = 1.8$, $d = 1.4$, $\lambda = 0.25$, $\delta = 0.5$, and $\theta = 1.25$, P2 exists and both retailers are better off under the agreement. □

B.15. Proof of Proposition 5

The decision trees and utilities for the "low" segment are the same as in our base model, given in Figures 1 and 2 under no agreement and agreement, respectively. We produce the decision trees and utilities for the "high" segment in Figures B.6 and B.7 under no agreement and agreement, respectively. As discussed in Appendix A.5, we are interested in the case where $\delta + \Delta > h > \delta$. Following the proof in Appendix B.1, under no agreement, we obtain the following thresholds corresponding to the two segments with low (l) and high (h) transportation cost: $x^{0l} = \frac{\theta-\lambda(\delta+h)-p_I+p_S}{2\theta}$ and $x^{0h} = \frac{\theta+\Delta-\lambda(\delta+h)-p_I+p_S}{2\theta}$, where $x^{0l} < x^{0h}$. Customers in Segment $i \in \{l, h\}$ located at $x < x^{0i}$ purchase online and located at $x \geq x^{0i}$ purchase offline.

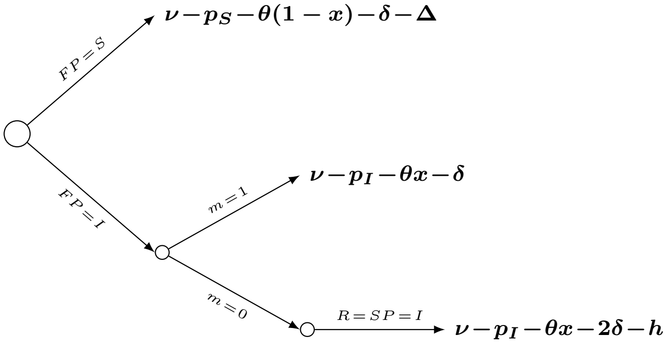

Figure B.6 Decision tree and utilities of a customer in Segment h under no agreement

Following the proof in Appendix B.2, under the agreement, we obtain the following thresholds: $x^{1l}_{SP} = \frac{\theta-\delta+p_S-p_I}{2\theta}$ and $x^{1l}_{FP} = x^{1l}_{SP} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$ and $x^{1h}_{SP} = \frac{\theta+\Delta-h+p_S-p_I}{2\theta}$ and $x^{1h}_{FP} = x^{1h}_{SP} + \frac{h-\lambda(h+\delta)}{2\theta(1-\lambda)}$, where $x^{1l}_{SP} < x^{1l}_{FP}$ and $x^{1h}_{SP} < x^{1h}_{FP}$ since $\lambda < 1/2$ and $h > \delta$. Customers in segment l located at $x < x^{1l}_{SP}$ follow Strategy ISI,

$x_{SP}^{1l} \leq x < x_{FP}^{1l}$ follow Strategy ISS, and follow Strategy S otherwise. Customers in segment h located at $x < x_{SP}^{1h}$ follow Strategy III (first purchase online, return online and repurchase online), $x_{SP}^{1h} \leq x < x_{FP}^{1h}$ follow Strategy ISS, and follow Strategy S otherwise.

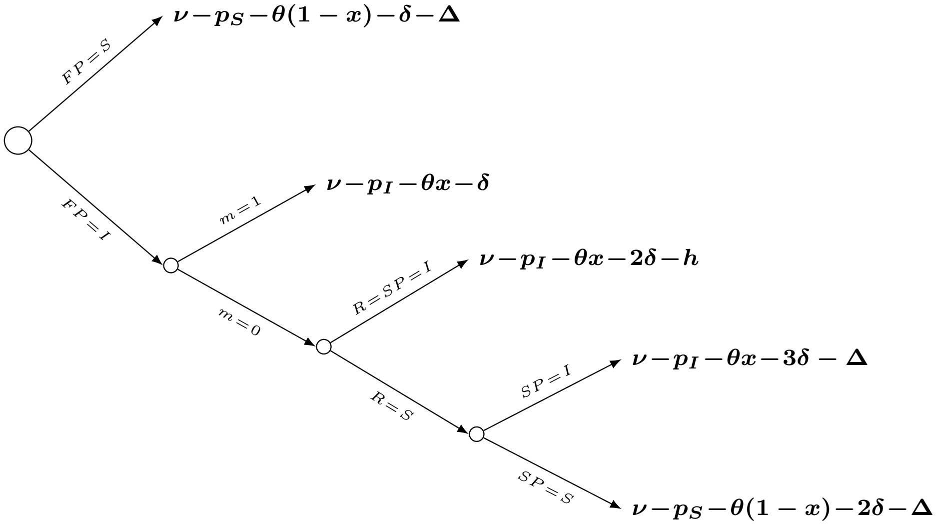

Figure B.7 **Decision tree and utilities of a customer in Segment h under the agreement**

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. Given that we are interested in the cases where, in equilibrium, each retailer has some customers from each segment who make their first purchase from it, we are left with one equilibrium under no agreement and three equilibria under the agreement (see Table B.12):

1. NP (No Agreement) such that $0 < x^{0l} < x^{0h} < 1$,
2. P1 (No Relinquishing) such that $0 < x_{SP}^{1l} < x_{FP}^{1l} < 1$ and $0 < x_{SP}^{1h} < x_{FP}^{1h} < 1$,
3. P2 (Relinquishing Low Type Returners) such that $x_{SP}^{1l} \leq 0 < x_{FP}^{1l} < 1$ and $0 < x_{SP}^{1h} < x_{FP}^{1h} < 1$,
4. P3 (Relinquishing All Returners) such that $x_{SP}^{1l} \leq 0 < x_{FP}^{1l} < 1$ and $x_{SP}^{1h} \leq 0 < x_{FP}^{1h} < 1$.

Note that returners from the segment that has a high offline shopping cost cannot be relinquished without relinquishing the returners from the "low" segment (since $x_{SP}^{1l} < x_{SP}^{1h}$ when $\delta + \Delta > h$).

Clearly, the profit of the offline retailer is always lower under P1 (no relinquishing) as compared to NP as $h > \delta$. Therefore, both retailers can be better off under the agreement only when returners from at least one

Table B.12

Eq Name	p_S	p_I	Π_S	Π_I
NP	$\frac{6\theta-\Delta+2\lambda(h+\delta)}{6}$	$\frac{6\theta+\Delta-2\lambda(h+\delta)}{6}$	$\frac{(6\theta-\Delta+2\lambda(h+\delta))^2}{72\theta}$	$\frac{(6\theta+\Delta-2\lambda(h+\delta))^2}{72\theta}$
P1	$\frac{6\theta-\Delta+(3\delta+h)\lambda}{6}$	$\frac{6\theta+\Delta-(3\delta+h)\lambda}{6}$	$\frac{(6\theta-\Delta+(3\delta+h)\lambda)^2}{72\theta}$	$\frac{(6\theta+\Delta-(3\delta+h)\lambda)^2}{72\theta}$
P2	$\frac{6\theta-\Delta+(\theta+2\delta+h)\lambda}{3(2-\delta)}$	$\frac{6\theta+\Delta-(\theta+2\delta+h)\lambda}{3(2-\delta)}$	$\frac{(6\theta-\Delta+(\theta+2\delta+h)\lambda)^2}{36\theta(2-\delta)}$	$\frac{(6\theta+\Delta-(\theta+2\delta+h)\lambda)^2}{36\theta(2-\delta)}$
P3	$\frac{6\theta-\Delta+(2\theta+2\delta+\Delta)\lambda}{6(1-\delta)}$	$\frac{6\theta+\Delta-(2\theta+2\delta+\Delta)\lambda}{6(1-\delta)}$	$\frac{(6\theta-\Delta+(2\theta+2\delta+\Delta)\lambda)^2}{72\theta(1-\delta)}$	$\frac{(6\theta+\Delta-(2\theta+2\delta+\Delta)\lambda)^2}{72\theta(1-\delta)}$

segment are relinquished (in P2 and P3). We provide instances where equilibria P2 and P3 exist, and the agreement benefits both retailers under those equilibria in Table A.2. □

B.16. Proof of Proposition 6

We produce the decision trees and utilities of customers in each segment $i \in \{l, h\}$ in Figures B.8 and B.9 under no agreement and agreement, respectively. Following the proof in Appendix B.1, under no agreement, we obtain the following thresholds: $x^{0i} = \frac{\theta-(h+\delta)\lambda+p_S-p_I+\sigma_i}{2\theta}$, where $x^{0l} < x^{0h}$. Customers in Segment i located at $x < x^{0i}$ purchase online and located at $x \geq x^{0i}$ purchase offline.

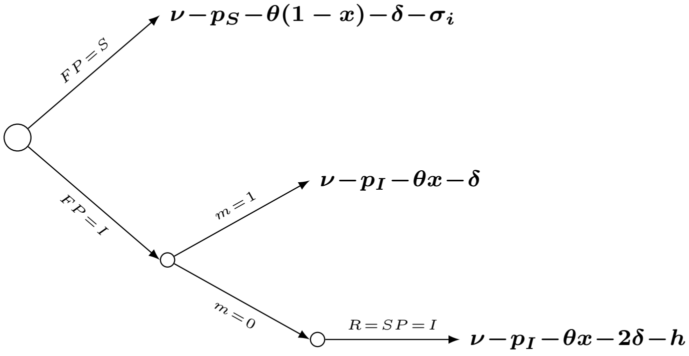

Figure B.8 **Decision tree and utilities of a customer in Segment $i \in \{l, h\}$ under no agreement**

Following the proof in Appendix B.2, under the agreement, we obtain the following thresholds: $x^{1i}_{SP} = \frac{\theta-\delta+p_S-p_I+\sigma_i}{2\theta}$ and $x^{1i}_{FP} = x^{1i}_{SP} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$, where $x^{1i}_{SP} < x^{1i}_{FP}$ since $\lambda < 1/2$ and $x^{1l}_k < x^{1h}_k, k \in \{SP, FP\}$ since $\sigma_h > \sigma_l$. Customers in segment i follow Strategy ISI if located at $x < x^{1i}_{SP}$, follow Strategy ISS if located at $x^{1i}_{SP} \leq x < x^{1i}_{FP}$, and follow Strategy S otherwise.

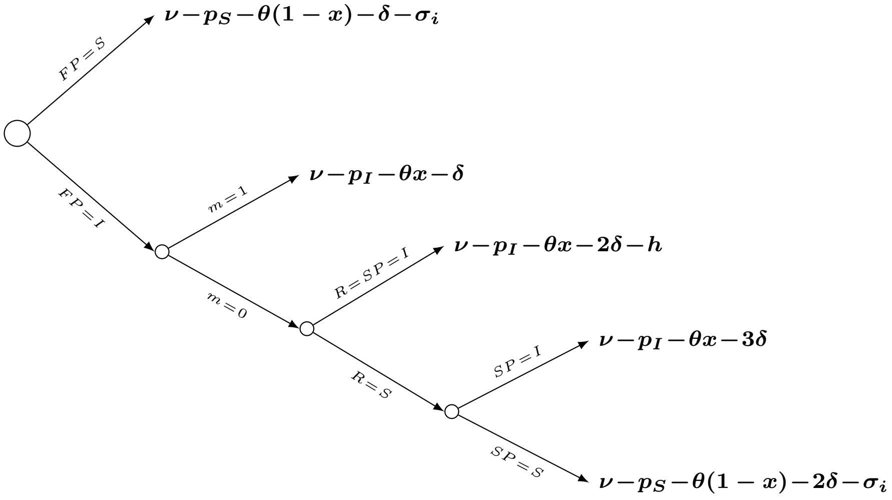

Figure B.9 Decision tree and utilities of a customer in Segment $i \in \{l, h\}$ under the agreement

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. Given that we are interested in the cases where, in equilibrium, each retailer has some customers from each segment who make their first purchase from it, we are left with one equilibrium under no agreement and three equilibria under the agreement (see Table B.13):

1. NP (No Agreement) such that $0 < x^{0l} < x^{0h} < 1$,
2. P1 (No Relinquishing) such that $0 < x^{1l}_{SP} < x^{1l}_{FP} < 1$ and $0 < x^{1h}_{SP} < x^{1h}_{FP} < 1$,
3. P2 (Relinquishing Low Type Returners) such that $x^{1l}_{SP} \leq 0 < x^{1l}_{FP} < 1$ and $0 < x^{1h}_{SP} < x^{1h}_{FP} < 1$,
4. P3 (Relinquishing All Returners) such that $x^{1l}_{SP} \leq 0 < x^{1l}_{FP} < 1$ and $x^{1h}_{SP} \leq 0 < x^{1h}_{FP} < 1$.

Note that returners from the segment that has a high in-store shopping cost cannot be relinquished without relinquishing the returners from the "low" segment (since $x^{1l}_{SP} < x^{1h}_{SP}$).

Clearly, the profit of the offline retailer is always lower under P1 (no relinquishing) as compared to NP as $h > \delta$. Therefore, both retailers can be better off under the agreement only when returners from at least one segment are relinquished (in P2 and P3). For instance, for parameter values $\nu = 20$, $h = 1.33$, $\lambda = 0.14$, $\delta = 0.77$, and $\sigma = 0.85$, the relinquishing only "low" type returners equilibrium (P2) exists when $\theta \in [0.669, 0.899]$ and both retailers are better off when $\theta \in [0.775, 0.899]$. For parameter values $\nu = 20$, $h = 3$, $\delta = 2$, $\lambda = 0.25$, and $\sigma = 0.6$, the relinquishing all returners equilibrium (P3) exists when $\theta \in [0.396, 0.914]$ and both retailers are better off when $\theta \in [0.840, 0.914]$. $\square$

Table B.13

Eq Name	p_S	p_I	Π_S	Π_I
NP	$\frac{6\theta+2\lambda(h+\delta)-\sigma}{6}$	$\frac{6\theta-2\lambda(h+\delta)+\sigma}{6}$	$\frac{(6\theta+2(h+\delta)\lambda-\sigma)^2}{72\theta}$	$\frac{(6\theta-2(h+\delta)\lambda+\sigma)^2}{72\theta}$
P1	$\frac{6\theta+4\delta\lambda-\sigma}{6}$	$\frac{6\theta-4\delta\lambda+\sigma}{6}$	$\frac{(6\theta+4\delta\lambda-\sigma)^2}{72\theta}$	$\frac{(6\theta-4\delta\lambda+\sigma)^2}{72\theta}$
P2	$\frac{3\delta\lambda+\theta(6+\lambda)-\sigma}{3(2-\lambda)}$	$\frac{\theta(6-\lambda)-3\delta\lambda+\sigma}{3(2-\lambda)}$	$\frac{(3\delta\lambda+\theta(6+\lambda)-\sigma)^2}{36\theta(2-\lambda)}$	$\frac{(\theta(6-\lambda)-3\delta\lambda+\sigma)^2}{36\theta(2-\lambda)}$
P3	$\frac{2\delta\lambda+2\theta(3+\lambda)-(1-\lambda)\sigma}{6(1-\lambda)}$	$\frac{2\theta(3-\lambda)-2\delta\lambda+(1-\lambda)\sigma}{6(1-\lambda)}$	$\frac{(2\delta\lambda+2\theta(3+\lambda)-(1-\lambda)\sigma)^2}{72\theta(1-\lambda)}$	$\frac{(2\theta(3-\lambda)-2\delta\lambda+(1-\lambda)\sigma)^2}{72\theta(1-\lambda)}$

B.17. Proof of Proposition 7

As in the main model, we solve for the different potential equilibria with and without the BORS agreement. The no agreement model is the same as in the main model and hence, we obtain the same equilibrium (NP). Under the agreement, we solve the model the same way as in the main model, and we obtain two equilibria under the agreement: one where the online retailer does not relinquish returners (P1) and one where the online retailer relinquishes returners (P2). We list these equilibria in the following table:

Table B.14

Eq Name	p_S	p_I	Π_S	Π_I
NP	$\frac{3\theta+\lambda(\delta+h)}{3}$	$\frac{3\theta-\lambda(\delta+h)}{3}$	$\frac{(3\theta+\lambda(\delta+h))^2}{18\theta}$	$\frac{(3\theta-\lambda(\delta+h))^2}{18\theta}$
P1	$\frac{3\theta+2\lambda(\delta-c)}{3}$	$\frac{3\theta-\lambda(c+2\delta)}{3}$	$\frac{\lambda^2\left(c^2(1-\lambda)+c\delta(14\lambda-5)-4\delta^2(\lambda-1)\right)+12\theta(\lambda-1)\lambda(c-\delta)-9\theta^2(\lambda-1)}{18\theta(1-\lambda)}$	$\frac{(3\theta-c\lambda-2\delta\lambda)^2}{18\theta}$
P2	$\frac{3\theta+\lambda(\delta+\theta-2c)}{3(1-\lambda)}$	$\frac{3\theta-\lambda(c+\delta+\theta)}{3(1-\lambda)}$	$\frac{\lambda^2(c+\delta+\theta)^2+6\theta\lambda(\delta+\theta-2c)+9\theta^2}{18\theta(1-\lambda)}$	$\frac{(3\theta-\lambda(c+\delta+\theta))^2}{18\theta(1-\lambda)}$

We can show that both retailers can be better off under the agreement only under Equilibrium P2 (relinquishing returners). Comparing the profits of both the online and the offline retailers under P2 and NP, we find that both are better off only when the offline retailer is better off, and that happens when $c < \bar{c}$, where $\bar{c} = \frac{\theta(6-\lambda)-\lambda\delta-\sqrt{9\theta^2(4-3\lambda)-6\theta\lambda(\delta(\lambda+2)-h(1-\lambda))+(1-\lambda)\lambda^2(\delta+h)^2}}{\lambda}$ such that at $c = \bar{c}$, the offline retailer has the same profit in NP and P2. For instance, for $v = 20$, $h = 1.7$, $\lambda = 0.09$, $\theta = 0.821$, and $\delta = 1.33$, both retailers benefit from the agreement when $c < 0.288$. □

B.18. Proof of Proposition 8

As in the main model, we solve for the different potential equilibria with and without the BORS agreement. We find one equilibrium under no agreement (NP) and two equilibria under the agreement: one where the online retailer does not relinquish returners (P1) and one where the online retailer relinquishes returners (P2). We list these equilibria in the following table:

Table B.15

Eq Name	p_S	p_I	Π_S	Π_I
NP	$\frac{1}{3}\lambda(\delta+h+r)+\theta$	$\theta-\frac{1}{3}\lambda(\delta+h-2r)$	$\frac{(3\theta+\lambda(\delta+h+r))^2}{18\theta}$	$\frac{(3\theta-\lambda(\delta+h+r))^2}{18\theta}$
P1	$\frac{1}{3}\lambda(2\delta+r)+\theta$	$\frac{2}{3}\lambda(r-\delta)+\theta$	$\frac{(3\theta+\lambda(2\delta+r))^2}{18\theta}$	$\frac{(1-\lambda)(3\theta-\lambda(2\delta+r))^2-9\delta(1-2\lambda)\lambda^2 r}{18\theta(1-\lambda)}$
P2	$\frac{3\theta+\lambda(\delta+\theta+r)}{3(1-\lambda)}$	$\frac{3\theta-\lambda(\delta+\theta-2r)}{3(1-\lambda)}$	$\frac{(3\theta+\lambda(\delta+\theta+r))^2}{18\theta(1-\lambda)}$	$\frac{(3\theta-\lambda(\delta+\theta+r))^2}{18\theta(1-\lambda)}$

Clearly, the profit of the offline retailer is always lower under P1 (no relinquishing) as compared to NP as $h > \delta$. Therefore, both retailers can be better off under the agreement only under Equilibrium P2 (relinquishing returners). Comparing the profits of the online retailer under P2 and NP, we find that the online retailer is better off under P2 only when $r < \bar{r}$, where $\bar{r} = \frac{h-\lambda(\delta+h)-\sqrt{1-\lambda}(\theta-h)+2\theta}{\lambda}$. Comparing the profits of the offline retailer under P2 and NP, we find that the offline retailer is better off under P2 only when $r > \underline{r}$, where $\underline{r} = \frac{h-\lambda(\delta+h)-\sqrt{1-\lambda}(\theta-h)-4\theta}{\lambda} = \bar{r} - 6\theta/\lambda$. Additionally, r must be positive. Therefore, both retailers can be better off under the agreement in P2 only when $\max\{0, \underline{r}\} < r < \bar{r}$. For instance, for $v = 20$, $\lambda = 0.3$, $\delta = 1.5$, $h = 1.55$, $\theta = 1.8$ and $r = 13$, both retailers benefits from the agreement. □

B.19. Proof of Proposition 9

Following the proof in Appendix B.1, under no agreement, we obtain the indifferent customers between the online and offline retailer: $x^0 = \frac{p_S-p_I+(1+a)\theta-\lambda(\delta+h)}{2\theta}$. Customers located at $x < x^0$ purchase online, and those located at $x \geq x^0$ purchase offline.

Following the proof in Appendix B.2, under the agreement, we obtain the following thresholds: $x^1_{SP} = \frac{p_S-p_I-\delta+(a+1)\theta}{2\theta}$ and $x^1_{FP} = x^1_{SP} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$, where $x^1_{SP} < x^1_{FP}$ since $\lambda < 1/2$. Customers located at $x < x^1_{SP}$ follow Strategy ISI, $x^1_{SP} \leq x < x^1_{FP}$ follow Strategy ISS, and follow Strategy S otherwise.

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. Given that we are interested in the cases where, in equilibrium, each retailer has some customers who make their first purchase from it, we are left with one equilibrium under no agreement and two equilibria under the agreement (see Table B.16):

1. NP (No Agreement) such that $0 < x^0 < 1$,
2. P1 (No Relinquishing) such that $0 < x^1_{SP} < x^1_{FP} < 1$,
3. P2 (Relinquishing Returners) such that $x^1_{SP} \leq 0 < x^1_{FP} < 1$.

Table B.16 Equilibria prices and profits

Eq Name	p_S	p_I	Π_S	Π_I
NP	$\frac{(3-a)\theta+\lambda(\delta+h)}{3}$	$\frac{(3+a)\theta-\lambda(\delta+h)}{3}$	$\frac{((3-a)\theta+\lambda(\delta+h))^2}{18\theta}$	$\frac{((3+a)\theta-\lambda(\delta+h))^2}{18\theta}$
P1	$\frac{\theta(3-a)+2\delta\lambda}{3}$	$\frac{\theta(3+a)-2\delta\lambda}{3}$	$\frac{(\theta(3-a)+2\delta\lambda)^2}{18\theta}$	$\frac{(\theta(3+a)-2\delta\lambda)^2}{18\theta}$
P2	$\frac{\lambda((1+a)\theta+\delta)+(3-a)\theta}{3(1-\lambda)}$	$\frac{(3+a)\theta-\lambda((1+a)\theta+\delta)}{3(1-\lambda)}$	$\frac{(\lambda((1+a)\theta+\delta)+(3-a)\theta)^2}{18\theta(1-\lambda)}$	$\frac{((3+a)\theta-\lambda((1+a)\theta+\delta))^2}{18\theta(1-\lambda)}$

Clearly, the profit of the offline retailer is always lower under P1 (no relinquishing) as compared to NP as $h > \delta$. Therefore, both retailers can be better off under the agreement only when returners are relinquished (P2). We find that the online retailer is always better off under the agreement but the offline retailer is better off only under P2 and $a > max\{0, \underline{a}\}$ such that $\underline{a} = \frac{(h-\theta)(1-\lambda)-\sqrt{(1-\lambda)(4\theta+(d+h)\lambda-h)^2}}{\theta(1-\lambda)}$ is the solution to $\Pi^{12}_S = \Pi^0_S$ under the condition of equilibrium P2. For instance, for parameters values $v = 20$, $\lambda = 0.25$, $\delta = 0.5$, $h = 1$ and $\theta = 0.3$, the relinquishing equilibrium (P2) exists when $a \in [0.09, 0.27]$. However, within P2, both retailers benefit from the BORS agreement only if $a > 0.12$.

B.20. Proof of Proposition 10

We produce customers' decision trees and utilities in Figures B.10 and B.11 under no agreement and agreement, respectively. In this proof, we focus on the case where the customer always ends up with her match from the online or the offline retailer. We solve all cases and use them to check for deviations but focus on the equilibria where all customers get their match.[3] Under no agreement, when a customer decides not to visit the offline store ($FV = I$), she would make the first purchase online ($FP = I$). When the customer visits the offline store ($FV = S$) and finds her match (with a probability ϕ), she would purchase the product offline ($FP = S$). However, if the customer does not find her match, she may purchase online ($FP = I$)

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[3] The customer derives a lower value equal to $v - \eta > 0$ from a non-matching product. We assume that $\eta > \max\{\frac{\delta(1-2\lambda^2\phi)}{\phi(1-\lambda)}, \delta + h\}$ as a necessary condition to ensure equilibria where all customers end up with their match under both agreement and no agreement.

or offline ($FP = S$). If the customer buys online, she is uncertain of the match when placing the order. With probability $1 - \lambda$, the ordered product would match her, and there is no return. With probability λ, the product does not match her. Since there is no agreement, she can only return the product through the regular return channels of the online retailer and repurchase online ($R = SP = I$). This leads to the thresholds $x_1^0 = \frac{p_S - p_I + \theta - \delta - \lambda(\delta + h)}{2\theta} + \frac{\delta}{2\theta\phi}$ and $x_2^0 = x_1^0 + \frac{\eta\phi - \delta}{2\theta\phi}$, where $x_1^0 < x_2^0$ such that customers located at $x < x_1^0$ purchase online, return and repurchase online, customers located at $x_1^0 < x < x_2^0$ visit the offline store and when there is no match, purchase online, and customers located at $x > x_2^0$ purchase offline regardless of the match (even if they do not find their match). As discussed, we focus on cases where, in equilibrium, customers purchase at the end a matching product ($x_2^0 > 1$).

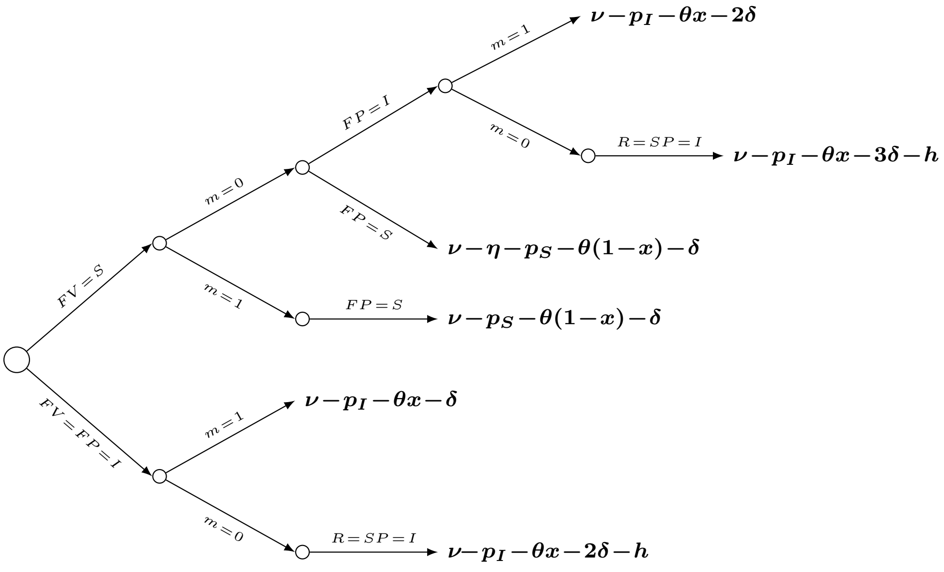

Figure B.10 Decision tree and utilities of a customer under no agreement

Under the agreement, the customer who purchases online can return offline in case of a mismatch and decide how to make the second purchase. We obtain the following thresholds: $x_{SP}^1 = \frac{p_S - p_I - \delta + \theta}{2\theta}$, $x_{FP1}^1 = x_{SP}^1 + \frac{\delta(1 - 2\lambda\phi)}{2\theta\phi(1 - \lambda)}$, and $x_{FP2}^1 = x_{SP}^1 + \frac{\eta - 2\delta\lambda}{2\theta}$ where $x_{SP}^1 < x_{FP1}^1 < x_{FP2}^1$. Customers located at $x < x_{SP}^1$ follow Strategy ISI, $x_{SP}^1 \leq x < x_{FP1}^1$ make their first purchase online, return offline and second purchase offline (if there is an offline match) or online (if there is no offline match), customers located at $x_{FP1}^1 < x < x_{FP2}^1$ visit the offline store and make their first purchase offline if there is an offline match and online otherwise, and customers located at $x^1_{FP2} < x$ make their first purchase offline regardless of the offline match. As discussed above, we focus on cases where, in equilibrium, customers only purchase a matching product (cases where $x^1_{FP2} > 1$).

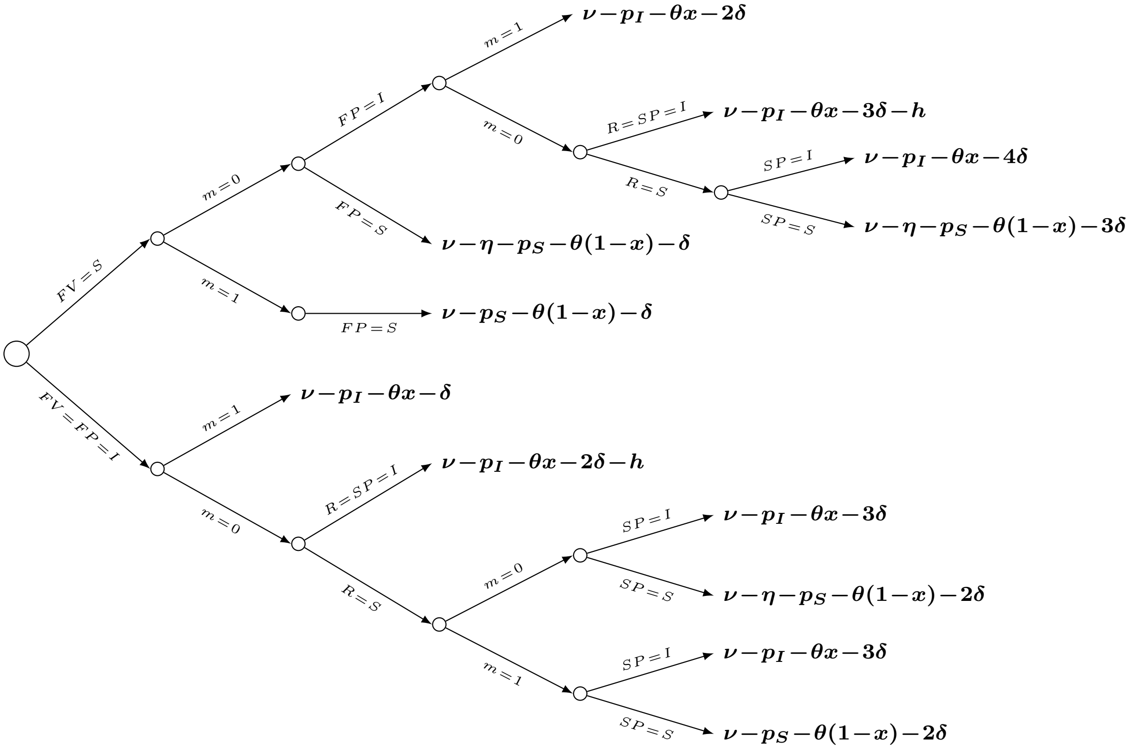

Figure B.11 Decision tree and utilities of a customer under the agreement

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. Given that we are interested in the cases where, in equilibrium, each retailer has some customers who make their first purchase from it, we are left with one equilibrium under no agreement and two equilibria under the agreement (see Table B.17):

1. NP (No agreement) such that $0 < x^0_1 < 1$,
2. P1 (No relinquishing) such that $0 < x^1_{SP} < x^1_{FP1} < 1 < x^1_{FP2}$,
3. P2 (Relinquishing returners who find offline match) such that $x^1_{SP} \leq 0 < x^1_{FP1} < 1 < x^1_{FP2}$.

Table B.17

Eq Name	p_S	p_I	Π_S	Π_I
NP	$\frac{2\theta-\delta+\phi(\delta+\theta+\lambda(\delta+h))}{3\phi}$	$\frac{4\theta+\delta-\phi(\delta+\theta+\lambda(\delta+h))}{3\phi}$	$\frac{(2\theta-\delta+\phi(\delta+\theta+\lambda(\delta+h)))^2}{18\theta\phi}$	$\frac{(4\theta+\delta-\phi(\delta+\theta+\lambda(\delta+h)))^2}{18\theta\phi}$
P1	$\frac{2\theta-\delta+\phi(2\delta\lambda+\delta+\theta)}{3\phi}$	$\frac{4\theta+\delta-\phi(2\delta\lambda+\delta+\theta)}{3\phi}$	$\frac{(2\theta-\delta+\phi(2\delta\lambda+\delta+\theta))^2}{18\theta\phi}$	$\frac{(4\theta+\delta-\phi(2\delta\lambda+\delta+\theta))^2}{18\theta\phi}$
P2	$\frac{2\theta-\delta+\phi(1+\lambda)(\delta+\theta)}{3(1-\lambda)\phi}$	$\frac{4\theta+\delta-\phi(1+\lambda)(\delta+\theta)}{3(1-\lambda)\phi}$	$\frac{(2\theta-\delta+\phi(1+\lambda)(\delta+\theta))^2}{18\theta(1-\lambda)\phi}$	$\frac{(4\theta+\delta-\phi(1+\lambda)(\delta+\theta))^2}{18\theta(1-\lambda)\phi}$

Clearly, the profit of the offline retailer is always lower under P1 (no relinquishing) as compared to NP since $h > \delta$. Therefore, both retailers can be better off under the agreement only when returners are relinquished (P2). We find that the online retailer is always better off under the agreement, but the offline retailer is better off only under P2. For instance, for parameters values $\nu = 20$, $\lambda = 0.114$, $\delta = 1.556$, $h = 1.712$, $\phi = 0.534$, and $\eta = 4.334$ the relinquishing equilibrium (P2) and the no agreement equilibrium (NP) exist when $\theta \in [0.608, 0.856]$. However, both retailers benefit from the BORS agreement only if $\theta \in [0.645, 0.856]$. $\square$

B.21. Proof of Proposition 11

We produce the decision trees and utilities of customers in each segment $i \in \{l, h\}$ in Figures B.12 and B.13 under no agreement and agreement, respectively. Segment i incurs a transportation cost of $\delta + \Delta_i$ to travel to the multichannel retailer's offline store, where $\Delta_l = 0$, $\Delta_h = \Delta > 0$ and $h > \delta + \Delta$. Following the proof in Appendix B.1, under no agreement, we obtain the following thresholds: $x^{0h} = \frac{p_S-p_I+\theta+\lambda(\Delta-h)}{2\theta}$ and $x^{0l} = \frac{p_S-p_I+\theta-\lambda(\delta+h)}{2\theta}$, where $x^{0l} < x^{0h}$. Customers in Segment l located at $x < x^{0l}$ purchase from the pure online retailer (I), and those located at $x \geq x^{0l}$ purchase at the offline store of the multichannel retailer (M_S). Customers in Segment h located at $x < x^{0h}$ purchase from I, and those located at $x \geq x^{0h}$ purchase from the online channel of the multichannel retailer (M_I).

Following the proof in Appendix B.2, under the agreement, we obtain the following thresholds: $x^{1l}_{SP} = \frac{p_S-p_I-\delta+\theta}{2\theta}$ and $x^{1l}_{FP} = x^{1l}_{SP} + \frac{\delta(1-2\lambda)}{2\theta(1-\lambda)}$ while $x^{1h}_{SP} = x^{1l}_{SP} = x^{1}_{SP} = \frac{p_S-p_I-\delta+\theta}{2\theta}$ and $x^{1h}_{FP} = x^{1}_{SP} + \frac{\delta}{2\theta}$, where $x^{1i}_{SP} < x^{1i}_{FP}$ since $\lambda < 1/2$ and $x^{1l}_{FP} < x^{1h}_{FP}$. Customers in segment i located at $x < x^{1i}_{SP}$ follow Strategy 'IM_SI', $x^{1i}_{SP} \leq x < x^{1i}_{FP}$ follow Strategy 'IM_SM_S', and follow Strategy 'M' otherwise. Strategy 'IM_SI' involves purchasing from I, returning at the offline store of M, and repurchasing from I. Strategy 'IM_SM_S' involves purchasing from I, returning and repurchasing at the offline store of M. Strategy 'M' involves purchasing at either the offline or the online channel of M (depending on the customer's offline transportation cost).

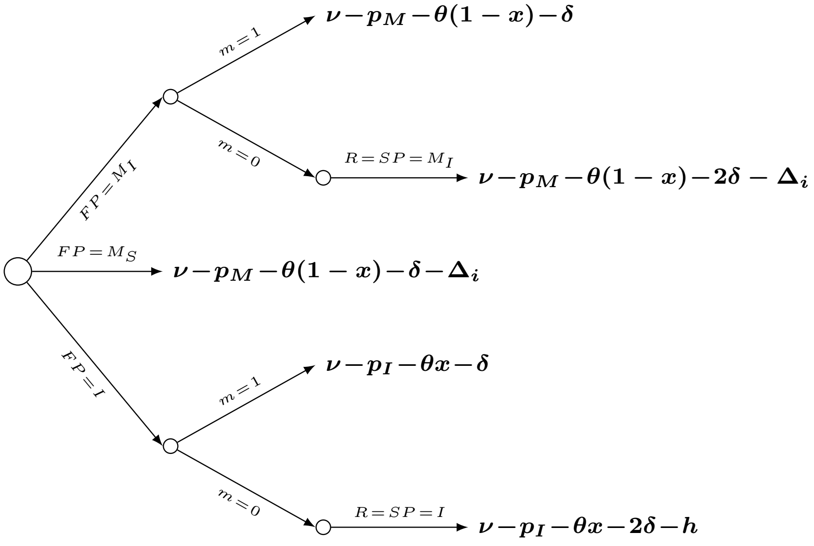

Figure B.12 Decision tree and utilities of a customer in Segment $i \in \{l,h\}$ for Retailers under no agreement

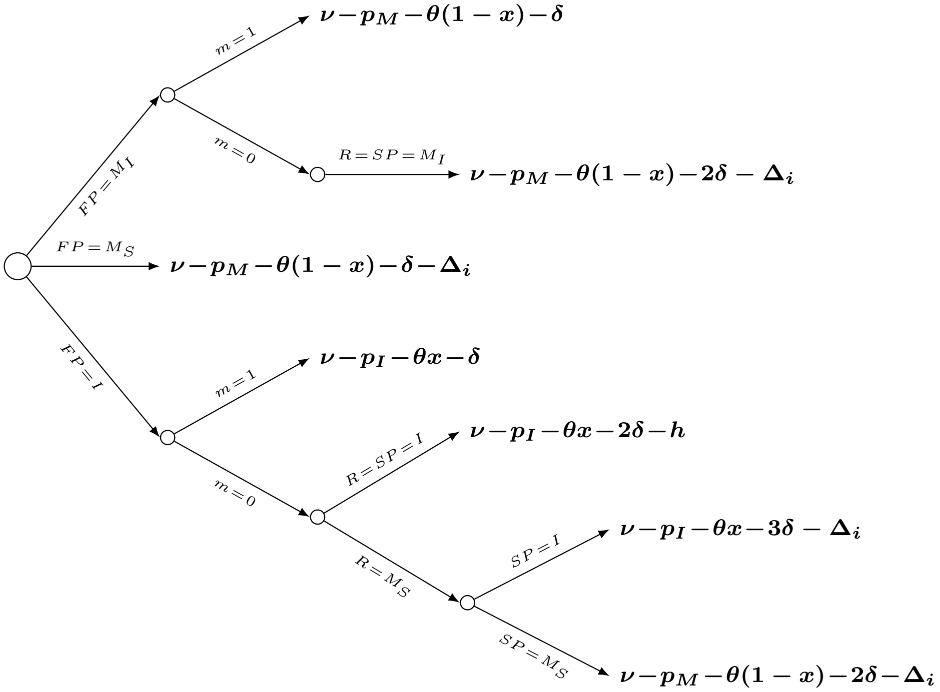

Figure B.13 Decision tree and utilities of a customer in Segment $i \in \{l,h\}$ for Retailers under the agreement

As in the main model, we solve for the different potential equilibria with and without the BORS agreement, checking for all possible non-local deviations. Given that we are interested in the cases where, in equilibrium, each retailer has some customers from each segment who make their first purchase from it, and customers end up with their match, we are left with one equilibrium under no agreement and two equilibria under the agreement (see Table B.18):

1. NP (No Agreement) such that $0 < x^{0l} < x^{0h} < 1$,
2. P1 (No Relinquishing) such that $0 < x^{1}_{SP} < x^{1l}_{FP} < x^{1h}_{FP} < 1$,
3. P2 (Relinquishing Returners) such that $x^{1}_{SP} \leq 0 < x^{1l}_{FP} < x^{1h}_{FP} < 1$.

Note that as the decision where to repurchase after making an offline return does not depend on the offline transportation cost (sunk cost). Therefore, returners from both segments are either relinquished or not relinquished together.

Table B.18 Equilibria Prices and Profits

Eq Name	p_S	p_I	Π_S	Π_I
NP	$\frac{3\theta+\lambda(\alpha\delta-(1-\alpha)\Delta+h)}{3}$	$\frac{3\theta-\lambda(\alpha\delta-(1-\alpha)\Delta+h)}{3}$	$\frac{(3\theta+\lambda(\alpha\delta-(1-\alpha)\Delta+h))^2}{18\theta}$	$\frac{(3\theta-\lambda(\alpha\delta-(1-\alpha)\Delta+h))^2}{18\theta}$
P1	$\frac{3\theta+\delta\lambda(1+\alpha)}{3}$	$\frac{3\theta-\delta\lambda(1+\alpha)}{3}$	$\frac{(3\theta+\delta\lambda(1+\alpha))^2}{18\theta}$	$\frac{(3\theta-\delta\lambda(1+\alpha))^2}{18\theta}$
P2	$\frac{\theta(3+\lambda)+\alpha\delta\lambda}{3(1-\lambda)}$	$\frac{\theta(3-\lambda)-\alpha\delta\lambda}{3(1-\lambda)}$	$\frac{(\theta(3+\lambda)+\alpha\delta\lambda)^2}{18\theta(1-\lambda)}$	$\frac{(\theta(3-\lambda)-\alpha\delta\lambda)^2}{18\theta(1-\lambda)}$

Clearly, the profit of the multichannel retailer is always lower under P1 (no relinquishing) as compared to NP as $h > \delta + \Delta$. Therefore, both retailers can be better off under the agreement only when returners are relinquished (P2). We find that the online retailer is always better off under the agreement but the multichannel retailer is better off only under P2 and $\alpha < \bar{\alpha}$ such that $\bar{\alpha} = \frac{(1-\lambda)(\delta+\Delta)(\lambda(\delta+h)+3\theta)-\lambda(1-\lambda)(\delta+\Delta)^2-\delta\theta(\lambda+3)-\sqrt{(1-\lambda)(\Delta\lambda(\delta+\theta)+3\Delta\theta+\delta\lambda(\theta-h))^2}}{\lambda\left(\delta^2-(1-\lambda)(\delta+\Delta)^2\right)}$ is the solution to $\Pi^{12}_S = \Pi^0_S$ under the condition of equilirium P2. For instance, for parameters values $\nu = 20$, $\lambda = 0.1$, $\delta = 0.5$, $\Delta = 0.25$, $h = 1$ and $\theta = 0.44$, the relinquishing equilibrium (P2) exists when $\alpha \in [0, 1]$. However, both retailers benefit from the BORS agreement only if $\alpha < 0.238$. $\square$