Supplemental Appendices

Buying from a Competitor: A Model of Knowledge Spillover and Innovation

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Abstract

Appendix A presents formal proofs of results from the main body of the paper. Appendix B contains additional model extensions and variations.

A Appendix A: Proofs

A.1 Proof of Lemma 1

We consider two cases: $|v_A - v_S| < 3t$ (implying positive equilibrium market shares), and $|v_A - v_S| \geq 3t$ (implying one firm may serve the entire market). We drop the period subscript i for simpler exposition.

- **$|v_A - v_S| < 3t$:**

  If Firm A buys from Firm T, equilibrium prices and profits are as in the standard Hoteling model. If A and S both have positive demand, the indifferent customer on the Hoteling line is at location: $x = \frac{1}{2} - \frac{p_A - p_S + v_S - v_A}{2t}$. Profits for Firm A and S are: $\pi_A = p_A(\frac{1}{2} - \frac{p_A - p_S + v_S - v_A}{2t})$ and $\pi_S = p_S(\frac{1}{2} + \frac{p_A - p_S + v_S - v_A}{2t})$. Solving the first-order conditions, equilibrium prices are: $p_A = t + \frac{v_A - v_S}{3}$ and $p_S = t + \frac{v_S - v_A}{3}$. Equilibrium quantities are: $q_A = \frac{1}{2} - \frac{v_S - v_A}{6t}$ and $q_S = \frac{1}{2} + \frac{v_S - v_A}{6t}$. Equilibrium profits are: $\pi_A = \frac{1}{18t}(3t + v_A - v_S)^2$ and $\pi_S = \frac{1}{18t}(3t - v_A + v_S)^2$. Industry profits are: $\Pi = \pi_A + \pi_S = t + \frac{1}{9t}(v_S - v_A)^2$. Full market coverage requires that the indifferent customer x to have positive utility, so, $v_A - p_A - tx > 0$, that is, $v_A + v_S > p_A + p_S + t$. Inserting prices yields, $v_A + v_S > 3t$, which holds because $t < \frac{2}{3}$. Moreover, both firms should have positive demands, which results in the constraint $|v_A - v_S| < 3t$.

  If Firm A buys from Firm S, the profit functions become: $\pi_A = (p_A - w_S)(\frac{1}{2} - \frac{p_A - p_S + v_S - v_A}{2t}) - F_S$ and $\pi_S = p_S(\frac{1}{2} + \frac{p_A - p_S + v_S - v_A}{2t}) + w_S(\frac{1}{2} - \frac{p_A - p_S + v_S - v_A}{2t}) + F_S$. Then, $\frac{\partial \pi_A}{\partial p_A} = \frac{-2p_A + p_S + t - v_S + v_A + w_S}{2t}$ and $\frac{\partial \pi_S}{\partial p_S} = \frac{p_A - 2p_S + t + v_S - v_A + w_S}{2t}$. Thus, equilibrium prices become: $p_A = t + w_S + \frac{v_A - v_S}{3}$ and $p_S = t + w_S + \frac{v_S - v_A}{3}$. Equilibrium demands are: $q_A = \frac{1}{2} - \frac{v_S - v_A}{6t}$ and $q_S = \frac{1}{2} + \frac{v_S - v_A}{6t}$. Equilibrium profits are: $\pi_A = \frac{1}{18t}(3t + v_A - v_S)^2 - F_S$ and $\pi_S = \frac{1}{18t}(3t - v_A + v_S)^2 + w_S + F_S$. Industry profits are: $\Pi = t + \frac{1}{9t}(v_S - v_A)^2 + w_S$. The indifferent customer x should have a positive utility, thus, $v_A - p_A - tx > 0$, that is, $v_A + v_S > p_A + p_S + t$. Inserting equilibrium prices, $v_A + v_S > 2w_S + 3t > 3t$. Moreover, for both demands to be positive, we should have $|v_A - v_S| < 3t$.

- $|v_A - v_S| > 3t$**:**

  If Firm A buys from Firm T, then,

  If $v_A > v_S + 3t$, then Firm A serves the entire market. Firm A sets its price to make the $x = 1$ customer indifferent between buying from Firm A at p_A or from Firm S at $p_S = 0$. It follows $p_A = v_A - v_S - t$ and $p_S = 0$. Firm A and S profits are: $\pi_A = v_A - v_S - t$ and $\pi_S = 0$. Industry profits are: $\Pi = v_A - v_S - t$.

  If $v_S > v_A + 3t$, then, Firm S serves the entire market. We have $p_S = v_S - v_A - t$ and $p_A = 0$. Firm A and S profits are: $\pi_A = 0$ and $\pi_S = v_S - v_A - t$. Industry profits are: $\Pi = v_S - v_A - t$.

  If Firm A buys from Firm S, then,

  If $v_A > v_S + 3t$, Firm A serves the entire market. We have $p_A = v_A - v_S - t$ and $p_S = 0$. Profits are $\pi_A = v_A - v_S - t - w_S - F_S$ and $\pi_S = w_S + F_S$. Industry profits are: $\Pi = v_A - v_S - t$.

  If $v_S > v_A + 3t$, Firm S serves the entire market. We have $p_S = v_S - v_A - t$ and $p_A = 0$. Firm A and S profits are: $\pi_A = -F_S$ and $\pi_S = v_S - v_A - t + F_S$. Industry profits are: $\Pi = v_S - v_A - t$.

A.2 Proof of Lemma 2

Per Lemma 1, if $|v_A - v_S| < 3t$ and $2w_S < v_A + v_S - 3t$, the market is fully covered and the industry profits are: $\Pi = t + \frac{1}{9t}(v_S - v_A)^2 + w_S$, which is increasing in w_S. The maximum possible wholesale price would then be $w_S^{max} = \frac{1}{2}(v_A + v_S - 3t)$, which fully extracts the surplus of the indifferent customer. So the maximum possible industry profits are: $\Pi^{max} = \frac{1}{2}(v_A + v_S) + \frac{1}{9t}(v_S - v_A)^2 - \frac{1}{2}t$.

Now consider the case $2w_S > v_A + v_S - 3t$, so the market is not fully covered. In this case, each firm is a local monopolist. Profits are $\pi_A = (p_A - w_S)(\frac{v_A - p_A}{t}) - F_S$ and $\pi_S = p_S(\frac{v_S - p_S}{t}) + w_S(\frac{v_A - p_A}{t}) + F_S$. Thus, equilibrium prices are: $p_A = \frac{v_A + w_S}{2}$ and $p_S = \frac{v_S}{2}$.

Equilibrium demands are: $q_A = \frac{v_A - w_S}{2t}$ and $q_S = \frac{v_S}{2t}$. Equilibrium profits are: $\pi_A = \frac{1}{4t}(v_A - w_S)^2 - F_S$ and $\pi_S = \frac{1}{4t}(v_S)^2 + w_S(\frac{v_A - w_S}{2t}) + F_S$. Industry profits are: $\Pi = \frac{1}{4t}(v_A^2 + v_S^2 - w_S^2)$. Thus, industry profits decrease with w_S, implying that firms would not increase the wholesale price above $w_S^{max} = \frac{1}{2}(v_A + v_S - 3t)$.

If $|v_A - v_S| > 3t$, Lemma 1 shows that the industry profits are $|v_A - v_S| - t$, which is independent of the wholesale price.

A.3 Proof of Lemma 3

With $v_{A,2} = v_{S,2} = 1$, per Lemma 1, if Firm A buys from Firm T in period 2, we have $\pi_{A,2} = \pi_{S,2} = \frac{1}{2}t$ and so the industry profit becomes $\Pi_2^{AT} = t$. Per Lemma 1 and 2, if Firm A buys from Firm S, then $\pi_{A,2} = \frac{1}{2}t - F_{S,2}$, $\pi_{S,2} = \frac{1}{2}t + w_{S,2} + F_{S,2}$ and so the industry profit becomes $\Pi_2^{AS} = t + w_{S,2}$ where $w_{S,2} = \frac{1}{2}(2 - 3t)$. So, Firm A buys from Firm S because $\Pi_2^{AS} > \Pi_2^{AT}$.

To derive $F_{S,2}$, we need to take into account firms' outside options. If Firm A and S fail to reach an agreement, then Firm A buys from Firm T resulting in $\pi_{A,2}^O = \pi_{S,2}^O = \frac{t}{2}$. Then, the fixed payment is derived from $\pi_{A,2} - \pi_{A,2}^O = \pi_{S,2} - \pi_{S,2}^O$ (firms will have equal surplus over their outside options). Solving for $F_{S,2}$ yields $F_{S,2} = -\frac{1}{2}w_{S,2} = \frac{1}{4}(3t - 2)$. Inserting $w_{S,2}$ and $F_{S,2}$ back into firm profits, we have, $\pi_{A,2} = \pi_{S,2} = \frac{1}{4}(2 - t)$.

A.4 Proof of Lemma 4

We first consider the $\Delta < 3t$ case. If Firm A buys from Firm S in period 1 and invests c, then Firm A has 2 options: to continue with Firm S in period or to switch to Firm T.

If Firm A continues sourcing from Firm S in period 2, we have $v_{A,2} = v_{S,2} = 1 + \Delta$. Inserting these product values into Lemmas 1 and 2, in period 2 we have firm profits $\pi_{A,2} = \frac{t}{2} - F_{S,2}$ and $\pi_{S,2} = \frac{t}{2} + w_{S,2} + F_{S,2}$, the wholesale price $w_{S,2} = \Delta + 1 - \frac{3t}{2}$, and industry profits $\Pi_2 = 1 + \Delta - \frac{t}{2}$.

However, if in period 2 Firm A switches to Firm T, we have $v_{A,2} = 1$ and $v_{S,2} = 1 + \Delta$.

Inserting these values into Lemma 1, firm profits in period 2 are $\pi_{A,2} = \frac{1}{18t}(3t - \Delta)^2$ and $\pi_{S,2} = \frac{1}{18t}(3t + \Delta)^2$, leading to industry profits of $\Pi_2 = t + \frac{1}{9t}\Delta^2$.

We have, $t + \frac{1}{9t}\Delta^2 < 1 + \Delta - \frac{t}{2} \Leftrightarrow (\frac{3t}{2} - 1) + \Delta(\frac{\Delta}{9t} - 1) < 0$, given that $\frac{\Delta}{3} < t < \frac{2}{3}$. Thus, industry profits are always higher if Firm A continues sourcing from Firm S. Thus, Firm A continues sourcing from Firm S with the wholesale price $w_{S,2} = \Delta + 1 - \frac{3t}{2}$.

We now calculate the fixed payment $F_{S,2}$. The fixed payment is derived from $\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2}$ where superscript O denotes outside options (when Firm A and S fail to agree and hence Firm A contracts with Firm T). Inserting $\pi^O_{A,2} = \frac{1}{18t}(3t - \Delta)^2$ and $\pi^O_{S,2} = \frac{1}{18t}(3t + \Delta)^2$ into the above equation yields $F_{S,2} = -(\frac{1+\Delta}{2} - \frac{3t}{4}) + \frac{\Delta}{3}$. Inserting $F_{S,2}$ back into the profit functions, we have: $\pi_{A,2} = \frac{1}{12}(2\Delta - 3t + 6)$ and $\pi_{S,2} = \frac{1}{12}(10\Delta - 3t + 6)$.

We now consider the case $\Delta > 3t$.

Suppose in period 2 Firm A switches to Firm T. We have $v_{A,2} = 1$ and $v_{S,2} = 1 + \Delta$ with $\Delta > 3t$. In this case, Firm S serves the entire market, leading to $\pi_{A,2} = 0$ and $\pi_{S,2} = \Delta - t$, and industry profits of $\Pi_2 = \Delta - t$, which is less than $1 + \Delta - \frac{t}{2}$. Thus, Firm A continues sourcing from Firm S with the wholesale price $w_{S,2} = \Delta + 1 - \frac{3t}{2}$.

We now calculate the fixed payment $F_{S,2}$ for the $\Delta > 3t$ case. The fixed payment is derived from $\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2}$ where $\pi^O_{A,2} = 0$ and $\pi^O_{S,2} = \Delta - t$. Hence, $(\frac{t}{2} - F_{S,2}) - 0 = (\frac{t}{2} + \Delta + 1 - \frac{3t}{2} + F_{S,2}) - (\Delta - t)$, which results in: $F_{S,2} = -\frac{1}{2} + \frac{t}{4}$. Inserting F_S back into the profit functions, we have: $\pi_{A,2} = \frac{1}{2} + \frac{t}{4}$ and $\pi_{S,2} = \frac{1}{2} + \Delta - \frac{3t}{4}$.

A.5 Proof of Lemma 5

We first consider the case $\Delta < 3t$. If Firm A buys from Firm T in period 1 and invests c, then Firm A has 2 options: to continue with Firm T in period 2 or to switch to Firm S.

If Firm A continues sourcing from Firm T in period 2, we have $v_{A,2} = 1 + \Delta$ and $v_{S,2} = 1$. Inserting these product values in Lemma 1, in period 2 we have $\pi_{A,2} = \frac{1}{18t}(3t + \Delta)^2$ and $\pi_{S,2} = \frac{1}{18t}(3t - \Delta)^2$, leading to industry profits of $\Pi_2 = t + \frac{1}{9t}\Delta^2$.

However, if in period 2 Firm A switches to Firm S, we have $v_{A,2} = v_{S,2} = 1$. Inserting these values into Lemma 1, firm profits in period 2 are $\pi_{A,2} = \frac{t}{2} - F_{S,2}$ and $\pi_{S,2} = \frac{t}{2} + w_{S,2} + F_{S,2}$, the wholesale price is $w_{S,2} = 1 - \frac{3t}{2}$, and industry profits are $\Pi_2 = 1 - \frac{t}{2}$.

We have, $t + \frac{1}{9t}\Delta^2 > 1 - \frac{t}{2} \Leftrightarrow \frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$. Thus, in period 2, if $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$ Firm A buys the input from the third-party, and otherwise it buys the input from its competitor. In the latter case, $w_{S,2} = 1 - \frac{3t}{2}$. The fixed payment is derived from $\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2}$. Inserting $\pi^O_{A,2} = \frac{1}{18t}(3t + \Delta)^2$ and $\pi^O_{S,2} = \frac{1}{18t}(3t - \Delta)^2$ into the above equation yields $F_{S,2} = -\frac{1}{2}w_{S,2} - \frac{1}{3}\Delta = -(\frac{1}{2} - \frac{3t}{4}) - \frac{\Delta}{3}$. This leads to firm profits $\pi_{A,2} = \frac{1}{2} - \frac{1}{4}t + \frac{1}{3}\Delta$ and $\pi_{S,2} = \frac{1}{2} - \frac{1}{4}t - \frac{1}{3}\Delta$.

We now consider the case $\Delta > 3t$. If Firm A buys from Firm T in period 1 and invests c, then Firm A has 2 options: to continue with Firm T in period 2 or to switch to Firm S.

If Firm A continues sourcing from Firm T in period 2, we have $v_{A,2} = 1 + \Delta$ and $v_{S,2} = 1$ where $\Delta > 3t$. Then we have $\pi_{A,2} = \Delta - t$ and $\pi_{S,2} = 0$, leading to industry profits of $\Delta - t$.

However, if in period 2 Firm A switches to Firm S, we have $v_{A,2} = v_{S,2} = 1$. Inserting these values in Lemma 1, firm profits in period 2 are $\pi_{A,2} = \frac{t}{2} - F_{S,2}$ and $\pi_{S,2} = \frac{t}{2} + w_{S,2} + F_{S,2}$, the wholesale price is $w_{S,2} = 1 - \frac{3t}{2}$, and industry profits are $\Pi_2 = 1 - \frac{t}{2}$.

We have, $\Delta - t > 1 - \frac{t}{2} \Leftrightarrow \Delta > 1 + \frac{t}{2}$. Thus, in period 2, if $\Delta > 1 + \frac{t}{2}$ Firm A buys the input from the third-party, and otherwise, it buys the input from its competitor. In the latter case, $w_{S,2} = 1 - \frac{3t}{2}$. The fixed payment is derived from $\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2}$. Inserting $\pi^O_{A,2} = \Delta - t$ and $\pi^O_{S,2} = 0$ into the preceding equation yields $(\frac{t}{2} - F_{S,2}) - (\Delta - t) = (\Delta + 1 - t + F_{S,2}) - 0$, yielding, $F_{S,2} = -\frac{1}{2} + \frac{5t}{4} - \Delta$. This leads to firm profits $\pi_{A,2} = \frac{1}{2} - \frac{3}{4}t + \Delta$ and $\pi_{S,2} = \frac{1}{2} + \frac{1}{4}t$.

A.6 Proof of Lemma 6

First, we consider the case $\Delta < 3t$. If Firm A buys from Firm S in period 1, its profits in period 2 are $\frac{\Delta}{6} + \frac{1}{4}(2 - t)$ if it invests, and $\frac{1}{4}(2 - t)$ if it does not (see Lemmas 3 and 4). Therefore, Firm A will invest c if and only if $\frac{\Delta}{6} > c$.

If Firm A buys from Firm T in period 1, its profits in period 2 are equal to $\frac{1}{2}t + \frac{\Delta^2}{18t} + \frac{1}{3}\Delta$

if it invests and $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$, equal to $\frac{1}{2} - \frac{1}{4}t + \frac{1}{3}\Delta$ if it invests and $\frac{\Delta^2}{9t} < 1 - \frac{3t}{2}$, and equal to $\frac{1}{2} - \frac{1}{4}t$ if it does not invest (see Lemmas 3 and 5).

Thus, Firm A invests if and only if $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$ and $c < \frac{1}{2}(\frac{\Delta^2}{9t} - (1 - \frac{3t}{2})) + \frac{\Delta}{3}$, or $\frac{\Delta^2}{9t} < 1 - \frac{3t}{2}$ and $c < \frac{\Delta}{3}$. These two conditions are combined together into $c < max\{\frac{2\Delta^2 - 18t + 27t^2}{36t}, 0\} + \frac{\Delta}{3}$.

Next, we consider the case $\Delta > 3t$. If Firm A buys from Firm S in period 1, its profits in period 2 are $\frac{1}{4}(2+t)$ if it invests and $\frac{1}{4}(2-t)$ if it does not (see proofs of Lemma 3 and 4). Therefore, if $\Delta > 3t$, and Firm A buys from Firm S in period 1, Firm A will invest if $c < \frac{t}{2}$.

If Firm A buys from Firm T in period 1, its profit in period 2 are equal to $\Delta - t$ if it invests and $\Delta > 1 + \frac{t}{2}$, equal to $\frac{1}{2} - \frac{3}{4}t$ if it invests and $\Delta < 1 + \frac{t}{2}$, and equal to $\frac{1}{2} - \frac{1}{4}t$ if it does not invest (see proofs of Lemma 3 and 5). Thus, if Firm A buys from Firm T in period 1, it invests if and only if $\Delta > 1 + \frac{t}{2}$ and $\Delta - t - c > \frac{1}{2} - \frac{1}{4}t \Leftrightarrow c < \Delta - \frac{3t}{4} - \frac{1}{2}$.

A.7 Proof of Proposition 1

We first consider the case $\Delta < 3t$. Per Lemma 6, if the Period 1 supplier is Firm S, Firm A will invest c if and only if $c < \frac{\Delta}{6}$. On the other hand, if $c > \frac{\Delta}{6}$, we need to compare the following two industry profits: (i) If Firm A buys From S in both periods and does not invest in the input, total industry profits are $\Pi = 2 - t$ (see Lemma 3) (ii) If Firm A buys From T in both periods and invests, total industry profits are: $\Pi = 2t + \frac{1}{9t}\Delta^2 - c$ (see Lemma 5). The former is larger if and only if $c > \frac{\Delta^2 - 18t + 27t^2}{9t}$. Thus, Firm A buys from Firm S in both periods and does not invest iff $c > \frac{\Delta^2 - 18t + 27t^2}{9t}$ and $c > \frac{\Delta}{6}$ whereas Firm A buys from Firm T in both periods and invests in Firm T's input iff $c < \frac{\Delta^2 - 18t + 27t^2}{9t}$ and $c > \frac{\Delta}{6}$.

Next, we derive the period 1 fixed payment when Firm A and Firm S contract, $F_{S,1}$. The fixed payment is derived from $\pi_A - \pi_A^O = \pi_S - \pi_S^O$, where π_A is focal firm's total profit across the two periods including any investment cost and π_S is Firm S's total profit across he two periods. The superscript O denotes outside options (when Firm A and Firm S fail to agree in period 1 and hence Firm A contracts with Firm T). As we showed above, there are three cases:

1. If $c < \frac{\Delta}{6}$, Firm A buys from Firm S in both periods and invests.

2. If $c > \frac{\Delta}{6}$ and $c > \frac{\Delta^2-18t+27t^2}{9t}$, Firm A buys from Firm S in both periods and does not invest.

3. If $\frac{\Delta}{6} < c < \frac{\Delta^2-18t+27t^2}{9t}$, Firm A buys from Firm T in both periods and invests.

We need to calculate the fixed payment in the first two cases in which Firm A and Firm S contract. Consider the first case, $c < \frac{\Delta}{6}$. In this case we have:

$$\pi_A = \underbrace{(\frac{t}{2} - F_{S,1})}_{Period\ 1} - \underbrace{c}_{investment\ cost} + \underbrace{\frac{1}{12}(2\Delta - 3t + 6)}_{Period\ 2},$$

$$\pi_S = \underbrace{(\frac{t}{2} + (1 - \frac{3t}{2}) + F_{S,1})}_{Period\ 1} + \underbrace{\frac{1}{12}(10\Delta - 3t + 6)}_{Period\ 2}$$

If Firm A and Firm S fail to agree in period 1, Firm A buys input from Firm T in period 1, and per Lemma 6, it will invest. Then, per Lemma 5, in period 2, if $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$, Firm A buys the input from the third party, resulting in period 2 profits $\pi_{A,2} = \frac{t}{2} + \frac{\Delta}{3} + \frac{\Delta^2}{18t}$ and $\pi_{S,2} = \frac{t}{2} - \frac{\Delta}{3} + \frac{\Delta^2}{18t}$, and industry profits $\Pi_2 = t + \frac{\Delta^2}{9t}$. Otherwise, Firm A buys the input from its competitor, with input contract $w_{S,2} = 1 - \frac{3t}{2}$ and $F_{S,2} = -(\frac{1}{2} - \frac{3t}{4}) - \frac{\Delta}{3}$, which results in profits $\pi_{A,2} = \frac{1}{2} - \frac{t}{4} + \frac{\Delta}{3}$ and $\pi_{S,2} = \frac{1}{2} - \frac{t}{4} - \frac{\Delta}{3}$, and industry profits $\Pi_2 = 1 - \frac{t}{2}$.

Therefore, the disagreement payoffs, π_A^O and π_S^O, depend on whether or not $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$. In particular, if $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$, then

$$\pi_A^O = (\frac{t}{2}) - c + (\frac{t}{2} + \frac{\Delta}{3} + \frac{\Delta^2}{18t}),$$

$$\pi_S^O = (\frac{t}{2}) + (\frac{t}{2} - \frac{\Delta}{3} + \frac{\Delta^2}{18t})$$

Whereas if $\frac{\Delta^2}{9t} < 1 - \frac{3t}{2}$, then

$$\pi_A^O = (\frac{t}{2}) - c + (\frac{1}{2} - \frac{t}{4} + \frac{\Delta}{3}),$$

$$\pi_S^O = (\frac{t}{2}) + (\frac{1}{2} - \frac{t}{4} - \frac{\Delta}{3})$$

The fixed payment is derived from $\pi_A - \pi_A^O = \pi_S - \pi_S^O$. Regardless of whether $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$ or not, we have $F_{S,1} = \frac{1}{12}(9t - 8\Delta - 6)$. Recall from Lemma 4, $F_{S,2} = \frac{1}{2}w_{S,2} + \frac{\Delta}{3} = \frac{1}{12}(9t - 2\Delta - 6)$. Thus, $F_{S,1} = F_{S,2} - \frac{\Delta}{2}$, implying $F_{S,1} < F_{S,2} < 0$.

Next we consider $c > \frac{\Delta}{6}$ and $c > \frac{\Delta^2-18t+27t^2}{9t}$, or, $c > \max\{\frac{\Delta}{6}, \frac{\Delta^2-18t+27t^2}{9t}\}$ and so Firm A buys from Firm S in both periods and does not invest. In this case we have:

$\pi_A = (\frac{t}{2} - F_{S,1}) + \frac{1}{4}(2-t)$,

$\pi_S = (\frac{t}{2} + (1 - \frac{3t}{2}) + F_{S,1}) + \frac{1}{4}(2-t)$

If Firm A and Firm S fail to agree in period 1, Firm A contracts with Firm T in period 1. Then, per Lemma 6, it invests only if $c < \max\{\frac{2\Delta^2-18t+27t^2}{36t}, 0\} + \frac{\Delta}{3} = \hat{c}$. Thus, if $\max\{\frac{\Delta}{6}, \frac{\Delta^2-18t+27t^2}{9t}\} < c < \hat{c}$, Firm A invests in cases of disagreement and hence the disagreement payoffs would be the same as $c < \frac{\Delta}{6}$ case. In particular, $\pi_A^O - \pi_S^O = \frac{2\Delta}{3} - c$. Moreover, $\pi_A - \pi_S = -2F_{S,1} - (1 - \frac{3t}{2})$. Thus, $F_{S,1} = \frac{c}{2} - \frac{\Delta}{3} - 1 + \frac{3t}{2}$.

On the other hand, if $c > \max\{\frac{\Delta}{6}, \frac{\Delta^2-18t+27t^2}{9t}, \hat{c}\}$, the investment would not occur in case of first-period disagreement. As a result, $\pi_A^O - \pi_S^O = 0$, because $\pi_A^O = \pi_S^O = (\frac{t}{2}) + (\frac{1}{4}(2-t))$. Thus, $F_{S,1} = -\frac{1}{2}(1 - \frac{3t}{2})$. Table 2 summarizes the outcomes (contract terms, firm profits, and industry profits) for $\Delta < 3t$.

Next, we consider the case $\Delta > 3t$. We will show that:

1. Firm A buys from Firm S in both periods and invests if $c < \frac{t}{2}$.

2. Firm A buys from Firm S in both periods and does not invest if $\max\{\frac{t}{2}, \Delta + t - 2\} < c$.

3. Firm A buys from Firm T in both periods and invests if $\frac{t}{2} < c < \Delta + t - 2$.

1. If $c < \frac{t}{2}$, then if the Period 1 supplier is Firm S, Firm A will invest (see proof of Lemma 6); thus, $\Pi = (1 - \frac{t}{2}) - c + (1 - \frac{t}{2} + \Delta) = \Delta - t + 2 - c$. However, if the Period 1 supplier is Firm T, the total industry profits are $(t) - c + (\Delta - t) = \Delta - c$ if Firm A invests ($T \to invest \to T$), and are $(t) + (1 - \frac{t}{2}) = 1 + \frac{t}{2}$ if it does not invest ($T \to Not\ invest \to S$). Since $\Delta - t + 2 - c$ is larger than both $\Delta - c$ and $1 + \frac{t}{2}$, Firm A buys from Firm S in both periods and invests.

2. If $\max\{\frac{t}{2}, \Delta + t - 2\} < c$, if the Period 1 supplier is Firm S, Firm A will not invest, resulting in $\Pi = (1 - \frac{t}{2}) + (1 - \frac{t}{2}) = 2 - t$. However, if the Period 1 supplier is Firm T and Firm A invests, the total industry profits are $\Pi = (t) - c + (\Delta - t) = \Delta - c$. The

Table 2: Equilibrium outcomes of the main model ($\Delta < 3t$).

Equilibrium Outcomes		**Condition**		
		$c < \frac{\Delta}{6}$	$\max\{\frac{\Delta}{6}, \frac{\Delta^2-18t+27t^2}{9t}\} < c$	$\frac{\Delta}{6} < c < \frac{\Delta^2-18t+27t^2}{9t}$
Firm A Sources from ...		Firm S (both periods)	Firm S (both periods)	Firm T (both periods)
Period 1	$w_{S,1}$	$1 - \frac{3t}{2}$	$1 - \frac{3t}{2}$	–
	$F_{S,1}$	$\frac{1}{12}(9t - 8\Delta - 6)$	$\frac{1}{6}(9t - 2\Delta - 6 + 3c)$ if $c < \hat{c}$ $\frac{1}{4}(3t - 2)$ if $c > \hat{c}$	–
	$\pi_{A,1}$	$\frac{1}{12}(-3t + 8\Delta + 6)$	$\frac{1}{6}(-6t + 2\Delta + 6 - 3c)$ if $c < \hat{c}$ $\frac{1}{4}(-t + 2)$ if $c > \hat{c}$	$\frac{t}{2}$
	$\pi_{S,1}$	$\frac{1}{12}(-3t - 8\Delta + 6)$	$\frac{1}{6}(3t - 2\Delta + 3c)$ if $c < \hat{c}$ $\frac{1}{4}(-t + 2)$ if $c > \hat{c}$	$\frac{t}{2}$
	Π_1	$1 - \frac{t}{2}$	$1 - \frac{t}{2}$	t
Firm A invests?		Yes	No	Yes
Period 2	$w_{S,2}$	$\Delta + 1 - \frac{3t}{2}$	$1 - \frac{3t}{2}$	–
	$F_{S,2}$	$\frac{1}{12}(9t - 2\Delta - 6)$	$\frac{1}{4}(3t - 2)$	–
	$\pi_{A,2}$	$\frac{1}{12}(-3t + 2\Delta + 6)$	$\frac{1}{4}(-t + 2)$	$\frac{1}{18t}(3t + \Delta)^2$
	$\pi_{S,2}$	$\frac{1}{12}(-3t + 10\Delta + 6)$	$\frac{1}{4}(-t + 2)$	$\frac{1}{18t}(3t - \Delta)^2$
	Π_2	$\Delta + 1 - \frac{t}{2}$	$1 - \frac{t}{2}$	$t + \frac{\Delta^2}{9t}$
Across Periods	π_A	$\frac{1}{6}(-3t + 5\Delta + 6) - c$	$\frac{1}{12}(-15t + 4\Delta + 18 - 6c)$ if $c < \hat{c}$ $\frac{1}{2}(-t + 2)$ if $c > \hat{c}$	$t + \frac{\Delta^2}{18t} + \frac{\Delta}{3} - c$
	π_S	$\frac{1}{6}(-3t + \Delta + 6)$	$\frac{1}{12}(3t - 4\Delta + 6 + 6c)$ if $c < \hat{c}$ $\frac{1}{2}(-t + 2)$ if $c > \hat{c}$	$t + \frac{\Delta^2}{18t} - \frac{\Delta}{3}$
	Π	$\Delta + 2 - t - c$	$2 - t$	$2t + \frac{\Delta^2}{9t} - c$

latter is smaller because $c > \Delta + t - 2$. If the Period 1 supplier is Firm T and Firm A does not invest, total industry profits are $\Pi = (t) + (1 - \frac{t}{2}) = 1 + \frac{t}{2}$, which is again smaller than $2 - t$. Consequently, Firm A buys from Firm S in both periods and does not invest if $\max\{\frac{t}{2}, \Delta + t - 2\} < c$.

3. Finally, if $\frac{t}{2} < c < \Delta + t - 2$, it implies $c < \Delta - \frac{3t}{4} - \frac{1}{2}$ and $1 + \frac{t}{2} < \Delta$. If the Period 1 supplier is Firm S, per Lemma 6's proof, Firm A will not invest, resulting in $\Pi = (1 - \frac{t}{2}) + (1 - \frac{t}{2}) = 2 - t$. However, it invests if the Period 1 supplier is Firm T, yielding a

total industry profit of $\Pi = (t) - c + (\Delta - t) = \Delta - c$. The latter is larger because $c < \Delta + t - 2$. Hence, Firm A buys from Firm T in both periods and invests if $\frac{t}{2} < c < \Delta + t - 2$.

We now calculate the fixed payment. Consider $c < \frac{t}{2}$. Then,

$$\pi_A = \underbrace{(\frac{t}{2} - F_{S,1})}_{Period\ 1} - \underbrace{c}_{investment\ cost} + \underbrace{\frac{1}{2} + \frac{t}{4}}_{Period\ 2},$$

$$\pi_S = \underbrace{(\frac{t}{2} + (1 - \frac{3t}{2}) + F_{S,1})}_{Period\ 1} + \underbrace{\Delta + \frac{1}{2} - \frac{3t}{4}}_{Period\ 2}$$

$$\pi_S - \pi_A = 2F_{S,1} + c + \Delta - \tfrac{5t}{2} + 1$$

If Firm A and Firm S fail to agree in period 1, Firm A contracts with Firm T in period 1. Then, per Lemma 6, it invests if and only if $\Delta > 1 + \frac{t}{2} = \hat{\Delta}$ and $c < \Delta - \frac{3t}{4} - \frac{1}{2}$. Therefore, the disagreement payoffs, π_A^O and π_S^O, depend on whether or not $\Delta > \hat{\Delta}$. In particular, if $\Delta > \hat{\Delta}$, then $c < \frac{t}{2}$ implies $c < \Delta - \frac{3t}{4} - \frac{1}{2}$. Therefore,

$$\pi_A^O = (\tfrac{t}{2}) - c + (\Delta - t),$$

$$\pi_S^O = (\tfrac{t}{2}) + (0)$$

$$\pi_S^O - \pi_A^O = c + t - \Delta$$

Whereas if $\Delta < \hat{\Delta}$, investment does not occur:

$$\pi_A^O = (\tfrac{t}{2}) + (\tfrac{1}{2} - \tfrac{t}{4}),$$

$$\pi_S^O = (\tfrac{t}{2}) + (\tfrac{1}{2} - \tfrac{t}{4})$$

$$\pi_S^O - \pi_A^O = 0$$

The fixed payment is derived from $\pi_S - \pi_A = \pi_S^O - \pi_A^O$. Hence, $F_{S,1} = -\Delta - \frac{1}{2} + \frac{7t}{4}$ if $\Delta > 1 + \frac{t}{2}$ and $F_{S,1} = -\frac{\Delta}{2} - \frac{1}{2} + \frac{5t}{4} - \frac{c}{2}$, otherwise.

Consider $\max\{\frac{t}{2}, \Delta + t - 2\} < c$. Then,

$$\pi_A = \underbrace{(\frac{t}{2} - F_{S,1})}_{Period\ 1} + \underbrace{\frac{1}{4}(-t + 2)}_{Period\ 2},$$

$$\pi_S = \underbrace{(\frac{t}{2} + (1 - \frac{3t}{2}) + F_{S,1})}_{Period\ 1} + \underbrace{\frac{1}{4}(-t + 2)}_{Period\ 2}$$

$$\pi_S - \pi_A = 2F_{S,1} + 1 - \tfrac{3t}{2}$$

If Firm A and Firm S fail to agree in period 1, Firm A contracts with Firm T in period 1. Then, per Lemma 6, it invests if and only if $\Delta > 1 + \frac{t}{2} = \hat{\Delta}$ and $c < \Delta - \frac{3t}{4} - \frac{1}{2}$. Therefore, if $\Delta > \hat{\Delta}$ and $c < \Delta - \frac{3t}{4} - \frac{1}{2}$, or, $\Delta > \max\{\hat{\Delta}, c + \frac{3t}{4} + \frac{1}{2}\} = \bar{\Delta}$,

$\pi_A^O = (\frac{t}{2}) - c + (\Delta - t)$,

$\pi_S^O = (\frac{t}{2}) + (0)$

$\pi_S^O - \pi_A^O = c + t - \Delta$

Otherwise, if $\Delta < \bar{\Delta}$, investment does not occur, and:

$\pi_A^O = (\frac{t}{2}) + (\frac{1}{2} - \frac{t}{4})$,

$\pi_S^O = (\frac{t}{2}) + (\frac{1}{2} - \frac{t}{4})$

$\pi_S^O - \pi_A^O = 0$

The fixed payment is derived from $\pi_S - \pi_A = \pi_S^O - \pi_A^O$. Hence, $F_{S,1} = \frac{1}{4}(5t - 2\Delta - 2 + 2c)$ if $\Delta > \bar{\Delta}$ and $F_{S,1} = -\frac{1}{2} + \frac{3t}{4}$, otherwise. Table 3 summarizes the game outcomes for the case of $\Delta > 3t$.

A.8 Proof of Proposition 2

The condition for the focal firm to source from the third party based on Proposition 1 is $\frac{\Delta}{6} < c < \frac{\Delta^2 - 18t + 27t^2}{9t}$, which can hold only if $\frac{\Delta}{6} < \frac{\Delta^2 - 18t + 27t^2}{9t}$. After rearranging terms, the latter condition is equivalent to: $2\Delta^2 - 3t\Delta - 36t + 54t^2 > 0$. Applying the quadratic equation and given $t < \frac{2}{3}$, we find that this inequality holds if and only if $\Delta > [3t + \sqrt{9t^2 + 4(72t - 108t^2)}]/4$, which is equivalent to $\Delta > \frac{3}{4}(t + \sqrt{32t - 47t^2})$. For $\Delta < 3t$, this inequality can hold only if $\frac{3}{4}(t + \sqrt{32t - 47t^2}) < 3t$, which is equivalent to $32t - 47t^2 < 9t^2$ and to $t > \frac{4}{7}$. Furthermore, we have $\frac{\partial}{\partial t}[\frac{\Delta^2 - 18t + 27t^2}{9t}] = 3 - \frac{\Delta^2}{9t^2}$, which is strictly positive for $\Delta < 3t$. Therefore, as t increases, the region of sourcing from the third party expands.

A.9 Proof of Lemma 7

In the model extension, the joint profits for Firms A and T if they reach an agreement are the same as the profits for Firm A in the main version of the model when it buys from Firm

Table 3: Equilibrium outcomes of the main model ($\Delta > 3t$).

Equilibrium Outcomes		Condition		
		$c < \frac{t}{2}$	$\max\{\frac{t}{2}, \Delta + t - 2\} < c$	$\frac{t}{2} < c < \Delta + t - 2$
Firm A Sources from ...		Firm S (both periods)	Firm S (both periods)	Firm T (both periods)
Period 1	$w_{S,1}$	$1 - \frac{3t}{2}$	$1 - \frac{3t}{2}$	–
Period 1	$F_{S,1}$	$\frac{1}{4}(7t - 4\Delta - 2)$ if $\Delta > \hat{\Delta}$ $\frac{1}{4}(5t - 2\Delta - 2 - 2c)$ if $\Delta < \hat{\Delta}$	$\frac{1}{4}(5t - 2\Delta - 2 + 2c)$ if $\Delta > \bar{\Delta}$ $\frac{1}{4}(3t - 2)$ if $\Delta < \bar{\Delta}$	–
Period 1	$\pi_{A,1}$	$\frac{1}{4}(-5t + 4\Delta + 2)$ if $\Delta > \hat{\Delta}$ $\frac{1}{4}(-3t + 2\Delta + 2 + 2c)$ if $\Delta < \hat{\Delta}$	$\frac{1}{4}(-3t + 2\Delta + 2 - 2c)$ if $\Delta > \bar{\Delta}$ $\frac{1}{4}(-t + 2)$ if $\Delta < \bar{\Delta}$	$\frac{t}{2}$
Period 1	$\pi_{S,1}$	$\frac{1}{4}(3t - 4\Delta + 2)$ if $\Delta > \hat{\Delta}$ $\frac{1}{4}(t - 2\Delta + 2 - 2c)$ if $\Delta < \hat{\Delta}$	$\frac{1}{4}(t - 2\Delta + 2 + 2c)$ if $\Delta > \bar{\Delta}$ $\frac{1}{4}(-t + 2)$ if $\Delta < \bar{\Delta}$	$\frac{t}{2}$
Period 1	Π_1	$1 - \frac{t}{2}$	$1 - \frac{t}{2}$	t
Firm A invests?		Yes	No	Yes
Period 2	$w_{S,2}$	$\Delta + 1 - \frac{3t}{2}$	$1 - \frac{3t}{2}$	–
Period 2	$F_{S,2}$	$\frac{1}{4}(t - 2)$	$\frac{1}{4}(3t - 2)$	–
Period 2	$\pi_{A,2}$	$\frac{1}{2} + \frac{t}{4}$	$\frac{1}{4}(-t + 2)$	$\Delta - t$
Period 2	$\pi_{S,2}$	$\Delta + \frac{1}{2} - \frac{3t}{4}$	$\frac{1}{4}(-t + 2)$	0
Period 2	Π_2	$\Delta + 1 - \frac{t}{2}$	$1 - \frac{t}{2}$	$\Delta - t$
Across Periods	π_A	$-t + \Delta + 1 - c$ if $\Delta > \hat{\Delta}$ $\frac{1}{2}(-t + \Delta + 2 - c)$ if $\Delta < \hat{\Delta}$	$\frac{1}{4}(-4t + 2\Delta + 4 - 2c)$ if $\Delta > \bar{\Delta}$ $\frac{1}{2}(-t + 2)$ if $\Delta < \bar{\Delta}$	$\Delta - \frac{t}{2} - c$
Across Periods	π_S	1 if $\Delta > \hat{\Delta}$ $\frac{1}{2}(-t + \Delta + 2 - c)$ if $\Delta < \hat{\Delta}$	$\frac{1}{4}(-2\Delta + 4 + 2c)$ if $\Delta > \bar{\Delta}$ $\frac{1}{2}(-t + 2)$ if $\Delta < \bar{\Delta}$	$\frac{t}{2}$
Across Periods	Π	$\Delta + 2 - t - c$	$2 - t$	$\Delta - c$

T. Conditional on any given product price for Firm S, these joint profits of Firms A and T are maximized when Firm A chooses a product price to set its first-order condition equal to zero given a wholesale input price of $w_{T,i} = 0$ (see the proof of Lemma 1). Any higher value of $w_{T,i}$ results in Firm A setting a product price higher than the level that maximizes these joint profits.

A.10 Proof of Lemma 8

Similar to the main model (see Lemma 5), Firm A can continue with Firm T or switch to Firm S. Period 2 industry profits if it continues with Firm T are $\Pi_2 = t + \frac{\Delta^2}{9t}$. Period 2 industry profits if it switches to Firm S are $\Pi_2 = 1 - \frac{t}{2}$. We have, $t + \frac{\Delta^2}{9t} > 1 - \frac{t}{2} \Leftrightarrow \frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$.

We now derive the fixed payment when Firm A contracts with Firm T (when $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$). We derive the disagreement payoffs. If Firm A contracts with Firm T, then Firm S makes $\pi_{S,2} = \frac{1}{18t}(3t - \Delta)^2$, so this is the lowest profit Firm S would accept. Therefore, if Firm A and T contract, Firm A's disagreement payoff is $\pi^O_{A,2} = 1 - \frac{t}{2} - \frac{1}{18t}(3t - \Delta)^2$. Firm T's disagreement payoff is zero.

We have: $\pi_{A,2} - \pi^O_{A,2} = \pi_{T,2} - \pi^O_{T,2} \Rightarrow (\frac{1}{18t}(3t+\Delta)^2 - F_{T,2}) - (1 - \frac{t}{2} - \frac{1}{18t}(3t-\Delta)^2) = (F_{T,2}) - 0$. Thus, $F_{T,2} = \frac{1}{2}(\frac{\Delta^2}{9t} - (1 - \frac{3t}{2}))$. Inserting the fixed fee back into Firm A and Firm T profits results in the expressions in the Lemma.

A.11 Proof of Lemma 9

If Firm A buys from Firm S in period 1, its profits in period 2 are $\pi_{A,2} = \frac{1}{12}(2\Delta - 3t + 6)$ if it invests (see Lemma 4) and $\pi_{A,2} = \frac{1}{4}(2 - t)$ if it does not invest (see Lemma 3). Therefore, Firm A will invest c if and only if the difference between these two is larger than investment cost c, that is, $c < \frac{\Delta}{6}$.

If Firm A buys from Firm T in period 1, its profits in period 2 are $\pi_{A,2} = \frac{1}{4}(2-t) + \frac{1}{3}\Delta$ if it invests (see Lemma 8) and $\pi_{A,2} = \frac{1}{4}(2 - t)$ if it does not invest (see Lemma 3). Therefore, Firm A will invest c if and only if $c < \frac{\Delta}{3}$.

A.12 Proof of Proposition 3

If $c < \frac{\Delta}{6}$, Firm A will invest if it contracts with Firm S (see Lemma 9). Therefore, Firm A buys from Firm S in both periods and invests.

If $c > \frac{\Delta}{3}$, Firm A never invests (see Lemma 9). Thus, Firm A buys from Firm S in both periods.

If $\frac{\Delta}{6} < c < \frac{\Delta}{3}$, Firm A invests only if the period 1 supplier is Firm T. In this case, industry profits across two periods are:

$$\Pi^{AT \to I \to AT} = t + t + \frac{\Delta^2}{9t} - c$$

However, if Firm A contracts with Firm S in both periods and does not invest:

$\Pi^{AS\to N\to AS} = 1 - \frac{t}{2} + 1 - \frac{t}{2}$

The former is larger if and only if $c < \frac{\Delta^2 - 18t + 27t^2}{9t}$. Thus, Firm A buys from Firm T in both periods and invests in Firm T's input if $\frac{\Delta}{6} < c < \min\{\frac{\Delta}{3}, \frac{\Delta^2-18t+27t^2}{9t}\} = \frac{\Delta^2-18t+27t^2}{9t}$. Otherwise, if $\max\{\frac{\Delta}{6}, \frac{\Delta^2-18t+27t^2}{9t}\} < c$, Firm A buys from Firm S in both periods without investing in the input. The regions are the same as those in Proposition 1.

B Appendix B: Model Extensions and Variations

B.1 Multiparty Negotiation with $w_{T,i} > 0$

In the multiparty negotiation model in the body of the paper, we require the input contract with the third party to be renegotiation-proof, which implies $w_{T,i} = 0$. To illustrate the importance of renegotiation-proof contracts, this appendix presents a model of multiparty negotiation that allows $w_{T,i} > 0$ and shows how the equilibrium regions for different supplier choices and investment decisions change under this alternative set-up. In this case, Firm A may agree to a high per-unit input price with Firm T in order to compel Firm S to set a higher product price.

We follow the same backwards induction process as in the main model. We drive the equilibrium prices. We then derive the period 2 supply contract. After that, we derive the innovation investment. Finally, we derive the period 1 supply contract, leading to equilibrium of the entire game.

B.1.1 Pricing

If Firm A buys Firm S, outcomes are the same as in main model (Lemma 1). However, if Firm A contracts with Firm T, it buys each unit of the input at $w_{T,i}$, and this per-unit fee may be strictly positive. We drop the subscript i in the following lemma for clarity of exposition.

Lemma 10. *(i) If Firm A buys from Firms S, product prices are:* $p_A = t + \frac{v_A - v_S}{3} + w_S$ *and* $p_S = t + \frac{v_S - v_A}{3} + w_S$*, profits are:* $\pi_A = \frac{1}{18t}(3t + v_A - v_S)^2 - F_S$ *and* $\pi_S = \frac{1}{18t}(3t - v_A + v_S)^2 + (w_S + F_S)$*, and industry profits are:* $\Pi_i = t + \frac{1}{9t}(v_A - v_S)^2 + w_S$ *for all* $w_S \in [0, \frac{1}{2}(v_A + v_S - 3t)]$*.*

(ii) If Firm A buys from Firm T, prices are: $p_A = t + \frac{v_A - v_S}{3} + \frac{2}{3}w_T$ *and* $p_S = t + \frac{v_S - v_A}{3} + \frac{1}{3}w_T$*, profits are:* $\pi_A = \frac{1}{18t}(3t - w_T + v_A - v_S)^2 - F_T$*,* $\pi_S = \frac{1}{18t}(3t + w_T - v_A + v_S)^2$*, and* $\pi_T = \frac{1}{6t}w_T(3t - w_T + v_A - v_S) + F_T$ *and industry profits are:* $\Pi_i = t + \frac{1}{9t}(v_A - v_S - w_T)^2 + \frac{1}{6t}w_T(3t - w_T + v_A - v_S)$ *for all* $w_T \in [0, (v_A + v_S - 3t)]$*.*

Proof: The first part is the same as Lemma 1. The second part is the standard Hotelling model with Firm A's marginal cost w_T. In particular, the indifferent customer is at location $x = \frac{1}{2} - \frac{p_A - p_S + v_S - v_A}{2t}$. Firm A and S profits are: $\pi_A = (p_A - w_T)(\frac{1}{2} - \frac{p_A - p_S + v_S - v_A}{2t})$ and $\pi_S = p_S(\frac{1}{2} + \frac{p_A - p_S + v_S - v_A}{2t})$. Solving the first-order conditions, equilibrium prices are: $p_A = t + \frac{v_A - v_S}{3} + \frac{2}{3}w_T$ and $p_S = t + \frac{v_S - v_A}{3} + \frac{1}{3}w_T$. Equilibrium quantities are: $q_A = \frac{1}{2} - \frac{v_S - v_A}{6t} - \frac{1}{6t}w_T$ and $q_S = \frac{1}{2} + \frac{v_S - v_A}{6t} + \frac{1}{6t}w_T$. Hence, for positive market shares we require $-3t + v_A - v_S < w_T < 3t + v_A - v_S$. Equilibrium profits are: $\pi_A = \frac{1}{18t}(3t - w_T + v_A - v_S)^2$ and $\pi_S = \frac{1}{18t}(3t + w_T - v_A + v_S)^2$. Full market coverage requires that the indifferent customer x to have positive utility, so, $v_A - p_A - tx > 0$, that is, $v_A + v_S - 3t > w_T$. We note that although prices are increasing in w_T, buying from the rival mitigates price competition to a larger extent because both prices increase by w_S.

Next, similar to Lemma 2, we derive the wholesale price that maximizes total profits of the negotiating firms. We drop the subscript i in the following lemma.

Lemma 11. *(i) If Firm A purchases the input from Firm S in period* i*, total Firm A and Firm S profits are maximized by setting wholesale input price* $w_S = \frac{1}{2}(v_A + v_S - 3t)$*, resulting in* $\pi_A = \frac{1}{18t}(3t + v_A - v_S)^2 - F_S$*,* $\pi_S = \frac{1}{18t}(3t - v_A + v_S)^2 + \frac{1}{2}(v_A + v_S - 3t + F_S)$*,* $\pi_T = 0$*, and total industry profits* $\Pi_i = \frac{1}{2}(v_A + v_S) + \frac{1}{9t}(v_S - v_A)^2 - \frac{1}{2}t$*.*

(ii) If Firm A purchases the input from Firm T in period i*, total Firm A and Firm T profits are maximized by setting wholesale input price* $w_T = \frac{1}{4}(v_A - v_S + 3t)$*, resulting in* $\pi_A = \frac{1}{32t}(3t + v_A - v_S)^2 - F_T$*,* $\pi_S = \frac{1}{32t}(5t - v_A + v_S)^2$*, and* $\pi_T = \frac{1}{32t}(3t + v_A - v_S)^2 + F_T$ *and*

the total industry profits $\Pi_i = \frac{1}{32t}(43t^2 + 2t(v_A - v_S) + 3(v_S - v_A)^2)$.

Proof. The first part of lemma is the same as Lemma 2. For the second part, we derive the first order condition:

$$\frac{d}{dw_T}(\pi_A + \pi_T) = \frac{d}{dw_T}(\frac{1}{18t}(3t - w_T + v_A - v_S)^2 + \frac{1}{6t}w_T(3t - w_T + v_A - v_S)) = 0,$$

yielding $w_T = \frac{1}{4}(v_A - v_S + 3t)$. To make sure market is fully covered we require $w_T = \frac{1}{4}(v_A - v_S + 3t) \in [0, (v_A + v_S - 3t)]$. Thus, $15t < 3v_A + 5v_S$. The valuations are equal or greater than one. Thus, a sufficient condition that guarantees interior solution would be $15t < 3 + 5$, or $t < \frac{8}{15}$, a stricter requirement than the condition $t < \frac{2}{3}$ from our main model.

B.1.2 Supply Contract in period 2

Similar to our base model, there are three sub-games: No investment in period 1, investment in the Firm S input, and investment in the Firm T input. The following three lemmas characterize the equilibrium for these three sub-games.

Lemma 12. *If Firm A did not invest in innovation in period 1, in period 2 it buys the input from the competitor, with contract* $w_{S,2} = 1 - \frac{3t}{2}$ *and* $F_{S,2} = \frac{55}{64}t - \frac{1}{2}$, *which results in profits* $\pi_{A,2} = \frac{1}{64}(32 - 23t)$, $\pi_{S,2} = \frac{1}{64}(32 - 9t)$, *and industry profit* $\Pi_2 = 1 - \frac{t}{2}$.

Proof. Thus, similar to main model, Firm A contracts with Firm S in period 2 if there was no investment in period 1. To prove this, we show that total industry profits are higher with this A-S contract. Using Lemma 11 with $v_{A,2} = v_{S,2} = 1$, the A-S contract results in $\Pi_2 = 1 - \frac{1}{2}t$ whereas the A-T contract results in $\Pi_2 = \frac{43}{32}t$. The former is higher because $t < \frac{8}{15} < \frac{32}{59}$. Thus, without period 1 investment, Firm A contracts with Firm S.

We now derive the the fixed payment ($F_{S,2}$). We first need to determine disagreement payoffs. Firm A's disagreement payoff ($\pi^O_{A,2}$) is the total maximum Firm A and Firm T combined profits at the maximizing wholesale price $w_{T,2} = \frac{1}{4}(v_{A,2} - v_{S,2} + 3t) = \frac{3t}{4}$. Inserting this wholesale price into Firm A and Firm T profits in Lemma 11 yields:

$$\pi^O_{A,2} = (\pi_{A,2} + \pi_{T,2}) \text{ at } \{w_{T,2} = \tfrac{3t}{4}\} = \frac{9t}{32} + \frac{9t}{32} = \frac{18t}{32}.$$

Moreover, at this level of wholesale price ($w_{T,2} = \frac{3t}{4}$), Firm S would make $\frac{25}{32}t$, which would be its disagreement payoff ($\pi^O_{S,2} = \frac{25}{32}t$). The fixed payment is derived from:

$$\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2} \Rightarrow (\tfrac{t}{2} - F_{S,2}) - (\tfrac{18}{32}t) = (1 - t + F_{S,2}) - (\tfrac{25}{32}t) \Rightarrow F_{S,2} = \tfrac{55}{64}t - \tfrac{1}{2}.$$

Inserting this fixed payment into each firm's profits results in the expressions in Lemma 12. Next, we solve the second sub-game.

Lemma 13. *If Firm A buys input from the competitor in period 1 and invests in innovation, then in period 2 it again buys the input from the competitor with contract* $w_{S,2} = 1 + \Delta - \frac{3t}{2}$ *and* $F_{S,2} = \frac{1}{192t}(-3\Delta^2 - 96t - 46\Delta t + 165t^2)$, *leading to period 2 profits* $\pi_{A,2} = \frac{1}{64t}(\Delta^2 + 32t + 10\Delta t - 23t^2)$ *and* $\pi_{S,2} = \frac{1}{64t}(-\Delta^2 + 32t + 54\Delta t - 9t^2)$, *and industry profits* $\Pi_2 = 1 + \Delta - \frac{t}{2}$.

Proof. Thus, similar to the main model, Firm A continues with Firm S in period 2. To prove this, we show that total industry profit is higher with the A-S contract. Per Lemma 11 with $v_{A,2} = v_{S,2} = 1 + \Delta$, the A-S contract would result in $\Pi_2 = 1 - \frac{1}{2}t + \Delta$ whereas the A-T contract results in $\Pi_2 = \frac{1}{32t}(43t^2 - 2t\Delta + 3\Delta^2)$. The former is larger since $t < \frac{8}{15}$. Thus, Firm A continues with Firm S in period 2.

We now derive the the fixed payment ($F_{S,2}$). Firm A's disagreement payoff ($\pi^O_{A,2}$) is: $\pi^O_{A,2} = (\pi_{A,2} + \pi_{T,2})$ *at* $\{w_{T,2} = \frac{1}{4}(3t - \Delta)\} = \frac{1}{16t}(3t - \Delta)^2$ (see Lemma 11). Moreover, Firm S's disagreement payoff is $\pi^O_{S,2} = \frac{1}{32t}(5t + \Delta)^2$. The fixed payment is derived from: $\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2}$, leading to $F_{S,2} = \frac{1}{192t}(-3\Delta^2 - 96t - 46\Delta t + 165t^2)$, and period 2 profits $\pi_{A,2} = \frac{1}{64t}(\Delta^2 + 32t + 10\Delta t - 23t^2)$ and $\pi_{S,2} = \frac{1}{64t}(-\Delta^2 + 32t + 54\Delta t - 9t^2)$.

Lemma 14. *Suppose in period 1 Firm A buys input from Firm T and invests in improving its input. In period 2,*

(i) if $59t^2 + 2t\Delta + 3\Delta^2 > 32t$, *Firm A buys the input from the third party with supply arrangement* $w_{T,2} = \frac{1}{4}(3t + \Delta)$ *and* $F_{T,2} = \frac{1}{64t}(\Delta^2 - 10\Delta t - 32t + 41t^2)$. *Firms make* $\pi_{A,2} = \frac{1}{64t}(\Delta^2 + 32t + 22t\Delta - 23t^2)$ *and* $\pi_{S,2} = \frac{1}{32t}(5t - \Delta)^2$, *and* $\pi_{T,2} = \frac{1}{64t}(3\Delta^2 - 32t + 2t\Delta + 59t^2)$, *leading to industry profits of* $\Pi_2 = \frac{1}{32t}(3\Delta^2 + 2t\Delta + 43t^2)$.

(ii) if $59t^2 + 2t\Delta + 3\Delta^2 < 32t$, *Firm A buys the input from its competitor, with input contract* $w_{S,2} = 1 - \frac{3t}{2}$ *and* $F_{S,2} = \frac{1}{64t}(\Delta^2 - 22\Delta t - 32t + 55t^2)$. *Firm profits are* $\pi_{A,2} =$

$\frac{1}{64t}(\Delta^2 + 32t + 22t\Delta - 23t^2)$, $\pi_{S,2} = \frac{1}{64t}(-\Delta^2 + 32t - 22t\Delta - 9t^2)$, *and* $\pi_{T,2} = 0$, *with industry profits* $\Pi_2 = 1 - \frac{t}{2}$.

Proof: Thus, similar to the main model (see Lemma 5), Firm A might continue with Firm T or switch to Firm S. Total period 2 industry profits if it continues with Firm T are $\Pi_2 = \frac{1}{32t}(43t^2+2t\Delta+3\Delta^2)$ (we insert $v_{A,2} = 1+\Delta, v_{S,2} = 1$ into Lemma 11). The total period 2 industry profits if it switches to Firm S are $\Pi_2 = 1 - \frac{t}{2}$. Then, $\frac{1}{32t}(43t^2 + 2t\Delta + 3\Delta^2) > 1 - \frac{t}{2} \Leftrightarrow 59t^2 + 2t\Delta + 3\Delta^2 > 32t$. The derivation of fixed payments and equilibrium profits are similar to the previous two lemmas.

B.1.3 Focal Firm's Investment Decision

We derive the focal firm's investment decision given the outcome of the negotiation in period 1.

Lemma 15. *In period 1, if Firm A buys the input from Firm S, it invests if and only if* $c < \frac{\Delta(\Delta+10t)}{64t}$. *If Firm A buys from Firm T, it invests if and only if* $c < \frac{(\Delta+22t)}{64t}$.

Proof: If Firm A buys from Firm S in period 1, its profits in period 2 are $\pi_{A,2} = \frac{1}{64t}(\Delta^2 + 10t\Delta + 32t - 23t^2)$ if it invests (see Lemma 13) and $\pi_{A,2} = \frac{1}{64}(32 - 23t)$ if it does not invest (see Lemma 12). Therefore, Firm A will invest c if and only if the difference between these two profits is larger than investment cost c, that is, $c < \frac{\Delta(\Delta+10t)}{64t}$.

If Firm A buys from Firm T in period 1, its profit in period 2 are $\pi_{A,2} = \frac{1}{64t}(\Delta^2 + 22t\Delta + 32t - 23t^2)$ if it invests (see Lemma 14) and $\pi_{A,2} = \frac{1}{64}(32 - 23t)$ if it does not invest (see Lemma 12). Therefore, Firm A will invest c if and only if $c < \frac{\Delta(\Delta+22t)}{64t}$.

B.1.4 Supply Contract in period 1

Next we derive the negotiation outcome in period 1. Product values are low in this period ($v_{A,1} = v_{S,1} = 1$). Therefore, industry profits in period 1 are $1 - \frac{t}{2}$ if Firm A contracts with Firm S, and $\frac{43t}{32}$ if Firm A contracts with Firm T. Note that when firms engage in bilateral negotiation in period 1, they account for the impact of the period 1 negotiation on the profits they earn in period 2. The winner of the negotiation in period 1 is the supplier that leads to higher total industry profits across the two periods.

Proposition 7. *(i) Firm A buys from Firm S in both periods and invests in Firm S's input if* $c < \frac{\Delta(\Delta+10t)}{64t}$*. (ii) Firm A buys from Firm S in both periods without investing in the input if* $\max\{\frac{\Delta(\Delta+10t)}{64t}, \frac{3\Delta^2-64t+2\Delta t+118t^2}{32t}\} < c$*. (iii) Firm A buys from Firm T in both periods and invests in Firm T's input if* $\frac{\Delta(\Delta+10t)}{64t} < c < \frac{3\Delta^2-64t+2\Delta t+118t^2}{32t}$*.*

Proof: If $c < \frac{\Delta(\Delta+10t)}{64t}$, Firm A will invest if it contracts with Firm S in period 1 (see Lemma 15). Therefore, Firm A buys from Firm S in both periods and invests.

If $c > \frac{\Delta(\Delta+22t)}{64t}$, Firm A never invests (see Lemma 15). Thus, Firm A buys from Firm S in both periods.

When $\frac{\Delta(\Delta+10t)}{64t} < c < \frac{\Delta(\Delta+22t)}{64t}$, Firm A invests only if the period 1 supplier is Firm T. If so, industry profits across the two periods are:

$$\Pi^{AT\to I\to AT} = \frac{43t}{32} + \frac{1}{32t}(3\Delta^2 + 2t\Delta + 43t^2) - c$$

However, if Firm A contracts with Firm S in both periods and does not invest:

$$\Pi^{AS\to N\to AS} = 1 - \frac{t}{2} + 1 - \frac{t}{2}$$

The former is larger if and only if $c < \frac{3\Delta^2-64t+2\Delta t+118t^2}{32t}$. Thus, Firm A buys from Firm T in both periods and invests in Firm T's input if $\frac{\Delta(\Delta+10t)}{64t} < c < Min\{\frac{3\Delta^2-64t+2\Delta t+118t^2}{32t}, \frac{\Delta(\Delta+22t)}{64t}\} = \frac{3\Delta^2-64t+2\Delta t+118t^2}{32t}$. This condition requires $t > \frac{128}{263}$, which is a wider condition compared to the main model $t > \frac{4}{7}$ (see Proposition 2). Thus, Firm T's bargaining power over the supply arrangement and ability to commit to a positive per-unit input price expands the region of parameters in which Firm A contracts with the third-party across the two periods.

B.2 Proof of Proposition 4 (Investment by the Competitive Supplier)

We follow the same backward induction process as in the main model. We first derive the equilibrium prices. Then we derive the period 2 supply contract. Next we derive the investment decisions by Firm A and Firm S. Finally, we derive the period 1 supply contract.

B.2.1 Pricing

Pricing remains the same as in the main model. Lemma 1 and Lemma 2 continue to hold.

B.2.2 Supply Contract in period 2

We now derive the equilibrium contract in the second period (corresponding to Lemmas 3, 4, and 5 in the main text). There are eight sub-games depending on the period 1 supplier and whether Firm A and Firm S decide to invest ($2 \times 2 \times 2$). When neither firm invests, the outcomes derived in Lemma 3 continue to hold. If only Firm S invests in innovation, then in period 2 Firm A buys the input from the competitor, and the outcomes are the same as in Lemma 4. Similarly, if in period 1 Firm A buys the input from Firm S and invests in improving its input, but Firm S does not invest, then in period 2, Firm A buys the input from the competitor, and the outcomes are the same as Lemma 4. Moreover, suppose in period 1 Firm A buys the input from Firm T and invests in improving its input, and Firm S does not invest in its input. In this case, the period 2 contract is the same as in Lemma 5.

There are two additional sub-games that we need to examine, described in the following two lemmas.

Lemma 16. *Suppose in period 1, Firm A buys the input from Firm S and invests in improving its input, and Firm S also invests in its input. In period 2, Firm A buys the input from the competitor with contract* $w_{S,2} = 1 + 2\Delta - \frac{3t}{2}$ *and* $F_{S,2} = -\frac{1}{2}w_{S,2} + \frac{2\Delta}{3}$*, leading to period 2 profits* $\pi_{A,2} = \frac{1}{12}(4\Delta - 3t + 6)$ *and* $\pi_{S,2} = \frac{1}{12}(20\Delta - 3t + 6)$*, and industry profits*

$\Pi_2 = 1 - \frac{t}{2} + 2\Delta$.

Proof. We replace Δ by 2Δ in the main text's Lemma 4. Firm A prefers to continue sourcing from Firm S because, if it switches to Firm T, it loses all input improvement while Firm S uses the improved input.

Lemma 17. *Suppose in period 1 Firm A buys the input from Firm T and invests in its input, and Firm S also invests in its own input. In period 2, Firm A will switch to Firm S, with input contract* $w_{S,2} = 1 + \Delta - \frac{3t}{2}$ *and* $F_{S,2} = -\frac{1}{2}(1 + \Delta - \frac{3t}{2})$*, leading to profits,* $\pi_{A,2} = \pi_{S,2} = \frac{1}{2} - \frac{t}{4} + \frac{\Delta}{2}$*, and industry profits* $\Pi_2 = 1 - \frac{t}{2} + \Delta$.

Proof. If Firm A sources from Firm T in period 1 and invests c_A, and Firm S also invest c_S, then Firm A has 2 options: to continue with Firm T in period 2 or to switch to Firm S.

If Firm A continues sourcing from Firm T in period 2, we will have $v_{A,2} = 1 + \Delta$ and $v_{S,2} = 1 + \Delta$. Inserting these product values into Lemma 1, in period 2 we have $\pi_{A,2} = \frac{t}{2}$ and $\pi_{S,2} = \frac{t}{2}$, leading to an industry profits of $\Pi_2 = t$.

However, if in period 2 Firm A switches to Firm S, we have $v_{A,2} = v_{S,2} = 1+\Delta$. Inserting these values into Lemma 1, industry profits are $\Pi_2 = 1 + \Delta - \frac{t}{2}$.

Given that $t < \frac{2}{3}$, we have $1 + \Delta - \frac{t}{2} > t$, so in period 2 Firm A will switch to Firm S. In this case, $w_{S,2} = 1 + \Delta - \frac{3t}{2}$. The fixed payment is derived from $\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2}$. Inserting $\pi^O_{A,2} = \frac{t}{2}$ and $\pi^O_{S,2} = \frac{t}{2}$ yields: $\pi_{A,2} = \frac{t}{2} - F_{S,2} = \pi_{S,2} = \frac{t}{2} + w_{S,2} + F_{S,2}$. Thus, $F_{S,2} = -\frac{1}{2}w_{S,2} = -\frac{1}{2}(1 + \Delta - \frac{3t}{2})$. This leads to firm profits $\pi_{A,2} = \pi_{S,2} = \frac{t}{2} - F_{S,2} = \frac{t}{2} + \frac{1}{2}(1 + \Delta - \frac{3t}{2}) = \frac{1}{2} + \frac{\Delta}{2} - \frac{t}{4}$.

B.2.3 Firms' Investment Decisions

Having solved the equilibrium supply contract in period 2, we derive firms' investment decisions given the period 1 supplier. We first summarize the period 2 profits (derived above) net of investment cost in a 2×2 game payoff matrix. In the these tables, let $X = \frac{1}{2} - \frac{t}{4}$.

Table 4: When Period 1 supplier is Firm S

	Firm S invests = Yes	Firm S invests = No
Firm A invests = Yes	$(X + \frac{\Delta}{3} - c_A \ , \ X + \frac{5\Delta}{3} - c_S)$	$(X + \frac{\Delta}{6} - c_A \ , \ X + \frac{5\Delta}{6})$
Firm A invests = No	$(X + \frac{\Delta}{6} \ , \ X + \frac{5\Delta}{6} - c_S)$	$(X \ , \ X)$

Table 5: When Period 1 Supplier is Firm T ($\frac{\Delta^2}{9t} < 1 - \frac{3t}{2}$).

	Firm S invests = Yes	Firm S invests = No
Firm A invests = Yes	$(X + \frac{\Delta}{2} - c_A \ , \ X + \frac{\Delta}{2} - c_S)$	$(X + \frac{\Delta}{3} - c_A \ , \ X - \frac{\Delta}{3})$
Firm A invests = No	$(X + \frac{\Delta}{6} \ , \ X + \frac{5\Delta}{6} - c_S)$	$(X \ , \ X)$

Table 6: When Period 1 Supplier is Firm T ($\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$).

	Firm S invests = Yes	Firm S invests = No
Firm A invests = Yes	$(X + \frac{\Delta}{2} - c_A \ , \ X + \frac{\Delta}{2} - c_S)$	$(\frac{t}{2} + \frac{\Delta}{3} + \frac{\Delta^2}{18t} - c_A \ , \ \frac{t}{2} - \frac{\Delta}{3} + \frac{\Delta^2}{18t})$
Firm A invests = No	$(X + \frac{\Delta}{6} \ , \ X + \frac{5\Delta}{6} - c_S)$	$(X \ , \ X)$

The following three lemmas derive the Firm A and Firm S investment strategies.

Lemma 18. *If Firm A buys the input from Firm S in period 1, Firm A invests if and only if $c_A < \frac{\Delta}{6}$ whereas Firm S invests if and only if $c_S < \frac{5\Delta}{6}$.*

Proof. See Table 4.

Lemma 19. *If Firm A buys the input from Firm T in period 1 and $\frac{\Delta^2}{9t} < 1 - \frac{3t}{2}$, Firm A invests if and only if $c_A < \frac{\Delta}{3}$ whereas Firm S invests if and only if $c_S < \frac{5\Delta}{6}$.*

Proof. See Table 5.

Lemma 20. *If Firm A buys the input from Firm T in period 1 and $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$, investment strategies are as shown in Table 7.*

Table 7: Firm A and Firm S Investment strategies when Period 1 Supplier is Firm T and $A = \frac{\Delta^2}{9t} - (1 - \frac{3t}{2}) > 0$.

c_A and c_S	$c_S < \frac{5\Delta}{6} - \frac{A}{2}$	$\frac{5\Delta}{6} - \frac{A}{2} < c_S < \frac{5\Delta}{6}$	$c_S > \frac{5\Delta}{6}$
$c_A < \frac{\Delta}{3}$	Both invest	Only A invests	Only A invests
$\frac{\Delta}{3} < c_A < \frac{\Delta}{3} + \frac{A}{2}$	Only S invests	Only A invests or Only S invests	Only A invests
$c_A > \frac{\Delta}{3} + \frac{A}{2}$	Only S invests	Only S invests	Neither invests

Proof. Using Table 6, if Firm S does not invest, Firm A invests if $\frac{t}{2} + \frac{\Delta}{3} + \frac{\Delta^2}{18t} - c_A > \frac{1}{2} - \frac{t}{4}$, or, $c_A < \frac{1}{2}(\frac{\Delta^2}{9t} - (1 - \frac{3t}{2})) + \frac{\Delta}{3}$. If Firm S invests, Firm A invests if $c_A < \frac{\Delta}{3}$.

If Firm A does not invest, Firm S invests if $c_S < \frac{5\Delta}{6}$. If Firm A invests, Firm S also invests if $\frac{1}{2} - \frac{t}{4} + \frac{\Delta}{2} - c_S > \frac{t}{2} - \frac{\Delta}{3} + \frac{\Delta^2}{18t}$, or, $c_S < \frac{5\Delta}{6} - \frac{1}{2}(\frac{\Delta^2}{9t} - (1 - \frac{3t}{2}))$.

Thus, letting $A = \frac{\Delta^2}{9t} - (1 - \frac{3t}{2})$, there are two cut-offs for c_A ($\frac{\Delta}{3}$ and $\frac{\Delta}{3} + \frac{A}{2}$), and two cut-offs for c_S ($\frac{5\Delta}{6} - \frac{A}{2}$ and $\frac{5\Delta}{6}$). The equilibrium investment strategies are then summarized in Table 7. We note that when $\frac{\Delta}{3} < c_A < \frac{\Delta}{3} + \frac{A}{2}$ and $\frac{5\Delta}{6} - \frac{A}{2} < c_S < \frac{5\Delta}{6}$ there are two equilibria: Only Firm A invests or only Firm S invests.

B.2.4 Supply Contract in period 1

In this section, we derive the equilibrium supply contracts in period 1. We consider two cases separately: When $A = \frac{\Delta^2}{9t} - (1 - \frac{3t}{2}) < 0$, and when $A > 0$.

$A < 0$**:** In this case, per Lemmas 18 and 19, Firm S invests if and only if $c_S < \frac{5\Delta}{6}$. Firm A's investment decision depends on the period 1 supplier. If the period 1 supplier is Firm S, Firm A invests if $c_A < \frac{\Delta}{6}$. If the period 1 supplier is Firm T, Firm A invests if $c_A < \frac{\Delta}{3}$. Table 8 summarizes total industry profits for different Firm A and Firm S costs.

Proposition 8. *If* $\frac{\Delta^2}{9t} - (1 - \frac{3t}{2}) < 0$, *Firm A contracts with Firm S in both periods. Firm*

Table 8: Industry profit (Π) across two periods ($A < 0$).

Costs		Period 1 Supplier	
c_A	c_S	Firm S	Firm T
$c_A < \frac{\Delta}{6}$	$c_S < \frac{5\Delta}{6}$	$(1-\frac{t}{2})+(1-\frac{t}{2}+2\Delta)-c_A-c_S$	$(t)+(1-\frac{t}{2}+\Delta)-c_A-c_S$
$c_A < \frac{\Delta}{6}$	$c_S > \frac{5\Delta}{6}$	$(1-\frac{t}{2})+(1-\frac{t}{2}+\Delta)-c_A-0$	$(t)+(1-\frac{t}{2})-c_A-0$
$\frac{\Delta}{6} < c_A < \frac{\Delta}{3}$	$c_S < \frac{5\Delta}{6}$	$(1-\frac{t}{2})+(1-\frac{t}{2}+\Delta)-0-c_S$	$(t)+(1-\frac{t}{2}+\Delta)-c_A-c_S$
$\frac{\Delta}{6} < c_A < \frac{\Delta}{3}$	$c_S > \frac{5\Delta}{6}$	$(1-\frac{t}{2})+(1-\frac{t}{2})-0-0$	$(t)+(1-\frac{t}{2})-c_A-0$
$c_A > \frac{\Delta}{3}$	$c_S < \frac{5\Delta}{6}$	$(1-\frac{t}{2})+(1-\frac{t}{2}+\Delta)-0-c_S$	$(t)+(1-\frac{t}{2}+\Delta)-0-c_S$
$c_A > \frac{\Delta}{3}$	$c_S > \frac{5\Delta}{6}$	$(1-\frac{t}{2})+(1-\frac{t}{2})-0-0$	$(t)+(1-\frac{t}{2})-0-0$

A invests if and only if $c_A < \frac{\Delta}{6}$, and Firm S invests if and only if $c_S < \frac{5\Delta}{6}$.

Proof. The proof follows directly from the comparison of industry profits in each case in Table 8.

$A > 0$**:** Per Lemmas 18 and 20, there are 12 cases to be considered, depending on the cut-offs for c_A or c_S. Table 9 summarizes these 12 cases and the total industry profits in each case. By comparing industry profits in each case, we derive the following proposition.

Proposition 9. *If $\frac{\Delta^2}{9t} - (1 - \frac{3t}{2}) > 0$, Firm A contracts with Firm T in both periods if and only if $\frac{\Delta}{6} < c_A < \frac{\Delta^2+27t^2-18t}{9t}$ and $c_S > \frac{5\Delta}{6}$. Otherwise, Firm A contracts with Firm S in both periods.*

Proof. Consider Table 9. The column between Firm A and Firm S indicates that in all cases except Case 6, sourcing from Firm S produces higher industry profits. Case 6 ($\frac{\Delta}{6} < c_A < \frac{\Delta}{3}$ and $c_S > \frac{5\Delta}{6}$) is similar to our base model; Firm A buys from Firm S in period

Table 9: Industry profit (Π) across two periods ($A > 0$).

	Costs		Period 1 Supplier		
	c_A	c_S	Firm S		Firm T
1	$c_A < \frac{\Delta}{6}$	$c_S < \frac{5\Delta}{6} - \frac{A}{2}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + 2\Delta) - c_A - c_S$	$>$	$(t) + (1 - \frac{t}{2} + \Delta) - c_A - c_S$
2	$c_A < \frac{\Delta}{6}$	$\frac{5\Delta}{6} - \frac{A}{2} < c_S < \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + 2\Delta) - c_A - c_S$	$>$	$(t) + (t + \frac{\Delta^2}{9t}) - c_A - 0$
3	$c_A < \frac{\Delta}{6}$	$c_S > \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + \Delta) - c_A - 0$	$>$	$(t) + (t + \frac{\Delta^2}{9t}) - c_A - 0$
4	$\frac{\Delta}{6} < c_A < \frac{\Delta}{3}$	$c_S < \frac{5\Delta}{6} - \frac{A}{2}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$	$>$	$(t) + (1 - \frac{t}{2} + \Delta) - c_A - c_S$
5	$\frac{\Delta}{6} < c_A < \frac{\Delta}{3}$	$\frac{5\Delta}{6} - \frac{A}{2} < c_S < \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$	$>$	$(t) + (t + \frac{\Delta^2}{9t}) - c_A - 0$
6	$\frac{\Delta}{6} < c_A < \frac{\Delta}{3}$	$c_S > \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2}) - 0 - 0$	$<>$	$(t) + (t + \frac{\Delta^2}{9t}) - c_A - 0$
7	$\frac{\Delta}{3} < c_A < \frac{\Delta}{3} + \frac{A}{2}$	$c_S < \frac{5\Delta}{6} - \frac{A}{2}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$	$>$	$(t) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$
8	$\frac{\Delta}{3} < c_A < \frac{\Delta}{3} + \frac{A}{2}$	$\frac{5\Delta}{6} - \frac{A}{2} < c_S < \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$	$>$ or $>$	$(t) + (t + \frac{\Delta^2}{9t}) - c_A$ or $(t) + (1 - \frac{t}{2} + \Delta) - c_S$
9	$\frac{\Delta}{3} < c_A < \frac{\Delta}{3} + \frac{A}{2}$	$c_S > \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2}) - 0 - 0$	$>$	$(t) + (t + \frac{\Delta^2}{9t}) - c_A - 0$
10	$c_A > \frac{\Delta}{3} + \frac{A}{2}$	$c_S < \frac{5\Delta}{6} - \frac{A}{2}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$	$>$	$(t) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$
11	$c_A > \frac{\Delta}{3} + \frac{A}{2}$	$\frac{5\Delta}{6} - \frac{A}{2} < c_S < \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$	$>$	$(t) + (1 - \frac{t}{2} + \Delta) - 0 - c_S$
12	$c_A > \frac{\Delta}{3} + \frac{A}{2}$	$c_S > \frac{5\Delta}{6}$	$(1 - \frac{t}{2}) + (1 - \frac{t}{2}) - 0 - 0$	$>$	$(t) + (1 - \frac{t}{2}) - 0 - 0$

1 if $2 - t > 2t + \frac{\Delta^2}{9t} - c_A$, or, $c_A > 3t - 2 + \frac{\Delta^2}{9t} = \frac{\Delta^2 + 27t^2 - 18t}{9t}$. Otherwise, it buys from Firm T in period 1 and invests. Also note in Case 6, $\min\{\frac{\Delta}{3}, \frac{\Delta^2+27t^2-18t}{9t}\} = \frac{\Delta^2+27t^2-18t}{9t}$. This completes the proof.

Finally, combining propositions 8 and 9 yields proposition 4 in the main text.

B.3 Proof of Proposition 5 (Incomplete Knowledge Spillover)

B.3.1 Pricing

The pricing (Lemma 1 and 2) remains unchanged.

B.3.2 Supply Contract in period 2

Lemma 3 (when Firm A does not invest) and Lemma 5 (when the period 1 supplier is Firm T and the focal firm invests) remain unchanged.

Lemma 4 changes as follows. If Firm A buys from Firm S in period 1 and invests c, then Firm A has 2 options: to continue with Firm S or to switch to Firm T.

If Firm A continues sourcing from Firm S in period 2, we have $v_{A,2} = 1+\Delta, v_{S,2} = 1+k\Delta$. The difference between valuations is $(1-k)\Delta$. If $(1-k)\Delta < 3t$, then, in period 2, both firms have positive market shares with firm profits $\pi_{A,2} = \frac{1}{18t}(3t + (1-k)\Delta)^2 - F_{S,2}$ and $\pi_{S,2} = \frac{1}{18t}(3t-(1-k)\Delta)^2 + w_{S,2} + F_{S,2}$, the wholesale price $w_{S,2} = \frac{1+k}{2}\Delta + 1 - \frac{3t}{2}$, and industry profits are $\Pi_2 = 1 + \frac{1+k}{2}\Delta + \frac{(1-k)^2}{9t}\Delta^2 - \frac{t}{2}$.

However, if $(1-k)\Delta > 3t$, then Firm A serves the entire market in period 2, and we have firm profits $\pi_{A,2} = (1-k)\Delta - t - w_{S,2} - F_{S,2}$ and $\pi_{S,2} = w_{S,2} + F_{S,2}$, and industry profits are $\Pi_2 = (1-k)\Delta$. Note that industry profits do not depend on the wholesale price $w_{S,2}$.

If in period 2, Firm A switches to Firm T, we have $v_{A,2} = 1$ and $v_{S,2} = 1 + k\Delta$. The difference between valuations is $k\Delta$. If $k\Delta < 3t$, then firm profits in period 2 are $\pi_{A,2} = \frac{1}{18t}(3t - k\Delta)^2$ and $\pi_{S,2} = \frac{1}{18t}(3t + k\Delta)^2$, leading to industry profits $\Pi_2 = t + \frac{k^2}{9t}\Delta^2$. However, if $k\Delta > 3t$, then Firm S serves the entire market, and we have: $\pi_{A,2} = 0$ and $\pi_{S,2} = k\Delta - t$, leading to industry profits of $\Pi_2 = k\Delta - t$.

Hence, we need to consider 4 different cases: (1) $k\Delta < 3t$ and $(1-k)\Delta < 3t$, (2) $k\Delta > 3t$ and $(1-k)\Delta > 3t$, (3) $k\Delta < 3t$ and $(1-k)\Delta > 3t$, and (4) $k\Delta > 3t$ and $(1-k)\Delta < 3t$.

Consider case 1: $k\Delta < 3t$ and $(1-k)\Delta < 3t$. Then industry profits are higher sourcing from Firm S if and only if $1 + \frac{1+k}{2}\Delta + \frac{(1-k)^2}{9t}\Delta^2 - \frac{t}{2} > t + \frac{k^2}{9t}\Delta^2$. This condition is always true. Thus, industry profits are always higher if Firm A continues sourcing from Firm S (similar to our main model). Thus, Firm A continues sourcing from Firm S with wholesale price $w_{S,2} = \frac{1+k}{2}\Delta + 1 - \frac{3t}{2}$. We now calculate the fixed payment $F_{S,2}$. The fixed payment is derived from $\pi_{A,2} - \pi^O_{A,2} = \pi_{S,2} - \pi^O_{S,2}$. Inserting $\pi^O_{A,2} = \frac{1}{18t}(3t - k\Delta)^2$ and $\pi^O_{S,2} = \frac{1}{18t}(3t + k\Delta)^2$ yields $F_{S,2} = -\frac{1}{2} + \frac{3t}{4} - \frac{3k-1}{12}\Delta$. Inserting F_S back into profits, we have: $\pi_{A,2} = \frac{1}{12}(\frac{2(1-k)^2}{3t}\Delta^2 +$

$(3-k)\Delta - 3t + 6)$ and $\pi_{S,2} = \frac{1}{12}(\frac{2(1-k)^2}{3t}\Delta^2 + (7k+3)\Delta - 3t + 6)$.

Consider case 2: $k\Delta > 3t$ and $(1-k)\Delta > 3t$. Then industry profits are higher sourcing from Firm S if and only if $(1-k)\Delta > k\Delta - t$. Both sourcing outcomes are possible here.

Consider case 3: $k\Delta < 3t$ and $(1-k)\Delta > 3t$. Then industry profits are higher sourcing from Firm S if and only if $(1-k)\Delta > t + \frac{k^2}{9t}\Delta^2$. This condition is always true because LHS is larger than $3t$ and RHS is smaller than $2t$.

Consider case 4: $k\Delta > 3t$ and $(1-k)\Delta < 3t$. Then industry profits are higher sourcing from Firm S if and only if $1 + \frac{1+k}{2}\Delta + \frac{(1-k)^2}{9t}\Delta^2 - \frac{t}{2} > k\Delta - t$. This condition is always true because $1 + \frac{1-k}{2}\Delta + \frac{(1-k)^2}{9t}\Delta^2 + \frac{t}{2} > 0$.

B.3.3 Focal Firm's Investment Decision

We consider the case $k\Delta < 3t$ and $(1-k)\Delta < 3t$. If Firm A buys from Firm S in period 1, its profits in period 2 are $\frac{1}{12}(\frac{2(1-k)^2}{3t}\Delta^2 + (3-k)\Delta - 3t + 6)$ if it invests and $\frac{1}{4}(2-t)$ if it does not invest. Therefore, Firm A will invest c if and only if $c < \frac{\Delta}{12}(\frac{2(1-k)^2}{3t}\Delta + (3-k))$.

If Firm A buys from Firm T in period 1, Firm A invests if and only if $c < max\{\frac{2\Delta^2 - 18t + 27t^2}{36t}, 0\} + \frac{\Delta}{3}$ (see proof of Lemma 6).

B.3.4 Supply Contract in period 1

We consider the case $k\Delta < 3t$ and $(1-k)\Delta < 3t$. In this case, suppose $c < \frac{\Delta}{12}(\frac{2(1-k)^2}{3t}\Delta + (3-k))$. If Firm A buys from Firm S in period 1, it will invest and continue sourcing from Firm S. Therefore, total industry profits are $\Pi = (1 - \frac{t}{2}) - c + (1 + \frac{1+k}{2}\Delta + \frac{(1-k)^2}{9t}\Delta^2 - \frac{t}{2})$. These profits are higher than the total industry profits if Firm A sources from Firm T in period 1.

On the other hand, if $c > \frac{\Delta}{12}(\frac{2(1-k)^2}{3t}\Delta + (3-k))$, then if Firm A buys from Firm S without investment, industry profits are $2 - t$ whereas if it buys from Firm T and invests, industry profits are $t - c + (t + \frac{k^2}{9t}\Delta^2)$. The latter is larger if and only if $c < 3t + \frac{k^2}{9t}\Delta^2 - 2 = \frac{k^2\Delta^2 - 18t + 27t^2}{9t}$. Thus, Firm A buys from Firm S in both periods and invests in Firm S's input if $c < \frac{2(1-k)^2\Delta^2+3(3-k)\Delta t}{36t}$. Firm A buys from Firm S in both periods without investing in the input if $\max\{\frac{2(1-k)^2\Delta^2+3(3-k)\Delta t}{36t}, \frac{k^2\Delta^2-18t+27t^2}{9t}\} < c$. Finally, Firm A buys from Firm T in both periods and invests in Firm T's input if $\frac{2(1-k)^2\Delta^2+3(3-k)\Delta t}{36t} < c < \frac{k^2\Delta^2-18t+27t^2}{9t}$. This completes the proof of Proposition 5.

To prove Corollary 2, note that when k is small enough, $c < \frac{k^2\Delta^2-18t+27t^2}{9t}$ cannot hold because $-18t+27t^2 < 0$ (given $t < \frac{2}{3}$). Therefore, if k is small enough, Firm A never buys from Firm T. Moreover, $\frac{2(1-k)^2\Delta^2+3(3-k)\Delta t}{36t} < c < \frac{k^2\Delta^2-18t+27t^2}{9t}$, implies $2(1-k)^2\Delta^2 + 3(3-k)\Delta t < 4(k^2\Delta^2 - 18t + 27t^2)$. This inequality, together with $k\Delta < 3t$ and $(1-k)\Delta < 3t$, implies $t > \frac{8k^2}{15k^2+k-2}$. Note that with $k = 1$, we have $t > \frac{4}{7}$, which was previously derived in Proposition 2. Thus, t and k must both be sufficiently large for Firm A to contract with Firm T.

B.4 Proof of Proposition 6 (Contracts with no Fixed Transfers)

In the main model, we allow the contract to be a two-part tariff. In this variation, we require fixed fees to be zero. Moreover, the third party's input can be purchased by Firm A at $w_T \geq 0$ per unit. All other aspects of the model remain the same as our main model.

B.4.1 Pricing

Given that the third party charges w_T for the input, the following lemma, corresponding to Lemma 1 in the manuscript, characterizes equilibrium prices.

Lemma 21. *Equilibrium goods prices are:* $p_{A,i} = t + \frac{v_{A,i}-v_{S,i}}{3} + I_i w_{S,i} + (1-I_i)\frac{2w_T}{3}$ *and* $p_{S,i} = t + \frac{v_{S,i}-v_{A,i}}{3} + I_i w_{S,i} + (1-I_i)\frac{w_T}{3}$*. Equilibrium profits are:* $\pi_{A,i} = \frac{1}{18t}(3t + v_{A,i} - v_{S,i} - (1-I_i)w_T)^2$ *and* $\pi_{S,i} = \frac{1}{18t}(3t - v_{A,i} + v_{S,i} + (1-I_i)w_T)^2 + I_i w_{S_i}$*, leading to industry profit* $\Pi_i = t + \frac{1}{9t}(v_{A,i} - v_{S,i} - (1-I_i)w_T)^2 + I_i w_{S,i}$ *for all* $w_{S,i} \in [0, \frac{1}{2}(v_{A,i} + v_{S,i} - 3t)]$*, where* $I_i = 1$ *if Firm A buys the input from its competitor in period* i *and* $I_i = 0$ *otherwise.*

Proof: The proof is the same as Lemma 1 with two differences: (1) we let $F = 0$, and (2) when Firm A contracts with the third party (i.e., $I = 0$), Firm A pays w_T (in the main model we set $w_T = 0$). Hence, the w_T appears in the price and profits. Note that Firm A's profits do not depend on w_S because a small increase of size ϵ in this per-unit input fee causes both equilibrium goods prices to increase by ϵ while also causing firm A's per-unit cost to increase by ϵ, thus leaving Firm A profits unchanged. If Firm A purchases the input from Firm S, then w_S is set to the maximum level for which the market is fully covered, and all of the additional profits from increased goods prices go to Firm S.

Lemma 22. *If Firm A purchases the input from Firm S in period* i*, equilibrium industry profit for the period* (Π_i) *is maximized by setting wholesale input price* $w_{S,i} = \frac{1}{2}(v_{A,i}+v_{S,i}-3t)$*, resulting in* $\Pi_i = \frac{1}{2}(v_{A,i} + v_{S,i}) + \frac{1}{9t}(v_{S,i} - v_{A,i})^2 - \frac{1}{2}t$.

B.4.2 Supply Contract in period 2

We now derive the equilibrium contract in the second period (corresponding to Lemmas 3, 4, and 5 in the main text).

Lemma 23. *If Firm A did not invest in innovation in period 1, then in period 2, it buys from Firm S with contract* $w_{S,2} = 1-\frac{3t}{2}$*, leading to period 2 profits* $\pi_{A,2} = \frac{t}{2}$ *and* $\pi_{S,2} = 1-t$*, and industry profit* $\Pi_2 = 1 - \frac{t}{2}$.

Proof. If Firm A buys from T, it earns $\pi_{A,2} = \frac{1}{18t}(3t - w_T)^2$, which is less than $t/2$ for any $w_T > 0$. Thus, it buys from Firm S.

Lemma 24. *If Firm A buys the input from the competitor in period 1 and invests in innovation, then in period 2 it again buys from the competitor with contract* $w_{S,2} = 1+\Delta-\frac{3t}{2}$*, leading to period 2 profits* $\pi_{A,2} = \frac{t}{2}$ *and* $\pi_{S,2} = 1-t+\Delta$*, and industry profits* $\Pi_2 = 1-\frac{t}{2}+\Delta$.

Proof. In this case, there is no reason not to continue with Firm S. Switching to Firm T not only leads to a loss of improved value but also imposes input cost w_T.

Lemma 25. *Suppose in period 1 Firm A buys the input from Firm T and invests in improving its input. In period 2, if* $w_T < \Delta$ *Firm A buys the input from the third-party, and otherwise, it buys the input from its competitor, with input contract* $w_{S,2} = 1 - \frac{3t}{2}$.

Proof. If Firm A sources from Firm T in period 1 and invests c, then Firm A has 2 options: to continue with Firm T in period 2 or to switch to Firm S.

If Firm A continues sourcing from Firm T in period 2, we have $v_{A,2} = 1 + \Delta$ and $v_{S,2} = 1$. Then, we have $\pi_{A,2} = \frac{1}{18t}(3t + \Delta - w_T)^2$ and $\pi_{S,2} = \frac{1}{18t}(3t - \Delta + w_T)^2$, leading to industry profits of $\Pi_2 = t + \frac{1}{9t}(\Delta - w_T)^2$.

However, if in period 2 Firm A switches to Firm S, we have $v_{A,2} = v_{S,2} = 1$. Firm profits in period 2 would be $\pi_{A,2} = \frac{t}{2}$ and $\pi_{S,2} = \frac{t}{2} + w_{S,2}$. Thus, Firm A contracts with Firm T if $w_T < \Delta$.

B.4.3 Focal Firm's Investment Decision

Having solved the equilibrium supply contract in period 2, we derive the focal firm's investment decision given its period 1 supplier. The following lemma summarizes the result.

Lemma 26. *If Firm A buys the input from Firm S in period 1, it does not invest in innovation. If Firm A buys the input from Firm T in period 1, it invests in innovation if and only if* $w_T < \Delta$ *and* $c < \frac{1}{18t}(\Delta - w_T)(6t + \Delta - w_T)$.

Proof. If Firm A sources from Firm S in period 1, its profit in period 2 are $\frac{t}{2}$ if it invests and also $\frac{t}{2}$ if it does not invest. Therefore, Firm A will not invest c.

If Firm A sources from Firm T in period 1, its profits in period 2 are equal to $\frac{1}{18t}(3t + \Delta - w_T)^2$ if it invests and $w_T < \Delta$, equal to $\frac{t}{2}$ if it invests and $w_T > \Delta$, and equal to $\frac{t}{2}$ if it does not invest.

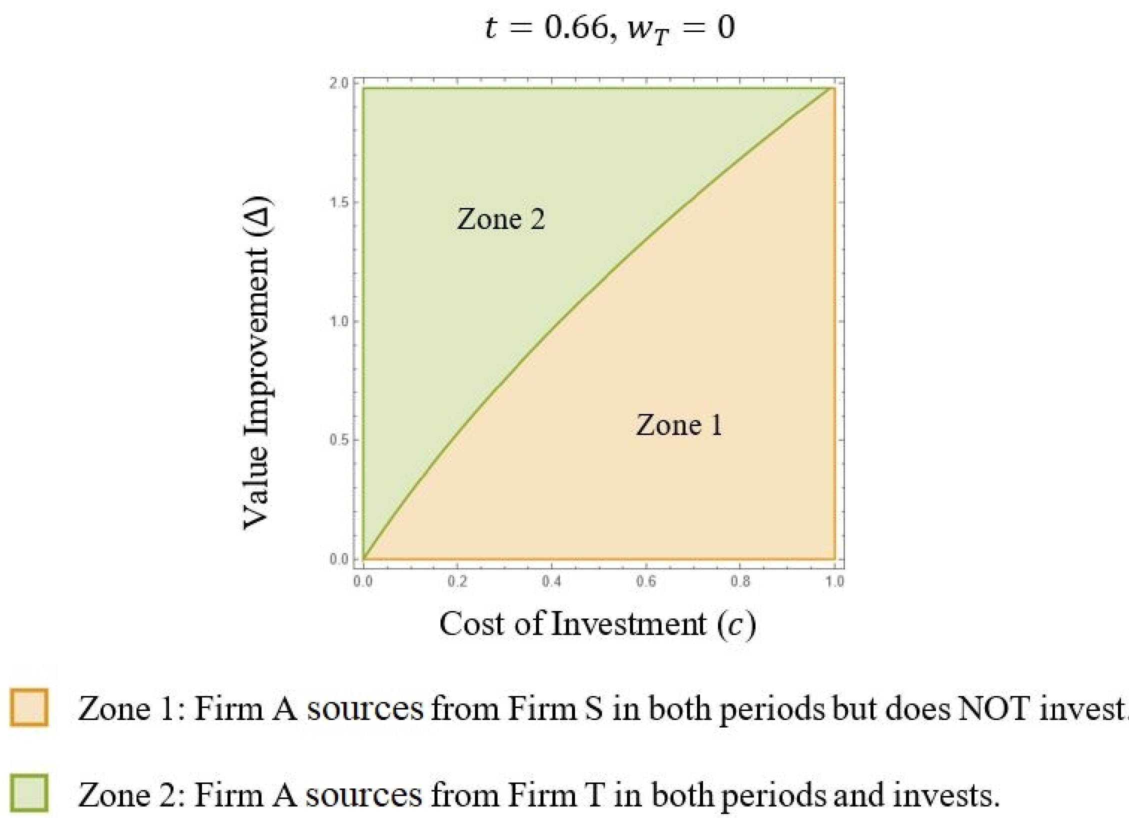
Thus, Firm A invests if and only if $w_T < \Delta$ and $c < \frac{1}{18t}(3t + \Delta - w_T)^2 - \frac{t}{2} = \frac{1}{18t}(\Delta - w_T)(6t + \Delta - w_T)$.

B.4.4 Supply Contract in period 1

In this section, we derive the equilibrium supply contracts in period 1.

Proposition 10. *Firm A sources from Firm T in both periods and invests in Firm T's input*

Figure 5: Equilibrium Regions for Firm A Supply Strategy (No Fixed Transfers).



if $w_T < \Delta$ *and* $c < \frac{1}{18t}(2w_T^2 + \Delta^2 - 2\Delta w_T + 6t\Delta - 12tw_T)$. *Otherwise, Firm A sources from Firm S in both periods without investing in the input.*

Proof. As shown in the preceding lemma, if the Period 1 supplier is Firm S, Firm A will not invest. Therefore, if Firm A selects Firm S in Period 1, its total profit across the two periods are $\Pi_A = t$. However, if it buys from Firm T in period 1, its total profit across the two periods are $\Pi_A = \frac{1}{18t}(3t - w_T)^2 - c + \frac{1}{18t}(3t + \Delta - w_T)^2$ if $\Delta > w_T$ and $c < \frac{1}{18t}(\Delta - w_T)(6t + \Delta - w_T)$ and $\Pi_A = \frac{1}{18t}(3t - w_T)^2 + \frac{t}{2}$ otherwise. The latter strategy is always dominated for any $w_T > 0$ because it is more profitable to buy from Firm S in the first period and not invest rather than to buy from Firm T and not invest. Thus, the only two possible outcomes are buying from Firm S in both periods and not investing or buying from Firm T in both periods and investing. The latter strategy occurs in equilibrium if

$t < \frac{1}{18t}(3t - w_T)^2 - c + \frac{1}{18t}(3t + \Delta - w_T)^2$, which is equivalent to $c < \frac{1}{18t}(2w_T^2 + \Delta^2 - 2\Delta w_T + 6t\Delta - 12tw_T)$. Note that letting $w_T = 0$ yields $c < \frac{1}{18t}(\Delta^2 + 6t\Delta)$, which is the condition in Proposition 6.

B.5 One-Period Models

In this section we analyze one-period models to demonstrate they cannot fully capture the dynamics and outcomes of our two-period model.

Model 1

Consider a one-period model with the following timing.

(1) Firm A decides whether to invest (and the resulting knowledge can be transferred to either supplier).

(2) Firms A and S negotiate over the supply contract.

(3) Firms A and S set product prices. Note that in Stage 1, Firm A's investment decision is not specific to a particular supplier; that is, Firm A commits to investing regardless of the identity of the supplier, which may occur because there is no history or relationship with any supplier. However, Firm A would rationally expect the equilibrium supplier choice when making the investment decision. If the supplier is going to be Firm T, the investment boosts Firm A's value only. However, if the supplier is going to be Firm S, the investment boosts both Firm A's and Firm S's product values.

As before, lemma 1 characterizes the pricing strategies. Suppose in Stage 1, Firm A decides not to invest. Then, in Stage 2, the supplier will be the competitive supplier. If the focal firm commits to invest, buying from Firm S yields an industry profit of $1 - \frac{t}{2} + \Delta$ whereas buying from Firm T yields an industry profit of $t + \frac{\Delta^2}{9t}$. The latter is always smaller. Thus, regardless of the commitment to the investment in Stage 1, the focal firm will buy from Firm S.

We now calculate the contract fixed payment F_S. The fixed payment is derived from

$\pi_A - \pi_A^O = \pi_S - \pi_S^O$. Inserting $\pi_A^O = \frac{1}{18t}(3t+\Delta)^2$ and $\pi_S^O = \frac{1}{18t}(3t-\Delta)^2$ into the above equation yields $F_S = -\frac{1}{2}w_S - \frac{2}{3}\Delta = -\frac{1}{2}(\Delta + 1 - \frac{3t}{2}) - \frac{2}{3}\Delta = -\frac{1}{2} + \frac{3t}{4} - \frac{7}{6}\Delta$. Inserting F_S back in profits, we get: $\pi_{A,2} = \frac{1}{2} - \frac{t}{4} + \frac{7}{6}\Delta$ and $\pi_{S,2} = \frac{1}{2} - \frac{t}{4} - \frac{1}{6}\Delta$.

Hence, in Stage 1, the focal firm invests if and only if $c < (\frac{1}{2} - \frac{t}{4} + \frac{7}{6}\Delta) - \frac{1}{2}(1 - \frac{t}{2}) = \frac{7}{6}\Delta$. To summarize, in Model 1, the focal firm will always buy from Firm S, and invests if $c < \frac{7}{6}\Delta$.

Model 2

Consider the following timing and strategies:

(1) Firm A decides whether to invest in the input of Firm S, or invest in the input of Firm T, or invest in neither input.

(2) Firms A and S negotiate over the supply contract.

(3) Firms A and S set product prices.

Please note that the Model 2 timing is similar to the Model 1 timing with one difference: Firm A commits to investing in a specific supplier in advance. The solution to Stage 2 is similar to our main model. In particular, Lemmas 3, 4, and 5 characterize the negotiation outcome in Stage 2.

In Stage 1, Firm A compares its profit across three different strategies it has. If it does not invest, its profit is $\frac{1}{2} - \frac{t}{4}$. If it invests in Firm S, its profit is $\frac{1}{2} - \frac{t}{4} + \frac{\Delta}{6} - c$. Finally, if it invests in Firm T:

-If $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$, it contracts with the third party, resulting in profit $\frac{t}{2} + \frac{\Delta}{3} + \frac{\Delta^2}{18t} - c$.

-Otherwise, it buys from its competitor, which results in profits $\frac{1}{2} - \frac{t}{4} + \frac{\Delta}{3} - c$.

Therefore, investing in Firm S in Stage 1 is always dominated: if $c > \frac{\Delta}{6}$, it is better not to invest. If $c < \frac{\Delta}{6}$ and $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$, it is better to invest in Firm T and buy from it because $c < \frac{1}{2}(\frac{\Delta^2}{9t} - 1 + \frac{3t}{2}) + \frac{\Delta}{6}$. If $c < \frac{\Delta}{6}$ and $\frac{\Delta^2}{9t} < 1 - \frac{3t}{2}$, it is better to invest in Firm T but then buy from the competitor to get a profit increase of $\frac{\Delta}{3}$ instead of $\frac{\Delta}{6}$.

To summarize, if $\frac{\Delta^2}{9t} > 1 - \frac{3t}{2}$ and $c < \frac{1}{2}(\frac{\Delta^2}{9t} - 1 + \frac{3t}{2}) + \frac{\Delta}{3}$ then Firm A invests in Firm T and buys from it. If $\frac{\Delta^2}{9t} < 1 - \frac{3t}{2}$ and $c < \frac{\Delta}{3}$, then Firm A invests in Firm T but buys from Firm S. Otherwise, Firm A does not invest and buys from the competitor.

B.6 Commitment not to Renegotiate

In this section, we allow Firm A and Firm S to agree on an input contract for both periods, and they commit not to renegotiate the supply contract. In particular, the two parties engage in negotiation only once, at the beginning of period 1. If they reach an agreement, they honor the contract and follow it in periods 1 and 2. If they fail to reach an agreement, the do not negotiate again in period 2, and the focal firm buys from the third party in both periods. Moreover, if the firms reach an agreement, they can commit to wholesale prices for both periods. In other words, at the beginning of period 1, they set period-1 wholesale price w_1 and period-2 wholesale price w_2 to maximize the industry profits across the two periods. Alternatively, we could restrict them to set a single wholesale price for both periods. The analysis of this alternative model, while more complicated, leads to qualitatively similar results.

Suppose Firm A and Firm S fail to reach an agreement in period 1. Firm A then buys again from Firm T in both periods regardless of its investment decision. If the focal firm invests in Firm T, its second-period profits will be $\pi_A = \frac{1}{18t}(3t+\Delta)^2$ (see Lemma 1 in the manuscript). Otherwise, its second-period profits will be $\frac{t}{2}$. Thus, Firm A will invest in Firm T if and only if $c < \frac{1}{18t}(3t+\Delta)^2 - \frac{t}{2} = \frac{\Delta}{18t}(\Delta + 6t)$.

Suppose Firm A and Firm S reach an agreement in period 1, with contract terms w_1, w_2 and F. Note that firms commit to the fixed fees, which implies distinguishing between F_1 and F_2 is unnecessary. The firms are then committed to honor this contract in period 2 and not renegotiate it. Thus, in period 2, Firm A buys from Firm S, paying w_2 per unit. Note that because the period-1 supplier is Firm S, the period-2 valuations for the product are equal. That is, both valuations are $1+\Delta$ with investment and 1 without investment. We derive equilibrium goods prices and profits when Firm A buys input from Firm S at wholesale price w, where we drop the period index for w and the fixed fee F for parsimony.

As in our main model (Lemma 1), if w is low ($w < v - \frac{3t}{2}$), firms price at $p = t + w$, split the market equally, and Firm A and Firm S profits are $\frac{t}{2}$ and $\frac{t}{2} + w$, respectively, leading to industry profit of $t + w$.

Second, at a higher wholesale price ($v - \frac{3t}{2} < w < v - t$), in equilibrium both firms charge price $p = v - \frac{t}{2}$, with each selling to half of the market. In this parameter range, equilibrium prices are independent of w. We then have $\pi_A = \frac{1}{2}(v - \frac{t}{2} - w)$, $\pi_S = \frac{1}{2}(v - \frac{t}{2} + w)$ and industry profits are $(v - \frac{t}{2})$. In this case, industry profits do not depend on wholesale price, while Firm A profits decrease and Firm S profits increase with the wholesale price.

Finally, when wholesale price increases further to $w > v - t$, Firm A has an incentive to deviate from the above equilibrium and charge its local monopoly price, $p_A = \frac{v+w}{2}$ and sells to $q_A = \frac{v-w}{2t}$ customers. Firm S will then set its price to fully cover the market, that is, $v - p_S - t(1 - q_A) = 0$, or, $p_S = \frac{1}{2}(3v - 2t - w)$. In this case, profits are $\pi_A = \frac{(v-w)^2}{4t}$, $\pi_S = 2v - t - w - \frac{3(v-w)^2}{4t}$, leading to industry profits $\Pi = 2v - t - w - \frac{(v-w)^2}{2t}$. This completes the characterization of prices and profits for any $w < v$. The following lemma summarizes these results.

Lemma 27. *Suppose Firm A buys an input from Firm S at wholesale price w.*

(i) If $w < v - \frac{3t}{2}$ then prices are $p_A = p_S = t + w$ and profits are $\pi_A = \frac{t}{2}$ and $\pi_S = \frac{t}{2} + w$.

(ii) If $v - \frac{3t}{2} < w < v - t$ then prices are $p_A = p_S = v - \frac{t}{2}$ and profits are $\pi_A = \frac{1}{2}(v - \frac{t}{2} - w)$ and $\pi_S = \frac{1}{2}(v - \frac{t}{2} + w)$.

(iii) If $w > v - t$, then prices are $p_A = \frac{v+w}{2}$ and $p_S = \frac{1}{2}(3v - 2t - w)$, and profit are $\pi_A = \frac{(v-w)^2}{4t}$ and $\pi_S = 2v - t - w - \frac{3(v-w)^2}{4t}$.

We now consider Firm A's investment decision. This decision depends on the wholesale price w_2 that Firm A will face in period 2. If the second-period wholesale price w_2 is sufficiently low ($w_2 < 1 - \frac{3t}{2} < 1 + \Delta - \frac{3t}{2}$), then Firm A's profits with or without investment are $\frac{t}{2}$. Thus, Firm A will not invest.

Consider now a higher wholesale price $1 - \frac{3t}{2} < w_2 < 1 + \Delta - \frac{3t}{2}$. As we showed above, Firm A's profits are lower than $\frac{t}{2}$ if it does not invest. In particular, without investment, Firm A's profits are $\frac{1}{2}(1 - \frac{t}{2} - w_2)$ if $w_2 < 1 - t$ and $\frac{(1-w_2)^2}{4t}$ if $w_2 > 1 - t$ (see the Lemma above). Firm A invests if its incremental benefit of investment, which is $\frac{t}{2}-\frac{1}{2}(1-\frac{t}{2}-w_2)$ if $w_2 < 1-t$ and $\frac{t}{2}-\frac{(1-w_2)^2}{4t}$ if $w_2 > 1-t$, is higher than c. Note that these incremental benefits become larger as w_2 becomes larger until it reaches $w_2 = 1+\Delta-\frac{3t}{2}$. Raising wholesale prices above this level cannot increase Firm A's incentives to invest because it will start to decrease Firm A's profit with investment.

Consequently, the second-period wholesale price w_2 should be set at $w_2 = 1+\Delta-\frac{3t}{2}$ whenever this price leads Firm A to invest; otherwise, $w_2 = 1-\frac{3t}{2}$. In particular, with $w_2 = 1+\Delta-\frac{3t}{2}$, the benefit of investment for Firm A is $\frac{t}{2}-\frac{1}{2}(1-\frac{t}{2}-1-\Delta+\frac{3t}{2})$ if $\Delta < \frac{t}{2}$, $\frac{t}{2}-\frac{(1-1-\Delta+\frac{3t}{2})^2}{4t}$ if $\frac{t}{2} < \Delta < \frac{3t}{2}$, and $\frac{t}{2}$ if $\Delta > \frac{3t}{2}$. In the last case, Firm A loses the entire market to Firm S if it does not invest. Thus, the wholesale price w_2 should be set at $w_2 = 1+\Delta-\frac{3t}{2}$ if $c < \frac{\Delta}{2}$ and $\Delta < \frac{t}{2}$, or if $c < \frac{8t^2-(3t-2\Delta)^2}{16t}$ and $\frac{t}{2} < \Delta < \frac{3t}{2}$, or if $c < \frac{t}{2}$ and $\Delta > \frac{3t}{2}$. Otherwise, the wholesale price w_2 should be set at $w_2 = 1-\frac{3t}{2}$ and investment will not occur. We proved the following lemma.

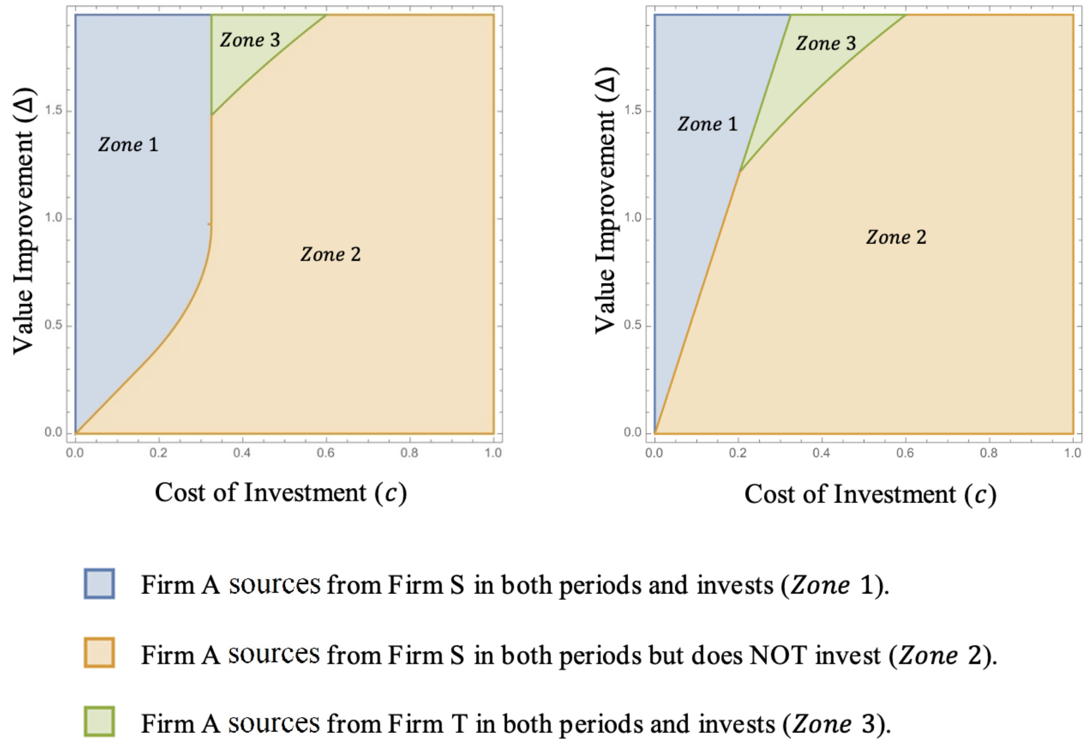
Lemma 28. *If Firm A and Firm S agree on a supply contract in Period 1 and commit not to renegotiate it, they set $w_1 = 1-\frac{3t}{2}$, and a higher period-2 wholesale price $w_2 = 1+\Delta-\frac{3t}{2}$ only if $\Delta < \frac{t}{2}$ and $c < \frac{\Delta}{2}$, or if $\frac{t}{2} < \Delta < \frac{3t}{2}$ and $c < \frac{8t^2-(3t-2\Delta)^2}{16t}$, or if $\Delta > \frac{3t}{2}$ and $c < \frac{t}{2}$.*

We are now ready to derive the equilibrium of the game with commitment not to renegotiate. If Firm A buys from Firm T, industry profits across two periods are $2t+\frac{\Delta^2}{9t}-c$ if $c < \frac{\Delta}{18t}(\Delta+6t)$ and $2t$, otherwise. On the other hand, if Firm A buys from Firm S, industry profits across the two periods are $(1-\frac{t}{2})+(1+\Delta-\frac{t}{2}-c)$ if $\Delta < \frac{t}{2}$ and $c < \frac{\Delta}{2}$, or if $\frac{t}{2} < \Delta < \frac{3t}{2}$ and $c < \frac{8t^2-(3t-2\Delta)^2}{16t}$, or if $\Delta > \frac{3t}{2}$ and $c < \frac{t}{2}$. Otherwise, the investment will not occur, and industry profits across two periods are $(1-\frac{t}{2})+(1-\frac{t}{2})$.

Comparing industry profits across these two sub-games (when Firm A buys from Firm S or Firm T) leads to the following proposition.

Proposition 11. *Firm A buys from Firm T and invests if $\frac{t}{2} < c < \frac{\Delta^2-18t+27t^2}{9t}$. Firm A buys from Firm S and invests if $\Delta < \frac{t}{2}$ and $c < \frac{\Delta}{2}$, or $\frac{t}{2} < \Delta < \frac{3t}{2}$ and $c < \frac{8t^2-(3t-2\Delta)^2}{16t}$, or if $\Delta > \frac{3t}{2}$ and $c < \frac{t}{2}$. Otherwise, Firm A buys from Firm S and does not invest.*

Figure 6: Equilibrium Regions for Firm A Supply Strategy ($t = 0.65$). Left panel: with commitment not to renegotiate. Right panel: Base model.



The above figure illustrates the proposition (outcomes of the game with a commitment not to renegotiate) side-by-side with those of our main model. We restrict our attention to the more interesting case of $\Delta < 3t$. Even when firms can commit not to renegotiate the contract in period 2, Firm A still may buy the input from the third party when Δ is large and c is in an intermediate range. Even though the firms commit to a supply contact, Firm A cannot commit to investment. Competition between Firm A and Firm S in the product market can lead to a relatively low benefit of investment for Firm A, which leads to the firm buying from the third party as a possible equilibrium outcome. Nonetheless, committing not to renegotiate expands the region of parameters in which Firm A buys from Firm S and invests while shrinking the region in which Firm A buys from the third party. Thus, although the commitment can improve channel outcomes and decrease the under-investment and hold-up issues, it does not fully eliminate them.

B.7 Uncertain Innovation Outcome

Suppose in our main model, if Firm A invests in its supplier, the product value increases by Δ only with probability q. If Firm A decides to invest, the value of the product (whether it is increased to v_H or not) is revealed after the investment occurs but before Period 2 negotiations. Our main model is obtained by letting $q = 1$.

Lemmas 1-2 do not change. In Lemmas 3-5, the statement “if Firm A invests,...” changes to “if Firm A invests and it is successful,...”. In Lemma 6 (Investment decision), we replace c by $\frac{c}{q}$ because the investment occurs if the expected benefit of investments outweighs the cost, c. The benefit of a successful investment is the same as in the main model, whereas the benefit of an unsuccessful investment is zero.

Period 1 Equilibrium

There are three cases. If $\frac{c}{q} < \frac{\Delta}{6}$, similar to the main model, Firm A contracts with Firm S in both Periods and invests (which may or may not succeed).

On the other hand, if $\frac{c}{q} > max\{\frac{2\Delta^2-18t+27t^2}{36t}, 0\} + \frac{\Delta}{3}$, the investment would not occur (regardless of the Period 1 supplier). Thus, the Period 1 supplier is Firm S for the price softening benefit.

Finally, consider $\frac{\Delta}{6} < \frac{c}{q} < max\{\frac{2\Delta^2-18t+27t^2}{36t}, 0\} + \frac{\Delta}{3}$. Per Lemma 6, the investment decision depends on who the Period 1 supplier is. In particular,

- If the Period 1 supplier is Firm S, the investment does not happen, and industry profits across the two periods are $\Pi = 2 - t$
- If the Period 1 supplier is Firm T, the investment does occur, and the expected industry profit across the two periods is $E\Pi = t - c + q(t + \frac{1}{9t}\Delta^2) + (1-q)(1-\frac{t}{2})$

The first term is larger (Firm contracts Firm S) if and only if $c > 2t - 2 + q(t + \frac{1}{9t}\Delta^2) + (1-q)(1-\frac{t}{2})$. Otherwise, the Firm contracts with Firm T and invests. It is possible Firm

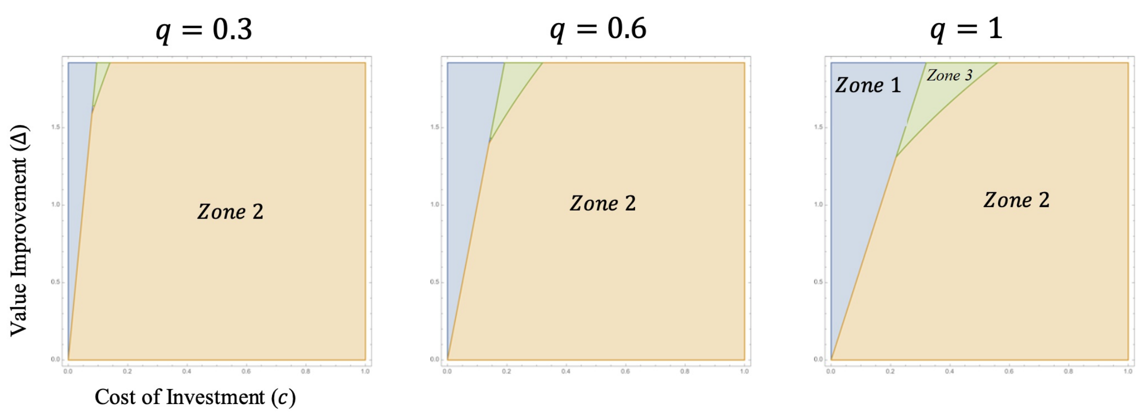
A contacts with Firm T and invests, but the investment fails. In this situation, Firm A switches to buying from Firm S in period 2.

To summarize:

- Firm A buys from Firm S in both periods and invests if $\frac{c}{q} < \frac{\Delta}{6}$
- Firm A buys from Firm S in both periods without investing if $\frac{c}{q} > max\{\frac{2\Delta^2-18t+27t^2}{36t}, 0\} + \frac{\Delta}{3}$, or $\frac{\Delta}{6} < \frac{c}{q} < max\{\frac{2\Delta^2-18t+27t^2}{36t}, 0\} + \frac{\Delta}{3}$ and $c > 2t - 2 + q(t + \frac{1}{9t}\Delta^2) + (1-q)(1-\frac{t}{2})$.
- Firm A buys from Firm T in Period one and invests if $\frac{\Delta}{6} < \frac{c}{q} < max\{\frac{2\Delta^2-18t+27t^2}{36t}, 0\} + \frac{\Delta}{3}$ and $c < 2t - 2 + q(t + \frac{1}{9t}\Delta^2) + (1-q)(1-\frac{t}{2})$.

As the probability of successful investment (q) decreases from one, Firm A's incentive to work with the third party shrinks because working with Firm T is beneficial only if it is followed by a successful investment in innovation.

Figure 7: Equilibrium Regions with Uncertain Innovation Outcome



Firm A sources from Firm S in both periods and invests (*Zone* 1).

Firm A sources from Firm S in both periods but does NOT invest (*Zone* 2).

Firm A sources from Firm T in period one and invests (*Zone* 3).