Technical Appendix

In our Technical Appendix, to simplify notations, we denote the L-type firm's product failure, profit, and price as functions of its advertising strategy $\lambda \in [0,1]$, which represents the probability the L-type firm truthfully reports m_L. In particular, $\lambda = 0$ indicates a pooling equilibrium, while $\lambda = 1$ implies a separating equilibrium. Hence, we drop "m_H" and replace $G(\lambda, m_H)$, $\pi_i^j(\lambda, m_H)$ and $p^j(\lambda, m_H)$ by $G(\lambda)$, $\pi_i^j(\lambda)$ and $p^j(\lambda)$, respectively for the liability rule $j \in \{S, C\}$ and firm type $i \in \{L, H\}$, where S indicates the strict liability and C indicates the comparative negligence rule.

Appendix A

Proof of Lemma 1

Proof. In the first benchmark model where the product quality is publicly observable and precaution effort is endogenized, under the strict liability rule, the consumer's optimal precaution effort $\widetilde{e}_i^S$ is the solution of

$$\frac{\partial F_i(e)}{\partial e}(l - s) + c'(e) = 0;$$

and under the comparative negligence rule, the consumer's optimal precaution effort $\widetilde{e}_i^C$ is the solution of

$$\frac{\partial F_i(e)}{\partial e} l + c'(e) = 0.$$

Given that $\frac{\partial^2 F_i(e)}{\partial^2 e} = 0$, it is obvious that $\frac{\partial \widetilde{e}_i^S}{\partial s} < 0$ and $\frac{\partial \widetilde{e}_i^C}{\partial s} = 0$. Since $\widetilde{e}_i^S = \widetilde{e}_i^C$ when $s = 0$, it must be that $\widetilde{e}_i^C > \widetilde{e}_i^S$ for any positive s.

The type i firm's profit under both product liability rules shares the same function form:

$$\widetilde{\pi}_i = v - F_i(\widetilde{e}_i)l - c(\widetilde{e}_i).$$

It is obvious that the comparative negligence rule yields the first best consumer effort by the first order condition. As the consumer's optimal precaution effort is always lower under the strict liability rule, then $\widetilde{\pi}_i^C > \widetilde{\pi}_i^S$. Thus, it must be that firm's expected profit is higher under the comparative negligence rule, i.e., $\widetilde{\Pi}^C > \widetilde{\Pi}^S$.

Finally, we discuss the impact of s on firm's expected profit. Under the comparative negligence rule, since precaution effort is invariant with s, the expected profit would not be affected by s as well. Under the strict liability rule, by the first order condition of consumer optimization,

$$\frac{\partial \tilde{\pi}_i^S}{\partial s} = -\frac{\partial F_i(\tilde{e}_i^S)}{\partial \tilde{e}_i^S}\frac{\partial \tilde{e}_i^S}{\partial s}s < 0.$$

Therefore, the increasing penalty would hurt the firm under the strict liability rule. □

Proof of Lemma 2

Proof. In the proof of this lemma, we keep precaution effort e in $G(\cdot)$, as e can take two possible exogenous values e_b and $\bar{e}$.

In the second benchmark model where the consumer's precaution effort is fixed at $\bar{e} \leq e_b$, under the comparative negligence rule, given the quality message m_H, consumer's expected utility is $U^C(m_H, \bar{e}) = v - G(\lambda, \bar{e})\left(l - \frac{G(\lambda, e_b)}{G(\lambda, \bar{e})}s\right) - p - c(\bar{e})$, where $G(\lambda, \bar{e}) = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha+(1-\alpha)}F_L(\bar{e}) + \frac{(1-\alpha)}{(1-\lambda)\alpha+(1-\alpha)}F_H(\bar{e}), G(\lambda, e_b) = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha+(1-\alpha)}F_L(e_b) + \frac{(1-\alpha)}{(1-\lambda)\alpha+(1-\alpha)}F_H(e_b)$. As the consumer purchases the product if and only if $U^C(m_H, \bar{e}) \geq 0$, then the optimal retail price charged by the firm is $\overline{p}^C(\lambda) = v - G(\lambda, \bar{e})l + G(\lambda, e_b)s - c(\bar{e})$, and the optimal profit achieved by the L-type firm is $\overline{\pi}_L^C(\lambda) = \overline{p}^C(\lambda) - F_L(e_b)s$. If instead, the L-type firm reports m_L, its profit is $\overline{\pi}_L^C(m_L) = \overline{p}^C(m_L) - F_L(e_b)s$, where $\overline{p}^C(m_L) = v - F_L(\bar{e})l + F_L(e_b)s - c(\bar{e})$.

$$\begin{aligned}
\overline{\pi}_L^C(\lambda) - \overline{\pi}_L^C(m_L) &= \overline{p}^C(\lambda) - \overline{p}^C(m_L) \\
&= [F_L(\bar{e}) - G(\lambda, \bar{e})]l - [F_L(e_b) - G(\lambda, e_b)]s \\
&> [F_L(e_b) - G(\lambda, e_b)](l - s) \\
&\geq 0.
\end{aligned}$$

Thus, in equilibrium, the L-type firm would always send quality message m_H (i.e., $\lambda = 0$), and the firm's expected profit is

$$\overline{\Pi}^C = \alpha\overline{\pi}_L^C(0) + (1-\alpha)\overline{\pi}_H^C(0) = v - G(0, \bar{e})\,l - c(\bar{e}).$$

Under the strict liability rule, it can be proved that the L-type firm would prefer to report m_H in a similar fashion. As the profit difference is

$$\overline{\pi}_L^S(\lambda) - \overline{\pi}_L^S(m_L) = [F_L(\bar{e}) - G(\lambda, \bar{e})](l - s) \geq 0.$$

The corresponding expected profit is

$$\overline{\Pi}^S = \alpha\overline{\pi}_L^S(0) + (1-\alpha)\overline{\pi}_H^S(0) = v - G(0, \bar{e})\,l - c(\bar{e}).$$

Obviously, $\overline{\Pi}^S = \overline{\Pi}^C$. In addition, as s has no impact on firm's advertising strategy or consumer's precaution effort, it would not affect the firm's expected profit. □

Proof of Proposition 1

Proof. As the proof of this proposition is lengthy, we provide an outline of the proof, which contains three main steps.

First, we prove that if k is sufficiently large, the separating (pooling) equilibrium holds when the penalty s is relatively large (small). The separating equilibrium is sustained when the L-type firm has no incentive to report m_H even if consumers have the most optimistic belief, i.e, $\pi_L^S(m_L) > \pi_L^S(1)$, while the pooling equilibrium holds when $\pi_L^S(0) - \pi_L^S(m_L) > 0$. Then, intuitively, our argument is valid when two inequalities hold

$$\frac{\partial\left[\pi_L^S(1) - \pi_L^S(0)\right]}{\partial s} < 0 \quad \text{and} \quad \frac{\partial\left[\pi_L^S(0) - \pi_L^S(m_L)\right]}{\partial s} < 0. \tag{1}$$

Second, we show that a unique hybrid equilibrium is supported for intermediate s. The key to establishing this argument is that the L-type's profit is a U-shape (convex) function of λ, so that there exists a unique λ that makes the L-type firm is indifferent between truth-telling and lying, i.e., $\pi_L^S(m_L) = \pi_L^S(\lambda)$.

Finally, we show that the H-type never lies. In this version, we provide more explanations, intermediary steps and intuitions during the proof process.

Let's start with separating and pooling equilibrium, we try to prove that (1) hold if k is sufficiently large. These two inequalities indicate that separating (pooling) equilibrium is more likely to be sustained when s is sufficiently large (small). From (11) in the main text,

$$\pi_L^S(\lambda) = v - G(\lambda)(l-s) - c(e^S) - F_L(e^S)s,$$

$$\frac{\partial \pi_L^S(\lambda)}{\partial s} = \left[-\frac{\partial G(\lambda)}{\partial e^S}(l-s) - c'(e^S)\right]\frac{\partial e^S}{\partial s} + G(\lambda) - F_L(e^S) - \frac{\partial F_L(e^S)}{\partial e^S}\frac{\partial e^S}{\partial s}s.$$

From the FOC of consumer's optimal effort that determines $e^S \equiv e^S(\lambda)$,

$$-\frac{\partial G(\lambda)}{\partial e^S}(l-s) - c'(e^S) = 0. \tag{2}$$

Hence, we have

$$\frac{\partial \pi_L^S(\lambda)}{\partial s} = \underbrace{G(\lambda) - F_L(e^S)}_{direct\ effect} - \underbrace{\frac{\partial F_L(e^S)}{\partial e^S}\frac{\partial e^S}{\partial s}s}_{indirect\ effect},$$

thus,

$$\frac{\partial\left[\pi_L^S(1)-\pi_L^S(0)\right]}{\partial s} = G(1)-G(0)-\left[F_L(e^S(1))-F_L(e^S(0))\right] - \left[\frac{\partial F_L(e^S(1))}{\partial e^S}\frac{\partial e^S(1)}{\partial s}-\frac{\partial F_L(e^S(0))}{\partial e^S}\frac{\partial e^S(0)}{\partial s}\right]s.$$

As penalty s increases, there are two effects on the L-type firm's profit. On the one hand, the firm has to pay higher compensation and its profit decreases, which is defined as the direct effect. On the other hand, consumer's precaution effort decreases with s, which then increases failure rate. First, we show that the difference of direct effect is negative. From (3) in the main text,

$$\begin{aligned} & G(1)-G(0)-\left[F_L(e^S(1))-F_L(e^S(0))\right] \\ =& \underbrace{F_H(e^S(1))-F_L(e^S(1))}_{<0}+\underbrace{(1-\alpha)\left[F_L(e^S(0))-F_H(e^S(0))\right]}_{>0} \\ <& F_H(e^S(1))-F_L(e^S(1))+\left[F_L(e^S(0))-F_H(e^S(0))\right] \\ =& \int_{e^S(0)}^{e^S(1)}\left[\frac{\partial F_H(e)}{\partial e}-\frac{\partial F_L(e)}{\partial e}\right]de<0. \end{aligned}$$

The last inequality holds as $e^S(1)<e^S(0)$ and $\frac{\partial F_H(e)}{\partial e}>\frac{\partial F_L(e)}{\partial e}$. In terms of the difference between indirect effect, from (2), it can be derived that

$$\frac{\partial e^S}{\partial s}=\frac{1}{k}\frac{\partial G(\lambda)}{\partial e^S},$$

which implicates the indirect effect would be small if k is sufficiently large. In that case, the direct effect dominates and we have

$$\frac{\partial\left[\pi_L^S(1)-\pi_L^S(0)\right]}{\partial s}<0.$$

In a similar fashion, it is true that

$$\frac{\partial\left[\pi_L^S(0)-\pi_L^S(m_L)\right]}{\partial s}<0.$$

As a result,

$$\frac{\partial\left[\pi_L^S(1)-\pi_L^S(m_L)\right]}{\partial s}<0.$$

From Equation (11) in the main text, it is clear that $\pi_L^S(m_L)>\pi_L^S(1)$ if s is sufficiently close to l, as the penalty is too strong that dominates the gain from cheating. Combining with the fact that $\pi_L^S(1)-\pi_L^S(m_L)$ monotonically decreases with s, there must be a unique

$\bar{s}$, such that $\pi_L^S(m_L) > \pi_L^S(1)$ if $s \geq \bar{s}$ and a separating equilibrium can be supported. Similarly, (10) implies that $\pi_L^S(0) - \pi_L^S(m_L) > 0$ if s is close to 0. Thus, a pooling equilibrium can be sustained when $s \leq \underline{s}$.

Then, we turn to the existence and uniqueness of hybrid equilibrium. There are two parts that need to be proved. (i) $\bar{s} \geq \underline{s}$; (ii) for $s \in (\underline{s}, \bar{s})$, there is a unique λ^* that makes $\pi_L^S(m_L) = \pi_L^S(\lambda^*)$, which supports a hybrid equilibrium. Both parts can be proved by showing that $\frac{\partial^2 \pi_L^S(\lambda)}{\partial \lambda^2} > 0$. Since $\frac{\partial^2 F(q,e)}{\partial e^2} = \frac{\partial^2 F(q,e)}{\partial q^2} = 0$,

$$\frac{\partial^2 \pi_L^S(\lambda)}{\partial \lambda^2} = -\left[\frac{\partial^2 G(\lambda)}{\partial \lambda \partial e^S}\frac{\partial e^S}{\partial \lambda}\right](l-s) - \frac{\partial F_L(e^S)}{\partial e^S}\frac{\partial^2 e^S}{\partial \lambda^2}s.$$

$\frac{\partial^2 F(q,e)}{\partial q^2} = 0$ implies that $\frac{\partial^2 e^S}{\partial \lambda^2} = 0$. In addition, $\frac{\partial^2 G(\lambda)}{\partial \lambda \partial e^S} > 0$ and $\frac{\partial e^S}{\partial \lambda} < 0$. Therefore,

$$\frac{\partial^2 \pi_L^S(\lambda)}{\partial \lambda^2} > 0.$$

This condition indicates that $\pi_L^S(\lambda)$ is a convex function of λ. The maximum value of $\pi_L^S(\lambda)$ must be reached at two end points, either $\pi_L^S(m_L)$ or $\pi_L^S(1)$. For (i), if instead $\underline{s} > \bar{s}$. For $s \in [\bar{s}, \underline{s}]$, by our argument above, it must be that $\pi_L^S(0) > \max\{\pi_L^S(m_L), \pi_L^S(1)\}$, which yields a contradiction. Then, if $s \in (\underline{s}, \bar{s})$, we have $\pi_L^S(0) < \pi_L^S(m_L) < \pi_L^S(1)$. Given that $\pi_L^S(\lambda)$ is U-shape function, there is a unique λ^* that makes $\pi_L^S(m_L) = \pi_L^S(\lambda^*)$.

Finally, we show that the H-type has no incentive to cheat under the strict liability rule. The proof of this part contains two steps.

i) We show that the H-type firm has a higher incentive to report m_H than the L-type. In a hybrid or a pooling equilibrium that the L-type firm sends m_L with probability λ^*,

$$\pi_L^S(\lambda^*) - \pi_L^S(m_L) = p^S(\lambda^*) - p^S(m_L) + \left[F_L(e^S(m_L)) - F_L(e^S(\lambda^*))\right]s \geq 0.$$

For the H-type firm,

$$\pi_H^S(\lambda^*) - \pi_H^S(m_L) = p^S(\lambda^*) - p^S(m_L) + \left[F_H(e^S(m_L)) - F_H(e^S(\lambda^*))\right]s.$$

Thus,

$$\begin{aligned}
&\left[\pi_H^S(\lambda^*) - \pi_H^S(m_L)\right] - \left[\pi_L^S(\lambda^*) - \pi_L^S(m_L)\right] \\
=&\left[F_H(e^S(m_L)) - F_H(e^S(\lambda^*))\right]s - \left[F_L(e^S(m_L)) - F_L(e^S(\lambda^*))\right]s \\
=&s\int_{e^S(\lambda^*)}^{e^S(m_L)}\left[\frac{\partial F_H(e)}{\partial e} - \frac{\partial F_L(e)}{\partial e}\right]de > 0.
\end{aligned}$$

The last inequality follows the assumption that product quality and precaution effort are strategic substitutes. Given this inequality, the H-type firm must report m_H in hybrid or pooling equilibrium in which the L-type firm has an incentive to report m_H.

ii) We pin down the condition that the H-type firm is truth-telling in a separate equilibrium.

$$\pi_H^S(1) - \pi_H^S(m_L) = \underbrace{p^S(1) - p^S(m_L)}_{>0} + \underbrace{\left[F_H(e^S(m_L)) - F_H(e^S(1))\right] s}_{<0}.$$

As we mentioned earlier, the equilibrium price increases with λ, i.e., $p^S(1) - p^S(m_L) > 0$. Therefore, if the absolute value of the second part on the RHS is not too large, we would have $\pi_H^S(1) - \pi_H^S(m_L) > 0$. Note that when q_H is sufficiently close to 1, or k is sufficiently large such that the difference between $e^S(m_L)$ and $e^S(1)$ is small, the desired result is obtained.

Under the comparative negligence rule, only pooling equilibrium is sustained, and the H-type firm has no incentive to report m_L as well. It can be easily shown that the H-type firm has a higher incentive to report m_H than the L-type firm, as the H-type firm pays a lower penalty ex-post.

$$\left[\pi_H^C(\lambda^*) - \pi_H^C(m_L)\right] - \left[\pi_L^C(\lambda^*) - \pi_L^C(m_L)\right] = [F_L(e_b) - F_H(e_b)]\, s \geq 0.$$

In sum, the H-type would have no incentive to cheat under both liability rules. □

Proof of Proposition 2

Proof. First, we define $\phi \equiv \beta^2 (1 - q_L)\, l$ to simplify notations in the following discussion. To avoid the trivial discussion that hybrid equilibrium never happens, we require that $\varepsilon > \frac{2(1-q_L)(\phi-1)}{1-\alpha}$ and $1 < \phi < \frac{3-\alpha}{2}$.

Based on Proposition 1, we find that the hybrid equilibrium arises under the strict liability rule when $s \in (\underline{s}, \bar{s})$, the L-type firm sends message m_L with probability λ^* and sends message m_H with probability $1 - \lambda^*$, where

$$\begin{aligned}
\underline{s} &= l - \frac{2(\phi - 1)}{(1-\alpha)\beta^2\varepsilon}, \\
\bar{s} &= l - \frac{2(\phi - 1)}{\beta^2\varepsilon}, \\
\lambda^* &= \frac{1}{\alpha} - \frac{(1-\alpha)\beta^2\varepsilon(l - s)}{2\alpha(\phi - 1)}.
\end{aligned}$$

Under the comparative negligence rule, firm's expected profit in the pooling equilibrium is

$$\Pi^C = v - [1 - q_L - (1-\alpha)\varepsilon]\, l + \frac{1}{2}\beta^2 [1 - q_L - (1-\alpha)\varepsilon]^2 l^2. \tag{3}$$

It is clear that firm's expected profit is invariant with s under the comparative negligence rule.

Under the strict liability rule, in the pooling equilibrium, i.e., $s \in (0, \underline{s}]$, firm's expected profit is

$$\Pi^S(s) = v - [1 - q_L - (1-\alpha)\varepsilon]\, l + \frac{1}{2}\beta^2 [1 - q_L - (1-\alpha)\varepsilon]^2 (l^2 - s^2). \tag{4}$$

In the separating equilibrium, i.e., $s \in [\overline{s}, l)$, firm's expected profit is

$$\begin{aligned} \Pi^S(s) =& v - [1 - q_L - (1-\alpha)\varepsilon]\, l \\ &+ \frac{1}{2}\beta^2 \left[(1-\alpha)(1 - q_L - \varepsilon)^2 + \alpha (1 - q_L)^2\right] (l^2 - s^2). \end{aligned} \tag{5}$$

Therefore, firm's expected profit monotonically decreases with s in the pooling and separating equilibrium under the strict liability rule. In the hybrid equilibrium, firm's expected profit is

$$\begin{aligned} \Pi^S(s) =& v - [1 - q_L - \phi(1-\alpha)\varepsilon]\, l + [(\phi - 1)(1-\alpha)\varepsilon]\, s \\ &+ \frac{1}{2}\beta^2 (1 - q_L)[(1 - q_L) - 2(1-\alpha)\varepsilon](l^2 - s^2). \end{aligned} \tag{6}$$

The first order condition of s with respect to $\Pi^S(s)$ is

$$\frac{\partial \Pi^S(s)}{\partial s} = \beta^2 (1 - q_L)[2(1-\alpha)\varepsilon - (1 - q_L)]\, s + (\phi - 1)(1-\alpha)\varepsilon.$$

The solution of $\frac{\partial \Pi^S(s)}{\partial s} = 0$ is $s_0 = \frac{(\phi-1)(1-\alpha)\varepsilon}{\beta^2(1-q_L)[(1-q_L)-2(1-\alpha)\varepsilon]}$. Let $\Delta s_1 = s_0 - \underline{s}$, then

$$\Delta s_1 = \frac{(3\phi - 1)(1-\alpha)^2 \varepsilon^2 - (5\phi - 4)(1 - q_L)(1-\alpha)\varepsilon + 2(\phi - 1)(1 - q_L)^2}{\beta^2 (1 - q_L)[(1 - q_L) - 2(1-\alpha)\varepsilon](1-\alpha)\varepsilon}.$$

Let $\Delta s_2 = s_0 - \overline{s}$, then

$$\Delta s_2 = \frac{(3\phi - 1)(1-\alpha)\varepsilon^2 - [\phi(5 - 4\alpha) - 4(1-\alpha)](1 - q_L)\varepsilon + 2(\phi - 1)(1 - q_L)^2}{\beta^2 (1 - q_L)[(1 - q_L) - 2(1-\alpha)\varepsilon]\varepsilon}.$$

Next, we let $\underline{\varepsilon} = \frac{2(1-q_L)(\phi-1)}{(1-\alpha)}$, $\widehat{\varepsilon} = \frac{1-q_L}{2(1-\alpha)}$, $\overline{\varepsilon} = 1 - q_L$, and define two threshold values ε_1 and ε_2, where $\varepsilon_1, \varepsilon_2 \in [\underline{\varepsilon}, \overline{\varepsilon}]$, and $\varepsilon_1 \leq \varepsilon_2$.

(I) If $\alpha \leq \frac{1}{2}$ (i.e., $\widehat{\varepsilon} \leq \overline{\varepsilon}$), we can have that (i) $\Pi^S(s)$ decreases with s when $\varepsilon \leq \varepsilon_1$, (ii) $\Pi^S(s)$ first increases then decreases with s when $\varepsilon_1 < \varepsilon < \varepsilon_2$, (iii) $\Pi^S(s)$ increases with s

when $\varepsilon \geq \varepsilon_2$, where

$$\varepsilon_1 = \begin{cases} \underline{\varepsilon}_0 & \text{if } \phi \leq 4 - 2\sqrt{2} \\ \underline{\varepsilon} & \text{if } \phi > 4 - 2\sqrt{2} \end{cases} \quad \text{and} \quad \varepsilon_2 = \begin{cases} \bar{\varepsilon}_0 & \text{if } \phi \leq \frac{5}{4} \text{ and } \alpha \leq \frac{21\phi - 12\phi^2 - 8}{4(\phi-1)} \\ \underline{\varepsilon} & \text{if } \phi \leq \frac{5}{4} \text{ and } \frac{21\phi - 12\phi^2 - 8}{4(\phi-1)} < \alpha \leq \frac{6\phi^2 - 13\phi + 7}{5 - 4\phi} \\ \bar{\varepsilon}_0 & \text{if } \phi \leq \frac{5}{4} \text{ and } \alpha > \frac{6\phi^2 - 13\phi + 7}{5 - 4\phi} \\ \underline{\varepsilon} & \text{if } \phi > \frac{5}{4} \end{cases},$$

$\underline{\varepsilon}_0$ is derived from $\Delta s_1 = 0$ and $\bar{\varepsilon}_0$ is derived from $\Delta s_2 = 0$, where

$$\underline{\varepsilon}_0 = \frac{(1-q_L)}{2(1-\alpha)} \left[\frac{5\phi - 4 + \sqrt{\phi^2 - 8\phi + 8}}{3\phi - 1} \right],$$

$$\bar{\varepsilon}_0 = \frac{(1-q_L)}{2(1-\alpha)} \left\{ \frac{(5-4\alpha)\phi - 4(1-\alpha) + \sqrt{(16\alpha^2 - 16\alpha + 1)\phi^2 + 8(1-\alpha)[(4\alpha - 1)\phi - (2\alpha - 1)]}}{3\phi - 1} \right\}.$$

(II) If $\alpha > \frac{1}{2}$ (i.e., $\widehat{\varepsilon} > \bar{\varepsilon}$), we can have that (i) $\Pi^S(s)$ decreases with s when $\varepsilon \leq \varepsilon_1$, (ii) $\Pi^S(s)$ first increases then decreases with s when $\varepsilon_1 < \varepsilon < \varepsilon_2$, (iii) $\Pi^S(s)$ increases with s when $\varepsilon \geq \varepsilon_2$, where

$$\varepsilon_1 = \begin{cases} \underline{\varepsilon}_0 & \text{if } \phi \leq 4 - 2\sqrt{2} \text{ and } \alpha \leq \frac{\phi + 2 - \sqrt{\phi^2 - 8\phi + 8}}{6\phi - 2} \\ \underline{\varepsilon} & \text{if } \phi \leq 4 - 2\sqrt{2} \text{ and } \alpha > \frac{\phi + 2 - \sqrt{\phi^2 - 8\phi + 8}}{6\phi - 2} \\ \underline{\varepsilon} & \text{if } \phi > 4 - 2\sqrt{2} \end{cases},$$

$$\varepsilon_2 = \begin{cases} \bar{\varepsilon} & \text{if } \phi \leq 4 - 2\sqrt{2} \text{ and } \alpha \leq \frac{\phi + 2 - \sqrt{\phi^2 - 8\phi + 8}}{6\phi - 2} \\ \underline{\varepsilon} & \text{if } \phi \leq 4 - 2\sqrt{2} \text{ and } \alpha > \frac{\phi + 2 - \sqrt{\phi^2 - 8\phi + 8}}{6\phi - 2} \\ \bar{\varepsilon}_0 & \text{if } \phi > 4 - 2\sqrt{2} \text{ and } \alpha \leq \frac{1}{3 - \phi} \\ \bar{\varepsilon} & \text{if } \phi > 4 - 2\sqrt{2} \text{ and } \alpha > \frac{1}{3 - \phi}. \end{cases}.$$

□

Proof of Proposition 3

Proof. Firm's expected profits under the two product liability rules are derived in the proof of Proposition 2 and denoted as Equations (3), (4), (5) and (6). First, from Equations (3) and (4), it is obvious that $\Pi^S(0) = \Pi^C$.

Let $\Delta_{SC}(s)$ denote the difference between firm's expected profit under the two product liability rules, i.e., $\Delta_{SC}(s) = \Pi^S(s) - \Pi^C$. When $s \in (0, \underline{s}]$,

$$\Delta_{SC}(s) = -\frac{1}{2}\beta^2 [1 - q_L - (1-\alpha)\varepsilon]^2 s^2;$$

when $s \in (\underline{s}, \bar{s})$,

$$\Delta_{SC}(s) = [(\phi - 1)(1-\alpha)\varepsilon](l+s) - \frac{1}{2}\beta^2 \left\{ \left[(1-\alpha)^2 \varepsilon^2\right] l^2 + \left[(1-q_L)^2 - 2(1-q_L)(1-\alpha)\varepsilon\right] s^2 \right\};$$

when $s \in [\bar{s}, l)$,

$$\Delta_{SC}(s) = \frac{1}{2}\beta^2 \left\{ \alpha(1-\alpha)\varepsilon^2 l^2 - \left[(1-\alpha)\varepsilon^2 - 2(1-q_L)(1-\alpha)\varepsilon + (1-q_L)^2\right] s^2 \right\}. \quad (7)$$

In the pooling equilibrium i.e., $s \in (0, \underline{s}]$, $\Delta_{SC}(s) < 0$, so that firm's expected profit is higher under the comparative negligence rule. In the hybrid equilibrium, i.e., $s \in (\underline{s}, \bar{s})$, if $\Pi^S(s)$ decreases with s, firm's expected profit is still higher under the comparative negligence rule as $\Pi^S(\underline{s}) < \Pi^S(0) = \Pi^C$ (i.e., $\Delta_{SC}(\underline{s}) < 0$). Recall Proposition 2, we can confirm that if the quality gap ε is relatively low, firm's expected profit will be higher under the comparative negligence rule.

Proposition 2 also indicates that firm's expected profit can increase with s if $\varepsilon > \varepsilon_2$ in the hybrid equilibrium, and decreases with s in the separating equilibrium. Under such a circumstance, the maximum of the expected profit difference is $\Delta_{SC}(\bar{s})$. If $\Delta_{SC}(\bar{s}) > 0$, there must be a range of (s_1, s_2) such that firm's expected profit is higher under the strict liability rule if $s \in (s_1, s_2)$. From Equation (7),

$$\begin{aligned} \Delta_{SC}(\bar{s}) &= \frac{1}{2}\beta^2 \left\{ \alpha(1-\alpha)\varepsilon^2(l^2 - \bar{s}^2) - \left[(1-\alpha)^2\varepsilon^2 - 2(1-q_L)(1-\alpha)\varepsilon + (1-q_L)^2\right] \bar{s}^2 \right\} \\ &= \frac{1}{2}\beta^2 \left\{ \alpha(1-\alpha)\varepsilon^2(l^2 - \bar{s}^2) - [(1-q_L) - (1-\alpha)\varepsilon]^2 \bar{s}^2 \right\}. \end{aligned}$$

Since $1 - q_L \geq \varepsilon$, then

$$\Delta_{SC}(\bar{s}) \geq \frac{1}{2}\beta^2 \left[\alpha(1-\alpha)\varepsilon^2(l^2 - \bar{s}^2) - \alpha^2\varepsilon^2\bar{s}^2\right] = \frac{1}{2}\beta^2\alpha\varepsilon^2\left[(1-\alpha)l^2 - \bar{s}^2\right].$$

Therefore, $\Delta_{SC}(\bar{s}) > 0$ if $(1-\alpha)l^2 > \bar{s}^2$, which is equivalent to

$$\alpha < 1 - \left[1 - \frac{2(\phi - 1)}{\beta^2 l \varepsilon}\right]^2.$$

Note that the RHS of this inequality lies between $(0, 1)$, which implies that there must be some α that are sufficiently small to make this inequality hold. Thus, under the strict liability rule, if the quality gap $\varepsilon > \varepsilon_2$ such that firm's expected profit monotonically increases in the hybrid equilibrium, firm's expected profit under the strict liability rule can be higher than that under the comparative negligence rule for a range of penalty $s \in (s_1, s_2)$ around $\bar{s}$, given that ϕ is relatively large, where $\underline{s} < s_1 < \bar{s} < s_2 < l$, $\Pi^S(s_1) = \Pi^S(s_2) = \Pi^C$. □

Proof of Proposition 4

Proof. Under the comparative negligence rule, the pooling equilibrium is supported for all s. Thus, the expected product failure rate is

$$EF^C = [1 - q_L - (1-\alpha)\varepsilon]\left[1 - \beta^2 l\,(1 - q_L - (1-\alpha)\varepsilon)\right].$$

Obviously, this rate is not affected by penalty s.

Under the strict liability rule, we can pin down the expected failure rate in the following three cases:

(I) If $s \in (0, \underline{s}]$ and the pooling equilibrium is supported,

$$\begin{aligned} EF^S(s) &= [1 - q_L - (1-\alpha)\varepsilon]\left[1 - \beta e^S(0)\right] \\ &= [1 - q_L - (1-\alpha)\varepsilon]\left[1 - \beta^2 (l - s)\,(1 - q_L - (1-\alpha)\varepsilon)\right]. \end{aligned}$$

It is clear that $EF^S(s) > EF^C$ for $s \in (0, \underline{s}]$.

(II) If $s \in [\overline{s}, l)$ and the separating equilibrium is sustained,

$$\begin{aligned} EF^S(s) &= \alpha F_L(e^S(m_L)) + (1-\alpha) F_H(e^S(1)) \\ &= \alpha(1 - q_L)\left[1 - \beta^2(l - s)(1 - q_L)\right] + (1-\alpha)(1 - q_H)\left[1 - \beta^2(l - s)(1 - q_H)\right]. \end{aligned}$$

In both pooling equilibrium and separating equilibrium, the expected failure rates monotonically increase with s.

(III) If $s \in (\underline{s}, \overline{s})$ and the hybrid equilibrium is supported,

$$\begin{aligned} EF^S(s) &= \alpha\lambda(1 - q_L)\left[1 - \beta^2(l - s)(1 - q_L)\right] + (1 - \alpha\lambda)(1 - q_\lambda)\left[1 - \beta^2(l - s)(1 - q_\lambda)\right] \\ &= 1 - q_L - (1-\alpha)\varepsilon - 2(1-\alpha)\varepsilon(\phi - 1) + \beta^2(l - s)(1 - q_L)\left[2(1-\alpha)\varepsilon - (1 - q_L)\right]. \end{aligned}$$

This equation implies that $EF^S(s)$ increases with s if and only if $\varepsilon \leq \frac{1-q_L}{2(1-\alpha)}$. If this condition holds, combined with the facts in (I) and (II), it must be that the expected failure rate monotonically increases with s under the strict liability rule, and is always higher than the failure rate under the comparative negligence rule. However, if instead $\varepsilon > \frac{1-q_L}{2(1-\alpha)}$, the expected failure rate would decrease with s in the hybrid equilibrium. We then compare the failure rate under the two product liability rules in this case.

It is straightforward that $EF^S(s)$ is continuous and reaches its minimum at $\overline{s}$. Thus, if $EF^S(\overline{s}) - EF^C(\overline{s}) < 0$, there must be a range (s_1', s_2') around $\overline{s}$, such that $EF^S(s) <$

$EF^C(s)$ in this region. Therefore, we focus on the sign of $EF^S(\bar{s}) - EF^C(\bar{s})$ in the following discussion. Let $\overline{\Delta}_{SC}(s) = EF^S(\bar{s}) - EF^C(\bar{s})$, then

$$\overline{\Delta}_{SC}(\bar{s}) = \beta^2 \bar{s} \left[1 - q_L - (1-\alpha)\varepsilon\right]^2 - 2(\phi - 1)\alpha(1-\alpha)\varepsilon.$$

By solving the inequality $\overline{\Delta}_{SC}(\bar{s}) < 0$, we can derive that $\alpha < \widehat{\alpha}$, where

$$\widehat{\alpha} = \frac{\beta^2\bar{s}\varepsilon^2 - \beta^2\bar{s}\left(1-q_L\right)\varepsilon + (\phi-1)\varepsilon + \sqrt{\left[2\beta^2\bar{s}\left(1-q_L\right) + (\phi-1)\right](\phi-1)\varepsilon^2 - 2\beta^2\bar{s}\left(1-q_L\right)^2(\phi-1)\varepsilon}}{\beta^2\bar{s}\varepsilon^2 + 2(\phi-1)\varepsilon}.$$

Recall the assumption in Proposition 2 (i.e., $\varepsilon > \frac{2(1-q_L)(\phi-1)}{(1-\alpha)}$), we can obtain that $EF^S(\bar{s}) - EF^C(\bar{s}) < 0$ if $\varepsilon > \frac{1-q_L}{2(1-\alpha)}$ and $\alpha < \overline{\alpha}$, where $\overline{\alpha} = \min\{\alpha_1, \widehat{\alpha}\}$ and $\alpha_1 = 1 - \frac{2(1-q_L)(\phi-1)}{\varepsilon}$. □

Proof of Lemma 3

Proof. Consumer's out-of-equilibrium belief plays an important role in the price signaling game. In this subsection, we assume that consumer holds the most pessimistic out-of-equilibrium belief, in which they believe any out-of-equilibrium price indicates a L-type product. Note that if consumers think the product is low-quality, the highest profit of the L-type firm is $\pi_L(m_L)$.

(I) Under the comparative negligence rule, as in the main model, the ex-post penalties for two types of firms are not affected by their advertising strategies. Therefore, even with price signaling, the L-type would always mimic and report m_H. The only possible equilibrium is the pooling equilibrium.

(II) Under the strict liability rule, we first characterize pooling equilibria. The L-type would report m_H, which yields a pooling equilibrium, if $\pi_L^S(0) \geq \pi_L^S(m_L)$, where

$$\pi_L^S(0) = p^{po} - F_L(e^S(0))s.$$

$\pi_L^S(m_L)$ measures the highest pay-off the L-type firm can obtain if it deviates from p^{po} under the most pessimistic out-of-equilibrium belief. Thus, it must be that

$$p^{po} \geq \pi_L^S(m_L) + F_L(e^S(0))s,$$

which pins down the lower bound of the equilibrium price. The price upper bound of pooling equilibria is the equilibrium price (pooling equilibrium) in our base model, i.e., $p^{po} \leq p^S(0)$, as that is the highest price extracting all consumer surplus. Hence, any price between the upper bound and lower bound can support a pooling equilibrium.

The next question is when pooling equilibria can be supported. Since $\pi_L^S(0) - \pi_L^S(m_L)$ decreases with s, the L-type has a lower incentive to cheat when s is large. Recall that in the base model, we show that pooling equilibrium can be sustained when $s \leq \underline{s}$. And the pooling equilibrium with highest price is the one in our base model. Therefore, with price signaling, pooling equilibria exist if $s \leq \underline{s}$.

Then, we move on to hybrid equilibria, which can be sustained if the L-type is indifferent between cheating and truth-telling, i.e., $\pi_L^S(\lambda) = \pi_L^S(m_L)$, or equivalently

$$p^{hy} - F_L(e^S(\lambda))s = \pi_L^S(m_L).$$

Hence, the equilibrium price in a hybrid equilibrium is $p^{hy} = F_L(e^S(\lambda))s + \pi_L^S(m_L)$. Note that the condition for a hybrid equilibrium remains the same as that in our base model without price signaling. In the base model, the hybrid equilibrium is the one that extracts all surplus, which renders it the one with the highest price. With price signaling, we now have multiple hybrid equilibria, as long as the price satisfies this indifference condition. It is obvious that hybrid equilibria can be sustained as long as $s \leq \bar{s}$.

In terms of separating equilibria, the L-type should have no incentive to report m_H, i.e., $\pi_L^S(m_L) \geq \pi_L^S(1)$. Thus, the equilibrium price should satisfy the following condition:

$$p^{se} \leq \pi_L^S(m_L) + F_L(e^S(0))s.$$

Separating equilibria can be supported when the price associated with m_H is not too large. The intuition is that the L-type would not mimic a slightly higher price by paying a much higher penalty. It is obvious that any s can sustain a series of separating equilibria.

Finally, the H-type firm would never report m_L with price signaling as well. The similar logic in the main model can be applied. The H-type must have higher incentive to report m_H than the L-type, since the H-type firm pays lower expected penalty. Thus, in pooling or hybrid equilibrium, the H-type must report m_H given that the L-type sends m_H with positive probability. In a separating equilibrium, a sufficient condition for the H-type is truth-telling is that its price is higher than the equilibrium price of L-type firm, i.e., $p^{se} \geq p^S(m_L)$, which holds trivially. □

Proof of Proposition 5

Proof. (I) Under the comparative negligence rule, only pooling equilibria are supported. Since LMSE selects the one with the highest payoff for the H-type firm. Hence, the pooling equilibrium in which the price extracts all consumer surplus would be chosen. This is exactly the pooling equilibrium we examined in the base model under the comparative negligence rule.

(II) Under the strict liability rule, as we mentioned in the main text, the candidates are the equilibrium with the highest price for the H-type. We first prove that the hybrid equilibrium is always dominated by the separating equilibrium, and thus should be eliminated by LMSE.

In the optimal hybrid equilibrium, the H-type's profit is

$$\pi_H^{hy*}(\lambda) = \pi_L^S(m_L) + \left[F_L(e^S(\lambda)) - F_H(e^S(\lambda))\right] s.$$

In the optimal separating equilibrium, the H-type's profit is

$$\pi_H^{se*}(1) = \pi_L^S(m_L) + \left[F_L(e^S(1)) - F_H(e^S(1))\right] s.$$

The profit difference between the two types of equilibria is

$$\begin{aligned}\Delta\pi =& \pi_H^{hy*}(\lambda) - \pi_H^{se*}(1) \\ =& \left\{ \left[F_L(e^S(\lambda)) - F_H(e^S(\lambda))\right] - \left[F_L(e^S(1)) - F_H(e^S(1))\right] \right\} s \\ =& - \int_{e^S(1)}^{e^S(\lambda)} \left[\frac{\partial F_H(e)}{\partial e} - \frac{\partial F_L(e)}{\partial e}\right] de < 0.\end{aligned}$$

The last inequality follows the facts that $\frac{\partial F_H(e)}{\partial e} > \frac{\partial F_L(e)}{\partial e}$ and $e^S(\lambda) > e^S(1)$. Therefore, when $s \in [\underline{s}, \bar{s}]$, such that separating and hybrid equilibria co-exist, LMSE would eliminate all hybrid equilibria and select the least-cost separating equilibrium with the highest price.

Then, we focus on the comparison between the optimal pooling and separating equilibrium when $s \leq \underline{s}$. The H-type's profit in the optimal pooling equilibrium is

$$\pi_H^{po*}(0) = p^{po*}(0) - F_H(e^S(0))s,$$

where $p^{po*}(0) = v - F(E(q), e^S(0))(l - s) - c(e^S(0))$ is the highest equilibrium price in a pooling equilibrium.

The optimal pooling equilibrium is selected if and only if the H-type firm obtains higher profit than its profit in the optimal separating equilibrium. We then investigate how the H-type's profit difference between two equilibria varies with s. By the envelope theorem,

$$\frac{\partial\left[\pi_H^{po*}(0) - \pi_H^{se*}(1)\right]}{\partial s} = \{F(E(q), e^S(0)) - F_L(e^S(0))\} + \{F_H(e^S(0)) - F_H(e^S(1))\} < 0.$$

The last inequality holds as $F(q, e)$ decreases with both q and e and $e(\lambda)$ decreases with λ. Thus, when s is low, the pooling equilibrium is sustained, while the separating equilibrium with price distortion is selected when s is close to $\underline{s}$.

Figure 1 and Figure 2 illustrate the equilibria supported under different s and the comparison of firm's expected profit under the strict liability rule and the comparative negligence rule respectively.[1] □

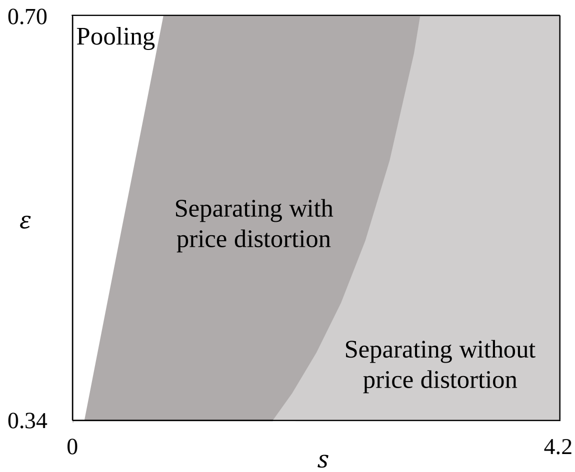

Figure 1: Equilibrium structure

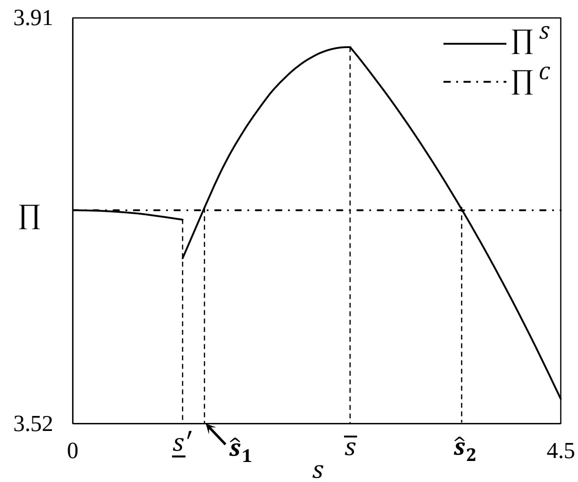

Figure 2: Comparison of expected profits

Proof of Lemma 4

Proof. Under the comparative negligence rule, given the extra penalty $P(\geq 0)$ for deceptive advertising, when the L-type firm advertises m_H, its profit $\pi_L^{C\prime}(\lambda)$ can be expressed as

$$\pi_L^{C\prime}(\lambda) = \pi_L^{C}(\lambda) - P = v - G(\lambda)\, l + G(\lambda, e_b)s - c(e^C(\lambda)) - F_L(e_b)\, s - P,$$

and when the L-type firm advertises m_L, its profit $\pi_L^{C\prime}(m_L)$ is

$$\pi_L^{C\prime}(m_L) = \pi_L^{C}(m_L) = v - G(m_L)l + G(m_L, e_b)s - c(e^C(m_L)) - F_L(e_b)s,$$

where $G(m_L) = F_L(e^C(m_L))$ and $G(m_L, e_b) = F_L(e_b)$. Since $\frac{\partial \pi_L^C(\lambda)}{\partial \lambda} = -\frac{\partial G(\lambda)}{\partial \lambda} l + \frac{\partial G(\lambda, e_b)}{\partial \lambda} s > 0$, then $\pi_L^C(\lambda)$ is increasing in λ and $\pi_L^C(1) > \pi_L^C(0) > \pi_L^C(m_L)$. Let $\Delta_L^C(\lambda) = \pi_L^C(\lambda) - \pi_L^C(m_L)$, we can further obtain that $\Delta_L^C(\lambda)$ is increasing in λ and $\Delta_L^C(0) < \Delta_L^C(\lambda) < \Delta_L^C(1)$, where $\Delta_L^C(0) = \pi_L^C(0) - \pi_L^C(m_L)$ and $\Delta_L^C(1) = \pi_L^C(1) - \pi_L^C(m_L)$. We define $\underline{P}^C = \Delta_L^C(0)$ and $\overline{P}^C = \Delta_L^C(1)$. Then, we can obtain the following results: if $P \leq \underline{P}^C$, $\pi_L^{C\prime}(m_L) \leq \pi_L^{C\prime}(0)$, the L-type firm always cheats, and a pooling equilibrium arises; if $\underline{P}^C < P < \overline{P}^C$, $\pi_L^{C\prime}(0) < \pi_L^{C\prime}(m_L) < \pi_L^{C\prime}(1)$, there exists a unique $\overline{\lambda}^*$ that makes $\pi_L^{C\prime}(\overline{\lambda}^*) = \pi_L^{C\prime}(m_L)$, and a hybrid

[1] In Figure 1, $F(q,e) = (1-q)(1-\beta e)$, $c(e) = \frac{e^2}{2}$, $v = 4.2$, $\alpha = 0.3$, $q_L = 0.3$, $\beta = 0.63$, $l = 4.2$. In Figure 2, $F(q,e) = (1-q)(1-\beta e)$, $c(e) = \frac{e^2}{2}$, $v = 4.5$ $\alpha = 0.25$, $q_L = 0.3$, $\varepsilon = 0.65$, $\beta = 0.63$, $l = 4.5$.

equilibrium can be supported; if $P \geq \overline{P}^C$, $\pi_L^{C\prime}(m_L) \geq \pi_L^{C\prime}(1)$, the L-type does not mimic the H-type, and a separating equilibrium arises.

Under the strict liability rule, given the penalty P for deceptive advertising, when the L-type firm advertises m_H, its profit $\pi_L^{S\prime}(\lambda)$ can be expressed as

$$\pi_L^{S\prime}(\lambda) = \pi_L^S(\lambda) - P = v - G(\lambda)(l-s) - c(e^S(\lambda)) - F_L(e^S(\lambda))s - P,$$

and when the L-type firm advertises m_L, its profit $\pi_L^{S\prime}(m_L)$ is

$$\pi_L^{S\prime}(m_L) = \pi_L^S(m_L) = v - G(m_L)(l-s) - c(e^S(m_L)) - F_L(e^S(m_L))s,$$

where $G(m_L) = F_L(e^S(m_L))$. Recall Proposition 1, when $s \leq \underline{s}$, $\pi_L^S(m_L) \leq \pi_L^S(0)$, then $\pi_L^{S\prime}(m_L) - \pi_L^{S\prime}(0) \geq 0$ if $P \geq \pi_L^S(0) - \pi_L^S(m_L)$, and $\pi_L^{S\prime}(m_L) - \pi_L^{S\prime}(1) \geq 0$ if $P \geq \pi_L^S(1) - \pi_L^S(m_L)$, as a result, a pooling equilibrium arises if $P \leq \pi_L^S(0) - \pi_L^S(m_L)$, a hybrid equilibrium arises if $\pi_L^S(0) - \pi_L^S(m_L) < P < \pi_L^S(1) - \pi_L^S(m_L)$, otherwise, a separating equilibrium arises. When $\underline{s} < s < \bar{s}$, $\pi_L^S(0) < \pi_L^S(m_L) < \pi_L^S(1)$, then if $P < \pi_L^S(1) - \pi_L^S(m_L)$, $\pi_L^{S\prime}(0) < \pi_L^{S\prime}(m_L) < \pi_L^{S\prime}(1)$, and a hybrid equilibrium can be supported; otherwise, a separating equilibrium arises. When $s \geq \underline{s}$, $\pi^S(m_L) \geq \pi^S(\lambda)$ for any $\lambda \in [0,1]$, which makes $\pi^{S\prime}(m_L)$ no less than $\pi^{S\prime}(\lambda, m_H)$ for any penalty $P \geq 0$, such that only a separating equilibrium exists.

Based on above, let $\underline{P}^S = \pi_L^S(0) - \pi_L^S(m_L)$ and $\overline{P}^S = \pi_L^S(1) - \pi_L^S(m_L)$, then the firms equilibrium advertising strategy under the strict liability rule can be summarized as follows: given the penalty for deceptive advertising, the L-type firm reports m_L with probability $\widetilde{\lambda}^*$ and it reports m_H with probability $1 - \widetilde{\lambda}^*$, in which

- when $s \leq \underline{s}$, i) if $P \leq \underline{P}^S$, $\widetilde{\lambda}^* = 0$, and a pooling equilibrium is sustained, ii) if $\underline{P}^S < P < \overline{P}^S$, $\widetilde{\lambda}^* \in (0,1)$, which uniquely solves $\pi_L^S(\widetilde{\lambda}^*) - P = \pi_L^S(m_L)$, and a hybrid equilibrium is sustained, iii) otherwise, a separating equilibrium is sustained;
- when $\underline{s} < s < \bar{s}$, i) if $P \leq \overline{P}^S$, $\widetilde{\lambda}^* \in (0,1)$, which uniquely solves $\pi_L^S(\widetilde{\lambda}^*) - P = \pi_L^S(m_L)$, and a hybrid equilibrium is sustained, ii) otherwise, $\widetilde{\lambda}^* = 1$, and a separating equilibrium is sustained;
- when $s \geq \bar{s}$, $\widetilde{\lambda}^* = 1$, and a separating equilibrium is sustained.

Figure 3 illustrates the trend of expected profit under two liability rules when s is low, medium, and large, respectively. □

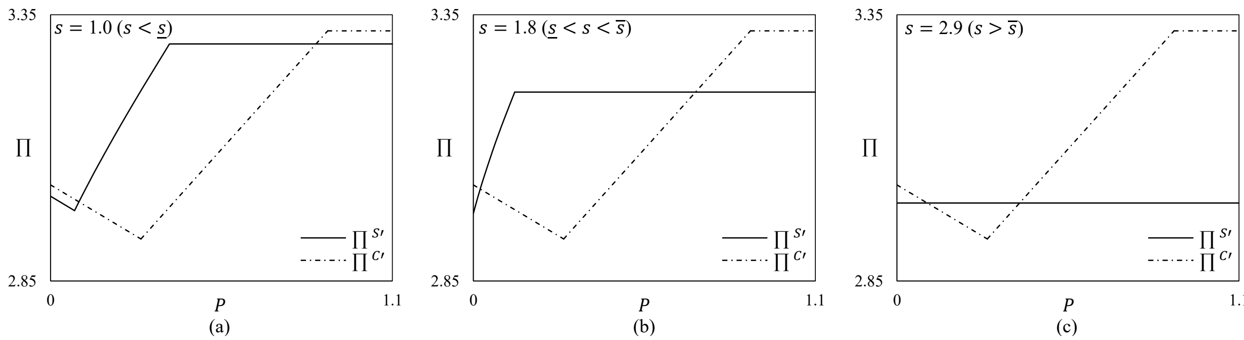

Figure 3: Comparison of firm's expected profits with advertising penalty ($v = 4$, $\alpha = 0.35$, $q_L = 0.25$, $\varepsilon = 0.65$, $\beta = 0.63$, $l = 4$, $\underline{s} = 1.73$, $\overline{s} = 2.52$, $F(q,e) = (1-q)(1-\beta e)$, $c(e) = \frac{e^2}{2}$)

Proof of Proposition 6

Proof. Under the comparative negligence rule, consumer (expected) surplus is

$$CS^C = \alpha CS_L^C(0, m_H) + (1-\alpha)CS_H^C(0, m_H),$$

where $CS_L^C(0, m_H)$ and $CS_H^C(0, m_H)$ are given by Equation (15) in the main text. Taking the derivative with respect to penalty s,

$$\begin{aligned}\frac{\partial CS^C}{\partial s} =& \alpha \frac{\partial CS_L^C(0, m_H)}{\partial s} + (1-\alpha)\frac{\partial CS_H^C(0, m_H)}{\partial s} \\ =& \alpha \frac{\left[V - F_L(e^C(0))l + F_L(e_b)s - c(e^C(0)) - p_L^C(0)\right]}{4V}(1-\alpha)\varepsilon(1-\beta e_b) \\ &- (1-\alpha)\frac{\left[V - F_H(e^C(0))l + F_H(e_b)s - c(e^C(0)) - p_H^C(0)\right]}{4V}\alpha\varepsilon(1-\beta e_b) \\ =& -\frac{\alpha(1-\alpha)\varepsilon(1-\beta e_b)}{4V}\left[2(1-\beta e^C(0))l - (1-\beta e_b)s\right]\varepsilon,\end{aligned}$$

where $p_i^C(0) = \frac{V - G(0)l + G(0,e_b)s - c(e^C(0)) + F_i(e_b)s}{2}$, $i \in \{H, L\}$. Since $l > s$ and $e^C(0) < e_b$, we have $2\left[1 - \beta e^C(0)\right] l - (1-\beta e_b)\, s > 0$. Thus, $\frac{\partial CS^C}{\partial s} < 0$, meaning consumer surplus decreases with s under the comparative negligence rule.

Under the strict liability rule, if $s \in [\overline{s}, l)$, separating equilibrium is supported, the consumer surplus is

$$CS^S = \alpha CS_L^S(m_L) + (1-\alpha)CS_H^S(1, m_H).$$

Given that in the separating equilibrium,

$$CS_L^S(m_L) = \frac{\left[V - F_L(e^S(m_L))(l-s) - c(e^S(m_L)) - F_L(e^S(m_L))s\right]^2}{8V},$$

$$CS_H^S(1, m_H) = \frac{\left[V - F_H(e^S(1))(l-s) - c(e^S(1)) - F_H(e^S(1))s\right]^2}{8V},$$

we have

$$\begin{aligned}\frac{\partial CS^S}{\partial s} =& -\alpha\frac{V - F_L(e^S(m_L))(l-s) - c(e^S(m_L)) - F_L(e^S(m_L))s}{8V}\frac{\partial F_L(e^S(m_L))}{\partial s}s \\ &- (1-\alpha)\frac{V - F_H(e^S(1))(l-s) - c(e^S(1)) - F_H(e^S(1))}{s}8V\frac{\partial F_H(e^S(1))}{\partial s}s.\end{aligned}$$

Since $\frac{\partial F_L(e^S(m_L))}{\partial s} > 0$, $\frac{\partial F_H(e^S(1))}{\partial s} > 0$ and V is sufficiently large, it must be that $\frac{\partial CS^S}{\partial s} < 0$. Thus, consumer surplus monotonically decreases with s in the separating equilibrium under the strict liability rule.

If $s \in (0, \underline{s}]$, consumer surplus in the pooling equilibrium is

$$CS^S = \alpha CS_L^S(0, m_H) + (1-\alpha)CS_H^S(0, m_H).$$

Since in the pooling equilibrium,

$$CS_i^S(0, m_H) = \frac{\left\{V - 2\left[F_i(e^S(0)) - G(0)\right](l-s) - c(e^S(0)) - F_i(e^S(0))s\right\}^2}{8V},$$

then

$$\begin{aligned}\frac{\partial CS^S}{\partial s} =& \alpha T_L\left[(1-3\beta e^S(0))(1-\alpha)\varepsilon - \beta^2(1-q_L)(1-E(q))s\right] \\ &+ (1-\alpha)T_H\left[-(1-3\beta e^S(0))\alpha\varepsilon - \beta^2(1-q_L)(1-E(q)))s\right],\end{aligned}$$

where $T_i = \frac{V - 2\left[F_i(e^S(0)) - G(0)\right](l-s) - c(e^S(0)) - F_i(e^S(0))s}{4V}$ and $E(q) = 1 - q_L - (1-\alpha)\varepsilon$. Now, we consider the extreme point where $s = 0$. Starting from zero penalty,

$$\frac{\partial CS^S}{\partial s}|_{s=0} = \alpha(1-\alpha)\varepsilon\left[1 - 3\beta e^S(0)\right]\frac{\left[F_H(e^S(0)) - F_L(e^S(0))\right](2l-s)}{4V}.$$

If $1 - 3\beta e^S(0) = 1 - 3\beta^2\left[1 - q_L - (1-\alpha)\varepsilon\right]l < 0$, then combining with the fact that $F_H(e^S(0)) - F_L(e^S(0)) < 0$, we have $\frac{\partial CS^S}{\partial s}|_{s=0} > 0$. Hence, consumer surplus increases starting with $s = 0$.

Recall that when $s = 0$, the two product liability rules yield identical consumer surplus as there is no penalty for the firm. We have shown that consumer surplus monotonically decreases under the comparative negligence rule, while it increases under the strict liability rule at $s = 0$. Therefore, there must be a range of s such that the strict liability rule provides higher consumer surplus. □

Proof of Proposition 7

Proof. First, when product quality and precaution effort are complements, a higher perceived quality would increase consumer effort. As $G'(\lambda, m_H) > G'(m_L)$, for all $\lambda \in [0,1]$, the L-type firm can charge a higher price if it reports m_H. In addition, consumer effort would be higher with m_H. Therefore, under both liability rules, reporting m_H is the dominating strategy for a L-type firm.

Given that the firm always sends m_H, the expected profit under both rules is

$$\Pi' = v - G'(0, e)l - c(e).$$

It is obvious that comparative negligence yields the first best consumer effort by the first order condition, and as consumer's optimal precaution effort is always lower under the strict liability rule, i.e. $e^{S\prime}(0) < e^{C\prime}(0)$, then $\Pi^{C\prime} > \Pi^{S\prime}$. □

Appendix B

Alternative product liability rules

We have analyzed the two widely used product liability rules, strict liability and comparative negligence. In the United States, sometimes comparative negligence is implemented in variant forms, such as modified comparative negligence and contributory negligence. In this subsection, we further examine these two forms.

Modified Comparative Negligence. Under the modified comparative negligence rule, the consumer can recover compensation based on her degree of fault only if the firm was at least 50% or at least 51% responsible for causing the product failure (see Bieber and Ramirez 2023). When the L-type sends message m_H, the consumer's expected utility can be expressed as follows:

$$U^{MC}(m_H, e) = \begin{cases} v - G(\lambda)\left[l - \frac{G(\lambda, e_b)}{G(\lambda)}s\right] - p - c(e) & \text{if } \frac{G(\lambda, e_b)}{G(\lambda)} > 0.5 \text{ or } 0.51 \\ v - G(\lambda)l - p - c(e) & \text{if } \frac{G(\lambda, e_b)}{G(\lambda)} < 0.5 \text{ or } 0.51 \end{cases},$$

where $G(\lambda) = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha+(1-\alpha)}F_L(e) + \frac{(1-\alpha)}{(1-\lambda)\alpha+(1-\alpha)}F_H(e)$, $G(\lambda, e_b) = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha+(1-\alpha)}F_L(e_b) + \frac{(1-\alpha)}{(1-\lambda)\alpha+(1-\alpha)}F_H(e_b)$. We define $\widehat{e}_1$ satisfies that $\frac{G(\lambda,e_b)}{G(\lambda,\widehat{e}_1)} = 0.5$ or 0.51,where $G(\lambda, \widehat{e}_1) = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha+(1-\alpha)}F_L(\widehat{e}_1) + \frac{(1-\alpha)}{(1-\lambda)\alpha+(1-\alpha)}F_H(\widehat{e}_1)$. Then, the consumer's expected utility can be rewritten as

$$U^{MC}(m_H, e) = \begin{cases} v - G(\lambda)l + G(\lambda, e_b)s - p - c(e) & \text{if } e > \widehat{e}_1 \\ v - G(\lambda)l - p - c(e) & \text{if } e < \widehat{e}_1 \end{cases},$$

The first order condition of e with respect to $U^{MC}(m_H, e)$ is $\frac{\partial U^{MC}(m_H,e)}{\partial e} = -\frac{\partial G(\lambda,m_H)}{\partial e}l - c'(e)$, so that the consumer's optimal precaution effort $e^{MC}(\lambda)$ satisfies the condition that $\frac{\partial G(\lambda,m_H)}{\partial e^{MC}}l + c'(e^{MC}) = 0$. Then, the optimal price set by the L-type firm is

$$p^{MC}(\lambda) = \begin{cases} v - G(\lambda)l + G(\lambda, e_b)s - c(e^{MC}(\lambda)) & \text{if } e^{MC}(\lambda) > \widehat{e}_1 \\ v - G(\lambda)l - c(e^{MC}(\lambda)) & \text{if } e^{MC}(\lambda) < \widehat{e}_1 \end{cases},$$

where $G(\lambda) = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha+(1-\alpha)}F_L(e^{MC}(\lambda)) + \frac{(1-\alpha)}{(1-\lambda)\alpha+(1-\alpha)}F_H(e^{MC}(\lambda))$. If $\frac{F_L(e_b)}{F_L(e^{MC}(\lambda))} > 0.5$ or 0.51, then the L-type firm's profit when advertising m_H is

$$\pi_L^{MC}(\lambda) = v - G(\lambda)l + G\left(\lambda, e_b\right)s - c(e^{MC}(\lambda)) - F_L(e_b)s,$$

and $\frac{\partial \pi_L^{MC}(\lambda)}{\partial \lambda} = -\frac{\partial G(\lambda)}{\partial \lambda}l + \frac{\partial G(\lambda,e_b)}{\partial \lambda}s > 0$. If $\frac{F_L(e_b)}{F_L(e^{MC}(\lambda))} < 0.5$ or 0.51, then the L-type firm's profit when reporting m_H is

$$\pi_L^{MC}(\lambda) = v - G(\lambda)l - c(e^{MC}(\lambda)),$$

and $\frac{\partial \pi_L^{MC}(\lambda)}{\partial \lambda} = -\frac{\partial G(\lambda,)}{\partial \lambda} l > 0$. Thus, $\pi_L^{MC}(\lambda)$ is increasing in λ.

Next, we define $\widehat{e}_2$ satisfies that $\frac{F_L(e_b)}{F_L(\widehat{e}_2)} = 0.5$ or 0.51. When the L-type firm sends message m_L, the consumer's expected utility can be expressed as

$$U^{MC}(m_L, e) = \begin{cases} v - F_L(e)l + F_L(e_b)s - p - c(e) & \text{if } e > \widehat{e}_2 \\ v - F_L(e)l - p - c(e) & \text{if } e < \widehat{e}_2 \end{cases}.$$

The consumer's optimal precaution effort $e^{MC}(m_L)$ satisfies that $\frac{\partial F_L(e^{MC})}{\partial e^{MC}} l + c'(e^{MC}) = 0$, and the optimal price charged by the L-type firm is

$$p^{MC}(m_L) = \begin{cases} v - F_L(e^{MC}(m_L))l + F_L(e_b)s - c(e^{MC}(m_L)) & \text{if } e^{MC}(m_L) > \widehat{e}_2 \\ v - F_L(e^{MC}(m_L))l - c(e^{MC}(m_L)) & \text{if } e^{MC}(m_L) < \widehat{e}_2 \end{cases}.$$

If $\frac{F_L(e_b)}{F_L(e^{MC}(m_L))} > 0.5$ or 0.51, then the L-type firm's profit when reporting m_L is

$$\pi_L^{MC}(m_L) = v - F_L(e^{MC}(m_L))l + F_L(q, e_b)\, s - c(e^{MC}(m_L)) - F_L(e_b)\, s.$$

If $\frac{F_L(e_b)}{F_L(e^{MC}(m_L))} < 0.5$ or 0.51, then the L-type firm's profit reporting m_L is

$$\begin{aligned} \pi_L^{MC}(m_L) &= v - F_L(e^{MC}(m_L))l - c(e^{MC}(m_L)) \\ &= v - F_L(e^{MC}(m_L))l + F_L(e_b)s - c(e^{MC}(m_L)) - F_L(e_b)s. \end{aligned}$$

By comparing $\pi_L^{MC}(\lambda)$ with $\pi_L^{MC}(m_L)$, we can obtain that under the modified comparative negligence rule, if the L-type firm truthfully reports m_L, its profit (i.e., $\pi_L^{MC}(m_L)$) is lower than the profit with m_H (i.e., $\pi_L^{MC}(\lambda)$) for any belief $\lambda \in [0,1]$. Thus, only a pooling equilibrium is sustained (i.e., $\lambda = 0$), and in equilibrium, firm's expected profit is $\Pi^{MC} = v - G(0)l - c(e^{MC}(0))$, where $G(0) = \alpha F_L(e^{MC}(0)) + (1-\alpha)F_H(e^{MC}(0))$, $e^{MC}(0) = e^C(0)$. As a result, the firm's equilibrium expected profit under the modified comparative negligence rule is equal to that under the comparative negligence rule (i.e., $\Pi^{MC} = \Pi^C$).

Contributory Negligence. Under the contributory negligence rule, the consumer who is even partly at fault for causing product failure is barred from pursuing a claim from compensation at all, which would mean the consumer who was 1% responsible for product failure would be barred from obtaining monetary compensation from the firm who was 99% to blame (see Bieber and Ramirez 2023). We implement the contributory negligence rule in the setup of the main model by assuming that the firm's penalty is $s(>0)$ if the consumer spends the precaution effort level e_b, but is zero if the consumer spends a lower level of effort $e < e_b$.

(I) If the level of precaution effort spent by the consumer is lower than e_b, when the L-type firm advertises m_H, the consumer's expected utility is $U^{CN}(m_H, e) = v - G(\lambda)\, l - p - c(e)$, and the optimal effort level $e^{CN}(\lambda)$ exerted by the consumer satisfies the condition that $\frac{\partial G(\lambda)}{\partial e^{CN}} l + c'(e^{CN}) = 0$. The optimal retail price set by the L-type firm is $p^{CN}(\lambda) = v - G(\lambda)l - c(e^{CN}(\lambda))$, and the optimal profit achieved by the L-type firm is $\pi_L^{CN}(\lambda) = p^{CN}(\lambda)$, where $G(\lambda) = \frac{(1-\lambda)\alpha}{(1-\lambda)\alpha+(1-\alpha)} F_L(e^{CN}(\lambda)) + \frac{(1-\alpha)}{(1-\lambda)\alpha+(1-\alpha)} F_H(e^{CN}(\lambda))$. As $\frac{\partial \pi_L^{CN}(\lambda)}{\partial \lambda} = -\frac{\partial G(\lambda)}{\partial \lambda} l > 0$, then $\pi_L^{CN}(\lambda)$ is also increasing in λ, the L-type firm will always report m_H.

(II) If the consumer spends the precaution effort level e_b, when the L-type firm advertises m_H, the consumer's expected utility is $U^{CN}(\lambda, e_b) = v - G(\lambda, e_b)(l-s) - p - c(e_b)$, the optimal retail price charged by the L-type firm is $p^{CN}(\lambda, e_b) = v - G(\lambda, e_b)(l-s) - c(e_b)$, and the L-type firm's profit is $\pi_L^{CN}(\lambda, e_b) = v - G(\lambda, e_b)(l-s) - c(e_b) - F_L(e_b)\, s$. Since $\pi_L^{CN}(\lambda, e_b) - \pi_L^{CN}(m_L, e_b) = [F_L(e_b) - G(\lambda, e_b)]\,(l-s) > 0$, then the L-type firm will not report its true type even when the consumer exerts the proper level of precaution effort.

Based on above, under the contributory negligence rule, only pooling equilibrium is sustained, i.e, $\lambda = 0$, and in equilibrium, the optimal precaution effort exerted by the consumer is $e^{CN}(0) = e^{C}(0)$, and firm's expected profit is $\Pi^{CN} = v - G(0)l - c(e^{CN}(0))$, where $G(0) = \alpha F_L(e^{CN}(0)) + (1-\alpha)F_H(e^{CN}(0))$. Obviously, firm's expected profit under the contributory negligence rule is equal to that under the comparative negligence rule (i.e., $\Pi^{CN} = \Pi^{C}$).

In conclusion, under either modified comparative negligence rule or contributory negligence rule, both types of firms advertise m_H, and only a pooling equilibrium is sustained, which is similar to the firms equilibrium advertising strategy under the pure comparative negligence rule in the main model. The intuition is that under the modified comparative negligence rule, the high-quality message sent by the firm may induce the consumer to spend less effort, which increases the consumer's degree of fault in the product failure and reduces the firm's compensation to the consumer; under the contributory negligence rule, the consumer cannot recover any compensation if they are partially responsible for the product failure, in other words, the contributory negligence rule sharply reduces the penalty, and in this circumstance, it is optimal for the L-type firm to advertise deceptively.

References

Bieber C, Ramirez A (2023) What Is Comparative Negligence? https://www.forbes.com/advisor/legal/personal-injury/comparative-negligence/.