Appendix A Proofs of Results

A.1 Proof of Equilibrium Under No Restriction Policy

The profit maximization problem of E and R can be written as

$$\max_{e_{Ep},\, e_{Er}} \pi_E = \max_{e_{Ep},\, e_{Er}} \alpha_E \left(N_p \frac{e_{Ep}}{e_{Ep}+e_{Rp}} + N_r \frac{e_{Er}}{e_{Er}+e_{Rr}} \right) - (e_{Ep}+e_{Er}), \tag{13}$$

$$\max_{e_{Rp},\, e_{Rr}} \pi_R = \max_{e_{Rp},\, e_{Rr}} \left(\alpha_{Rp} N_p \frac{e_{Rp}}{e_{Ep}+e_{Rp}} + \alpha_{Rr} N_r \frac{e_{Rr}}{e_{Er}+e_{Rr}} \right) - (e_{Rp}+e_{Rr}). \tag{14}$$

First-order conditions are:

$$\frac{\partial \pi_E}{\partial e_{Ep}} = \alpha_E N_p \frac{e_{Rp}}{(e_{Ep}+e_{Rp})^2} - 1 = 0, \tag{15}$$

$$\frac{\partial \pi_E}{\partial e_{Er}} = \alpha_E N_r \frac{e_{Rr}}{(e_{Er}+e_{Rr})^2} - 1 = 0, \tag{16}$$

$$\frac{\partial \pi_R}{\partial e_{Rp}} = \frac{\alpha_{Rp} N_p e_{Ep}}{(e_{Ep}+e_{Rp})^2} - 1 = 0, \tag{17}$$

$$\frac{\partial \pi_R}{\partial e_{Rr}} = \frac{\alpha_{Rr} N_r e_{Er}}{(e_{Er}+e_{Rr})^2} - 1 = 0. \tag{18}$$

From the following first-order conditions, we can obtain the equilibrium levels of ad expense by substituting $e_{Rp} = -e_{Ep} + \sqrt{\alpha_{Rp} N_p e_{Ep}}$ (from equation (17)) into (15) and $e_{Rr} = -e_{Er} + \sqrt{\alpha_{Rr} N_r e_{Er}}$ (from (18)) into (16).

$$e_{Ep}^{NR} = \frac{\alpha_E^2 \alpha_{Rp} N_p}{(\alpha_E + \alpha_{Rp})^2},\quad e_{Er}^{NR} = \frac{\alpha_E^2 \alpha_{Rr} N_r}{(\alpha_E + \alpha_{Rr})^2},$$

$$e_{Rp}^{NR} = \frac{\alpha_E \alpha_{Rp}^2 N_p}{(\alpha_E + \alpha_{Rp})^2},\quad e_{Rr}^{NR} = \frac{\alpha_E \alpha_{Rr}^2 N_r}{(\alpha_E + \alpha_{Rr})^2}.$$

To make sure the solution is a global maximizer over the assumed parameter ranges, we check the definiteness of the Hessians:

$$H_E = \begin{pmatrix} -\frac{2\alpha_E N_p e_{Rp}}{(e_{Ep}+e_{Rp})^3} & 0 \\ 0 & -\frac{2\alpha_E N_r e_{Rr}}{(e_{Er}+e_{Rr})^3} \end{pmatrix}$$

$$H_R = \begin{pmatrix} -\frac{2\alpha_{Rp} N_p e_{Ep}}{(e_{Ep}+e_{Rp})^3} & 0 \\ 0 & -\frac{2\alpha_{Rr} N_r e_{Er}}{(e_{Er}+e_{Rr})^3} \end{pmatrix}$$

It is obvious that both Hessians are negative definite over the entire parameter ranges. Using the equilibrium ad-budgets, we can obtain the profit of the platform under the no restriction policy:

$$\begin{aligned} \pi_P^{NR} &= e_{Ep}^{NR} + e_{Er}^{NR} + e_{Rp}^{NR} + e_{Rr}^{NR}, \\ &= \alpha_E \left(\frac{\alpha_{Rp} N_p}{\alpha_E + \alpha_{Rp}} + \frac{\alpha_{Rr} N_r}{\alpha_E + \alpha_{Rr}} \right). \end{aligned} \tag{19}$$

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A.2 Proof of Equilibrium Under Equal Treatment Policy

With the fairness constraint $\frac{e_{Ep}}{N_p} = \frac{e_{Er}}{N_r}$, we have $e_{Ep} = \frac{N_p}{N_r} e_{Er}$. To simplify the notations, we define $n = \frac{N_p}{N_r}$ and substitute the constraint into the advertisers' profit maximization problem:

$$\max_{e_{Er}} \pi_E = \max_{e_{Er}} \alpha_E \left(N_p \frac{n e_{Er}}{n e_{Er} + e_{Rp}} + N_r \frac{e_{Er}}{e_{Er} + e_{Rr}} \right) - (\frac{N_p}{N_r} e_{Er} + e_{Er}),$$
$$\max_{e_{Rp},\ e_{Rr}} \pi_R = \max_{e_{Rp},\ e_{Rr}} \left(\alpha_{Rp} N_p \frac{e_{Rp}}{n e_{Er} + e_{Rp}} + \alpha_{Rr} N_r \frac{e_{Rr}}{e_{Er} + e_{Rr}} \right) - (e_{Rp} + e_{Rr}).$$

The first-order conditions are:

$$\frac{\partial \pi_E}{\partial e_{Er}} = \alpha_E \left[N_p \frac{n e_{Rp}}{(n e_{Er} + e_{Rp})^2} + N_r \frac{e_{Rr}}{(e_{Er} + e_{Rr})^2} \right] - (n+1) = 0 \tag{20}$$
$$\frac{\partial \pi_R}{\partial e_{Rp}} = \frac{\alpha_{Rp} N_p n e_{Er}}{(n e_{Er} + e_{Rp})^2} - 1 = 0 \tag{21}$$
$$\frac{\partial \pi_R}{\partial e_{Rr}} = \frac{\alpha_{Rr} N_r e_{Er}}{(e_{Er} + e_{Rr})^2} - 1 = 0 \tag{22}$$

From the conditions (21) and (22), we obtain that $e_{Rp} = -n e_{Er} + \sqrt{\alpha_{Rp} N_p n e_{Er}}$ and $e_{Rr} = -e_{Er} + \sqrt{\alpha_{Rr} N_r e_{Er}}$. After substituting these back into (15), it is straightforward to solve for the solutions, with the notation $B_1 = \sqrt{\alpha_{Rp}} \alpha_{Rr} N_p + \sqrt{\alpha_{Rr}} \alpha_{Rp} N_r$ and $B_2 = (\alpha_E + \alpha_{Rp}) \alpha_{Rr} N_p + \alpha_{Rp} (\alpha_E + \alpha_{Rr}) N_r$:

$$e_{Ep}^{ET} = N_p \left(\frac{B_1}{B_2} \alpha_E \right)^2, \ e_{Er}^{ET} = N_r \left(\frac{B_1}{B_2} \alpha_E \right)^2,$$
$$e_{Rp}^{ET} = \frac{B_1 \left[\sqrt{\alpha_{Rp}} \alpha_{Rr} (N_p + N_r) + \alpha_E \left(\sqrt{\alpha_{Rp}} - \sqrt{\alpha_{Rr}} \right) N_r \right]}{B_2^2} \alpha_E \alpha_{Rp} N_p,$$
$$e_{Rr}^{ET} = \frac{B_1 \left[\sqrt{\alpha_{Rr}} \alpha_{Rp} (N_p + N_r) + \alpha_E \left(\sqrt{\alpha_{Rr}} - \sqrt{\alpha_{Rp}} \right) N_p \right]}{B_2^2} \alpha_E \alpha_{Rr} N_r.$$

To make sure the solution is an optimal solution over the assumed parameter ranges, we check the second-order conditions.

$$\frac{\partial^2 \pi_E}{\partial e_{Er}^2} = -2\alpha_E \left[N_p \frac{n^2 e_{Rp}}{(n e_{Er} + e_{Rp})^3} + N_r \frac{e_{Rr}}{(e_{Er} + e_{Rp})^3} \right] < 0$$
$$H_R = \begin{pmatrix} -\frac{2\alpha_{Rp} N_p n e_{Er}}{(n e_{Er} + e_{Rp})^3} & 0 \\ 0 & -\frac{2\alpha_{Rr} N_r e_{Er}}{(e_{Er} + e_{Rr})^3} \end{pmatrix}$$

Using the equilibrium values of ad budgets, we can obtain the profit of the platform under the equal treatment policy:

$$\pi_P^{ET} = e_{Ep}^{ET} + e_{Er}^{ET} + e_{Rp}^{ET} + e_{Rr}^{ET}$$
$$= \frac{\alpha_E \alpha_{Rp} \alpha_{Rr} (N_p^2 + N_r^2) + \alpha_E \sqrt{\alpha_{Rp} \alpha_{Rr}} N_p N_r (\alpha_{Rp} + \alpha_{Rr})}{(\alpha_E + \alpha_{Rp}) \alpha_{Rr} N_p + \alpha_{Rp} (\alpha_E + \alpha_{Rr}) N_r}. \tag{23}$$

It is straightforward to see that the second-order conditions are satisfied for the optimal solution.

We now proceed to show that under the no restriction (NR) and equal treatment (ET) policies, protected users will see fewer economic opportunity ads than regular users. That is, $f_{Ep} < f_{Er}$, under both NR and ET.

Showing that $f_{Ep}^{NR} < f_{Er}^{NR}$:

Following the definition of ad share f_{ij}, we have:

$$f_{Ep}^{NR} = \frac{e_{Ep}^{NR}}{e_{Ep}^{NR} + e_{Rp}^{NR}} = \frac{\alpha_E}{\alpha_E + \alpha_{Rp}},$$
$$f_{Er}^{NR} = \frac{e_{Er}^{NR}}{e_{Er}^{NR} + e_{Rr}^{NR}} = \frac{\alpha_E}{\alpha_E + \alpha_{Rr}}.$$

Comparison of ad expense levels on *protected* users between advertisers:

$$\begin{aligned} & e_{Rp}^{NR} - e_{Ep}^{NR} \\ =& \frac{\alpha_E \alpha_{Rp}^2 N_p}{(\alpha_E + \alpha_{Rp})^2} - \frac{\alpha_E^2 \alpha_{Rp} N_p}{(\alpha_E + \alpha_{Rp})^2}, \\ =& \frac{\alpha_E \alpha_{Rp} N_p}{(\alpha_E + \alpha_{Rp})^2} (\alpha_{Rp} - \alpha_E) > 0 \text{ when } \alpha_{Rp} > \alpha_E. \end{aligned}$$

Comparison of ad expense levels on *regular* users between advertisers:

$$\begin{aligned} & e_{Rr}^{NR} - e_{Er}^{NR} \\ =& \frac{\alpha_E \alpha_{Rr}^2 N_r}{(\alpha_E + \alpha_{Rr})^2} - \frac{\alpha_E^2 \alpha_{Rr} N_r}{(\alpha_E + \alpha_{Rr})^2}, \\ =& \frac{\alpha_E \alpha_{Rr} N_r}{(\alpha_E + \alpha_{Rr})^2} (\alpha_{Rr} - \alpha_E) < 0 \text{ when } \alpha_{Rr} < \alpha_E. \end{aligned}$$

Showing that $f_{Ep}^{ET} < f_{Er}^{ET}$:

Follow the definition of ad share f_{ij}, we have:

$$f_{Ep}^{ET} = \frac{e_{Ep}^{ET}}{e_{Ep}^{ET} + e_{Rp}^{ET}} = \frac{\alpha_E B_1}{\sqrt{\alpha_{Rp}} B_2},$$
$$f_{Er}^{ET} = \frac{e_{Er}^{ET}}{e_{Er}^{ET} + e_{Rr}^{ET}} = \frac{\alpha_E B_1}{\sqrt{\alpha_{Rr}} B_2}.$$

Compare two advertisers' expenses on the protected users

$$\begin{aligned} e_{Ep}^{ET} - e_{Rp}^{ET} =& N_p \left(\frac{B_1}{B_2}\alpha_E\right)^2 - \frac{B_1 \left[\sqrt{\alpha_{Rp}}\alpha_{Rr}(N_p + N_r) + \alpha_E \left(\sqrt{\alpha_{Rp}} - \sqrt{\alpha_{Rr}}\right) N_r\right]}{B_2^2} \alpha_E \alpha_{Rp} N_p, \\ =& \frac{\alpha_E N_p B_1}{B_2^2} \left(\alpha_E B_1 - \alpha_{Rp} \left[\sqrt{\alpha_{Rp}}\alpha_{Rr}(N_p + N_r) + \alpha_E \left(\sqrt{\alpha_{Rp}} - \sqrt{\alpha_{Rr}}\right) N_r\right]\right), \\ =& \frac{\alpha_E N_p B_1}{B_2^2} \left[\alpha_E \left(\sqrt{\alpha_{Rp}}\alpha_{Rr} N_p + 2\sqrt{\alpha_{Rr}}\alpha_{Rp} N_r - \sqrt{\alpha_{Rp}}\alpha_{Rp} N_r\right) - \sqrt{\alpha_{Rp}}\alpha_{Rp}\alpha_{Rr}(N_p + N_r)\right]. \end{aligned}$$

Therefore, we can see that $e_{Ep}^{ET} < e_{Rp}^{ET}$ when $\alpha_E < \frac{\alpha_{Rp}\alpha_{Rr}(N_p+N_r)}{2\sqrt{\alpha_{Rp}\alpha_{Rr}}N_r + \alpha_{Rr}N_p - \alpha_{Rp}N_r}$. We denote this upper threshold for α_E as $\hat{\alpha}_E^{ET-h}$. Notably, we find that $\hat{\alpha}_E^{ET-h}$ is larger than α_{Rp} with the following comparison:

$$\begin{aligned}\hat{\alpha}_E^{ET-h} - \alpha_{Rp} =& \frac{\alpha_{Rp}\alpha_{Rr}(N_p+N_r)}{2\sqrt{\alpha_{Rp}\alpha_{Rr}}N_r + \alpha_{Rr}N_p - \alpha_{Rp}N_r} - \alpha_{Rp},\\ =& \alpha_{Rp}\left[\frac{\alpha_{Rr}(N_p+N_r) - 2\sqrt{\alpha_{Rp}\alpha_{Rr}}N_r - \alpha_{Rr}N_p + \alpha_{Rp}N_r}{2\sqrt{\alpha_{Rp}\alpha_{Rr}}N_r + \alpha_{Rr}N_p - \alpha_{Rp}N_r}\right],\\ =& \alpha_{Rp}\left[\frac{\left(\alpha_{Rp} - 2\sqrt{\alpha_{Rp}\alpha_{Rr}} + \alpha_{Rr}\right)N_r}{2\sqrt{\alpha_{Rp}\alpha_{Rr}}N_r + \alpha_{Rr}N_p - \alpha_{Rp}N_r}\right] > 0.\end{aligned}$$

Compare two advertisers' expenses on the regular users:

$$\begin{aligned}e_{Er}^{ET} - e_{Rr}^{ET} =& N_r\left(\frac{B_1}{B_2}\alpha_E\right)^2 - \frac{B_1\left[\sqrt{\alpha_{Rr}}\alpha_{Rp}(N_p+N_r) + \alpha_E\left(\sqrt{\alpha_{Rr}} - \sqrt{\alpha_{Rp}}\right)N_p\right]}{B_2^2}\alpha_E\alpha_{Rr}N_r,\\ =& \frac{\alpha_E N_r B_1}{B_2^2}\left(\alpha_E B_1 - \alpha_{Rr}\left[\sqrt{\alpha_{Rr}}\alpha_{Rp}(N_p+N_r) + \alpha_E\left(\sqrt{\alpha_{Rr}} - \sqrt{\alpha_{Rp}}\right)N_p\right]\right),\\ =& \frac{\alpha_E N_r B_1}{B_2^2}\left[\alpha_E\left(\sqrt{\alpha_{Rr}}\alpha_{Rr}N_p + 2\sqrt{\alpha_{Rp}}\alpha_{Rr}N_p + \alpha_{Rp}\sqrt{\alpha_{Rr}}N_r\right) - \alpha_{Rp}\alpha_{Rr}\sqrt{\alpha_{Rr}}(N_p+N_r)\right].\end{aligned}$$

Therefore, we can see that $e_{Er}^{ET} > e_{Rr}^{ET}$ when $\alpha_E < \frac{\alpha_{Rp}\alpha_{Rr}(N_p+N_r)}{2\sqrt{\alpha_{Rp}\alpha_{Rr}}N_p - \alpha_{Rr}N_p + \alpha_{Rp}N_r}$. We denote this upper threshold for α_E as $\hat{\alpha}_E^{ET-l}$.

Compare ad E's market share between user groups

With $f_{Ep}^{ET} - f_{Er}^{ET} = \frac{\alpha_E B_1}{\sqrt{\alpha_{Rp}}B_2} - \frac{\alpha_E B_1}{\sqrt{\alpha_{Rr}}B_2}$, we can easily see that $f_{Ep}^{ET} < f_{Er}^{ET}$ as long as $\alpha_{Rr} < \alpha_{Rp}$. ∎

A.3 Proof of Equilibrium Under Equal Exposure with Equal Treatment Policy

We now solve the problem under the equal exposure with equal treatment (EET) policy and derive the optimal result. We know that under equal exposure with equal treatment, the platform announces a budget increase of E by $\Delta e \geqslant 0$ for the protected user in order to close any exposure gap. Thus, the number of impressions received by advertisers E and R, respectively, are as follows:

$$n_p^E = N_p\frac{e_{Ep} + \Delta e}{e_{Ep} + \Delta e + e_{Rp}}, \tag{24}$$

$$n_p^R = N_p\frac{e_{Rp}}{e_{Ep} + \Delta e + e_{Rp}}, \tag{25}$$

$$n_r^E = N_r\frac{e_{Er}}{e_{Er} + e_{Rr}}, \tag{26}$$

$$n_r^R = N_r\frac{e_{Rr}}{e_{Er} + e_{Rr}}. \tag{27}$$

Under equal exposure with equal treatment policy, we need to solve the problems of advertisers E and R under the following two constraints:

$$\text{Equal Treatment Constraint: } \frac{e_{Ep}}{N_p} = \frac{e_{Er}}{N_r},$$

$$\text{Equal Exposure Constraint: } \frac{n_p^E}{N_p} = \frac{n_r^E}{N_r}.$$

Substituting the values of n_p^E and n_r^E from (24) and (26), the above constraints can be re-written as follows:

$$\text{Equal Treatment Constraint: } e_{Ep} = \frac{N_p}{N_r} e_{Er}, \tag{28}$$

$$\text{Equal Exposure Constraint: } \Delta e = \frac{e_{Er}e_{Rp} - e_{Ep}e_{Rr}}{e_{Rr}}. \tag{29}$$

The profit functions of E and R can be written as follows:

$$\pi_R = \alpha_{Rp} n_p^R + \alpha_{Rr} n_r^R - e_{Rp} - e_{Rr}, \tag{30}$$

$$\pi_E = \alpha_{E} n_p^E + \alpha_{E} n_r^E - e_{Ep} - e_{Er}. \tag{31}$$

We solve for a rational expectations equilibrium. Thus, both E and R rationally internalize the constraints in (28) and (29) in their optimization problem. Thus, the problem of E can be written as

$$\max_{e_{Ep}, e_{Er}} \pi_E,$$
$$\text{Subject to: (28) and (29).}$$

Similarly, the problem of R can be written as

$$\max_{e_{Rp}, e_{Rr}} \pi_R,$$
$$\text{Subject to: (28) and (29).}$$

We now proceed to solve the problems of E and R. Since the constraints in (28) and (29) are equality constraints, we can directly substitute these constraints in the objective function. Substituting the values of Δe and e_{Ep} from constraints (28) and (29) in π_R, we get

$$\pi_R = \frac{e_{Rr}\left(N_p\alpha_{Rp} + N_r\alpha_{Rr} - e_{Rp} - e_{Rr}\right) - e_{Er}\left(e_{Rp} + e_{Rr}\right)}{e_{Er} + e_{Rr}}. \tag{32}$$

Taking derivative of π_R with respect to e_{Rp}, we get

$$\frac{\partial \pi_R}{\partial e_{Rp}} = -1.$$

Since $\frac{\partial \pi_R}{\partial e_{Rp}} < 0$, the optimal value of e_{Rp} is the minimum possible value. In equation (29), we see that $\Delta e = \frac{e_{Er}e_{Rp} - e_{Ep}e_{Rr}}{e_{Rr}}$ is increasing in e_{Rp}. Since R wants to choose the minimum value of e_{Rp}, it will choose e_{Rp} such that $\Delta e = 0$. Thus, solving $\Delta e = 0$ for e_{Rp}, we get

$$e_{Rp} = \frac{e_{Ep}e_{Rr}}{e_{Er}}. \tag{33}$$

From ET constraint in (28), we know that $\frac{e_{Ep}}{e_{Er}} = \frac{N_p}{N_r}$. Substituting this in the above equation, we get

$$e_{Rp} = \frac{N_p e_{Rr}}{N_r}. \tag{34}$$

Substituting the optimal value of e_{Rp} from above in the expression of π_R in (32) and taking derivatives with respect to e_{Rr}, we get

$$\frac{\partial \pi_R}{\partial e_{Rr}} = -\frac{-e_{Er}N_r\left(N_p\alpha_{Rp}+N_r\alpha_{Rr}\right)+2e_{Er}e_{Rr}\left(N_p+N_r\right)+e_{Er}^2\left(N_p+N_r\right)+e_{Rr}^2\left(N_p+N_r\right)}{N_r\left(e_{Er}+e_{Rr}\right)^2}, \tag{35}$$

$$\frac{\partial^2 \pi_R}{\partial e_{Rr}^2} = -\frac{2e_{Er}\left(N_p\alpha_{Rp}+N_r\alpha_{Rr}\right)}{\left(e_{Er}+e_{Rr}\right)^3} \tag{36}$$

Solving $\frac{\partial \pi_R}{\partial e_{Rr}} = 0$ for e_{Rr}, we get

$$e_{Rr} = \frac{\sqrt{e_{Er}N_r\left(N_p+N_r\right)\left(N_p\alpha_{Rp}+N_r\alpha_{Rr}\right)}}{N_p+N_r} - e_{Er}. \tag{37}$$

Since $\frac{\partial^2 \pi_R}{\partial e_{Rr}^2} < 0$, the above solution is optimal.

We now proceed to solve the problem of E. Similar to the above, substituting Δe and e_{Ep} from constraints (28) and (29) in the expression of π_E in (31) we get,

$$\pi_E = e_{Er}\left(N_p+N_r\right)\left(\frac{\alpha_E}{e_{Er}+e_{Rr}} - \frac{1}{N_r}\right). \tag{38}$$

Taking the derivative of π_E with respect to e_{Er}, we get

$$\frac{\partial \pi_E}{\partial e_{Er}} = \left(N_p+N_r\right)\left(\frac{\alpha_E e_{Rr}}{\left(e_{Er}+e_{Rr}\right)^2} - \frac{1}{N_r}\right),$$

$$\frac{\partial^2 \pi_E}{\partial e_{Er}^2} = -\frac{2\alpha_E e_{Rr}\left(N_p+N_r\right)}{\left(e_{Er}+e_{Rr}\right)^3}.$$

We see that $\frac{\partial^2 \pi_E}{\partial e_{Er}^2} < 0$. Thus, π_E is concave in e_{Er}, and the solution to the first-order condition will be optimal. Solving $\frac{\partial \pi_E}{\partial e_{Er}} = 0$ for e_{Er}, we get

$$e_{Er} = \sqrt{\alpha_E}\sqrt{e_{Rr}}\sqrt{N_r} - e_{Rr}. \tag{39}$$

Having obtained the optimal responses of both E and R, we now solve these decisions simultaneously to obtain the equilibrium decisions. Solving the above equation and (37) simultaneously for e_{Er} and e_{Rr}, we get

$$e_{Er}^* = \frac{\alpha_E^2 N_r\left(N_p+N_r\right)\left(N_p\alpha_{Rp}+N_r\alpha_{Rr}\right)}{\left(N_p\left(\alpha_E+\alpha_{Rp}\right)+N_r\left(\alpha_E+\alpha_{Rr}\right)\right)^2}, \tag{40}$$

$$e_{Rr}^* = \frac{\alpha_E N_r\left(N_p\alpha_{Rp}+N_r\alpha_{Rr}\right)^2}{\left(N_p\left(\alpha_E+\alpha_{Rp}\right)+N_r\left(\alpha_E+\alpha_{Rr}\right)\right)^2}. \tag{41}$$

Substituting the value of e_{Rr}^* in (34), we get

$$e_{Rp}^* = \frac{\alpha_E N_p\left(N_p\alpha_{Rp}+N_r\alpha_{Rr}\right)^2}{\left(N_p\left(\alpha_E+\alpha_{Rp}\right)+N_r\left(\alpha_E+\alpha_{Rr}\right)\right)^2}. \tag{42}$$

Using $e_{Ep} = \frac{N_p}{N_r}e_{Er}$ from (28), we get

$$e_{Ep}^* = \frac{\alpha_E^2 N_p\left(N_p+N_r\right)\left(N_p\alpha_{Rp}+N_r\alpha_{Rr}\right)}{\left(N_p\left(\alpha_E+\alpha_{Rp}\right)+N_r\left(\alpha_E+\alpha_{Rr}\right)\right)^2}. \tag{43}$$

Having obtained the optimal decisions of advertisers, we can now write the platform's profit.

$$\begin{aligned}\pi_{P}^{EET} &= e_{Er}^{*} + e_{Rr}^{*} + e_{Rp}^{*} + e_{Ep}^{*},\\ &= \frac{\alpha_{E}\left(N_p + N_r\right)\left(N_p\alpha_{Rp} + N_r\alpha_{Rr}\right)}{N_p\left(\alpha_{E} + \alpha_{Rp}\right) + N_r\left(\alpha_{E} + \alpha_{Rr}\right)}.\end{aligned} \tag{44}$$

We now proceed to show that under the equal exposure with equal treatment policy, both advertisers allocate their bidding expenses proportional to the population size of user groups. That is,

$$\frac{e_{Ep}^{EET}}{e_{Er}^{EET}} = \frac{e_{Rp}^{EET}}{e_{Rr}^{EET}} = \frac{N_p}{N_r}.$$

From the equilibrium ad-budgets under the equal exposure with equal treatment policy, we can easily obtain that $\frac{e_{Ep}^{EET}}{e_{Er}^{EET}} = \frac{e_{Rp}^{EET}}{e_{Rr}^{EET}} = \frac{N_p}{N_r}$. Next, we compare the ad budget between the two advertisers:

$$\begin{aligned}& e_{Rp}^{EET} - e_{Ep}^{EET}\\ =& \frac{\alpha_{E}(N_p + N_r)\left(\alpha_{Rp}N_p + \alpha_{Rr}N_r\right)N_p}{\left[\left(\alpha_{E} + \alpha_{Rp}\right)N_p + \left(\alpha_{E} + \alpha_{Rr}\right)N_r\right]^2}\left[\left(\alpha_{Rp}N_p + \alpha_{Rr}N_r\right) - \alpha_{E}\left(N_p + N_r\right)\right]\\ & e_{Rr}^{EET} - e_{Er}^{EET}\\ =& \frac{\alpha_{E}(N_p + N_r)\left(\alpha_{Rp}N_p + \alpha_{Rr}N_r\right)N_r}{\left[\left(\alpha_{E} + \alpha_{Rp}\right)N_p + \left(\alpha_{E} + \alpha_{Rr}\right)N_r\right]^2}\left[\left(\alpha_{Rp}N_p + \alpha_{Rr}N_r\right) - \alpha_{E}\left(N_p + N_r\right)\right]\end{aligned}$$

We can see that these two comparisons share the main term. Therefore, to have $e_{Ep}^{EET} < e_{Rp}^{EET}$ and $e_{Er}^{EET} < e_{Rr}^{EET}$, we require α_{E} satisfies:

$$\alpha_{E} \leqslant \bar{\alpha}_{R} = \frac{\alpha_{Rp}N_p + \alpha_{Rr}N_r}{N_p + N_r}.$$

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A.4 Proof of Theorem 1

(i) EET vs. NR: We first compare the profits of the platform under the equal exposure with equal treatment (EET) and the no restriction (NR) policies. Using the values of π_{P}^{EET} and π_{P}^{NR} from (44) and (19), we obtain:

$$\begin{aligned}& \pi_{P}^{EET} - \pi_{P}^{NR}\\ =& \frac{\alpha_{E}\left(N_p + N_r\right)\left(\alpha_{Rp}N_p + \alpha_{Rr}N_r\right)}{\left[\left(\alpha_{E} + \alpha_{Rp}\right)N_p + \left(\alpha_{E} + \alpha_{Rr}\right)N_r\right]} - \alpha_{E}\left(\frac{\alpha_{Rp}N_p}{\alpha_{E} + \alpha_{Rp}} + \frac{\alpha_{Rr}N_r}{\alpha_{E} + \alpha_{Rr}}\right)\\ =& \alpha_{E}\frac{\begin{array}{c}\left(N_p + N_r\right)\left(\alpha_{Rp}N_p + \alpha_{Rr}N_r\right)\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right)\\ -\left[\alpha_{Rp}N_p\left(\alpha_{E} + \alpha_{Rr}\right) + \alpha_{Rr}N_r\left(\alpha_{E} + \alpha_{Rp}\right)\right]\left[\left(\alpha_{E} + \alpha_{Rp}\right)N_p + \left(\alpha_{E} + \alpha_{Rr}\right)N_r\right]\end{array}}{\left[\left(\alpha_{E} + \alpha_{Rp}\right)N_p + \left(\alpha_{E} + \alpha_{Rr}\right)N_r\right]\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right)}\\ =& \alpha_{E}\frac{\begin{array}{r}\left[\alpha_{Rp}N_p^2 + \left(\alpha_{Rp} + \alpha_{Rr}\right)N_pN_r + \alpha_{Rr}N_r^2\right]\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right) - \alpha_{Rp}N_p^2\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right)\\ -\alpha_{Rp}N_pN_r\left(\alpha_{E} + \alpha_{Rr}\right)^2 - \alpha_{Rr}N_pN_r\left(\alpha_{E} + \alpha_{Rp}\right)^2 - \alpha_{Rr}N_r^2\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right)\end{array}}{\left[\left(\alpha_{E} + \alpha_{Rp}\right)N_p + \left(\alpha_{E} + \alpha_{Rr}\right)N_r\right]\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right)}\\ =& \alpha_{E}\frac{N_pN_r\left[\left(\alpha_{Rp} + \alpha_{Rr}\right)\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right) - \alpha_{Rr}\left(\alpha_{E} + \alpha_{Rp}\right)^2 - \alpha_{Rp}\left(\alpha_{E} + \alpha_{Rr}\right)^2\right]}{\left[\left(\alpha_{E} + \alpha_{Rp}\right)N_p + \left(\alpha_{E} + \alpha_{Rr}\right)N_r\right]\left(\alpha_{E} + \alpha_{Rp}\right)\left(\alpha_{E} + \alpha_{Rr}\right)}\end{aligned}$$

$$=\alpha_E \frac{N_p N_r \left(\alpha_{Rp}-\alpha_{Rr}\right)^2 \alpha_E}{\left[\left(\alpha_E+\alpha_{Rp}\right) N_p+\left(\alpha_E+\alpha_{Rr}\right) N_r\right]\left(\alpha_E+\alpha_{Rp}\right)\left(\alpha_E+\alpha_{Rr}\right)} \geqslant 0.$$

Thus, $\pi_P^{EET} \geqslant \pi_P^{NR}$.

(ii) EET vs. ET: Now we compare the platform's profits under the equal exposure with equal treatment (EET) and the equal treatment (ET) policies. Using the values of π_P^{EET} and π_P^{ET} from (44) and (23), we obtain:

$$\pi_P^{EET}-\pi_P^{ET}=\frac{\alpha_E\left(N_p+N_r\right)\left(\alpha_{Rp} N_p+\alpha_{Rr} N_r\right)}{\left[\left(\alpha_E+\alpha_{Rp}\right) N_p+\left(\alpha_E+\alpha_{Rr}\right) N_r\right]}-\frac{\alpha_E \alpha_{Rp} \alpha_{Rr}(N_p^2+N_r^2)+\alpha_E \sqrt{\alpha_{Rp} \alpha_{Rr}} N_p N_r(\alpha_{Rp}+\alpha_{Rr})}{(\alpha_E+\alpha_{Rp}) \alpha_{Rr} N_p+\alpha_{Rp}(\alpha_E+\alpha_{Rr}) N_r}$$

The sign of the above equation is determined by the numerators; thus, we focus on:

$$\begin{aligned}
&\left(N_p+N_r\right)\left(\alpha_{Rp} N_p+\alpha_{Rr} N_r\right)\left[(\alpha_E+\alpha_{Rp}) \alpha_{Rr} N_p+\alpha_{Rp}(\alpha_E+\alpha_{Rr}) N_r\right] \\
&\quad-\left[\alpha_{Rp} \alpha_{Rr}(N_p^2+N_r^2)+\sqrt{\alpha_{Rp} \alpha_{Rr}} N_p N_r(\alpha_{Rp}+\alpha_{Rr})\right]\left[\left(\alpha_E+\alpha_{Rp}\right) N_p+\left(\alpha_E+\alpha_{Rr}\right) N_r\right] \\
=&\alpha_E N_p N_r(\sqrt{\alpha_{Rp}}-\sqrt{\alpha_{Rr}})^2\left[\alpha_E\left(\alpha_{Rp}+\alpha_{Rr}+\sqrt{\alpha_{Rp} \alpha_{Rr}}\right)\left(N_p+N_r\right)-\sqrt{\alpha_{Rp} \alpha_{Rr}}\left(\alpha_{Rp} N_p+\alpha_{Rr} N_r\right)\right]
\end{aligned}$$

Therefore, when α_E satisfies the following condition, equal exposure with equal treatment strategy dominates equal treatment for the platform's profit ($\pi_P^{EET} \geqslant \pi_P^{ET}$):

$$\alpha_E \geqslant \hat{\alpha}_E^{\pi_P}=\frac{\sqrt{\alpha_{Rp} \alpha_{Rr}}\left(\alpha_{Rp} N_p+\alpha_{Rr} N_r\right)}{\left(\alpha_{Rp}+\alpha_{Rr}+\sqrt{\alpha_{Rp} \alpha_{Rr}}\right)\left(N_p+N_r\right)}.$$

Thus, $\pi_P^{EET} \geqslant \pi_P^{ET}$ when $\alpha_E \geqslant \hat{\alpha}_E^{\pi_P}$.

(iii) NR vs. ET: We now compare the profits of the platform under the no restriction (NR) and the equal treatment (ET) policies. Using the values of π_P^{NR} and π_P^{ET} from (19) and (23), we obtain:

$$\begin{aligned}
\pi_P^{NR}-\pi_P^{ET}&=\alpha_E\left(\frac{\alpha_{Rp} N_p}{\alpha_E+\alpha_{Rp}}+\frac{\alpha_{Rr} N_r}{\alpha_E+\alpha_{Rr}}\right)-\frac{\alpha_E \alpha_{Rp} \alpha_{Rr}(N_p^2+N_r^2)+\alpha_E \sqrt{\alpha_{Rp} \alpha_{Rr}} N_p N_r(\alpha_{Rp}+\alpha_{Rr})}{(\alpha_E+\alpha_{Rp}) \alpha_{Rr} N_p+\alpha_{Rp}(\alpha_E+\alpha_{Rr}) N_r} \\
&=\frac{N_p N_r \alpha_E(\sqrt{\alpha_{Rp}}-\sqrt{\alpha_{Rr}})^2(\alpha_E-\sqrt{\alpha_{Rp} \alpha_{Rr}})\left[\alpha_{Rp} \alpha_{Rr}+\alpha_E(\alpha_{Rp}+\sqrt{\alpha_{Rp} \alpha_{Rr}}+\alpha_{Rr})\right]}{(\alpha_E+\alpha_{Rp})(\alpha_E+\alpha_{Rr})\left[(\alpha_E+\alpha_{Rp}) \alpha_{Rr} N_p+\alpha_{Rp}(\alpha_E+\alpha_{Rr}) N_r\right]}
\end{aligned}$$

It is easy see that $\pi_P^{NR} \geqslant \pi_P^{ET}$, when $\alpha_E \geqslant \sqrt{\alpha_{Rp} \alpha_{Rr}}$. ■

A.5 Proof of Proposition 1

We compare the ad-exposure gap between NR and ET. Let $\Delta f^{NR}=|f_{Ep}-f_{Er}|^{NR}$ and $\Delta f^{ET}=|f_{Ep}-f_{Er}|^{ET}$. Then, we can write

$$\begin{aligned}
|f_{Ep}-f_{Er}|^{NR}-|f_{Ep}-f_{Er}|^{ET}=&\frac{\alpha_E(\alpha_{Rp}-\alpha_{Rr})}{(\alpha_E+\alpha_{Rp})(\alpha_E+\alpha_{Rr})}-\frac{\left(\sqrt{\alpha_{Rp}}-\sqrt{\alpha_{Rr}}\right)\left(\sqrt{\alpha_{Rr}} N_p+\sqrt{\alpha_{Rp}} N_r\right)}{(\alpha_E+\alpha_{Rp}) \alpha_{Rr} N_p+\alpha_{Rp}(\alpha_E+\alpha_{Rr}) N_r}, \\
=&\frac{\alpha_E(\sqrt{\alpha_{Rp}}-\sqrt{\alpha_{Rr}})\left(\sqrt{\alpha_{Rp} \alpha_{Rr}}-\alpha_E\right)}{(\alpha_E+\alpha_{Rp})(\alpha_E+\alpha_{Rr})\left((\alpha_E+\alpha_{Rp}) \alpha_{Rr} N_p+\alpha_{Rp}(\alpha_E+\alpha_{Rr}) N_r\right)}.
\end{aligned}$$

Thus, $|f_{Ep}-f_{Er}|^{NR}<|f_{Ep}-f_{Er}|^{ET}$ when $\alpha_E \geqslant \sqrt{\alpha_{Rp} \alpha_{Rr}}$. Therefore, the equal treatment (ET) policy results in a lower level of fairness than the baseline no restriction (NR) policy. ■

A.6 Proof of Proposition 2

To compare the total number of ads E exposure, we first compare the ads E's share (f_{Ep} and f_{Er}) among three policies.

(i) EET vs. NR: We compare E's market share between no restriction (NR) and equal exposure with equal treatment (EET) policies in each user group. For the protected group, we check the sign of $f_{Ep}^{EET} - f_{Ep}^{NR}$:

$$\begin{aligned}
& f_{Ep}^{EET} - f_{Ep}^{NR} \\
= & \frac{\alpha_E (N_p + N_r)}{(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r} - \frac{\alpha_E}{\alpha_E + \alpha_{Rp}} \\
= & \alpha_E \frac{(N_p + N_r)(\alpha_E + \alpha_{Rp}) - (\alpha_E + \alpha_{Rp}) N_p - (\alpha_E + \alpha_{Rr}) N_r}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r](\alpha_E + \alpha_{Rp})} \\
= & \alpha_E \frac{(\alpha_E + \alpha_{Rp}) N_r - (\alpha_E + \alpha_{Rr}) N_r}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r](\alpha_E + \alpha_{Rp})} \\
= & \frac{\alpha_E (\alpha_{Rp} - \alpha_{Rr}) N_r}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r](\alpha_E + \alpha_{Rp})} > 0;
\end{aligned}$$

Similarly, for the regular users, we check:

$$\begin{aligned}
& f_{Er}^{EET} - f_{Er}^{NR} \\
= & \frac{\alpha_E (N_p + N_r)}{(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r} - \frac{\alpha_E}{\alpha_E + \alpha_{Rr}} \\
= & \alpha_E \frac{(N_p + N_r)(\alpha_E + \alpha_{Rr}) - (\alpha_E + \alpha_{Rp}) N_p - (\alpha_E + \alpha_{Rr}) N_r}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r](\alpha_E + \alpha_{Rr})} \\
= & \frac{\alpha_E (\alpha_{Rr} - \alpha_{Rp}) N_p}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r](\alpha_E + \alpha_{Rr})} < 0.
\end{aligned}$$

Thus, we see that

$$f_{Ep}^{EET} > f_{Ep}^{NR}, \quad f_{Er}^{EET} > f_{Er}^{NR}.$$

Next, we compare the overall consumer surplus in terms of the total number of users who saw ads E:

$$\begin{aligned}
& N_p(f_{Ep}^{EET} - f_{Ep}^{NR}) + N_r(f_{Er}^{EET} - f_{Er}^{NR}) \\
= & \frac{\alpha_E (\alpha_{Rp} - \alpha_{Rr}) N_p N_r}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r](\alpha_E + \alpha_{Rp})} + \frac{\alpha_E (\alpha_{Rr} - \alpha_{Rp}) N_p N_r}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r](\alpha_E + \alpha_{Rr})} \\
= & \frac{\alpha_E (\alpha_{Rp} - \alpha_{Rr}) N_p N_r}{[(\alpha_E + \alpha_{Rp}) N_p + (\alpha_E + \alpha_{Rr}) N_r]} \left(\frac{1}{\alpha_E + \alpha_{Rp}} - \frac{1}{\alpha_E + \alpha_{Rr}} \right) < 0.
\end{aligned}$$

(ii) EET vs. ET: Following the same process as above, we compare E's market share between equal exposure with equal treatment (EET) and equal treatment (ET) policies. Among the protected users, we calculate:

$$
\begin{aligned}
& f_{Ep}^{EET} - f_{Ep}^{ET} \\
= & \frac{\alpha_E(N_p+N_r)}{\left[(\alpha_E+\alpha_{Rp})N_p + (\alpha_E+\alpha_{Rr})N_r\right]} - \frac{\alpha_E B_1}{\sqrt{\alpha_{Rp}}B_2} \\
= & \frac{\alpha_E\left(\sqrt{\alpha_{Rp}}-\sqrt{\alpha_{Rr}}\right)N_r\left[(\alpha_E+\alpha_{Rr})(N_p+N_r)\sqrt{\alpha_{Rp}} - \sqrt{\alpha_{Rr}}N_p(\alpha_{Rp}-\alpha_{Rr})\right]}{\left[(\alpha_E+\alpha_{Rp})N_p + (\alpha_E+\alpha_{Rr})N_r\right]B_2}.
\end{aligned}
$$

In the above, we can see that $f_{Ep}^{EET} > f_{Ep}^{ET}$, when $\alpha_E \geqslant \frac{N_p(\alpha_{Rp}-\alpha_{Rr})\sqrt{\alpha_{Rr}}}{(N_p+N_r)\sqrt{\alpha_{Rp}}} - \alpha_{Rr}$.

For the regular users, we check:

$$
\begin{aligned}
& f_{Er}^{EET} - f_{Er}^{ET} \\
= & \frac{\alpha_E(N_p+N_r)}{\left[(\alpha_E+\alpha_{Rp})N_p + (\alpha_E+\alpha_{Rr})N_r\right]} - \frac{\alpha_E B_1}{\sqrt{\alpha_{Rr}}B_2} \\
= & \frac{-\alpha_E\left(\sqrt{\alpha_{Rp}}-\sqrt{\alpha_{Rr}}\right)N_p\left[(\alpha_E+\alpha_{Rp})(N_p+N_r)\sqrt{\alpha_{Rr}} + \sqrt{\alpha_{Rp}}N_r(\alpha_{Rp}-\alpha_{Rr})\right]}{\left[(\alpha_E+\alpha_{Rp})N_p + (\alpha_E+\alpha_{Rr})N_r\right]B_2}.
\end{aligned}
$$

We observe that under the normal parameter condition of $\alpha_{Rp} > \alpha_{Rr}$, we have $f_{Er}^{EET} < f_{Er}^{ET}$.

Next, the overall consumer comparison $N_p(f_{Ep}^{EET} - f_{Ep}^{ET}) + N_r(f_{Er}^{EET} - f_{Er}^{ET})$ is equivalent to:

$$
\begin{aligned}
& \left[(\alpha_E+\alpha_{Rr})(N_p+N_r)\sqrt{\alpha_{Rp}} - \sqrt{\alpha_{Rr}}N_p(\alpha_{Rp}-\alpha_{Rr})\right] \\
& \quad - \left[(\alpha_E+\alpha_{Rp})(N_p+N_r)\sqrt{\alpha_{Rr}} + \sqrt{\alpha_{Rp}}N_r(\alpha_{Rp}-\alpha_{Rr})\right] \\
= & \left(\alpha_E - \sqrt{\alpha_{Rp}\alpha_{Rr}}\right)(N_p+N_r)\left(\sqrt{\alpha_{Rp}}-\sqrt{\alpha_{Rr}}\right) - \left(\sqrt{\alpha_{Rr}}N_p + \sqrt{\alpha_{Rp}}N_r\right)(\alpha_{Rp}-\alpha_{Rr})
\end{aligned}
$$

Thus, the equal exposure with equal treatment policy leads to lower overall exposure to ads E when $\alpha_E < \hat{\alpha}_E^{CS} = 2\sqrt{\alpha_{Rp}\alpha_{Rr}} + \frac{\alpha_{Rr}N_p + \alpha_{Rp}N_r}{N_p+N_r}$.

(iii) NR vs. ET: Following the same process as above, we compare E's market share between the no restriction (NR) and equal treatment (ET) policies. Among the protected users, we calculate:

$$
\begin{aligned}
& f_{Ep}^{NR} - f_{Ep}^{ET} \\
= & \frac{\alpha_E}{\alpha_E+\alpha_{Rp}} - \frac{\alpha_E B_1}{\sqrt{\alpha_{Rp}}B_2} \\
= & \frac{(\alpha_E+\alpha_{Rp})\alpha_{Rr}N_p + \alpha_{Rp}(\alpha_E+\alpha_{Rr})N_r - \left(\alpha_{Rr}N_p + \sqrt{\alpha_{Rp}\alpha_{Rr}}N_r\right)(\alpha_E+\alpha_{Rp})}{(\alpha_E+\alpha_{Rp})B_2}\alpha_E \\
= & \frac{\alpha_E N_r(\alpha_E - \sqrt{\alpha_{Rp}\alpha_{Rr}})(\alpha_{Rp} - \sqrt{\alpha_{Rp}\alpha_{Rr}})}{(\alpha_E+\alpha_{Rp})B_2}.
\end{aligned}
$$

For the regular users, we check:

$$
\begin{aligned}
& f_{Er}^{NR} - f_{Er}^{ET} \\
= & \frac{\alpha_E}{\alpha_E+\alpha_{Rr}} - \frac{\alpha_E B_1}{\sqrt{\alpha_{Rr}}B_2} \\
= & \frac{(\alpha_E+\alpha_{Rp})\alpha_{Rr}N_p + \alpha_{Rp}(\alpha_E+\alpha_{Rr})N_r - \left(\alpha_{Rp}N_r + \sqrt{\alpha_{Rp}\alpha_{Rr}}N_p\right)(\alpha_E+\alpha_{Rr})}{(\alpha_E+\alpha_{Rr})B_2}\alpha_E \\
= & \frac{\alpha_E N_p(\sqrt{\alpha_{Rp}\alpha_{Rr}} - \alpha_E)(\sqrt{\alpha_{Rp}\alpha_{Rr}} - \alpha_{Rr})}{(\alpha_E+\alpha_{Rr})B_2}.
\end{aligned}
$$

We can observe that $f_{Ep}^{NR} > f_{Ep}^{ET}$ and $f_{Er}^{NR} < f_{Er}^{ET}$ when $\alpha_E > \sqrt{\alpha_{Rp}\alpha_{Rr}}$.

Next, the overall consumer comparison $N_p(f_{Ep}^{NR} - f_{Ep}^{ET}) + N_r(f_{Er}^{NR} - f_{Er}^{ET})$ is equivalent to:

$$\begin{aligned}
&(\alpha_E - \sqrt{\alpha_{Rp}\alpha_{Rr}})\left[(\alpha_{Rp} - \sqrt{\alpha_{Rp}\alpha_{Rr}})(\alpha_E + \alpha_{Rr}) - (\sqrt{\alpha_{Rp}\alpha_{Rr}} - \alpha_{Rr})(\alpha_E + \alpha_{Rp})\right] \\
&= (\alpha_E - \sqrt{\alpha_{Rp}\alpha_{Rr}})^2(\sqrt{\alpha_{Rp}} - \sqrt{\alpha_{Rr}})^2 > 0.
\end{aligned}$$

■