Entrepreneurs and Investors: Funding-Induced Distortions in Lean Startup Product Experiments and Innovation

Online Appendix

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B Proofs of Results in the Main Text

For convenience, we expand the labels for elements in Ω to $\mathbb{Z}$ such that, for $n_1, n_2 \in \mathbb{Z}$, the two labels represent the same element in Ω if and only if $n_1 - n_2 = aN$ for some $a \in \mathbb{Z}$.

B.1 Proof of Theorem 1

By Assumption 3(a), when the entrepreneur's type is low, the payoff from the final product is always less than K. Therefore, it is optimal for the low-type entrepreneur to abort the venture.

For the sake of generality, let $s \in \Omega$ be the realized signal received by the high-type entrepreneur. We let $V_{ij}(\lambda, \rho)$, where $i, j \in \{0, 1\}$, be the high-type entrepreneur's expected payoff (including the investment cost K) from the venture by the following strategy: 1. The test product is (λ, ρ); 2. When the test product sells, the optimal final product is launched if and only if $i = 1$; 3. When the test product does not sell, the optimal final product is launched if and only if $j = 1$. Given these definitions, the optimal test product can be characterized as the solution to the following problem:

$$\max_{\lambda \in \Omega, \rho \in [0,V], i,j \in \{0,1\}} V_{ij}(\lambda, \rho). \tag{B.1}$$

We analyze the high-type entrepreneur's expected payoff for each possible choice of λ.

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As there is no information asymmetry between the entrepreneur and the "investor", the expected payment to the "investor" from the entrepreneur is always exactly K if the venture is funded. Moreover, since the high-type entrepreneur's belief is always supported on $\{s, s+1\}$, whenever the final product is launched, the high-type entrepreneur always chooses $\Lambda \in \{s, s+1\}$. Consequently, there are only two possible payoffs for the entrepreneur from the final product: $P^{tp} - K$ when $W = \Lambda$ and $P^{fp} - K$ when $|W - \Lambda| = 1$.

Case $\lambda = s$.

If the final product is launched regardless of the test outcome, then the high-type entrepreneur's payoff would be $P^{tp} - K$ with probability $rf^{tp}(\rho) + \max\{r(1 - f^{tp}(\rho)), (1-r)(1 - f^{fp}(\rho))\}$ and $P^{fp} - K$ with the remaining probability. If $r(1-f^{tp}(\rho)) \geq (1-r)(1-f^{fp}(\rho))$, then $\Lambda = s$ regardless of the test outcome, and the high-type entrepreneur's expected payoff is equivalent to launching the final product optimal with respect to the high-type entrepreneur's prior belief, which we later show is strictly dominated. Assume $r(1 - f^{tp}(\rho)) < (1-r)(1 - f^{fp}(\rho))$, which means that the high-type entrepreneur chooses $\Lambda = s$ when the test product sells and $\Lambda = s+1$ when it does not. In this case, the high-type entrepreneur's expected payoff is

$$V_{11}(s, \rho) := [rf^{tp} + (1-r)(1-f^{fp})](P^{tp} - K) + [r(1-f^{tp}) + (1-r)f^{fp}](P^{fp} - K). \tag{B.2}$$

By the definitions of f^{tp} and f^{fp}, we obtain that

$$\frac{dV_{11}(s,\rho)}{d\rho} = \begin{cases} \frac{1-r}{2\epsilon}(P^{tp} - P^{fp}) & \rho \in [0, V-\epsilon) \\ -\frac{3r-1}{2\epsilon}(P^{tp} - P^{fp}) & \rho \in (V-\epsilon, V-1+\epsilon) \\ -\frac{r}{\epsilon}(P^{tp} - P^{fp}) & \rho \in (V-1+\epsilon, V]. \end{cases}$$

Since $P^{tp} > P^{fp}$ by Assumption 1 and $r > 1/2$, $V_{11}(s,\rho)$ is maximized at $\rho^* = V - \epsilon$ to $V_{11}(s,\rho^*) = (r + \frac{1-r}{2\epsilon})(P^{tp} - K) + \frac{(1-r)(2\epsilon-1)}{2\epsilon}(P^{fp} - K)$.

We show that $V_{11}(s,\rho^*) > u_H$. Observe that by Assumption 2 and that $r \in (1/2, 1)$, we have

$$\frac{r}{(1-r)(2\epsilon-1)/(2\epsilon)} > \max\left\{\frac{1-r}{r(2\epsilon-1)/(2\epsilon)}, \frac{r(2\epsilon-1)/(2\epsilon)}{1-r}\right\}.$$

That is, in $V_{11}(s,\rho^*)$, the weight on $P^{tp} - K$ is larger than that in Assumption 3(b). Thus, we have $V_{11}(s,\rho^*) > u_H$. Never launching the final product is therefore strictly dominated.

If the entrepreneur launches the final product only if the test product sells, the entrepreneur's payoff is $P^{tp} - K$ with probability rf^{tp}, $P^{fp} - K$ with probability $(1-r)f^{fp}$, and u_H with the remaining probability. In this case, overall, the high-type entrepreneur's expected payoff is

$$V_{10}(s,\rho) := rf^{tp}\cdot(P^{tp}-K)+(1-r)f^{fp}\cdot(P^{fp}-K)+\left[r(1-f^{tp})+(1-r)(1-f^{fp})\right]u_H.$$

If $\rho \leq V-\epsilon$, then in comparison to $V_{11}(s,\rho)$, $V_{10}(s,\rho)$ takes away weight from $P^{tp}-K$ to $P^{fp}-K$ and u_H. Thus, $V_{10}(s,\rho)<V_{11}(s,\rho)$. Assume $\rho > V-\epsilon$. Since $V_{10}(S,\rho)$ is linear in $\rho \in (V-\epsilon, V]$, it suffices to compare $V_{10}(s,\rho^{**})$ and $V_{11}(s,\rho^{*})$, where $\rho^{**} = V-1+\epsilon$. If $P^{fp} \geq K+u_H$, then clearly $V_{10}(s,\rho^{**}) < V_{10}(s,\rho^{*})$, and therefore $V_{10}(s,\rho^{**}) < V_{11}(s,\rho^{*})$. If $P^{fp} < K+u_H$, then we observe that

$$V_{11}(s,\rho^{*})-V_{10}(s,\rho^{**}) = \left[\frac{r(2\epsilon-1)}{\epsilon}+\frac{1-r}{2\epsilon}\right](P^{tp}-K-u_H)+\frac{(1-r)(2\epsilon-1)}{2\epsilon}(P^{fp}-K-u_H). \quad \text{(B.3)}$$

By Assumption 2 and that $r\in(1/2,1)$, we have

$$\frac{2r(2\epsilon-1)+(1-r)}{(1-r)(2\epsilon-1)} > \max\left\{\frac{1-r}{r(2\epsilon-1)/(2\epsilon)}, \frac{r(2\epsilon-1)/(2\epsilon)}{1-r}\right\}. \quad \text{(B.4)}$$

That is, the relative weight on $P^{tp}-K-u_H$ is larger and that on $P^{fp}-K-u_H$ is smaller in (B.3) than in Assumption 3(b). Thus, we have $V_{11}(s,\rho^{*}) > V_{10}(s,\rho^{**})$, and hence $V_{11}(s,\rho^{*}) > V_{10}(s,\rho)$ for $\rho \in [0,V]$.

If the final product is launched only when the test product does not sell, the entrepreneur's payoff is $P^{tp}-K$ with probability $\max\{r(1-f^{tp}),(1-r)(1-f^{fp})\}$, $P^{fp}-K$ with probability $\min\{r(1-f^{tp}),(1-r)(1-f^{fp})\}$, and u_H with the remaining probability. The high-type entrepreneur's expected payoff is

$$\begin{aligned} V_{01}(s,\rho) := {} & \max\{r(1-f^{tp}),(1-r)(1-f^{fp})\}(P^{tp}-K) \\ & + \min\{r(1-f^{tp}),(1-r)(1-f^{fp})\}(P^{fp}-K)+\left[rf^{tp}+(1-r)f^{fp}\right]u_H. \end{aligned}$$

If $\rho \geq V-\epsilon$, when the test product sells, the conditional expected payoff of launching the optimal final product is more than u_H by Assumption 3(b). Thus, $V_{01}(s,\rho) < V_{11}(s,\rho)$. Assume $\rho < V-\epsilon$. Since $V_{01}(s,\rho)$ is linear in $\rho \in [0, V-\epsilon)$, it suffices to compare $V_{01}(s,0)$ and $V_{11}(s,\rho^{*})$. We have

$$\begin{aligned} V_{11}(s,\rho^{*})-V_{01}(s,0) = {} & \left[r+\frac{1-r}{2\epsilon}-\frac{(1-r)(1+\epsilon-V)}{2\epsilon}\right](P^{tp}-K-u_H) \\ & + \frac{(1-r)(2\epsilon-1)}{2\epsilon}(P^{fp}-K-u_H). \quad \text{(B.5)} \end{aligned}$$

By Assumptions 1, 2, and that $r \in (1/2, 1)$, we have

$$\frac{2r\epsilon + (1-r)(V-\epsilon)}{(1-r)(2\epsilon-1)} > \max\left\{\frac{1-r}{r(2\epsilon-1)/(2\epsilon)}, \frac{r(2\epsilon-1)/(2\epsilon)}{1-r}\right\}. \tag{B.6}$$

That is, the relative weight on $P^{tp} - K - u_H$ is larger and that on $P^{fp} - K - u_H$ is smaller in (B.5) than in Assumption 3(b). Thus, we have $V_{11}(s, \rho^*) > V_{01}(s, 0)$ and thus $V_{11}(s, \rho^*) > V_{01}(s, \rho)$ for $\rho \in [0, V]$.

We have shown that conditional $\lambda = s$, the high-type entrepreneur's optimal test product is $(s, V - \epsilon)$ and the optimal final product is launched regardless of the test outcome.

Case $\lambda = s + 1$.

If the final product is launched regardless of the test outcome, then the high-type entrepreneur's expected payoff is $P^{tp} - K$ with probability $\max\{(1-r)f^{tp}, rf^{fp}\} + \max\{(1-r)(1-f^{tp}), r(1-f^{fp})\}$ and $P^{fp} - K$ with the remaining probability. By Assumption 2 and $r > 1/2$, we have $r + \frac{1-r}{2\epsilon} > \max\{(1-r)f^{tp}, rf^{fp}\} + \max\{(1-r)(1-f^{tp}), r(1-f^{fp})\}$. Thus, $V_{11}(s, \rho^*) > V_{11}(s+1, \rho)$ for $\rho \in [0, V]$.

If the final product is launched only if the test product sells, then the high-type entrepreneur's expected payoff is $P^{tp} - K$ with probability $\max\{(1-r)f^{tp}, rf^{fp}\}$, $P^{fp} - K$ with probability $\min\{(1-r)f^{tp}, rf^{fp}\}$, and u_H with the remaining probability. We observe that the probability of the test product selling is strictly higher for test product (s, ρ) than for $(s+1, \rho)$. Moreover, the posterior belief of the test product (s, ρ) when the test product sells is always more informative than the posterior belief of the test product $(s+1, \rho)$, for every $\rho \in [0, V]$. Thus, we have $V_{10}(s+1, \rho) < V_{10}(s, \rho)$, and hence $V_{10}(s+1, \rho) < V_{11}(s, \rho^*)$.

Lastly, if the final product is launched only if the test product does not sell, then the high-type entrepreneur's expected payoff is $P^{tp} - K$ with probability $\max\{(1-r)(1-f^{tp}), r(1-f^{fp})\}$, $P^{fp} - K$ with probability $\min\{(1-r)(1-f^{tp}), r(1-f^{fp})\}$, and u_H with the remaining probability. If $\rho \geq V - \epsilon$, then by Assumption 3(b), the high-type entrepreneur's expected payoff of launching the final product conditional on the test product selling is strictly above u_H. That is, $V_{10}(s+1, \rho) < V_{11}(s+1, \rho) < V_{11}(s, \rho^*)$. Assume $\rho < V - \epsilon$. Since $V_{01}(s+1, \rho)$ is linear in $\rho \in [0, V - \epsilon)$, it suffices to look at $V_{01}(s+1, 0)$ for $\rho \in [0, V - \epsilon)$. As the posterior belief of the entrepreneur when the test product does not sell is the same as the posterior belief for test product $(s+1, \rho^*)$, and the probability of the test product not selling is strictly higher with test product $(s+1, \rho^*)$, we therefore have $V_{01}(s+1, 0) < V_{11}(s, \rho^*)$.

We have shown that every test product with $\lambda = s + 1$ is not optimal.

Case $\lambda \notin \{s-1,\dots,s+2\}$.

If $\lambda \notin \{s-1,\dots,s+2\}$, then the test product never sells by Assumption 1. When the test product does not sell, the high-type entrepreneur's posterior belief is the same as his prior. The expected payoff in this case is equal to either $V_{11}(s,V)$ or $V_{10}(s,V)$, both of which are strictly smaller than $V_{11}(s,\rho^*)$. We have shown that every test product with $\lambda \notin \{s-1,\dots,s+2\}$ is not optimal.

Case $\lambda = s-1$.

If $\lambda = s-1$ and $\rho \geq V-1+\epsilon$, then the test product never sells. For this case, we have shown in the case $\lambda \notin \{s-1,s,s+1\}$ that the expected payoff is strictly smaller than $V_{11}(s,\rho^*)$.

Assume $\rho < V-1+\epsilon$. If the final product is launched regardless of the test outcome, then the high-type entrepreneur's expected payoff is $P^{tp}-K$ with probability $rf^{fp}+\max\{r(1-f^{fp}),1-r\}$ and $P^{fp}-K$ with the remaining probability. As $r+\frac{1-r}{2\epsilon} > rf^{fp}+\max\{r(1-f^{fp}),1-r\}$ by Assumption 1, we have $V_{11}(s-1,\rho) < V_{11}(s,\rho^*)$ for every $\rho \in [0,V]$.

If the final product is launched only if the test product sells, then the high-type entrepreneur's expected payoff is $P^{tp}-K$ with probability $rf^{fp}(\rho)$ and u_H with the remaining probability. Thus, in this case, the high-type entrepreneur's expected payoff is maximized at $\rho = 0$. By Assumption 2, we have $\frac{V-1+\epsilon}{2\epsilon} < \frac{1-\epsilon}{\epsilon}$. That is, $V_{10}(s,V-1+\epsilon)$ puts more weight on $P^{tp}-K$. Thus, $V_{10}(s-1,0) < V_{10}(s,V-1+\epsilon)$.

If the final product is launched only if the test product does not sell, then the high-type entrepreneur's expected payoff is clearly dominated by the payoff when the final product is launched regardless of the test outcome since the posterior belief is the most informative when the test product sells.

We have shown that every test product with $\lambda = s-1$ is not optimal.

Case $\lambda = s+2$.

If $\lambda = s+2$ and $\rho \geq V-1+\epsilon$, then the test product never sells, and the expected payoff is strictly smaller than $V_{11}(s,\rho^*)$ as we have shown in the case $\lambda = s-1$.

Assume $\rho < V-1+\epsilon$. If the final product is launched regardless of the test outcome, then the high-type entrepreneur's expected payoff is $P^{tp}-K$ with probability $(1-r)f^{fp}+r$ and $P^{fp}-K$ with the remaining probability. Thus, $V_{11}(s+2,\rho)$ is maximized at $\rho = 0$, and

$$V_{11}(s+2,0) = \left[r + \frac{(1-r)(V-1+\epsilon)}{2\epsilon}\right](P^{tp}-K) + \frac{(1-r)(1+\epsilon-V)}{2\epsilon}(P^{fp}-K).$$

As $V-1+\epsilon < 1$ by Assumption 1, the weight on $P^{tp}-K$ is larger in $V_{11}(s,\rho^*)$ than in $V_{11}(s+2,0)$. Thus, we have $V_{11}(s+2,0) < V_{11}(s,\rho^*)$.

If the final product is launched only if the test product sells, the high-type entrepreneur's expected payoff is $P^{tp} - K$ with probability $(1-r)f^{fp}$ and u_H with the remaining probability. As $r > 1/2$, the probability of the test product selling is strictly lower than that of $(s-1, \rho)$. Thus, we have $V_{10}(s+2, \rho) < V_{10}(s-1, \rho)$.

If the final product is launched only if the test product does not sell, then the high-type entrepreneur's expected payoff is clearly dominated by the payoff when the final product is launched regardless of the test outcome since the posterior belief is the most informative when the test product sells. We have shown that every test product with $\lambda = s+2$ is not optimal.

To summarize, we have shown that solution to (B.1) is unique and the objective function's maximum is $V_{11}(s, V-\epsilon)$. Therefore, the high-type entrepreneur's optimal test product is $(s, V-\epsilon)$, and the optimal final product is launched regardless of the test outcome. When the test product sells, the high-type entrepreneur chooses $\Lambda = s$, and chooses $\Lambda = s+1$ otherwise.

B.2 Proof of Theorem 2

Fix a pure-strategy label-independent equilibrium where the high-type entrepreneur's expected payoff is strictly above u_H. Assume that the equilibrium is not pooling. There are two cases where the equilibrium is not pooling: the two types differ in their choice of test product conditional on some realized signal, or they give different equity share offers conditional on some test outcome.

First assume that the two types of entrepreneurs choose different test products for some realized signal S. By the definition of label independence, their test product choices must be different for every signal. As the investor rejects any equity share $\alpha \in [0,1]$ offered by a low-type entrepreneur by Assumption 3(a), the low-type entrepreneur's payoff is 0. As the high-type entrepreneur's expected payoff is strictly above u_H, he must offer some equity share $\alpha < 1$ for some test outcome that happens with a positive probability. Moreover, by Assumption 1 and the type-dependent signal structure, for every finite N, the probability of this test outcome is also positive for the low type. Thus, if the low-type entrepreneur imitates the high-type entrepreneur's strategy, then by the strict positivity of Π^* in (2), the low-type entrepreneur's expected payoff is strictly above 0, contradicting the equilibrium definition.

Now assume that the two types of entrepreneurs offer different equity shares for some test outcome on a supported path. In this case, the low-type entrepreneur's offer is rejected since $mP^{tp} < K$ by Assumption 3(a). If the high-type entrepreneur's offer is also rejected on this equilibrium path where the two types offer different equity shares, then by assumption, they both offer $\alpha = 0$, a contradiction. If the high-type entrepreneur's offer is accepted, then the high-type entrepreneur's expected payoff conditional on the test outcome must be strictly above u_H by the tie-breaking assumption. Thus, the high-type entrepreneur must offer some $\alpha \in (0,1)$.

By imitating the high-type entrepreneur's strategy, the low-type entrepreneur obtains a positive expected payoff, contradicting the equilibrium definition.

Therefore, every pure-strategy label-independent equilibrium must be pooling. The proof is complete.

B.3 Proof of Theorem 3

For the sake of generality, let $s \in \Omega$ be the realized signal received by the high-type entrepreneur.

The following lemma is useful for the proof of the theorem. To aid its presentation, we make a new definition: a strategy profile is *individually rational* if every player obtains at least the player's reservation payoff on every supported path.

Lemma B.1. *Every pooling and individually rational strategy profile where the investor's belief mapping is consistent with Bayes' rule on very supported path and assigns zero probability to the entrepreneur's type being high for every off-path strategy of the entrepreneur is a pooling equilibrium.*

Proof. Proof of Lemma B.1. Fix a pooling and individually rational strategy profile where the test product is (λ, ρ) and the investor's belief mapping assigns to every off-path strategy of the entrepreneur the probability of the entrepreneur's type being high to 0.

If the investor rejects an equity share offer $\alpha > 0$ when it is on some support path, then the investor's expected payoff is 0. Since the strategy profile is individually rational, accepting the equity share offer gives the investor at least her reservation payoff. Hence, there is no profitable deviation for the investor.

If the entrepreneur deviates to a different test product or chooses a different equity share offer after some test outcome that happens with a positive probability, then by the assumption about the investor's belief mapping, the investor believes the entrepreneur's type to be low with probability 1. In this case, by Assumption 3(a), the investor rejects any equity share offer $\alpha \in [0, 1]$, and the entrepreneur obtains his reservation payoff. Again, since the strategy profile is individually rational, the deviation is not profitable for the entrepreneur.

We have shown that no player has an incentive to deviate. Lastly, by assumption, the investor's belief mapping is consistent with Bayes' rule on every supported path. Therefore, the pooling strategy profile is a pooling equilibrium. The proof of the lemma is complete. □

Let $\hat{V}_{ij}(\lambda, \rho; N)$ be the high-type entrepreneur's expected payoff of the following strategy: 1. The test product is (λ, ρ); 2. When the final product is launched, Λ is optimized with respect to the high-type entrepreneur's posterior belief conditional on the test outcome; 3. When the test product sells, the final product is launched if and only if $i = 1$; 4. When the test product does not sell, the final product is launched if and only if $j = 1$; 5. The investor's belief mapping is the same as the belief mapping in a pooling equilibrium; 6. When the optimal final product is launched, the entrepreneur pays the investor just enough for the investor to accept the equity share offer. With these definitions, the optimal test product can be characterized as the solution to the following problem:

$$\max_{\lambda\in\Omega,\rho\in[0,V],i,j\in\{0,1\}} \quad \hat{V}_{ij}(\lambda,\rho;N), \tag{B.7}$$

provided that in the strategy profile characterized by its solution, every player obtains at least their reservation payoffs. To see that such a strategy profile characterizes the high-type-optimal pooling equilibrium, it is sufficient to show that this strategy profile is a pooling equilibrium, which is implied by Lemma B.1 in the proof of Theorem 2.

There are two parts to the rest of the proof. In the first part, we will show that given the fixed parameter set, the equilibrium test product converges after some finite N. Based on the first part, we then proceed to characterize the equilibrium test product in the asymptotic case.

Finite Convergence of Equilibrium Test Product.

In this part, we show that $\hat{V}_{ij}$ is uniformly convergent to some piecewise linear function for $\rho\in[0,V]$. Moreover, its first-order derivative (where it exists) is also uniformly convergent except on some set which we will show does not contain the optimal test product for large enough N. Let π denote the random variable for the payoff of the optimal final product launched by the entrepreneur. We first analyze the minimum equity share the investor would demand.

Expected payoff when test product sells. If the entrepreneur's type is low, the test product sells with probability $\frac{1}{N}(f^{tp}+2f^{fp})$, which is piecewise linear in ρ. If the entrepreneur is to launch the optimal final product by solving (2), he will choose $\Lambda=\lambda$, and the unconditional expected payoff of the final product is $\frac{m}{N}(f^{tp}P^{tp}+2f^{fp}P^{fp})$. When the entrepreneur's type is high, the test product sells with probability of the form $ra_1+(1-r)c_1$, where $a_1,c_1\in\{0,f^{fp},f^{tp}\}$. Since the high-type entrepreneur's belief is always between s and $s+1$, the unconditional expected payoff of the optimal final product launched by the high-type entrepreneur is of the form $E(\pi\,\mathbf{1}(\theta=1)|X=1,\lambda,\rho)=ra_1b_1+(1-r)c_1d_1$, where $b_1,d_1\in\{P^{fp},P^{tp}\}$. Therefore, using Bayes' rule, the investor's assessment of the optimal final product's total expected payoff is

$$E(\pi|X=1,\lambda,\rho)=\frac{q[ra_1b_1+(1-r)c_1d_1]+\frac{(1-q)m}{N}(f^{tp}P^{tp}+2f^{fp}P^{fp})}{q[ra_1+(1-r)c_1]+\frac{1-q}{N}(f^{tp}+2f^{fp})}.$$

Thus, the maximum equity share the entrepreneur can keep to himself is

$$\begin{aligned}1-\alpha_1^*(\lambda,\rho) :=& \frac{K}{E(\pi|X=1,\lambda,\rho)}\\ =& \frac{q[ra_1(b_1-K)+(1-r)c_1(d_1-K)]+\frac{1-q}{N}(f^{tp}(mP^{tp}-K)+2mf^{fp}(P^{fp}-K))}{q[ra_1b_1+(1-r)c_1d_1]+\frac{(1-q)m}{N}(f^{tp}P^{tp}+2f^{fp}P^{fp})}.\end{aligned}$$

If a high-type entrepreneur launches the optimal final product given the entrepreneur's posterior belief, then the entrepreneur's expected payoff joint with the test product selling is $E(\pi\,\mathbf{1}(X=1)|\lambda,\rho,\theta=1)=ra_1b_1+(1-r)c_1d_1$, which is 0 if $a_1=c_1=0$. If $a_1\neq 0$ or $c_1\neq 0$, the expected payoff of the high-type entrepreneur joint with the event of the test product selling is

$$\begin{aligned}U_1(\lambda,\rho) :=& E(\pi\,\mathbf{1}(X=1)|\lambda,\rho,\theta=1)[1-\alpha_1^*(\lambda,\rho)]\\ =& \frac{q[ra_1(b_1-K)+(1-r)c_1(d_1-K)]+\frac{1-q}{N}(f^{tp}(mP^{tp}-K)+2f^{fp}(mP^{fp}-K))}{q+\frac{(1-q)m}{N}\frac{f^{tp}P^{tp}+2f^{fp}P^{fp}}{ra_1b_1+(1-r)c_1d_1}}.\end{aligned}$$

Let A_1, B_1, and C_1 be such that the following holds as an identity:

$$U_1(\lambda,\rho)=\frac{qA_1+\frac{1-q}{N}B_1}{q+\frac{(1-q)m}{N}C_1}. \tag{B.8}$$

Note that A_1, B_1, and C_1 are all nonnegative; A_1 and B_1 are piecewise linear in ρ; C_1 is the ratio of two piecewise linear functions in ρ. As for every $\rho\in(0,V]$, we have $B_1=o(N)$ and $C_1=o(N)$. Thus, we have

$$\lim_{N\to\infty}U_1(\lambda,\rho)=A_1 \tag{B.9}$$

which is piecewise linear in ρ. As $\frac{A_1}{\Pr(X=1|\lambda,\rho,\theta=1)}$ is the high-type entrepreneur's expected payoff conditional on the test product selling in the benchmark case with self-funding, in the limiting case of $N\to\infty$, the investor becomes certain that the entrepreneur's type is high, and from the high-type entrepreneur's perspective, the entrepreneur expects to pay the investor exactly K to obtain the funding.

We claim that the convergence of $U_1(\lambda,\rho)$ to A_1 is uniform for every fixed test product (λ,ρ). Let D be the set of test products such that $a_1=c_1=0$.[1] For every test product $(\lambda,\rho)\in D$, we have $U_1(\lambda,\rho)=A_1=0$. The claim holds immediately. Given $\delta>0$, let $\mathscr{B}(D,\delta)$ be the set of products such that for every (λ,ρ) in the set, $(\lambda,\rho)\notin D$ and there exists $(\hat{\lambda},\hat{\rho})\in D$ such that $\lambda=\hat{\lambda}$ and

[1] Specifically, the set D consists of (λ,ρ) for $\lambda\in\{s-1,s+2\}$ and $\rho>V-1+\epsilon$; (λ,V) for every $\lambda\in\Omega$; and (λ,ρ) for every $\lambda\notin\{s-1,s,s+1,s+2\}$ and $\rho\in[0,V]$.

$|\rho - \hat{\rho}| < \delta$. The uniform convergence of U_1 is clear on the set of test products not in $\mathscr{B}(D,\delta)$. For test products in $\mathscr{B}(D,\delta)$, we first observe that

$$U_1(\lambda,\rho) \le A_1 + \frac{1-q}{qN} B_1,$$

where $\lim_{N\to\infty} \frac{1-q}{qN} B_1 = 0$ uniformly. Moreover, as A_1 is invariant in N and $\lim_{\delta\to 0} \sup_{(\lambda,\rho)\in\mathscr{B}(D,\delta)} A_1 = 0$ uniformly, we have $\lim_{\delta\to 0} \sup_{(\lambda,\rho)\in\mathscr{B}(D,\delta)} |U_1(\lambda,\rho) - A_1| = 0$ uniformly. We have shown that $\lim_{N\to\infty} U_1(\lambda,\rho) = A_1$ uniformly for every test product (λ,ρ).

If (λ,ρ) is in the interior of D, then U_1 has zero first-order derivative as it is locally constant at 0. Assume $(\lambda,\rho) \notin D$. In this case, note that $U_1(\lambda,\rho)$ is differentiable in ρ on $(0, V-\epsilon)$, $(V-\epsilon, V-1+\epsilon)$, and $(V-1+\epsilon, V)$. If we differentiate $U_1(\lambda,\rho)$ with respect to ρ, we get

$$\frac{dU_1(\lambda,\rho)}{d\rho} = \frac{q\frac{\partial A_1}{\partial\rho} + \frac{1-q}{N}\frac{\partial B_1}{\partial\rho}}{q + \frac{(1-q)m}{N} C_1} - \frac{\left[qA_1 + \frac{1-q}{N} B_1\right]\frac{1-q}{N}\frac{\partial C_1}{\partial\rho}}{\left[q + \frac{(1-q)m}{N} C_1\right]^2}.$$

Given our earlier analysis of A_1, B_1, and C_1, we see that $\frac{dU_1(\lambda,\rho)}{d\rho}$ converges to $\lim_{N\to\infty} \frac{dU_1(\lambda,\rho)}{d\rho} = \frac{\partial A_1}{\partial\rho}$ pointwise in ρ on $[0, V-\epsilon)$, $(V-\epsilon, V-1+\epsilon)$, and $(V-1+\epsilon, V)$, respectively. Moreover, for every $\delta > 0$, if we restrict the set of test products to those not in $\mathscr{B}(D,\delta)$, the convergence is uniform on $[0, V-\epsilon)$, $(V-\epsilon, V-1+\epsilon)$, and $(V-1+\epsilon, V-\delta)$, respectively.

Expected payoff when test product does not sell. If the entrepreneur's type is low, the test product does sell with probability $\frac{N-f^{tp}-2f^{fp}}{N}$. If the entrepreneur is to launch the optimal final product by solving (2), he will choose $\Lambda = \lambda - 3$, and the unconditional expected payoff of the final product is $\frac{m(P^{tp}+2P^{fp})}{N}$. If the entrepreneur's type is high, the test product does not sell with probability of the form $ra_0 + (1-r)c_0$, where $a_0, c_0 \in \{0, 1-f^{tp}, 1-f^{fp}, 1\}$. Since the high-type entrepreneur's belief is always between s and $s+1$, the unconditional expected payoff of the optimal final product launched by the high-type entrepreneur is of the form $E(\pi\,\mathbf{1}(\theta=1)|X = 0, \lambda, \rho) = ra_0 b_0 + (1-r)c_0 d_0$, where $b_0, d_0 \in \{P^{fp}, P^{tp}\}$. Therefore, using Bayes' rule, the investor's assessment of the optimal final product's total expected payoff is

$$E(\pi|X=0,\lambda,\rho) = \frac{q[ra_0 b_0 + (1-r)c_0 d_0] + \frac{(1-q)m}{N}(P^{tp} + 2P^{fp})}{q[ra_0 + (1-r)c_0] + \frac{(1-q)}{N}(N - f^{tp} - 2f^{fp})}.$$

Thus, the maximum equity share the entrepreneur can keep to himself is

$$\begin{aligned}
1-\alpha_0^*(\lambda,\rho)\ :=&\frac{K}{E(\pi|X=0,\lambda,\rho)}\\
=&\frac{q[ra_0(b_0-K)+(1-r)c_0(d_0-K)]+\frac{1-q}{N}[mP^{tp}+2mP^{fp}-K(N-f^{tp}-2f^{fp})]}{q[ra_0b_0+(1-r)c_0d_0]+\frac{(1-q)m}{N}(P^{tp}+2P^{fp})}.
\end{aligned}$$

If a high-type entrepreneur launches the optimal final product given the entrepreneur's posterior belief, then the entrepreneur's expected payoff joint with the test product not selling is $E(\pi\,\mathbf{1}(X=0)|\lambda,\rho,\theta=1)=ra_0b_0+(1-r)c_0d_0$. Thus, based on the calculations, the expected payoff of the high-type entrepreneur joint with the event of the test product not selling is

$$\begin{aligned}
U_0(\lambda,\rho)\ :=&E(\pi\,\mathbf{1}(X=0)|\lambda,\rho,\theta=1)[1-\alpha_0^*(\lambda,\rho)]\\
=&\frac{q[ra_0(b_0-K)+(1-r)c_0(d_0-K)]+\frac{1-q}{N}[mP^{tp}+2mP^{fp}-K(N-f^{tp}-2f^{fp})]}{q+\frac{(1-q)m}{N}\frac{P^{tp}+2P^{fp}}{ra_0b_0+(1-r)c_0d_0}}.
\end{aligned}$$

Let A_0, B_0, and C_0 be such that the following holds as an identity:

$$U_0(\lambda,\rho)=\frac{qA_0+\frac{1-q}{N}B_0}{q+\frac{(1-q)m}{N}C_0}. \tag{B.10}$$

Note that A_0 and C_0 are nonnegative; A_0 and B_0 are piecewise linear in ρ; C_0 is the ratio of two piecewise linear functions in ρ. By Assumptions 1 and 2, there exists some $\eta>0$ such that $ra_0+(1-r)c_0>\eta$ for every test product. As $\lim_{N\to\infty}B_0/N=-K$ and $C_0=o(N)$ for every $\rho\in(0,V]$, we have

$$\lim_{N\to\infty}U_0(\lambda,\rho)=A_0-\frac{(1-q)K}{q}, \tag{B.11}$$

which is piecewise linear in ρ. The convergence is uniform for test product (λ,ρ) since the denominator in (B.10) is bounded away from zero. We also observe that, in the benchmark case with self-funding, $\frac{A_0}{\Pr(X=0|\lambda,\rho,\theta=1)}$ is the high-type entrepreneur's expected payoff conditional on the test product not selling. Thus, in the limiting case of $N\to\infty$, from the high-type entrepreneur's perspective, the high-type entrepreneur expects to pay the investor $K+\frac{1-q}{q\Pr(X=0|\lambda,\rho,\theta=1)}K$ to obtain the funding, which is strictly more than K.

Note that $U_0(\lambda,\rho)$ is differentiable in ρ on $[0,V-\epsilon)$, $(V-\epsilon,V-1+\epsilon)$, and $(V-1+\epsilon,V]$. If we differentiate $U_0(\lambda, \rho)$ with respect to ρ, we get

$$\frac{dU_0(\lambda,\rho)}{d\rho} = \frac{q\frac{\partial A_0}{\partial \rho} + \frac{1-q}{N}\frac{\partial B_0}{\partial \rho}}{q + \frac{(1-q)m}{N}C_0} - \frac{\left[qA_0 + \frac{1-q}{N}B_0\right]\frac{1-q}{N}\frac{\partial C_0}{\partial \rho}}{\left[q + \frac{(1-q)m}{N}C_0\right]^2}.$$

As A_0 and B_0 are both linear in ρ and C_0 is uniformly bounded in ρ, we have that $\frac{dU_0(\lambda,\rho)}{d\rho}$ converges to $\lim_{N\to\infty}\frac{dU_0(\lambda,\rho)}{d\rho} = \frac{\partial A_0}{\partial \rho}$ uniformly on $[0, V-\epsilon)$, $(V-\epsilon, V-1+\epsilon)$, and $(V-1+\epsilon, V]$, respectively.

Convergence of Maximizer. Note that for each $\hat{V}_{ij}(\lambda, \rho; N)$, it can be written as

$$\begin{aligned}\hat{V}_{ij}(\lambda,\rho;N) =& \mathbf{1}(i=1)U_1(\lambda,\rho) + [1-\mathbf{1}(i=1)]\Pr(X=1|\lambda,\rho,\theta=1)u_H \\ &+ \mathbf{1}(j=1)U_0(\lambda,\rho) + [1-\mathbf{1}(j=1)]\Pr(X=0|\lambda,\rho,\theta=1)u_H.\end{aligned}$$

Through some abuse of notation, we define $\hat{V}_{ij}(\lambda,\rho) = \lim_{N\to\infty}\hat{V}_{ij}(\lambda,\rho;N)$. Consider the maximization problem for the asymptotic case below:

$$\max_{\lambda\in\Omega,\rho\in[0,V],i,j\in\{0,1\}} \hat{V}_{ij}(\lambda,\rho) \tag{B.12}$$

As $\lim_{N\to\infty}\tilde{V}_{ij}(\lambda,\rho;N) = \tilde{V}_{ij}(\lambda,\rho)$ uniformly, if there is no solution to (B.12) in $\mathscr{B}(D,\delta)$ for some $\delta > 0$, then there is no solution to (B.7) after some finite N. Moreover, by our analysis so far, $\hat{V}_{ij}(\lambda,\rho)$ is piecewise linear in ρ, and the first-order derivative of $\hat{V}_{ij}(\lambda,\rho;N)$ converges to that of $V_{ij}(\lambda,\rho)$ uniformly in ρ except for test products in $\mathscr{B}(D,\delta)$. Thus, on condition that every solution to (B.12) is not in $\mathscr{B}(D,\delta)$, each solution to (B.7) converges to that to (B.12). Moreover, in this case, the piecewise linearity of $\tilde{V}_{ij}$ ensures that the solution to (B.7) stabilizes at that to (B.12) after some finite N.

Asymptotic Equilibrium Test Product.

On condition that the solution to (B.7) stabilizes after some finite N, we characterize the optimal test product for large enough N by analyzing the maximization problem in the asymptotic case as in (B.12). To characterize the solution to (B.12), we first show the optimality of $\lambda = s$, and then we will analyze the optimal test product for different values of u_H and q.

To show that $\lambda \neq s$ is always suboptimal, there are three groups of cases to consider: $\lambda = s-1$, $\lambda = s+2$, and $\lambda \notin \{s-1,\ldots,s+2\}$. Moreover, as $V_{i1}(\lambda,\rho) = \hat{V}_{i1}(\lambda,\rho) - (1-q)K/q$ by (B.11), which is a constant, Theorem 1 implies that it suffices to show the suboptimality of $\lambda \neq s$ when the final product is launched only if the test product sells. Another observation that will be useful in the rest of the proof is $V_{i0}(\lambda, \rho) = \hat{V}_{i0}(\lambda, \rho)$.

Suboptimality of case $\lambda \notin \{s-1, \ldots, s+2\}$. In this case, the test product never sells. If the final product is not launched when the test product does not sell, then the expected payoff to the high-type entrepreneur is u_H, which is strictly dominated by $\hat{V}_{10}(s, V-1+\epsilon)$ by Assumption 3(b) and the observation $\hat{V}_{10}(s, V-1+\epsilon) = V_{10}(s, V-1+\epsilon)$. Thus, every test product with $\lambda \notin \{s-1, \ldots, s+2\}$ is not optimal.

Suboptimality of case $\lambda = s-1$. If the final product is launched only if the test product sells, the expected payoff to the high-type entrepreneur is $\hat{V}_{10}(s-1, \rho) = V_{10}(s-1, \rho)$. As $\hat{V}_{10}(s, V-1+\epsilon) = V_{10}(s, V-1+\epsilon)$ and we have shown in the proof of Theorem 1 (case $\lambda = s-1$) that $V_{10}(s-1, \rho) < V_{10}(s, V-1+\epsilon)$, every test product with $\lambda = s-1$ is not optimal.

Suboptimality of case $\lambda = s+2$. If the final product is launched only if the test product sells, the expected payoff to the high-type entrepreneur is $\hat{V}_{10}(s+2, \rho) = V_{10}(s+2, \rho)$. As $\hat{V}_{10}(s-1, \rho) = V_{10}(s-1, \rho)$ and we have shown in the proof of Theorem 1 (case $\lambda = s+2$) that $V_{10}(s+2, \rho) < V_{10}(s-1, \rho)$, every test product with $\lambda = s+2$ is not optimal.

Suboptimality of case $\lambda = s+1$. If the final product is launched only if the test product sells, the expected payoff to the high-type entrepreneur is $\hat{V}_{10}(s+1, \rho) = V_{10}(s+1, \rho)$. As $\hat{V}_{10}(s, \rho) = V_{10}(s, \rho)$ and we have shown in the proof of Theorem 1 (case $\lambda = s+1$) that $V_{10}(s+1, \rho) < V_{10}(s, \rho)$, every test product with $\lambda = s+1$ is not optimal. To summarize, every test product with $\lambda \neq s$ is not optimal. That is, the optimal test product must have $\lambda = s$.

Optimal test product with $\lambda = s$. Assume the final product is launched regardless of the test outcome. As $\hat{V}_{11}(s, \rho)$ differs from $V_{11}(s, \rho)$ by a constant, $\hat{V}_{11}(\lambda, \rho)$ is uniquely maximized at $(s, V-\epsilon)$ by the proof of Theorem 1. That is, the optimal test product is $(s, V-\epsilon)$, and we have

$$\hat{V}_{11}(s, V-\epsilon) = V_{11}(s, V-\epsilon) - \frac{1-q}{q}K = \left[r + \frac{1-r}{2\epsilon}\right](P^{tp}-K) + \frac{(1-r)(2\epsilon-1)}{2\epsilon}(P^{fp}-K) - \frac{(1-q)K}{q}. \tag{B.13}$$

If the final product is launched only when the test product sells, the expected payoff to the high-type entrepreneur is

$$V_{10}(s, \rho) = rf^{tp}(P^{tp}-K) + (1-r)f^{fp}(P^{fp}-K) + \left[r(1-f^{tp}) + (1-r)(1-f^{fp})\right]u_H,$$

which is piecewise linear and differentiable in ρ on $[0, V-\epsilon)$, $(V-\epsilon, V-1+\epsilon]$, and $(V-1+\epsilon, V]$.

Specifically, we have

$$\frac{dV_{10}(s,\rho)}{d\rho} = \begin{cases} -\frac{(1-r)}{2\epsilon}(P^{fp}-K-u_H) & \rho \in [0, V-\epsilon), \\ -\frac{1}{2\epsilon}\left[2r(P^{tp}-K-u_H)+(1-r)(P^{fp}-K-u_H)\right] & \rho \in (V-\epsilon, V-1+\epsilon), \\ -\frac{1}{\epsilon}(P^{tp}-K-u_H) & \rho \in (V-1+\epsilon, V]. \end{cases}$$

Let $\underline{u}_H$ be the unique solution of u_H to $P^{fp} = K + u_H$, and $\bar{u}_H$ the unique solution to $2r(P^{tp} - K - u_H) + (1-r)(P^{fp} - K - u_H) = 0$. We observe that both $\underline{u}_H$ and $\bar{u}_H$ are invariant in q. Moreover, $V_{10}(s,\rho)$ is maximized at $\rho = 0$ if $u_H < \underline{u}_H$; the maximizer is $\rho = V - \epsilon$ if $\underline{u}_H < u_H < \bar{u}_H$; the maximizer is $\rho = V - 1 + \epsilon$ if $u_H > \bar{u}_H$.

Therefore, the maximum of (B.12) is between $\hat{V}_{11}(s, V-\epsilon)$ and $\max_{\rho\in[0,V]} \hat{V}_{10}(s,\rho)$, which needs to be analyzed in each of the three cases above.

Case $u_H < \underline{u}_H$. In this case, $\hat{V}_{10}(s,\rho)$ is maximized at $\rho = 0$. We have

$$\hat{V}_{11}(s,V-\epsilon)-\hat{V}_{10}(s,0) = \frac{1-r}{2\epsilon}(P^{tp}-K-u_H) - \frac{(1-r)(V-\epsilon)}{2\epsilon}(P^{fp}-K-u_H) - \frac{(1-q)K}{q}, \tag{B.14}$$

which is positive for q close to 1 and negative for q close to 0. Thus, there exists $q_1(u_H)^* \in (0,1)$ such that if $q = q_1^*(u_H)$, then $\hat{V}_{11}(s, V-\epsilon) - \hat{V}_{10}(s,0) = 0$. If $q > q_1^*(u_H)$, then the optimal test product is $(s, V-\epsilon)$, and the optimal final product is launched regardless of the test outcome. If $q < q_1^*(u_H)$, then the optimal test product is $(s, 0)$, and the final product is launched only if the test product sells.

Case $\underline{u}_H < u_H < \bar{u}_H$. In this case, $\hat{V}_{10}(s,\rho)$ is maximized at $\rho = V - \epsilon$. We have

$$\hat{V}_{11}(s,V-\epsilon)-\hat{V}_{10}(s,V-\epsilon) = \frac{1-r}{2\epsilon}(P^{tp}-K-u_H) - \frac{(1-q)K}{q}, \tag{B.15}$$

which is positive for q close to 1 and negative for q close to 0. Thus, there exists $q_2^*(u_H) \in (0,1)$ such that if $q = q_2^*(u_H)$, then $\hat{V}_{11}(s, V-\epsilon) - \hat{V}_{10}(s, V-\epsilon) = 0$. If $q > q_2^*(u_H)$, then the optimal test product is $(s, V-\epsilon)$, and the optimal final product is launched regardless of the test outcome. If $q < q_2^*(u_H)$, then the optimal test product is the same, but the final product is launched only if the test product sells.

Case $u_H > \bar{u}_H$. In this case, $\hat{V}_{10}(s, \rho)$ is maximized at $\rho = V - 1 + \epsilon$. We have

$$\begin{aligned}\hat{V}_{11}(s, V - \epsilon) - \hat{V}_{10}(s, V - 1 + \epsilon) = \frac{2r(2\epsilon - 1) + 1 - r}{2\epsilon}(P^{tp} - K - u_H) \\ + \frac{(1 - r)(2\epsilon - 1)}{2\epsilon}(P^{fp} - K - u_H) - \frac{(1 - q)K}{q},\end{aligned} \tag{B.16}$$

which is positive for q close to 1 and negative for q close to 0. Thus, there exists $q_3^* \in (0, 1)$ such that if $q = q_3^*(u_H)$, then $\hat{V}_{11}(s, V - \epsilon) - \hat{V}_{10}(s, V - 1 + \epsilon) = 0$. If $q > q_3^*(u_H)$, then the optimal test product is $(s, V - \epsilon)$, and the optimal final product is launched regardless of the test outcome. If $q < q_3^*(u_H)$, then the optimal test product is $(s, V - 1 + \epsilon)$, and the final product is launched only if the test product does not sell.

We let $q^*(u_H) = \max\{q_1^*(u_H), q_2^*(u_H), q_3^*(u_H)\}$. Clearly, when $q > q^*(u_H)$, the optimal test product is $(s, V - \epsilon)$, and the optimal final product is launched regardless of the test outcome. When $q < q^*(u_H)$, the optimal final product is launched only if the test product sells. Moreover, if additionally $u_H < \underline{u}_H$, then the optimal test product is $(s, 0)$; if $\underline{u}_H < u_H < \bar{u}_H$, then the optimal test product is $(s, V - \epsilon)$; if $u_H > \bar{u}_H$, then the optimal test product is $(s, V - 1 + \epsilon)$. Lastly, as none of the possible solutions to (B.12) is in $\mathscr{B}(D, \delta)$ for some $\delta > 0$, the solution to (B.7) becomes invariant in N and stays at the solution to (B.12) after some finite N. The proof is complete.

B.4 Proof of Proposition 2

Let s be the realized signal received by the entrepreneur.

Proof of part (a)

Assume $q > q^*(u_H)$. By letting $N \to \infty$ in (B.8) of the proof of Theorem 3, we see that only the high-type entrepreneur's test product sells in the asymptotic case. Therefore, conditional on the test product seeling, the investor and the high-type entrepreneur have the same belief about W. Thus, from the high-type entrepreneur's perspective, the investor's expected payoff is K.

When the test product does not sell, by letting $N \to \infty$ in (B.10) of the proof of Theorem 3, the investor's expected payoff joint with the test product not selling from the high-type entrepreneur's perspective is $\Pr(X = 0|\lambda, \rho, \theta = 1)K + \frac{1-q}{q}K$. Thus, the investor's expected payoff conditional on the test product not selling from the high-type entrepreneur's perspective in the asymptotic case is $K + \frac{1-q}{q \Pr(X=0|\lambda,\rho,\theta=1)}K$, which is strictly larger than K.

Proof of part (b)

By Theorem 3, the final product is launched only if the test product sells when $q < q^*(u_H)$. For the same reasoning about the investor's payoff when the test product sell in the case of $q > q^*(u_H)$, from the high-type entrepreneur's perspective, the investor's expected payoff conditional on the test product selling when $q < q^*(u_H)$ is exactly K. As the demand of the final product conditional on funding is an increasing step function in u_H by Proposition 4, the equity share is therefore a decreasing step function.

B.5 Proof of Proposition 3

Proof for Part (a)

Assume $q > q^*(u_H)$. By Theorem 3, the venture is always funded.

Proof for Part (b)

Assume $q < q^*(u_H)$. By Theorem 3, the venture is funded only if the test product sells. Moreover, by our analysis of (B.8) in the proof of Theorem 3, only the high-type entrepreneur's test product sells in the asymptotic case. Thus, given (λ, ρ), the funding probability is $\Pr(X = 1|\lambda, \rho, \theta = 1)$, which is strictly less than 1 by Assumption 1.

As there are only three possible equilibrium test products for the three cases of u_H, the funding probability is a step function in u_H. It remains to show that the probability is decreasing in u_H.

For the sake of generality, let s be the realized signal received by the entrepreneur. When $u_H < \underline{u}_H$, the optimal test product is $(s, 0)$. Conditional on $W = s$, the test product sells with probability 1; conditional on $W = s + 1$, the test product sells with probability $\frac{V-1+\epsilon}{2\epsilon}$. Therefore, the funding probability is

$$\Pr(X = 1|s, 0, \theta = 1) = r + (1 - r)\frac{V - 1 + \epsilon}{2\epsilon}. \tag{B.17}$$

When $\underline{u}_H < u_H < \bar{u}_H$, the optimal test product is $(s, V - \epsilon)$. In this case, conditional on $W = s$, the test product sells with probability 1; conditional on $W = s + 1$, the test product sells with probability $\frac{2\epsilon-1}{2\epsilon}$. Therefore, the funding probability is

$$\Pr(X = 1|s, V - \epsilon, \theta = 1) = r + (1 - r)\frac{2\epsilon - 1}{2\epsilon}. \tag{B.18}$$

When $u_H > \bar{u}_H$, the optimal test product is $(s, V - 1 + \epsilon)$. In this case, conditional on $W = s$, the test product sells with probability $\frac{1-\epsilon}{\epsilon}$; conditional on $W = s + 1$, the test product sells with probability 0. Therefore, the funding probability is

$$\Pr(X = 1|s, V - 1 + \epsilon, \theta = 1) = r\frac{1-\epsilon}{\epsilon}. \tag{B.19}$$

By comparing (B.18) and (B.19), we have $\Pr(X = 1|s, V-\epsilon, \theta = 1) > r > \Pr(X = 1|s, V-1+\epsilon, \theta = 1)$. By Assumption 1, we compare (B.17) and (B.18) to get $\Pr(X = 1|s, 0, \theta = 1) > \Pr(X = 1|s, V - \epsilon, \theta = 1)$. Therefore, the funding probability is a decreasing step function in u_H. Lastly, we have shown that $\Pr(X = 1|s, 0, \theta = 1) < 1$.

B.6 Proof of Proposition 4

For the sake of generality, let s be the realized signal received by the entrepreneur.

Proof of Part (a)

Assume $q > q^*(u_H)$. By Theorem 3, the venture is always funded. As the entrepreneur chooses $(s, V - \epsilon)$ as the test product in this case, when the test product does not sell, the high-type entrepreneur will know $W = s + 1$ as the test product sells with probability 1 conditional on $W = s$. In this case, the expected demand of the final product is the maximum demand P^{tp}.

Proof of Part (b)

Assume $q < q^*(u_H)$. By Theorem 3, in equilibrium, the venture is funded only if the test product sells, and the equilibrium test product is fixed for $u_H < \underline{u}_H$, $\underline{u}_H < u_H < \bar{u}_H$, and $u_H > \bar{u}_H$. As the high-type entrepreneur's posterior belief is completely determined by the choice of the test product and the test outcome, the expected demand of the final product conditional on funding is constant in u_H in each interval of $u_H < \underline{u}_H$, $\underline{u}_H < u_H < \bar{u}_H$, and $u_H > \bar{u}_H$. That is, the expected demand of the final product conditional on funding is a step function in u_H.

It remains to show that the expected demand of the final product conditional on funding is increasing in u_H and has maximum P^{tp}. When $u_H < \underline{u}_H$, the optimal test product is $(s, 0)$. In this case, when the test product sells, to the high-type entrepreneur, the posterior belief of the event $W = s$ is $\gamma_1 := \frac{r}{r+(1-r)\frac{V-1+\epsilon}{2\epsilon}}$. Moreover, as $0 < \frac{V-1+\epsilon}{2\epsilon} < 1$ by Assumption 1 and $r > 1 - r$, we have $\gamma_1 > 1/2$.

When $\underline{u}_H < u_H < \bar{u}_H$, the optimal test product is $(s, V - \epsilon)$. In this case, when the test product sells, to the high-type entrepreneur, the posterior belief of the event $W = s$ is $\gamma_2 := \frac{r}{r+(1-r)\frac{2\epsilon-1}{2\epsilon}}$.

When $u_H > \bar{u}_H$, the optimal test product is $(s, V - 1 + \epsilon)$. In this case, when the test product sells, to the high-type entrepreneur, the posterior belief of the event $W = s$ is $\gamma_3 := 1$.

By Assumption 1, we have $1/2 < \gamma_1 < \gamma_2 < \gamma_3$. Thus, when the test product sells, if $u_H > \bar{u}_H$, the high-type entrepreneur's posterior belief, which is characterized by γ_3, is more informative than the posterior belief if $u_H < \underline{u}_H$, which is characterized by γ_2; if $\underline{u}_H < u_H < \bar{u}_H$, the posterior belief is more informative than the posterior belief if $u_H < \underline{u}_H$, which is characterized by γ_1. Therefore, the expected demand of the final product conditional on funding is increasing in u_H. Lastly, as γ_3 characterizes the most informative belief, the expected demand of the final product conditional on funding is P^{tp}.

B.7 Proof of Proposition 5

For the sake of generality, let s be the realized signal received by the entrepreneur. In the benchmark case with self-funding, the final product is launched regardless of the test outcome. Thus, the expected demand generated by the venture is at least $\Pi^*(r)$.

In the case with external funding, when $q < q^*(u_H)$, the final product is launched only if the test product sells, and the product launch always has $\Lambda = s$. If the high-type entrepreneur launches the final product with $\Lambda = s$ when the test product does not sell, the expected demand of the final product is strictly positive and the venture generates expected demand $\Pi^*(r)$. Thus, the expected demand generated by the venture when $q < q^*(u_H)$ in the external-funding case is strictly less than that in the benchmark case with self-funding.

As the final product launched always has $\Lambda = s$ when $q < q^*(u_H)$, the expected demand generated by the venture is proportional to the funding probability. As the funding probability is a decreasing step function in u_H by Proposition 3, the expected demand of the final product is also a decreasing step function in u_H.

B.8 Proof of Proposition 6

Let s be the realized signal received by the entrepreneur. In the benchmark case with sef-funding, only the high-type entrepreneur launches the final product, and by Theorem 1, the test product is $(s, V-\epsilon)$ and the final product is launched regardless of the test outcome. Thus, the venture's expected total payoff is

$$q\left[\left(r + \frac{1-r}{2\epsilon}\right)P^{tp} + \frac{(1-r)(2\epsilon-1)}{2\epsilon}P^{fp}\right] + (1-q)K \tag{B.20}$$

In the case with external funding, when $q > q^*(u_H)$, by Theorem 3, the venture is always funded, and the equilibrium test product is $(s, V-\epsilon)$. Thus, the venture's expected payoff is

$$q\left[\left(r + \frac{1-r}{2\epsilon}\right)P^{tp} + \frac{(1-r)(2\epsilon-1)}{2\epsilon}P^{fp}\right]. \tag{B.21}$$

By comparing (B.20) and (B.21), we see that the equilibrium inefficiency is

$$(1-q)K, \tag{B.22}$$

which is strictly positive, invariant in u_H, and linear and decreasing in q.

When $q < q^*(u_H)$ and $u_H < \underline{u}_H$, by Theorem 3, the equilibrium test product is $(s, 0)$, and the venture is funded only if the test product sells. Thus, the venture's expected payoff is

$$q\left[rP^{tp} + \frac{(1-r)(V-1+\epsilon)}{2\epsilon}P^{fp} + \frac{(1-r)(1+\epsilon-V)}{2\epsilon}(K+u_H)\right] + (1-q)K. \tag{B.23}$$

By comparing (B.20) and (B.23), we see that the equilibrium inefficiency is

$$q\left[\frac{1-r}{2\epsilon}(P^{tp}-K-u_H) - \frac{(1-r)(V-\epsilon)}{2\epsilon}(P^{fp}-K-u_H)\right], \tag{B.24}$$

which is strictly positive as $P^{tp} > P^{fp}$ and $V-\epsilon \leq 1$ by Assumption 1 and 2. Thus, (B.24) is linear and increasing in q. By the same reasoning about $V-\epsilon \leq 1$, we have that (B.24) is decreasing in u_H.

When $q < q^*(u_H)$ and $\underline{u}_H < u_H < \bar{u}_H$, by Theorem 3, the equilibrium test product is $(s, V-\epsilon)$, and the venture is funded only if the test product sells. Thus, the venture's expected payoff is

$$q\left[rP^{tp} + \frac{(1-r)(2\epsilon-1)}{2\epsilon}P^{fp} + \frac{1-r}{2\epsilon}(K+u_H)\right] + (1-q)K. \tag{B.25}$$

By comparing (B.20) and (B.25), we see that the equilibrium inefficiency is

$$q\left[\frac{1-r}{2\epsilon}(P^{tp}-K-u_H)\right], \tag{B.26}$$

which is strictly positive, and linear and increasing in q. We also observe that (B.26) is decreasing in u_H.

When $q < q^*(u_H)$ and $u_H > \bar{u}_H$, by Theorem 3, the equilibrium test product is $(s, V-1+\epsilon)$, and the venture is funded only if the test product sells. Thus, the venture's expected payoff is

$$q\left[\frac{r(1-\epsilon)}{\epsilon}P^{tp} + \left(\frac{r(2\epsilon-1)}{\epsilon} + 1 - r\right)(K+u_H)\right]. \tag{B.27}$$

By comparing (B.20) and (B.27), we see that the equilibrium inefficiency is

$$q\left[\frac{2r(2\epsilon-1)+1-r}{2\epsilon}(P^{tp}-K-u_H) + \frac{(1-r)(2\epsilon-1)}{2\epsilon}(P^{fp}-K-u_H)\right], \tag{B.28}$$

which is strictly positive by our analysis around (B.3) in the proof of Theorem 1. Thus, (B.28) is linear increasing in q. Lastly, clearly (B.28) is decreasing in u_H.

We have shown that the equilibrium inefficiency is decreasing in q for $q > q^*(u_H)$ and increasing in q for $q < q^*(u_H)$. Moreover, the inefficiency is invariant in u_H for $q > q^*(u_H)$ and decreasing in u_H for $q < q^*(u_H)$.

B.9 Proof of Proposition 7

To compare different efficiency, we will rely on our inefficiency calculations in (B.22), (B.24), (B.26), and (B.28). Specifically, we observe that (B.24) subtracted by (B.22) is equal to (B.14) multiplied by q; (B.26) subtracted by (B.22) is equal to (B.15) multiplied by q; (B.28) subtracted by (B.22) is equal to (B.16) multiplied by q. Moreover, by the definition of q, each of the calculations is positive if and only if $q > q^*(u_H)$.

Proof of part (a).

First assume that $u_H < \underline{u}_H$. In this case, the equilibrium outcome is changed from the case with $q > q^*(u_H)$ to the case with $q < q^*(u_H)$ and $u_H < \underline{u}_H$. The change in inefficiency is (B.24) subtracted by (B.22), which is positive since $q > q^*(u_H)$.

Assume that $\underline{u}_H < u_H < \bar{u}_H$. In this case, the equilibrium outcome is changed from the case with $q > q^*(u_H)$ to the case with $q < q^*(u_H)$ and $\underline{u}_H < u_H < \bar{u}_H$. The change in inefficiency is (B.26) subtracted by (B.22), which is positive since $q > q^*(u_H)$.

Assume that $u_H > \bar{u}_H$. In this case, the equilibrium outcome is changed from the case with $q > q^*(u_H)$ to the case with $q < q^*(u_H)$ and $u_H > \bar{u}_H$. The change in inefficiency is (B.28) subtracted by (B.22), which is positive since $q > q^*(u_H)$.

We have shown that the negative stereotype makes the inefficiency higher.

When $q > q^*(u_H)$, the expected payoff of the investor is $q(K + \frac{1-q}{q}K) = K$. When $q < q^*(u_H)$, the expected payoff of the investor is also K since only the high-type entrepreneur gets funded. Moreover, as the inefficiency becomes higher and the low-type entrepreneurs' payoff is always 0, the increased inefficiency must come from a decrease to the high-type entrepreneurs' payoff.

Proof of part (b).

First assume that $u_H < \underline{u}_H$. In this case, the equilibrium outcome is changed from the case with $q < q^*(u_H)$ and $u_H < \underline{u}_H$ to the case with $q > q^*(u_H)$. Thus, by the proof of Proposition 6, the change in inefficiency is (B.22) subtracted by (B.24) with q changed to q_0, which is positive since $q_0 < q^*(u_H)$.

Assume that $\underline{u}_H < u_H < \bar{u}_H$. In this case, the equilibrium outcome is changed from the case with $q < q^*(u_H)$ and $\underline{u}_H < u_H < \bar{u}_H$ to the case with $q > q^*(u_H)$. The change in inefficiency is (B.22) subtracted by (B.26), with q changed to q_0, which is positive since $q_0 < q^*(u_H)$.

Assume that $u_H > \bar{u}_H$. In this case, the equilibrium outcome is changed from the case with $q < q^*(u_H)$ and $u_H > \bar{u}_H$ to the case with $q > q^*(u_H)$. The change in inefficiency is (B.22) subtracted by (B.28), with q changed to q_0, which is positive since $q_0 < q^*(u_H)$.

We have shown that the positive stereotype makes the inefficiency higher.

When $q > q^*(u_H)$, the total payoff to the high-type entrepreneurs must become higher as the extra payoff to the investor to launch the final product when the test product does not sell becomes lower. As the high-type entrepreneurs' payoff is higher and the low-type entrepreneurs' payoff is always 0, the increased inefficiency must come from a decrease in the investor's payoff.

The proof is complete.

C Proofs of Results in Appendix A

We show the proofs of the results in Appendix A, which is about the case where Assumption 3(a) does not hold. Towards this objective, a few intermediate results are needed.

Firstly, to establish whether a given strategy profile is a separating equilibrium, we need to characterize the low-type entrepreneur's strategic consideration. The following result shows the product launch pattern by the low-type entrepreneur in a separating equilibrium.

Lemma C.1 (Low-type funding in separating equilibrium)**.** *For large enough N, in every separating equilibrium, the low-type entrepreneur gets funded if and only if the test product sells.*

Proof. Proof of Lemma C.1. Fix a separating equilibrium and let (λ, ρ) denote the low-type entrepreneur's test product choice in the equilibrium. If the low-type entrepreneur does not get funded regardless of the test outcome, then by imitating the high-type entrepreneur's strategy, he is guaranteed a strictly positive payoff and thus a profitable deviation since the high-type entrepreneur gets funded with a positive probability. Therefore, the low-type entrepreneur must get funded after some test outcome.

Moreover, the low-type entrepreneur's posterior belief is more informative when the test product sells than when it does not. To see this, note that, by Assumption 1, conditional on W, the probability of the test product selling is f^{tp} if $W = \lambda$, f^{fp} if $|W - \lambda| = 1$, and 0 if otherwise. Hence, when the test product sells, the posterior belief about W is $\frac{f^{tp}}{f^{tp}+2f^{fp}}$ for $W = \lambda$, $\frac{f^{fp}}{f^{tp}+2f^{fp}}$ for $|W - \lambda| = 1$, and 0 otherwise. When the test product does not sell, the posterior belief about W is $\frac{1-f^{tp}}{N-f^{tp}-2f^{fp}}$ for $W = \lambda$, $\frac{1-f^{fp}}{N-f^{tp}-2f^{fp}}$ for $|W - \lambda| = 1$, and 1 otherwise. For large N, the posterior belief about W is more informative when the test product sells than when it does not. Hence, if there exists an equity share offer that is acceptable to both the low-type entrepreneur and the investor when the test product does not sell, then such an offer must also exist when the test product sells. That is, the low-type entrepreneur must get funded when the test product sells.

It remains to show that the low-type entrepreneur does not get funded when the test product does not sell. Based on our analysis of the low-type entrepreneur's posterior belief in the previous paragraph, if the final product is launched after a negative test outcome, the low-type entrepreneur solves (2) by choosing $\Lambda = \lambda - 3$.[2] Thus, the expected demand of the final product conditional on the negative test outcome is $\frac{P^{tp}+2P^{fp}}{N-f^{tp}-2f^{fp}}$, which converges to 0 as N tends to infinity. Hence, for large enough N, there exists no equity share contract that is acceptable to both the low-type entrepreneur and the investor.

The proof of the lemma is complete. □

When N is large and the test product does not sell, the low-type entrepreneur's posterior belief improves little from his prior belief because of his very diffuse prior. Moreover, as his type is revealed in a separating equilibrium, the investor will reject any equity share offer $\alpha \in [0, 1]$ by the low-type entrepreneur after a negative test outcome. On the other hand, if the test product sells, then the low-type entrepreneur's posterior belief can become very informative. For example, if he chooses test product $(0, V - 1 + \epsilon)$, then he knows that $W = 1$ with certainty if the test product sells, in which case there exists an equity share offer $\alpha \in [0, 1]$ that is mutually acceptable to the investor and the low-type entrepreneur.

Now that the low-type entrepreneur gets funded when the test product sells, his payoff is also affected by the demand of the final product. Therefore, the low-type entrepreneur also faces the learning-funding tradeoff in separating equilibria. The following result fully characterizes the low-type entrepreneur's strategic tradeoff in his experimentation strategy.

Lemma C.2 (Low-type entrepreneur's strategy)**.** *Let ρ_L be the low-type entrepreneur's test product's vertical attribute. For large enough N, we have the following characterizations of ρ_L in every separating equilibrium:*

(a) If $mP^{fp} > K$, then $\rho_L = 0$.

(b) If $mP^{fp} < K$ and $m(P^{tp} + P^{fp}) > 2K$, then $\rho_L = V - \epsilon$.

(c) If $m(P^{tp} + P^{fp}) < 2K$, then $\rho_L = V - 1 + \epsilon$.

Proof. Proof of Lemma C.2. By Lemma C.1, the low-type entrepreneur gets funded if and only if the test product sells. In this case, by our analysis of the low-type entrepreneur's posterior belief

[2] Recall that we have two integers λ_1 and λ_2 denote the same product location on Ω if $\lambda_1 - \lambda_2 = \alpha N$ for some $\alpha \in \mathbb{Z}$.

in the proof of Lemma C.1, the low-type entrepreneur's (unconditional) expected payoff is

$$\frac{f^{tp}(mP^{tp}-K)+2f^{fp}(mP^{fp}-K)}{N}. \tag{C.29}$$

Recall that f^{tp} is linear and decreasing in $\rho \in (V-\epsilon, V)$ (with derivative $-\frac{1}{\epsilon}$) and constant for $\rho \in (0, V-\epsilon)$; f^{fp} is constant for $\rho \in (V-1+\epsilon, V)$ and linear and decreasing in $\rho \in (0, V-1+\epsilon)$ (with derivative $-\frac{1}{2\epsilon}$).

For $\rho \in (V-1+\epsilon, V)$, since $mP^{tp} > K$, (C.29) is linear and decreasing in ρ. For $\rho \in (V-\epsilon, V-1+\epsilon)$, (C.29) is linear in ρ, with the derivative having the opposite sign as $m(P^{tp}+P^{fp})-2K$. For $\rho \in (0, V-\epsilon)$, (C.29) is linear in ρ, with the derivative having the opposite sign as $mP^{fp}-K$.

Hence, we have shown that (C.29) is decreasing in $\rho \in (V-1+\epsilon, V)$; decreasing in $\rho \in (V-\epsilon, V-1+\epsilon)$ if and only if $m(P^{tp}+P^{fp}) > 2K$; and decreasing in $\rho \in (0, V-\epsilon)$ if and only if $mP^{fp} > K$.

Therefore, if $mP^{fp} > K$, then (C.29) is maximized at $\rho_L = 0$; if $mP^{fp} < K$ and $m(P^{tp}+P^{fp}) > 2K$, then (C.29) is maximized at $\rho_L = V-\epsilon$; if $m(P^{tp}+P^{fp}) < 2K$, then (C.29) is maximized at $\rho_L = V-1+\epsilon$.

The proof of lemma is complete. □

The intuition is quite similar to that in the low investor confidence case (i.e., $q < q^*(u_H)$) where the final product is launched only if the test product sells. When the value of K is small such that $P^{fp} > K$, the low-type entrepreneur prioritizes funding over learning, since the low investment cost affords him a large share of the pie if the final product is launched. Hence, he sets $\rho_L = 0$ to maximize the probability of the test product selling, and therefore the probability of funding. When the value of K is moderate such that $P^{fp} < K$ but $m(P^{tp}+P^{fp}) > 2K$, the low-type entrepreneur trades off learning about the product-market fit and his chances of getting funded, which leads him to set ρ_L to the intermediate level $V-\epsilon$. Lastly, when the value of K is large such that $m(P^{tp}+P^{fp}) < 2K$ (but $mP^{tp} > K$ still holds), the low-type entrepreneur prioritizes making his posterior belief after a positive test outcome as informative as possible, which is achieved by setting $\rho_L = V-1+\epsilon$ since he would know $W = 1$ with certainty after a positive test outcome.

Remark C.1. Lemma C.2 also characterizes the low-type entrepreneur's optimal strategy in the case with self-funding: when deviating to an off-path strategy leads the investor to believe the entrepreneur's type to be low with probability one, the low-type entrepreneur's payoff from each off-path strategy is the same as that in the case with self-funding. With the same reasoning, we see that Lemma C.1 also holds for the low-type entrepreneur in the benchmark case with self-funding.

With the intermediate results we have established, we are ready to show the proofs of the results in Appendix A.

C.1 Proof of Proposition A1

Let π denote the random variable denoting the demand of the final product. Based on our derivation for the value of $E(\pi|X=1,\lambda,\rho)$ in the proof of Theorem 3, we have

$$U_1(\rho) := \lim_{N\to\infty} E(\pi\mathbf{1}(X=1)|\lambda=0,\rho,\theta=1) = rf^{tp}(\rho)P^{tp} + (1-r)f^{fp}(\rho)P^{fp}.$$

Similarly, based on our discussion in the proof of Theorem 3 regarding the (unconditional) expected demand of the final product launched by the low-type entrepreneur when the test product sells, we have

$$U_0(\rho) := NE(\pi\mathbf{1}(X=1)|,\lambda=0,\rho,\theta=0) = f^{tp}(\rho)P^{tp} + 2f^{fp}(\rho)P^{fp},$$

where the multiplication by N allows us to compare the expected demand of the final product for different values of ρ.

The following technical lemma shows that the proportional growth of $U_1(\rho)$ is less than that of $U_0(\rho)$.

Lemma C.3. $\log\frac{U_1(\rho)}{U_0(\rho)}$ *is non-decreasing in* $\rho\in(0,V)$.

Proof. Proof of Lemma C.3. We need to show that $\frac{\partial\log U_1(\rho)}{\partial\rho}\geq\frac{\partial\log U_0(\rho)}{\partial\rho}$. Towards this objective, we observe that $\frac{\partial\log U_0(\rho)}{\partial\rho}=\frac{\partial\log U_1(\rho)}{\partial\rho}|_{r=1/3}$. Since $r>1/2$ by assumption, it suffices to show that $\frac{\partial\log U_1(\rho)}{\partial\rho}$ is non-decreasing in r at every $\rho\in(0,V)$ (except for $\rho\in\{V-\epsilon,V-1+\epsilon\}$, where $U_1(\rho)$ is not differentiable).

When $\rho>V-1+\epsilon$, $f^{fp}(\rho)=0$ and $\log\frac{U_1(\rho)}{U_0(\rho)}=\log r$, which is constant in ρ.

When $\rho\in(V-\epsilon,V-1+\epsilon)$, both $f^{tp}(\rho)$ and $f^{fp}(\rho)$ are linear in ρ, with $\partial f^{tp}/\partial\rho=-1/\epsilon$ and $\partial f^{fp}/\partial\rho=-1/(2\epsilon)$. Hence, we have $\partial U_1(\rho)/\partial\rho=-\left[2rP^{tp}+(1-r)P^{fp}\right]/(2\epsilon)$ and therefore

$$\begin{aligned}
\frac{\partial^2\log U_1(\rho)}{\partial r\partial\rho} &= -\frac{1}{2\epsilon}\frac{\partial}{\partial r}\left[\frac{2rP^{tp}+(1-r)P^{fp}}{rf^{tp}(\rho)P^{tp}+(1-r)f^{fp}(\rho)P^{fp}}\right]\\
&= -\frac{(2P^{tp}-P^{fp})(rf^{tp}P^{tp}+(1-r)f^{fp}P^{fp})-(2rP^{tp}+(1-r)P^{fp})(f^{tp}P^{tp}-f^{fp}P^{fp})}{2\epsilon(rf^{tp}P^{tp}+(1-r)f^{fp}P^{fp})^2}\\
&= \frac{(f^{tp}-2f^{fp})P^{tp}P^{fp}}{2\epsilon(rf^{tp}P^{tp}+(1-r)f^{fp}P^{fp})^2},
\end{aligned}$$

which is positive since $2\epsilon(f^{tp}-2f^{fp})=1-\epsilon>0$ for $\rho\in(V-\epsilon,V-1+\epsilon)$ by Assumption 2.

When $\rho \in (0, V-\epsilon)$, we have $f^{tp}(\rho) = 1$ and $f^{fp}(\rho)$ is linear and decreasing in ρ with the rate of change being $-1/(2\epsilon)$. Hence, we have $\frac{\partial U_1(\rho)}{\partial \rho} = -\frac{(1-r)P^{fp}}{2\epsilon}$ and therefore

$$\frac{\partial \log U_1(\rho)}{\partial \rho} = -\frac{(1-r)P^{fp}}{2\epsilon(rP^{tp} + (1-r)f^{fp}(\rho)P^{fp})} = -\frac{P^{fp}}{2\epsilon\left(\frac{r}{1-r}P^{tp} + f^{fp}(\rho)P^{fp}\right)},$$

which is increasing in r.

We have shown that $\frac{\partial \log U_1(\rho)}{\partial \rho}$ is non-decreasing in r at every $\rho \in (0, V)$. Therefore, $\log \frac{U_1(\rho)}{U_0(\rho)}$ is non-decreasing in $\rho \in (0, V)$. The proof of the lemma is complete. □

Let $\alpha^*(\rho)$ be the optimal equity share offer to the investor when the test product $(0, \rho)$ sells. Since $q < q^*(u_H)$ by assumption, it is without loss of generality to assume that the final product is not launched after a negative test outcome. Thus, the high-type entrepreneur's expected payoff from pooling with the low-type entrepreneur with test product $(0, \rho)$ is $\overline{V}_1(\rho) := U_1(\rho)(1 - \alpha^*(\rho)) + \Pr(X = 0|\lambda = 0, \rho, \theta = 1)u_H$; for the low-type entrepreneur, it is $U_0(\rho)(1 - \alpha^*(\rho))$. By Lemma C.3, for any $\rho \in (0, V)$, $\partial \log U_1(\rho)/\partial\rho < 0$ implies that $\partial \log U_0(\rho)/\partial\rho < 0$ holds. Therefore, $U_0(\rho)(1 - \alpha^*(\rho))$ is decreasing at ρ if $U_1(\rho)(1 - \alpha^*(\rho))$ is decreasing at ρ.

Let ρ_H^{**} be the minimum maximizer of $\overline{V}_1(\rho)$ and ρ_H^* be the minimum maximizer of $U_1(\rho)(1 - \alpha^*(\rho))$. Let ρ_L^* be the maximum maximizer of the low-type entrepreneur's expected payoff in the self-funding case, which is characterized by Lemma C.2 via Remark C.1.

We will show that neither types of entrepreneurs have an incentive to deviate from the pooling strategy profile with test product $(0, \rho_H^{**})$ for small enough u_H. The high-type entrepreneur has no incentive to deviate since ρ_H^{**} is the maximizer of $\overline{V}_1(\rho)$ by definition. For the low-type entrepreneur, by the proof of Lemma C.2 and our choice of ρ_L^*, the low-type entrepreneur's expected payoff is non-decreasing in $\rho \in (0, \rho_L^*)$ and decreasing in $\rho \in (\rho_L^*, V)$. Meanwhile, we observe that $\overline{V}_1(\rho) = U_1(\rho)(1 - \alpha^*(\rho))$ if $u_H = 0$. Hence, by our proof of Theorem 3, for large enough N, we have that $U_1(\rho)(1 - \alpha^*(\rho))$ is non-increasing in $\rho \in (\rho_H^*, V)$. In this case, by Lemma C.3, if $\rho_H^* \leq \rho_L^*$, then $U_0(\rho)(1 - \alpha^*(\rho))$ is decreasing in $\rho \in (\rho_H^*, V)$. Thus, the low-type entrepreneur has no incentive to deviate in a pooling strategy profile with test product $(0, \rho)$ where $\rho \in [\rho_L^*, \rho_H^*]$.

We now show that $\rho_H^* \leq \rho_L^*$. Based on the derivations of $\underline{u}_H$ and $\bar{u}_H$ in the proof of Theorem 3, for large enough N, we have

$$\rho_H^{**} = \begin{cases} 0 & \text{if } P^{fp} - K - u_H > 0 \\ V - \epsilon & \text{if } P^{fp} - K - u_H < 0 \text{ and } 2r(P^{tp} - K - u_H) + (1-r)(P^{fp} - K - u_H) > 0 \\ V - 1 + \epsilon & \text{if } 2r(P^{tp} - K - u_H) + (1-r)(P^{fp} - K - u_H) < 0. \end{cases}$$

The values of ρ_H^* can be derived similarly by setting $u_H = 0$. Since $u_H \geq 0$, we immediately have

$\rho_H^* \leq \rho_H^{**}$.

Meanwhile, by Lemma C.2, we have that

$$\rho_L^* = \begin{cases} 0 & \text{if } mP^{fp} > K \\ V - \epsilon & \text{if } mP^{fp} < K \text{ and } m(P^{tp} + P^{fp}) > 2K \\ V - 1 + \epsilon & \text{if } m(P^{tp} + P^{fp}) < 2K. \end{cases}$$

Since $r > 1/2$ and $m \leq 1$ by assumption, we have $\rho_H^* \leq \rho_L^*$ for large enough N.

To prove the proposition, it suffices to show that $\rho_H^{**} \leq \rho_L^*$ for small enough $u_H > 0$ and large enough N.

Assume $mP^{fp} > K$, in which case we have $\rho_L^* = 0$. Since $m \leq 1$, we have $P^{fp} > K$, and can find $\bar{u}_{H1}^* > 0$ such that $P^{fp} - K - u_H > 0$ for $u_H < \bar{u}_{H1}^*$, in which case, for large enough N, we have $\rho_H^{**} = 0$ and hence $\rho_H^{**} = \rho_L^*$.

Assume $mP^{fp} \leq K$ and $m(P^{tp} + P^{fp}) > 2K$. In this case, $\rho_L^* = V - \epsilon$. Since $m \leq 1$ and $r > 1/2$ by assumption, we have $2r(P^{tp} - K) + (1 - r)(P^{fp} - K) > 0$, and can thus find $\bar{u}_{H2}^* > 0$ such that $2r(P^{tp} - K - u_H) + (1 - r)(P^{fp} - K - u_H) > 0$ for $u_H < \bar{u}_{H2}^*$, in which case, for large enough N, we have $\rho_H^{**} \leq V - \epsilon$ and hence $\rho_H^{**} \leq \rho_L^*$.

Assume $m(P^{tp} + P^{fp}) \leq 2K$. In this case, $\rho_L^* = V - 1 + \epsilon$. Since $\rho_H^* \leq V - 1 + \epsilon$ by Theorem 3 and its proof, we have $\rho_H^* \leq \rho_L^*$ for large enough N.

Let $\bar{u}_H^* = \min\{\bar{u}_{H1}^*, \bar{u}_{H2}^*\} > 0$. We have shown that for $u_H < \bar{u}_H^*$ and large enough N, we have $\rho_H^{**} \in [\rho_H^*, \rho_L^*]$. Hence, the low-type entrepreneur has no incentive to deviate from the pooling strategy profile with test product $(0, \rho_H^{**})$.

The proof is complete.

C.2 Proof of Proposition A2

Assume the low-type entrepreneur sets the test product's horizontal attribute to 0. Let ρ_L denote the low-type entrepreneur's choice of his test product's vertical attribute. By Lemma C.2, since $m(P^{tp} + P^{fp}) > 2K$ by assumption, we can focus on $\rho_L \leq V - \epsilon$.

Let $\underline{u}_{H1}^*$ be the unique solution for u_H to

$$\max\left\{\frac{r(2\epsilon - 1)}{\epsilon}, 1 - r\right\}(P^{tp} - K - u_H) + \min\left\{\frac{r(2\epsilon - 1)}{\epsilon}, 1 - r\right\}(P^{fp} - K - u_H) = 0,$$

whose left-hand side is the expected payoff of the high-type entrepreneur if he offers the investor just enough equity share to launch the final product when test product $(0, V - 1 + \epsilon)$ does not sell.

When $u_H > \underline{u}_{H1}^*$ and the test product is $(0, V - 1 + \epsilon)$, the high-type entrepreneur aborts the venture after a negative test outcome even if his type is known to the investor. Let $\underline{u}_H^* = \max\{\underline{u}_{H1}^*, \overline{u}_H\}$, where $\overline{u}_H$ is as defined in Theorem 3. When $u_H > \underline{u}_H^*$, conditional on launching the final product only after a positive test outcome, by the proof of Theorem 3, the high-type entrepreneur's expected payoff is maximized at test product $(0, V-1+\epsilon)$.

Consider the strategy profile where the high-type entrepreneur sets test product $(0, V-1+\epsilon)$ and the low-type entrepreneur sets test product $(0, \rho_L)$, both types of entrepreneurs launch the final product if and only if the test product sells. It remains to show that neither type of entrepreneur has an incentive to deviate from this strategy profile.

For the high-type entrepreneur, his deviation payoff must be weakly lower than his payoff from the separating strategy profile, since by Lemma C.1, he would not get funded after a negative test outcome for any deviation. Thus, the high-type entrepreneur has no incentive to deviate.

For the low-type entrepreneur, if he mimics the high-type entrepreneur's strategy with test product $(0, V-1+\epsilon)$, his payoff is the same as his payoff if $m = 1$ and his type is revealed to the investor, since when $m = 1$ and the test product $(0, V-1+\epsilon)$ sells, the investor is indifferent between the two types of entrepreneurs. Moreover, when his type is known, the low-type entrepreneur's expected payoff from setting test product $(0, V-1+\epsilon)$ is strictly dominated by that from setting test product $(0, \rho_L)$ for $\rho_L < V-1+\epsilon$. Thus, there exists $\underline{m} < 1$ such that, if $m > \underline{m}$, then the low-type entrepreneur's imitation payoff is strictly lower than his payoff from choosing test product $(0, \rho_L)$ for some $\rho_L \le V-\epsilon$ and revealing his type to the investor. Hence, the low-type entrepreneur has no incentive to deviate for large enough m.

We have shown that, for large enough N, there exists $\underline{u}_H^*$ and $\underline{m}$ such that, if $u_H > \underline{u}_H^*$ and $m > \underline{m}$, then a separating equilibrium where the high-type entrepreneur sets test product $(0, V-1+\epsilon)$ exists. The proof is complete.

C.3 Proof of Proposition A3

Fix a separating equilibrium where the high-type entrepreneur chooses test product $(0, \rho_H)$. Such a separating equilibrium exists for some parameter values, e.g., the numerical example in Figure A1. We want to show that $\rho_H > V-\epsilon$. Towards a contradiction, assume instead $\rho_H \le V-\epsilon$. Let (λ_L, ρ_L) denote the low-type entrepreneur's test product choice in equilibrium. Based on our analysis of the low-type entrepreneur's posterior belief in the proof of Lemma C.1, the expected demand of the optimal final product joint with the test product selling is

$$\frac{f^{tp}(\rho_L) m P^{tp} + 2 f^{fp}(\rho_L) m P^{fp}}{N}. \tag{C.30}$$

The expected demand of the optimal final product joint with the test product not selling is

$$\frac{m(P^{tp} + 2P^{fp})}{N}. \tag{C.31}$$

Since $f^{tp}(\rho_L) \leq 1$ and $f^{fp}(\rho_L) \leq 1$, (C.31) is (weakly) larger than (C.30).

Let α^* be the minimum equity share offer acceptable to the investor when she knows that the entrepreneur's type is high and the entrepreneur knows W with certainty (i.e., his posterior belief is degenerate). Hence, α^* is the minimum equity share offer that could occur on every equilibrium path. Since $\rho_H \leq V - \epsilon$, which leads to the most informative posterior belief about a negative test outcome, the high-type entrepreneur offers α^* after a negative test outcome.

By Lemma C.1, for large enough N, the low-type entrepreneur is not funded when the test product does not sell in every separating equilibrium. Hence, the low-type entrepreneur's equilibrium expected payoff is bounded above by

$$(1-\alpha^*)\frac{f^{tp}(\rho_L)mP^{tp} + 2f^{fp}(\rho_L)mP^{fp}}{N}, \tag{C.32}$$

since the equity share offered to the investor cannot be lower than α^*.

If the low-type entrepreneur imitates the high-type entrepreneur's test product choice, then the low-type entrepreneur offers equity share offer α^* to get funded after a negative test outcome. Let $\alpha_{H1} \in (0, 1)$ denote the equity share the high-type entrepreneur offers when the test product sells. If the low-type entrepreneur imitates the high-type entrepreneur's strategy, by Theorem 1, the low-type entrepreneur is always funded, and his expected payoff is

$$(1-\alpha_{H1})\frac{f^{tp}(V-\epsilon)mP^{tp} + 2f^{fp}(V-\epsilon)mP^{fp}}{N} + (1-\alpha^*)\frac{m(P^{tp} + 2P^{fp})}{N}. \tag{C.33}$$

The difference between (C.33) and (C.32) is

$$(1-\alpha_{H1})\frac{f^{tp}(V-\epsilon)mP^{tp} + 2f^{fp}(V-\epsilon)mP^{fp}}{N} + (1-\alpha^*)\left(\frac{m(P^{tp} + 2P^{fp})}{N} - \frac{f^{tp}(\rho_L)mP^{tp} + 2f^{fp}(\rho_L)mP^{fp}}{N}\right),$$

which is positive since the first term is positive and the second term is also positive by our analysis of the difference between (C.31) and (C.30). Thus, the low-type entrepreneur becomes strictly better off by imitating the high-type entrepreneur's strategy, a contradiction. Therefore, for large enough N, we must have $\rho_H > V - \epsilon$.

The proof is complete.