WEB APPENDIX

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WEB APPENDIX A

PROOF OF LEMMA 1. Consider the subgame where $(R_1, R_2) = (1,1)$. We solve the game by backward induction using the guess-and-verify approach. Specifically, we start with a guess that in the first period, the market is fully covered, and hence, no consumer chooses to wait till the second period to buy a product (we will later verify our guess).[1] Mathematically, this is equivalent to

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} \geq \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\} \quad \text{(A1)}$$

for all $d \in [0,1]$, where p_{12}^e and p_{22}^e are the prices that first-period consumers expect firms 1 and 2 to charge in the second period, respectively.

If the inequality (A1) is satisfied, then in the second period, the firms compete for the $(1-\alpha)$ measure of new consumers who enter the market. Given the firms' second-period prices p_{12} and p_{22}, a consumer located at a distance d from firm 1 will prefer firm 1's product over firm 2's if and only if $V - p_{12} - \theta d > V - p_{22} - \theta(1-d)$, or equivalently, $d < \frac{\theta - p_{12} + p_{22}}{2\theta}$. Hence, firm i will choose its second-period price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22} = (1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}) p_{22}$. Solving the first-order conditions ($\frac{\partial \pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial \pi_{22}}{\partial p_{22}} = 0$), we find that $p_{12}^{(1,1)} = p_{22}^{(1,1)} = \theta$. The corresponding second-period profits are given by $\pi_{12}^{(1,1)} = \pi_{22}^{(1,1)} = \frac{(1-\alpha)\theta}{2}$. Rational-expectations condition requires that $p_{12}^e = p_{22}^e = \theta$. Plugging these prices into (A1), the inequality (A1) becomes

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} \geq \max\{\delta(V - \theta - \theta d), \delta(V - \theta - \theta(1-d))\} \quad \text{(A2)}$$

Hence, if a given pair (p_{11}, p_{21}) of first-period prices satisfies the inequality (A2), then *a*) $p_{12}^e = p_{22}^e = \theta$ are rational expectations about the second-period prices, and *b*) all first-period consumers rationally choose to buy in the first period rather than wait till the second period.

In the first period, the firms choose their prices to maximize the sum of their profits over the two periods. If prices p_{11} and p_{21} satisfy (A2), then the firms' profits are given by $\pi_1 = \pi_{11} + \pi_{12}^{(1,1)} =$

[1] In Web Appendix B, we show that there does not exist an alternative equilibrium in which some of the consumers who enter the market in the first period prefer to wait till the second period to buy a product.

$\alpha \frac{\theta - p_{11} + p_{21}}{2\theta} p_{11} + \frac{(1-\alpha)\theta}{2}$ and $\pi_2 = \pi_{21} + \pi_{22}^{(1,1)} = \alpha(1 - \frac{\theta - p_{11} + p_{21}}{2\theta}) p_{21} + \frac{(1-\alpha)\theta}{2}$. Solving the first-order conditions, we find that $p_{11}^{(1,1)} = p_{21}^{(1,1)} = \theta$. Firm i's corresponding profits in each period are given by $\pi_{i1}^{(1,1)} = \frac{\alpha\theta}{2}$ and $\pi_{i2}^{(1,1)} = \frac{(1-\alpha)\theta}{2}$.

Because $\delta < 1$, it is easy to see that the price pair $(p_{11}, p_{21}) = (\theta, \theta)$ indeed satisfies the inequality (A2). Hence, our initial guess that first-period consumers will not wait till the second period is verified.

Finally, let us check that firm i does not have any non-local profitable deviations from its first-period price $p_{i1}^{(1,1)} = \theta$. Given firm 2's first-period price $p_{21}^{(1,1)} = \theta$, suppose that firm 1 deviates to a price p'_{11}. Because $\theta < \bar{\theta} = \frac{V(1-\delta)}{2-\delta}$, one can show that the inequality (A2) is satisfied for any pair (p'_{11}, θ). If first-period prices satisfy (A2), then as we discussed earlier, all first-period consumers would rationally choose to purchase in the first period instead of waiting till the second period. It follows that firm 1's deviation to p'_{11} affects firm 1's first-period market share but does not induce any consumer to wait till the second period instead of buying in the first period. If none of the first-period consumers waits till the second period, then we know that firm 1's second-period subgame equilibrium profits are $\pi_{12} = \frac{(1-\alpha)\theta}{2}$. Hence, firm 1's deviation profit is given by $\pi'_1 = \alpha \frac{\theta - p'_{11} + \theta}{2\theta} p'_{11} + \frac{(1-\alpha)\theta}{2}$. From earlier analysis, we know that π'_1 is maximized when $p'_{11} = \theta$. Therefore, firm 1's deviation to a price $p'_{11} \neq \theta$ cannot be profitable.

The analysis for the subgame $(R_1, R_2) = (2, 2)$ is provided in the main text. ■

PROOF OF LEMMA 2. We solve the game using backward induction. In the second period, the firms compete for $(1-\alpha)$ new consumers entering the market, as well as those first-period consumers who had decided not to buy in the first period but wait. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize those first-period consumers who will prefer not to buy in the first period but wait. Specifically, a consumer will choose to wait if and only if

$$\max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\} > V - p_{11} - \theta d. \quad \text{(A3)}$$

The left-hand-side of the inequality shows the utility that a consumer anticipates to receive if she chooses to wait till the second period to buy. The right-hand-side is the utility that the consumer can obtain by purchasing firm 1's product in the first period. One can readily show that $\max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\} - (V - p_{11} - \theta d)$ is increasing in d. Therefore, if the inequality (A3) is satisfied for some $d \in [0,1]$, then there must exist a unique $\hat{d}$ such that the inequality holds if and only if $d \in (\hat{d}, 1]$. Hence, to know which first-period consumers will still be shopping in the second period, we need to find $\hat{d}$. There are four possible situations that may happen in equilibrium.

i. *No consumer chooses to wait*. In this case, $\hat{d} = 1$. Therefore, it must be that even the consumer located furthest away from firm 1 prefers to buy firm 1's product in the first period rather than wait till the second period: $V - p_{11} - \theta > \max\{\delta(V - p_{12}^e - \theta), \delta(V - p_{22}^e)\}$. Because, in equilibrium, firm 2 must have a positive market share in the second period and consumers' price expectations must be rational, we have $V - p_{12}^e - \theta < V - p_{22}^e$, i.e., the consumer with $d = 1$ would buy from firm 2 in the second period.[2] Hence, $V - p_{11} - \theta > \max\{\delta(V - p_{12}^e - \theta), \delta(V - p_{22}^e)\} = \delta(V - p_{22}^e)$, which is equivalent to $p_{11} < V(1-\delta) + \delta p_{22}^e - \theta$.

ii. *A positive measure of consumers chooses to wait and all of them anticipate purchasing firm 2's product in the second period*. Hence, the consumer with $d = \hat{d}$ must be indifferent between buying firm 1's product in the first period and buying firm 2's product in the second period. Solving the equality $V - p_{11} - \theta\hat{d} = \delta(V - p_{22}^e - \theta(1-\hat{d}))$, we obtain $\hat{d} = \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)}$. It must be that $0 < \hat{d} < 1$ and $V - p_{22}^e - \theta(1-\hat{d}) \geq V - p_{12}^e - \theta\hat{d}$. These inequalities hold when $V(1-\delta) + \delta p_{22}^e - \theta < p_{11} \leq \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2}$.

iii. *A positive measure of consumers chooses to wait; some of these consumers anticipate purchasing firm 1's product in the second period, while others expect to buy firm 2's product*. In this case, the marginal consumer with $d = \hat{d}$ must be indifferent between purchasing firm 1's product in the first period and purchasing it in the second period. Solving the equality $V - p_{11} - \theta\hat{d} =$

[2] More formally, we know that both firms will have positive market share in the second period. Therefore, one can show that their second-period prices will satisfy $0 < \frac{\theta - p_{12}^* + p_{22}^*}{2\theta} < 1$, where $\frac{\theta - p_{12}^* + p_{22}^*}{2\theta}$ is the location of the marginal consumer who is indifferent between the two products. Rational expectations require that $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$, which implies that $0 < \frac{\theta - p_{12}^e + p_{22}^e}{2\theta} < 1$. Finally, the last inequality implies that $V - p_{12}^e - \theta < V - p_{22}^e$.

$\delta(V - p_{12}^e - \theta\hat{d})$, we obtain $\hat{d} = \frac{V(1-\delta)-p_{11}+\delta p_{12}^e}{\theta(1-\delta)}$. It must be that $0 < \hat{d} < 1$ and $V - p_{12}^e - \theta\hat{d} > V - p_{22}^e - \theta(1-\hat{d})$. These inequalities are equivalent to $\frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e$.

iv. *All consumers choose to wait till the second period.* In this case, $\hat{d} = 0$. That is, the consumer with $d = 0$ prefers to wait instead of buying firm 1's product in the first period which implies that all other consumers also prefer to wait. Mathematically, $V - p_{11} \leq \max\{\delta(V - p_{12}^e), \delta(V - p_{22}^e - \theta)\}$. Because firm 1 will have positive market share in the second period and consumers' price expectations are rational, it must be that $V - p_{12}^e > V - p_{22}^e - \theta$, i.e., the consumer with $d = 0$ would buy from firm 1 in the second period. Therefore, $V - p_{11} \leq \delta(V - p_{12}^e)$, which is equivalent to $p_{11} \geq V(1-\delta) + \delta p_{12}^e$.

To summarize the above, we considered the four possible situations that may happen in equilibrium and characterized $\hat{d}$ in each situation. Further, in each case, we obtained the conditions on p_{11}, p_{12}^e and p_{22}^e to ensure that consumers' decisions to wait are optimal given their expectations about second-period prices. Specifically, $\hat{d}$ is given by

$$\hat{d} \equiv \begin{cases} 1 & \text{if } p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta \\ \frac{V(1-\delta)-p_{11}+\delta p_{22}^e+\theta\delta}{\theta(1+\delta)} & \text{if } V(1-\delta) + \delta p_{22}^e - \theta < p_{11} \leq \frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} \\ \frac{V(1-\delta)-p_{11}+\delta p_{12}^e}{\theta(1-\delta)} & \text{if } \frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e \\ 0 & \text{if } p_{11} \geq V(1-\delta) + \delta p_{12}^e \end{cases} \tag{A4}$$

Outline of solution steps. To find the second-period subgame equilibrium prices that satisfy the rational expectations conditions, we will use the standard "guess and verify" approach. *Step 1*: we start by making a guess that p_{11} satisfies the conditions corresponding to one of the four possible cases outlined above. *Step 2*: solving the first-order conditions, we obtain the firms' second-period subgame equilibrium prices p_{12}^* and p_{22}^* as a function of p_{12}^e, p_{22}^e and p_{11}. *Step 3*: because we are looking for a rational-expectations equilibrium, we solve the equations $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ to find the price pair (p_{12}^e, p_{22}^e) that satisfies the condition for rational-expectations: $(p_{12}^e, p_{22}^e) = (p_{12}^*, p_{22}^*)$. *Step 4*: to verify the guess that we started with, we plug the prices $(p_{12}^e, p_{22}^e) = (p_{12}^*, p_{22}^*)$ back into the condition that we assumed to hold in Step 1. This allows us to find conditions on p_{11} under which the triplet

$(p_{11}, p_{12}^e, p_{22}^e) = (p_{11}, p_{12}^*, p_{22}^*)$ satisfies the conditions of the case, verifying our initial guess. *Step 5*: given the conditions on p_{11} that we obtained in Step 4, we check whether any firm has a profitable non-local deviation. We repeat Steps 1-5 for each of the four possible cases.

Case i: $p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta$. In this case, $\hat{d} = 1$, i.e., all first-period consumers buy firm 1's product in the first period. Hence, in the second-period, firms compete for the new consumers entering the market. Firms 1 and 2 choose prices to maximize $(1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $(1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}) p_{22}$, respectively. Solving the first-order conditions, we find that $p_{12}^* = p_{22}^* = \theta$. Rational expectations require that $p_{i2}^e = p_{i2}^* = \theta$. Note that Case *i* assumes that $p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta$. We need to check that this inequality is satisfied. Substituting $p_{22}^e = \theta$ into the inequality $p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta$, it follows that the condition for case *i* is satisfied when $p_{11} \leq \underline{\rho}$, where $\underline{\rho} \equiv (1-\delta)(V-\theta)$. It is straightforward to show that the firms' profit functions are quasi-concave, and thus, the firms do not have any profitable non-local deviations.

Case ii: $V(1-\delta) + \delta p_{22}^e - \theta < p_{11} \leq \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2}$. In this case, we have $\hat{d} = \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)}$. In the first period, consumers with $d \leq \hat{d}$ bought firm 1's product, while the remaining consumers with $d > \hat{d}$ preferred to wait till the second period, anticipating to buy firm 2's product. Since we are solving for a rational-expectations equilibrium, the firms' second-period subgame equilibrium prices must indeed be such that consumers with $d > \hat{d}$ buy firm 2's product. This happens when firm 2's second-period price is sufficiently low: $p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$. In addition, to have a positive market share, firm 1's price must not be too high: $p_{12} < p_{22} + \theta$, or equivalently, $p_{22} > p_{12} - \theta$. When $p_{12} - \theta \leq p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$, the firms' second-period profits are

$$\pi_{12} = (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$$

$$\pi_{22} = (\alpha(1-\hat{d}) + (1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}))p_{22}$$

Solving the first-order conditions ($\frac{d\pi_{12}}{dp_{12}} = 0$ and $\frac{d\pi_{22}}{dp_{22}} = 0$), we obtain the candidate equilibrium prices: $p_{12}^* = \frac{2\alpha(p_{11} - V - p_{22}^e\delta + V\delta) + \theta(3(1+\delta) - \alpha - 3\alpha\delta)}{3(1-\alpha)(1+\delta)}$ and $p_{22}^* = \frac{4\alpha(p_{11} - V - p_{22}^e\delta + V\delta) + \theta(3(1+\delta) + \alpha(1-3\delta))}{3(1-\alpha)(1+\delta)}$. To ensure that the rational-expectations condition is satisfied, we need that $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$. Solving these equations, we find that $p_{12}^e = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^e = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$. Plugging p_{12}^e and p_{22}^e into the expressions for p_{12}^* and p_{22}^*, we verify $p_{12}^* = p_{12}^e = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^* = p_{22}^e = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$. For future reference, using p_{22}^e, we find that $\hat{d} = \frac{V(1-\delta)-p_{11}+\delta p_{22}^e+\theta\delta}{\theta(1+\delta)} = \frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))}$.

Next, we need to verify the assumptions that we used in the analysis. First, case *ii* assumes that p_{11} satisfies $V(1-\delta)+\delta p_{22}^e-\theta < p_{11} \leq \frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2}$. Plugging in the expressions for p_{12}^e and p_{22}^e, we find that $V(1-\delta)+\delta p_{22}^e-\theta < p_{11} \leq \frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2}$ is equivalent to $\underline{\rho} < p_{11} \leq \frac{2V(3-2\alpha)(1-\delta)+\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$, where $\underline{\rho} = (1-\delta)(V-\theta)$. Second, recall that to ensure that both firms have positive market shares and that consumers with $d > \hat{d}$ indeed buy firm 2's product, we assumed that the firms' prices would satisfy $p_{12}-\theta \leq p_{22} \leq p_{12}-\theta+2\theta\hat{d}$. One can readily show that when $\underline{\rho} < p_{11} \leq V(1-\delta)+\frac{\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$, the equilibrium prices p_{12}^* and p_{22}^* indeed satisfy $p_{12}^*-\theta \leq p_{22}^* \leq p_{12}^*-\theta+2\theta\hat{d}$.

Firm 1's profit function is not necessarily quasi-concave because firm 1 may potentially want to make a non-local deviation to a much lower price in order to capture some of the consumers with $d > \hat{d}$. Hence, we need to rule out such deviations. To serve some of the consumers located on $(\hat{d}, 1]$, firm 1 will need to deviate to a price $p'_{12} < p_{22}^*+\theta-2\theta\hat{d}$, in which case firm 1's deviation profit will be $\pi'_{12} = (\alpha(\frac{\theta-p'_{12}+p_{22}^*}{2\theta}-\hat{d})+(1-\alpha)\frac{\theta-p'_{12}+p_{22}^*}{2\theta})p'_{12}$. Note that π'_{12} is concave. Evaluating the left-derivative of π'_{12} at the point $p'_{12} = p_{22}^*+\theta-2\theta\hat{d}$, we find that $\frac{\partial_-\pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p_{22}^*+\theta-2\theta\hat{d}} = \frac{V(6-7\alpha+3\alpha^2)(1-\delta)-p_{11}(6-7\alpha+3\alpha^2)-\theta(3-\alpha)(1-3\delta+2\alpha\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))}$. Next, $\frac{\partial_-\pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p_{22}^*+\theta-2\theta\hat{d}} \geq 0$ if and only if $p_{11} \leq \tilde{\rho}$, where $\tilde{\rho} \equiv (1-\delta)V+\frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ satisfies $\underline{\rho} < \tilde{\rho} < V(1-\delta)+\frac{\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$. Since π'_{12} is concave, $\frac{\partial_-\pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p_{22}^*+\theta-2\theta\hat{d}} \geq 0$ implies that π'_{12} is increasing in p'_{12} at any $p'_{12} < p_{22}^*+\theta-2\theta\hat{d}$. Since firm 1's profit function is continuous at the point $p_{22}^*+\theta-2\theta\hat{d}$, it follows that a deviation to $p'_{12} < p^*_{22} + \theta - 2\theta\hat{d}$ cannot be profitable whenever $\underline{\rho} < p_{11} \le \tilde{\rho}$. Because $\tilde{\rho} < V(1-\delta) + \frac{\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$, our initial guess holds. Therefore, the prices $p^*_{12} = p^e_{12} = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p^*_{22} = p^e_{22} = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$ constitute a rational-expectations subgame equilibrium for any given $p_{11} \in (\underline{\rho}, \tilde{\rho}]$.

Case iii: $\frac{(2V-p^e_{22}-\theta)(1-\delta)+p^e_{12}(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p^e_{12}$. In this case, $\hat{d} = \frac{V(1-\delta)-p_{11}+\delta p^e_{12}}{\theta(1-\delta)}$. In the first period, consumers with $d < \hat{d}$ bought firm 1's product. The marginal consumer with $d = \hat{d}$ preferred to wait because the consumer anticipated that buying firm 1's product in the second period will give greater utility. So, in a rational-expectations equilibrium, firm 1 must have a positive market share among consumers who are located on the interval $(\hat{d}, 1]$, which happens when firm 1's second-period price is low enough: $p_{12} \le p_{22} + \theta - 2\theta\hat{d}$. Also, firm 2 will have a positive market share only if $p_{22} < p_{12} + \theta$, or equivalently, if $p_{12} > p_{22} - \theta$. When $p_{22} - \theta < p_{12} \le p_{22} + \theta - 2\theta\hat{d}$, the firms' second-period profit functions are as follows:

$$\pi_{12} = (\alpha(\frac{\theta-p_{12}+p_{22}}{2\theta} - \hat{d}) + (1-\alpha)\frac{\theta-p_{12}+p_{22}}{2\theta})p_{12}$$

$$\pi_{22} = (\alpha\frac{\theta-p_{22}+p_{12}}{2\theta} + (1-\alpha)\frac{\theta-p_{22}+p_{12}}{2\theta})p_{22}$$

Solving the first-order conditions ($\frac{\partial\pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial\pi_{22}}{\partial p_{22}} = 0$) and plugging in $\hat{d} = \frac{V(1-\delta)-p_{11}+\delta p^e_{12}}{\theta(1-\delta)}$, we obtain the subgame equilibrium prices: $p^*_{12} = \theta - \frac{4\alpha(V-V\delta-p_{11}+p^e_{12}\delta)}{3(1-\delta)}$ and $p^*_{22} = \theta - \frac{2\alpha(V-V\delta-p_{11}+p^e_{12}\delta)}{3(1-\delta)}$. To ensure that the rational-expectations condition is satisfied, we need that $p^e_{12} = p^*_{12}$ and $p^e_{22} = p^*_{22}$. Solving these equations, we find that $p^e_{12} = \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{3-(3-4\alpha)\delta}$ and $p^e_{22} = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta}$. Plugging p^e_{12} and p^e_{22} back into the expressions for p^*_{12} and p^*_{22}, we verify that $p^*_{12} = p^e_{12} = \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{3-(3-4\alpha)\delta}$ and $p^*_{22} = p^e_{22} = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta}$. For future reference, plugging p^e_{12} into the expression for $\hat{d}$, we find that $\hat{d} = \frac{3(V-V\delta-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)}$.

Next, we need to verify the assumptions that we used in the analysis. First, case *iii* assumes that p_{11} satisfies $\frac{(2V-p^e_{22}-\theta)(1-\delta)+p^e_{12}(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p^e_{12}$. Plugging in the expressions for p^e_{12} and

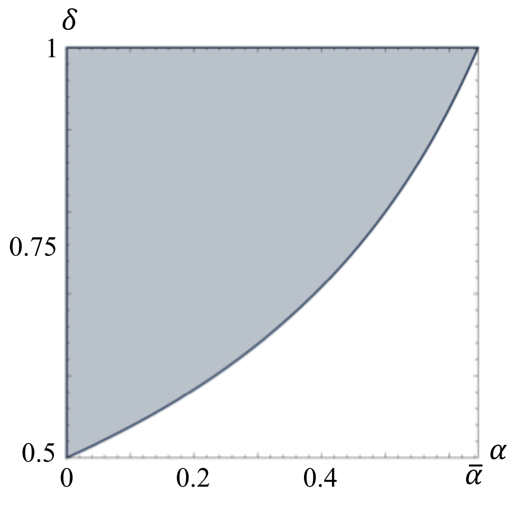
p^e_{22}, we find that $\frac{(2V-p^e_{22}-\theta)(1-\delta)+p^e_{12}(1+\delta)}{2} < p_{11} < V(1-\delta)+\delta p^e_{12}$ is equivalent to $\frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{p}$, where $\bar{p} \equiv (1-\delta)V + \delta\theta$. Second, recall that to ensure that both firms have positive market shares and that firm 1 serves some of the consumers located on $(\hat{d}, 1]$ we assumed that the firms' prices would satisfy $p_{22} - \theta < p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$. Using the expressions for p^*_{12} and p^*_{22}, one can readily show that when $\frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{p}$, we indeed have $p^*_{22} - \theta < p^*_{12} \leq p^*_{22} + \theta - 2\theta\hat{d}$.

As in the previous case, firm 1's profit function is not necessarily quasi-concave, and hence, we need to check that there are no non-local deviations. It is easy to check that deviations to lower prices will not be profitable. However, firm 1 may potentially deviate to a higher price to serve only the new consumers who entered the market in the second period, giving up its market share in the segment of first-period consumers who waited to buy a product in the second period. Specifically, if $p'_{12} > p^*_{22} + \theta - 2\theta\hat{d}$, then firm 1's deviation profit is $\pi'_{12} = (1-\alpha)\frac{\theta - p'_{12} + p^*_{22}}{2\theta} p'_{12}$. Notice that π'_{12} is concave. One can show that the right derivative of π'_{12} at $p'_{12} = p^*_{22} + \theta - 2\theta\hat{d}$ is given by $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12} = p^*_{22}+\theta-2\theta\hat{d}} = \frac{(1-\alpha)(V(6+\alpha)(1-\delta) - p_{11}(6+\alpha) - 3\theta(1-(3-\alpha)\delta))}{\theta(3-(3-4\alpha)\delta)}$. Straightforward algebra shows that $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12} = p^*_{22}+\theta-2\theta\hat{d}} \leq 0$ if and only if $p_{11} \geq \hat{p}$, where $\hat{p} \equiv (1-\delta)V + \frac{3\theta((3-\alpha)\delta-1)}{6+\alpha}$ satisfies $\frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)} < \hat{p} < \bar{p}$. Since π'_{12} is concave, $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12} = p^*_{22}+\theta-2\theta\hat{d}} \leq 0$ implies that π'_{12} is decreasing in p'_{12} when $p'_{12} > p^*_{22} + \theta - 2\theta\hat{d}$. Hence, continuity of firm 1's profit function at the point $p^*_{22} + \theta - 2\theta\hat{d}$ implies that deviations to $p'_{12} > p^*_{22} + \theta - 2\theta\hat{d}$ will not be profitable for firm 1 provided that $\hat{p} \leq p_{11} < \bar{p}$. Because $\hat{p} > \frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)}$, our initial guess holds when $p_{11} \in (\hat{p}, \bar{p})$. Therefore, the prices $p^*_{12} = p^e_{12} = \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{3-(3-4\alpha)\delta}$ and $p^*_{22} = p^e_{22} = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta}$ constitute a rational-expectations subgame equilibrium for any given $p_{11} \in (\hat{p}, \bar{p})$.

Case iv: $p_{11} \geq V(1-\delta) + \delta p^e_{12}$. In this case, $\hat{d} = 0$, i.e., all first-period consumers decided not to purchase a product in the first period but wait. In the second period, firm i will choose its second-period price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = \frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22} = \frac{\theta - p_{22} + p_{12}}{2\theta} p_{22}$. Solving the first-order conditions, we find that $p_{12}^* = p_{22}^* = \theta$. Rational expectations imply that $p_{i2}^e = p_{i2}^* = \theta$. Note that Case *iv* assumes that $p_{11} \geq V(1-\delta) + \delta p_{12}^e$. We need to check that this inequality is satisfied. Substituting $p_{12}^e = \theta$ into the inequality $p_{11} \geq V(1-\delta) + \delta p_{12}^e$, it follows that the condition for case *iv* is satisfied when $p_{11} \geq \bar{\rho}$, where recall that $\bar{\rho} \equiv (1-\delta)V + \delta\theta$. It is straightforward to show that the firms' profit functions are quasi-concave, and thus, the firms do not have any profitable non-local deviations.

Since firm 1 chooses $p_{11} \geq 0$ in the first period to maximize the sum of its first- and second-period profits, we need to know the second-period subgame equilibrium outcome corresponding to each $p_{11} \geq 0$. Our analysis in cases *i*, *ii*, *iii* and *iv* characterizes and shows the existence of the second-period subgame equilibrium outcome for p_{11} contained in the intervals $[0, \underline{\rho}]$, $(\underline{\rho}, \tilde{\rho}]$, $(\hat{\rho}, \bar{\rho})$, and $[\bar{\rho}, \infty)$, respectively. Notice that $[0, \underline{\rho}] \cup (\underline{\rho}, \tilde{\rho}] \cup (\hat{\rho}, \bar{\rho}) \cup [\bar{\rho}, \infty) = [0, \infty]$ if and only if $\tilde{\rho} \geq \hat{\rho}$, where recall that $\tilde{\rho} = (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta - 1)}{6-\alpha(7-3\alpha)}$ and $\hat{\rho} = (1-\delta)V + \frac{3\theta((3-\alpha)\delta - 1)}{6+\alpha}$. The inequality $\tilde{\rho} \geq \hat{\rho}$ is satisfied if and only if $\underline{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$ where $\underline{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$. The shaded region in Figure A1 graphically illustrates the parameter conditions on δ and α.

Figure A1 Parameter Region where $\underline{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$



Note that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$, there are two possible second-period rational-expectations equilibria corresponding to the cases *ii* and *iii* that we analyzed earlier. The selection of which equilibrium occurs will not have a qualitative impact on our results. Since both firms' profits are higher when the equilibrium from case *ii* is played (i.e., it is the focal equilibrium), we assume that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$, the subgame equilibrium prices will be the ones from case *ii*, i.e., $p_{12}^* = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^* = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$.

Hence, to summarize our analysis, it follows that when $(R_1, R_2) = (1,2)$, given any $p_{11} \geq 0$, the firms' second-period prices in a rational-expectations subgame equilibrium are as follows:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{A5}$$

$$p_{22}^{(1,2)} = p_{22}^e = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{A6}$$

where $\underline{\rho} = (1-\delta)(V-\theta)$, $\tilde{\rho} \equiv (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv (1-\delta)V + \delta\theta$. This finishes the proof of Lemma 2. For future reference, let us derive the firms' equilibrium profits and sales. Specifically, upon plugging the prices in (A5) and (A6) into the respective profit functions (explicitly provided in earlier cases *i–iv*), we obtain the firms' second-period subgame equilibrium profits when $(R_1, R_2) = (1,2)$:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{A7}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4\alpha(V(1-\delta)-p_{11})-\theta(3(1+\delta)+\alpha(1-3\delta)))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{A8}$$

Using the prices in (A5)-(A6), we can also find the firms' second-period unit sales:

$$S_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{2\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{A9}$$

$$S_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(\theta(3+\alpha)+4\alpha(p_{11}-V(1-\delta))+3\theta(1-\alpha)\delta)}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{2\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{A10}$$

To find firm 1's first-period unit sales, $S_{11}^{(1,2)}$, recall that consumers with $d < \hat{d}$ buy firm 1's product in the first period, where $\hat{d}$ is given in the equation (A4). Since consumers are uniformly distributed, $S_{11}^{(1,2)} = \alpha\hat{d}$. Plugging $p_{12}^e = p_{12}^{(1,2)}$ and $p_{22}^e = p_{22}^{(1,2)}$ into the expression for $\hat{d}$, we find that

$$S_{11}^{(1,2)} = \begin{cases} \alpha & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha\frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{A11}$$

■

<u>PROOF OF LEMMA 3</u>. In the first period, firm 1 chooses its price to maximize its overall profit over the two periods: $\pi_1 = S_{11}^{(1,2)}p_{11} + \pi_{12}^{(1,2)}$, where $S_{11}^{(1,2)}$ is firm 1's first-period unit sales as in equation (A11) and $\pi_{12}^{(1,2)}$ is firm 1's second-period subgame equilibrium profit as in equation (A7).

$$\pi_1 = \begin{cases} \alpha\, p_{11} + \frac{(1-\alpha)\theta}{2} & \text{if } p_{1_1} \in [0, \underline{\rho}) \\ \alpha\frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))}p_{1_1} + \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{1_1} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)}p_{1_1} + \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{1_1} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{1_1} \in [\bar{\rho}, \infty) \end{cases}$$

(A12)

where $\underline{\rho} = (1-\delta)(V-\theta)$, $\tilde{\rho} \equiv (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv (1-\delta)V + \delta\theta$.

It is easy to see that π_1 is increasing in p_{11} at any $p_{11} < \underline{\rho}$. Since π_1 is continuous at $\underline{\rho}$, it must be that the optimal price is above $\underline{\rho}$. Further, if $p_{11} > \bar{\rho}$, then π_1 is constant. Since π_1 is continuous at $\bar{\rho}$, the optimal price will not be above $\bar{\rho}$. Hence, the optimal price is either in $I_1 \equiv [\underline{\rho}, \tilde{\rho}]$ or $I_2 \equiv (\tilde{\rho}, \bar{\rho}]$. Let p_{I_k} denote the locally optimal price within the interval I_k, where $k = 1{,}2$. We will first find p_{I_1} and p_{I_2}, after which we will compare the corresponding profits to determine the globally optimal price.

First, when $p_{11} \in I_2$, we have

$$\pi_1 = \alpha \frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} p_{11} + \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2}$$

It is easy to show that $\frac{d^2\pi_1}{d(p_{11})^2} < 0$, i.e., π_1 is concave.

- If $0 < \theta \le \dot{\theta}$, then π_1 is decreasing in p_{11} at any $\underline{\rho} < p_{11} < \tilde{\rho}$, where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Hence, the locally optimal price is at the left corner: $p_{I_1} = \underline{\rho} = (1-\delta)(V-\theta)$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{2V\alpha(1-\delta)+\theta(1-\alpha(3-2\delta))}{2}$.
- If $\dot{\theta} < \theta < \ddot{\theta}$, then the local maximizer is in the interior of the interval I_1, where $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$. Solving $\frac{d\pi_1}{dp_{11}} = 0$, we find the interior maximizer: $p_{I_1} = \frac{V(1-\alpha)(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{2(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}$. The corresponding profit is
$$\pi_1(p_{I_1}) = \frac{9V^2(1-\alpha)^2\alpha(1-\delta)^2-4\theta V(3-\alpha)(1-\alpha)\alpha(1-\delta)(1-3\delta)+2\theta^2(3-\alpha)^2(1+\delta-\alpha(1-\delta)(1+2\delta))}{4\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}.$$
- If $\ddot{\theta} \le \theta < \bar{\theta}$, then π_1 is increasing in p_{11} at any $\underline{\rho} < p_{11} < \tilde{\rho}$, and hence, the locally optimal price is at the right corner: $p_{I_1} = \tilde{\rho} = (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{(3-\alpha)(2V\alpha(6-\alpha(7-3\alpha))(1-\delta)+\theta(3-\alpha)(4-\alpha(10-6\delta-\alpha(5-\alpha-4\delta))))}{2(6-\alpha(7-3\alpha))^2}$.

Second, when $p_{1_1} \in I_2$, we have

$$\pi_1 = \alpha \frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} p_{1_1} + \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2}$$

Because $\underline{\delta} < \delta < 1$, $0 < \alpha < \bar{\alpha}$ and $\theta < \bar{\theta}$, one can readily show that $\frac{d\pi_1}{dp_{11}} = \frac{\alpha(V(1-\delta)(9-9\delta-4\alpha(4-3\delta))-2p_{11}(9(1-\delta)-4\alpha(2-3\delta))+3\theta(4-\delta(1+3\delta-4\alpha\delta)))}{\theta(3-(3-4\alpha)\delta)^2} < 0$ for all $p_{11} \in I_2$. That is, π_1 is decreasing in p_{11} on I_2. Since $I_2 = (\tilde{\rho}, \bar{\rho}]$, π_1 does not have a local maximizer on I_2 because the point $\tilde{\rho}$ is not included in I_2. However, this does not create problems for the existence of a global maximizer of π_1. Specifically, one can readily show that $\pi_1(\tilde{\rho}) > \lim_{p_{11}\to\tilde{\rho}^+} \pi_1$, i.e., the right limit of π_1 at $p_{11} = \tilde{\rho}$ is strictly lower than $\pi_1(\tilde{\rho})$. In other words, π_1 discretely jumps down at the point $\tilde{\rho}$. Since $\frac{d\pi_1}{dp_{11}} < 0$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$, it follows that $\pi_1(\tilde{\rho}) > \pi_1(p_{11})$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Since $\pi_1(p_{I_1}) \geq \pi_1(\tilde{\rho})$, it follows that $\pi_1(p_{I_1}) > \pi_1(p_{11})$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Hence, the optimal price within the interval I_1 yields greater profit than any price $p_{11} \in I_2$. Therefore, we conclude that the global maximizer of π_1 is contained in the interval I_1. That is, $p_{11}^{(1,2)} = p_{I_1}$. Hence,

$$p_{11}^{(1,2)} = \begin{cases} (1-\delta)(V-\theta) & \text{if } \theta \leq \dot{\theta} \\ \frac{V(1-\alpha)(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{2(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \tag{A13}$$

where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$.

For future reference, let us use the expression of $p_{11}^{(1,2)}$ in (A13) to obtain the firms' profits and second-period prices on the equilibrium path.

Upon plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$ from Lemma 2, we can find the firms' second-period prices on the equilibrium path.

$$p_{12}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \leq \dot{\theta} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3V(1-\alpha)\alpha(1-\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \tag{A14}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \leq \dot{\theta} \\ \frac{9\theta(1+\delta)-6V(1-\alpha)\alpha(1-\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \tag{A15}$$

Using the equilibrium prices in (A13)-(A15), we obtain the firms' equilibrium profits in each period.

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(1-\delta)(V-\theta) & \text{if } \theta \le \dot{\theta} \\ \frac{\alpha(9V(1-\alpha-\delta+\alpha\delta)-2\theta(3-\alpha)(1-3\delta))A}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{(3-\alpha)\alpha(V(6-\alpha(7-3\alpha))(1-\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases} \tag{A16}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \le \dot{\theta} \\ \frac{(3V\alpha(1-\alpha-\delta+\alpha\delta)-\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-5\alpha+\alpha^2)^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases} \tag{A17}$$

$$\pi_{21}^{(1,2)} = 0 \tag{A18}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \le \dot{\theta} \\ \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases} \tag{A19}$$

where $A \equiv \frac{V(1-\alpha)(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$. Note that since the firms are symmetric, when $(R_1, R_2) = (2,1)$, the firms' profits are reversed, i.e., $\pi_{1t}^{(2,1)} = \pi_{2t}^{(1,2)}$ and $\pi_{2t}^{(2,1)} = \pi_{1t}^{(1,2)}$. ■

PROOF OF PROPOSITION 1. We will first prove part b) of Proposition 1 and then proceed to the proof of part a).

When $(R_1, R_2) = (1,1)$, Lemma 1 showed that the firms' equilibrium prices will be $p_{i2}^{(1,1)} = \theta$. When $(R_1, R_2) = (1,2)$, the firms' equilibrium second-period prices are given in (A14)-(A15).

Clearly, if $\theta \le \dot{\theta}$, then $p_{i2}^{(1,2)} = p_{i2}^{(1,1)} = \theta$ for each firm $i = 1,2$.

Next, if $\dot{\theta} < \theta < \ddot{\theta}$, we have $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3V(1-\alpha)\alpha(1-\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}$ and $p_{22}^{(1,2)} = \frac{9\theta(1+\delta)-6V(1-\alpha)\alpha(1-\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}$. Straightforward algebra shows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$ if and only if $\theta > \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Recall that $\dot{\theta} \equiv \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Hence, when $\dot{\theta} < \theta < \ddot{\theta}$, we have $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$.

Finally, if $\ddot{\theta} \leq \theta < \bar{\theta}$, then $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)}$ and $p_{22}^{(1,2)} = \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)}$. Since $(3-\alpha)(2-\alpha) > 6-\alpha(7-3\alpha)$ and $6-\alpha(3+\alpha) > 6-\alpha(7-3\alpha)$ for any $\alpha \in [0,\bar{\alpha})$, it readily follows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$.

The above shows that $p_{i2}^{(1,2)} \geq p_{i2}^{(1,1)}$. Next, let us prove part a) of Proposition 1. For future reference, recall that to ensure the existence of a rational-expectations equilibrium with full market coverage, we are assuming that $\underline{\delta} < \delta < 1$, $0 \leq \alpha < \bar{\alpha}$, and $\theta < \bar{\theta}$, where $\underline{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$, $\bar{\alpha} \approx 0.64$ and $\bar{\theta} = \frac{V(1-\delta)}{2-\delta}$.

Firm i chooses its product release period $R_i \in \{1,2\}$ to maximize its overall profit $\pi_i^{(R_1,R_2)} = \pi_{i1}^{(R_1,R_2)} + \pi_{i2}^{(R_1,R_2)}$. The pair (R_1,R_2) is an equilibrium if no firm has any profitable deviations.

First, one can readily show that $(R_1,R_2) = (2,2)$ is not an equilibrium because firm 1 will benefit by deviating to $R_1 = 1$, i.e., $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, where $\pi_1^{(2,2)} = \frac{\theta}{2}$. To see this, notice that if firm 1 deviates to $R_1 = 1$ and sets an extremely large price ($p_{11} \geq \bar{\rho}$), then its overall profit will be $\pi_1(\bar{\rho}) = \frac{\theta}{2}$, which is given in equation (A12). However, we know from Lemma 3 that firm 1's unique optimal price is $p_{11}^{(1,2)} < \bar{\rho}$ with a corresponding profit of $\pi_1^{(1,2)}$. Hence, $\pi_1^{(1,2)} > \pi_1(\bar{\rho})$. Since $\pi_1(\bar{\rho}) = \pi_1^{(2,2)} = \frac{\theta}{2}$, it follows that $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Thus, in equilibrium, either one firm releases its product in the first period and the other firm does so in the second period, or both firms release their products in the first period.

Next, let us demonstrate that a pure strategy equilibrium exists. If $(R_1,R_2) = (1,2)$ is an equilibrium, then the proof is finished. If $(R_1,R_2) = (1,2)$ is not an equilibrium, then let us show that $(R_1,R_2) = (1,1)$ must be an equilibrium. Since $(R_1,R_2) = (1,2)$ is not an equilibrium, one of the firms must have a profitable deviation. Firm 1 will not want to deviate to $R_1 = 2$ because $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Hence, it must be that firm 2 has a profitable deviation from $R_2 = 2$ to $R_2 = 1$, which means that $\pi_2^{(1,1)} > \pi_2^{(1,2)}$. By symmetry between firms 1 and 2, we know that $\pi_2^{(1,2)} = \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} = \pi_1^{(1,1)}$. Hence, $\pi_2^{(1,1)} > \pi_2^{(1,2)}$ implies that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$. Then, it must be $(R_1,R_2) = (1,1)$ is an equilibrium because we found that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} > \pi_2^{(1,2)}$.

Now, let us find conditions under which $(R_1, R_2) = (1, 2)$ is an equilibrium. If $R_1 = 1$, then firm 2's best response is $R_2 = 2$ if and only if $\pi_2^{(1,2)} - \pi_2^{(1,1)} > 0$, where $\pi_2^{(1,1)} = \frac{\theta}{2}$ and $\pi_2^{(1,2)} = \pi_{21}^{(1,2)} + \pi_{22}^{(1,2)}$ is given in equations (A18)-(A19). Define $\Delta\pi_2 \equiv \pi_2^{(1,2)} - \pi_2^{(1,1)}$.

$$\Delta\pi_2 = \begin{cases} \frac{(1-\alpha)\theta}{2} - \frac{\theta}{2} & \text{if } \theta \leq \dot{\theta} \\ \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{\theta}{2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{\theta}{2} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases}$$

where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$. Note that $\Delta\pi_2$ is a continuous.

When $\theta \leq \dot{\theta}$, clearly, $\Delta\pi_2 < 0$. Hence, consider $\theta > \dot{\theta}$. The following two results about the sign of $\frac{d\Delta\pi_2}{d\theta}$ are important for the analysis.

First, when $\dot{\theta} < \theta < \ddot{\theta}$, the function $\Delta\pi_2$ is increasing in θ, i.e., $\frac{d\Delta\pi_2}{d\theta} > 0$. To see this, note that $\frac{d^2\Delta\pi_2}{d\theta^2} = \frac{36V^2(1-\alpha)\alpha^2(1-\delta)^2}{\theta^3(9-11\alpha+9\delta+3\alpha\delta)^2} > 0$, i.e., $\Delta\pi_2$ is convex. Further, $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\dot{\theta}} = \frac{\alpha(23-9\delta-\alpha(21-13\delta))}{2(9-11\alpha+9\delta+3\alpha\delta)} > 0$, where the inequality follows because $\delta > \underline{\delta} \geq 0.5$. The inequalities $\frac{d^2\Delta\pi_2}{d\theta^2} > 0$ and $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\dot{\theta}} > 0$ imply that $\frac{d\Delta\pi_2}{d\theta} > 0$ for any $\dot{\theta} < \theta < \ddot{\theta}$.

Second, when $\ddot{\theta} \leq \theta < \bar{\theta}$, we have $\frac{d\Delta\pi_2}{d\theta} = \frac{(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{1}{2} > 0$ if and only if $\alpha < \hat{\alpha}$, where $\hat{\alpha} \approx 0.32$ is the solution to $\frac{(1-\hat{\alpha})(6-\hat{\alpha}(3+\hat{\alpha}))^2}{2(6-\hat{\alpha}(7-3\hat{\alpha}))^2} - \frac{1}{2} = 0$, or equivalently, $12 - \hat{\alpha}(52 - (3-\hat{\alpha})\hat{\alpha}(17 + \hat{\alpha})) = 0$.

We will separately analyze the cases with $\alpha \leq \hat{\alpha}$ and $\alpha > \hat{\alpha}$. Specifically, we will show that if $\alpha > \hat{\alpha}$, then $\Delta\pi_2 < 0$, but if $0 < \alpha \leq \hat{\alpha}$, then $\Delta\pi_2 > 0$ for θ large enough.

Case 1: $\alpha > \hat{\alpha}$. When $\alpha > \hat{\alpha}$, one can show that $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$.[3] We showed earlier that $\Delta\pi_2$ is increasing when $\dot{\theta} < \theta < \ddot{\theta}$ and decreasing when $\ddot{\theta} \leq \theta < \bar{\theta}$. Since $\Delta\pi_2$ is

[3] To see this, note that $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ if and only if $\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta} < 0$. Using the fact that $\delta > \underline{\delta} \geq 0.5$, one can show that the function $\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta}$ is decreasing in α. Plugging in $\alpha = 0.3$, one can show that $(\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta})|_{\alpha=0.3} < 0$.

continuous, it follows that $\Delta\pi_2|_{\theta=\bar{\theta}} > \Delta\pi_2$ for any $\dot{\theta} \leq \theta < \bar{\theta}$ and $\theta \neq \ddot{\theta}$. Straightforward algebra shows that $\Delta\pi_2|_{\theta=\ddot{\theta}} = \frac{3V(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))(1-\delta)}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, and $\Delta\pi_2|_{\theta=\ddot{\theta}} < 0$ if and only if $12 - \alpha(52-(3-\alpha)\alpha(17+\alpha)) > 0$. The last inequality is equivalent to $\alpha > \hat{\alpha}$, where recall that $\hat{\alpha} \approx 0.32$ satisfies $12 - \hat{\alpha}(52-(3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$. Hence, when $\alpha > \hat{\alpha}$, we have $\Delta\pi_2 \leq \Delta\pi_2|_{\theta=\ddot{\theta}} < 0$ for any $\dot{\theta} \leq \theta < \bar{\theta}$.

Case 2: $\alpha < \hat{\alpha}$. When $\alpha < \hat{\alpha}$, we showed earlier that $\Delta\pi_2$ is increasing in θ at any $\dot{\theta} \leq \theta < \bar{\theta}$. Further, we can show that $\Delta\pi_2|_{\theta=\ddot{\theta}} > 0$, where recall that $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$. Specifically, if $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, then we already know from Case 1 that $\Delta\pi_2|_{\theta=\ddot{\theta}} = \frac{3V(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))(1-\delta)}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} > 0$ when $\alpha < \hat{\alpha}$. Next, if $\ddot{\theta} = \frac{V(1-\delta)}{2-\delta}$, then $\Delta\pi_2|_{\theta=\ddot{\theta}} = \frac{V\alpha(1-\delta)((3+\alpha)^3\delta^2+2(1-\alpha)(3+\alpha)(3+7\alpha)\delta-9+63\alpha-103\alpha^2+49\alpha^3)}{2(1-\alpha)(2-\delta)(9-11\alpha+3(3+\alpha)\delta)^2} > 0$, where the inequality follows because $\delta > \underline{\delta} \geq \frac{1}{2}$. Because $\Delta\pi_2$ is increasing in θ, $\Delta\pi_2|_{\theta=\ddot{\theta}} > 0$ implies that $\Delta\pi_2 > 0$ for all $\ddot{\theta} \leq \theta < \bar{\theta}$. Further, because $\Delta\pi_2|_{\theta=\dot{\theta}} = -\frac{3V(1-\alpha)\alpha(1-\delta)}{8(2-\alpha(2-\delta))} < 0$, $\Delta\pi_2|_{\theta=\ddot{\theta}} > 0$ and $\Delta\pi_2$ is a continuous increasing function, it follows that there exists a unique $\dot{\theta} < \hat{\theta} < \ddot{\theta}$, such that $\Delta\pi_2|_{\theta=\hat{\theta}} = 0$ and $\Delta\pi_2 < 0$ if $\dot{\theta} < \theta < \hat{\theta}$ and $\Delta\pi_2 > 0$ if $\hat{\theta} < \theta < \ddot{\theta}$. Solving $\Delta\pi_2 = 0$, we find that

$$\hat{\theta} = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))} \tag{A20}$$

To summarize the above, we found that if $0 < \alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $\Delta\pi_2 > 0$. Thus, in this parameter region, firm 2 will not deviate from $R_2 = 2$. Since $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, firm 1 also has no incentives to deviate from $R_1 = 1$. It follows that $(R_1, R_2) = (1, 2)$ is a Nash equilibrium. ■

Hence, $\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta} < 0$ for all $\alpha > 0.3$. Since $\hat{\alpha} > 0.3$, it follows that $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ when $\alpha > \hat{\alpha}$.

PROOF OF PROPOSITION 2. First, let us show that firm 2's equilibrium unit sales, $S_2^{(1,2)}$, are decreasing in α. Note that $S_2^{(1,2)} = S_{22}^{(1,2)}$, where $S_{22}^{(1,2)}$ is given in the equation (A10). Upon plugging in firm 1's first-period equilibrium price $p_{11}^{(1,2)}$, we find firm 2's sales on the equilibrium path:

$$S_2^{(1,2)} = \begin{cases} \frac{1-\alpha}{2} & \text{if } \theta \leq \dot{\theta} \\ \frac{9\theta(1+\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)-6V\alpha(1-\delta)(1-\alpha)}{2\theta(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{(1-\alpha)(6-\alpha(3+\alpha))}{2(6-\alpha(7-3\alpha))} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \tag{A21}$$

where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$. Note that $S_2^{(1,2)}$ is continuous. We first need to show that if $0 < \alpha < \hat{\alpha}$, then $\frac{dS_2^{(1,2)}}{d\alpha} < 0$.

When $\theta < \dot{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{1}{2} < 0$.

When $\dot{\theta} < \theta < \ddot{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = \frac{\Psi}{\theta(9-11\alpha+3(3+\alpha)\delta)^2}$, where

$\Psi = \frac{6V(1-\delta)(\alpha(18-11\alpha+3(6+\alpha)\delta)-9(1+\delta))+\theta(5\alpha^2(1-\delta)(11-3\delta)+9(1+\delta)(7-9\delta)-90\alpha(1-\delta^2))}{2}$. Clearly, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ if and only if $\Psi < 0$. Hence, we need to show that $\Psi < 0$. First, note that Ψ is concave in α because $\frac{d^2\Psi}{d\alpha^2} = -2(1-\delta)(6V-5\theta(1-\delta))(11-3\delta) < 0$.[4] Second, because $\theta < \bar{\theta}$, one can show that $\Psi|_{\alpha=0} = -\frac{9(6V(1-\delta)-\theta(7-9\delta))(1+\delta)}{2} < 0$ and $\Psi|_{\alpha=0.5} = -\frac{6V(1-\delta)(11-3\delta)-\theta(127-\delta(142+129\delta))}{8} < 0$. Concavity of Ψ, together with $\Psi|_{\alpha=0} < 0$ and $\Psi|_{\alpha=0.5} < 0$, implies that $\Psi < 0$ for any $0 < \alpha < 0.5$. Since $\hat{\alpha} \approx 0.32 < 0.5$, it follows that $\Psi < 0$ for any $0 < \alpha < \hat{\alpha}$. Thus, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ when $\dot{\theta} < \theta < \ddot{\theta}$.

When $\ddot{\theta} \leq \theta < \bar{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{12+\alpha(12-\alpha(31-\alpha(14-3\alpha)))}{2(6-\alpha(7-3\alpha))^2}$. Straightforward algebra shows that $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ for any $0 < \alpha < \hat{\alpha}$.

[4] To see that $\frac{d^2\Psi}{d\alpha^2} < 0$, note that $11 - 3\delta > 0$ and $6V - 5\theta(1-\delta) > 0$ because $\theta < \bar{\theta} = \frac{V(1-\delta)}{2-\delta}$.

To prove the second part of the proposition, we will show that there exists $\tilde{\alpha} < \hat{\alpha}$, such that if $\alpha < \tilde{\alpha}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$. Since we are focusing on $\theta > \hat{\theta}$ and since $\dot{\theta} < \hat{\theta} < \ddot{\theta}$, it follows from the equation (A19) that $\pi_2^{(1,2)} = \pi_{22}^{(1,2)}$ is given by

$$\pi_2^{(1,2)} = \begin{cases} \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \hat{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases}$$

Upon differentiating $\pi_2^{(1,2)}$ with respect to α and evaluating at $\alpha = 0$, we find that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{\theta(23-9\delta)-12V(1-\delta)}{18(1+\delta)}$ when $\hat{\theta} < \theta < \ddot{\theta}$, and $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{\theta}{6}$ when $\ddot{\theta} < \theta < \bar{\theta}$. Clearly, when $\ddot{\theta} < \theta < \bar{\theta}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$. Further, when $\hat{\theta} < \theta < \ddot{\theta}$, it is easy to show that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$ if and only if $\theta > \frac{12V(1-\delta)}{23-9\delta}$. Note that $\hat{\theta}|_{\alpha=0} = \frac{12V(1-\delta)}{23-9\delta}$. By continuity of $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}$, it follows that there exists $\tilde{\alpha} \in (0, \hat{\alpha}]$ such that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$ if $\alpha < \tilde{\alpha}$ and $\theta > \hat{\theta}$. ■

PROOF OF PROPOSITION 3. For a given $0 < \alpha < \hat{\alpha}$, the cutoff $\hat{\theta}$ satisfies $\pi_2^{(1,2)}|_{\theta=\hat{\theta}} - \pi_2^{(1,1)}|_{\theta=\hat{\theta}} = 0$. Totally differentiating with respect to α, we find that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} + \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}} \frac{d\hat{\theta}}{d\alpha} - \frac{d\pi_2^{(1,1)}}{d\alpha}|_{\theta=\hat{\theta}} - \frac{d\pi_2^{(1,1)}}{d\theta}|_{\theta=\hat{\theta}} \frac{d\hat{\theta}}{d\alpha} = 0$. Because $\pi_2^{(1,1)} = \frac{\theta}{2}$, we have $\frac{d\pi_2^{(1,1)}}{d\alpha} = 0$ and $\frac{d\pi_2^{(1,1)}}{d\theta} = \frac{1}{2}$. Hence, $\frac{d\hat{\theta}}{d\alpha} = \frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} \left(\frac{1}{2} - \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}}\right)^{-1}$. To show that $\frac{d\hat{\theta}}{d\alpha} < 0$, we will demonstrate that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}} > \frac{1}{2}$.

As we showed in the proof of Proposition 1, $\hat{\theta}$ is given by $\hat{\theta} = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$ and satisfies $\dot{\theta} < \hat{\theta} < \ddot{\theta}$. Further, $\pi_2^{(1,2)}$ is given by $\pi_2^{(1,2)} = \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2}$. One can readily show that

$\frac{d\pi_2^{(1,2)}}{d\theta} = \frac{(6V(1-\alpha)\alpha(1-\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)+9\theta(1+\delta))(9\theta(1+\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)-6V\alpha(1-\alpha-\delta+\alpha\delta))}{2\theta^2(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2}$, and

$\frac{d\pi_2^{(1,2)}}{d\theta} > \frac{1}{2}$ if and only if $\theta > \frac{6V(1-\alpha)(1-\delta)\sqrt{\alpha}}{\sqrt{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}}$. Using $\alpha < \hat{\alpha} \approx 0.32$, one can show that $\frac{6V(1-\alpha)(1-\delta)\sqrt{\alpha}}{\sqrt{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}} < \dot{\theta}$.

Because $\hat{\theta} > \dot{\theta}$, it follows that $\frac{d\pi_2^{(1,2)}}{d\theta} > \frac{1}{2}$ when $\theta = \hat{\theta}$.

Next,

$\frac{d\pi_2^{(1,2)}}{d\alpha} = \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))}{(1-\alpha)^2}\left(\frac{6V(1-\alpha)(1-\delta)(18(1+\delta)-\alpha(27-11\alpha+3(9+\alpha)\delta))}{2\theta(9-11\alpha+3(3+\alpha)\delta)^3} + \frac{(5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta)))}{2(9-11\alpha+3(3+\alpha)\delta)^3}\right)$. Denote $A = \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))}{(1-\alpha)^2}$ and $B = \frac{6V(1-\alpha)(1-\delta)(18(1+\delta)-\alpha(27-11\alpha+3(9+\alpha)\delta))}{2\theta(9-11\alpha+3(3+\alpha)\delta)^3} + \frac{(5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta)))}{2(9-11\alpha+3(3+\alpha)\delta)^3}$. Hence, $\frac{d\pi_2^{(1,2)}}{d\alpha} = AB$. We will show that if $\theta = \hat{\theta}$, then $A < 0$ and $B < 0$, which will imply that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$. First, one can show that $A < 0$ if and only if $\theta > \frac{6V(1-\alpha)\alpha(1-\delta)}{9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)}$. Because $\hat{\theta} > \dot{\theta} > \frac{6V(1-\alpha)\alpha(1-\delta)}{9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)}$, it follows that $A|_{\theta=\hat{\theta}} < 0$. Second, $B < 0$ if and only if

$$\theta > \frac{6V(1-\alpha)(1-\delta)(-18(1+\delta)+\alpha(27-11\alpha+3(9+\alpha)\delta))}{5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta))}.$$

Because $\hat{\theta} > \frac{6V(1-\alpha)(1-\delta)(-18(1+\delta)+\alpha(27-11\alpha+3(9+\alpha)\delta))}{5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta))}$ when $\alpha < \hat{\alpha}$, it follows that $B|_{\theta=\hat{\theta}} < 0$. The inequalities $A|_{\theta=\hat{\theta}} < 0$ and $B|_{\theta=\hat{\theta}} < 0$ imply that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} = (AB)|_{\theta=\hat{\theta}} > 0$.

We showed that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\theta} > \frac{1}{2}$. Hence, $\frac{d\hat{\theta}}{d\alpha} = \frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} \left(\frac{1}{2} - \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}}\right)^{-1} < 0$.

■

WEB APPENDIX B

1. UNIQUENESS OF EQUILIBRIUM IN LEMMA 1

Lemma 1 shows the existence of a rational-expectations equilibrium in which the market is fully covered in the first period, i.e., in equilibrium, first-period consumers buy a product in the first period instead of waiting. In this section, we consider the possibility that other equilibria may exist where, on the equilibrium path, some first-period consumers prefer not to purchase in the first period but wait. We will show that such equilibria do not exist when the differentiation level between the firms is not too high (i.e., $\theta < \bar{\theta}$). Hence, the equilibrium from Lemma 1 is unique.

Given the firms' first-period prices, a consumer located at $d \in [0,1]$ will prefer to wait till the second period to buy a product if and only if the following inequality holds:

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} < \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\}, \text{(B1)}$$

where p_{12}^e and p_{22}^e are the second-period prices that the consumer expects the firms to charge. One can readily show that if the above inequality is satisfied for some $d \in [0,1]$, then the set of such d's must constitute a connected set, i.e., an interval $[\hat{d}_1, \hat{d}_2]$.[5] There are three possible cases.

i) $\delta(V - p_{12}^e - \theta\hat{d}_1) > \delta(V - p_{22}^e - \theta(1-\hat{d}_1))$ and $\delta(V - p_{12}^e - \theta\hat{d}_2) < \delta(V - p_{22}^e - \theta(1-\hat{d}_2))$, i.e., some of the consumers who prefer not to buy in the first period expect to purchase firm 1's product in the second period, while others expect to purchase firm 2's product.

ii) $\delta(V - p_{12}^e - \theta\hat{d}_1) < \delta(V - p_{22}^e - \theta(1-\hat{d}_1))$ and $\delta(V - p_{12}^e - \theta\hat{d}_2) < \delta(V - p_{22}^e - \theta(1-\hat{d}_2))$, i.e., all consumers who prefer not to buy in the first period expect to purchase firm 2's product in the second period.

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[5] This can be shown using a proof by contradiction. Specifically, if the set of d's satisfying inequality (B1) is not connected, then there must exist $d_1 < d_2 < d_3$ such that d_1 and d_3 satisfy (B1) but d_2 does not. There are two possible cases: $d_2 < \frac{\theta - p_{11} + p_{21}}{2\theta}$ or $d_2 \geq \frac{\theta - p_{11} + p_{21}}{2\theta}$. Without loss of generality, assume that $d_2 < \frac{\theta - p_{11} + p_{21}}{2\theta}$. Then, $\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} = V - p_{11} - \theta d$ for $d = d_1, d_2$. Because $\delta < 1$, notice that $V - p_{11} - \theta d - \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\}$ is decreasing in d. Therefore, if $V - p_{11} - \theta d_2 > \max\{\delta(V - p_{12}^e - \theta d_2), \delta(V - p_{22}^e - \theta(1-d_2))\}$, then it must be that $V - p_{11} - \theta d_1 > \max\{\delta(V - p_{12}^e - \theta d_1), \delta(V - p_{22}^e - \theta(1-d_1))\}$. This contradicts the assumption that d_1 satisfies (B1).

iii) $\delta(V - p_{12}^e - \theta\hat{d}_1) > \delta(V - p_{22}^e - \theta(1 - \hat{d}_1))$ and $\delta(V - p_{12}^e - \theta\hat{d}_2) > \delta(V - p_{22}^e - \theta(1 - \hat{d}_2))$, i.e., all consumers who prefer not to buy in the first period expect to purchase firm 1's product in the second period.

We will search for an equilibrium using the guess-and-verify approach. Specifically, we start by guessing that the inequalities characterizing one of the above three cases are satisfied. Then, using first-order conditions, we will find the firms' second- and first-period prices. Using these prices, we will check whether our initial guess is indeed satisfied, as well as other necessary and sufficient conditions for the prices to constitute an equilibrium.

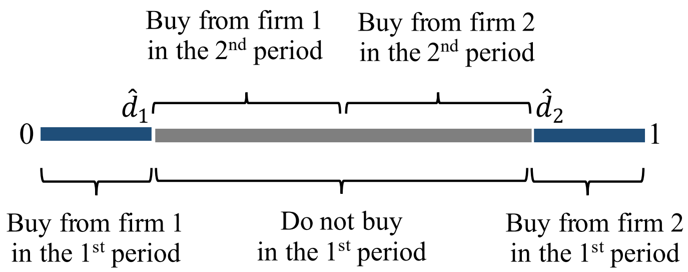
***Case i*)**. We guess that the following inequalities will be satisfied in equilibrium:

$$\delta(V - p_{12}^e - \theta\hat{d}_1) > \delta(V - p_{22}^e - \theta(1 - \hat{d}_1)) \tag{B2}$$

$$\delta(V - p_{12}^e - \theta\hat{d}_2) < \delta(V - p_{22}^e - \theta(1 - \hat{d}_2)) \tag{B3}$$

The above inequalities imply that among first-period consumers who choose to wait till the second period to buy, some expect to purchase from firm 1 in the second period, while others expect to purchase from firm 2. In a rational-expectations equilibrium, the decision that a consumer expects to make in the second period must be consistent with the consumer's equilibrium choice. Figure B1 graphically illustrates first-period consumers' decisions.

Figure B1 Purchase Decisions of Consumers Entering the Market in the 1st Period



Note that the consumer located at $\hat{d}_1$ must be indifferent between buying firm 1's product in the first period and buying it in the second period. Mathematically, $V - p_{11} - \theta\hat{d}_1 = \delta(V - p_{12}^e - \theta\hat{d}_1)$, which implies that $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + \delta p_{12}^e}{\theta(1-\delta)}$. Similarly, the consumer located at $\hat{d}_2$ must be indifferent between buying firm 2's product in the first period and doing so in the second period. Mathematically, $V - p_{21} - \theta(1-\hat{d}_2) = \delta(V - p_{22}^e - \theta(1-\hat{d}_2))$, which implies $\hat{d}_2 = \frac{p_{21} - \delta p_{22}^e - (V-\theta)(1-\delta)}{\theta(1-\delta)}$.

In the second period, the firms compete for the $(1-\alpha)$ new consumers entering the market, as well as first-period consumers located on $[\hat{d}_1, \hat{d}_2]$ who did not buy a product in the first period. If a rational-expectations equilibrium exists, then the firms' second-period prices, p_{12} and p_{22}, must be such that the consumer with $d = \hat{d}_1$ prefers to buy firm 1's product, whereas the consumer with $d = \hat{d}_2$ prefers to buy firm 2's product.[6] Therefore, the firms' second-period profit functions are as follows:

$$\pi_{12} = \left(\alpha\left(\frac{\theta - p_{12} + p_{22}}{2\theta} - \hat{d}_1\right) + (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta}\right)p_{12}, \tag{B4}$$

$$\pi_{22} = \left(\alpha\left(\hat{d}_2 - \frac{\theta - p_{12} + p_{22}}{2\theta}\right) + (1-\alpha)\left(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}\right)\right)p_{12}, \tag{B5}$$

where recall that $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + \delta p_{12}^e}{\theta(1-\delta)}$ and $\hat{d}_2 = \frac{p_{21} - \delta p_{22}^e - (V-\theta)(1-\delta)}{\theta(1-\delta)}$. Using the first-order conditions, we find the firms' profit-maximizing second-period prices: $p_{12}^* = \theta - \frac{2\alpha(3V(1-\delta) + (2p_{12}^e + p_{22}^e)\delta - 2p_{11} - p_{21})}{3(1-\delta)}$ and $p_{22}^* = \theta - \frac{2\alpha(3V(1-\delta) + (p_{12}^e + 2p_{22}^e)\delta - p_{11} - 2p_{21})}{3(1-\delta)}$. Rational-expectations condition requires that $p_{12}^* = p_{12}^e$ and $p_{22}^* = p_{22}^e$. Solving these equalities, we find that $p_{12}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(2p_{11}(1-(1-\alpha)\delta) - (1-\delta)(3V - V(3-2\alpha)\delta - p_{21}))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$ and

$p_{22}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(p_{11}(1-\delta) + 2p_{21}(1-(1-\alpha)\delta) - V(1-\delta)(3-3\delta+2\alpha\delta))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$. Upon plugging p_{12}^e and p_{22}^e back into p_{12}^* and p_{22}^*, one can verify that

$$p_{12}^* = p_{12}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(2p_{11}(1-(1-\alpha)\delta) - (1-\delta)(3V - V(3-2\alpha)\delta - p_{21}))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}, \tag{B6}$$

$$p_{22}^* = p_{22}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(p_{11}(1-\delta) + 2p_{21}(1-(1-\alpha)\delta) - V(1-\delta)(3-3\delta+2\alpha\delta))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}. \tag{B7}$$

The firms' corresponding second-period profits are given by

$$\pi_{12}^* = \frac{(\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(2p_{11}(1-(1-\alpha)\delta) - (1-\delta)(3V - V(3-2\alpha)\delta - p_{21})))^2}{2\theta(3-(3-2\alpha)\delta)^2(1-(1-2\alpha)\delta)^2}, \tag{B8}$$

$$\pi_{22}^* = \frac{(\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(p_{11}(1-\delta) + 2p_{21}(1-(1-\alpha)\delta) - V(1-\delta)(3-3\delta+2\alpha\delta)))^2}{2\theta(3-(3-2\alpha)\delta)^2(1-(1-2\alpha)\delta)^2}. \tag{B9}$$

[6] If otherwise, then the prices p_{12} and p_{22} would violate either the conditions (B2)-(B3) or the rational-expectations condition $p_{12} = p_{12}^e$ and $p_{22} = p_{22}^e$.

Next, let us obtain the conditions under which our initial guess is satisfied. Upon plugging p^e_{12} and p^e_{22} into the inequalities (B2) and (B3), these inequalities reduce to the following conditions, respectively:

$$p_{11} > \frac{\theta(1-(3-2\alpha)\delta)(3-(3-2\alpha)\delta)+2(p_{21}\alpha(1-3\delta+2\alpha\delta)-V(1-\delta)(3-3\delta+2\alpha\delta))}{2(\alpha-3+(1-\alpha)(3-2\alpha)\delta)} \quad \text{(B10)}$$

$$p_{21} > \frac{\theta(1-(3-2\alpha)\delta)(3-(3-2\alpha)\delta)+2(p_{11}\alpha(1-3\delta+2\alpha\delta)-V(1-\delta)(3-3\delta+2\alpha\delta))}{2(\alpha-3+(1-\alpha)(3-2\alpha)\delta)} \quad \text{(B11)}$$

Hence, after finding the firms' first-period equilibrium prices p^*_{11} and p^*_{21}, we will need to verify that they satisfy (W10) and (W11). In the first period, the firms maximize the sum of their profits over the two periods:

$$\pi_1 = \pi_{11} + \pi^*_{12},$$

$$\pi_2 = \pi_{21} + \pi^*_{22},$$

where $\pi_{11} = \alpha\, \hat{d}_1 p_{11}$, $\pi_{21} = \alpha(1-\hat{d}_2)p_{21}$, $\hat{d}_1 = \frac{(1-\delta)V-p_{11}+\delta p^e_{12}}{\theta(1-\delta)}$ and $\hat{d}_2 = \frac{p_{21}-\delta p^e_{22}-(V-\theta)(1-\delta)}{\theta(1-\delta)}$.

Upon plugging in the expressions for p^e_{12} and p^e_{22} , we have $\hat{d}_1 = \frac{3V-p_{11}(3-(3-4\alpha)\delta)+\delta(3\theta-6V+2(p_{21}+V)\alpha+(V-\theta)(3-2\alpha)\delta)}{\theta(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$ and

$\hat{d}_2 = \frac{(6V-2(p_{11}+V)\alpha-p_{21}(3-4\alpha)-\theta(9-8\alpha))\delta-3(V-p_{21}-\theta)-(V-2\theta(1-\alpha))(3-2\alpha)\delta^2}{\theta(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$. Plugging $\hat{d}_1$ and $\hat{d}_2$ into the profit functions and solving the first-order conditions ($\frac{d\pi_1}{dp_{11}} = 0$ and $\frac{d\pi_2}{dp_{21}} = 0$), we obtain the firms' first-period prices:

$$p^*_{11} = p^*_{21} = \frac{V(1-\delta)(3(1-\delta)^2-8\alpha(1-\delta)^2-4\alpha^2(2-\delta)\delta)+\theta(4+\delta(4\alpha-5-2\delta+4\alpha\delta+(3-4(2-\alpha)\alpha)\delta^2))}{2(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}. \quad \text{(B12)}$$

Using the expression for p^*_{11} and p^*_{21}, we find that

$$\hat{d}_1 = \frac{(1-\delta)(3V-4\theta-(V-\theta)(3-4\alpha)\delta)}{2\theta(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}, \quad \text{(B13)}$$

$$\hat{d}_2 = 1 - \frac{(1-\delta)(3V-4\theta-(V-\theta)(3-4\alpha)\delta)}{2\theta(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}, \quad \text{(B14)}$$

$$\pi^*_{11} = \pi^*_{21} =$$

$$\frac{\alpha(1-\delta)(3V-4\theta-(V-\theta)(3-4\alpha)\delta)(V(1-\delta)(3(1-\delta)^2-8\alpha(1-\delta)^2-4\alpha^2(2-\delta)\delta)+\theta(4+\delta(4\alpha-5-2\delta+4\alpha\delta+(3-4(2-\alpha)\alpha)\delta^2)))}{4\theta(1-(1-\alpha)\delta)^2(3-3\delta-\alpha(4-6\delta))^2},$$

(B15)

$$\pi_{12}^* = \pi_{22}^* = \frac{(V\alpha(1-\delta)(3-(3-4\alpha)\delta)-\theta(3-6(1-\alpha)\delta+(3-2(3-\alpha)\alpha)\delta^2))^2}{2\theta(1+(1-\alpha)\delta)^2(3-3\delta-\alpha(4-6\delta))^2}. \tag{B16}$$

The firms' overall profits are $\pi_1^* = \pi_{11}^* + \pi_{12}^*$ and $\pi_2^* = \pi_{21}^* + \pi_{22}^*$.

The prices p_{11}^* and p_{21}^* in (B12) must satisfy the inequalities (B10) and (B11) because otherwise, our initial guess would be violated. Moreover, a necessary condition for the existence of this equilibrium is that $0 < \hat{d}_1 < \hat{d}_2 < 1$. We find that the inequalities (B10), (B11) and $0 < \hat{d}_1 < \hat{d}_2 < 1$ are satisfied if and only if $\frac{V(1-\delta)(3-(3-4\alpha)\delta)}{7-4\alpha-13\delta+(17-4\alpha)\alpha\delta+(6-\alpha(13-6\alpha))\delta^2} < \theta < \bar{\theta}$, where recall that $\bar{\theta} = \frac{V(1-\delta)}{2-\delta}$.

Denote $\tilde{\theta} = \frac{V(1-\delta)(3-(3-4\alpha)\delta)}{7-4\alpha-13\delta+(17-4\alpha)\alpha\delta+(6-\alpha(13-6\alpha))\delta^2}$. Note that the inequality $\tilde{\theta} < \bar{\theta}$ can hold if and only if $0 < \delta < \frac{1}{3}$ and $0 < \alpha < \frac{9(1-\delta)\delta-4+\sqrt{16-(1-\delta)\delta(56-9(1-\delta)\delta)}}{4\delta(2-3\delta)}$.

Because our main analysis focuses on $\delta > \underline{\delta} \geq \frac{1}{2}$, it follows that the equilibrium from Case *i*) cannot exist as an alternative equilibrium on the parameter region that we analyze. Furthermore, as we demonstrate below, even when $0 < \delta < \frac{1}{3}$, the equilibrium from Case *i*) still fails to exist due to the existence of non-local profitable price deviations in the first period.

Let us show that when $\tilde{\theta} < \theta < \bar{\theta}$, firm 2 has a profitable non-local deviation in the first period. Specifically, suppose that firm 2 deviates to a lower price: $p_{12}' = \theta$. Under the price pair (p_{11}^*, θ), first-period consumers rationally choose to buy a product in the first period instead of waiting.[7] Hence, after its deviation to $p_{12}' = \theta$, firm 2's first-period sales are given by $\alpha(1-\tilde{d})$, where $\tilde{d}$ satisfies $V - p_{11}^* - \theta\tilde{d} = V - \theta - \theta(1-\tilde{d})$ and is given by

$$\tilde{d} = \frac{V(1-\delta)(3(1-\delta)^2-8\alpha(1-\delta)^2-4\alpha^2(2-\delta)\delta)+\theta(4-\delta(5-4\alpha+2\delta-4\alpha\delta-(3-4(2-\alpha)\alpha)\delta^2))}{4\theta(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}.$$

Because none of the first-period consumers waits till the second period to buy, in the second-period, the firms' equilibrium prices are $p_{12} = p_{22} = \theta$, with corresponding profits of $\pi_{12} = \pi_{22} = \frac{(1-\alpha)\theta}{2}$

[7] We showed this in the proof of Lemma 1. Namely, for a given pair of first-period prices (p_{11}, θ), consumers' rational beliefs about second-period prices are $p_{12}^e = p_{22}^e = \theta$. Given these beliefs, buying in the first period is preferable because $\max\{V - p_{11} - \theta d, V - \theta - \theta(1-d)\} \geq \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\}$, where the inequality holds because $\theta < \bar{\theta} = \frac{V(1-\delta)}{2-\delta}$. If all consumers buy in the first period, then we know that the equilibrium second-period prices are $p_{12}^* = p_{22}^* = \theta$. Hence, consumers' price expectations and purchase decisions are indeed rational.

(see the proof of Lemma 1). Therefore, by deviating to $p'_{12} = \theta$, firm 2 obtains a profit of $\pi'_2 = \alpha(1 - \tilde{d})\theta + \frac{(1-\alpha)\theta}{2}$. It remains to demonstrate that $\pi'_2 > \pi^*_2 = \pi^*_{21} + \pi^*_{22}$, where π^*_{21} and π^*_{22} are given in equations (B15) and (B16), respectively. Denote $\Delta\pi = \pi'_2 - \pi^*_2$. Then, $\Delta\pi$ is concave because $\frac{d^2\Delta\pi}{d\theta^2} = -\frac{V^2\alpha(1-\delta)^2(3-(3-4\alpha)\delta)(3-2\alpha-2(3-5\alpha)\delta+(3-4(2-\alpha)\alpha)\delta^2)}{2\theta^3(1-(1-\alpha)\delta)^2(3-3\delta-\alpha(4-6\delta))^2} < 0$. Further,

$$\Delta\pi|_{\theta=\tilde{\theta}} = \frac{V(1-\alpha)\alpha(1-\delta)(3-(3-4\alpha)\delta)}{2(7-4\alpha-13\delta+(17-4\alpha)\alpha\delta+(6-\alpha(13-6\alpha))\delta^2)} > 0 \text{ and}$$

$$\Delta\pi|_{\theta=\bar{\theta}} = \frac{V\alpha(1-\delta)(2(1-\delta)^3(4-3\delta)+4\alpha^4(2-\delta)\delta^2(2-3\delta)+8\alpha^3(1-\delta)\delta(4-\delta(13-6\delta))-\alpha(1-\delta)^2(24-\delta(65-33\delta))+\Omega)}{2(2-\delta)(1-(1-\alpha)\delta)^2(3-3\delta-\alpha(4-6\delta))^2}$$

where $\Omega = \alpha^2(1-\delta)(16-\delta(108-\delta(163-63\delta)))$. Recall that we must have $0 < \delta < \frac{1}{3}$, $0 < \alpha < \frac{9(1-\delta)\delta-4+\sqrt{16-(1-\delta)\delta(56-9(1-\delta)\delta)}}{4\delta(2-3\delta)}$ and $\tilde{\theta} < \theta < \bar{\theta}$ to ensure that the necessary condition for the existence of the equilibrium is satisfied. Using the conditions $0 < \delta < \frac{1}{3}$ and $0 < \alpha < \frac{9(1-\delta)\delta-4+\sqrt{16-(1-\delta)\delta(56-9(1-\delta)\delta)}}{4\delta(2-3\delta)}$, one can show that $\Delta\pi|_{\theta=\bar{\theta}} > 0$. Given the concavity of $\Delta\pi$ at any $\theta \in (\tilde{\theta}, \bar{\theta})$ and $\Delta\pi|_{\theta=\tilde{\theta}} > 0$ and $\Delta\pi|_{\theta=\bar{\theta}} > 0$, it follows that $\Delta\pi > 0$ for any $\theta \in (\tilde{\theta}, \bar{\theta})$. Hence, this implies that firm 2's deviation to $p'_{21} = \theta$ is profitable, which, in turn, implies that p^*_{12} and p^*_{21} in (B12) cannot constitute an equilibrium.

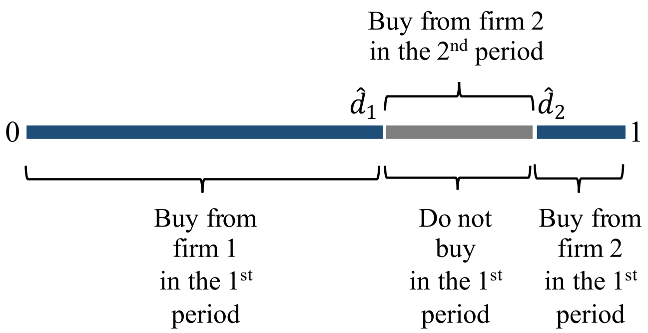
Case ii). We guess that the following inequalities will be satisfied in equilibrium:

$$\delta(V - p^e_{12} - \theta\hat{d}_1) < \delta(V - p^e_{22} - \theta(1 - \hat{d}_1)), \quad \text{(B17)}$$

$$\delta(V - p^e_{12} - \theta\hat{d}_2) < \delta(V - p^e_{22} - \theta(1 - \hat{d}_2)). \quad \text{(B18)}$$

In words, first-period consumers who do not buy in the first period expect to purchase firm 2's product in the second period. In a rational-expectations equilibrium, the decision that a consumer expects to make in the second period must be consistent with the consumer's equilibrium choice. Figure B2 illustrates first-period consumers' decisions.

Figure B2 Purchase Decisions of Consumers Entering the Market in the 1st Period



The consumer located at $\hat{d}_1$ is indifferent between purchasing firm 1's product in the first period and purchasing firm 2's product in the second period, whereas the consumer located at $d = \hat{d}_2$ is indifferent between purchasing firm 2's product in the first period and purchasing it in the second period. Mathematically, $V - p_{11} - \theta\hat{d}_1 = \delta(V - p_{22}^e - \theta(1 - \hat{d}_1))$ and $V - p_{21} - \theta(1 - \hat{d}_2) = \delta(V - p_{22}^e - \theta(1 - \hat{d}_2))$. Solving these equations, we find $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + (p_{22}^e + \theta)\delta}{\theta(1+\delta)}$ and $\hat{d}_2 = \frac{p_{21} - (1-\delta)(V-\theta) - p_{22}^e\delta}{\theta(1-\delta)}$. In a rational-expectations equilibrium, the firms' second-period prices must be such that all consumers located on $[\hat{d}_1, \hat{d}_2]$ purchase firm 2's product because otherwise the inequalities (B17) and (B18) will be violated. The firms' second-period profit functions are as follows:

$$\pi_{12} = (1 - \alpha)\frac{\theta - p_{12} + p_{22}}{2\theta}p_{12}, \tag{B19}$$

$$\pi_{22} = \left(\alpha\left(\hat{d}_2 - \hat{d}_1\right) + (1 - \alpha)\left(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}\right)\right)p_{22}, \tag{B20}$$

where $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + (p_{22}^e + \theta)\delta}{\theta(1+\delta)}$ and $\hat{d}_2 = \frac{p_{21} - (1-\delta)(V-\theta) - p_{22}^e\delta}{\theta(1-\delta)}$. Solving the first-order conditions, we obtain $p_{12}^* = \theta + \frac{2(p_{21} - p_{22}^e)\alpha}{3(1-\alpha)(1-\delta)} + \frac{2(p_{11} + p_{22}^e + \theta - 2V)\alpha}{3(1-\alpha)(1+\delta)}$ and $p_{22}^* = \theta + \frac{4(p_{21} - p_{22}^e)\alpha}{3(1-\alpha)(1-\delta)} + \frac{4(p_{11} + p_{22}^e + \theta - 2V)\alpha}{3(1-\alpha)(1+\delta)}$. The rational-expectations condition requires that $p_{12}^* = p_{12}^e$ and $p_{22}^* = p_{22}^e$. Solving these equalities using the expressions for p_{12}^* and p_{22}^*, we find that $p_{12}^e = \frac{2\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1+\delta)(3 - \alpha - 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}$ and $p_{22}^e = \frac{4\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1-\delta)(3 + \alpha + 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}$. Upon plugging the expression for p_{12}^e and p_{22}^e back into p_{12}^* and p_{22}^*, we verify that

$$p_{12}^* = p_{12}^e = \frac{2\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1+\delta)(3 - \alpha - 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}, \tag{B21}$$

$$p_{22}^* = p_{22}^e = \frac{4\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1-\delta)(3 + \alpha + 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}. \tag{B22}$$

The firms' second-period profits are given by

$$\pi_{12}^* = \frac{(1-\alpha)(2\alpha(p_{21}(1+\delta)-(2V-p_{11})(1-\delta))+\theta(1+\delta)(3-\alpha-3(1-\alpha)\delta))^2}{2\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))^2}, \tag{B23}$$

$$\pi_{22}^* = \frac{(1-\alpha)(4\alpha(p_{21}(1+\delta)-(2V-p_{11})(1-\delta))+\theta(1-\delta)(3+\alpha+3(1-\alpha)\delta))^2}{2\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))^2}. \tag{B24}$$

Using the expressions for p_{12}^e and p_{22}^e, we obtain the cutoffs $\hat{d}_1 = \frac{(1-\delta)V-p_{11}+(p_{22}^e+\theta)\delta}{\theta(1+\delta)}$ and $\hat{d}_2 = \frac{p_{21}-(1-\delta)(V-\theta)-p_{22}^e\delta}{\theta(1-\delta)}$ as functions of the first-period prices:

$$\hat{d}_1 = \frac{3V(1-\alpha)(1-\delta)^2+4\alpha\delta p_{21}+2\theta\delta(3-\alpha-3(1-\alpha)\delta)-p_{11}(3-3\alpha-3\delta+7\alpha\delta)}{\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))}, \tag{B25}$$

$$\hat{d}_2 = \frac{3(p_{21}+\theta-V)(1-\alpha)+\delta(p_{21}(3+\alpha)-4p_{11}\alpha-\theta(3-7\alpha))+3(V-2\theta)(1-\alpha)\delta^2}{\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))}. \tag{B26}$$

A necessary condition for the existence of the equilibrium is that $\hat{d}_1 < \hat{d}_2$. Further, p_{12}^e and p_{22}^e must satisfy the inequalities (B17) and (B18). We find that these conditions can be satisfied only if the firms' first-period prices satisfy the following inequality:

$$\max\left\{\frac{3(V-\theta)(1-\alpha)\delta^2-3(V-p_{21})(1-\alpha)+(p_{21}-\theta)(3+\alpha)\delta}{4\alpha\delta}, \frac{(1-\delta)(2V-3\theta+p_{21})-2(p_{21}-\theta)}{1-\delta}\right\} < p_{11} < \frac{(2V(1-\delta)-\theta(1-3\delta))(3-\alpha-3(1-\alpha)\delta)-2p_{21}\alpha(1-3\delta)}{6(1-\delta)-4\alpha(1-3\delta)} \tag{B27}$$

After finding the firms' first-period prices, we will need to verify that these prices satisfy the inequality (B27).

In the first-period, consumers with $d < \hat{d}_1$ buy firm 1's product, whereas consumers with $d > \hat{d}_2$ buy firm 2's product. The firms' equilibrium first-period prices must maximize the sum of their profits over the two periods:

$$\pi_1 = \alpha\hat{d}_1 p_{11} + \pi_{12}^*, \tag{B28}$$

$$\pi_2 = \alpha(1-\hat{d}_2)p_{21} + \pi_{22}^*. \tag{B29}$$

We assume that $\delta > \frac{1}{2}$ to ensure that the profit functions are concave. Solving the first-order conditions ($\frac{d\pi_1}{dp_{11}} = 0$ and $\frac{d\pi_2}{dp_{21}} = 0$), we obtain

$$p_{11}^* = \frac{\theta(9-(16-7\alpha)\alpha+\alpha(17-5\alpha)\delta-9(1-\alpha)\delta^2)((1-\delta)(1+\delta)(1+3\delta)-\alpha(1+3\delta(1-\delta(3+\delta))))+\Sigma_1}{3(1-\alpha)^2(9-19\alpha)+6(1-\alpha)\alpha(21-31\alpha)\delta-6(1-\alpha)(9-\alpha(23+16\alpha))\delta^2-2\alpha(63-\alpha(116-5\alpha))\delta^3+9(1-\alpha)(3-\alpha(6+\alpha))\delta^4}$$

$$p_{21}^* = \frac{\frac{\theta(9-\alpha(4+5\alpha)+2\alpha(5+7\alpha)\delta-9(1-\alpha)^2\delta^2)((1-\delta)(1+\delta)(4+3\delta)-\alpha(4+3\delta(1-\delta(4+\delta))))}{2}+\Sigma_2}{3(1-\alpha)^2(9-19\alpha)+6(1-\alpha)\alpha(21-31\alpha)\delta-6(1-\alpha)(9-\alpha(23+16\alpha))\delta^2-2\alpha(63-\alpha(116-5\alpha))\delta^3+9(1-\alpha)(3-\alpha(6+\alpha))\delta^4}$$

where $\Sigma_1 = \frac{V(1-\alpha)(1-\delta)(27(1-\delta^2)^2-12\alpha(1-\delta)(1+\delta)(8-3\delta(3+\delta))+\alpha^2(69-\delta(204-\delta(34+60\delta+9\delta^2))))}{2}$ and $\Sigma_2 = \frac{V(1-\alpha)(1-\delta)(27(1-\delta^2)^2-12\alpha(1-\delta)(1+\delta)(11-9\delta-6\delta^2)+\alpha^2(105-\delta(252+\delta(118-3\delta(52+15\delta)))))}{2}$.

A necessary condition for the prices p_{11}^* and p_{21}^* to constitute an equilibrium is that the inequality (B27) is a satisfied. Upon plugging the prices p_{11}^* and p_{21}^* into the inequality (B27) and simplifying the expressions, one can show that $\frac{(1-\delta)(2V-3\theta+p_{21}^*)-2(p_{21}^*-\theta)}{1-\delta} > p_{11}^*$ for any $\delta > \frac{1}{2}$ and $\theta < \bar{\theta}$. Therefore, the prices p_{11}^* and p_{21}^* cannot satisfy (B27). It follows that the necessary conditions for p_{11}^* and p_{21}^* to be an equilibrium fail to hold. Intuitively, to induce some of the first-period consumers to wait till the second period, the firms' first-period prices need to be high. However, if firm 1 sets a high first-period price, then firm 2 gains more profit by reducing its first-period price and capturing more customers in the first period. As a result, our initial guess that some of the first-period consumers would wait till the second period to buy fails to hold on the "equilibrium" path.

Case iii). The analysis of Case *iii*) is similar to that in Case *ii*) because of the symmetry between the firms.

The above analysis suggests that on the parameter region that our analysis focus on, the equilibrium in Lemma 1 is unique. However, for very large values of θ, there can also exist an equilibrium where the firms avoid competitive friction and stay as local monopolists. In this situation, the firms would want to enter the market early instead of delaying. Thus, high level of differentiation between the firms acts as a boundary condition for our results; we discuss this in the Conclusion section in the main paper.

2. Profit Comparison when $(R_1, R_2) = (1, 2)$

As we point out in the main text, when $(R_1, R_2) = (1, 2)$, firm 1's equilibrium profit exceeds firm 2's profit: $\pi_1^{(1,2)} > \pi_2^{(1,2)}$. Below we prove this.

Using firm 1's first-period equilibrium price $p_{11}^{(1,2)}$ in Lemma 3, we can plug it into the equations (A9), (A10), (A11), (A14) and (A15) to obtain the firms' first- and second-period sales and prices *on the equilibrium path*. To simplify notation, we will write $S_{it}^{*(1,2)}$ to denote firm i's sales in period t on the equilibrium path, i.e., $S_{it}^{*(1,2)} = S_{it}^{(1,2)}|_{p_{11}=p_{11}^{(1,2)}}$. Similarly, we will write $p_{i2}^{*(1,2)}$ to denote firm i's second-period price on the equilibrium path: $p_{i2}^{*(1,2)} = p_{i2}^{(1,2)}|_{p_{11}=p_{11}^{(1,2)}}$.

Using the above notation, the firms' equilibrium profits can be written as follows: $\pi_1^{(1,2)} = S_{11}^{*(1,2)} p_{11}^{(1,2)} + S_{12}^{*(1,2)} p_{12}^{*(1,2)}$ and $\pi_2^{(1,2)} = S_{22}^{*(1,2)} p_{22}^{*(1,2)}$. Note that $S_{11}^{*(1,2)} + S_{12}^{*(1,2)} + S_{22}^{*(1,2)} = 1$ because the measure of consumers entering the market is one. Then, we can write $\pi_1^{(1,2)} - \pi_2^{(1,2)} = S_{11}^{*(1,2)} p_{11}^{(1,2)} - (\alpha - S_{11}^{*(1,2)}) p_{22}^{*(1,2)} + S_{12}^{*(1,2)} p_{12}^{*(1,2)} - (1 - \alpha - S_{12}^{*(1,2)}) p_{22}^{*(1,2)}$. One can readily show that $S_{11}^{*(1,2)} > \frac{\alpha}{2}$ and $S_{12}^{*(1,2)} \geq \frac{1-\alpha}{2}$. Further, $p_{11}^{(1,2)} > p_{22}^{*(1,2)}$.[8] Hence, we must have $S_{11}^{*(1,2)} p_{11}^{(1,2)} > (\alpha - S_{11}^{*(1,2)}) p_{22}^{*(1,2)}$. Now, given firm 1's first-period price $p_{11}^{(1,2)}$, we know that $p_{12}^{*(1,2)}$ and $p_{22}^{*(1,2)}$ are the firms' second-period prices on the equilibrium path. Suppose that in the second-period, firm 1 deviates to $p_{12}^{d} = p_{22}^{*(1,2)}$, i.e., charges the same price as firm 2. Then, firm 1 will capture exactly half of the new consumers entering the market in the second period, obtaining a second-period profit of $\frac{1-\alpha}{2} p_{22}^{*(1,2)}$. This deviation profit must be less than what firm 1 earns by charging its equilibrium price $p_{12}^{*(1,2)}$, i.e., $\frac{1-\alpha}{2} p_{22}^{*(1,2)} < S_{12}^{*(1,2)} p_{12}^{*(1,2)}$. Because $S_{12}^{*(1,2)} \geq \frac{1-\alpha}{2}$, it follows that $S_{12}^{*(1,2)} p_{12}^{*(1,2)} > \frac{1-\alpha}{2} p_{22}^{*(1,2)} > (1 - \alpha - S_{12}^{*(1,2)}) p_{22}^{*(1,2)}$. The inequalities $S_{11}^{*(1,2)} p_{11}^{(1,2)} > (\alpha - S_{11}^{*(1,2)}) p_{22}^{*(1,2)}$ and $S_{12}^{*(1,2)} p_{12}^{*(1,2)} > (1 - \alpha -$

[8] To see this, note that for a given $p_{11} \in [\underline{\rho}, \tilde{\rho}]$, Lemma 2 shows that $p_{22}^{(1,2)} = \frac{4\alpha(p_{11} - V(1-\delta)) + \theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta) - \alpha(3-\delta)}$. Because $\theta < \bar{\theta}$, straightforward algebra shows $\frac{4\alpha(p_{11} - V(1-\delta)) + \theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta) - \alpha(3-\delta)} < p_{11}$ for all $p_{11} \in [\underline{\rho}, \tilde{\rho}]$. From Lemma 3, we know that $p_{11}^{(1,2)} \in [\underline{\rho}, \tilde{\rho}]$. Hence, on the equilibrium path, $p_{11}^{(1,2)} > p_{22}^{*(1,2)}$.

$S_{12}^{*(1,2)})p_{22}^{*(1,2)}$ imply that $\pi_1^{(1,2)} - \pi_2^{(1,2)} = S_{11}^{*(1,2)}p_{11}^{(1,2)} - (\alpha - S_{11}^{*(1,2)})p_{22}^{*(1,2)} + S_{12}^{*(1,2)}p_{12}^{*(1,2)} - (1 - \alpha - S_{12}^{*(1,2)})p_{22}^{*(1,2)} > 0$. ■

3. Effect of Vertical Differentiation

We will first obtain the firms' equilibrium pricing strategies and profits in each subgame $(R_1, R_2) \in \{(1,1),(2,2),(1,2),(2,1)\}$. Then, we will characterize market conditions under which $(R_1^*, R_2^*) = (1,2)$ and/or $(R_1^*, R_2^*) = (2,1)$ constitute an equilibrium. We assume that $\mu < 3\theta$ to ensure that, in equilibrium, firm 1 has a positive market share.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{1}, \mathbf{2})$

The analysis follows similar steps as in the proof of Lemma 2. We solve the game by backward induction. In the second period, the firms compete for the new consumers entering the market, as well as consumers who entered the market in the first period but did not buy a product in that period. Therefore, to obtain the firms' second-period prices, we first characterize those first-period consumers who would still be shopping in the second period. Namely, in the first period, a consumer prefers to wait till the second period to buy if and only if

$$V - p_{11} - \theta d < \max\{\delta(V - p_{12}^e - \theta d), \delta(V + \mu - p_{22}^e - \theta(1-d))\}. \tag{B30}$$

Similar analysis as in the main model shows that (B30) reduces to

$$d > \hat{d}. \tag{B31}$$

The cutoff $\hat{d}$ is given by

$$\hat{d} \equiv \begin{cases} 1 & \text{if } p_{11} \leq V(1-\delta) + \delta(p_{22}^e - \mu) - \theta \\ \frac{V(1-\delta) - p_{11} + (p_{22}^e + \theta - \mu)\delta}{\theta(1+\delta)} & \text{if } V(1-\delta) + \delta(p_{22}^e - \mu) - \theta < p_{11} \leq \frac{(2V + \mu - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)} & \text{if } \frac{(2V + \mu - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e \\ 0 & \text{if } p_{11} \geq V(1-\delta) + \delta p_{12}^e \end{cases} \tag{B32}$$

As in the proof of Lemma 2, we separately analyze the four possible cases. We find that the second period equilibrium prices are as follows:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \theta - \frac{\mu}{3} & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{2\alpha(p_{11} - V(1-\delta)) + \theta(3-\alpha)(1+\delta) - \mu(1-\alpha)(1+\delta)}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11} - V(1-\delta)) + 3\theta(1-\delta) - \mu(1-\delta)}{3 - (3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B33}$$

$$p_{22}^{(1,2)} = p_{22}^{e} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{4\alpha(p_{11} - V(1-\delta)) + \theta(3+\alpha+3\delta(1-\alpha)) + \mu(1+\delta-\alpha(1-3\delta))}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3\theta+\mu+2\alpha(p_{11}-V)+(2(\theta+V+\mu)\alpha-3\theta-\mu)\delta}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \quad \text{(B34)}$$

where $\underline{\rho} = (1-\delta)(V-\theta) - \frac{2\mu\delta}{3}$, $\tilde{\rho} \equiv \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv V(1-\delta) + \theta\delta - \frac{\mu\delta}{3}$. Consistent with the main model, to ensure the existence of the second-period subgame equilibrium for any given first-period price $p_{11} \geq 0$, we assume that $\alpha < \bar{\alpha}$ and $\delta > \underline{\delta}$, where $\bar{\alpha} \approx 0.64$, $\underline{\delta} \equiv \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$.

For future reference, we use the prices in (B33) and (B34) to obtain the firms' second-period subgame equilibrium profits when $(R_1, R_2) = (1, 2)$:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)-\mu(1-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)-\mu(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{(3\theta-\mu)^2}{18} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B35)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)+\mu(1-\alpha+\delta+3\alpha\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(3\theta+\mu+2\alpha(p_{11}-V)+(2(\theta+V+\mu)\alpha-3\theta-\mu)\delta)^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{(3\theta+\mu)^2}{18} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B36)}$$

Firm 1's first-period sales are given by $S_{11}^{(1,2)} = \alpha\hat{d}$. Plugging $p_{12}^{e} = p_{12}^{(1,2)}$ and $p_{22}^{e} = p_{22}^{(1,2)}$ into the expression for $\hat{d}$, we find that

$$S_{12}^{(1,2)} = \begin{cases} \alpha & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha\frac{3(V-p_{11})(1-\alpha)+2\theta(3-\alpha)\delta-(3V+2\mu)(1-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3V-3p_{11}-(3V-3\theta+\mu)\delta}{\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B37)}$$

In the first period, firm 1 chooses its price to maximize its overall profit over the two periods: $\pi_1 = S_{11}^{(1,2)}p_{11} + \pi_{12}^{(1,2)}$. One can show that the price $p_{11}^{(1,2)}$ maximizing π_1 is contained in the interval $[\underline{\rho}, \tilde{\rho}]$. Define $\dot{\mu}_a = \min\{\frac{12\theta(2-\alpha(2-\delta))+9V(-1+\alpha+\delta-\alpha\delta)}{2(1-\alpha)(1-3\delta)}; 3\theta\}$ and $\ddot{\mu}_a = \min\{\max\{0, \frac{\theta(3-\alpha)}{1-\alpha} +$

$\frac{3V(6-\alpha(7-3\alpha))(1-\delta)}{6\delta+2\alpha(6-8\delta-\alpha(1-3\delta))-10}\}; 3\theta\}$. Further, let $\Sigma_{a1} = \{(\theta,\mu)|\ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\mu < \dot{\mu}_a\}$, $\Sigma_{a2} = \{(\theta,\mu)|\ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\mu < \ddot{\mu}_a\}$ and $\Sigma_{a3} = \{(\theta,\mu)|\ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \leq \mu < 3\theta\} \cup \{(\theta,\mu)|\ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\ddot{\mu}_a \leq \mu < 3\theta\}$. Then, we find that

$$p_{11}^{(1,2)} = \begin{cases} (1-\delta)(V-\theta) - \frac{2\mu\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{2\theta(3-\alpha)(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2))+(1-\alpha)(V(1-\delta)(9-13\alpha+9\delta+3\alpha\delta)-2\mu(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2)))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B38}$$

Upon plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$, we find the firms' second-period prices on the equilibrium path.

$$p_{12}^{(1,2)} = \begin{cases} \theta - \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)(\theta(3-\alpha)-\mu+\mu\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-(1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B39}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\theta(6-\alpha(3+\alpha))+\mu(2-(1-\alpha)\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{(1-\alpha)(3\mu(1+\delta)-6V\alpha(1-\delta)-5\mu\alpha(1-\delta))+\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B40}$$

Using the equilibrium prices in (B38)-(B40), we obtain the firms' equilibrium profits in each period.

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(1-\delta)(V-\theta) - \frac{2\mu\alpha\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\alpha(\theta(3-\alpha)+\mu\alpha-\mu)(V(1-\delta)(6-\alpha(7-3\alpha))+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\alpha((1-\alpha)(9V(1-\delta)+2\mu-6\mu\delta)-2\theta(3-\alpha)(1-3\delta))A}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B41}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)^2(1-\alpha)(\theta(3-\alpha)+\mu\alpha-\mu)^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))-\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B42}$$

$$\pi_{21}^{(1,2)} = 0 \tag{B43}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \quad \text{(B44)}$$

where $A \equiv \frac{(1-\alpha)(V(1-\delta)(9-13\alpha+3(3+\alpha)\delta)-2\mu(1-\alpha+4(1-\alpha)\delta+(3+\alpha)\delta^2))+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$.

Using the profits obtained in each period in equations (B41)-(B44), we can readily find firm i's overall profit in the subgame with $(R_1, R_2) = (1,2)$: $\pi_i^{(1,2)} = \pi_{i1}^{(1,2)} + \pi_{i2}^{(1,2)}$.

Subgame with $(R_1, R_2) = (2,1)$

The analysis is very similar to that in the subgame with $(R_2, R_2) = (1,2)$. Therefore, we will directly provide the equilibrium prices and profits to avoid repetitiveness.

Define $\dot{\mu}_b = \max\{\frac{12\theta(2-\alpha(2-\delta))-9V(1-\alpha-\delta+\alpha\delta)}{(1-\alpha)(7-3\delta)}; 0\}$ and $\ddot{\mu}_b = \max\{0, \frac{2\theta(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))-3V(1-\alpha)(1-\delta)(6-\alpha(7-3\alpha))}{(1-\alpha)(8-12\delta-\alpha(9-5\delta-\alpha(7-3\delta)))}\}$. Further, let $\Sigma_{b1} = \{(\theta,\mu) |\, 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))}$ and $\dot{\mu}_b < \mu < 3\theta\} \cup \{(\theta,\mu) |\, \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))} < \theta < \bar{\theta}$ and $3\theta(1-\alpha) \leq \mu < 3\theta\}$, $\Sigma_{b2} = \{(\theta,\mu) |\, \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))} < \theta < \bar{\theta}$ and $\ddot{\mu}_b < \mu < 3\theta(1-\alpha)\}$ and $\Sigma_{b3} = \{(\theta,\mu) |\, 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))}$ and $0 \leq \mu < \dot{\mu}_b\} \cup \{(\theta,\mu) |\, \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))} < \theta < \bar{\theta}$ and $0 \leq \mu < \ddot{\mu}_b\}$. We find that firm 2's optimal first-period price is given by

$$p_{21}^{(2,1)} = \begin{cases} (V-\theta)(1-\delta) + \frac{3-\delta}{3}\mu & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(5-\alpha(2-\alpha)(3-\delta)-3\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{2\theta(3-\alpha)(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2))+(1-\alpha)(V(1-\delta)(9-13\alpha+9\delta+3\alpha\delta)+\mu(11+8\delta-3\delta^2-\alpha(15-8\delta+\delta^2)))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases}$$

(B45)

The firms' second-period prices on the equilibrium path are given by

$$p_{12}^{(2,1)} = \begin{cases} \theta - \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{6\theta - \theta\alpha(3+\alpha) - \mu(2-\alpha+\alpha^2)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta))-(1-\alpha)(\mu(3+\alpha)+6V\alpha(1-\delta)+\mu(3-\alpha)\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B46)}$$

$$p_{22}^{(2,1)} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{(\theta(3-\alpha)+\mu(1-\alpha))(2-\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-(1-\alpha)(2\mu\alpha(3-\delta)+3V\alpha(1-\delta)-3\mu(1+\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B47)}$$

Using the equilibrium prices, we obtain the firms' equilibrium profits in each period.

$$\pi_{11}^{(2,1)} = 0 \quad \text{(B48)}$$

$$\pi_{12}^{(2,1)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))-\mu(2-(1-\alpha)\alpha))^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+\mu(3+\alpha)+\mu(3-\alpha)\delta)-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B49)}$$

$$\pi_{21}^{(2,1)} =$$

$$\begin{cases} \alpha(V-\theta)(1-\delta) + \alpha\mu\frac{3-\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{\alpha(\theta(3-\alpha)-\mu\alpha+\mu)(V(1-\delta)(6-\alpha(7-3\alpha))+\mu(5-3\delta-(2-\alpha)\alpha(3-\delta))-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{\alpha((1-\alpha)(9V(1-\delta)+\mu(7-3\delta))-2\theta(3-\alpha)(1-3\delta))B}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B50)}$$

$$\pi_{22}^{(2,1)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{(2-\alpha)^2(1-\alpha)(\theta(3-\alpha)-\mu\alpha+\mu)^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{((1-\alpha)(3V\alpha(1-\delta)-3\mu(1+\delta)+2\mu\alpha(3-\delta))-\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B51)}$$

where $B \equiv \frac{(1-\alpha)(V(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+\mu(11-15\alpha+8(1+\alpha)\delta-(3+\alpha)\delta^2)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$.

Using the profits obtained in each period in the equations (B48)-(B51), we can readily obtain firm i's overall profit in the subgame with $(R_1, R_2) = (2, 1)$: $\pi_i^{(2,1)} = \pi_{i1}^{(2,1)} + \pi_{i2}^{(2,1)}$.

Subgame with $(R_1, R_2) = (1, 1)$

We solve the game by backward induction, following similar steps as in the main model. Specifically, we start with a guess that in the first period, the market is fully covered, and hence, no consumer will choose to wait till the second period to buy a product (we will later verify our guess).[9] Mathematically, this is equivalent to

$$\max\{V - p_{11} - \theta d, V + \mu - p_{21} - \theta(1-d)\} \geq \delta \max\{V - p_{12}^e - \theta d, V + \mu - p_{22}^e - \theta(1-d)\} \quad \text{(B52)}$$

for all $d \in [0,1]$, where p_{12}^e and p_{22}^e are the prices that first-period consumers expect firms 1 and 2 to charge in the second period, respectively.

If the inequality (B52) is satisfied, then in the second period, the firms compete for the $(1-\alpha)$ measure of new consumers who enter the market. Given the firms' second-period prices p_{12} and p_{22}, a consumer located at a distance d from firm 1 will prefer firm 1's product over firm 2's if and only if $V - p_{12} - \theta d > V + \mu - p_{22} - \theta(1-d)$, or equivalently, $d < \frac{\theta-\mu-p_{12}+p_{22}}{2\theta}$. Hence, firm i will choose its second-period price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = (1-\alpha)\frac{\theta-\mu-p_{12}+p_{22}}{2\theta}p_{12}$ and $\pi_{22} = (1-\alpha)(1-\frac{\theta-\mu-p_{12}+p_{22}}{2\theta})p_{22}$. Solving the first-order conditions ($\frac{\partial \pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial \pi_{22}}{\partial p_{22}} = 0$), we find that $p_{12}^{(1,1)} = \theta - \frac{\mu}{3}$ and $p_{22}^{(1,1)} = \theta + \frac{\mu}{3}$. The corresponding second-period profits are given by $\pi_{12}^{(1,1)} = \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta}$ and $\pi_{22}^{(1,1)} = \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta}$. Rational-expectations condition implies that $p_{12}^e = \theta - \frac{\mu}{3}$ and $p_{22}^e = \theta + \frac{\mu}{3}$. Plugging these prices into (B52), the inequality (B52) becomes

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} \geq \delta \max\{(V - \theta + \frac{\mu}{3} - \theta d), \delta(V + \frac{2}{3}\mu - \theta - \theta(1-d))\} \quad \text{(B53)}$$

Hence, if a given pair (p_{11}, p_{21}) of first-period prices satisfies the inequality (B53), then *a*) $p_{12}^e = \theta - \frac{\mu}{3}$ and $p_{22}^e = \theta + \frac{\mu}{3}$ are rational expectations about the second-period prices, and *b*) all first-period consumers rationally choose to buy in the first period rather than wait till the second period.

In the first period, the firms choose their prices to maximize the sum of their profits over the two periods. If prices p_{11} and p_{21} satisfy (B53), then the firms' profits are given by $\pi_1 = \pi_{11} + \pi_{12}^{(1,1)} = \alpha\frac{\theta-\mu-p_{11}+p_{21}}{2\theta}p_{11} + \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta}$ and $\pi_2 = \pi_{21} + \pi_{22}^{(1,1)} = \alpha(1-\frac{\theta-\mu-p_{11}+p_{21}}{2\theta})p_{21} + \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta}$. Solving the first-order conditions, we find that $p_{11}^{(1,1)} = \theta - \frac{\mu}{3}$ and $p_{21}^{(1,1)} = \theta + \frac{\mu}{3}$. It follows that the

[9] Using a similar argument as in the main model, one can show that this equilibrium is unique when $\theta < \bar{\theta}$.

firms' first-period profits are given by $\pi_{11}^{(1,1)} = \frac{\alpha(3\theta-\mu)^2}{18\theta}$ and $\pi_{21}^{(1,1)} = \frac{\alpha(3\theta+\mu)^2}{18\theta}$. Their overall profits across the two periods are $\pi_{1}^{(1,1)} = \frac{(3\theta-\mu)^2}{18\theta}$ and $\pi_{2}^{(1,1)} = \frac{(3\theta+\mu)^2}{18\theta}$.

Because $\delta < 1$, it is easy to see that the price pair $(p_{11}, p_{21}) = (\theta - \frac{\mu}{3}, \theta + \frac{\mu}{3})$ indeed satisfies the inequality (B53). Hence, our initial guess that first-period consumers will not wait till the second period is verified.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{2}, \mathbf{2})$

The analysis is straightforward. One can show that $p_{12}^{(2,2)} = \theta - \frac{\mu}{3}$ and $p_{22}^{(2,2)} = \theta + \frac{\mu}{3}$, and the firms' profits are given by $\pi_{1}^{(2,2)} = \frac{(3\theta-\mu)^2}{18\theta}$ and $\pi_{2}^{(2,2)} = \frac{(3\theta+\mu)^2}{18\theta}$.

Having obtained the firms' equilibrium profits in each subgame, we can find their equilibrium product release times.

<u>**PROOF OF PROPOSITION 4**</u>. First, let us characterize market conditions under which the high-quality firm (firm 2) releases its product later than firm 1.

Equilibrium where $(\boldsymbol{R_1^*}, \boldsymbol{R_2^*}) = (\mathbf{1}, \mathbf{2})$

$(R_1^*, R_2^*) = (1, 2)$ is an equilibrium if and only if $\pi_1^{(1,2)} > \pi_1^{(2,2)}$ and $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. Using a similar argument as in the proof of Proposition 1, one can easily show that $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. It remains to check whether $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. Define $\Delta\pi_2 \equiv \pi_2^{(1,2)} - \pi_2^{(1,1)}$.

If $(\theta, \mu) \in \Sigma_{a1} = \{(\theta, \mu) | \; 0 < \theta \le \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\mu < \dot{\mu}_a\}$, then $\Delta\pi_2 = \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} - \frac{(3\theta+\mu)^2}{18\theta}$. Clearly, $\Delta\pi_2 < 0$. Therefore, $(R_1, R_2) = (1, 2)$ cannot be an equilibrium.

Next, consider $(\theta, \mu) \in \{(\theta, \mu) | \; 0 < \theta \le \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \le \mu < 3\theta\} \subseteq \Sigma_{a3}$. This subset of Σ_{a3} is non-empty (i.e., $\dot{\mu}_a < 3\theta$) if and only if $\frac{3V(1-\alpha)(1-\delta)}{6(1+\delta)-2\alpha(3+\delta)} < \theta \le \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Assuming θ satisfies these conditions, $\Delta\pi_2$ is given by

$\Delta\pi_2 = \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{(3\theta+\mu)^2}{18\theta}$. We find that $\Delta\pi_2 < 0$ when $\frac{3V(1-\alpha)(1-\delta)}{6(1+\delta)-2\alpha(3+\delta)} < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \leq \mu < 3\theta$.[10]

The above analysis suggests that if $\theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$, then $\Delta\pi_2 < 0$ for all $\mu < 3\theta$. Hence, we may have $\Delta\pi_2 > 0$ only if $\theta \geq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Denote $\dot{\theta}_{a1} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Conditional on $\dot{\theta}_{a1} \leq \theta < \bar{\theta}$, the expression for $\Delta\pi_2$ is as follows:

$$\Delta\pi_2 = \begin{cases} \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} - \frac{(3\theta+\mu)^2}{18\theta} & \text{if } 0 < \mu \leq \ddot{\mu}_a \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{(3\theta+\mu)^2}{18\theta} & \text{if } \ddot{\mu}_a \leq \mu < 3\theta \end{cases}.$$

Note that $\Delta\pi_2$ is a continuous function. Using the expression of $\ddot{\mu}_a$, one can readily show that

$$\ddot{\mu}_a = \begin{cases} 0 & \text{if } \dot{\theta}_{a1} \leq \theta \leq \dot{\theta}_{a2} \\ \frac{\theta(3-\alpha)}{1-\alpha} + \frac{3V(6-\alpha(7-3\alpha))(1-\delta)}{6\delta+2\alpha(6-8\delta-\alpha(1-3\delta))-10} & \text{if } \dot{\theta}_{a2} \leq \theta \leq \dot{\theta}_{a3} \\ 3\theta & \text{if } \dot{\theta}_{a3} \leq \theta < \bar{\theta} \end{cases}$$

where $\dot{\theta}_{a2} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}; \bar{\theta}\}$ and $\dot{\theta}_{a3} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{4\alpha(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}; \bar{\theta}\}$.

We will separately analyze the cases with *a*) $\dot{\theta}_{a1} \leq \theta \leq \dot{\theta}_{a2}$, *b*) $\dot{\theta}_{a2} \leq \theta \leq \dot{\theta}_{a3}$, and *c*) $\dot{\theta}_{a3} \leq \theta < \bar{\theta}$.

a) Consider $\dot{\theta}_{a1} \leq \theta \leq \dot{\theta}_{a2}$. On this parameter range,

$\Delta\pi_2 = \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{(3\theta+\mu)^2}{18\theta}$ for all $\mu \in [0, 3\theta)$.

Because $\alpha < \bar{\alpha}$ and $\delta > \underline{\delta}$, one can show that $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$, i.e., $\Delta\pi_2$ is concave in μ. Further, if $\Delta\pi_2|_{\mu=0} < 0$, then one can show that $\frac{d\Delta\pi_2}{d\mu}|_{\mu=0} < 0$, which implies—due to concavity of $\Delta\pi_2$—that $\Delta\pi_2 < 0$ for all $\mu \in [0, 3\theta)$. Therefore, we may have $\Delta\pi_2 > 0$ only if $\Delta\pi_2|_{\mu=0} \geq 0$. The latter inequality is equivalent to $\theta > \hat{\theta}_2$, where

$\hat{\theta}_2 = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$. On a side note, notice that $\hat{\theta}_2$ is the same as $\hat{\theta}$ in Proposition 1. One can readily show that $\Delta\pi_2|_{\mu\to\infty} < 0$. Therefore, when

[10] To demonstrate this, first note that $\Delta\pi_2$ is concave as a function of μ, i.e., $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$. Further, we find that $\frac{d\Delta\pi_2}{d\mu}|_{\mu=\dot{\mu}_a} < 0$. The inequalities $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$ and $\frac{d\Delta\pi_2}{d\mu}|_{\mu=\dot{\mu}_a} < 0$ imply that $\frac{d\Delta\pi_2}{d\mu} < 0$ for all $\mu \geq \dot{\mu}_a$. Finally, because $\Delta\pi_2|_{\mu=\dot{\mu}_a} < 0$, it follows that $\Delta\pi_2 < 0$ for all $\mu \geq \dot{\mu}_a$.

$\hat{\theta}_2 < \theta \leq \dot{\theta}_{a2}$, the inequalities $\Delta\pi_2|_{\mu=0} \geq 0$ and $\Delta\pi_2|_{\mu\to\infty} < 0$, together with the concavity of $\Delta\pi_2$, imply that there exists a unique $\hat{\mu}_{2a} > 0$, such that $\Delta\pi_2 > 0$ if and only if $\mu \in [0, \hat{\mu}_{2a}]$. Solving $\Delta\pi_2 = 0$, we find that

$$\hat{\mu}_{2a} = \frac{3(-6V(1-\alpha)(1-\delta)(9(1+\sqrt{1-\alpha})(1+\delta)+\alpha(3\delta+15\sqrt{1-\alpha}\delta-11-15\sqrt{1-\alpha}))+\theta(9(1+\delta)(20+3\sqrt{1-\alpha}-3(4-\sqrt{1-\alpha})\delta)+F_1+F_2))}{\sqrt{1-\alpha}(153-374\alpha+225\alpha^2(1-\delta)^2+9(2-15\delta)\delta+6\alpha\delta(64+9\delta))},$$

where $F_1 = 5\alpha^2(1-\delta)(44+15\sqrt{1-\alpha}-3(4+5\sqrt{1-\alpha})\delta)$ and $F_2 = 2\alpha(-200-53\sqrt{1-\alpha}+48(2+\sqrt{1-\alpha})\delta+9(8-3\sqrt{1-\alpha})\delta^2)$. Because we are focusing on $\mu < 3\theta$, it follows that when $\hat{\theta}_2 < \theta \leq \dot{\theta}_{a2}$ and $\mu \leq \min\{\hat{\mu}_{2a}, 3\theta\}$, we have $\Delta\pi_2 > 0$. Note that the interval $(\hat{\theta}_2, \dot{\theta}_{a2}]$ is non-empty (i.e., $\hat{\theta}_2 < \dot{\theta}_{a2}$) if and only if $\alpha < \hat{\alpha}$, where recall from the proof of Proposition 1 that $\hat{\alpha} \approx 0.32$ is the solution to the equality $12 - \hat{\alpha}(52-(3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$.

b) Consider $\dot{\theta}_{a2} \leq \theta \leq \dot{\theta}_{a3}$. Let $\tilde{\alpha}$ be the solution to the equality $6 - 11\tilde{\alpha} + \left(3-2\sqrt{1-\tilde{\alpha}}\right)\tilde{\alpha}^2 = 0$. One can show that $\tilde{\alpha} \approx 0.6$. We will separately analyze three subcases: $\alpha < \hat{\alpha}$, $\hat{\alpha} < \alpha < \tilde{\alpha}$ and $\tilde{\alpha} < \alpha < \bar{\alpha}$.

First, if $\alpha < \hat{\alpha}$, then $\Delta\pi_2|_{\mu=0} > 0$ and $\frac{d\Delta\pi_2}{d\mu}|_{\mu=0} > 0$ for $\mu \in [0, \ddot{\mu}_a)$. Because $\frac{d^2\Delta\pi_2}{d\mu^2} > 0$ at any $\mu \in [0, \ddot{\mu}_a)$, the inequality $\frac{d\Delta\pi_2}{d\mu}|_{\mu=0} > 0$ implies that $\Delta\pi_2$ is increasing in μ on the interval $[0, \ddot{\mu}_a)$. Therefore, the inequalities $\Delta\pi_2|_{\mu=0} > 0$ and $\frac{d\Delta\pi_2}{d\mu} > 0$ imply that $\Delta\pi_2 > 0$ for all $\mu \in [0, \ddot{\mu}_a)$. Next, for $\mu \in [\ddot{\mu}_a, 3\theta)$, $\Delta\pi_2$ takes the same functional form as in *a*). Using the same argument, we can show that $\Delta\pi_2 > 0$ if $\ddot{\mu}_a < \mu < \min\{\hat{\mu}_{2a}, 3\theta\}$, and $\Delta\pi_2 < 0$ if $\min\{\hat{\mu}_{2a}, 3\theta\} < \mu < 3\theta$. Note that $\ddot{\mu}_a < \hat{\mu}_{2a}$. To summarize, when $\alpha < \hat{\alpha}$, we have $\Delta\pi_2 > 0$ if and only if $\mu \in [0, \min\{\hat{\mu}_{2a};\ 3\theta\})$.

Second, if $\hat{\alpha} < \alpha < \tilde{\alpha}$, then $\Delta\pi_2|_{\mu=0} < 0$. Define $\tilde{\theta} = \frac{3V(1-\alpha)(6-\alpha(5-6\sqrt{1-\alpha}+(3+6\sqrt{1-\alpha})\alpha))(1-\delta)}{4(3-4\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$. One can show that if $\dot{\theta}_{a2} < \theta < \tilde{\theta}$, then $\Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$. Convexity of $\Delta\pi_2$ on the interval $[0, \ddot{\mu}_a)$ implies that $\Delta\pi_2 < 0$ for all $\mu \in [0, \ddot{\mu}_a)$. Moreover, because $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$ on $(\ddot{\mu}_a, 3\theta)$ and $\frac{d\Delta\pi_2}{d\mu}|_{\mu=\ddot{\mu}_a} < 0$, it follows that $\frac{d\Delta\pi_2}{d\mu} < 0$ for all $\mu \in [\ddot{\mu}_a, 3\theta)$. Hence, $\Delta\pi_2 < \Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$ for all $\mu \in (\ddot{\mu}_a, 3\theta)$. In sum, when $\dot{\theta}_{a2} < \theta < \tilde{\theta}$, we have $\Delta\pi_2 < 0$ for any $\mu \in [0, 3\theta)$. Next, if $\tilde{\theta} < \theta \leq \dot{\theta}_{a3}$, then

$\Delta\pi_2|_{\mu=\ddot{\mu}_a} > 0$. Hence, because $\Delta\pi_2$ is a strictly convex function on $[0, \ddot{\mu}_a)$, there must exist a unique $\tilde{\mu}_2 \in (0, \ddot{\mu}_a)$ such that $\Delta\pi_2 > 0$ if $\mu \in (\tilde{\mu}_2, \ddot{\mu}_a)$ and $\Delta\pi_2 < 0$ if $\mu \in [0, \tilde{\mu}_2)$. Solving $\Delta\pi_2 = 0$, we find that

$$\tilde{\mu}_2 = \frac{3\theta(-12+4\sqrt{1-\alpha}\alpha(6-\alpha(7-3\alpha))-\alpha(-28+3\alpha(5-(2-\alpha)\alpha)))}{12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha))}.$$

On the interval $(\ddot{\mu}_a, 3\theta)$, we know that $\Delta\pi_2$ is concave. Because $\Delta\pi_2|_{\mu=\ddot{\mu}_a} > 0$, it follows that $\Delta\pi_2 > 0$ if $\mu \in [\ddot{\mu}_a, \min\{\hat{\mu}_{2a}, 3\theta\})$ and $\Delta\pi_2 < 0$ if $\mu \in [\min\{\hat{\mu}_{2a}, 3\theta\};\ 3\theta)$, where recall that $\hat{\mu}_{2a}$ satisfies $\Delta\pi_2|_{\mu=\hat{\mu}_{2a}} = 0$. In sum, when $\tilde{\theta} < \theta \leq \dot{\theta}_{a3}$, we have $\Delta\pi_2 > 0$ if and only if $\mu \in (\tilde{\mu}_2, \min\{\hat{\mu}_{2a}, 3\theta\})$.

Third, if $\tilde{\alpha} < \alpha < \bar{\alpha}$, then $\Delta\pi_2|_{\mu=0} < 0$ and $\Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$. Convexity of $\Delta\pi_2$ on the interval $[0, \ddot{\mu}_a)$ implies that $\Delta\pi_2 < 0$ for all $\mu \in [0, \ddot{\mu}_a)$. Moreover, $\Delta\pi_2$ is decreasing on the interval $[\ddot{\mu}_a, 3\theta)$. Therefore, $\Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$ implies that $\Delta\pi_2 < \Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$ for all $\mu \in (\ddot{\mu}_a, 3\theta)$. In sum, $\Delta\pi_2 < 0$ for any $\mu \in [0, 3\theta)$.

c) Consider $\dot{\theta}_{a3} < \theta < \bar{\theta}$. On this interval, we have $\Delta\pi_2 = \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} - \frac{(3\theta+\mu)^2}{18\theta}$ for all $\mu \in [0, 3\theta)$. Similar analysis as in *b*) shows that $\Delta\pi_2 > 0$ provided that $\mu \in (\min\{\tilde{\mu}_2, 3\theta\}, 3\theta)$. Note that the interval $(\min\{\tilde{\mu}_2, 3\theta\}, 3\theta)$ is non-empty (i.e., $\tilde{\mu}_2 < 3\theta$) if and only if $\alpha < \tilde{\alpha}$, where recall that $\tilde{\alpha} \approx 0.6$.

Define $\hat{\mu}_2 = \min\{\hat{\mu}_{2a}, 3\theta\}$. The analyses in *a*), *b*), and *c*) can be summarized as follows: $\Delta\pi_2 > 0$ (and thus, $(R_1^*, R_2^*) = (1, 2)$ is an equilibrium) if and only if

i. $0 < \alpha < \hat{\alpha}$, $\hat{\theta}_2 < \theta < \bar{\theta}$ and $0 \leq \mu \leq \hat{\mu}_2$ or

ii. $\hat{\alpha} \leq \alpha < \tilde{\alpha}$, $\tilde{\theta} < \theta < \bar{\theta}$ and $\tilde{\mu}_2 \leq \mu \leq \hat{\mu}_2$

where $\hat{\alpha} \approx 0.32$ is the same as in Proposition 1, $\tilde{\alpha} \approx 0.6$ is the solution to the equality $6 - 11\tilde{\alpha} + \left(3 - 2\sqrt{1-\tilde{\alpha}}\right)\tilde{\alpha}^2 = 0$, $\hat{\theta}_2 = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$,

$\tilde{\theta} = \frac{3V(1-\alpha)(6-\alpha(5-6\sqrt{1-\alpha}+(3+6\sqrt{1-\alpha})\alpha))(1-\delta)}{4(3-4\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, $\hat{\mu}_2 = \min\{\hat{\mu}_{2a}, 3\theta\}$,

$\hat{\mu}_{2a} =$

$$\frac{3(-6V(1-\alpha)(1-\delta)(9(1+\sqrt{1-\alpha})(1+\delta)+\alpha(3\delta+15\sqrt{1-\alpha}\delta-11-15\sqrt{1-\alpha}))+\theta(9(1+\delta)(20+3\sqrt{1-\alpha}-3(4-\sqrt{1-\alpha})\delta)+F_1+F_2))}{\sqrt{1-\alpha}(153-374\alpha+225\alpha^2(1-\delta)^2+9(2-15\delta)\delta+6\alpha\delta(64+9\delta))},$$

$$\tilde{\mu}_2 = \frac{3\theta(-12+4\sqrt{1-\alpha}\alpha(6-\alpha(7-3\alpha))-\alpha(-28+3\alpha(5-(2-\alpha)\alpha)))}{12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha))}.$$

Equilibrium where $(\boldsymbol{R_1^*}, \boldsymbol{R_2^*}) = (\mathbf{2}, \mathbf{1})$

The analysis follows similar steps as for $(R_1^*, R_2^*) = (1, 2)$. Therefore, we directly provide the conditions under which $(R_1, R_2) = (2, 1)$ is an equilibrium. Namely, $(R_1^*, R_2^*) = (2, 1)$ if and only if the following conditions are satisfied: $0 < \alpha < \hat{\alpha}$, $0 < \mu < \hat{\mu}_1$ and $\hat{\theta}_1 < \theta < \bar{\theta}$, where

$$\hat{\theta}_1 = \begin{cases} \hat{\theta}_{b1} & if\ 0 < \mu < \hat{\mu}_{b1} \\ \hat{\theta}_{b2} & if\ \hat{\mu}_{b1} < \mu < \hat{\mu}_1 \end{cases}$$

$$\hat{\mu}_1 = \frac{3V(12-4\alpha\sqrt{1-\alpha}(6-\alpha(7-3\alpha))-\alpha(28-3\alpha(5-(2-\alpha)\alpha)))(1-\delta)}{(12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha)))(2-\delta)},$$

$\hat{\mu}_{b1} =$

$$\frac{9V(1-\alpha)(-1+\delta)(12-9\alpha^5(1-\delta)+36\delta-\alpha^2\left(315-184\sqrt{1-\alpha}-\left(203-216\sqrt{1-\alpha}\right)\delta\right)-\alpha^4\left(81-8\sqrt{1-\alpha}-\left(9-24\sqrt{1-\alpha}\right)\delta\right)+C_2)}{81\alpha^6(1-\delta)^2-12(1+3\delta)^2-4\alpha(1+3\delta)(59-51\delta)+\alpha^2(153+(926-375\delta)\delta)+4\alpha^5(97+33(2-3\delta)\delta)+8\alpha^3(125-\delta(18+91\delta))-C_1}$$

$C_1 = 2\alpha^4(687 + \delta(154 - 489\delta))$, $C_2 = 4\alpha(23 - 30\sqrt{1-\alpha} - 9(5 - 2\sqrt{1-\alpha})\delta) + \alpha^3(301 - 72\sqrt{1-\alpha} - (93 - 136\sqrt{1-\alpha})\delta)$,

$\hat{\theta}_{b1} =$

$$\frac{\mu\sqrt{1-\alpha}(166\alpha+9\alpha^2(1-\delta)^2-6\alpha\delta(8+9\delta)-9(1+\delta)(19-21\delta))+18V(1-\alpha)(1-\delta)(-9(1+\sqrt{1-\alpha})(1+\delta)+\alpha(11-3\sqrt{1-\alpha}+3(-1+\sqrt{1-\alpha})\delta))}{3(\alpha(40+178\sqrt{1-\alpha}-6\delta((6+9\sqrt{1-\alpha})\delta-10(1-\sqrt{1-\alpha})))-9(1+\delta)(2+21\sqrt{1-\alpha}+3(2-5\sqrt{1-\alpha})\delta)-\alpha^2(1-\delta)(22-15\sqrt{1-\alpha}-3(2-5\sqrt{1-\alpha})\delta))},$$

$$\hat{\theta}_{b2} = \frac{\mu(12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha)))}{36+3\alpha(-4(7+6\sqrt{1-\alpha})+\alpha(15+28\sqrt{1-\alpha}+3\alpha(-2-4\sqrt{1-\alpha}+\alpha)))}. \blacksquare$$

4. Product Improvement over Time

We extend our main model to study situations in which a firm can improve its product to deliver a greater value, but this necessitates a delayed entry.[11] To capture such product improvement effects in this extension we assume that by delaying its product release till the second period, a firm can increase the valuation of its product to $V + \mu$, where $\mu \geq 0$ represents the product improvement effect. We assume that $\mu < 3\theta$ to ensure that each firm has a positive market share regardless of its product release time. To keep the model as simple as possible, without loss of generality, the product improvement is assumed to be costless for the firm. For example, the firm may have a technology that is already developed but requires additional time to be finalized and integrated into the product. By contrast, if the firm releases its product in the first period, then consumers' product valuation stays constant at the level V, i.e., the firm is unable to sell an improved version of the product in the second period (Bayus et al. 1997). This is reasonable because, in practice, modifying a product that is already released to the market is likely to require substantial time and resources to change the product design, manufacturing process, supply chain, and marketing strategy. Finally, for a more interesting analysis, we assume that the product improvement effect is not excessively high ($\mu < \bar{\mu}$) to avoid both firms choosing to wait till the second period in order to improve their products.

We provide the technical analysis below. We find that, as expected, the ability to improve its product over time gives a firm even more incentives to release it in the second period because by doing so a) the firm can gain a competitive advantage in the second period due to its improved product, b) the firm does not lose so many sales because, in the first period, consumers anticipate the firm's product improvement, becoming more willing to wait for it, and c) the increased number of consumers who wait for the firm's product tends to reinforce the price-competition alleviation effect, providing an additional benefit to the firm. Thus, the possibility of product improvement over time expands the parameter region where, in equilibrium, one firm prefers to wait till the second period to release its product whereas the competitor releases in the first period.

[11] Note that even if there is no improvement in the product's functionality, a product that is released later may be perceived by consumers as newer or more state-of-the-art, and such perceptions may increase consumers' willingness to pay for the product.

We will first obtain the firms' equilibrium pricing strategies and profits in each subgame $(R_1, R_2) \in \{(1,1), (2,2), (1,2), (2,1)\}$. Then, we will characterize market conditions under which $(R_1^*, R_2^*) = (1,2)$ is an equilibrium.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{1}, \mathbf{2})$

In this subgame, firm 2 is able to improve its product, increasing consumers' valuation for the product to $V + \mu$. The analysis of this subgame is identical to that in the previous section with vertically differentiated firms. Hence, we will directly provide the equilibrium prices and profits.

Recall that $\dot{\mu}_a = \min\{\frac{12\theta(2-\alpha(2-\delta))+9V(-1+\alpha+\delta-\alpha\delta)}{2(1-\alpha)(1-3\delta)}; 3\theta\}$ and $\ddot{\mu}_a = \min\{\max\{0, \frac{\theta(3-\alpha)}{1-\alpha} + \frac{3V(6-\alpha(7-3\alpha))(1-\delta)}{6\delta+2\alpha(6-8\delta-\alpha(1-3\delta))-10}\}; 3\theta\}$. Further, $\Sigma_{a1} = \{(\theta,\mu) |\ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\mu < \dot{\mu}_a\}$, $\Sigma_{a2} = \{(\theta,\mu) |\ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\mu < \ddot{\mu}_a\}$ and $\Sigma_{a3} = \{(\theta,\mu) |\ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \leq \mu < 3\theta\} \cup \{(\theta,\mu) |\ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\ddot{\mu}_a \leq \mu < 3\theta\}$. Then, from (B38), we know that firm 1's equilibrium first-period price is given by

$$p_{11}^{(1,2)} = \begin{cases} (1-\delta)(V-\theta) - \frac{2\mu\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{2\theta(3-\alpha)(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2))+(1-\alpha)(V(1-\delta)(9-13\alpha+9\delta+3\alpha\delta)-2\mu(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2)))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

The firms' second-period prices on the equilibrium path are given in (B39)-(B40):

$$p_{12}^{(1,2)} = \begin{cases} \theta - \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)(\theta(3-\alpha)-\mu+\mu\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-(1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\theta(6-\alpha(3+\alpha))+\mu(2-(1-\alpha)\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{(1-\alpha)(3\mu(1+\delta)-6V\alpha(1-\delta)-5\mu\alpha(1-\delta))+\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

The firms' equilibrium profits in each period are given in (B41)-(B44).

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(1-\delta)(V-\theta) - \frac{2\mu\alpha\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\alpha(\theta(3-\alpha)+\mu\alpha-\mu)(V(1-\delta)(6-\alpha(7-3\alpha))+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\alpha((1-\alpha)(9V(1-\delta)+2\mu-6\mu\delta)-2\theta(3-\alpha)(1-3\delta))A}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)^2(1-\alpha)(\theta(3-\alpha)+\mu\alpha-\mu)^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))-\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

$$\pi_{21}^{(1,2)} = 0$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

where $A \equiv \frac{(1-\alpha)(V(1-\delta)(9-13\alpha+3(3+\alpha)\delta)-2\mu(1-\alpha+4(1-\alpha)\delta+(3+\alpha)\delta^2))+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$.

Using the profits obtained in each period, we can readily find firm i's overall profit in the subgame with $(R_1, R_2) = (1,2)$: $\pi_i^{(1,2)} = \pi_{i1}^{(1,2)} + \pi_{i2}^{(1,2)}$.

Subgame with $(R_1, R_2) = (2,1)$

Due to symmetry between the firms, the equilibrium profits in this subgame are the reverse of profits in the subgame with $(R_1, R_2) = (1,2)$. That is, $\pi_1^{(2,1)} = \pi_2^{(1,2)}$ and $\pi_2^{(2,1)} = \pi_1^{(1,2)}$.

Subgame where $(R_1, R_2) = (1,1)$

Since both firms release their products in the first period, neither firm's product is improved. Therefore, the analysis of this subgame is identical to the one in the main model (see Lemma 1). Specifically, firm i's equilibrium price in period t is given by $p_{it}^{(1,1)} = \theta$, where $i = 1,2$ and $t = 1,2$. Firm i's corresponding profits in the first and second periods are given by $\pi_{i1}^{(1,1)} = \frac{\alpha\theta}{2}$ and $\pi_{i2}^{(1,1)} = \frac{(1-\alpha)\theta}{2}$, respectively.

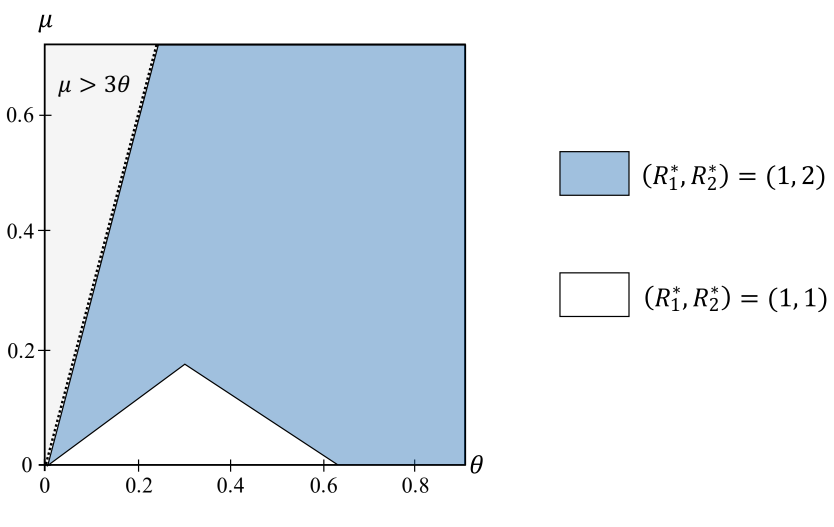
Subgame where $(R_1, R_2) = (2, 2)$

Because both firms wait till the second period to release their products, each firm improves its product, increasing consumers' valuation to $V + \mu$. Since the firms are symmetric, one can easily show that second-period equilibrium prices and profits are given by $p_{12}^{(2,2)} = p_{22}^{(2,2)} = \theta$ and $\pi_{12}^{(2,2)} = \pi_{22}^{(2,2)} = \frac{\theta}{2}$. Because first-period profits are zero, it follows that the firms' overall profits are the same as what they receive in the second period: $\pi_1^{(2,2)} = \pi_2^{(2,2)} = \frac{\theta}{2}$.

Equilibrium Product Release Times

$(R_1^*, R_2^*) = (1, 2)$ is an equilibrium if and only if $\pi_1^{(1,2)} \geq \pi_1^{(2,2)}$ and $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$. $(R_1^*, R_2^*) = (1, 1)$ is an equilibrium if and only if $\pi_1^{(1,1)} \geq \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} \geq \pi_2^{(1,2)}$. Using the subgame equilibrium profits that we characterized in the earlier analysis, we graphically illustrate the firms' equilibrium product release times in Figure B3.

Figure B3 Firms' Equilibrium Product Release Times (R_1^*, R_2^*) [12]



Notice from Figure B3 that as μ increases above zero, in equilibrium, firm 2 releases its product later than firm 1 (i.e., $(R_1^*, R_2^*) = (1, 2)$) not only when the level of horizontal differentiation between the firms is high (i.e., θ is large) but also when the differentiation level is low. Intuitively, when θ is

[12] Figure 5 is drawn using the following parameter values: $V = 10$, $\alpha = 0.3$ and $\delta = 0.9$. Under these values, $\mu < \bar{\mu} \approx 0.73$ ensures that, in equilibrium, at least one of the firms releases its product in the first period. Further, we focus on $\mu < 3\theta$ to guarantee that each firm has a positive market share regardless of product release times.

small, consumers tend to care more about vertical attributes (value differences) than horizontal ones (distance from the firm). Therefore, in such a situation, the increase in product value that firm 2 obtains due to its late product release allows firm 2 to significantly increase its market share, as well as its price. In other words, even a small value of μ can make a big difference when there is not much horizontal differentiation between the firms. For this reason, even when θ is very small, firm 2 may optimally choose to release its product in the second period in order to improve its product and gain an advantage over firm 1. However, it is important to note that when θ is in the lower region, firm 2's late product release is more driven by the direct benefit from product improvement than by the strategic benefit from alleviated price competition. This is because when θ is small, in the first period, firm 1 exploits the lack of horizontal differentiation between the firms and captures a very large fraction of the marker. Hence, very few first-period consumers choose to wait till the second period to buy firm 2's product, which dampens firm 2's incentives to charge a high price, thus weakening the competition-alleviation effect. By contrast, when θ is large, the competition-alleviation-effect is much stronger and plays an important role in inducing firm 2 to release its product later than firm 1 even when the product improvement effect is very weak (i.e., μ is small). Hence, we can make the following distinction: in markets where horizontal differentiation between the firms is low, product improvement effect has a stronger influence on product release timing decisions than price competition alleviation effect. By contrast, when the differentiation level is high, the price competition alleviation effect tends to play a more important role, making late product release preferable even in the absence of any product improvement effects.

5. Consumer Preference Uncertainty

In practice, firms often provide detailed information about their products before launching them. For example, tech companies typically present the products at special events and tech conventions (e.g., the International Consumer Electronics Show). Popular magazines, bloggers and other media outlets closely follow such events, providing consumers with detailed information about the firms' products. However, in some other markets (especially for new product categories), consumers may still lack full knowledge about their tastes or preferences prior to purchasing a product (e.g., Gu and Xie 2013, Shulman et al. 2015). So, we extend our core model by assuming that consumers' knowledge about their own preferences is "noisy." In particular, at the aggregate level, consumers' true preferences are assumed to be uniformly distributed on the unit line $[0,1]$, which is consistent with our core model. However, a randomly chosen consumer does not know her exact location on the line. Instead, each consumer receives a signal $s \in [0,1]$ about her location. Provided that the consumer's true location is $l \in [0,1]$, the distribution of s is as follows: $s = l$ with probability γ, and with probability $1-\gamma$, the signal s is randomly drawn from uniform distribution on $[0,1]$. The parameter γ captures the level of uncertainty that a given consumer faces.

Proposition WA5.1 shows the robustness of our main results in the presence of consumers' preference uncertainty and discusses how such uncertainty affects the firms' decisions and profits.

Proposition WA5.1.

a) There exist $0 < \hat{\alpha} < \bar{\alpha}$, $0 < \hat{\theta} < \bar{\theta}$ and $0 < \hat{\gamma} < 1$ such that if $0 < \alpha < \hat{\alpha}$, $\hat{\theta} < \theta < \bar{\theta}$ and $\hat{\gamma} < \gamma \leq 1$, then, in equilibrium, firm 2 will release its product later than firm 1: $(R_1^*, R_2^*) = (1,2)$.

b) Consumers' preference uncertainty reduces firm 2's gains from releasing its product later, i.e., $\frac{d(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma} \geq 0$.

c) Both firms' equilibrium profits are increasing in γ.

The first part of the proposition shows that firm 2 may prefer to release its product late even if consumers face some uncertainty about their preferences. However, as the second part of the proposition suggests, consumers' preference uncertainty makes it less likely that firm 2 will choose to release its product in the second period rather than in the first period. Put differently, preference uncertainty makes the parameter region where $(R_1^*, R_2^*) = (1, 2)$ smaller. To see the intuition, note that if consumers face preference uncertainty, then fewer consumers will be inclined to wait for firm 2's product. For example, even if a consumer receives a signal $s = 1$, there is still some chance that firm 2's product is not a good fit with the consumer's true preference. The existence of such uncertainty makes the consumer less willing to wait for firm 2's product. Fewer consumers waiting for firm 2's product has two negative effects on firm 2's profits. First, firm 2's equilibrium sales decrease. Second, price competition becomes more intense in the second period because firm 2 gains more incentives to compete for the segment of new consumers entering the market rather than focus on the consumers who have been waiting for its product release. For these reasons, consumers' preference uncertainty makes it less attractive for firm 2 to release its product in the second period. Finally, the third part of Proposition WA5.1 shows that a reduction in consumers' preference uncertainty will benefit not only firm 2 but also firm 1. One reason is that an increase in γ can induce firm 2 to release its product in the second period instead of the first period, which will alleviate price competition and allow firm 1 to capture a bigger market share. Another reason is that an increase in γ makes consumers less price sensitive as consumers become more certain in their preferences for one or the other firm – this makes price competition less intense, allowing both firms to charge higher prices. Therefore, our results suggest that well before releasing their products to the market, the firms would benefit from revealing information about their planned products and provide opportunities to different media outlets to try early samples and share information about these products with consumers, allowing them to better understand their needs and preferences.

PROOF OF PROPOSITION WA5.1. We will first solve for the firms' subgame equilibrium profits for each $(R_1, R_2) \in \{(1,1), (2,2), (1,2), (2,1)\}$.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{1}, \mathbf{2})$

We solve the game by backward induction, starting with the second period. In the second period, the firms compete for the $(1 - \alpha)$ new consumers entering the market, as well as any first-period consumers who decided not to buy a product in the first period. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize the segment of first-period consumers who wait till the second period to buy. In the first period, a consumer who received a signal $s \in [0,1]$ will prefer to wait till the second period to purchase a product if and only if the following condition holds:

$$\delta \max\{V - p_{12}^e - \theta\mathbb{E}(d|s)), V - p_{22}^e - \theta\big(1 - \mathbb{E}(d|s)\big)\} > V - p_{11} - \theta\mathbb{E}(d|s) \tag{B54}$$

Using $\mathbb{E}(d|s) = \gamma s + (1-\gamma)\frac{1}{2}$, one can readily show that (B54) is equivalent to

$$s > \hat{s} \tag{B55}$$

where

$$\hat{s} \equiv \begin{cases} 1 & \text{if } p_{11} \le \frac{2((1-\delta)V + p_{22}^e\delta) - \theta(1+\gamma-(1-\gamma)\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)} & \text{if } \frac{2((1-\delta)V + p_{22}^e\delta) - \theta(1+\gamma-(1-\gamma)\delta)}{2} < p_{11} \le \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)} & \text{if } \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} < p_{11} < \frac{2((1-\delta)V + p_{12}^e\delta) + \theta(\delta(1-\gamma)-1+\gamma)}{2} \\ 0 & \text{if } p_{11} \ge \frac{2((1-\delta)V + p_{12}^e\delta) + \theta(\delta(1-\gamma)-1+\gamma)}{2} \end{cases}$$

Having characterized the first-period consumers who wait till the second period to buy a product, we can now solve for the second-period subgame equilibrium prices that satisfy the rational-expectations condition. The analysis is very similar to the cases *i-iv* in the proof of Lemma 2. To avoid repetitiveness, we directly provide the second-period rational-expectations subgame equilibrium prices:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \theta\gamma & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty] \\ \frac{2p_{11}\alpha - 2V\alpha(1-\delta) + \theta(3\gamma(1+\delta) + \alpha(1-2\gamma-\delta))}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4p_{11}\alpha - (4V\alpha - \theta(2\alpha(1-\gamma)+3\gamma))(1-\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B56}$$

$$p_{22}^{(1,2)} = p_{22}^e = \begin{cases} \theta\gamma & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty] \\ \frac{4p_{11}\alpha - 4V\alpha(1-\delta) + \theta(3\gamma(1+\delta) + \alpha(2-\gamma-2\delta-\gamma\delta))}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2p_{11}\alpha - 2V\alpha(1-\delta) + \theta(\alpha+3\gamma-\alpha\gamma-\alpha\delta-3\gamma\delta+3\alpha\gamma\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B57}$$

where $\underline{\rho} \equiv \frac{(2V-\theta-\theta\gamma)(1-\delta)}{2}$,

$\tilde{\rho} \equiv \frac{2V(6-\alpha(7-3\alpha))(1-\delta) + \theta(\alpha(7-3\alpha(1-\gamma)-5\gamma) - 6(1-\delta) + (12\gamma - \alpha(7+11\gamma-\alpha(3+\gamma)))\delta)}{2(6-\alpha(7-3\alpha))}$,

$\hat{\rho} \equiv \frac{2V(6+\alpha)(1-\delta)-\theta(\alpha(1-\gamma-\delta+7\gamma\delta)+6(1-\delta-2\gamma\delta))}{2(6+\alpha)}$ and $\bar{\rho} \equiv \frac{2V(1-\delta)-\theta(1-\gamma-\delta-\gamma\delta)}{2}$. Note that similar to the main model, we assume that $\underline{\delta} < \delta < 1$ and $0 \le \alpha < \bar{\alpha}$ to ensure the existence of a subgame equilibrium for every $p_{11} \ge 0$, where $\underline{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

Using the second-period subgame equilibrium prices in (B56)-(B57), we obtain the firms' second-period subgame equilibrium profits:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{\theta\gamma(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2V\alpha(1-\delta)-2p_{11}\alpha-3\theta\gamma(1+\delta)-\theta\alpha(1-2\gamma-\delta))^2}{2\theta\gamma(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4p_{11}\alpha-(1-\delta)(4V\alpha-\theta(2\alpha(1-\gamma)+3\gamma)))^2}{2\theta\gamma(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta\gamma}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B58}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{\theta\gamma(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4V\alpha(1-\delta)-4p_{11}\alpha-3\theta\gamma(1+\delta)-\theta\alpha(2-\gamma-(2+\gamma)\delta))^2}{2\theta\gamma(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2p_{11}\alpha-2V\alpha(1-\delta)+\theta(\alpha+3\gamma(1-\delta)-\alpha\delta-\alpha\gamma(1-3\delta)))^2}{2\theta\gamma(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta\gamma}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B59}$$

Firm 1's first-period profit is given by $\pi_{11} = \alpha\, \hat{s}\, p_{11}$. Plugging the prices from (B56)-(B57) into the expression for $\hat{s}$, we can obtain π_{11}:

$$\pi_{11} = \begin{cases} \alpha\, p_{11} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha \frac{6V(1-\alpha)(1-\delta)-6p_{11}(1-\alpha)+\theta(\alpha(3-3\gamma-3\delta-\gamma\delta)+3(\gamma+\delta+3\gamma\delta-1))}{2\theta\gamma(3(1+\delta)-\alpha(3-\delta))} p_{11} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha \frac{3(2V(1-\delta)-2p_{11}+\theta(\gamma+\delta+\gamma\delta-1))}{2\theta\gamma(3-(3-4\alpha)\delta)} p_{11} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B60}$$

In the first period, firm 1 chooses its price to maximize its overall profit $\pi_1 = \pi_{11} + \pi_{12}^{(1,2)}$. Similar analysis as in the proof of Lemma 3 shows that firm 1's optimal price belongs to the interval $[\underline{\rho}, \tilde{\rho}]$ and is given by:

$$p_{11}^{(1,2)} = \begin{cases} \frac{(2V-\theta-\theta\gamma)(1-\delta)}{2} & \text{if } 0 < \gamma \le \underline{\gamma} \\ \frac{(1-\alpha)\left((2V-\theta)(9-13\alpha)+\theta(21-17\alpha)\gamma\right)+16(1-\alpha)(2V\alpha+3\theta\gamma-\theta\alpha)\delta-(3+\alpha)(3(2V-\theta)(1-\alpha)-\theta(9-\alpha)\gamma)\delta^2}{4(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \underline{\gamma} < \gamma \le \bar{\gamma} \\ \frac{2V(6-\alpha(7-3\alpha))(1-\delta)+\theta(\alpha(7-3\alpha(1-\gamma)-5\gamma)-6(1-\delta)+(12\gamma-\alpha(7+11\gamma-\alpha(3+\gamma)))\delta)}{2(6-\alpha(7-3\alpha))} & \text{if } \bar{\gamma} < \gamma \le 1 \end{cases} \tag{B61}$$

where $\underline{\gamma} = \min\{1, \frac{3(2V-\theta)(1-\alpha)(1-\delta)}{\theta(13+3\delta-\alpha(13-5\delta))}\}$ and $\bar{\gamma} = \min\{1, \frac{3(2V-\theta)(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{\theta(42-18\delta+\alpha(69\delta-53+\alpha(6-38\delta+\alpha(5+3\delta))))}\}$.

Upon plugging $p_{11}^{(1,2)}$ into equations (B58)-(B60), we find the firms' profits on the equilibrium path.

$$\pi_{11}^{(1,2)} =$$

$$\begin{cases} \frac{\alpha(2V-\theta-\theta\gamma)(1-\delta)}{2} & \text{if } 0<\gamma\leq\underline{\gamma} \\ \frac{\alpha((1-\alpha)(16(\theta\alpha-2V\alpha-3\theta\gamma)\delta-(2V-\theta)(9-13\alpha)-\theta(21-17\alpha)\gamma)+(3+\alpha)(3(2V-\theta)(1-\alpha)-\theta(9-\alpha)\gamma)\delta^2)E}{(1-\alpha)\gamma(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\gamma}<\gamma\leq\bar{\gamma} \\ \frac{(3-\alpha)\alpha(2V(6-7\alpha+3\alpha^2)(1-\delta)-\theta(\alpha^2(3-3\gamma-3\delta-\gamma\delta)+6(1-\delta-2\gamma\delta)-\alpha(7-5\gamma-7\delta-11\gamma\delta)))}{2(6-7\alpha+3\alpha^2)^2} & \text{if } \bar{\gamma}<\gamma\leq 1 \end{cases}$$

(B62)

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{\theta(1-\alpha)\gamma}{2} & \text{if } 0<\gamma\leq\underline{\gamma} \\ \frac{(6V(1-\alpha)\alpha(1-\delta)+\theta(\alpha^2(3-\gamma(9-\delta)-3\delta)-18\gamma(1+\delta)-3\alpha(1-\delta-3\gamma(3+\delta))))^2}{8\theta(1-\alpha)\gamma(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\gamma}<\gamma\leq\bar{\gamma} \\ \frac{\theta(1-\alpha)(6-5\alpha+\alpha^2)^2\gamma}{2(6-7\alpha+3\alpha^2)^2} & \text{if } \bar{\gamma}<\gamma\leq 1 \end{cases} \tag{B63}$$

$$\pi_{21}^{(1,2)} = 0 \tag{B64}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta\gamma}{2} & \text{if } 0<\gamma\leq\underline{\gamma} \\ \frac{(6V(1-\alpha)\alpha(1-\delta)-\theta(1-\alpha)(9\gamma+\alpha(3+2\gamma))-\theta\delta(9\gamma-\alpha(3-3\alpha+3\gamma-2\alpha\gamma)))^2}{2\theta(1-\alpha)\gamma(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\gamma}<\gamma\leq\bar{\gamma} \\ \frac{\theta(1-\alpha)(6-3\alpha-\alpha^2)^2\gamma}{2(6-7\alpha+3\alpha^2)^2} & \text{if } \bar{\gamma}<\gamma\leq 1 \end{cases} \tag{B65}$$

where $E = \frac{-18V(1-\alpha)(1-\delta)+\theta(9-9\alpha+3\gamma+5\alpha\gamma-3(3+9\gamma-\alpha(3+\gamma))\delta)}{16\theta}$. The firms' overall profits in the subgame $(R_1, R_2) = (1, 2)$ are $\pi_1^{(1,2)} = \pi_{11}^{(1,2)} + \pi_{12}^{(1,2)}$ and $\pi_2^{(1,2)} = \pi_{21}^{(1,2)} + \pi_{22}^{(1,2)} = \pi_{22}^{(1,2)}$. Note that since the firms are symmetric, their profits in the subgame $(R_1, R_2) = (2, 1)$ are given by $\pi_1^{(2,1)} = \pi_2^{(1,2)}$ and $\pi_2^{(2,1)} = \pi_1^{(1,2)}$.

***Subgame with* $(R_1, R_2) = (1, 1)$ or $(R_1, R_2) = (2, 2)$**

The analysis of the subgames with $(R_1, R_2) = (1, 1)$ or $(R_1, R_2) = (2, 2)$ follows similar steps as in the main model. Namely, if $(R_1, R_2) = (2, 2)$, then consumers who enter the market in the first period wait till the second period to buy a product. Hence, in the second period, firms compete for a unit measure of consumers. A consumer who received a signal $s \in [0,1]$ will prefer to buy firm 1's product if and only if

$$V - p_{12} - \theta\mathbb{E}(d|s) > V - p_{22} - \theta\big(1 - \mathbb{E}(d|s)\big) \tag{B66}$$

where $\mathbb{E}(d|s)$ is the consumer's expected location on the Hotelling line conditional on the signal s. Since the consumer does not know the actual value of l, she makes her purchase decision to maximize expected utility. Note that $\mathbb{E}(d|s) = \gamma s + (1-\gamma)\frac{1}{2}$. Using this expression in (B66), it follows that the consumer will buy from firm 1 if and only if $s \le \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}$. Note that the probability of $s \le \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}$ is $\frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}$. Hence, firms' profits are given by $\pi_{12} = \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma} p_{12}$ and $\pi_{22} = (1 - \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}) p_{22}$. Solving the first-order conditions, we find the firms' second-period equilibrium prices: $p_{12}^{(2,2)} = p_{22}^{(2,2)} = \theta\gamma$. The firms' equilibrium profits are

$$\pi_1^{(2,2)} = \pi_2^{(2,2)} = \frac{\theta\gamma}{2} \tag{B67}$$

The analysis of the subgame with $(R_1, R_2) = (1,1)$ follows similar steps as in the main model. Specifically, one can show that firm i's equilibrium price are given by $p_{i1}^{(1,1)} = p_{i2}^{(1,1)} = \theta\gamma$, with corresponding profits of $\pi_{i1}^{(1,1)} = \alpha\frac{\theta\gamma}{2}$ and $\pi_{i2}^{(1,1)} = (1-\alpha)\frac{\theta\gamma}{2}$ in the first and second periods, respectively. Thus, the firms' overall profits in the $(R_1, R_2) = (1,1)$ subgame are given by

$$\pi_1^{(1,1)} = \pi_2^{(1,1)} = \frac{\theta\gamma}{2} \tag{B68}$$

Having characterized the firms' subgame equilibrium profits, we proceed to finding the firms' equilibrium product release times. A similar proof as in Proposition 1 shows that a pure strategy equilibrium in product release times exists and in equilibrium, at least one of the firms will release its product in the first period. Hence, in equilibrium, either $(R_1, R_2) = (1,1)$ or $(R_1, R_2) = (1,2)$.

Let us first demonstrate that $\frac{d\pi_i^{(R_1,R_2)}}{d\gamma} > 0$ for all (R_1, R_2). If $(R_1, R_2) = (1,1)$, then $\pi_i^{(1,1)} = \frac{\theta\gamma}{2}$, and it is clear that $\frac{d\pi_i^{(1,1)}}{d\gamma} = \frac{\theta}{2} > 0$. Similarly, if $(R_1, R_2) = (2,2)$, then $\frac{d\pi_i^{(2,2)}}{d\gamma} = \frac{\theta}{2} > 0$. Next, if $(R_1, R_2) = (1,2)$, then we will analyze each of the possible intervals to which γ can belong: $(0, \underline{\gamma}]$, $(\underline{\gamma}, \bar{\gamma}]$ and $(\bar{\gamma}, 1]$.

First, if $\gamma \in (0, \underline{\gamma}]$, then $\frac{d\pi_1^{(1,2)}}{d\gamma} = \frac{\theta(1-\alpha(2-\delta))}{2} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\gamma} = \frac{\theta(1-\alpha)}{2} > 0$, where the first inequality follows because $\bar{\delta} < \delta < 1$ and $0 \le \alpha < \bar{\alpha}$.

Second, if $\gamma \in (\underline{\gamma}, \bar{\gamma}]$, then note that $\frac{d^2\pi_1^{(1,2)}}{d\gamma^2} = \frac{9(2V-\theta)^2(1-\alpha)\alpha(1-\delta)^2}{8\theta\gamma^3(9-11\alpha+3(3+\alpha)\delta)} > 0$ and $\frac{d\pi_1^{(1,2)}}{d\gamma}|_{\gamma=\underline{\gamma}} = \frac{\theta(1-\alpha(2-\delta))}{2} > 0$, which implies that $\frac{d\pi_1^{(1,2)}}{d\gamma} > 0$ for all $\gamma \in (\underline{\gamma}, \bar{\gamma}]$. Next, $\frac{d^2\pi_2^{(1,2)}}{d\gamma^2} = \frac{9(2V-\theta)^2(1-\alpha)\alpha^2(1-\delta)^2}{\theta\gamma^3(9-11\alpha+3(3+\alpha)\delta)^2} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\gamma}|_{\gamma=\underline{\gamma}} = \frac{\theta(9(1+\delta)+\alpha(6-\alpha(15-7\delta)))}{18-22\alpha+6(3+\alpha)\delta} > 0$, which implies that $\frac{d\pi_2^{(1,2)}}{d\gamma} > 0$ for all $\gamma \in (\underline{\gamma}, \bar{\gamma}]$.

Third, if $\gamma \in (\bar{\gamma}, 1]$, then $\frac{d\pi_1^{(1,2)}}{d\gamma} = \frac{\theta(3-\alpha)(12+\alpha(4(-7+3\delta)+\alpha(18-11\delta-\alpha(5-\alpha-\delta))))}{2(6-\alpha(7-3\alpha))^2} > 0$, where the inequality follows because $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$. Similarly, $\frac{d\pi_2^{(1,2)}}{d\gamma} = \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} > 0$.

Hence, the above three cases all indicate that $\frac{d\pi_i^{(1,2)}}{d\gamma} > 0$. Since $\pi_i^{(1,2)}$ is a continuous function that is differentiable at all but a finite number of points, it follows that $\pi_i^{(1,2)}$ is increasing in γ. The proof for the subgame $(R_1, R_2) = (2, 1)$ is identical to the one for $(R_1, R_2) = (1, 2)$.

Next, we will show that if $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then there exists $\hat{\gamma} \in (0,1)$ such that $(R_1, R_2) = (1, 2)$ is an equilibrium if and only if $\gamma > \hat{\gamma}$. As we discussed earlier, firm 1 will not want to deviate from $R_1 = 1$. Hence, we need to show that firm 2 will not want to deviate from $R_2 = 2$, which is the case when $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. Note that if $\gamma \in [0, \underline{\gamma}]$, then $\pi_2^{(1,2)} < \pi_2^{(1,1)}$. Hence, let us focus on $\gamma \in (\underline{\gamma}, 1]$. When $\gamma \in (\underline{\gamma}, \bar{\gamma}]$, then $\frac{d(\pi_2^{(1,2)}-\pi_2^{(1,1)})}{d\gamma} > 0$ because $\frac{d^2(\pi_2^{(1,2)}-\pi_2^{(1,1)})}{d\gamma^2} = \frac{9(2V-\theta)^2(1-\alpha)\alpha^2(1-\delta)^2}{\theta\gamma^3(9-11\alpha+3(3+\alpha)\delta)^2} > 0$ and $\frac{d(\pi_2^{(1,2)}-\pi_2^{(1,1)})}{d\gamma}|_{\gamma=\underline{\gamma}} = \frac{\theta\alpha(17-3\delta-\alpha(15-7\delta))}{18-22\alpha+6(3+\alpha)\delta} > 0$, where the last inequality holds since $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$. When $\gamma \in (\bar{\gamma}, 1]$, we have $\frac{d(\pi_2^{(1,2)}-\pi_2^{(1,1)})}{d\gamma} = \frac{\theta}{2}(\frac{(1-\alpha)(6-\alpha(3+\alpha))^2}{(6-\alpha(7-3\alpha))^2} - 1) > 0$ if and only if $\alpha < \hat{\alpha}$, where $\hat{\alpha} \approx 0.32$ satisfies $12 - \hat{\alpha}(52 - (3 - \hat{\alpha})\hat{\alpha}(17 + \hat{\alpha})) = 0$. Note that this cutoff is the same as in Proposition 1 (see the proof). Let us analyze the two possible cases: $\alpha < \hat{\alpha}$ and $\hat{\alpha} < \alpha < \bar{\alpha}$.

If $\alpha < \hat{\alpha}$, then we showed in the previous paragraph that $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing in γ at any $\gamma \in (\underline{\gamma}, 1]$. Hence, the maximal value is obtained at $\gamma = 1$. We find that $(\pi_2^{(1,2)} - \pi_2^{(1,1)})_{\gamma=1} > 0$ if and only if $\theta > \hat{\theta}$, where

$\hat{\theta} = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$ is the same as in Proposition 1. Assuming that $\theta > \hat{\theta}$, it is straightforward to show that $\pi_2^{(1,2)} - \pi_2^{(1,1)} > 0$ if and only if $\gamma > \hat{\gamma}$, where

$$\hat{\gamma} = \frac{3(2V-\theta)(1-\delta)((1-\alpha)^{3/2}(9-11\alpha+3(3+\alpha)\delta)+(1-\alpha)(\alpha(-7-2\alpha(1-\delta)-3\delta)+9(1+\delta)))}{\theta(4\alpha^3(1-\delta)^2+9(1+\delta)(17-3\delta)+\alpha^2(149-\delta(82+3\delta))+18\alpha(-17+\delta(-2+5\delta)))} \quad \text{(B69)}$$

Note that $\hat{\gamma} < 1$.

If $\hat{\alpha} < \alpha < \bar{\alpha}$, then $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing in γ at $\gamma \in (\underline{\gamma}, \bar{\gamma}]$ and decreasing at $(\underline{\gamma}, 1]$. Hence, the maximal value is obtained when $\gamma = \bar{\gamma}$. One can show that $(\pi_2^{(1,2)} - \pi_2^{(1,1)})_{\gamma=\bar{\gamma}} < 0$ when $\alpha > \hat{\alpha}$. Therefore, it follows that $\pi_2^{(1,2)} - \pi_2^{(1,1)} < 0$ for all $\gamma \in [0,1]$.

Finally, let us prove that $\frac{d(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma} \geq 0$. We already proved that $(R_1^*, R_2^*) = (1,2)$ if and only if $\alpha < \hat{\alpha}$, $\theta > \hat{\theta}$ and $\gamma > \hat{\gamma}$. Further, we showed that if $\alpha < \hat{\alpha}$, then $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing in γ at any point $\gamma \in (\underline{\gamma}, 1]$. Since $\hat{\gamma} \in (\underline{\gamma}, 1]$, it follows that $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing at any $\gamma > \hat{\gamma}$. Hence, when $(R_1^*, R_2^*) = (1,2)$, firm 2 is better off as γ increases. ■

6. Firms' discount factor

To simplify the analysis, in our main model, we assumed that the firms' discount factor is one. We will demonstrate that our qualitative results can still hold even if the firms' discount factor is below one. Let us denote the firms' discount factor by $\delta_f \in (0,1)$. To make the algebra less cumbersome, we consider a numerical example with $V = 10$, $\delta = 0.95$ and $\alpha = 0.2$. To find the equilibrium outcome, we need to find the firms' equilibrium profits in each subgame corresponding to the firms' product release times $(R_1, R_2) \in \{(1,1),(2,2),(1,2),(2,2)\}$. Let $\pi_i^{(R_1,R_2)}$ represent the discounted sum of firm i's first- and second-period profits in the subgame with (R_1, R_2), i.e., $\pi_i^{(R_1,R_2)} = \pi_{i1}^{(R_1,R_2)} + \delta_f \pi_{i2}^{(R_1,R_2)}$.

First, let us analyze the subgame where $(R_1, R_2) = (1,2)$. The discount factor δ_f clearly does not affect the firms' second-period pricing decisions, and the analysis for finding the second-period subgame equilibrium prices is the same as in our core model. More specifically, the firms' second-period prices and profits in a rational-expectations subgame equilibrium are the same as in Lemma 2 in the main paper. The corresponding second-period profits are provided in the equations (A7) and (A8) in the Web Appendix A, which we reproduce below for convenience:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4\alpha(V(1-\delta)-p_{11})-\theta(3(1+\delta)+\alpha(1-3\delta)))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}$$

where recall that $\underline{\rho} = (1-\delta)(V-\theta)$, $\tilde{\rho} \equiv (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv (1-\delta)V + \delta\theta$.

In the first period, firm 1 chooses its first-period price p_{11} to maximize the discounted sum of its first- and second-period profits, $\pi_1^{(1,2)} = S_{11}^{(1,2)} p_{11} + \delta_f \pi_{12}^{(1,2)}$, where $S_{11}^{(1,2)}$ is firm 1's first-period unit sales as in the equation (A11) in the Web Appendix A.

$\pi_1^{(1,2)} =$

$$\begin{cases} \alpha\, p_{1_1} + \delta_f \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha \frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} p_{1_1} + \delta_f \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha \frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} p_{1_1} + \delta_f \frac{(2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \delta_f \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}$$

After plugging in the numerical values for the parameters, $\pi_1^{(1,2)}$ becomes

$$\pi_1^{(1,2)} = \begin{cases} \frac{1}{4} p_{11} + \delta_f \frac{3}{8}\theta & \text{if } p_{11} \in [0, \frac{10-\theta}{20}) \\ \frac{3p_{11}(10-20p_{11}+19\theta)}{88\,\theta} p_{11} + \delta_f \frac{(10-20p_{11}-3\theta)^2}{968\,\theta} & \text{if } p_{11} \in [\frac{10-\theta}{20}, \frac{71+121\,\theta}{142}] \\ \frac{45-90p_{11}+209\theta}{854\,\theta} p_{11} + \delta_f \frac{3(20-40p_{11}-429\theta)^2}{1458632\,\theta} & \text{if } p_{11} \in (\frac{71+121\,\theta}{142}, \frac{10+19\,\theta}{20}] \\ \delta_f \frac{1}{2}\theta & \text{if } p_{11} \in [\frac{10+19\,\theta}{20}, \infty) \end{cases}$$

Firm 1 chooses its first-period price p_{11} to maximize $\pi_1^{(1,2)}$. Consistent with the main model, we focus on $\theta < \frac{V(1-\delta)}{2-\delta} = \frac{10}{21}$. One can show that the optimal p_{11} is as follows:

$$p_{11}^* = \begin{cases} \frac{10-\theta}{20} & \text{if } \theta \in (0, \frac{45}{218+60\,\delta_f}) \\ \frac{427(45+209\,\theta)-60(20-429\,\theta)\delta_f}{76860-2400\,\delta_f} & \text{if } \theta \in [\frac{45}{218+60\,\delta_f}, \min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}) \\ \frac{71+121\,\theta}{142} & \text{if } \theta \in [\min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}, \frac{10}{21}) \end{cases}$$

Upon plugging p_{11}^* into $\pi_1^{(1,2)}$ and $\pi_2^{(1,2)} = \delta_f \pi_{22}^{(1,2)}$, we find the firms' subgame equilibrium profits:

$$\pi_1^{(1,2)} = \begin{cases} \frac{10-\theta(1-30\,\delta_f)}{80} & \text{if } \theta \in (0, \frac{45}{218+60\,\delta_f}) \\ \frac{(45+209\theta)^2-330\theta(20-429\theta)\delta_f}{240\theta(1281-40\,\delta_f)} & \text{if } \theta \in [\frac{45}{218+60\,\delta_f}, \min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}) \\ \frac{11(284+11\theta(44+147\,\delta_f))}{40328} & \text{if } \theta \in [\min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}, \frac{10}{21}) \end{cases}$$

$$\pi_2^{(1,2)} = \begin{cases} \frac{3\theta\,\delta_f}{8} & \text{if } \theta \in (0, \frac{45}{218+60\,\delta_f}) \\ \frac{25\delta_f(36-\theta(943+24\,\delta_f))^2}{24\theta(1281-40\delta_f)^2} & \text{if } \theta \in [\frac{45}{218+60\,\delta_f}, \min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}) \\ \frac{20667\,\theta\,\delta_f}{40328} & \text{if } \theta \in [\min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}, \frac{10}{21}) \end{cases}$$

When $(R_1, R_2) = (2, 1)$, the symmetry between the firms implies that $\pi_1^{(2,1)} = \pi_2^{(1,2)}$ and $\pi_2^{(2,1)} = \pi_1^{(1,2)}$.

When $(R_1, R_2) = (1, 1)$, given our focus on θ not too high, a similar argument as in the main model can show that, in equilibrium, consumers who enter the market in the first period will buy a product in that period instead of waiting till the second period to buy. In this situation, the firms' subgame equilibrium profits are given by $\pi_{11}^{(1,1)} = \pi_{12}^{(1,1)} = \alpha \frac{\theta}{2}$ and $\pi_{12}^{(1,1)} = \pi_{22}^{(1,1)} = (1-\alpha)\frac{\theta}{2}$. The discounted sum of the profits is given by $\pi_1^{(1,1)} = \pi_2^{(1,1)} = \alpha \frac{\theta}{2} + \delta_f (1-\alpha)\frac{\theta}{2}$.

Finally, when $(R_1, R_2) = (2, 2)$, clearly, the firms' discount factor does not influence their pricing decisions in the second period. Therefore, the subgame equilibrium prices are the same as in the main model. The corresponding profits are given by $\pi_1^{(2,2)} = \pi_2^{(2,2)} = \delta_f \frac{\theta}{2}$.

Now, let us find the firms' equilibrium product release times. $(R_1, R_2) = (1, 2)$ is an equilibrium if and only if $\pi_1^{(1,2)} \geq \pi_1^{(2,2)}$ and $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$. Because $\pi_1^{(1,2)} \geq \pi_1^{(2,2)}$, firm 1 will not benefit by deviating from $R_1 = 1$ to $R_2 = 2$. It remains to check whether $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$. Straightforward algebra shows that $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$ holds when $\hat{\delta}_f < \delta_f \leq 1$ and $\hat{\theta} < \theta < \frac{10}{21}$, where $\hat{\theta} \equiv \frac{180\delta_f}{-1281\sqrt{3\delta_f(1+3\delta_f)}+5\delta_f(943+24\delta_f+8\sqrt{3}\sqrt{\delta_f(1+3\delta_f)})}$ and $\hat{\delta}_f \equiv \frac{5041}{5544} \approx 0.91$.

Recall that we assumed the consumers' discount factor is $\delta = 0.95$. Since $\hat{\delta}_f < \delta$, the above analysis shows that late product release can be optimal for firm 2 even if the firms' discount factor is less than that of consumers. ■

7. ALTERNATIVE MODELING OF CONSUMPTION VALUE

As a robustness check, we show that our main results continue to hold when we explicitly model the stream of consumption values that a durable product provides to consumers. Specifically, in section 7.A., we assume that upon purchasing the firm's product in period t, the cosumer obtains a value v in each period up to some period $T > 2$. Thus, the utility from purchasing the firm's product is given by $V_{it} - p_{it} - \theta d_i$, where $V_{it} = \sum_{j=t}^{T} \delta^{j-t} v$. In section 7.B., we show that our results will hold if the consumer receives $v - \theta d_i$ in each period, i.e., horizontal preferences influence the consumer's consumption value in each period. All remaining assumption are the same as in our main model. Further, we focus on $\theta < \frac{V_2}{2}$ to ensure that the market is fully covered so that, in equilibrium, each consumer makes a purchase either in the first or in the second period.

7.A. $V_{it} = \sum_{j=t}^{T} \delta^{j-t} v$

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{1}, \mathbf{1})$ **or** $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{2}, \mathbf{2})$

The analyses of these subgames is very similar to those in the main model. One can show that the firms' subgame equilibrium profits are given by $\pi_i^{(1,1)} = \pi_i^{(2,2)} = \frac{\theta}{2}$ for each firm $i = 1, 2$.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{1}, \mathbf{2})$

We solve the game using backward induction. In the second period, the firms compete for $(1 - \alpha)$ new consumers entering the market, as well as those first-period consumers who had decided not to buy in the first period but wait. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize those first-period consumers who will prefer not to buy in the first period but wait. In the main text, we showed that a consumer will choose to wait if

$$\max\{\delta(V_2 - p_{12}^e - \theta d), \delta(V_2 - p_{22}^e - \theta(1 - d))\} > V_1 - p_{11} - \theta d,$$

which reduces to

$$d > \hat{d} \tag{B69}$$

The cutoff $\hat{d}$ is given by

$$\hat{d} \equiv \begin{cases} 1 & \text{if } p_{11} \leq v + \delta p_{22}^e - \theta \\ \frac{v - p_{11} + \delta p_{22}^e + \delta\theta}{\theta(1+\delta)} & \text{if } v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2} \\ \frac{v - p_{11} + p_{12}^e \delta}{\theta(1-\delta)} & \text{if } \frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2} < p_{11} < v + \delta p_{12}^e \\ 0 & \text{if } p_{11} \geq v + \delta p_{12}^e \end{cases} \quad \text{(B70)}$$

Note that $\hat{d}$ depends on the prices p_{12}^e and p_{22}^e that consumers expect the firms to charge in the second period, and it takes a different functional form depending on p_{11}. We will separately analyze each of the four intervals into which p_{11} can fall. In each case, we will first find the firms' second-period subgame equilibrium prices p_{12}^* and p_{22}^* as a function of p_{12}^e, p_{22}^e and p_{11}. Since we are looking for a rational-expectations equilibrium, we will solve the equations $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ to find the price pair (p_{12}^e, p_{22}^e) that satisfies the condition for rational-expectations: $(p_{12}^e, p_{22}^e) = (p_{12}^*, p_{22}^*)$.

Case i: $p_{11} \leq v + \delta p_{22}^e - \theta$. In this case, $\hat{d} = 1$, i.e., all first-period consumers buy firm 1's product in the first period. Hence, in the second-period, firms compete for the new consumers entering the market. Firms 1 and 2 choose prices to maximize $(1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta}p_{12}$ and $(1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta})p_{22}$, respectively. Solving the first-order conditions, we find that $p_{12}^* = p_{22}^* = \theta$. Rational expectations require that $p_{i2}^e = p_{i2}^* = \theta$. Substituting $p_{i2}^e = \theta$ into inequality $p_{11} \leq v + \delta p_{22}^e - \theta$, it follows that the condition for case *i* is satisfied when $p_{11} \leq \underline{\rho}$, where $\underline{\rho} \equiv v - \theta(1-\delta)$.

Case ii: $v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2}$. In this case, we have $x = \frac{v - p_{11} + \delta p_{22}^e + \delta\theta}{\theta(1+\delta)}$. In the first period, consumers with $d \leq \hat{d}$ bought firm 1's product, while the remaining consumers with $d > \hat{d}$ preferred to wait till the second period in order to buy firm 2's product. Since we are solving for a rational-expectations equilibrium, the firms' second-period subgame equilibrium prices must indeed be such that consumers with $d > \hat{d}$ actually buy firm 2's product. This happens when firm 2's second-period price is sufficiently low: $p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$. In addition, to have a positive market share, firm 1's price must not be too high: $p_{12} < p_{22} + \theta$, or equivalently, $p_{22} > p_{12} - \theta$. When $p_{12} - \theta \leq p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$, the firms' second-period profits are

$$\pi_{12} = (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta}p_{12}$$

$$\pi_{22} = (\alpha(1-\hat{d}) + (1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}))p_{22}$$

Solving the first-order conditions ($\frac{d\pi_{12}}{dp_{12}} = 0$ and $\frac{d\pi_{22}}{dp_{22}} = 0$), we obtain the equilibrium prices: $p_{12}^* = \frac{\theta(3(1+\delta)-\alpha-3\alpha\delta)-2\alpha(v-p_{11}+p_{22}^e\delta)}{3(1-\alpha)(1+\delta)}$ and $p_{22}^* = \frac{\theta(3+\alpha)+3\theta(1-\alpha)\delta+4\alpha(p_{11}-v-p_{22}^e\delta)}{3(1-\alpha)(1+\delta)}$. Solving $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ together, we find that $p_{12}^e = \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^e = \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$. Plugging p_{12}^e and p_{22}^e into the expressions for p_{12}^*, p_{22}^* and $\hat{d}$, we verify that $p_{12}^* = p_{12}^e = \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$, $p_{22}^* = p_{22}^e = \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $\hat{d} = \frac{3v(1-\alpha)+2\theta(3-\alpha)\delta-3p_{11}(1-\alpha)}{\theta(3(1+\delta)-\alpha(3-\delta))}$.

We need to check that case *ii*'s condition $v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v-(1-\delta)(p_{22}^e+\theta)+(1+\delta)p_{12}^e}{2}$ is satisfied. Using the expressions for p_{12}^e and p_{22}^e, one can readily show that $v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v-(1-\delta)(p_{22}^e+\theta)+(1+\delta)p_{12}^e}{2}$ is equivalent to $\underline{\rho} < p_{11} \leq \frac{6v-4v\alpha-\theta(3-\alpha)(1-3\delta)}{6-4\alpha}$, where recall that $\underline{\rho} = v - \theta(1-\delta)$. Note that when $\underline{\rho} < p_{11} \leq \frac{6v-4v\alpha-\theta(3-\alpha)(1-3\delta)}{6-4\alpha}$, one can show that the prices p_{12}^* and p_{22}^* satisfy $p_{12}^* - \theta \leq p_{22}^* \leq p_{12}^* - \theta + 2\theta\hat{d}$, which ensures that both firms have positive market shares and that consumers with $d > \hat{d}$ indeed buy firm 2's product.

Firm 1's profit function is not necessarily quasi-concave because firm 1 may potentially want to decrease its price to capture some of the consumers with $d > \hat{d}$. Specifically, to capture some of the consumers located on $(\hat{d}, 1]$, firm 1 will need to deviate to a price $p_{12}' < p_{22}^* + \theta - 2\theta\hat{d}$, in which case firm 1's deviation profit will be $\pi_{12}' = (\alpha(\frac{\theta-p_{12}'+p_{22}^*}{2\theta} - \hat{d}) + (1-\alpha)\frac{\theta-p_{12}'+p_{22}^*}{2\theta})p_{12}'$. Note that π_{12}' is concave. Evaluating the left-derivative of π_{12}' at the point $p_{12}' = p_{22}^* + \theta - 2\theta\hat{d}$, we find that $\frac{\partial_-\pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} = \frac{v(6-7\alpha+3\alpha^2)-p_{11}(6-7\alpha+3\alpha^2)-\theta(3-\alpha)(1-3\delta+2\alpha\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))}$. One can readily show that $\frac{\partial_-\pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} > 0$ if and only if $p_{11} < \tilde{\rho}$, where $\tilde{\rho} \equiv v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ satisfies $\underline{\rho} < \tilde{\rho} < \frac{6v-4v\alpha-\theta(3-\alpha)(1-3\delta)}{6-4\alpha}$. Since π_{12}' is concave, $\frac{\partial_-\pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} > 0$ implies that π_{12} is increasing in p_{12}' at any $p_{12}' < p_{22}^* + \theta - 2\theta\hat{d}$. Since firm 1's profit function is continuous at the point $p_{22}^* + \theta - 2\theta\hat{d}$, it follows that a deviation to $p_{12}' < p_{22}^* + \theta - 2\theta\hat{d}$ cannot be profitable as long as $p_{11} < \tilde{\rho}$.

Case iii: $\frac{2v-(1-\delta)(p_{22}^e+\theta)+(1+\delta)p_{12}^e}{2} < p_{11} < v + \delta p_{12}^e$. In this case, $\hat{d} = \frac{v-p_{11}+p_{12}^e\delta}{\theta(1-\delta)}$. In the first period, consumers with $d < \hat{d}$ bought firm 1's product. The marginal consumer with $d = \hat{d}$ preferred to wait till the second period to buy firm 1's product. So, in a rational-expectations equilibrium, firm 1 must have a positive market share among consumers who are located on the interval $(\hat{d}, 1]$, which happens when firm 1's second-period price is low enough: $p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$. Also, firm 2 will have a positive market share only if $p_{22} < p_{12} + \theta$, or equivalently, if $p_{12} > p_{22} - \theta$. When $p_{22} - \theta < p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$, the firms' second-period profit functions are as follows:

$$\pi_{12} = (\alpha(\frac{\theta - p_{12} + p_{22}}{2\theta} - \hat{d}) + (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta})p_{12}$$

$$\pi_{22} = (\alpha\frac{\theta - p_{22} + p_{12}}{2\theta} + (1-\alpha)\frac{\theta - p_{22} + p_{12}}{2\theta})p_{22}$$

Solving the first-order conditions ($\frac{\partial \pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial \pi_{22}}{\partial p_{22}} = 0$) and plugging in $\hat{d} = \frac{v - p_{11} + p_{12}^e \delta}{\theta(1-\delta)}$, we obtain the subgame equilibrium prices: $p_{12}^* = \theta - \frac{4\alpha(v - p_{11} + p_{12}^e \delta)}{3(1-\delta)}$ and $p_{22}^* = \theta - \frac{2\alpha(v - p_{11} + p_{12}^e \delta)}{3(1-\delta)}$. Solving $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ together, we find that $p_{12}^e = \frac{3\theta(1-\delta) + 4\alpha(p_{11} - v)}{3 - (3 - 4\alpha)\delta}$ and $p_{22}^e = \frac{2(p_{11} - v)\alpha + \theta(3 - (3 - 2\alpha)\delta)}{3 - (3 - 4\alpha)\delta}$. Plugging p_{12}^e and p_{22}^e back into the expressions for p_{12}^*, p_{22}^* and $\hat{d}$, we verify that $p_{12}^* = p_{12}^e = \frac{3\theta(1-\delta) + 4\alpha(p_{11} - v)}{3 - (3 - 4\alpha)\delta}$ and $p_{22}^* = p_{22}^e = \frac{2(p_{11} - v)\alpha + \theta(3 - (3 - 2\alpha)\delta)}{3 - (3 - 4\alpha)\delta}$ and $\hat{d} = \frac{3(v - p_{11} + \theta\delta)}{\theta(3 - (3 - 4\alpha)\delta)}$.

Using the expressions for p_{12}^e and p_{22}^e, we find that *case iii*'s condition $\frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2} < p_{11} < v + \delta p_{12}^e$ is satisfied if and only if $\frac{2(3-\alpha)v - 3(\theta - \theta(3 - 2\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{p}$, where $\bar{p} \equiv v + \delta\theta$. Note that when $\frac{2(3-\alpha)v - 3(\theta - \theta(3 - 2\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{p}$, the prices p_{12}^* and p_{22}^* indeed satisfy $p_{22}^* - \theta < p_{12}^* < p_{22}^* + \theta - 2\theta\hat{d}$, which ensures that both firms have positive market shares and that the marginal first-period consumer located at $\hat{d} = \frac{3(v - p_{11} + \theta\delta)}{\theta(3 - (3 - 4\alpha)\delta)}$ buys firm 1's product in the second period.

As in the previous case, firm 1's profit function is not necessarily quasi-concave, and hence, we need to check that there are no non-local deviations. It is easy to check that deviations to lower prices will not be profitable. However, firm 1 may potentially deviate to a higher price to serve only the new consumers who entered the market in the second period, giving up its market share in the segment of first-period consumers who waited to buy a product in the second period. Specifically, if $p'_{12} > p_{22}^* + \theta - 2\theta\hat{d}$, then firm 1's deviation profit is $\pi'_{12} = (1-\alpha)\frac{\theta - p'_{12} + p_{22}^*}{2\theta}p'_{12}$. Notice that π'_{12} is concave. One can show that the right derivative of π'_{12} at $p'_{12} = p^*_{22} + \theta - 2\theta\hat{d}$ is given by $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p^*_{22}+\theta-2\theta\hat{d}} = -\frac{(1-\alpha)(p_{11}(6+\alpha)-v(6+\alpha)+3\theta(1-(3-\alpha)\delta))}{\theta(3-(3-4\alpha)\delta)}$. Straightforward algebra shows that $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p^*_{22}+\theta-2\theta\hat{d}} < 0$ if and only if $p_{11} > \hat{\rho}$, where $\hat{\rho} \equiv \frac{v(6+\alpha)-3\theta(1-(3-\alpha)\delta)}{6+\alpha}$ satisfies $\frac{2(3-\alpha)v-3(\theta-\theta(3-2\alpha)\delta)}{2(3-\alpha)} < \hat{\rho} < \bar{\rho}$. Since π'_{12} is concave, $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p^*_{22}+\theta-2\theta\hat{d}} < 0$ implies that π'_{12} is decreasing in p'_{12} when $p'_{12} > p^*_{22} + \theta - 2\theta\hat{d}$. Hence, if $p_{11} > \hat{\rho}$, then continuity of firm 1's profit function at the point $p^*_{22} + \theta - 2\theta\hat{d}$ implies that deviations to $p'_{12} > p^*_{22} + \theta - 2\theta\hat{d}$ will not be profitable because firm 1 will gain by reducing its price.

Case iv: $p_{11} \geq v + \delta p^e_{12}$. In this case, $\hat{d} = 0$, i.e., all first-period consumers will decide not to purchase a product in the first period, but wait. In the second period, firm i will choose its second-period price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = \frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22} = \frac{\theta - p_{22} + p_{12}}{2\theta} p_{22}$. Solving the first-order conditions, we find that $p^*_{12} = p^*_{22} = \theta$. Rational expectations imply that $p^e_{i2} = p^*_{i2} = \theta$. Substituting $p^e_{i2} = \theta$ into the inequality $p_{11} \geq v + \delta p^e_{12}$, it follows that the condition for case *iv* is satisfied if and only if $p_{11} \geq \bar{\rho}$, where recall that $\bar{\rho} = v + \delta\theta$.

Our analysis in cases *i*, *ii*, *iii* and *iv* characterizes the second-period subgame equilibrium prices for every p_{11} contained in the intervals $[0, \underline{\rho}]$, $(\underline{\rho}, \tilde{\rho}]$, $(\hat{\rho}, \bar{\rho})$, and $[\bar{\rho}, \infty]$, respectively. To find the first-period equilibrium price, we need to know the second-period subgame equilibrium outcome for each $p_{11} \geq 0$. That is, to have a full characterization of second-period subgame equilibria, we need that $[0, \underline{\rho}] \cup (\underline{\rho}, \tilde{\rho}] \cup (\hat{\rho}, \bar{\rho}) \cup [\bar{\rho}, \infty] = [0, \infty]$, which can happen only if $\tilde{\rho} \geq \hat{\rho}$, where recall that $\tilde{\rho} = v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\hat{\rho} = \frac{v(6+\alpha)-3\theta(1-(3-\alpha)\delta)}{6+\alpha}$. The inequality $\tilde{\rho} \geq \hat{\rho}$ is satisfied if and only if $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$ where $\bar{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

Note that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$, there are two possible second-period rational-expectations equilibria corresponding to the cases *ii* and *iii* that we analyzed earlier. The selection of which equilibrium occurs will not have a qualitative impact on our results. Since both firms' profits are higher when the equilibrium from case *ii* is played (i.e., it is the focal equilibrium), we assume that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$, the subgame equilibrium prices will be the ones from case *ii*, i.e., $p_{12}^* = \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^* = \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$.

Hence, to summarize our analysis, it follows that when $(R_1, R_2) = (1, 2)$, the firms' second-period prices in a rational-expectations subgame equilibrium are as follows:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3\theta(1-\delta)+4\alpha(p_{11}-v)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B71}$$

$$p_{22}^{(1,2)} = p_{22}^e = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2(p_{11}-v)\alpha+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B72}$$

where $\underline{\rho} = v - \theta(1-\delta)$, $\tilde{\rho} \equiv v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv v + \delta t$. For future reference, let us derive the firms' equilibrium profits and sales. Specifically, upon plugging the prices in (B71) and (B72) into the respective profit functions (explicitly provided in earlier cases *i*–*iv*), we obtain the firms' second-period subgame equilibrium profits when $(R_1, R_2) = (1,2)$:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4(p_{11}-v)\alpha+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B73}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2(p_{11}-v)\alpha+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B74}$$

We can also find the firms' second-period unit sales:

$$S_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11}-v)+3\theta(1-\delta)}{6\theta(1-\delta)+8\theta\alpha\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B75)}$$

$$S_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4(p_{11}-v)\alpha+3\theta(1+\delta)+\theta\alpha(1-3\delta))}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3\theta(1-\delta)+2\alpha(p_{11}-v)+2\theta\alpha\delta}{6\theta(1-\delta)+8\theta\alpha\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B76)}$$

To find firm 1's first-period unit sales, $S_{11}^{(1,2)}$, recall that consumers with $d < \hat{d}$ buy firm 1's product in the first period, where $\hat{d}$ is given in the equation (B70). Since consumers are uniformly distributed, $S_{11}^{(1,2)} = \alpha\hat{d}$. Plugging $p_{12}^{e} = p_{12}^{(1,2)}$ and $p_{22}^{e} = p_{22}^{(1,2)}$ into the expression for $\hat{d}$, we find that

$$S_{11}^{(1,2)} = \begin{cases} \alpha & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha \frac{(3v(1-\alpha)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha \frac{3(v-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B77)}$$

In the first period, firm 1 chooses its price to maximize $\pi_1 = \pi_{11}^{(1,2)} + \pi_{12}^{(1,2)}$, where $\pi_{11}^{(1,2)} = S_{11}^{(1,2)} p_{11}$. Note that $\pi_{12}^{(1,2)}$ and $S_{11}^{(1,2)}$ are given in equations (B73) and (B77), respectively.

$$\pi_1 = \begin{cases} \alpha\, p_{11} + \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha \frac{(3v(1-\alpha)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))} p_{11} + \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha \frac{3(v-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} p_{11} + \frac{(4(p_{11}-v)\alpha+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}, \quad \text{(B78)}$$

where $\underline{\rho} = v - \theta(1-\delta)$, $\tilde{\rho} = v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} = v + \delta\theta$.

Note that π_1 is continuous everywhere except at $p_{11} = \tilde{\rho}$, where π_1 jumps down. It is easy to see that π_1 is increasing in p_{11} at any $p_{11} < \underline{\rho}$, and hence, the optimal price will be above $\underline{\rho}$. Further, if $p_{11} > \bar{\rho}$, then π_1 is constant. Since π_1 is continuous at $\bar{\rho}$, the optimal price will not be above $\bar{\rho}$ either.

Hence, the optimal price is either in $I_1 \equiv [\underline{\rho}, \tilde{\rho}]$ or $I_2 \equiv (\tilde{\rho}, \bar{\rho}]$. Let p_{I_k} denote the locally optimal price within the interval I_k. We will first find p_{I_1} and p_{I_2}, after which we will compare the corresponding profits to determine the globally optimal price.

First, consider $p_{11} \in I_1$. On this interval, we have

$$\pi_1 = \alpha \frac{(3v(1-\alpha)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))} p_{11} + \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2}.$$

One can readily show that π_1 is concave because $\frac{d^2\pi_1}{dp_{11}^2} < 0$.

If $0 < \theta \leq \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$, then π_1 is decreasing in p_{11} at any $\underline{\rho} < p_{11} < \tilde{\rho}$, and hence, the locally optimal price is at the left corner: $p_{I_1} = \underline{\rho}$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{2v\alpha+\theta(1+\alpha(-3+2\delta))}{2}$.

If $\frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta} < \theta < \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$, then the local maximizer is in the interior of the interval $[\underline{\rho}, \tilde{\rho}]$. Using the fist-order condition ($\frac{d\pi_1}{dp_{1_1}} = 0$), we find the interior maximizer: $p_{I_1} = \frac{v(1-\alpha)(9-13\alpha+9\delta+3\alpha\delta)-2\theta(4\alpha(1+4\delta)+\alpha^2(-1-4\delta+\delta^2)-3(1+4\delta+3\delta^2))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)}$. The profit is $\pi_1(p_{I_1}) = \frac{\alpha(9v(1-\alpha)-2\theta(3-\alpha)(1-3\delta))(v(1-\alpha)(9-13\alpha+9\delta+3\alpha\delta)-2\theta(4\alpha(1+4\delta)-\alpha^2(1+4\delta-\delta^2)-3(1+4\delta+3\delta^2)))}{4\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2}$.

If $\min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\} \leq \theta < \frac{V_2}{2}$, then π_1 is increasing in p_{1_1} at any $\underline{\rho} < p_{1_1} < \tilde{\rho}$, and hence, the locally optimal price is at the right corner: $p_{I_1} = \tilde{\rho}$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{(3-\alpha)(2v\alpha(6-\alpha(7-3\alpha))+\theta(3-\alpha)(4-\alpha(10-6\delta-\alpha(5-\alpha-4\delta))))}{2(6+\alpha(-7+3\alpha))^2}$.

Second, consider $p_{11} \in I_2$. On this interval, we have

$$\pi_1 = \alpha \frac{3(v-p_{11}+t\delta)}{\theta(3-(3-4\alpha)\delta)} p_{11} + \frac{(4(p_{11}-v)\alpha+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2}.$$

It is easy to show that $\frac{d^2\pi_1}{dp_{11}^2} < 0$, implying that π_1 is concave.

If $\theta \leq \tilde{\theta}$, then π_1 is decreasing in p_{11} at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$, where $\tilde{\theta} = \frac{3v(6-\alpha(7-3\alpha))(3-(3-4\alpha)\delta)}{2(63-\alpha(75-26\alpha))-(234-\alpha(417-(213-32\alpha)\alpha))\delta+3(3-4\alpha)(12-(11-\alpha)\alpha)\delta^2}$. Hence, a local maximum does not exist.

If $\tilde{\theta} < \theta < \frac{3v}{4-3\delta}$, then π_1 has a local maximizer in the interior of the interval $(\tilde{\rho}, \bar{\rho}]$. Using the first-order condition, we find that $p_{I_2} = \frac{3\theta(4-\delta-(3-4\alpha)\delta^2)+v(9-9\delta-4\alpha(4-3\delta))}{2(9-9\delta-4\alpha(2-3\delta))}$. The corresponding profit is $\pi_1(p_{I_2}) = \frac{9v^2\alpha-6\theta v\alpha(4-3\delta)+9\theta^2(2-2\delta+\alpha\delta^2)}{4\theta(9-9\delta-4\alpha(2-3\delta))}$.

If $\frac{3v}{4-3\delta} < \theta < \frac{V_2}{2}$, then π_1 is decreasing in p_{11} at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Hence, the local maximizer is at $p_{I_2} = \bar{\rho}$, with a corresponding profit of $\pi_1(p_{I_2}) = \frac{\theta}{2}$.

Next, let us find the globally optimal price, $p_{11}^{(1,2)}$.

When $\theta \leq \tilde{\theta}$, π_1 is decreasing at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Since π_1 jumps down at $p_{11} = \tilde{\rho}$ (i.e., $\lim_{p_{11}\to\tilde{\rho}^-} \pi_1 > \lim_{p_{11}\to\tilde{\rho}^+} \pi_1$) and $\frac{d\pi_1}{dp_{11}} < 0$ at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$, it follows that $\pi_1(\tilde{\rho}) > \pi_1(p_{11})$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Since $\tilde{\rho} \in I_1$ and $\pi_1(p_{I_1}) \geq \pi_1(\tilde{\rho})$, it follows that $p_{11}^{(1,2)} = p_{I_1}$.

When $\tilde{\theta} < \theta < \frac{3v}{4-3\delta}$, we have $\pi_1(p_{I_2}) = \frac{9v^2\alpha-6\theta v\alpha(4-3\delta)+9\theta^2(2-2\delta+\alpha\delta^2)}{4\theta(9-9\delta-4\alpha(2-3\delta))}$. One can show that $\pi_1(\tilde{\rho}) > \pi_1(p_{I_2})$.[13] Since $\tilde{\rho} \in I_1$, it must be that $\pi_1(p_{I_1}) \geq \pi_1(\tilde{\rho})$. Hence, by transitivity, it follows that $\pi_1(p_{I_1}) > \pi_1(p_{I_2})$, which implies that $p_{11}^{(1,2)} = p_{I_1}$.

Finally, when $\frac{3v}{4-3\delta} < \theta < \frac{V_2}{2}$, we have $\pi_1(p_{I_2}) = \frac{\theta}{2}$. Again, straightforward algebra shows that $\pi_1(\tilde{\rho}) > \pi_1(p_{I_2})$, and by the same argument as in the previous paragraph, it follows that $p_{11}^{(1,2)} = p_{I_1}$.

Hence, we found that firm 1's optimal first-period price belongs to the interval I_1, and is given by

$$p_{11}^{(1,2)} = \begin{cases} \underline{\rho} & \text{if } \theta \leq \underline{\theta} \\ \frac{v(1-\alpha)(9-13\alpha+9\delta+3\alpha\delta)-2\theta(4\alpha(1+4\delta)-\alpha^2(1+4\delta-\delta^2)-3(1+4\delta+3\delta^2))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \tilde{\rho} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases} \quad \text{(B79)}$$

where $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$, $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$, $\underline{\rho} = v - \theta(1-\delta)$ and $\tilde{\rho} = v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$.

[13] To see this, note that the function $\pi_1(\tilde{\rho}) - \pi_1(p_{I_2})$ is concave in θ because $\frac{d^2(\pi_1(\tilde{\rho})-\pi_1(p_{I_2}))}{d\theta^2} = -\frac{9v^2\alpha}{2\theta^3(9-9\delta-4\alpha(2-3\delta))} < 0$ for any $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$. Further, $(\pi_1(\tilde{\rho}) - \pi_1(p_{I_2}))|_{\theta=\tilde{\theta}} > 0$ and $(\pi_1(\tilde{\rho}) - \pi_1(p_{I_2}))|_{\theta=\frac{3v}{4-3\delta}} > 0$. Hence, it must be that $\pi_1(\tilde{\rho}) - \pi_1(p_{I_2}) > 0$ for any $\tilde{\theta} < \theta < \frac{3v}{4-3\delta}$.

For future reference, by plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$ from Lemma 2, we can find the firms' second-period prices on the equilibrium path:

$$p_{12}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \leq \underline{\theta} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases} \tag{B80}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \leq \underline{\theta} \\ \frac{\theta(9(1+\delta)-5\alpha^2(1-\delta)-\alpha(4+6\delta))-6v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases} \tag{B81}$$

Using the equilibrium prices, we obtain the firm's equilibrium profits in each period. Specifically,

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(v-\theta(1-\delta)) & \text{if } \theta \leq \underline{\theta} \\ \frac{\alpha(9v(1-\alpha)-2\theta(3-\alpha)(1-3\delta))(v(1-\alpha)(9-13\alpha+3(3+\alpha)\delta)+A)}{4\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{(3-\alpha)\alpha(v(6-\alpha(7-3\alpha))-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases} \tag{B82}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{(3(1-\alpha)(v\alpha-\theta(3-\alpha))-\theta(3-\alpha)^2\delta)^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-5\alpha+\alpha^2)^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases} \tag{B83}$$

$$\pi_{21}^{(1,2)} = 0 \tag{B84}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{(6v(1-\alpha)\alpha+\theta(5\alpha^2(1-\delta)-9(1+\delta)+2\alpha(2+3\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases} \tag{B85}$$

where $A \equiv 2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))$. Note that since the firms are symmetric, when $(R_1, R_2) = (2,1)$, the firms' profits are reversed, i.e., $\pi_{1t}^{(2,1)} = \pi_{2t}^{(1,2)}$ and $\pi_{2t}^{(2,1)} = \pi_{1t}^{(1,2)}$.

PROPOSITION 7.1. *If firm 2 releases its product later than firm 1, then second-period price competition is less intense than when firm 2 also releases its product in the first period, i.e.,* $p_{12}^{(1,2)} \geq p_{12}^{(1,1)}$ *and* $p_{22}^{(1,2)} \geq p_{22}^{(1,1)}$.

PROOF OF PROPOSITION 7.1. Before we proceed to the proof, recall that to ensure the existence of a rational-expectations equilibrium with full market coverage, we are assuming that $t < \frac{V_2}{2}$, $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$, where $\bar{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

When $(R_1, R_2) = (1,1)$, Lemma 1 showed that the firms' equilibrium prices will be $p_{i2}^{(1,1)} = \theta$.

When $(R_1, R_2) = (1,2)$, the firms' equilibrium second-period prices are given in equations (B80)-(B81). We need to show that $p_{i2}^{(1,2)} \geq p_{i2}^{(1,1)}$. Clearly, if $\theta \leq \underline{\theta}$, then $p_{i2}^{(1,2)} = p_{i2}^{(1,1)} = \theta$ for each firm $i = 1,2$.

Next, if $\underline{\theta} < \theta < \bar{\theta}$, we have $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)}$ and $p_{22}^{(1,2)} = \frac{\theta(9(1+\delta)-5\alpha^2(1-\delta)-\alpha(4+6\delta))-6v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)}$. Straightforward algebra shows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$ if and only if $\theta > \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$. Recall that $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$. Hence, $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$ when $\underline{\theta} < \theta < \bar{\theta}$.

Finally, if $\bar{\theta} \leq \theta < \frac{V_2}{2}$, then $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)}$ and $p_{22}^{(1,2)} = \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)}$. Since $(3-\alpha)(2-\alpha) > 6-\alpha(7-3\alpha)$ and $6-\alpha(3+\alpha) > 6-\alpha(7-3\alpha)$ for any $\alpha \in [0, \bar{\alpha})$, it readily follows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$. ■

PROPOSITION 7.2. *There exist* $0 < \hat{\alpha} < \bar{\alpha}$ *and* $0 < \hat{\theta} < \frac{V_2}{2}$, *such that if* $\alpha < \hat{\alpha}$ *and* $\theta > \hat{\theta}$, *then in equilibrium, firm 2 will release its product later than firm 1, i.e.,* $(R_1^*, R_2^*) = (1,2)$.

PROOF OF PROPOSITION 7.2. For future reference, recall that to ensure the existence of a rational-expectations equilibrium with full market coverage, we are assuming that $t < \frac{V_2}{2}$, $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$, where $\bar{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

Firm i chooses its product release period $R_i \in \{1,2\}$ to maximize its overall profit $\pi_i^{(R_1,R_2)} = \pi_{i1}^{(R_1,R_2)} + \pi_{i2}^{(R_1,R_2)}$. The pair (R_1, R_2) is an equilibrium if no firm has any profitable deviations.

First, one can readily show that $(R_1, R_2) = (2,2)$ is not an equilibrium because firm 1 will benefit by deviating to $R_1 = 1$, i.e., $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, where $\pi_1^{(2,2)} = \frac{\theta}{2}$. To see this, notice from (B78) that if firm 1 deviates to $R_1 = 1$ and sets an extremely large price ($p_{11} \geq \bar{\rho}$), then its overall profit will be $\pi_1(\bar{\rho}) = \frac{\theta}{2}$. However, we know from Lemma 3 that firm 1's unique optimal price is $p_{11}^{(1,2)} < \bar{\rho}$ with a corresponding profit of $\pi_1^{(1,2)}$. Hence, $\pi_1^{(1,2)} > \pi_1(\bar{\rho})$. Since $\pi_1(\bar{\rho}) = \pi_1^{(2,2)} = \frac{\theta}{2}$, it follows that $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Therefore, in equilibrium, either one firm releases its product in the first period and the other firm does so in the second period, or both firms release their products in the first period.

Next, let us demonstrate that a pure strategy equilibrium exists. If $(R_1, R_2) = (1,2)$ is an equilibrium, then the proof is finished. If $(R_1, R_2) = (1,2)$ is not an equilibrium, then let us show that $(R_1, R_2) = (1,1)$ must be an equilibrium. Since $(R_1, R_2) = (1,2)$ is not an equilibrium, one of the firms must have a profitable deviation. Firm 1 will not want to deviate to $R_1 = 2$ because $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Hence, it must be that firm 2 has a profitable deviation from $R_2 = 2$ to $R_2 = 1$, which means that $\pi_2^{(1,1)} > \pi_2^{(1,2)}$. By symmetry between firms 1 and 2, we know that $\pi_2^{(1,2)} = \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} = \pi_1^{(1,1)}$. Hence, $\pi_2^{(1,1)} > \pi_2^{(1,2)}$ implies that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$. Then, it must be $(R_1, R_2) = (1,1)$ is an equilibrium because we found that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} > \pi_2^{(1,2)}$.

Now, let us find conditions under which $(R_1, R_2) = (1,2)$ is an equilibrium. If $R_1 = 1$, then firm 2's best response is $R_2 = 2$ if and only if $\pi_2^{(1,2)} - \pi_2^{(1,1)} > 0$, where $\pi_2^{(1,1)} = \frac{\theta}{2}$ and $\pi_2^{(1,2)} = \pi_{21}^{(1,2)} + \pi_{22}^{(1,2)}$ is given in equations (B84)-(B85). Define $\Delta\pi_2 \equiv \pi_2^{(1,2)} - \pi_2^{(1,1)}$.

$$\Delta\pi_2 = \begin{cases} \frac{(1-\alpha)\theta}{2} - \frac{\theta}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{(6v(1-\alpha)\alpha+\theta(5\alpha^2(1-\delta)-9(1+\delta)+2\alpha(2+3\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2} - \frac{\theta}{2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{\theta}{2} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases},$$

where $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$ and $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$. Note that $\Delta\pi_2$ is a continuous function. When $\theta \leq \underline{\theta}$, clearly, $\Delta\pi_2 < 0$. Hence, consider $\theta > \underline{\theta}$.

First, let us show that when $\underline{\theta} < \theta < \bar{\theta}$, the function $\Delta\pi_2$ is increasing in θ, i.e., $\frac{d\Delta\pi_2}{d\theta} > 0$. To see this, note that $\frac{d^2\Delta\pi_2}{d\theta^2} = \frac{36v^2(1-\alpha)\alpha^2}{\theta^3(9-11\alpha+9\delta+3\alpha\delta)^2} > 0$, i.e., $\Delta\pi_2$ is convex. Further, $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\underline{\theta}} = \frac{\alpha(23-9\delta-\alpha(21-13\delta))}{2(9-11\alpha+9\delta+3\alpha\delta)} > 0$, where the inequality follows because $\delta > \bar{\delta} \geq 0.5$. The inequalities $\frac{d^2\Delta\pi_2}{d\theta^2} > 0$ and $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\underline{\theta}} > 0$ imply that $\frac{d\Delta\pi_2}{d\theta} > 0$ for any $\underline{\theta} < \theta < \bar{\theta}$.

Second, when $\bar{\theta} \leq \theta < \frac{V_2}{2}$, we have $\frac{d\Delta\pi_2}{d\theta} = \frac{(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{1}{2} > 0$ if and only if $\alpha < \hat{\alpha}$, where $\hat{\alpha} \approx 0.32$ satisfies $\frac{(1-\hat{\alpha})(6-\hat{\alpha}(3+\hat{\alpha}))^2}{2(6-\hat{\alpha}(7-3\hat{\alpha}))^2} - \frac{1}{2} = 0$, or equivalently, $12 - \hat{\alpha}(52 - (3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$.

We will separately analyze the cases with $\alpha \leq \hat{\alpha}$ and $\alpha > \hat{\alpha}$. Specifically, we will show that if $\alpha > \hat{\alpha}$, then $\Delta\pi_2 < 0$, but if $\alpha \leq \hat{\alpha}$, then $\Delta\pi_2 > 0$ for θ large enough.

Case 1: $\alpha > \hat{\alpha}$. When $\alpha > \hat{\alpha}$, one can show that $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$.[14] We showed earlier that $\Delta\pi_2$ is increasing when $\underline{\theta} < \theta < \bar{\theta}$ and decreasing when $\bar{\theta} \leq \theta < \frac{V_2}{2}$. Since $\Delta\pi_2$ is continuous, it follows that $\Delta\pi_2|_{\theta=\bar{\theta}} > \Delta\pi_2$ for any $\underline{\theta} < \theta < \frac{V_2}{2}$ and $\theta \neq \bar{\theta}$. Straightforward algebra shows that $\Delta\pi_2|_{\theta=\bar{\theta}} = \frac{3v(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, and $\Delta\pi_2|_{\theta=\bar{\theta}} < 0$ if and only if $12 - \alpha(52 - (3-\alpha)\alpha(17+\alpha)) > 0$. The last inequality is equivalent to $\alpha > \hat{\alpha}$, where recall that $\hat{\alpha} \approx 0.32$ satisfies $12 - \hat{\alpha}(52 - (3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$. Hence, when $\alpha > \hat{\alpha}$, we have $\Delta\pi_2 \leq \Delta\pi_2|_{\theta=\bar{\theta}} < 0$ for any $\underline{\theta} < \theta < \frac{V_2}{2}$.

Case 2: $\alpha < \hat{\alpha}$. When $\alpha < \hat{\alpha}$, we showed that $\Delta\pi_2$ is increasing in θ at any $\underline{\theta} < \theta < \frac{V_2}{2}$. Further, we can show that $\Delta\pi_2|_{\theta=\bar{\theta}} > 0$, where recall that $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$. Specifically, if $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, then we already know from Case 1 that $\Delta\pi_2|_{\theta=\bar{\theta}} =$

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[14] To see this, note that $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ if and only if $\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2} < 0$. Using the fact that $\delta > \bar{\delta} > 0.5$, one can show that the function $\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2}$ is decreasing in α. Plugging in $\alpha = 0.3$, one can show that $(\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2})|_{\alpha=0.3} < 0$. Hence, $\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2} < 0$ for all $\alpha > 0.3$. Since $\hat{\alpha} > 0.3$, it follows that $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ when $\alpha > \hat{\alpha}$.

$\frac{3v(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} > 0$ when $\alpha < \hat{\alpha}$. Next, if $\bar{\theta} = \frac{V_2}{2}$, then first note that $V_2 = \sum_{k=2}^{T} \delta^{k-2} v$ is increasing in T. Since $T \geq 3$, it follows that $V_2 \geq V_2|_{T=3} = v(1+\delta)$. Since we already showed that $\Delta\pi_2$ is increasing in θ at any $\theta > \underline{\theta}$ and since $\frac{v(1+\delta)}{2} > \underline{\theta}$, it follows that $\Delta\pi_2|_{\theta=\frac{V_2}{2}} > \Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}}$. $\Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}} = \frac{v(9(1+\delta)^2+\alpha^2(7+5\delta^2)-2\alpha(8+\delta(5+3\delta)))^2}{4(1-\alpha)(1+\delta)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{v(1+\delta)}{4}$, and since $\delta > \bar{\delta} > 0.5$, one can show that $\Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}} > 0$. Hence, $\Delta\pi_2|_{\theta=\bar{\theta}} = \Delta\pi_2|_{\theta=\frac{V_2}{2}} > \Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}} > 0$.

Since $\Delta\pi_2$ is increasing in θ, $\Delta\pi_2|_{\theta=\bar{\theta}} > 0$ implies that $\Delta\pi_2 > 0$ for all $\bar{\theta} \leq \theta < \frac{V_2}{2}$. Further, since $\Delta\pi_2|_{\theta=\underline{\theta}} = -\frac{3v(1-\alpha)\alpha}{8(2-\alpha(2-\delta))} < 0$, $\Delta\pi_2|_{\theta=\bar{\theta}} > 0$ and $\Delta\pi_2$ is a continuous increasing function, it follows that there exists a unique $\underline{\theta} < \hat{\theta} < \bar{\theta}$, such that $\Delta\pi_2|_{\theta=\hat{\theta}} = 0$ and $\Delta\pi_2 < 0$ if $\underline{\theta} < \theta < \hat{\theta}$ and $\Delta\pi_2 > 0$ if $\hat{\theta} < \theta < \bar{\theta}$. Solving $\Delta\pi_2 = 0$, we find that

$$\hat{\theta} = \frac{6(v(1-\alpha)^{3/2}(9-11\alpha+3(3+\alpha)\delta) - v(1-\alpha)(\alpha(4+5\alpha(1-\delta)+6\delta)-9(1+\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))} \quad \text{(B86)}$$

To summarize the above, we found that if $0 < \alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $\Delta\pi_2 > 0$. Thus, in this parameter region, firm 2 will not deviate from $R_2 = 2$. Since $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, firm 1 also has no incentives to deviate from $R_1 = 1$. It follows that $(R_1, R_2) = (1,2)$ is a Nash equilibrium. ■

PROPOSITION 7.3. *Suppose that $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, so that in equilibrium $(R_1^*, R_2^*) = (1,2)$. Then, firm 2's equilibrium sales are decreasing in α. However, firm 2's profits are non-monotone in α, and an increase in α can make firm 2 better off as long as α is not too high.*

PROOF OF PROPOSITION 7.3. First, let us show that firm 2's equilibrium unit sales, $S_2^{(1,2)}$, are decreasing in α. Note that $S_2^{(1,2)} = S_{22}^{(1,2)}$, where $S_{22}^{(1,2)}$ is given in the equation (B76). Upon plugging in firm 1's first-period equilibrium price, we find firm 2's sales on the equilibrium path:

$$S_2^{(1,2)} = \begin{cases} \frac{1-\alpha}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{\theta(9(1+\delta)-5\alpha^2(1-\delta)-2\alpha(2+3\delta))-6v(1-\alpha)\alpha}{2\theta(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} \leq \theta \leq \bar{\theta} \\ \frac{(1-\alpha)(6-\alpha(3+\alpha))}{2(6-\alpha(7-3\alpha))} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases}$$

where $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$ and $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$. Note that $S_2^{(1,2)}$ is continuous. When $\theta < \underline{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{1}{2} < 0$. When $\underline{\theta} < \theta < \bar{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = \frac{\Psi}{\theta(9-11\alpha+3(3+\alpha)\delta)^2}$, where $\Psi = \frac{6v(\alpha(18-11\alpha+3(6+\alpha)\delta)-9(1+\delta))+\theta(5\alpha^2(1-\delta)(11-3\delta)+9(1+\delta)(7-9\delta)-90\alpha(1-\delta^2))}{2}$. Clearly, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ if and only if $\Psi < 0$. Hence, we need to show that $\Psi < 0$. First, note that Ψ is concave in α because $\frac{d^2\Psi}{d\alpha^2} = -(6v - 5\theta(1-\delta))(11-3\delta) < 0$.[15] Second, since $\theta < \frac{V_2}{2}$, one can show that $\Psi|_{\alpha=0} = -\frac{9(1+\delta)(6v-\theta(7-9\delta))}{2} < 0$ and $\Psi|_{\alpha=0.5} = -\frac{6v(11-3\delta)-\theta(127-142\delta-129\delta^2)}{4} < 0$.[16] Concavity of Ψ, together with $\Psi|_{\alpha=0} < 0$ and $\Psi|_{\alpha=0.5} < 0$, implies that $\Psi < 0$ for any $0 < \alpha < 0.5$. Since $\hat{\alpha} \approx 0.32 < 0.5$, it follows that $\Psi < 0$ for any $0 < \alpha < \hat{\alpha}$. Thus, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ when $\underline{\theta} < \theta < \bar{\theta}$. Finally, when $\bar{\theta} \leq \theta < \frac{V_2}{2}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{12+\alpha(12-\alpha(31-\alpha(14-3\alpha)))}{2(6-\alpha(7-3\alpha))^2}$. One can easily show that $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ for any $0 < \alpha < \hat{\alpha}$. This proves the first part of the proposition.

Next, let us show that there exists $\tilde{\alpha} < \hat{\alpha}$, such that if $\alpha < \tilde{\alpha}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$. Since we are focusing on $\theta > \hat{\theta}$ and since $\underline{\theta} < \hat{\theta} < \bar{\theta}$, it follows from the equation (B85) that $\pi_2^{(1,2)} = \pi_{22}^{(1,2)}$ is given by

$$\pi_2^{(1,2)} = \begin{cases} \frac{(6v(1-\alpha)\alpha+\theta(5\alpha^2(1-\delta)-9(1+\delta)+2\alpha(2+3\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2} & \text{if } \hat{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases}$$

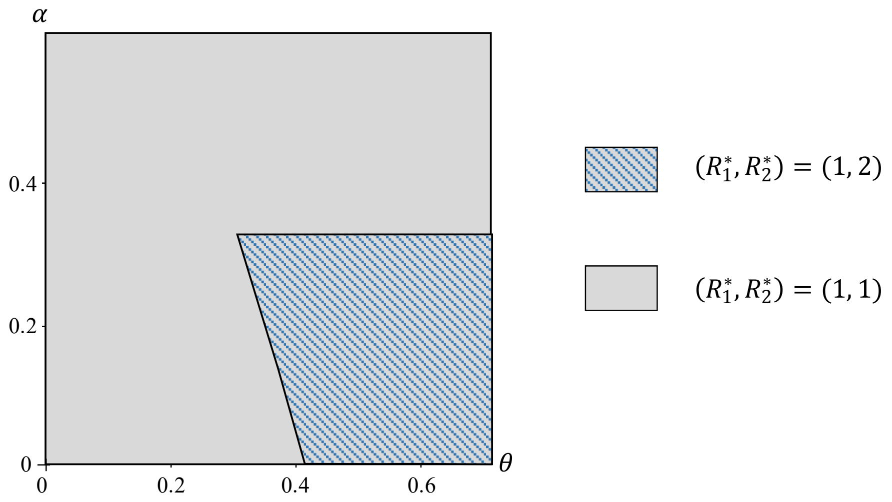
Differentiating $\pi_2^{(1,2)}$ with respect to α and evaluating at $\alpha = 0$, we find that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{23v-12v-9\theta\delta}{18(1+\delta)}$ when $\hat{\theta} < \theta < \bar{\theta}$, and $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{\theta}{6}$ when $\bar{\theta} < \theta < \frac{V_2}{2}$. It is clear that if $\bar{\theta} < \theta < \frac{V_2}{2}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$. It is also easy to see that if $\hat{\theta} < \theta < \bar{\theta}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$ if and only if $\theta > \frac{12v}{23-9\delta}$. Note that $\hat{\theta}|_{\alpha=0} = \frac{12v}{23-9\delta}$. By continuity of $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}$, it follows that there exists $\tilde{\alpha} \in (0, \hat{\alpha}]$ such that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$ if $\alpha < \tilde{\alpha}$ and $\theta > \hat{\theta}$. ■

[15] To see that $\frac{d^2\Psi}{d\alpha^2} < 0$, note that $11 - 3\delta > 0$ and $6v - 5t(1-\delta) > 0$ because $\theta < \frac{V_2}{2}$.

[16] This is easily shown using the fact that $\theta < \frac{V_2}{2} < \frac{v}{2(1-\delta)}$, where the last inequality follows from taking $T \to \infty$.

Figure B4 shows the firms' equilibrium product release times in the (θ, α) parameter space.

Figure B4 Firms' Equilibrium Product Release Times (R_1^*, R_2^*) [17]



7.B. $V_{it} = \sum_{j=t}^{T} \delta^{j-t}(v - \theta d_i)$

In this subsection, we consider the scenario where consumers' horizontal preferences influence the consumption value of the durable product in each use period. Purchasing the firm i's product in period t gives the consumer a utility of $V_{it} - p_{it}$, where $V_{it} = \sum_{j=t}^{T} \delta^{j-t}(v - \theta d_i)$. To ensure full market coverage, we focus on θ sufficiently small: $\theta < \frac{v}{2}$.

Subgame with $(R_1, R_2) = (1, 1)$ or $(R_1, R_2) = (2, 2)$

First, consider the subgame with $(R_1, R_2) = (2, 2)$. Consumers who enter the market in the first period wait till the second period to buy a product. Hence, in the second period, firms compete for a unit measure of consumers. A consumer who is located at a distance d from firm 1 will buy firm 1's product if and only if

$$V_{12} - p_{12} > V_{22} - p_{22} \tag{B87}$$

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[17] Figure B4 is drawn using the following parameter values: $v = 0.5$, $\delta = 0.95$ and $T = 4$. To ensure full market coverage, we focus on $\theta < \frac{V_2}{2} \approx 0.71$.

Using the expression for V_{12} and V_{22} and simplifying the above inequality, we find that a consumer will buy from firm 1 if and only if $d \leq \frac{\theta(\delta-\delta^T)-p_{12}(1-\delta)\delta+p_{22}(1-\delta)\delta}{2\theta(\delta-\delta^T)}$. Hence, the firms' profits are given by $\pi_{12} = \frac{\theta(\delta-\delta^T)-p_{12}(1-\delta)\delta+p_{22}(1-\delta)\delta}{2\theta(\delta-\delta^T)} p_{12}$ and $\pi_{22} = (1 - \frac{\theta(\delta-\delta^T)-p_{12}(1-\delta)\delta+p_{22}(1-\delta)\delta}{2\theta(\delta-\delta^T)}) p_{22}$. Solving the first-order conditions, we find the firms' second-period equilibrium prices: $p_{12}^{(2,2)} = p_{22}^{(2,2)} = \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta}$. The firms' equilibrium profits are

$$\pi_1^{(2,2)} = \pi_2^{(2,2)} = \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} \tag{B88}$$

The analysis of the subgame with $(R_1, R_2) = (1, 1)$ is similar. Specifically, one can show that firm i's equilibrium price are given by $p_{i1}^{(1,1)} = p_{i2}^{(1,1)} = \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta}$, with corresponding profits of $\pi_{i1}^{(1,1)} = \alpha \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta}$ and $\pi_{i2}^{(1,1)} = (1-\alpha)\frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta}$ in the first and second periods, respectively. Thus, the firms' overall profits in the $(R_1, R_2) = (1, 1)$ subgame are given by

$$\pi_1^{(1,1)} = \pi_2^{(1,1)} = \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} \tag{B89}$$

Subgame with $(R_1, R_2) = (1, 2)$

We solve the game by backward induction, starting with the second period. In the second period, the firms compete for the $(1-\alpha)$ new consumers entering the market, as well as any first-period consumers who decided not to buy a product in the first period. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize the segment of first-period consumers who wait till the second period to buy. Namely, in the first period, a consumer who is located at a distance d from firm 1 will prefer to wait till the second period to purchase a product if and only if

$$\delta \max\{V_{12} - p_{12}^e, V_{22} - p_{22}^e\} > V_{11} - p_{11} \tag{B90}$$

Using the expressions for V_{11}, V_{12}, and V_{22}, we find that (B90) is equivalent to

$$d > \hat{d} \tag{B91}$$

where

$\hat{d} \equiv$

$$\begin{cases} 1 & \text{if } p_{11} \leq \frac{(1-\delta)(v+p_{22}^e\delta)-\theta(1-\delta^T)}{1-\delta} \\ \frac{(v-p_{11})(1-\delta)+p_{22}^e(\delta-\delta^2)+\delta\theta-\theta\delta^T}{\theta(1+\delta-2\delta^T)} & \text{if } \frac{(1-\delta)(v+p_{22}^e\delta)-\theta(1-\delta^T)}{1-\delta} < p_{11} \leq \frac{(2v-\theta)(\delta-\delta^T)-p_{22}^e(1-\delta)\delta+p_{12}^e\delta(1+\delta-2\delta^T)}{2(\delta-\delta^T)} \\ \frac{v-p_{11}+p_{12}^e\delta}{\theta} & \text{if } \frac{(2v-\theta)(\delta-\delta^T)-p_{22}^e(1-\delta)\delta+p_{12}^e\delta(1+\delta-2\delta^T)}{2(\delta-\delta^T)} < p_{11} < v+\delta p_{12}^e \\ 0 & \text{if } p_{11} \geq v+\delta p_{12}^e \end{cases}$$

We can now solve for the second-period subgame equilibrium prices that satisfy the rational expectations condition. Since $\hat{d}$ takes different functional forms depending on the interval to which p_{11} belongs, we will separately analysis each of the four possible cases. The analysis is very similar to the cases *i-iv* in the proof of Lemma 2. To avoid repetitive analysis, we will directly provide the second-period subgame equilibrium prices. Specifically, one can show that if the following condition holds,

$$v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1-2(1-\alpha)\delta^T)}{(6-7\alpha+3\alpha^2)(1-\delta)} \geq v + \frac{\theta(3(3-\alpha)\delta-3-3(2-\alpha)\delta^T)}{(6+\alpha)(1-\delta)} \tag{B92}$$

then the prices in (B93) and (B94) constitute a rational-expectations subgame equilibrium:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta} & \text{if } p_{11} \in [0,\underline{\rho}) \cup [\bar{\rho},\infty] \\ \frac{(\delta-\delta^T)(\theta(3-\alpha)(1+\delta-2\delta^T)-2(v-p_{11})\alpha(1-\delta))}{(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)} & \text{if } p_{11} \in [\underline{\rho},\tilde{\rho}] \\ \frac{(3\theta-4(v-p_{11})\alpha)(\delta-\delta^T)}{\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)} & \text{if } p_{11} \in (\tilde{\rho},\bar{\rho}] \end{cases} \tag{B93}$$

$$p_{22}^{(1,2)} = p_{22}^e = \begin{cases} \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta} & \text{if } p_{11} \in [0,\underline{\rho}) \cup [\bar{\rho},\infty] \\ \frac{(\delta-\delta^T)(\theta(3+\alpha+3\delta-3\alpha\delta-2(3-\alpha)\delta^T)-4(v-p_{11})\alpha(1-\delta))}{(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta+2(-3+\alpha)\delta^T)} & \text{if } p_{11} \in [\underline{\rho},\tilde{\rho}] \\ \frac{(\delta-\delta^T)(\theta(3-(3-2\alpha)\delta-2\alpha\delta^T)-2(v-p_{11})\alpha(1-\delta))}{(1-\delta)\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)} & \text{if } p_{11} \in (\tilde{\rho},\bar{\rho}] \end{cases} \tag{B94}$$

where $\underline{\rho} = v - \theta$, $\tilde{\rho} = v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1-2(1-\alpha)\delta^T)}{(6-7\alpha+3\alpha^2)(1-\delta)}$, and $\bar{\rho} = v + \frac{\theta(\delta-\delta^T)}{1-\delta}$. The condition (B92) is akin to the existence condition that we found in Lemma 3 in the main model. (B92) is equivalent to $\bar{\delta} < \delta \leq 1$ and $0 < \alpha < \bar{\alpha}$, where $\bar{\alpha} = \frac{45\delta-10-35\delta^T-\sqrt{100-108\delta+441\delta^2-92\delta^T+433\delta^{2T}-774\delta^{1+T}}}{22(\delta-\delta^T)}$ and $\bar{\delta} \geq \frac{1}{2}$ is implicitly defined from the equation $\bar{\alpha} = 0$.

Using the prices in (B93)-(B94), we obtain the firms' second-period subgame equilibrium profits:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(\delta-\delta^T)(\theta(3-\alpha)(1+\delta-2\delta^T)-2(v-p_{11})\alpha(1-\delta))^2}{2\theta(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(3\theta+4(p_{11}-v)\alpha)^2(1-\delta)(\delta-\delta^T)}{2\theta\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \qquad \text{(B95)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(\delta-\delta^T)(\theta(3+\alpha+3\delta-3\alpha\delta-2(3-\alpha)\delta^T)-4(v-p_{11})\alpha(1-\delta))^2}{2\theta(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(\delta-\delta^T)(2(v-p_{11})\alpha(1-\delta)-\theta(3-(3-2\alpha)\delta-2\alpha\delta^T))^2}{2\theta(1-\delta)\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \qquad \text{(B96)}$$

Firm 1's first-period profit is given by $\pi_{11} = \alpha\, \hat{d}\, p_{11}$. Plugging the prices from (B93)-(B94) into the expression for $\hat{d}$, we can obtain π_{11}:

$$\pi_{11} = \begin{cases} \alpha\, p_{11} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{p_{11}\alpha(3v(1-\alpha-\delta+\alpha\delta)+2\theta(3-\alpha)(\delta-\delta^T)-3p_{11}(1-\alpha)(1-\delta))}{\theta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3p_{11}\alpha((v-p_{11})(1-\delta)+\theta(\delta-\delta^T))}{\theta(3-(3-4\alpha)\delta-4\alpha\delta^T)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \qquad \text{(B97)}$$

In the first period, firm 1 chooses its price to maximize its overall profit $\pi_1 = \pi_{11} + \pi_{12}^{(1,2)}$. To simplify the analysis, we assume that $T = 5$.[18] Similar analysis as in the proof of Lemma 3 shows that firm 1's optimal price belongs to the interval $[\underline{\rho}, \tilde{\rho}]$ and given by:

$$p_{11}^{(1,2)} = \begin{cases} v-\theta & \text{if } 0 < \theta \le \underline{\theta} \\ \frac{v(9-13\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(5+\delta(5+\delta(5+3\delta))))-G}{2(9-11\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(4+\delta(4+\delta(4+3\delta))))} & \text{if } \underline{\theta} < \theta \le \bar{\theta} \\ v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1-2(1-\alpha)\delta^5)}{(6-7\alpha+3\alpha^2)(1-\delta)} & \text{if } \bar{\theta} < \theta \le \frac{v}{2} \end{cases} \qquad \text{(B98)}$$

where $G = \frac{2\theta(3-\alpha)(1+\delta)(1+\delta^2)(\alpha-(1+3\delta)(1+2\delta(1+\delta)(1+\delta^2))+\alpha\delta(5+2\delta(2+\delta(2+\delta(2+\delta)))))}{1-\alpha}$,

$\underline{\theta} = \frac{3v(1-\alpha)}{8(1-\alpha)-2\alpha\delta(2+\delta(2+\delta(2+\delta)))+2\delta(4+\delta(4+\delta(4+3\delta)))}$ and

$\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-6\alpha+\alpha^2(1+\delta)(1-3\delta)(1+\delta^2)+2\delta(1+\delta+\delta^2)+\alpha\delta(2+\delta(2+\delta(2+5\delta))))}$.

[18] We have conducted the analysis using different values of $T \ge 3$ and results have been very similar. We provide the analysis for $T = 5$ as a representative case.

By plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$ from (B93) and (B94), we can find the firms' second-period prices on the equilibrium path. Specifically,

$$p_{12}^{(1,2)} = \begin{cases} \theta(1+\delta)(1+\delta^2) & \text{if } 0 < \theta \leq \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(\theta(3-\alpha)(3+6\delta(1+\delta)(1+\delta^2)-\alpha(3+4\delta(1+\delta)(1+\delta^2)))-3v(1-\alpha)\alpha)}{(1-\alpha)(9-11\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(4+\delta(4+\delta(4+3\delta))))} & \text{if } \underline{\theta} < \theta \leq \bar{\theta} \\ \frac{\theta(6-5\alpha+\alpha^2)(1+\delta+\delta^2+\delta^3)}{6-7\alpha+3\alpha^2} & \text{if } \bar{\theta} < \theta \leq \frac{v}{2} \end{cases} \quad \text{(B99)}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta(1+\delta)(1+\delta^2) & \text{if } 0 < \theta \leq \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta)))))-6v(1-\alpha)\alpha)}{(1-\alpha)(9-11\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(4+\delta(4+\delta(4+3\delta))))} & \text{if } \underline{\theta} < \theta \leq \bar{\theta} \\ \frac{\theta(6-3\alpha-\alpha^2)(1+\delta+\delta^2+\delta^3)}{6-7\alpha+3\alpha^2} & \text{if } \bar{\theta} < \theta \leq \frac{v}{2} \end{cases} \quad \text{(B100)}$$

Next, using the equilibrium prices, we can obtain the firm's equilibrium profits in each period. Specifically,

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(v-\theta) & \text{if } \theta \leq \underline{\theta} \\ \frac{\alpha(9v(1-\alpha)-2\theta(3-\alpha)(1+\delta)(1-3\delta)(1+\delta^2))(K-L)}{4\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{(3-\alpha)\alpha(v(6-\alpha(7-3\alpha))-\theta(3-\alpha)(1-2(1-\alpha)\delta(1+\delta)(1+\delta^2)))}{(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{v}{2} \end{cases} \quad \text{(B101)}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{\theta(1-\alpha)(1+\delta)(1+\delta^2)}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(3v(1-\alpha)\alpha-\theta(3-\alpha)(3+6\delta(1+\delta)(1+\delta^2)-\alpha(3+4\delta(1+\delta)(1+\delta^2))))^2}{2\theta(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(3-\alpha)^2(2-\alpha)^2(1-\alpha)(1+\delta)(1+\delta^2)}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{v}{2} \end{cases} \quad \text{(B102)}$$

$$\pi_{21}^{(1,2)} = 0 \quad \text{(B103)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{\theta(1-\alpha)(1+\delta)(1+\delta^2)}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(6v(1-\alpha)\alpha-\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta))))))^2}{2\theta(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2(1+\delta)(1+\delta^2)}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{v}{2} \end{cases} \quad \text{(B104)}$$

where $K = \frac{v(1-\alpha)(9-13\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(5+\delta(5+\delta(5+3\delta))))}{1-\alpha}$ and

$L = \frac{2\theta(3-\alpha)(1+\delta)(1+\delta^2)(\alpha-(1+3\delta)(1+2\delta(1+\delta)(1+\delta^2))+\alpha\delta(5+2\delta(2+\delta(2+\delta(2+\delta)))))}{1-\alpha}$. Note that since the firms are symmetric, when $(R_1, R_2) = (2,1)$, the firms' profits are reversed, i.e., $\pi_{1t}^{(2,1)} = \pi_{2t}^{(1,2)}$ and $\pi_{2t}^{(2,1)} = \pi_{1t}^{(1,2)}$.

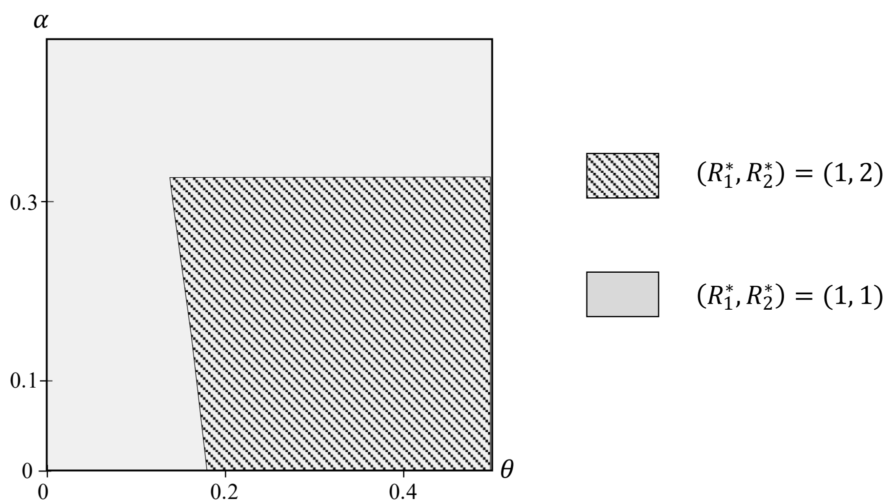
A similar proof as in Proposition 1 shows that a pure strategy equilibrium in product release times exists and in equilibrium, at least one of the firms will release its product in the first period. Hence, in equilibrium, either $(R_1, R_2) = (1,1)$ or $(R_1, R_2) = (1,2)$. Let us show that there exist $\hat{\alpha} \leq \bar{\alpha}$ and $\hat{\theta} < \frac{v}{2}$ such that if $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $(R_1, R_2) = (1,2)$ is an equilibrium.

To prove that $(R_1, R_2) = (1,2)$ is an equilibrium, we need to demonstrate that neither firm has a profitable deviation. If $R_2 = 2$, then firm 1 will not want to deviate from $R_1 = 1$. Hence, we need to show that firm 2 will not want to deviate from $R_2 = 2$, which is the case when $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. The technical analysis is very similar to that in the proof of Proposition 1. Specifically, one can show that if $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $\pi_2^{(1,2)} > \pi_2^{(1,1)}$, where $\hat{\alpha} \approx 0.32$ and

$$\hat{\theta} = \frac{6v((1-\alpha)^{3/2}(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))+(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta))))))}{\alpha^3(5-2\delta^4)^2+9(1+2\delta(1+\delta)(1+\delta^2))(23+2\delta(7+\delta(7+\delta(7+3\delta))))-3\alpha(131+4\delta(68+\delta(89+\delta(110+3\delta(40+\delta(22+\delta(3+\delta)(5+\delta)))))))+M}$$

and $M = \alpha^2(161 + 4\delta(69 + \delta(85 + \delta(101 + \delta(107 + \delta(46 + \delta(30 + (14 - 3\delta)\delta)))))))$. Note that $\underline{\theta} < \hat{\theta} < \bar{\theta}$ and satisfies $\frac{(1+\delta)(1+\delta^2)(6v(1-\alpha)\alpha-\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta))))))^2}{2\theta(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} = \frac{\theta(\delta-\delta^5)}{2(1-\delta)\delta}$ (i.e., $\pi_2^{(1,2)} = \pi_2^{(1,1)}$). Figure B5 graphically illustrates the region where $(R_1^*, R_2^*) = (1,2)$.

Figure B5 Firms' Equilibrium Product Release Times (R_1^*, R_2^*) [19]



[19] To obtain Figure B5, the following numerical values are used: $v = 1$, $\delta = 0.95$, and $T = 5$.

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