Online Appendix

A. Solving the optimization problem from section 4.2

First, notice that rebates for products targeted to type P consumers will never be larger than c. To see this, suppose that the product targeted to consumer-segment HP was offered with a rebate $x_{HP} > c$ (the argument for type LP is similar). Then, this consumer-segment would redeem the rebate; the cost incurred by type HP to purchase the product would be $p_{HP} - x_{HP} + c$. Instead, the firm could set price $p'_{HP} = p_{HP} - x_{HP} + c$ and set $x'_{HP} = 0$. These new prices do not change the cost incurred by type P consumers to purchase the product, and they make the products less appealing to type Z consumers, i.e. the new prices relax the incentive compatibility constraints. Moreover, the profit of the firm is larger with the new prices, because the cost incurred by type P consumers is entirely transferred to the firm (whereas if the rebate is larger than c, part of the cost incurred by type P consumers is wasted as a redemption cost). Because $x_{HP} \leq c$ and $x_{LP} \leq c$ we can write the profit function as follows.

$$\pi = \sum_{t \in \{LZ,HZ\}} \mu(t)\left(p_t - x_t - \frac{q_t^2}{2}\right) + \sum_{t \in \{LP,HP\}} \mu(t)\left(p_t - \frac{q_t^2}{2}\right)$$

The firm chooses products $\{(q_t, p_t, x_t)\}_{t \in \mathcal{T}}$ to maximize π subject to the constraints below.

$$IC(HP;HZ) \; : \; \theta_H q_{HP} - p_{HP} \geq \theta_H q_{HZ} - p_{HZ} \tag{C1}$$

$$IC(HP;LP) \; : \; \theta_H q_{HP} - p_{HP} \geq \theta_H q_{LP} - p_{LP} \tag{C2}$$

$$IC(HP;LZ) \; : \; \theta_H q_{HP} - p_{HP} \geq \theta_H q_{LZ} - p_{LZ} \tag{C3}$$

$$IR(HP) \; : \; \theta_H q_{HP} - p_{HP} \geq 0 \tag{C4}$$

$$IC(HZ;HP) \; : \; \theta_H q_{HZ} - p_{HZ} + x_{HZ} \geq \theta_H q_{HP} - p_{HP} + x_{HP} \tag{C5}$$

$$IC(HZ;LP) \; : \; \theta_H q_{HZ} - p_{HZ} + x_{HZ} \geq \theta_H q_{LP} - p_{LP} + x_{LP} \tag{C6}$$

$$IC(HZ;LZ) \; : \; \theta_H q_{HZ} - p_{HZ} + x_{HZ} \geq \theta_H q_{LZ} - p_{LZ} + x_{LZ} \tag{C7}$$

$$IR(HZ) \; : \; \theta_H q_{HZ} - p_{HZ} + x_{HZ} \geq 0 \tag{C8}$$

$$IC(LP;HZ) \; : \; \theta_L q_{LP} - p_{LP} \geq \theta_L q_{HZ} - p_{HZ} \tag{C9}$$

$$IC(LP;HP) \; : \; \theta_L q_{LP} - p_{LP} \geq \theta_L q_{HP} - p_{HP} \tag{C10}$$

$$IC(LP;LZ) \; : \; \theta_L q_{LP} - p_{LP} \geq \theta_L q_{LZ} - p_{LZ} \tag{C11}$$

$$IR(LP) \; : \; \theta_L q_{LP} - p_{LP} \geq 0 \tag{C12}$$

$$IC(LZ;HZ) \; : \; \theta_L q_{LZ} - p_{LZ} + x_{LZ} \geq \theta_L q_{HZ} - p_{HZ} + x_{HZ} \tag{C13}$$

$$IC(LZ;HP) \; : \; \theta_L q_{LZ} - p_{LZ} + x_{LZ} \geq \theta_L q_{HP} - p_{HP} + x_{HP} \tag{C14}$$

$$IC(LZ;LP) \; : \; \theta_L q_{LZ} - p_{LZ} + x_{LZ} \geq \theta_L q_{LP} - p_{LP} + x_{LP} \tag{C15}$$

$$IR(LZ) \; : \; \theta_L q_{LZ} - p_{LZ} + x_{LZ} \geq 0 \tag{C16}$$

Lemma A1 *The only individual rationality constraint that binds is that for type LP, (C12).*

Proof. For $t \in \{LZ, HZ, HP\}$, IC(t,LP) and IR(LP) imply IR(t). It follows that IR(LZ), IR(HZ) and IR(HP) are not binding.

It then follows that IR(LP) must bind, otherwise the firm could increase all prices by the same (small) amount, which would leave the ICs unaffected and increase profits. □

Lemma A2 $x^*_{LP} = x^*_{HP} = 0$

Proof. First suppose that $x_{LP} > 0$. Reducing x_{LP} relaxes constraints C6 and C15 and leaves the remaining constraints unaffected. Similarly, if $x_{HP} > 0$ then reducing x_{HP} relaxes constraints C5 and C14, while leaving the remaining constraints unaffected. □

Lemma A3 $q^*_{HZ} = q^*_{HP} = \theta_H$

Proof. Suppose, by contradiction, that $q_{HZ} < \theta_H$. The firm can increase q_{HZ} by ϵ and increase p_{HZ} by $\epsilon\theta_H$. This leaves (C1), (C5), (C6), (C7) unaffected, relaxes (C9) and (C13), and increases profits.

Similarly, if $q_{HH} < \theta_H$, the firm can increase q_{HP} by ϵ and increase p_{HP} by $\epsilon\theta_H$. This leaves (C1), (C2), (C3), (C5) unaffected, relaxes (C10) and (C14), and increases profits. □

I now denote $q_H \equiv \theta_H$.

Claim A1 *(C14) is not binding.*

Proof. (C13) and (C5) together with the result from Lemmas A2 and A3 imply (C14). □

Claim A2 *(C6) is not binding.*

Proof. (C2) and (C5) together with the result from Lemma A2 directly imply (C6). □

Claim A3 *(C9) is not binding.*

Proof. (C1) and (C10) together with the results from Lemmas A2 and A3 directly imply (C9). □

Claim A4 $q_{LP} \leq \theta_L$.

Proof. Suppose, by contradiction, that $q_{LP} > \theta_L$. The firm can decrease q_{LP} by ϵ and decrease p_{LP} by $\epsilon\theta_L$. This leaves (C10) and (C11) unaffected, relaxes (C2), and increases profits. □

Claim A5 *If (C2) is not binding then (C15) is binding.*

Proof. If both (C2) and (C15) are not binding, the firm can increase p_{HP}, p_{HZ} and p_{LZ} by ϵ. This leaves (C1), (C5), (C7), and (C13) unaffected, relaxes (C3), (C10), and (C11) and increases profits. □

Claim A6 *If (C5) is not binding then (C15) is binding.*

Proof. If both (C5) and (C15) are not binding, the firm can increase p_{HZ} and p_{LZ} by ϵ. This leaves (C7), and (C13) unaffected, relaxes (C1), (C3), and (C11) and increases profits. □

Claim A7 *If (C13) is not binding then (C15) is binding.*

Proof. If both (C13) and (C15) are not binding, the firm can increase p_{LZ} by ϵ. This relaxes (C3), (C7), and (C11), leaves the remaining constraints unaffected and increases profits. □

Claim A8 *One of the following constraints is not binding: (C2), (C5), (C13).*

Proof. Suppose all the constraints are binding. Then

$$\begin{aligned}
\theta_L q_{LZ} - p_{LZ} + x_{LZ} &= \theta_L q_H - p_{HZ} + x_{HZ} \\
&= \theta_L q_H - p_{HP} \\
&= \theta_L q_H - p_{LP} - \theta_H [q_H - q_{LP}] \\
&= \theta_L [q_H - q_{LP}] - \theta_H [q_H - q_{LP}] \\
&= (\theta_L - \theta_H)[q_H - q_{LP}] \\
&< 0
\end{aligned}$$

The first equality follows from (C13) binding, the second equality follows from (C5) binding, and the third equality follows from (C2) binding. The fourth equality follows from Lemma A1, the fifth equality follows from Lemma A3. Finally, the strict inequality follows from Lemma A3 and Claim A4. I find that if (C2), (C5), and (C13) are binding, then (C15) is violated, a contradiction. □

Corollary A1 *(C15) is binding.*

Proof. Follows directly from Claims A5, A6, A7 and A8. □

Claim A9 *(C10) is not binding.*

Proof. Using the result from Lemma A1, I can write (C10) as $p_{HP} \geq \theta_L q_H$. I will show that this result is implied by constraints (C5) and (C13).

Using the results from Lemmas A2 and A3, one can write (C5) as $p_{HP} \geq p_{HZ} - x_{HZ}$.

Using the results from Lemma A3 and Corollary A1, one can write (C13) as $p_{HZ} \geq \theta_L q_H + x_{HZ}$.

Combining the above two expressions, one obtains $p_{HP} \geq \theta_L q_H$, which is (C10). □

Claim A10 *(C11) is not binding.*

Proof. Using the results from Lemmas A2, A1, and Corollary A1, one can write (C11) as $x_{LZ} \geq 0$, which always holds. □

The maximization problem can be written as follows.

$$\max_{p_{HP},p_{HZ},x_{HZ},x_{LZ},q_{LZ},q_{LP}} \pi \equiv \mu(LZ)\left[\theta_L q_{LZ} - \frac{q_{LZ}^2}{2}\right] + \mu(LP)\left[\theta_L q_{LP} - \frac{q_{LP}^2}{2}\right] + \mu(HZ)\left[p_{HZ} - x_{HZ} - \frac{\theta_H^2}{2}\right] + \mu(HP)\left[p_{HP} - \frac{\theta_H^2}{2}\right]$$

s.t.

$$IC(HP;HZ) \quad : \quad p_{HZ} \geq p_{HP} \tag{C17}$$

$$IC(HP;LP) \quad : \quad p_{HP} \leq \theta_H q_H - (\theta_H - \theta_L) q_{LP} \tag{C18}$$

$$IC(HP;LZ) \quad : \quad p_{HP} \leq x_{LZ} + \theta_H q_H - (\theta_H - \theta_L) q_{LZ} \tag{C19}$$

$$IC(HZ;HP) \quad : \quad p_{HP} \geq p_{HZ} - x_{HZ} \tag{C20}$$

$$IC(HZ;LZ) \quad : \quad p_{HZ} \leq \theta_H q_H - (\theta_H - \theta_L) q_{LZ} + x_{HZ} \tag{C21}$$

$$IC(LZ;HZ) \quad : \quad p_{HZ} \geq \theta_L q_H + x_{HZ} \tag{C22}$$

Claim A11 $q_{LZ} \leq \theta_L$.

Proof. If $q_{LZ} > \theta_L$, the firm can reduce q_{LZ}, which relaxes the constraints and increases profit. □

Claim A12 *One of the following constraints is binding: (C19), (C21).*

Proof. I consider two cases.

Case 1: (C18) is not binding.

In this case, the firm can increase both p_{HP} and p_{HZ} by ϵ. Such increase relaxes (C22), leaves the remaining constraints unaffected and leads to higher profit.

Case 2: (C18) is binding.

Suppose, by contradiction, that neither (C19) nor (C21) bind. It follows that (C20) must bind (otherwise the firm could increase p_{HZ} which would lead to higher profit). Because both (C18) and (C20) bind, the maximization problem becomes:

$$\max_{x_{HZ},x_{LZ},q_{LZ},q_{LP}} \pi \equiv \mu(LZ)\left[\theta_L q_{LZ} - \frac{q_{LZ}^2}{2}\right] + \mu(LP)\left[\theta_L q_{LP} - \frac{q_{LP}^2}{2}\right]$$
$$+ \left\{\mu(HZ) + \mu(HP)\right\}\left[\theta_H q_H - (\theta_H - \theta_L)q_{LP} - \frac{\theta_H^2}{2}\right]$$

It then follows that it is optimal to set $q_{LZ} = \theta_L$ and $q_{LP} = \theta_L - (\theta_H - \theta_L)\frac{\mu(HZ)+\mu(HP)}{\mu(LP)}$.

This violates the constraints. Indeed, (C18), (C20) and (C21) imply that $q_{LZ} \leq q_{LP}$. □

Claim A13 *If (C19) is binding, then (C21) is also binding.*

Proof. If (C19) binds, then (C20) becomes $p_{HZ} \leq \theta_H q_H - (\theta_H - \theta_L)q_{LZ} + x_{HZ} + x_{LZ}$, which is implied by (C21). Hence, (C20) is not binding. If (C21) is also not binding, the firm can increase p_{HZ}, which relaxes the remaining constraints and increases profits. □

Corollary A2 *(C21) is binding.*

Proof. Follows directly from Claims A12 and A13. □

Corollary A3 *(C22) is not binding.*

Proof. Using the result from Corollary A2, one can write (C22) as $q_H \geq q_{LZ}$. It follows from Claim A11 and the definition of q_H that this always holds. □

Using the result above, one can write the monopolist's problem as follows.

$$\max_{p_{HP},x_{HZ},x_{LZ},q_{LZ},q_{LP}} \pi \equiv \mu(LZ)\left[\theta_L q_{LZ} - \frac{q_{LZ}^2}{2}\right] + \mu(LP)\left[\theta_L q_{LP} - \frac{q_{LP}^2}{2}\right]$$
$$+ \mu(HZ)\left[\theta_H q_H - (\theta_H - \theta_L)q_{LZ} - \frac{\theta_H^2}{2}\right] + \mu(HP)\left[p_{HP} - \frac{\theta_H^2}{2}\right]$$

subject to

$$IC(HP;HZ) \quad : \quad p_{HP} \leq \theta_H q_H - (\theta_H - \theta_L)q_{LZ} + x_{HZ} \tag{C23}$$

$$IC(HP;LP) \quad : \quad p_{HP} \leq \theta_H q_H - (\theta_H - \theta_L)q_{LP} \tag{C24}$$

$$IC(HP;LZ) \quad : \quad p_{HP} \leq \theta_H q_H - (\theta_H - \theta_L)q_{LZ} + x_{LZ} \tag{C25}$$

$$IC(HZ;HP) \quad : \quad p_{HP} \geq \theta_H q_H - (\theta_H - \theta_L)q_{LZ} \tag{C26}$$

Lemma A4 $x_{LZ} = x_{HZ}$.

Proof. Suppose that $x_{LZ} > x_{HZ}$ (the proof for the other case is identical). Then (C25) is not binding. The firm can then reduce x_{LZ} which leaves the constraints unaffected. □

Using the result from Lemma A4, I find that constraints (C23) and (C25) are equivalent, so I can drop one of them.

Claim A14 *(C24) is binding.*

Proof. If (C24) is not binding, then it is optimal to have $q_{LP} = \theta_L$. Moreover, combining (C26) with the fact that (C24) is not binding I obtain that $q_{LZ} > q_{LP}$, which then implies that $q_{LZ} > \theta_L$, which is a contradiction to Claim A11. □

Using the results from Lemma A4 and Claim A14, one can write the monopolist's problem as follows.

$$\max_{q_{LZ},q_{LP},x} \pi \equiv \mu(LZ)\left[\theta_L q_{LZ} - \frac{q_{LZ}^2}{2}\right] + \mu(LP)\left[\theta_L q_{LP} - \frac{q_{LP}^2}{2}\right]$$
$$+ \mu(HZ)\left[\theta_H q_H - (\theta_H - \theta_L)q_{LZ} - \frac{\theta_H^2}{2}\right] + \mu(HP)\left[\theta_H q_H - (\theta_H - \theta_L)q_{LP} - \frac{\theta_H^2}{2}\right]$$

subject to

$$IC(HP;LZ) \;:\; (\theta_H - \theta_L)(q_{LZ} - q_{LP}) \leq x \tag{C27}$$

$$IC(HZ;HP) \;:\; q_{LZ} \geq q_{LP} \tag{C28}$$

Notice that it was already shown that $x \leq c$.

Let q_{LZ}^I and q_{LP}^I denote the solutions to the unconstrained maximization problem, which are characterized in (19) and (18). If $c \leq (\theta_H - \theta_L)(q_{LZ}^I - q_{LP}^I)$, then (C27) is binding and the firm sets $x = c$. The solution to the above maximization problem is characterized in Lemma 7.

B. Details regarding the example in Section 6.2

Let $\mathcal{T}'$ denote the set of consumer types and let G_S and G_B denote, respectively, the cumulative distribution of redemption costs of types S and B.

$$\pi \equiv \sum_{t\in\{LZ,HZ\}} \mu(t)\left(p_t - x_t - \frac{q_t^2}{2}\right) + \sum_{t\in\{LS,HS\}} \mu(t)\left(p_t - G_S(x_t)x_t - \frac{q_t^2}{2}\right) + \sum_{t\in\{LB,HB\}} \mu(t)\left(p_t - G_B(x_t)x_t - \frac{q_t^2}{2}\right)$$

The firm chooses products $\{(q_t, p_t, x_t)\}_{t\in\mathcal{T}'}$ to maximize π subject to the individual rationality and incentive compatibility constraints.

Let us consider a relaxed problem, in which we do not consider the incentive compatibility constraints between redemption-segments. In this case, the firm chooses a product-line for each redemption-segment independently. It follows from Lemma 5 that the firm offers qualities $q_{HZ} = q_{HS} = q_{HB} = \theta_H = 30$ and $q_{LZ} = \theta_L - \frac{1-\lambda_Z}{\lambda_Z}(\theta_H - \theta_L) = 22.8$, $q_{LS} = \theta_L - \frac{1-\lambda_S}{\lambda_S}(\theta_H - \theta_L) = 21.5$, and $q_{LB} = \theta_L - \frac{1-\lambda_B}{\lambda_B}(\theta_H - \theta_L) = 20$. The prices paid by each consumer type are

$p^*_{LZ} = \theta_L q_{LZ} = 570$

$p^*_{HZ} = \theta_H q_{HZ} - (\theta_H - \theta_L)q_{LZ} = 786$

$p^*_{LS} = \theta_L q_{LS} = 537.5$

$p^*_{HS} = \theta_H q_{HS} - (\theta_H - \theta_L)q_{LS} = 792.5$

$p^*_{LB} = \theta_L q_{LB} = 500$

$p^*_{HB} = \theta_H q_{HB} - (\theta_H - \theta_L)q_{LB} = 800$

Now let us consider the general problem (in which all IC constraints are included). Consider the following products offered by the firm

$(q_{HZ}, p_{HZ}, x_{HZ}) = (30, p^*_{HZ} + x_{HZ} - 7.5\frac{\epsilon}{1-\epsilon}, \frac{6.5}{\epsilon} + \frac{7.5}{1-\epsilon})$

$(q_{LZ}, p_{LZ}, x_{LZ}) = (22.8, p^*_{LZ} + x_{LZ} - 7.5\frac{\epsilon}{1-\epsilon}, \frac{6.5}{\epsilon} + \frac{7.5}{1-\epsilon})$

$(q_{HS}, p_{HS}, x_{HS}) = (30, p^*_{HS} + (1-\epsilon)x_{HS}, \frac{7.5}{1-\epsilon})$

$(q_{LS}, p_{LS}, x_{LS}) = (21.5, p^*_{LS} + (1-\epsilon)x_{LS}, \frac{7.5}{1-\epsilon})$

$(q_{HB}, p_{HB}, x_{HB}) = (30, p^*_{HB}, 0)$

$(q_{LB}, p_{LB}, x_{LB}) = (20, p^*_{LB}, 0)$

It can be verified that the above products satisfy all the incentive compatibility and individual rationality constraints. Let us now inspect the effective prices that consumers pay when the firm offers the products above.

It is straightforward to see that the prices paid by consumers of type B are the same as the prices they pay in the relaxed problem, since the products targeted to them have no rebate. It is shown below that consumers of type S also pay the same effective price as in the relaxed problem. Notice that a consumer of type S redeems a rebate of size smaller than c with probability $1 - \epsilon$.

Price paid by type LS: $p_{LS} - (1 - \epsilon)x_{LS} = p^*_{LS}$

Price paid by type HS: $p_{HS} - (1 - \epsilon)x_{HS} = p^*_{HS}$

Finally, the prices paid by consumers of type Z are:

Price paid by type LZ: $p_{LZ} - x_{LZ} = p^*_{LZ} - 7.5\frac{\epsilon}{1-\epsilon}$

Price paid by type HZ: $p_{HZ} - x_{HZ} = p^*_{HZ} - 7.5\frac{\epsilon}{1-\epsilon}$

As ϵ converges to zero, the prices paid by consumers of type Z converge to the prices they pay in the relaxed problem. Therefore, as ϵ approaches zero, the profit that the firm obtains when offering the above products converges to the profit in the relaxed problem. It follows by continuity that, for small enough ϵ, the firm benefits from segmenting the market in three redemption-segments. By an argument similar to the one presented in Lemma A3, it follows that high valuation consumers are all served with the efficient quality, i.e. $q_{HZ} = q_{HS} = q_{HB} = \theta_H$. Therefore, segmenting the market in three redemption-segments is achieved by offering products of 4 different qualities and using different rebate sizes for products targeted to type Z and type S consumers.

C. Nonzero redemption costs

In this section, I consider an extension in which type Z consumers face redemption cost $c_Z > 0$, but still lower than the redemption cost faced by type P, i.e. $c_Z < c$. Similar to the analysis in online appendix A, only products targeted to type Z consumers will offer rebates. Hence, we can write the profit function as follows.

$$\pi = \sum_{t \in \{LZ, HZ\}} \mu(t) \left(p_t - x_t - \frac{q_t^2}{2} \right) + \sum_{t \in \{LP, HP\}} \mu(t) \left(p_t - \frac{q_t^2}{2} \right)$$

The firm chooses products $\{(q_t, p_t, x_t)\}_{t \in \mathcal{T}}$ to maximize π subject to the constraints below.

$$IC(HP;HZ) \quad : \quad \theta_H q_{HP} - p_{HP} \geq \theta_H q_{HZ} - p_{HZ} \tag{D1}$$

$$IC(HP;LP) \quad : \quad \theta_H q_{HP} - p_{HP} \geq \theta_H q_{LP} - p_{LP} \tag{D2}$$

$$IC(HP;LZ) \quad : \quad \theta_H q_{HP} - p_{HP} \geq \theta_H q_{LZ} - p_{LZ} \tag{D3}$$

$$IR(HP) \quad : \quad \theta_H q_{HP} - p_{HP} \geq 0 \tag{D4}$$

$$IC(HZ;HP) \quad : \quad \theta_H q_{HZ} - p_{HZ} + x_{HZ} - c_Z \geq \theta_H q_{HP} - p_{HP} \tag{D5}$$

$$IC(HZ;LP) \quad : \quad \theta_H q_{HZ} - p_{HZ} + x_{HZ} - c_Z \geq \theta_H q_{LP} - p_{LP} \tag{D6}$$

$$IC(HZ;LZ) \quad : \quad \theta_H q_{HZ} - p_{HZ} + x_{HZ} \geq \theta_H q_{LZ} - p_{LZ} + x_{LZ} \tag{D7}$$

$$IR(HZ) \quad : \quad \theta_H q_{HZ} - p_{HZ} + x_{HZ} - c_Z \geq 0 \tag{D8}$$

$$IC(LP;HZ) \quad : \quad \theta_L q_{LP} - p_{LP} \geq \theta_L q_{HZ} - p_{HZ} \tag{D9}$$

$$IC(LP;HP) \quad : \quad \theta_L q_{LP} - p_{LP} \geq \theta_L q_{HP} - p_{HP} \tag{D10}$$

$$IC(LP;LZ) \quad : \quad \theta_L q_{LP} - p_{LP} \geq \theta_L q_{LZ} - p_{LZ} \tag{D11}$$

$$IR(LP) \quad : \quad \theta_L q_{LP} - p_{LP} \geq 0 \tag{D12}$$

$$IC(LZ;HZ) \quad : \quad \theta_L q_{LZ} - p_{LZ} + x_{LZ} \geq \theta_L q_{HZ} - p_{HZ} + x_{HZ} \tag{D13}$$

$$IC(LZ;HP) \quad : \quad \theta_L q_{LZ} - p_{LZ} + x_{LZ} - c_Z \geq \theta_L q_{HP} - p_{HP} \tag{D14}$$

$$IC(LZ;LP) \quad : \quad \theta_L q_{LZ} - p_{LZ} + x_{LZ} - c_Z \geq \theta_L q_{LP} - p_{LP} \tag{D15}$$

$$IR(LZ) \quad : \quad \theta_L q_{LZ} - p_{LZ} + x_{LZ} - c_Z \geq 0 \tag{D16}$$

Lemmas A1 and A3, and Claims A1-A10 are still valid in this setting. The maximization problem can then be written as follows.

$$\max_{p_{HP}, p_{HZ}, x_{HZ}, x_{LZ}, q_{LZ}, q_{LP}} \pi \equiv \mu(LZ)\left[\theta_L q_{LZ} - c_Z - \frac{q_{LZ}^2}{2}\right] + \mu(LP)\left[\theta_L q_{LP} - \frac{q_{LP}^2}{2}\right]$$
$$+ \mu(HZ)\left[p_{HZ} - x_{HZ} - \frac{\theta_H^2}{2}\right] + \mu(HP)\left[p_{HP} - \frac{\theta_H^2}{2}\right]$$

s.t.

$$IC(HP;HZ) \quad : \quad p_{HZ} \geq p_{HP} \tag{D17}$$

$$IC(HP;LP) \quad : \quad p_{HP} \leq \theta_H q_H - (\theta_H - \theta_L) q_{LP} \tag{D18}$$

$$IC(HP;LZ) \quad : \quad p_{HP} \leq x_{LZ} - c_Z + \theta_H q_H - (\theta_H - \theta_L) q_{LZ} \tag{D19}$$

$$IC(HZ;HP) \quad : \quad p_{HP} \geq p_{HZ} - x_{HZ} + c_Z \tag{D20}$$

$$IC(HZ;LZ) \quad : \quad p_{HZ} \leq \theta_H q_H - (\theta_H - \theta_L) q_{LZ} + x_{HZ} - c_Z \tag{D21}$$

$$IC(LZ;HZ) \quad : \quad p_{HZ} \geq \theta_L q_H + x_{HZ} - c_Z \tag{D22}$$

Claims A11, A12 and A13 still hold, so we can write the maximization problem as follows

$$\max_{p_{HP}, p_{HZ}, x_{HZ}, x_{LZ}, q_{LZ}, q_{LP}} \pi \equiv \mu(LZ)\left[\theta_L q_{LZ} - c_Z - \frac{q_{LZ}^2}{2}\right] + \mu(LP)\left[\theta_L q_{LP} - \frac{q_{LP}^2}{2}\right]$$
$$+ \mu(HZ)\left[\theta_H q_H - (\theta_H - \theta_L) q_{LZ} - c_Z - \frac{\theta_H^2}{2}\right] + \mu(HP)\left[p_{HP} - \frac{\theta_H^2}{2}\right]$$

s.t.

$$IC(HP;HZ) \quad : \quad p_{HP} \leq \theta_H q_H - (\theta_H - \theta_L) q_{LZ} + x_{HZ} - c_Z \tag{D23}$$

$$IC(HP;LP) \quad : \quad p_{HP} \leq \theta_H q_H - (\theta_H - \theta_L) q_{LP} \tag{D24}$$

$$IC(HP;LZ) \quad : \quad p_{HP} \leq x_{LZ} - c_Z + \theta_H q_H - (\theta_H - \theta_L) q_{LZ} \tag{D25}$$

$$IC(HZ;HP) \quad : \quad p_{HP} \geq \theta_H q_H - (\theta_H - \theta_L) q_{LZ} \tag{D26}$$

Lemma A4 and Claim A14 still hold. Hence, we can write the maximization problem as follows.

$$
\begin{aligned}
\max_{p_{HP}, p_{HZ}, x_{HZ}, x_{LZ}, q_{LZ}, q_{LP}} \pi \equiv \mu(LZ)\left[\theta_L q_{LZ} - c_Z - \frac{q_{LZ}^2}{2}\right] + \mu(LP)\left[\theta_L q_{LP} - \frac{q_{LP}^2}{2}\right] \\
+ \mu(HZ)\left[\theta_H q_H - (\theta_H - \theta_L) q_{LZ} - c_Z - \frac{\theta_H^2}{2}\right] \\
+ \mu(HP)\left[\theta_H q_H - (\theta_H - \theta_L) q_{LP} - \frac{\theta_H^2}{2}\right]
\end{aligned}
$$

s.t.

$$
IC(HP; LZ) \quad : \quad (\theta_H - \theta_L)(q_{LZ} - q_{LP}) \leq x - c_Z \tag{D27}
$$

$$
IC(HZ; HP) \quad : \quad q_{LZ} \geq q_{LP} \tag{D28}
$$

Moreover, it was already shown that $x \leq c$, so that type P consumers do not redeem the rebate. Notice that if c is large enough, the firm can choose a large value of x so that ($D27$) is not binding. The firm's problem in this case is similar to the maximization problem in the main setting: notice that c_Z only impacts the profit function by a reduction of fixed amount. Moreover, even if c is not large, although the optimal qualities offered by the firm will be different from those offered in the setting of zero redemption costs, the role of rebates is still the same.