(For Online Publication)

Online Appendix

A Asymptotics

The asymptotics is built on $T \to \infty$ with the number of products J and the number of groups G being fixed. Let $\theta \equiv (\theta_1, \theta_2)$, where $\theta_1 \equiv (\tau, \omega, \rho, (\delta_2 - \delta_1)/(1-\beta), \ldots, (\delta_J - \delta_1)/(1-\beta), \gamma/(1-\beta), \alpha^{(1)})$, and $\theta_2 \equiv (\beta, \delta_1)$. We decompose θ into these two parts, because the estimation of θ_2 relies on the estimation of θ_1. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ denote the estimator of θ_1 and θ_2, respectively.

It is easier to derive the asymptotic distribution backwardly from $\theta_2 \equiv (\beta, \delta_1)$. We use eq. (Linear-Reg-2) to estimate θ_2. In the estimation of θ_2, we fix U at certain number. In particular, we let $U = 0$ here to simplify the discussion. For exposition simplicity, we ignore the dependence on U below. Let

$$\hat{W}^{(g)}(\hat{\theta}_1) \equiv T^{-1}\sum_{t=1}^{T} W_t^{(g)}(U=0, \hat{\theta}_1), \qquad \text{and} \qquad \hat{\ell}^{(g)}(\hat{\theta}_1) \equiv T^{-1}\sum_{t=1}^{T} \ln \sigma_{0t}^{(g)}(U=0; \hat{\theta}_1).$$

Then

$$\hat{\theta}_2 = [A(\hat{\theta}_1)'A(\hat{\theta}_1)]^{-1}A(\hat{\theta}_1)'Y_2(\hat{\theta}_1).$$

where $A(\hat{\theta}_1)$ is $G \times 2$ matrix, and $Y_2(\hat{\theta}_1)$ is $G \times 1$ vector defined below:

$$A(\hat{\theta}_1) \equiv \begin{bmatrix} 1 & \hat{W}^{(1)}(\hat{\theta}_1) + \hat{\ell}^{(1)}(\hat{\theta}_1) \\ \vdots & \vdots \\ 1 & \hat{W}^{(G)}(\hat{\theta}_1) + \hat{\ell}^{(G)}(\hat{\theta}_1) \end{bmatrix} \qquad \text{and} \qquad Y_2(\hat{\theta}_1) \equiv \begin{bmatrix} \hat{W}^{(1)}(\hat{\theta}_1) \\ \vdots \\ \hat{W}^{(G)}(\hat{\theta}_1) \end{bmatrix}.$$

They are defined in this way so that Y_2 is the "dependent variable" and A is the "design matrix" of eq. (Linear-Reg-2). Because $\hat{\theta}_2$ is analytical function of random variables $\hat{W}^{(g)}(\hat{\theta}_1)$ and $\hat{\ell}^{(g)}(\hat{\theta}_1)$, its variance can be easily obtained by simulation provided that we know the asymptotic distribution of $\hat{W}^{(g)}(\hat{\theta}_1)$ and $\hat{\ell}^{(g)}(\hat{\theta}_1)$.

Now, we derive the distribution of $\hat{W}^{(g)}(\hat{\theta}_1)$ and $\hat{\ell}^{(g)}(\hat{\theta}_1)$, whose definition requires $W_t^{(g)}(0; \hat{\theta}_1)$ and $\sigma_{0t}^{(g)}(0; \hat{\theta}_1)$. We have

$$\begin{aligned} W_t^{(g)}(0; \hat{\theta}_1) &= \ln\left[\frac{\hat{\sigma}_{1t}^{(g)}(0)}{\hat{\sigma}_{0t}^{(g)}(0)}\right] - X_{1t}'\widehat{\Big(\frac{\gamma}{1-\beta}\Big)} + (\hat{\alpha}^{(1)} + \hat{\tau}^{(g)})P_{1t} \\ &= \ln\left[\frac{\hat{\sigma}_{1t}^{(1)}(\hat{\tau}^{(g)}/\hat{\omega})}{\hat{\sigma}_{0t}^{(1)}(\hat{\tau}^{(g)}/\hat{\omega})}\right] - X_{1t}'\widehat{\Big(\frac{\gamma}{1-\beta}\Big)} + (\hat{\alpha}^{(1)} + \hat{\tau}^{(g)})P_{1t} \\ &= \Big[\hat{\rho}_{jt1} + \hat{\rho}_{jt2}\frac{\hat{\tau}^{(g)}}{\hat{\omega}} + \hat{\rho}_{jt3}\Big(\frac{\hat{\tau}^{(g)}}{\hat{\omega}}\Big)^2\Big] - X_{1t}'\widehat{\Big(\frac{\gamma}{1-\beta}\Big)} + (\hat{\alpha}^{(1)} + \hat{\tau}^{(g)})P_{1t}, \end{aligned}$$

and $\sigma_{0t}^{(g)}(0; \hat{\theta}_1) = \sigma_{0t}^{(1)}(\hat{\tau}^{(g)}/\hat{\omega}; \hat{\theta}_1)$ has the series logit expression. Both $W_t^{(g)}(0; \hat{\theta}_1)$ and $\sigma_{0t}^{(g)}(0; \hat{\theta}_1)$ are analytical functions of $\hat{\theta}_1$. We then can determine the variance of $\hat{W}^{(g)}(\hat{\theta}_1)$ and $\hat{\ell}^{(g)}(\hat{\theta}_1)$ by randomly drawing samples from the asymptotic distribution of $\hat{\theta}_1$.

Lastly, we derive the distribution of $\hat{\theta}_1$. We estimate θ_1 by

$$\hat{\theta}_1 \equiv \arg\min_{\theta_1 \in \Theta_1} T^{-1} \sum_{t=1}^{T} h_t(\theta)' h_t(\theta)$$

subject to constraints eq. (B.4) below,

where

$$h_t(\theta_1)' \equiv (h_{1t}(\theta_1)', h_{2t}(\theta_1)', \ldots, h_{Jt}(\theta_1)'),$$

and

$$h_{jt}(\theta_1) \equiv \begin{bmatrix} S_{jt}^{(1)} - \mathrm{GH}_{jt}^{(G)}(\tau, \omega, \rho) \\ \vdots \\ S_{jt}^{(G)} - \mathrm{GH}_{jt}^{(G)}(\tau, \omega, \rho) \\ X_{jt}^{IV} \left[Y_{jt} - \frac{\delta_j - \delta_1}{1-\beta} - (X_{jt} - X_{1t})' \frac{\gamma}{1-\beta} + \alpha^{(1)}(P_{jt} - P_{1t}) \right] \end{bmatrix}.$$

Here X_{jt}^{IV} is a vector of IV in eq. (Linear-Reg-1) satisfying $\mathrm{E}[X_{jt}^{IV}(\xi_{jt} - \xi_{1t})] = 0$. This is a standard M-estimation problem, so under the regularity conditions, $\sqrt{T}(\hat{\theta}_1 - \theta_1) \to_d \mathcal{N}(0, \Sigma_1)$. The asymptotic variance Σ_1 is readily reported by most statistical softwares, or computed using numerical score and Hessian matrices.

B Proofs

Proof of Proposition 1. First, without attrition, the pool of consumers does not change with time. So that $f_t^{(g)}(u) = f_1^{(g)}(u) = \phi(u)$ for any period t. In the rest, we focus on the case of attrition.

When $t = 1$, $f_1^{(g)}(u) = \phi(u)$ by Assumption 5. We will prove

$$f_t^{(g)}(u) = \phi(u) \times \prod_{s=1}^{t-1} \frac{\sigma_{0t}^{(g)}(u)}{S_{0t}^{(g)}}, \qquad t \geq 2. \tag{B.1}$$

by the induction. Define a few notations for exposition. Let A_{it} denote the discrete purchasing choice made by consumer i in period t. Particularly, $A_{it} = 0$ means not purchase in period t. Let $Z_t \equiv (X_t', P_t', \xi_t')'$ denote the vector of product characteristics in period t. Also recall that $D_i^{(g)} = 1$ denotes that i is from group g.

When we randomly draw a consumer i with unobserved price sensitivity U_i from group g, $f_t^{(g)}(u)$ is the probability that $U_i = u$ provided that consumer i still exists in period t. Because consumers leave the market after purchasing, a consumer would exist in period t if and only if she had chosen not to purchase in all the previous periods given the past product characteristics. In other words, $f_t^{(g)}(u)$ is the probability that $U_i = u$ conditional on that $D_i^{(g)} = 1$ (so i is from group g) and consumer i did not purchase from period 1 to $t-1$ with the past product characteristics $Z_1, \ldots, Z_{t-1}$. That is

$$f_t^{(g)}(u) = \Pr(U_i = u \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_{t-1}).$$

The above conditioning variables just restrict the population to be the remaining consumers after $t-1$ periods. Because all the remaining consumers in period t face the same product state variables Z_t, we also have the following conditional independence,

$$\begin{aligned} f_t^{(g)}(u) &= \Pr(U_i = u \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_{t-1}) \\ &= \Pr(U_i = u \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_{t-1}, Z_t). \end{aligned} \tag{B.2}$$

We now prove eq. (B.1) by the induction. Starting with period 2, we have

$$\begin{aligned} f_2^{(g)}(u) &= \Pr(U_i = u \mid D_i^{(g)} = 1, A_{i1} = 0, Z_1) \\ &= \frac{f(U_i = u \mid D_i^{(g)} = 1, Z_1) f(A_{i1} = 0 \mid U_i = u, D_i^{(g)} = 1, Z_1)}{f(A_{i1} = 0 \mid D_i^{(g)} = 1, Z_1)} \\ &= \phi(u) \times \frac{\sigma_{01}^{(g)}(u)}{S_{01}^{(g)}}. \end{aligned}$$

The third line used $f(U_i = u \mid D_i^{(g)} = 1, Z_1) = f(U_i = u \mid D_i^{(g)} = 1)$ because all consumers in group g face the same product characteristics Z_1 in the first period.

Suppose eq. (B.1) holds for period t. We will prove that this equation also holds for period $t+1$. We have

$$\begin{aligned} f_{t+1}^{(g)}(u) &= \Pr(U_i = u \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{i,t-1} = 0, A_{it} = 0, Z_1, \ldots, Z_t) \\ &= \Pr(U_i = u \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{it-1} = 0, Z_1, \ldots, Z_t) \\ &\quad \times \frac{f(A_{it} = 0 \mid U_i = u, D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{it-1} = 0, Z_1, \ldots, Z_t)}{f(A_{it} = 0 \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{it-1} = 0, Z_1, \ldots, Z_t)} \\ &= f_t^{(g)}(u) \times \frac{f(A_{it} = 0 \mid U_i = u, D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{it-1} = 0, Z_1, \ldots, Z_t)}{f(A_{it} = 0 \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{it-1} = 0, Z_1, \ldots, Z_t)} \quad \text{by eq. (B.2)} \\ &= f_t^{(g)}(u) \frac{\sigma_{0t}^{(g)}(u)}{S_{0t}^{(g)}} = \phi(u) \times \prod_{s=1}^{t} \frac{\sigma_{0s}^{(g)}(u)}{S_{0s}^{(g)}}. \end{aligned}$$

Note that the purchase choice A_{it} in period t depends only on the payoffs v_{ijt}, which are functions of $(U_i, D_i^{(g)}, Z_t)$ only. So that $A_{it} \perp\!\!\!\perp (A_{i1}, \ldots, A_{it-1}, Z_1, \ldots, Z_{t-1}) \mid (U_i, D_i^{(g)}, Z_t)$, and the last line follows. This completes the proof. ■

Proposition B.1 (Group composition due to attrition). *Suppose consumers leave the market after purchasing. Let $\pi_t^{(g)}$ denote the proportion of group g consumers in period t, and let S_{0t} denote the share of consumers who choose the outside option (not purchase) in period t. We have*

$$\pi_t^{(g)} = \pi_1^{(g)} \times \Big(\prod_{s=1}^{t-1} \frac{S_{0s}^{(g)}}{S_{0s}} \Big). \tag{B.3}$$

Proof. The proof is similar to the proof of Proposition 1 and we keep using the notation defined in that proof. We prove by induction. Starting from period 2, we have the following by definition,

$$\begin{aligned} \pi_2^{(g)} &= \Pr(D_i^{(g)} = 1 \mid A_{i1} = 0, Z_1) \\ &= \frac{\Pr(D_i^{(g)} = 1 \mid Z_1) \Pr(A_{i1} = 0 \mid D_i^{(g)} = 1, Z_1)}{\Pr(A_{i1} = 0, Z_1)} \\ &= \pi_1^{(g)} \times \frac{S_{01}^{(g)}}{S_{01}}. \end{aligned}$$

Suppose eq. (B.3) holds for period t. We want to show the statement holds for period $t+1$, we have

$$\begin{aligned}\pi_{t+1}^{(g)} &= \Pr(D_i^{(g)} = 1 \mid A_{i1} = 0, \ldots, A_{i,t-1} = 0, A_{it} = 0, Z_1, \ldots, Z_t) \\ &= \Pr(D_i^{(g)} = 1 \mid A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_{t-1}, Z_t) \\ &\quad \times \frac{\Pr(A_{it} = 0 \mid D_i^{(g)} = 1, A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_t)}{\Pr(A_{it} = 0 \mid A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_t)} \\ &= \pi_t^{(g)} \frac{S_{0t}^{(g)}}{S_{0t}}.\end{aligned}$$

The last line follows because the vector product characteristics Z_t is the same for different groups of remaining consumers after $t-1$ periods, so that $\Pr(D_i^{(g)} = 1 \mid A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_{t-1}, Z_t) = \Pr(D_i^{(g)} = 1 \mid A_{i1} = 0, \ldots, A_{i,t-1} = 0, Z_1, \ldots, Z_{t-1}) = \pi_t^{(g)}$. This completes the proof. ■

Proof of Proposition 2. We claim that the dynamic model implies the following constraints on the parameters ρ_t in the CCP function:

$$\rho_{jt2} - \rho_{1t2} = -\omega(P_{jt} - P_{1t}) \qquad \text{and} \qquad \rho_{jt3} - \rho_{1t3} = 0, \qquad j = 2, \ldots, J. \tag{B.4}$$

The above constraints eliminate many parameters, leaving the following to estimate in the NLS problem, eq. (5):

$$(\rho_{1t1}, \ldots, \rho_{Jt1})', \quad \rho_{1t2}, \quad \rho_{1t3}, \quad \omega, \quad \tau, \qquad \text{for } t = 1, \ldots, T.$$

The degree of freedom of the NLS problem is $JGT - JT - 2T - G$, where JGT is the number of observations, JT results from ρ_{jt1} for each $j = 1, \ldots, J$ and $t = 1, \ldots, T$, $2T$ comes from (ρ_{1t2}, ρ_{1t3}) for each $t = 1, \ldots, T$, and G refers to one ω plus $(G-1) \times 1$ vector τ. The most stringent case is when $G = 2$, in which we need at least three products ($J \geq 3$) and $(J-2)T > G$. In practice, such NLS with the above constraints takes very little time and is robust to the choice of initial guess.

To see how we obtain the above constraints, note that in dynamic model, we have $\ln[\sigma_{jt}^{(g)}(U)/\sigma_{1t}^{(g)}(U)] = v_{jt}^{(g)}(U) - v_{1t}^{(g)}(U)$. Using the definition of the payoffs, we can compute the derivative:

$$\frac{\mathrm{d} \ln\left[\sigma_{jt}^{(g)}(U)/\sigma_{1t}^{(g)}(U)\right]}{\mathrm{d}U} = -\omega(P_{jt} - P_{1t})$$

By the series logit approximation, we have

$$\ln\left[\frac{\sigma_{jt}^{(g)}(U)}{\sigma_{1t}^{(g)}(U)}\right] = (\rho_{jt1} - \rho_{1t1}) + \left[\left(\frac{\rho_{jt2}}{\omega}\right) - \left(\frac{\rho_{1t2}}{\omega}\right)\right](\omega U + \tau^{(g)}) + \left[\left(\frac{\rho_{jt3}}{\omega^2}\right) - \left(\frac{\rho_{1t3}}{\omega^2}\right)\right](\omega U + \tau^{(g)})^2, \tag{B.5}$$

which implies

$$\frac{\mathrm{d} \ln\left[\sigma_{jt}^{(g)}(U)/\sigma_{1t}^{(g)}(U)\right]}{\mathrm{d}U} = \omega\left[\left(\frac{\rho_{jt2}}{\omega}\right) - \left(\frac{\rho_{1t2}}{\omega}\right)\right] + 2\left[\left(\frac{\rho_{jt3}}{\omega^2}\right) - \left(\frac{\rho_{1t3}}{\omega^2}\right)\right](\omega U + \tau^{(g)})\omega.$$

Equalizing the two formulas of the same derivative gives rise to the conclusion in eq. (B.4).

A useful conclusion is the following. Applying the constraints about ρ to eq. (B.5) for the first group, $g = 1$, we have

$$\ln\left[\frac{\sigma_{jt}^{(1)}(U)}{\sigma_{1t}^{(1)}(U)}\right] = (\rho_{jt,1} - \rho_{1t,1}) - \omega U(P_{jt} - P_{1t}).$$

The dependent variable Y_{jt} of eq. (Linear-Reg-1), whose definition is copied below, has a simple expression,

$$Y_{jt} \equiv \int \ln\left[\frac{\sigma_{jt}^{(1)}(U)}{\sigma_{1t}^{(1)}(U)}\right] \mathrm{d}\, F_t^{(1)}(U) + \omega(P_{jt} - P_{1t}) \int U \ \mathrm{d}\, F_t^{(1)}(U) = \rho_{jt,1} - \rho_{1t,1}.$$

This is useful, because the NLS step will estimate $\rho_{jt,1} - \rho_{1t,1}$. After which, one can estimate $(\delta_j - \delta_1)/(1-\beta)$, $\gamma/(1-\beta)$ and $\alpha^{(1)}$ by running 2SLS of $(\rho_{jt,1} - \rho_{1t,1})$ on $(X_{jt} - X_{1t})$ and $(P_{jt} - P_{1t})$ according to eq. (Linear-Reg-1). This also proves Proposition 2. ■

Proof of Proposition 3. For expositional simplicity, we omit the market index m in this proof. Also note that $U = (U_1, U_2)'$ involves two-dimensional unobserved heterogeneity, and U_1 and U_2 are associated with a variable MPGe and price, respectively. Correspondingly, $\tau^{(g)} = (\tau_1^{(g)}, \tau_2^{(g)})'$. Thus, what is below is a generalized proof of for two dimensions of heterogenity and not just one. We begin by first specifying the log market share ratio under each policy where the current policy follows from eq. (E.1), and the new policy takes the form of

$$\ln\left[\frac{\breve{\sigma}_{jt}^{(g)}(U)}{\breve{\sigma}_{0t}^{(g)}(U)}\right] = \frac{\delta_j}{1-\beta} + X_{jt}'\frac{\gamma_i}{1-\beta} - (\alpha^{(1)} + \tau_2^{(g)} + \omega_2 U_2)\tilde{P}_{jt} + \frac{\xi_{jt}}{1-\beta} - \beta\, \mathrm{E}[\bar{V}_{t+1}^{(g)}(U) \mid X_t, P_t, \xi_t], \quad \text{(B.6)}$$

where $\tilde{P}_{jt}$ is the new price of the EV vehicle with the proposed tax credit. Its series representation is that

$$\ln\left[\frac{\breve{\sigma}_{jt}^{(g)}(U)}{\breve{\sigma}_{0t}^{(g)}(U)}\right] = \tilde{\rho}_{jt1} + \tilde{\rho}_{jt2}'\Omega^{-1}\Big(\Omega U + \tau^{(g)}\Big) + \Big(\Omega U + \tau^{(g)}\Big)'\Omega^{-1}\tilde{\rho}_{jt3}\Omega^{-1}\Big(\Omega U + \tau^{(g)}\Big),$$

with unknown coefficients $\tilde{\rho}_{jt1}, \tilde{\rho}_{jt2}$, and $\tilde{\rho}_{jt3}$. Ω is a diagonal matrix whose diagonal elements are ω_1 and ω_2. The objective is to solve for $\tilde{\rho}_{jt1}, \tilde{\rho}_{jt2}$, and $\tilde{\rho}_{jt3}$. Define $\tilde{P}_{jt} = P_{jt} + \Delta P_{jt}$. First note that by eq. (E.1), we have

$$\begin{aligned}
\ln\left[\frac{\breve{\sigma}_{jt}^{(g)}(U)}{\breve{\sigma}_{0t}^{(g)}(U)}\right] &= \ln\left[\frac{\sigma_{jmt}^{(g)}(U)}{\sigma_{0mt}^{(g)}(U)}\right] - (\alpha^{(1)} + \tau_2^{(g)} + \omega_2 U_2)\Delta P_{jt} \\
&= \rho_{jt1} + \rho_{jt2}'\Omega^{-1}\Big(\Omega U + \tau^{(g)}\Big) + \Big(\Omega U + \tau^{(g)}\Big)'\Omega^{-1}\rho_{jt3}\Omega^{-1}\Big(\Omega U + \tau^{(g)}\Big) \\
&\qquad - (\alpha^{(1)} + \tau_2^{(g)} + \omega_2 U_2)\Delta P_{jt} \\
&= \rho_{jt1} - (\alpha^{(1)} + \tau_2^{(g)})\Delta P_{jt} + \rho_{jt2}'\Omega^{-1}\Big(\Omega U + \tau^{(g)}\Big) + \Big(\Omega U + \tau^{(g)}\Big)'\Omega^{-1}\rho_{jt3}\Omega^{-1}\Big(\Omega U + \tau^{(g)}\Big) \\
&\qquad - \omega_2 U_2 \Delta P_{jt}.
\end{aligned}$$

Note that ρ_{jt2} is a two-dimensional vector:

$$\rho_{jt2} = \begin{pmatrix} \rho_{jt2,1} \\ \rho_{jt2,2} \end{pmatrix} \qquad \text{and} \qquad \rho_{jt2}'\Omega^{-1} = (\rho_{jt2,1}\omega_1^{-1}, \rho_{jt2,2}\omega_2^{-1}),$$

where $\rho_{jt2,1}$ is associated with MPGe, and $\rho_{jt2,2}$ is with price. Next, we write $\omega_2 U_2 \Delta P_{jt} = (\omega_2 U_2 + \tau_2^{(g)})\Delta P_{jt} - \tau_2^{(g)}\Delta P_{jt}$ so that

$$\ln\left[\frac{\breve{\sigma}_{jt}^{(g)}(U)}{\breve{\sigma}_{0t}^{(g)}(U)}\right] = \rho_{jt1} - \alpha^{(1)}\Delta P_{jt} + \begin{pmatrix} \rho_{jt2,1} \\ \rho_{jt2,2} - \Delta P_{jt}\omega_2 \end{pmatrix} \Omega^{-1}\Big(\Omega U + \tau^{(g)}\Big) + \frac{\rho_{jt3}}{\omega^2}\Big(\Omega U + \tau^{(g)}\Big)^2.$$

Thus,

$$\tilde{\rho}_{jt1} = \rho_{jt1} - \alpha^{(1)}\Delta P_{jt}, \qquad \tilde{\rho}_{jt2} \equiv \begin{pmatrix} \tilde{\rho}_{jt2,1} \\ \tilde{\rho}_{jt2,2} \end{pmatrix} = \begin{pmatrix} \rho_{jt2,1} \\ \rho_{jt2,2} - \Delta P_{jt}\omega_2 \end{pmatrix}, \qquad \tilde{\rho}_{jt3} = \rho_{jt3}.$$

■

C Initial values

Good initial values help solve the NLS in eq. (5). We need initial values of $\tau_{\text{init}} = (\tau_{\text{init}}^{(2)}, \ldots, \tau_{\text{init}}^{(G)})'$, ρ_{init}, and ω_{init}. We follow two steps to obtain the initial values, and these steps are based on the first order Taylor expansion of CCP function $\sigma_{jt}^{(g)}(U)$.

In the *first step*, we find τ_{init} by running 2SLS for the following linear regression,

$$\ln\left(\frac{S_{jt}^{(g)}}{S_{1t}^{(g)}}\right) = \frac{\delta_j - \delta_1}{1-\beta} + (X_{jt} - X_{1t})'\frac{\gamma}{1-\beta} - (\alpha^{(1)} + \tau^{(g)})(P_{jt} - P_{1t}) + \frac{\xi_{jt} - \xi_{1t}}{1-\beta}.$$

To see the rationale, recall the identity

$$S_{jt}^{(g)} = \mathrm{E}[\sigma_{jt}^{(g)}(U^*)\mathit{\Gamma}_t^{(g)}(U^*)], \qquad U^* \sim \mathcal{N}(0,1),$$

and consider the first order Taylor expansion of CCP function $\sigma_{jt}^{(g)}(U^*)\mathit{\Gamma}_t^{(g)}(U^*)$ at 0, which is the mean of $U^* \sim \mathcal{N}(0,1)$, for each group g. We have

$$\begin{aligned} S_{jt}^{(g)} &= \mathrm{E}[\sigma_{jt}^{(g)}(U^*)\mathit{\Gamma}_t^{(g)}(U^*)] \\ &\approx \sigma_{jt}^{(g)}(0)\mathit{\Gamma}_t^{(g)}(0) + \left(\frac{\mathrm{d}\sigma_{jt}^{(g)}(U^*)\mathit{\Gamma}_t^{(g)}(U^*)}{\mathrm{d}U^*}\right)_{U^*=0} \mathrm{E}(U^* - 0) \\ &= \sigma_{jt}^{(g)}(0)\mathit{\Gamma}_t^{(g)}(0). \end{aligned}$$

The first order Taylor expansion leads to $S_{jt}^{(g)} \approx \sigma_{jt}^{(g)}(0)\mathit{\Gamma}_t^{(g)}(0)$. Thus,

$$\frac{S_{jt}^{(g)}}{S_{1t}^{(g)}} \approx \frac{\sigma_{jt}^{(g)}(0)}{\sigma_{1t}^{(g)}(0)}.$$

The application of this conclusion to eq. (Linear-Reg-1) when $U = 0$ gives rise to the stated regression.

In the *second step*, we find $\rho_{jt1,\text{init}}$ for all $j = 1, \ldots, J$, and $(\rho_{1t2}/\omega)_{\text{init}}, (\rho_{1t3}/\omega^2)_{\text{init}}$ by running OLS for the following linear regression,

$$\ln\left(\frac{S_{jt}^{(g)}}{S_{0t}^{(g)}}\right) + (P_{jt} - P_{1t})\tau_{\text{init}}^{(g)} = \rho_{jt1} + \left(\frac{\rho_{1t2}}{\omega}\right)\tau_{\text{init}}^{(g)} + \left(\frac{\rho_{1t3}}{\omega^2}\right)(\tau_{\text{init}}^{(g)})^2.$$

We now explain how we got the above regression. It follows from series logit that

$$\begin{aligned} \ln\left[\frac{\sigma_{jt}^{(g)}(0)}{\sigma_{0t}^{(g)}(0)}\right] = \ln\left[\frac{\sigma_{jt}^{(1)}(\tau^{(g)}/\omega;\rho_t)}{\sigma_{0t}^{(1)}(\tau^{(g)}/\omega;\rho_t)}\right] &= \rho_{jt1} + \left(\frac{\rho_{jt2}}{\omega}\right)\tau^{(g)} + \left(\frac{\rho_{jt3}}{\omega^2}\right)(\tau^{(g)})^2 \\ &= \rho_{jt1} + \left(\frac{\rho_{1t2}}{\omega}\right)\tau^{(g)} - (P_{jt} - P_{1t})\tau^{(g)} + \left(\frac{\rho_{1t3}}{\omega^2}\right)(\tau^{(g)})^2. \end{aligned}$$

The second line follows from imposing the constraints eq. (B.4). By the approximation,

$$\ln\left(\frac{S_{jt}^{(g)}}{S_{0t}^{(g)}}\right) \approx \ln\left[\frac{\sigma_{jt}^{(g)}(0)}{\sigma_{0t}^{(g)}(0)}\right],$$

we have the stated regression.[1]

D Multidimensional Unobserved Heterogeneity

In this appendix, we extend our main results to include multidimensional unobserved heterogeneity. We have two observations. First, our estimation method works for multidimensional unobserved heterogeneity after some modification in the stage of CCP estimation. Second, a higher dimension of unobserved heterogeneity does not cause the curse of dimensionality for our CCP estimation that involves a series polynomial approximation of CCP as a function of multidimensional unobserved heterogeneity. This is because the structural model imposes certain restrictions that can eliminate a large number of parameters in CCP function.

D.1 Model

We now have the new dimension of unobserved heterogeneity γ_i associated with product characteristics X_{jt}. Particularly, when X_{jt} includes the product dummy variable, the above specification says that consumers could have heterogeneous valuation about the unobserved product characteristics (e.g. advertising).[2]

Using our group specification, we write

$$\begin{pmatrix}\gamma_i\\ \alpha_i\end{pmatrix} = \begin{pmatrix}\gamma^{(1)}\\ \alpha^{(1)}\end{pmatrix} + D_i^{(2)}\begin{pmatrix}\tau_1^{(2)}\\ \tau_2^{(2)}\end{pmatrix} + \cdots + D_i^{(G)}\begin{pmatrix}\tau_1^{(G)}\\ \tau_2^{(G)}\end{pmatrix} + \mathit{\Omega}\begin{pmatrix}U_{i1}\\ U_{i2}\end{pmatrix}, \tag{D.1}$$

where $\mathit{\Omega}$ is a diagonal matrix,

$$\mathit{\Omega} \equiv \begin{pmatrix}\mathit{\Omega}_1 & \\ & \omega_2\end{pmatrix}.$$

The diagonal elements $\mathit{\Omega}_1$, which is also a diagonal matrix, and ω_2 determine the within group variation of γ_i and α_i, respectively. Below, let $\tau^{(g)} \equiv (\tau_1^{(g)\prime}, \tau_2^{(g)})'$ and let $U_i \equiv (U_{i1}', U_{i2})'$. Use q to denote the dimension of $(X_{jt}', P_{jt})'$. Again, let $\tau^{(1)} \equiv \mathbf{0}$. Let $\boldsymbol{\phi}(U)$ denote the PDF of the multivariate normal distribution $\mathcal{N}(\mathbf{0},\mathbf{I})$.

Under the specification of (γ_i', α_i) in Equation (D.1), the expected lifetime payoff of purchasing product j at time t becomes

$$v_{jt}^{(g)}(U_i) = \frac{\delta_j + \gamma_i' X_{jt} + \xi_{jt}}{1-\beta} - \alpha_i P_{jt}, \qquad j = 1, \ldots, J.$$

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[1] We do not have a clever initial value of ω_{init}. In our simulation, we varied the initial value ω_{init} substantially, and our optimization routine seems very robust.

[2] For example, suppose X_{jt} is just the product dummy variable that equals 1 for product j and 0 otherwise. The expected payoff of product j in period t then reads $v_{ijt} = \delta_j + \gamma_i + \xi_{jt} - \alpha_i P_{jt}$, and γ_i serves as random effect that explains consumer heterogeneity in the valuation of unobserved product characteristics.

Correspondingly, the CCP of type-(g, U) is

$$\sigma_{jt}^{(g)}(U) = \frac{\exp(v_{jt}^{(g)}(U))}{\exp(v_{0t}^{(g)}(U)) + \sum_{k=1}^{J} \exp(v_{kt}^{(g)}(U))}.$$

Because $v_{jt}^{(g)}(U) = v_{jt}^{(1)}(U + \Omega^{-1}\tau^{(g)})$, we still have shifting formula $\sigma_{jt}^{(g)}(U) = \sigma_{jt}^{(1)}(U + \Omega^{-1}\tau^{(g)})$. So the CCP estimation can be based on the same NLS problem excepting for the constraints about the series approximation coefficients. Once the CCP functions are known, it will be easy to estimate the model structural parameters using our procedures in the post-CCP estimation section.

The constraints about ρ_t result from the comparison between the derivatives of the log CCP ratio with respect to U in the structural demand model and the same derivatives in the series approximation. Particularly, we have

$$\begin{aligned}
\ln\left[\frac{\sigma_{jt}^{(g)}(U)}{\sigma_{1t}^{(g)}(U)}\right] &= v_{jt}^{(g)}(U) - v_{1t}^{(g)}(U) \\
&= \frac{\delta_j - \delta_1}{1-\beta} + \frac{(\gamma^{(1)} + \tau_1^{(g)})'}{1-\beta}(X_{jt} - X_{1t}) - (\alpha^{(1)} + \tau_2^{(g)})(P_{jt} - P_{1t}) + \frac{\xi_{jt} - \xi_{1t}}{1-\beta} + \\
&\quad ((1-\beta)^{-1}\Omega_1 U_1)'(X_{jt} - X_{1t}) - \omega_2 U_2 (P_{jt} - P_{1t}).
\end{aligned}$$

This gives rise to

$$\frac{\mathrm{d}\ln[\sigma_{jt}^{(g)}(U)/\sigma_{1t}^{(g)}(U)]}{\mathrm{d}U} = \begin{pmatrix} (1-\beta)^{-1}\Omega_1(X_{jt} - X_{1t}) \\ -\omega_2(P_{jt} - P_{1t}) \end{pmatrix},$$

and any higher order derivatives are zeros. Comparing the above derivatives with the resulted derivatives from series logit form, we conclude

$$\rho_{jt2} - \rho_{1t2} = \begin{pmatrix} (1-\beta)^{-1}\Omega_1(X_{jt} - X_{1t}) \\ -\omega_2(P_{jt} - P_{1t} \end{pmatrix}, \quad (\rho_{jt3} - \rho_{1t3}) = (\rho_{jt4} - \rho_{1t4}) = \cdots = 0.$$

In the series approximation, we only consider the second order approximation. So for the dynamic model with multidimensional heterogeneity, the CCP estimation stage involves the following unknowns,

$$(\rho_{1t1}, \ldots, \rho_{Jt1})', \quad \rho_{1t2}, \quad \rho_{1t3}, \quad (1-\beta)^{-1}\Omega_1, \quad \omega_2, \quad \tau, \qquad \text{for all } t.$$

Note that in this step of CCP estimation, we are only able to estimate $(1-\beta)^{-1}\Omega_1$ as a whole and cannot separately estimate β from Ω_1. Recall q is the dimension of $(X_{jt}', P_{jt})'$, and ρ_{1t3} is a $q \times q$ triangular matrix. The degree of freedom of the NLS problem is $GJT - (JT + (3q + q^2)T/2 + 2q)$. In order to make this degree of freedom be positive, it is necessary to satisfy $(G-1)J - (3q+q^2)/2 > 0$.[3] Depending on the number of dimension of unobserved heterogeneity, we may or may not need a large number of groups or products. It is interesting to point out that even when the number of groups is small, we can ensure a positive degree of freedom by including a large number of products. This manifests one advantage of our method—the number of products, rather than causing the curse of dimensionality, helps solve the curse of dimensionality if we are willing to assume that purchasing is a terminal action in the dynamic model, which is reasonable for the market of durable goods.

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[3] We write $GJT - (JT + (3q+q^2)T/2 + 2q) = [(G-1)J - (3q+q^2)/2]T - 2q$. When T is relatively large, $(G-1)J - (3q+q^2)/2 > 0$ will also be sufficient to have positive degree of freedom.

Table D.1: Simulation Results: Multi Dimensional Heterogeneity
DGP: $M = 2$, $T = 48$ and $J = 8$

$\delta = -0.1$	-0.0923 (0.0097)
$\gamma = 0.03$	0.0289 (0.0017)
$\alpha^{(1)} = 0.10$	0.0963 (0.0175)
$\tau_X^{(2)} = 0.05$	0.0484 (0.0032)
$\tau_X^{(3)} = 0.10$	0.0968 (0.0063)
$\tau_X^{(4)} = 0.15$	0.1453 (0.0098)
$\tau_X^{(5)} = 0.20$	0.1942 (0.0137)
$\tau_X^{(6)} = 0.25$	0.2434 (0.0182)
$\tau_p^{(2)} = 0.05$	0.0507 (0.0016)
$\tau_p^{(3)} = 0.10$	0.1014 (0.0034)
$\tau_p^{(4)} = 0.15$	0.1519 (0.0053)
$\tau_p^{(5)} = 0.20$	0.2024 (0.0073)
$\tau_p^{(6)} = 0.25$	0.2528 (0.0092)
$\omega_X = 0.1$	0.0980 (0.0065)
$\omega_p = 0.075$	0.0754 (9.72e-5)
$\beta = 0.90$	0.9088 (0.0078)

Note: Mean and standard deviation (in parenthesis) for 50 simulations. Starting values follow the procedure in the appendix and vary with each simulation run. The starting value for $\omega = 0.25$ for all simulation runs.

D.2 Multi Dimensional Monte Carlo Simulation

In Table D.1, we report the results of Monte Carlo simulations where within group unobserved heterogeneity is present in both price and the X variable. The data generating process is identical to the uni-dimensional simulations with the exception of the Gaussian Hermite quadrature approximation of the normal distribution for the price and X coefficients uses 6 nodes rather than 12. This is done for computation time as with 12 we would have 144 individuals to simulation in the DGP process.

E Empirical Application: Estimation of $\delta_{make,1}$, β and $\gamma_{2k}/(1-\beta q_{cnty,k})$

The next step is to use

$$\begin{aligned}\ln\left[\frac{\sigma_{1t}^{(g)}(U)}{\sigma_{0t}^{(g)}(U)}\right] = \frac{\delta_{make,1}}{1-\beta} + \sum_{k=1}^{d_{ev}} X_{1mt,k}^{ev}\frac{\gamma_{1k}}{1-\beta q_{ev,k}} + \sum_{k=1}^{d_{cnty}} X_{mt,k}^{cnty}\frac{\gamma_{2k}}{1-\beta q_{cnty,k}} \\ + \left(\frac{\eta^{(1)}}{1-\beta} + \frac{\tau_{mpge}^{(g)}}{1-\beta} + \omega_{mpge}U_{mpge}\right)MPGe_{1mt} \\ - (\alpha^{(1)} + \tau_p^{(g)} + \omega_p U_p)P_{1mt} + \frac{\xi_{1mt}}{1-\beta} + \beta\,\mathrm{E}[\bar{V}_{t+1}^{(g)}(U) \mid X_{mt}, P_{mt}, \xi_{mt}]. \quad \text{(E.1)}\end{aligned}$$

to recover $\delta_{make,1}$, $\gamma_{2k}/(1-\beta q_{cnty,k})$ $(k = 1, \ldots, d_{cnty})$ and the discount factor β.

Letting

$$W_{mt}^{(g)}(U) \equiv \ln\left[\frac{\sigma_{1t}^{(g)}(U)}{\sigma_{0t}^{(g)}(U)}\right] -$$
$$\left[\sum_{k=1}^{d_{ev}} X_{1mt,k}^{ev} \frac{\gamma_{1k}}{1-\beta q_{ev,k}} + \left(\frac{\eta^{(1)}}{1-\beta} + \frac{\tau_{mpge}^{(g)}}{1-\beta} + \omega_{mpge} U_{mpge}\right) MPGe_{1mt} - (\alpha^{(1)} + \tau_p^{(g)} + \omega_p U_p) P_{1mt}\right], \quad \text{(E.2)}$$

we have the conclusion following the similar arguments in section 4.2:

$$W_{mt}^{(g)}(\tilde{U}) = \delta_{make,1} + \sum_{k=1}^{d_{cnty}} X_{mt,k}^{cnty} \frac{\gamma_{2k}}{1-\beta q_{cnty,k}} + \frac{\xi_{1mt}}{1-\beta} +$$
$$\beta\, \mathrm{E}\left(W_{m,t+1}^{(g)}(\tilde{U}) - \sum_{k=1}^{d_{cnty}} X_{m,t+1,k}^{cnty} \frac{\gamma_{2k}}{1-\beta q_{cnty,k}} + \ln \sigma_{0,m,t+1}^{(g)}(\tilde{U}) - \frac{\xi_{1,m,t+1}}{1-\beta} \,\middle|\, X_{mt}, P_{mt}, \xi_{mt} \right), \quad \text{(E.3)}$$

for a fixed $\tilde{U}$.

To estimate the discount factor β and γ_2, one possible approach is to consider the unconditional expectation again and to have the following conclusion:

$$\mathrm{E}[W_{mt}^{(g)}(\tilde{U})] = \delta_{make,1} + \beta\, \mathrm{E}[W_{m,t+1}^{(g)}(\tilde{U}) + \ln \sigma_{0,m,t+1}^{(g)}(\tilde{U})] - \sum_{k=1}^{d_{cnty}} \mathrm{E}[(X_{mt,k}^{cnty} - \beta X_{m,t+1,k}^{cnty})] \frac{\gamma_{2k}}{1-\beta q_{cnty,k}} = 0,$$

for a fixed $\tilde{U}$. Then we can solve β and $\gamma_{2k}/(1-\beta q_{cnty,k})$ from the above equation by the variation of groups. For this particular application, this approach is infeasible because some of our state variables are nonstationary as illustrated by the time series plot in Figure 3. For example, the number of charging stations is clearly non-stationary.

As an alternative, we note that in eq. (E.3), for a fixed $\tilde{U}$, the random variable $W_{mt}^{(g)}(\tilde{U})$ is a function of the vector $(X_{mt}, P_{mt}, \xi_{mt})$. So we have a conditional moment condition:

$$\mathrm{E}\left(W_{mt}^{(g)}(\tilde{U}) - \beta W_{m,t+1}^{(g)}(\tilde{U}) - \delta_{make,1} - \beta \ln \sigma_{0,m,t+1}^{(g)}(\tilde{U}) - \sum_{k=1}^{d_{cnty}} (X_{mt,k}^{cnty} - \beta X_{m,t+1,k}^{cnty}) \frac{\gamma_{2k}}{1-\beta q_{cnty,k}} - \frac{\xi_{1mt} - \beta \xi_{1,m,t+1}}{1-\beta} \,\middle|\, X_{mt}, P_{mt}, \xi_{mt} \right) = 0.$$

Suppose there is a vector of $X_{mt,IV}$, which are elements or functions of the conditioning variables $(X_{mt}, P_{mt}, \xi_{mt})$ and satisfy

$$\mathrm{E}(X_{mt,IV}\xi_{1mt}) = \mathrm{E}(X_{mt,IV}\xi_{1m,t+1}) = 0.$$

We have the moment conditions of

$$\mathrm{E}\left[X_{mt,IV} \left(W_{mt}^{(g)}(\tilde{U}) - \beta W_{m,t+1}^{(g)}(\tilde{U}) - \delta_{make,1} - \beta \ln \sigma_{0,m,t+1}^{(g)}(\tilde{U}) - \sum_{k=1}^{d_{cnty}} (X_{mt,k}^{cnty} - \beta X_{m,t+1,k}^{cnty}) \frac{\gamma_{2k}}{1-\beta q_{cnty,k}} \right) \right] = 0.$$

We then can estimate $\delta_{make,1}$, β and $\gamma_{2k}/(1-\beta q_{cnty,k})$ from the above equation using the moment estimator. In our empirical estimates, we let $\tilde{U} = 0$ and define $X_{mt,IV}$ as a vector of ones, gas_{mt}^{cnty}, the log battery price in period t, and the log number of EV makes in a given period t.