Online Appendix to Algorithmic Targeting and the Precision-Recall Trade-off

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Alternative Correlation Structure

We now consider an alternative correlation structure. Suppose firm one's prediction for a given consumer is p_1. Then, with probability ρ, firm two's prediction is also p_1, and with probability $1-\rho$, firm two's prediction is uniformly drawn from $f(p)$. Under this alternative correlation structure, the expected payoff of a firm from targeting a probability p consumer is $[\rho q(p) + (1-\rho)a]pw + [1 - \rho q(p) - (1-\rho)a]pv - c$ rather than $[\rho q(p) + (1-\rho)R]pw + [1 - \rho q(p) - (1-\rho)R]pv - c$.

All the main results in the paper hold under this alternative correlation structure. We present proof of those results below.

Proof of Lemma 1. The proof is the same as the proof in the appendix, except that one needs to replace R with a in the formulas. ■

Proof of Lemma 2.

Existence:

The proof is the same as the proof of Lemma 2 in the appendix.

Uniqueness:

1. Firms never target any consumer for sure ($\overline{p} = 1$)

$$
\begin{aligned}
a &= \int_{\underline{p}}^{1} f(p)q(p)dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)\frac{-c/p + (1-\rho)aw + [1-(1-\rho)a]v}{\rho(v-w)}dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)a]dp
\end{aligned}
$$

For any fixed ρ, define:

$$
G(a) := \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)a]dp - a
$$

Then, $G(0) = \int_{\frac{c}{v}}^{1} \frac{f(p)}{\rho(v-w)}[v - c/p]dp > 0$

$$
\begin{aligned}
G'(a) &= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)}dp - \\
&\quad c\left(-\frac{-(1-\rho)(v-w)}{[v-(1-\rho)(v-w)a]^2}\right)\left[v-(1-\rho)(v-w)a - \frac{c}{\frac{c}{v-(1-\rho)(v-w)a}}\right]\frac{f\left(\frac{c}{v-(1-\rho)(v-w)a}\right)}{\rho(v-w)} \\
&= -\frac{1-\rho}{\rho}\left[1 - F\left(\frac{c}{v-(1-\rho)(v-w)a}\right)\right] - 1 < 0
\end{aligned}
$$

Uniqueness then follows.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

$$
\begin{aligned}
&[\rho + (1-\rho)a(\rho)]\overline{p}w + [1-\rho-(1-\rho)a(\rho)]\overline{p}v - c = 0 \\
&\Rightarrow \overline{p} = \frac{c}{[\rho + (1-\rho)a(\rho)]w + [1-\rho-(1-\rho)a(\rho)]v} \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
a &= \int_{\overline{p}}^{1} f(p)dp + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp \\
&= 1 - F(\overline{p}) + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp
\end{aligned}
$$

For any fixing ρ, define:

$$
H(a) := 1 - F(\overline{p}) + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp - a
$$

Then, $H(0) = 1 - F(\overline{p}) + \int_{c/v}^{\overline{p}} f(p)q(p)dp > 0$

$$\begin{aligned}
H'(a) =& c\frac{-(1-\rho)(v-w)}{[v-(1-\rho)(v-w)a]^2} q(\frac{c}{v-(1-\rho)(v-w)a}) f(\frac{c}{v-(1-\rho)(v-w)a}) + \\
& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{\overline{p}} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)}dp - 1 \\
=& -(1-\rho)(v-w)[\frac{cq(\frac{c}{v-(1-\rho)(v-w)a})f(\frac{c}{v-(1-\rho)(v-w)a})}{[v-(1-\rho)(v-w)a]^2} + \frac{F(\overline{p}) - F(\frac{c}{v-(1-\rho)(v-w)a})}{\rho(v-w)}] - 1 \\
& < 0
\end{aligned}$$

Uniqueness then follows.

To show that there exists $\widehat{\rho} \in [0, \frac{v-c}{v-w}]$ such that firms never target any consumer for sure if and only if $\rho \geq \widehat{\rho}$, we just need to show the following claim:

If firms never target any consumer for sure for $\rho = \rho_s$, then they also never target any consumer for sure for any $\rho_l > \rho_s$.

Suppose not. When $\rho = \rho_l$, Lemma 1 implies that there exists $\overline{p}_l$ and $\underline{p}_l$ such that firms target consumers with probabilities $p \geq \overline{p}_l$ for sure and mix for consumers with probabilities between $\underline{p}_l$ and $\overline{p}_l$. Thus, $q_l(p) = 1 > q_s(p)$, $\forall p \geq \overline{p}_l$. The proof of the comparative statics results will show that $q_l'(p) < q_s'(p)$, $\forall p \in (\underline{p}_s, \overline{p}_l)$. So, $q_l(p) > q_s(p)$, $\forall p \in [\underline{p}_s, 1] \Rightarrow a_l > a_s$. A contradiction to the comparative statics that the overall targeting probability decreases in ρ, which will be shown in the proof of the comparative statics results. ■

Proof of Proposition 1. The same as the proof of Proposition 1 in the appendix.

■

Proof of the comparative statics results.

1. Firms never target any consumer for sure ($\overline{p} = 1$)

   Comparative statics of a w.r.t. ρ:

   In equilibrium, $G(a) = 0$. By the implicit function theorem, $\frac{\partial a}{\partial \rho} = -\frac{\frac{\partial G}{\partial \rho}}{\frac{\partial G}{\partial a}}$. We have shown that $\frac{\partial G}{\partial a}$ is negative.

$$\frac{\partial G}{\partial \rho} = \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{v-w}[-c/p + v - (1-\rho)(v-w)a](-1/\rho^2) + \frac{f(p)a}{\rho}dp$$

$$=\int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho^2(v-w)}[c/p - v + (1-\rho)(v-w)a + \rho a(v-w)]dp$$

$$\propto \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)[-q(p)+a]dp$$

$$=a\int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)dp - \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)q(p)dp$$

$$=a\int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)dp - a$$

$$=-F\left(\frac{c}{v-(1-\rho)(v-w)a}\right)a < 0.$$

Therefore, $\frac{\partial a}{\partial \rho} < 0$.

Comparative statics of $\underline{p}$ w.r.t. ρ:

We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v-(1-\rho)(v-w)a$ increases. Equation (**??**) then implies that $\underline{p}$ decreases in ρ.

Comparative statics of the precision wrt ρ :

The same as the proof in the appendix except that we use $\int_{\underline{p}_s}^1 q_s(p)f(p)dp = a_s > a_l = \int_{\underline{p}_l}^1 q_l(p)f(p)dp > \int_{\underline{p}_s}^1 q_l(p)f(p)dp$ and $q_l(\underline{p}_s) > q_l(\underline{p}_l) = 0 = q_s(\underline{p}_s)$ to argue that there must exist $\tilde{p} \in (\underline{p}_s, 1)$ such that $\begin{cases} q_s(p) > q_l(p), \ if \ p \in (\tilde{p}, 1] \\ \tilde{q} := q_s(\tilde{p}) = q_l(\tilde{p}) \\ q_s(p) < q_l(p), \ if \ p \in [\underline{p}_s, \tilde{p}) \end{cases}$.

Proof of the Comparative statics of the recall wrt ρ :

Suppose $\rho > \frac{v-c}{v-w}$. We have shown that $\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_l} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s}$. Since $a_s > a_l$, we have $\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_l} = \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_s}\frac{a_s}{a_l} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s} \Rightarrow \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_s} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s} \Rightarrow \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{\mu_0} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{\mu_0}$.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

   Comparative statics of a w.r.t. ρ:

   In equilibrium, $H(a) = 0$. By the implicit function theorem, $\frac{\partial a}{\partial \rho} = -\frac{\frac{\partial H}{\partial \rho}}{\frac{\partial H}{\partial a}}$. We have shown that $\frac{\partial H}{\partial a}$ is negative.

$$
\begin{aligned}
\frac{\partial H}{\partial \rho} =& - f(\overline{p})\frac{\partial \overline{p}}{\partial \rho} + \frac{\partial \overline{p}}{\partial \rho} q(\overline{p}) f(\overline{p}) - \\
& \frac{c}{[v-(1-\rho)(v-w)a]^2}(v-w)aq(\frac{c}{v-(1-\rho)(v-w)a})f(\frac{c}{v-(1-\rho)(v-w)a}) \\
=& - \frac{c}{[v-(1-\rho)(v-w)a]^2}(v-w)aq(\frac{c}{v-(1-\rho)(v-w)a})f(\frac{c}{v-(1-\rho)(v-w)a}) \\
<& 0
\end{aligned}
$$

Therefore, $\frac{\partial a}{\partial \rho} < 0$.

Comparative statics of $\underline{p}$ w.r.t. ρ:

We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v-(1-\rho)(v-w)a$ increases. Equation (**??**) then implies that $\underline{p}$ decreases in ρ.

Comparative statics of $\overline{p}$ w.r.t. ρ:

Consider any given $\rho_l > \rho_s$ such that the corresponding $\overline{p}_l$ and $\overline{p}_s$ are lower than 1. We prove by contradiction. suppose $\overline{p}_l \leq \overline{p}_s$. The overall targeting probability corresponding to ρ_l is $a_l = \int_{\overline{p}_l}^1 f(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)f(p)dp$. The overall targeting probability corresponding to ρ_s is $a_s = \int_{\overline{p}_s}^1 f(p)dp + \int_{\underline{p}_s}^{\overline{p}_s} q_s(p)f(p)dp$. We first show that $q_l(p) > q_s(p), \forall p \in (\underline{p}_s, \overline{p}_l)$.

Observe that $q_l(\overline{p}_l) = 1 > q_s(\overline{p}_l)$. For any $p \in (\underline{p}_s, \overline{p}_l)$, we have

$$
\begin{aligned}
q_l(p) =& q_l(\overline{p}_l) + \frac{c}{\rho_l(v-w)}(1/\overline{p}_l - 1/p) \\
>& q_s(\overline{p}_l) + \frac{c}{\rho_s(v-w)}(1/\overline{p}_l - 1/p) = q_s(\overline{p}_l)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
a_l =& \int_{\overline{p}_s}^1 f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} f(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)f(p)dp \\
>& \int_{\overline{p}_s}^1 f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)f(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_l(p)f(p)dp
\end{aligned}
$$

$$> \int_{\overline{p}_s}^{1} f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)f(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_s(p)f(p)dp = a_s$$

But we have shown that a decreases in ρ. A contradiction. Therefore, $\overline{p}_l > \overline{p}_s$.

Comparative statics of the profit w.r.t. ρ:

The same as the proof in the appendix, except that one needs to replace R with a in the formulas.

■

Proof of Proposition 2. The same as the proof in the appendix, except that one needs to replace R with a in the formulas. ■

Proof of Proposition 3. The threshold for a monopoly equilibrium to exist is now $\rho \geq \frac{a_m w + (1-a_m)v - c}{(1-a_m)(v-w)}$ rather than $\rho \geq \frac{R_m w + (1-R_m)v - c}{(1-R_m)(v-w)}$.

The proof is the same as the proof in the appendix,, except that one needs to replace R with a in the formulas. ■