

A Proof of Theorem 3.1

Proof. From (ii), there are $Q \geq 1$ pairs of states in $\mathcal{S}^{id}$. For the q th pair:

$$\mathbf{s}_{q,1} = [p_j, \mathbf{p}_{-j,1}, i], \mathbf{s}_{q,2} = [p_j, \mathbf{p}_{-j,2}, i] \quad (\text{A.1})$$

for some i and $\mathbf{p}_{-j,1}, \mathbf{p}_{-j,2}$, where $\mathbf{p}_{-j,1} \neq \mathbf{p}_{-j,2}$ for at least one element. We can then derive the exclusion restriction:

$$u_j(\mathbf{s}_{q,1}) - u_0(\mathbf{s}_{q,1}) = u_j(\mathbf{s}_{q,2}) - u_0(\mathbf{s}_{q,2}). \quad (\text{A.2})$$

From the q th exclusion restriction, we can construct a moment condition as in (3.5). Define the $1 \times L$ vector as:

$$\mathbf{D}_q(\beta) = \beta [\mathbf{F}_j(\mathbf{s}_{q,1}) - \mathbf{F}_0(\mathbf{s}_{q,1}) - \mathbf{F}_j(\mathbf{s}_{q,2}) + \mathbf{F}_0(\mathbf{s}_{q,2})] [\mathbf{I} - \beta \mathbf{F}_0]^{-1}. \quad (\text{A.3})$$

Next, write the left-hand side of (3.5) as:

$$\Delta y_q = \ln \left(\frac{\sigma_j(\mathbf{s}_{q,1})}{\sigma_0(\mathbf{s}_{q,1})} \right) - \ln \left(\frac{\sigma_j(\mathbf{s}_{q,2})}{\sigma_0(\mathbf{s}_{q,2})} \right). \quad (\text{A.4})$$

Now Equation (3.5) can be written compactly as:

$$\Delta y_q = \mathbf{D}_q(\beta)(\mathbf{m} + \mathbf{u}_0). \quad (\text{A.5})$$

By the consumption rule (i), the no-purchase flow utility depends only on inventory: $u_0(\mathbf{s}, c(\mathbf{p}, i)) = u_0(i)$. We show that $\mathbf{D}_q(\beta)\mathbf{u}_0 = 0$ for any $q \in \{1, \dots, Q\}$. We can expand Equation (A.3) and interact it with $\mathbf{u}_0$ as follows:

$$\mathbf{D}_q(\beta) \mathbf{u}_0 = \beta [\mathbf{F}_j(\mathbf{s}_{q,1}) - \mathbf{F}_0(\mathbf{s}_{q,1}) - \mathbf{F}_j(\mathbf{s}_{q,2}) + \mathbf{F}_0(\mathbf{s}_{q,2})] [\mathbf{I} + \beta \mathbf{F}_0 + \beta^2 \mathbf{F}_0 \mathbf{F}_0 + \dots] \mathbf{u}_0. \quad (\text{A.6})$$

From the consumption rule (i) and Assumption 1, not purchasing deterministically reduces inventory by $\bar{c}$ when $i \geq \bar{c}$ and by 0 if $i < \bar{c}$, so:

$$\mathbf{F}_0(\mathbf{p}, i) \underbrace{\mathbf{F}_0 \mathbf{F}_0 \dots \mathbf{F}_0}_{K \text{ times}} \mathbf{u}_0 = u_0(\tilde{i})$$

where $\tilde{i} = i - k\bar{c}$ and $\{k \in \mathbb{Z}_{\geq 0} : \exists \rho \in \mathbb{Z}_{\geq 0} \text{ s.t. } i - (\rho + 1)\bar{c} < 0 \leq i - \rho\bar{c}, k = \min\{K + 1, \rho\}\}$. Since $\mathbf{s}_{q,1}$ and $\mathbf{s}_{q,2}$ share the same inventory state, $\mathbf{F}_0(\mathbf{s}_{q,1})\mathbf{F}_0\mathbf{F}_0\dots\mathbf{F}_0\mathbf{u}_0 = \mathbf{F}_0(\mathbf{s}_{q,2})\mathbf{F}_0\mathbf{F}_0\dots\mathbf{F}_0\mathbf{u}_0$. Similarly, purchasing j in the current period deterministically shifts inventory by q_j from any state:

$$\mathbf{F}_j(\mathbf{p}, i) \underbrace{\mathbf{F}_0 \mathbf{F}_0 \dots \mathbf{F}_0}_{K \text{ times}} \mathbf{u}_0 = u_0(\tilde{i})$$

where $\tilde{i} = i + q_j - (k + 1)\bar{c}$ and $\{k \in \mathbb{Z}_{\geq 0} : \exists \rho \in \mathbb{Z}_{\geq 0} \text{ s.t. } i - (\rho + 1)\bar{c} < 0 \leq i - \rho\bar{c}, k = \min\{K, \rho\}\}$.

Therefore, $\mathbf{F}_j(\mathbf{s}_{q,1})\mathbf{F}_0\mathbf{F}_0\ldots\mathbf{F}_0\mathbf{u}_0 = \mathbf{F}_j(\mathbf{s}_{q,2})\mathbf{F}_0\mathbf{F}_0\ldots\mathbf{F}_0\mathbf{u}_0$. This gives:

$$\begin{aligned} \mathbf{D}_q(\beta)\mathbf{u}_0 &= \beta[\mathbf{F}_j(\mathbf{s}_{q,1}) - \mathbf{F}_j(\mathbf{s}_{q,2})][\mathbf{I} + \beta\mathbf{F}_0 + \beta^2\mathbf{F}_0\mathbf{F}_0 + \ldots]\mathbf{u}_0 \\ &\quad + \beta[\mathbf{F}_0(\mathbf{s}_{q,2}) - \mathbf{F}_0(\mathbf{s}_{q,1})][\mathbf{I} + \beta\mathbf{F}_0 + \beta^2\mathbf{F}_0\mathbf{F}_0 + \ldots]\mathbf{u}_0 \\ &= 0 \end{aligned} \tag{A.7}$$

and Equation (3.5) is reduced to:

$$\begin{aligned} \Delta y_q &= \mathbf{D}_q(\beta)(\mathbf{m} + \mathbf{u}_0) \\ &= \mathbf{D}_q(\beta)\mathbf{m}, \end{aligned} \tag{A.8}$$

which is independent of $\mathbf{u}_0$. This moment condition is equivalent to the case with state-independent reference utility, so Theorem 1 in Abbring and Daljord (2020) applies directly, implying that the identified set for β is a closed, discrete subset of $[0, 1)$. Moreover, Theorem 6 in Abbring et al. (2025) establishes that the identified set is finite, with cardinality at most L .

□

B Proof of Theorem 3.2

Proof. For choice j and state $\mathbf{s}$ we have:

$$\ln\left(\frac{\sigma_j(\mathbf{s})}{\sigma_0(\mathbf{s})}\right) = u_j(\mathbf{s}) - u_0(\mathbf{s}) + \beta[\mathbf{F}_j(\mathbf{s}) - \mathbf{F}_0(\mathbf{s})][\mathbf{I} - \beta\mathbf{F}_0]^{-1}(\mathbf{m} + \mathbf{u}_0). \tag{B.1}$$

Stack these as:

$$\begin{aligned} \ln\left(\frac{\sigma_j}{\sigma_0}\right) &= \mathbf{u}_j - \mathbf{u}_0 + \beta[\mathbf{F}_j - \mathbf{F}_0][\mathbf{I} - \beta\mathbf{F}_0]^{-1}(\mathbf{m} + \mathbf{u}_0) \\ \mathbf{y}_j &= \mathbf{u}_j + [\beta(\mathbf{F}_j - \mathbf{F}_0)[\mathbf{I} - \beta\mathbf{F}_0]^{-1} - \mathbf{I}]\mathbf{u}_0 \end{aligned} \tag{B.2}$$

where $\mathbf{y}_j = \ln\left(\frac{\sigma_j}{\sigma_0}\right) - \beta[\mathbf{F}_j - \mathbf{F}_0][\mathbf{I} - \beta\mathbf{F}_0]^{-1}\mathbf{m}$ contains only known features of the data. Stack these over choices to get the following $J \times L$ moment conditions that are linear in $J + 1 + 1 + R$ unknown parameters (brand intercepts, price coefficient, consumption utility, and a fully saturated inventory cost function which is the same across purchase options and the no-purchase option):

$$\mathbf{y} = \mathbf{X}[\mathbf{u}', \mathbf{u}_0']'. \tag{B.3}$$

Since $\mathbf{X}$ is full rank, the system is at least just identified, and the solution for $[\mathbf{u}', \mathbf{u}_0']'$ that satisfies Equation (B.3) is unique.

□

C Identification with Stochastic Inventory Evolution

We now show that the exclusion restrictions in Section 3 can also be used to identify the discount factor when inventory evolves stochastically, as in the case of stochastic consumption rates (e.g., Hendel and Nevo (2006)). When inventory is stochastic, the moment conditions in Equation (3.5) depend on both β and $\mathbf{u}_0$. Therefore, β must be identified jointly with $\mathbf{u}_0$ from these conditions. The following lemma shows that under some regularity conditions, we can solve $\mathbf{u}_0$ as a real analytic function of β from these moment conditions in closed form. Once we have $\mathbf{u}_0(\beta)$, we show that the discount factor is set-identified by the remaining moment conditions.

Lemma C.1. *Suppose u_0 takes $R < L$ unknown values, that inventory is observed, and that at least R unique exclusion restrictions of the kind in Equation (3.3) are satisfied. Subject to a rank condition, the associated moment conditions define u_0 as a known, real analytic function $u_0(s|\beta)$. For each $\beta \in [0, 1)$, $u_0(s|\beta)$ uniquely maps $\mathcal{S}$ to $\mathbb{R}$.*

Proof. Inventory takes values $\{0, 1, \dots, R\}$. Since the reference utility does not depend on prices, it follows that for any pair of price vectors $\mathbf{p}$ and $\mathbf{p}'$ we have $u_0(\mathbf{p}, i) = u_0(\mathbf{p}', i)$.¹⁶

□

Suppose we have R pairs of states in $\mathcal{S}^{id}$. Denote the r^{th} pair:

$$\mathbf{s}_{r,1} = [p_j, \mathbf{p}_{-j,1}, i] \tag{C.1}$$

$$\mathbf{s}_{r,2} = [p_j, \mathbf{p}_{-j,2}, i]. \tag{C.2}$$

In this pair, the prices of at least one of the $-j$ products shifts within each pair of states, while the j th price and the inventory remains fixed. Now the following exclusion restrictions hold for some choice $j \geq 1$:

$$u_j(\mathbf{s}_{r,1}) - u_0(\mathbf{s}_{r,1}) = u_j(\mathbf{s}_{r,2}) - u_0(\mathbf{s}_{r,2}). \tag{C.3}$$

¹⁶Note that a unit of inventory does not have to be a pack-size of detergents.

We now show that if β is known, the moment conditions in Equation (3.5) consist of a system of R linear equations in R unknowns, $\{u_0(\cdot, 1|\beta), \dots, u_0(\cdot, R|\beta)\}$.

Collecting terms to remove redundant notation, for each exclusion restriction define the $1 \times L$ vector:

$$\mathbf{D}_r(\beta) = \beta [\mathbf{F}_j(\mathbf{s}_{r,1}) - \mathbf{F}_0(\mathbf{s}_{r,1}) - \mathbf{F}_j(\mathbf{s}_{r,2}) + \mathbf{F}_0(\mathbf{s}_{r,2})] [\mathbf{I} - \beta \mathbf{F}_0]^{-1}, \quad r \in \{1, \dots, R\} \quad (\text{C.4})$$

and stack these in the $R \times L$ matrix $\mathbf{D}(\beta)$. Next, write the left-hand side of Equation (3.5) as:

$$\Delta y_r = \ln \left(\frac{\sigma_j(\mathbf{s}_{r,1})}{\sigma_0(\mathbf{s}_{r,1})} \right) - \ln \left(\frac{\sigma_j(\mathbf{s}_{r,2})}{\sigma_0(\mathbf{s}_{r,2})} \right) \quad (\text{C.5})$$

and stack all R moment conditions in $\mathbf{y}$. We can now write our system of moments more compactly as:

$$\mathbf{D}(\beta) \mathbf{u}_0 = \mathbf{y} - \mathbf{D}(\beta) \mathbf{m}. \quad (\text{C.6})$$

We want to solve for the R unique values of the $L \times 1$ vector $\mathbf{u}_0$. Let:

$$\mathbf{u}_0^* = [u_0(\cdot, 1), \dots, u_0(\cdot, R)]' \quad (\text{C.7})$$

be a vector of the R unique values $\mathbf{u}_0$ takes. We construct an $L \times R$ matrix $\mathbf{E}$ that collapses $\mathbf{D}$ to a square matrix:

$$\mathbf{D}^*(\beta) = \mathbf{D}(\beta) \mathbf{E} \quad (\text{C.8})$$

which allows us to solve for $\mathbf{u}_0$ in closed form. The matrix $\mathbf{E}$ has typical elements of:

$$e_{l,r} = \begin{cases} 1 & \text{if } u_0(\mathbf{s}_l) = u_0(.,r) \\ 0 & \text{otherwise.} \end{cases} \quad (\text{C.9})$$

Define:

$$\mathbf{y}^* = \mathbf{y} - \mathbf{D}(\beta)\mathbf{m}. \quad (\text{C.10})$$

If $\mathbf{D}^*(\beta)$ is invertible, then substituting Equations (C.7), (C.8), and (C.10) into Equation (C.6) gives:

$$\mathbf{u}_0^*(\beta) = \mathbf{D}^{*-1}(\beta)\mathbf{y}^*. \quad (\text{C.11})$$

It follows that for each $\beta \in [0, 1)$, $u_0(\mathbf{s}|\beta)$ uniquely maps $\mathcal{S}$ to $\mathbb{R}$.

The invertibility of $\mathbf{D}^*(\beta)$ can be verified in the data. For instance, $\mathbf{D}^*(\beta)$ is singular when inventory is deterministic and consumption is strictly positive whenever $i > 0$. In this case, one should find very high condition numbers, which lead to numerically unstable results. There may be other relevant cases.

Proposition 1 below establishes that we can concentrate $\mathbf{u}_0^*(\beta)$ from the moment conditions.

Proposition 1. *Suppose Lemma C.1 holds and that $\mathcal{S}^{id}$ contains at least $R+1$ unique pairs of states. Then, subject to a rank condition, the identified set of discount factors β and the reference utility function $\mathbf{u}_0$ is a closed, discrete subset of $[0, 1) \times \mathbb{R}^R$.*

Proof. From Lemma C.1, we have that $\mathbf{u}_0(\beta)$ is a real analytic function in β . We combine this result with Theorem 1 in Abbring and Daljord (2020) which shows that $\mathbf{u}$ and β are jointly identified up to a finite set for a known reference utility under one exclusion restriction. To simplify notation, we write the inverse matrix on the right-hand side of Equation (C.4) as:

$$\mathbf{B}^{-1} = [\mathbf{I} - \beta\mathbf{F}_0]^{-1}. \quad (\text{C.12})$$

This inverse matrix can be expressed in terms of its cofactor matrix $\mathbf{B}^{-1} = \frac{1}{\det(\mathbf{B})} \text{adj}(\mathbf{B})$, where $\text{adj}(\mathbf{B})$ is the adjugate matrix.¹⁷ Since $\mathbf{B}$ is invertible on $\beta \in [0, 1)$, the denominator $\det(\mathbf{B})$ is a non-zero scalar. The numerator has entries that are determinants of the minors of $\mathbf{B}$, which are polynomials of at most degree $L - 1$. We then have that $\mathbf{D}_{ij} = \beta[\mathbf{F}_a(\mathbf{s}_i) - \mathbf{F}_0(\mathbf{s}_i) - \mathbf{F}_a(\mathbf{s}_j) + \mathbf{F}_0(\mathbf{s}_j)]\mathbf{B}^{-1}$ is a $1 \times L$ vector with elements that are proper rational functions, and hence real analytic in β in its domain. The matrix $\mathbf{D}^*$ has entries that are sums of real analytic functions, which are themselves real analytic functions.

□

From Equation (C.11), $\mathbf{u}_0^*$ is a matrix product of real analytic elements in $\mathbf{D}$ and, if $\mathbf{D}^*$ is invertible, it is a matrix product of real analytic elements in $\mathbf{D}^*$. It follows that $\mathbf{u}_0^*$ is itself real analytic. Theorem 1 in Abbring and Daljord (2020) now applies to Equation (3.5), where we replace $\mathbf{u}_0$ with $\mathbf{u}_0(\beta)$.

A special case of some interest outside our application follows immediately from Theorem 1.

Corollary C.1.1. *If $\mathcal{S}^{id}$ contains at least one pair of states, then the discount factor is set-identified in the DDC model when the reference utility is independent of the state, subject to a rank condition.*

Proof. If u_0 is independent of the state, then $R = 0$ and Theorem 1 in Abbring and Daljord (2020) can be applied directly.

□

Standard discrete-choice implies that the discount factor is identified without additional assumptions in models such as Gowrisankaran and Rysman (2012).

D Proof of Lemma 3.1

Let:

$$\mathbf{u}^* \equiv (\tilde{u}(c_1), \dots, \tilde{u}(c_{R_c}), C(i_1), \dots, C(i_R))' \in \mathbb{R}^{R_c+R}$$

¹⁷The adjugate of $\mathbf{B}$ is the transpose of a matrix where the i, j th entry is the determinant of a matrix where the i th row and the j th column of $\mathbf{B}$ has been removed (the cofactor matrix of $\mathbf{B}$).

denote the vector of unknown consumption utilities and storage costs. The stacked reference utilities $\mathbf{u}_0$ can be written as:

$$\mathbf{u}_0 = \mathbf{E}\mathbf{u}^*$$

where $\mathbf{E} \in \{-1, 0, 1\}^{L \times (R_c + R)}$ is a fixed selection matrix mapping the primitive utilities (u, C) into the state-specific utilities $u_0(\mathbf{s})$ for $\mathbf{s} \in \mathcal{S}$.

For each triple $(\mathbf{s}_r, \mathbf{s}'_r, j) \in \mathcal{S}^{id}$, define:

$$\mathbf{D}_r(\beta) \equiv \beta [\mathbf{F}_j(\mathbf{s}_r) - \mathbf{F}_0(\mathbf{s}_r) - \mathbf{F}_j(\mathbf{s}'_r) + \mathbf{F}_0(\mathbf{s}'_r)] (I - \beta \mathbf{F}_0)^{-1}. \quad (\text{D.1})$$

The full matrix $\mathbf{D}(\beta)$ is obtained by stacking the rows $\mathbf{D}_r(\beta)$ over all triples in $\mathcal{S}^{id}$.

Proof. Stacking the moment conditions from $\mathcal{S}^{id}$ gives the linear system:

$$\mathbf{D}(\beta)\mathbf{E}\mathbf{u}^* = \mathbf{y} - \mathbf{D}(\beta)\mathbf{m} \quad (\text{D.2})$$

where $\mathbf{y}$, $\mathbf{m}$, $\mathbf{E}$ is known from data. If $\text{rank}(\mathbf{D}(\beta)\mathbf{E}) = (R_c + R)$, then (D.2) has the unique solution:

$$\mathbf{u}^*(\beta) = (\mathbf{E}'\mathbf{D}(\beta)'\mathbf{D}(\beta)\mathbf{E})^{-1}\mathbf{E}'\mathbf{D}(\beta)'(\mathbf{y} - \mathbf{D}(\beta)\mathbf{m}), \quad \mathbf{u}_0(\beta) = \mathbf{E}\mathbf{u}^*(\beta). \quad (\text{D.3})$$

Thus, $\mathbf{u}_0(\beta)$ is uniquely determined in closed form.

Since $\mathbf{D}(\beta)$ is analytic in β for $\beta \in (0, 1)$ and matrix inversion preserves analyticity on the nonsingular set, $\mathbf{u}_0(\beta)$ is real analytic in β . $\square$

E Proof of Theorem 3.3

Proof. By Lemma 3.1, $\mathbf{u}_0(\beta)$ is a real analytic function in $\beta \in (0, 1)$. For any exclusion triple $(\mathbf{s}, \mathbf{s}', j)$, define:

$$f_{(\mathbf{s}, \mathbf{s}', j)}(\beta) = y(\mathbf{s}, \mathbf{s}', j) - \beta [\mathbf{F}_j(\mathbf{s}) - \mathbf{F}_0(\mathbf{s}) - \mathbf{F}_j(\mathbf{s}') + \mathbf{F}_0(\mathbf{s}')] (\mathbf{I} - \beta \mathbf{F}_0)^{-1} [\mathbf{m} + \mathbf{u}_0(\beta)]. \quad (\text{E.1})$$

Any triple in $\mathcal{S}^u$ used to solve for $\mathbf{u}_0(\beta)$ produces $f_{(\mathbf{s}, \mathbf{s}', j)}(\beta) = 0$ for all β . Therefore, the identification of β requires at least one exclusion triple not used in forming the system in Lemma 3.1.

Note that $(\mathbf{I} - \beta \mathbf{F}_0)^{-1}$ exists for $\beta \in (0, 1)$ and equals:

$$(\mathbf{I} - \beta \mathbf{F}_0)^{-1} = \mathbf{I} + \beta \mathbf{F}_0 + \beta^2 \mathbf{F}_0^2 + \dots \quad (\text{E.2})$$

For $\beta \in (0, 1)$, since $\mathbf{F}_0$ is a Markov transition matrix with eigenvalues bounded by 1 in absolute value, we have $\beta^{-1} > 1$, so $\det(\mathbf{F}_0 - \beta^{-1} \mathbf{I}) \neq 0$ and the series converges. Hence $(\mathbf{I} - \beta \mathbf{F}_0)^{-1}$ is real analytic in β on $(0, 1)$. Because sums, products, and compositions of real-analytic functions are real analytic, each $f_{(\mathbf{s}, \mathbf{s}', j)}(\beta)$ is real analytic on $(0, 1)$.

- If there exists a triple with $y(\mathbf{s}, \mathbf{s}', j) \neq 0$, then $f_{(\mathbf{s}, \mathbf{s}', j)}(0) \neq 0$, so $f_{(\mathbf{s}, \mathbf{s}', j)}$ is not identically zero.
- If $y(\mathbf{s}, \mathbf{s}', j) = 0$ for the triple, but there is some $\beta^* \in (0, 1)$ with

$$[\mathbf{F}_j(\mathbf{s}) - \mathbf{F}_0(\mathbf{s}) - \mathbf{F}_j(\mathbf{s}') + \mathbf{F}_0(\mathbf{s}')] (\mathbf{I} - \beta^* \mathbf{F}_0)^{-1} [\mathbf{m} + \mathbf{u}_0(\beta^*)] \neq 0,$$

then $f_{(\mathbf{s}, \mathbf{s}', j)}(\beta^*) \neq 0$. Hence, again $f_{(\mathbf{s}, \mathbf{s}', j)}$ is not identically zero.

If at least one exclusion moment is not identically zero, then by Theorem 1 in Abbring and Daljord (2020), the moment condition has only finitely many solutions in β . Therefore, the identified set of $(\beta, \mathbf{u}_0)$ is a closed, discrete subset of $(0, 1) \times \mathbb{R}^L$.

Boundary case $\beta = 0$: At $\beta = 0$, $f_{(\mathbf{s}, \mathbf{s}', j)}(0) = y(\mathbf{s}, \mathbf{s}', j)$. Thus, $\beta = 0$ solves all moments if and only if $y(\mathbf{s}, \mathbf{s}', j) = 0$ for all triples in $\mathcal{S}^{id}$. But when $\beta = 0$, $\mathbf{D}(0) = 0$, so Lemma 3.1 does not determine $\mathbf{u}_0(0)$. Identification of $\mathbf{u}_0$ in this case reduces to the static Hotz-Miller equations, which recover utility components under the assumed normalizations.

□

F Proof of Corollary 3.1.1

Proof. For choice j and state $\mathbf{s}$ we have:

$$\ln \left(\frac{\sigma_j(\mathbf{s})}{\sigma_0(\mathbf{s})} \right) = u_j(\mathbf{s}) - u_0(\mathbf{s}) + \beta [\mathbf{F}_j(\mathbf{s}) - \mathbf{F}_0(\mathbf{s})] [\mathbf{I} - \beta \mathbf{F}_0]^{-1} (\mathbf{m} + \mathbf{u}_0). \quad (\text{F.1})$$

Stack these as:

$$\begin{aligned} \ln \left(\frac{\sigma_j}{\sigma_0} \right) &= \mathbf{u}_j - \mathbf{u}_0 + \beta [\mathbf{F}_j - \mathbf{F}_0] [\mathbf{I} - \beta \mathbf{F}_0]^{-1} (\mathbf{m} + \mathbf{u}_0) \\ \mathbf{y}_j &= \mathbf{u}_j + [\beta (\mathbf{F}_j - \mathbf{F}_0) [\mathbf{I} - \beta \mathbf{F}_0]^{-1} - \mathbf{I}] \mathbf{u}_0 \end{aligned} \quad (\text{F.2})$$

where $\mathbf{y}_j = \ln\left(\frac{\sigma_j}{\sigma_0}\right) - \beta[\mathbf{F}_j - \mathbf{F}_0][\mathbf{I} - \beta\mathbf{F}_0]^{-1}\mathbf{m}$ contains only known features of the data. Stack these over choices to get the following $J \times L$ moment conditions:

$$\mathbf{y} = \mathbf{X}[\mathbf{u}', \mathbf{u}_0']'. \quad (\text{F.3})$$

For $\beta \neq 0$, the moment conditions (F.3) are linear in $J + 1$ unknown parameters (brand intercepts and price coefficient). For $\beta = 0$ the moment conditions are linear in $J + 1 + R + R_c$ unknown parameters (brand intercepts, price coefficient, consumption utility, and storage costs).

If the stacked matrix $\mathbf{X}$ has full column rank with $J + 1$ parameters when $\beta \in (0, 1)$ or with $J + 1 + R + R_c$ parameters when $\beta = 0$, the system is at least just identified, and the solution for $[\mathbf{u}', \mathbf{u}_0']'$ in (F.3) is unique.

□

G Comparison to Past Approaches

G.1 Comparison to Ching and Osborne (2020)

Let $\mathbf{Z}_t \equiv ((q_{k_{t1}}, R_{t1}), \dots, (q_{k_{tM_t}}, R_{tM_t}))$ denote the consumer's period- t inventory of non-depleted, previously purchased bottles ordered from the most recently purchased ($m = 1$) to the oldest ($m = M_t$). Each bottle is indexed by its pack-size, $q_{k_{tm}} \in \mathcal{Q} \subset \mathbb{R}_+$, and remaining contents, $R_{tm} \leq q_{k_{tm}}$. Let $i^{(k)}(\mathbf{Z}_t) = \sum_{m:k_{tm}=k} R_{tm}$ denote the sum of available quantity in bottles of the k^{th} pack-size, and $i(\mathbf{Z}_t) = \sum_{k=1}^K i^{(k)}(\mathbf{Z}_t)$ denote the total units of inventory. Finally, let $\mathbf{B}(\mathbf{Z}_t) \in \mathbb{N}_0^K$ denote the number of packs of each of the K available pack-sizes, where:

$$B^{(k)}(\mathbf{Z}_t) = \lceil \frac{1}{q_k} i^{(k)}(\mathbf{Z}_t) \rceil, \quad \forall k = 1, \dots, K, \quad (\text{G.1})$$

where $\lceil \cdot \rceil$ returns the smallest integer greater than or equal to its argument.

The consumer's observed state vector is now $\mathbf{s} = (\mathbf{p}, \mathbf{Z})$, which expands the dimension of the state space from $J + 1$ dimensions to $J + M + 1$ dimensions, where M is the total length of the inventory queue.

After the consumer makes purchase decision d_t , the inventory queue $\mathbf{Z}_{t,d_t}^+$ evolves in two

stages. First, the post-purchase inventory $\mathbf{Z}_{t,d_t}^+$ is given by:

$$\mathbf{Z}_{t,d_t}^+ = \begin{cases} ((q_{d_t}, q_{d_t}), \mathbf{Z}_t) & \text{if } d_t \in \mathcal{J} \\ \mathbf{Z}_t & \text{if } d_t = 0, \end{cases}$$

where (q_{d_t}, q_{d_t}) denotes the addition of a full bottle of the purchased size, q_{d_t} , to the front of the queue.

As in Section 3.1, we assume the consumer has a fixed consumption need, $\bar{c}$, every period.¹⁸ The consumer's consumption decision is:

$$c(\mathbf{Z}_{t,d_t}^+) = \mathbb{1}_{[i(\mathbf{Z}_{t,d_t}^+) \geq \bar{c}]} \bar{c} \quad (\text{G.2})$$

which ensures consumption only occurs when there is sufficient inventory.

Second, after consumption, the end-of-period inventory is denoted $\tilde{\mathbf{Z}}_{t,d_t} = \Gamma(\mathbf{Z}_{t,d_t}^+)$, where $\Gamma(\cdot)$ represents an inventory depletion rule that determines how consumption depletes the inventory queue.

Without imposing any parametric restrictions on the flow utility, we can conceptually construct exclusion restrictions based on states that share the same purchase utility, consumption utility, and storage costs. However, constructing such exclusion sets relies on equalities in storage costs, which are not directly observable. To operationalize these exclusion restrictions, Ching and Osborne (2020) impose additional structure on the storage cost function, formalized as follows:

Assumption 7 (Discrete Storage Costs). *The storage costs depend only on the number of bottles in inventory, giving rise to the following step function:*

$$C(\mathbf{Z}) = C(\mathbf{B}^{(1)}(\mathbf{Z}), \dots, \mathbf{B}^{(K)}(\mathbf{Z})) \geq 0. \quad (\text{G.3})$$

¹⁸Ching and Osborne (2020) only provide an identification result under the assumption of exogenous consumption. We leave it to future research to determine whether the approach extends to the case of endogenous consumption.

Hence, storage costs take values from a finite set indexed by pack counts. This assumption implies that storage costs only change when entire packs are added or removed from inventory.

Assumption 8 (Inventory Management). *The consumer uses a FIFO inventory depletion rule:*

$$\Gamma(\mathbf{Z}) = \begin{cases} ((q_{k_1}, R_{k_1}), \dots, (q_{k_{m^*}}, R_{k_{m^*}} - x_{m^*})) & \text{if } R_{k_{m^*}} > x_{m^*} \\ ((q_{k_1}, R_{k_1}), \dots, (q_{k_{(m^*-1)}}, R_{k_{(m^*-1)}})) & \text{if } R_{k_{m^*}} = x_{m^*} \end{cases}, \quad (\text{G.4})$$

where $S_m = \sum_{n=m}^M R_n$ is the cumulative sum of inventory from the oldest to the m^{th} vintage, $m^* = \min\{m : S_m \geq \mathbb{1}_{[i(\mathbf{Z}) \geq \bar{c}]} \bar{c}\}$ denotes the position in the FIFO queue of the last bottle consumed in the current period, and $x_{m^*} = \mathbb{1}_{[i(\mathbf{Z}) \geq \bar{c}]} \bar{c} - S_{m^*+1}$ is the amount to be consumed from bottle m^* .

The FIFO conduct assumption is an accounting rule that pins down the law of motion $\Gamma(\cdot)$ for the inventory system. This assumption also implies $B^j(\tilde{\mathbf{Z}}_j) = B^j(\tilde{\mathbf{Z}}_0) + 1$.

Assumptions 7 and 8 generate the following exclusion set:

$$\begin{aligned} \mathcal{S}^{CO} = \Big\{ (\mathbf{s}, \mathbf{s}', j) : \mathbf{p} = \mathbf{p}', \mathbf{Z} \neq \mathbf{Z}', c(\mathbf{Z}) = c(\mathbf{Z}') = \bar{c}, c(\tilde{\mathbf{Z}}_j) = c(\tilde{\mathbf{Z}}'_j), \\ B^k(\tilde{\mathbf{Z}}_0) = B^k(\tilde{\mathbf{Z}}'_0), B^k \left(\Gamma \left(\left(\tilde{\mathbf{Z}}_0 \right)_j^+ \right) \right) = B^k \left(\Gamma \left(\left(\tilde{\mathbf{Z}}'_0 \right)_j^+ \right) \right) \text{ for all } k \Big\}. \end{aligned} \quad (\text{G.5})$$

This set consists of triples containing pairs of states for which (i) prices are unchanged, (ii) available inventory is sufficient for consumption, (iii) conditional on purchasing j today, the next periods consumption status under no-purchase is identical across the two states, (iv) the end-of-period storage cost from not purchasing is invariant across the two states, (v) conditional on not-purchasing today, the end-of-period storage cost from purchasing j next period is identical across the two states. Each triple in $\mathcal{S}^{CO}$ satisfies the following exclusion

restriction:

$$u_j(\mathbf{p}, \mathbf{Z}, c(\mathbf{Z}_j^+)) - u_0(\mathbf{p}, \mathbf{Z}, c(\mathbf{Z}_0^+)) = u_j(\mathbf{p}, \mathbf{Z}', c(\mathbf{Z}_j^+)) - u_0(\mathbf{p}, \mathbf{Z}', c(\mathbf{Z}_0^+)) \quad (\text{G.6})$$

Unlike the built-in exclusions in $\mathcal{S}^{id}$ from Equation (3.4), which use relative price variation to semi-parametrically identify β , the exclusions in (G.5) rely on the parametric restriction that the storage costs are a “step function” (Assumption 7) and the FIFO inventory management conduct assumption (Assumption 8). These restrictions ensure that storage costs are invariant to inventory changes that leave the number of bottles unchanged.¹⁹ While price variation does not enter the identifying moments for β , one still requires price variation to estimate $\mathbf{u}$.

The following proposition states the formal identification result. It also extends the results proved in Ching and Osborne (2020) by allowing a household to store more than a single pack-size in inventory in any period.

Proposition 2. *Suppose that Assumptions 1-3 and 7-8 hold; and*

- (i) *consumption satisfies Equation (G.2); and*
- (ii) *there exists at least one triple $(\mathbf{s}, \mathbf{s}', j) \in \mathcal{S}^{CO}$ such that:*

$$\mathbf{H}(\mathbf{p}) \left[\mathbf{m}(\tilde{\mathbf{Z}}_j) - \mathbf{m}(\tilde{\mathbf{Z}}_0) - \mathbf{m}(\tilde{\mathbf{Z}}'_j) + \mathbf{m}(\tilde{\mathbf{Z}}'_0) + \mathbf{y}(\tilde{\mathbf{Z}}_0, \tilde{\mathbf{Z}}'_0, j) \right] \neq 0, \quad (\text{G.7})$$

where $\mathbf{H}(\mathbf{p})$ denotes the transition probabilities associated with the independent evolution of prices.

The following moment condition associated with such a triple point identifies β :

$$y(\mathbf{s}, \mathbf{s}', j) = \beta \mathbf{H}(\mathbf{p}) \left[\mathbf{m}(\tilde{\mathbf{Z}}_j) - \mathbf{m}(\tilde{\mathbf{Z}}_0) - \mathbf{m}(\tilde{\mathbf{Z}}'_j) + \mathbf{m}(\tilde{\mathbf{Z}}'_0) + \mathbf{y}(\tilde{\mathbf{Z}}_0, \tilde{\mathbf{Z}}'_0, j) \right] \quad (\text{G.8})$$

Proof. See Appendix G.3. □

Our moment condition (G.8) is identical to Ching and Osborne (2020)’s moment condition Eq(7) under the special case in their Theorem 1 (see section G.3 for details). In contrast

¹⁹For instance, Ching and Osborne (2020)’s laundry detergent data do not track the contents of each pack in a household’s inventory. So the FIFO rule is necessary to determine when consumption would empty a pack, leading to a change in storage costs.

to Theorem 3.1, Proposition 2 requires the addition of Assumptions 7-8. By formulating the identification result through the exclusion restrictions in (G.5), we also clarify how the parametric assumptions about storage costs identify β . For instance, by exploiting the special structure of the stockpiling model, we show that point identification holds with a single exclusion triple. Under discrete storage costs and FIFO, this triple needs to satisfy two storage cost invariance conditions, as in Ching and Osborne (2020)'s Theorem 1. However, when storage costs are separable, point identification requires only one invariance condition, thereby relaxing the two-invariance requirement in Ching and Osborne (2020) under their maintained separability assumption.

The results in Section 3.1 indicate that Assumptions 7 and 8 are testable when price (or attribute) variation can be used to exploit the model's built-in restrictions in $\mathcal{S}^{id}$, as in Equation (3.4), and the corresponding moments satisfy the rank condition in Theorem 3.1. In this case, while not strictly necessary for identification, Assumptions 7 - 8 can also be used to point identify β and to improve the statistical efficiency of estimation.

The model structure used to derive the moments in Proposition 2 creates a potential curse of dimensionality since the dimension of the state space expands to $J + M + 1$ where M is the maximum possible number of bottles that can be stored and depends on both the unique bottle sizes available, K , and the quantity of each bottle type. To avoid this curse of dimensionality, Ching and Osborne (2020) estimate two simplified versions of their storage cost specification. In their Appendix, Ching and Osborne (2020) also estimate an alternative version of the model that allows $C(i) > 0$; but restricts the inventory to contain at most three of the five available pack-sizes and a total of six bottles, reducing the dimension of the state space to $J + 6 + 1$.

Their "preferred model" (Ching and Osborne, 2020, p.716) uses a special case of an alternative *linear* storage cost function, with $C(i) = 0 \forall i$, rather than the step function assumed in their identification proof. Linearity reduces the state vector back to $\mathbf{s} = (\mathbf{p}, i)$ with dimension $J + 1$. The following corollary formally proves that this alternative linear

storage cost assumption, with $C(i) = 0 \forall i$ as a special case, also point identifies β under the simplified set of exclusion restrictions:

$$\mathcal{S}_{linear}^{CO} = \{(\mathbf{s}, \mathbf{s}', j) \in \mathcal{S} \times \mathcal{S} \times \mathcal{J} : \mathbf{p} = \mathbf{p}', i \neq i', c(i) = c(i') = \bar{c}, \\ c(i + q_j - \bar{c}) = c(i' + q_j - \bar{c})\} \quad (\text{G.9})$$

Corollary G.0.1. *Suppose that Assumptions 1-3 hold, and that*

- (i) *consumption satisfies Equation (3.6);*
- (ii) *storage costs are linear in inventory units, $C(i) = \lambda i$;*
- (iii) *storage costs enter the choice-specific flow utilities additively and separably from purchase and consumption utility; and*
- (iv) *there exists at least one triple $(\mathbf{s}, \mathbf{s}', j) \in \mathcal{S}_{linear}^{CO}$ such that:*

$$\mathbf{H}(\mathbf{p}) [\mathbf{m}(i + q_j - \bar{c}) - \mathbf{m}(i - \bar{c}) - \mathbf{m}(i' + q_j - \bar{c}) + \mathbf{m}(i' - \bar{c}) + \mathbf{y}(i - \bar{c}, i' - \bar{c}, j)] \neq 0. \quad (\text{G.10})$$

Then the following moment condition, associated with such a triple, point identifies β :

$$y(\mathbf{s}, \mathbf{s}', j) = \beta \mathbf{H}(\mathbf{p}) \left[\mathbf{m}(i + q_j - \bar{c}) - \mathbf{m}(i - \bar{c}) - \mathbf{m}(i' + q_j - \bar{c}) + \mathbf{m}(i' - \bar{c}) \right. \\ \left. + \mathbf{y}(i - \bar{c}, i' - \bar{c}, j) \right] \quad (\text{G.11})$$

Proof. See Appendix G.4. □

Intuitively, under linearity, the incremental storage cost of buying product j versus not buying is λq_j , which is identical at all inventory levels. This invariance removes storage-cost differences from the flow utility contrasts in (3.3). The results in Section 3.1 can be used to test the linear storage cost assumption.

G.2 Comparison to Akca and Otter (2015)

We show that the alternative identification strategy proposed by Akca and Otter (2015) can also be characterized as a set of exclusion restrictions, under additional assumptions on

the FIFO inventory management conduct, the storage cost function, and the separability of storage costs.

Akca and Otter (2015) use a similar approach to Ching and Osborne (2020) except they allow for a more sophisticated inventory management behavior that tracks separate stocks for each differentiated product. This assumption increases the dimension of the state space even further.

We reuse the inventory queue, the FIFO rule, and the inventory counts from Section G.1, augmenting only by adding a brand index d_m to each vintage m : $\mathbf{Z} \equiv ((d_1, q_{k_1}, R_1), \dots, (d_M, q_{k_M}, R_M))$. For each product $j \in \mathcal{J}$, we define product-specific stock as $i^{(j)}(\mathbf{Z}) = \sum_{m:d_m=j} R_{tm}$ and the overall inventory $i(\mathbf{Z}) = \sum_{j \in \mathcal{J}} i^{(j)}(\mathbf{Z})$.

The consumer's observed state vector is still $\mathbf{s} = (\mathbf{p}, \mathbf{Z})$, with $J + M + 1$ dimensions, where M is the maximum possible bottles that can be stored. However, since each position in the inventory queue must also record brand identity, the number of support points mechanically expands, exacerbating the curse of dimensionality.

As in Section 3.1, the consumer faces a fixed consumption need $\bar{c}$ every period; see Equation (G.2).

Assumption 9 (Brand-specific consumption utility). *Consumption utility is brand-specific: $\tilde{u}(c(\mathbf{Z})) = \mathbb{1}_{[i^{(d_M)}(\mathbf{Z}) \geq \bar{c}]} c_{d_M}$.*

Under the FIFO rule (see Assumption 7), only the product identity of the most recent vintage matters for consumption utility, provided the product-level stock is sufficient.

Assumption 10 (No direct brand utility from purchase). $\bar{\psi}_j = \mathbf{0}$ for all j .

This assumption excludes intrinsic brand utility from the purchase utility so that brand differentiation only affects choices indirectly through the consumption utility.

Assumption 11 (Unit-Based Storage Costs). *The storage costs depend only on the total number of units in inventory: $C(\mathbf{Z}) = C(i(\mathbf{Z}))$.*

Under Assumptions 9 - 11, additive separability (Assumption 4), and FIFO (Assumption 7), the choice-specific value function admits a decomposition in which the current choice affects utility only through a deterministic, discounted stream of future consumption utility. Specifically, for a choice j in state $\mathbf{s} = (\mathbf{p}, \mathbf{Z})$,

$$v_j(\mathbf{s}) = U(\mathbf{p}) + C(i(\tilde{\mathbf{Z}}_j)) + S(\mathbf{Z}) + \sum_{\ell=\eta}^{\eta+\tau_j-1} \beta^\ell c_j + \beta \mathbf{F}_j(\mathbf{s}) \mathbf{v}^* \quad (\text{G.12})$$

where $S(\mathbf{Z})$ denotes the net present value of consumption utility from the initial inventory $\mathbf{Z}$, η is the number of periods until the initial inventory is depleted (assuming $i(\mathbf{Z})/\bar{c} \in \mathbb{N}$), τ_j is duration over which the product j purchased today is consumed, and $\mathbf{v}^*$ is the continuation value net of all consumption utility realized between t and $t + \eta + \tau_j - 1$.²⁰ Importantly, $\mathbf{v}^*$ depends only on aggregate inventory and prices, and not on the detailed brand composition of the inventory.

As a result, we can construct the following set of exclusion restrictions:

$$\begin{aligned} \mathcal{S}^{AO} = \Big\{ (\mathbf{s}, \mathbf{s}', j, k) \in \mathcal{S}^2 \times \mathcal{J}^2 : p_j = p'_j, p_k = p'_k, \mathbf{Z} \neq \mathbf{Z}', d_M = d'_{M'}, \\ i^{(d_M)}(\mathbf{Z}) \geq \bar{c}, i^{(d'_{M'})}(\mathbf{Z}') \geq \bar{c}, i(\mathbf{Z})/\bar{c} > i(\mathbf{Z}')/\bar{c}, q_j = q_k \Big\}. \end{aligned} \quad (\text{G.13})$$

For each $(\mathbf{s}, \mathbf{s}', j, k) \in \mathcal{S}^{AO}$,

$$\ln \sigma_j(\mathbf{s}) - \ln \sigma_k(\mathbf{s}) = \sum_{\ell=\eta}^{\eta+\tau-1} \beta^\ell (c_j - c_k), \quad \ln \sigma_j(\mathbf{s}') - \ln \sigma_k(\mathbf{s}') = \sum_{\ell=\eta'}^{\eta'+\tau'-1} \beta^\ell (c_j - c_k) \quad (\text{G.14})$$

where $\tau = i(\mathbf{Z})/\bar{c}$ and $\tau' = i(\mathbf{Z}')/\bar{c}$. We can then construct the moment condition

$$\frac{\ln \sigma_j(\mathbf{s}) - \ln \sigma_k(\mathbf{s})}{\ln \sigma_j(\mathbf{s}') - \ln \sigma_k(\mathbf{s}')} = \beta^{\eta-\eta'}. \quad (\text{G.15})$$

which point identifies the discount factor β using variation in time-to-consumption induced

²⁰For expositional simplicity, we restrict attention to inventory states satisfying $i(\mathbf{Z})/\bar{c} \in \mathbb{N}$.

by differences in inventory.

Once again, these auxiliary restrictions are not necessary for identification when prices (and/or product attributes) generate the model's built-in restrictions in $\mathcal{S}^{id}$, as in Equation 3.4, and the rank condition in Theorem 3.1 holds. However, in this case, the restrictions $\mathcal{S}^{AO}$ are testable and can potentially be imposed to improve efficiency.

G.3 Proof of Proposition 2

Proof. We begin by decomposing the moment condition (3.5) using $\mathbf{v} = \mathbf{v}_0 + \mathbf{m}$, where $\mathbf{v}_0 = [v_0(\mathbf{s}_1), \dots, v_0(\mathbf{s}_L)]'$ and $\mathbf{m} = [-\ln \sigma(\mathbf{s}_1), \dots, \ln \sigma(\mathbf{s}_L)]'$. Substituting this decomposition, the moment condition (3.5) can be rewritten as:

$$\begin{aligned} y(\mathbf{s}, \mathbf{s}', j) = \beta \mathbf{H}(\mathbf{p}) & \left[\mathbf{m}(\tilde{\mathbf{Z}}_j) - \mathbf{m}(\tilde{\mathbf{Z}}_0) - \mathbf{m}(\tilde{\mathbf{Z}}'_j) + \mathbf{m}(\tilde{\mathbf{Z}}'_0) \right. \\ & \left. + \mathbf{v}_0(\tilde{\mathbf{Z}}_j) - \mathbf{v}_0(\tilde{\mathbf{Z}}_0) - \mathbf{v}_0(\tilde{\mathbf{Z}}'_j) + \mathbf{v}_0(\tilde{\mathbf{Z}}'_0) \right]. \end{aligned} \quad (\text{G.16})$$

We exploit the independence of prices and inventory conditional on the purchase decision to factorize the expectation operator. In turn, we can decompose the moment conditions into (i) a component involving $\mathbf{H}(\mathbf{p})$ that depends only on price dynamics and (ii) a component involving $\tilde{\mathbf{Z}}$ that depends only on end-of-period inventory dynamics.

To simplify the value function terms, note that the FIFO rule implies that the post-purchase queue is identical whether j is purchased before or after one no-purchase period: starting from beginning inventory $\mathbf{Z}$ at t , the sequences (purchase j at t , purchase 0 at $t+1$) and (purchase 0 at t , purchase j at $t+1$) both yield the same ending queue $((q_j, q_j), \Gamma(\mathbf{Z}))$ prior to consumption in $t+1$. Thus:

$$\mathbf{F}_j(\mathbf{p}, \tilde{\mathbf{Z}}_0) = \mathbf{F}_0(\mathbf{p}, \tilde{\mathbf{Z}}_j), \quad \mathbf{F}_j(\mathbf{p}, \tilde{\mathbf{Z}}'_0) = \mathbf{F}_0(\mathbf{p}, \tilde{\mathbf{Z}}'_j).$$

It follows that:

$$\mathbf{v}_j(\mathbf{p}, \tilde{\mathbf{Z}}_0) - \mathbf{v}_0(\mathbf{p}, \tilde{\mathbf{Z}}_j) = u_j(\mathbf{p}, \tilde{\mathbf{Z}}_0) - u_0(\mathbf{p}, \tilde{\mathbf{Z}}_j), \quad \mathbf{v}_j(\mathbf{p}, \tilde{\mathbf{Z}}'_0) - \mathbf{v}_0(\mathbf{p}, \tilde{\mathbf{Z}}'_j) = u_j(\mathbf{p}, \tilde{\mathbf{Z}}'_0) - u_0(\mathbf{p}, \tilde{\mathbf{Z}}'_j).$$

By conditions (iii) and (v) in the definition of $\mathcal{S}^{CO}$, these utility gaps coincide, implying:

$$\mathbf{v}_0(\tilde{\mathbf{Z}}_j) - \mathbf{v}_0(\tilde{\mathbf{Z}}'_j) = \mathbf{v}_j(\tilde{\mathbf{Z}}_0) - \mathbf{v}_j(\tilde{\mathbf{Z}}'_0). \quad (\text{G.17})$$

Hence, the value function differences in (G.16) reduce to the Hotz-Miller condition:

$$\mathbf{v}_0(\tilde{\mathbf{Z}}_j) - \mathbf{v}_0(\tilde{\mathbf{Z}}_0) - \mathbf{v}_0(\tilde{\mathbf{Z}}'_j) + \mathbf{v}_0(\tilde{\mathbf{Z}}'_0) = \mathbf{y}(\tilde{\mathbf{Z}}_0, \tilde{\mathbf{Z}}'_0, j), \quad (\text{G.18})$$

so the moment simplifies to:

$$y(\mathbf{s}, \mathbf{s}', j) = \beta \mathbf{H}(\mathbf{p}) \left[\mathbf{m}(\tilde{\mathbf{Z}}_j) - \mathbf{m}(\tilde{\mathbf{Z}}_0) - \mathbf{m}(\tilde{\mathbf{Z}}'_j) + \mathbf{m}(\tilde{\mathbf{Z}}'_0) + \mathbf{y}(\tilde{\mathbf{Z}}_0, \tilde{\mathbf{Z}}'_0, j) \right] \quad (\text{G.19})$$

which delivers a linear moment that point identifies β provided that:

$$\mathbf{H}(\mathbf{p}) \left[\mathbf{m}(\tilde{\mathbf{Z}}_j) - \mathbf{m}(\tilde{\mathbf{Z}}_0) - \mathbf{m}(\tilde{\mathbf{Z}}'_j) + \mathbf{m}(\tilde{\mathbf{Z}}'_0) + \mathbf{y}(\tilde{\mathbf{Z}}_0, \tilde{\mathbf{Z}}'_0, j) \right] \neq 0.$$

□

The proof also shows how to simplify the identification result with additional structure on storage costs. In particular, when storage costs enter utility additively and separably from purchase and consumption utility, the storage cost invariance condition (v) in the definition of $\mathcal{S}^{CO}$ is no longer required. Separability of storage costs, together with condition (iii) of $\mathcal{S}^{CO}$, ensures that the utility gaps in (G.17) coincide, which in turn implies (G.18). The remainder of the proof then follows unchanged.

As a result, the exclusion set $\mathcal{S}^{CO}$ can be relaxed to the following less restrictive set,

denoted $\mathcal{S}^{CO,sep}$:

$$\mathcal{S}^{CO,sep} = \left\{ (\mathbf{s}, \mathbf{s}', j) : \mathbf{p} = \mathbf{p}', \mathbf{Z} \neq \mathbf{Z}', c(\mathbf{Z}) = c(\mathbf{Z}') = \bar{c}, c(\tilde{\mathbf{Z}}_j) = c(\tilde{\mathbf{Z}}'_j), \right. \\ \left. B^k(\tilde{\mathbf{Z}}_0) = B^k(\tilde{\mathbf{Z}}'_0) \text{ for all } k \right\}. \quad (\text{G.20})$$

Comparison to Eq(7) of Ching and Osborne (2020). Our moment condition (G.8) is identical to Eq(7) of Ching and Osborne (2020) under the special case in Theorem 1 for a single product ($J = 1$) with package size b , fixed price p , exogenous consumption need $\bar{c} = 1$ (when inventory is sufficient), and state vector $\mathbf{s} = (p, i)$. In addition, flow utility is assumed to be additively separable in the purchase utility, consumption utility and storage cost.

Consider the triple $(\mathbf{s}, \mathbf{s}', j = 1)$ with $\mathbf{s} = (p, I + 2)$ and $\mathbf{s}' = (p, I + 1)$. Under storage cost invariance $B(I + 1) = B(I) = B(I - 1)$, this triple belongs to $\mathcal{S}^{CO}$; under separability, it belongs to $\mathcal{S}^{CO,sep}$ with the weaker condition $B(I + 1) = B(I)$.

The inventory state evolution is:

$$\begin{aligned} Z = I + 2, \quad Z_j^+ = I + 2 + b, \quad Z_0^+ = I + 2, \quad \tilde{Z}_j = I + b + 1, \quad \tilde{Z}_0 = I + 1, \quad \text{for state } \mathbf{s} \\ Z' = I + 1, \quad Z_j'^+ = I + 1 + b, \quad Z_0'^+ = I + 1, \quad \tilde{Z}'_j = I + b, \quad \tilde{Z}'_0 = I, \quad \text{for state } \mathbf{s}'. \end{aligned} \quad (\text{G.21})$$

With fixed price, the RHS of (G.8) becomes

$$\beta [m(p, I + b + 1) - m(p, I + 1) - m(p, I + b) + m(p, I) + y(p, I + 1, I, j)].$$

In Ching and Osborne (2020)'s notation,

$$m(p, I + b + 1) - m(p, I + 1) - m(p, I + b) + m(p, I) = \hat{\Phi}(I) \quad (\text{G.22})$$

$$y(p, I + 2, I + 1, j) = \Delta \log(\hat{P}(I + 2)) - \Delta \log(\hat{P}(I + 1)) \quad (\text{G.23})$$

$$y(p, I + 1, I, j) = \Delta \log(\hat{P}(I + 1)) - \Delta \log(\hat{P}(I)) \quad (\text{G.24})$$

Therefore (G.8) equals Eq(7) in Ching and Osborne (2020):

$$\begin{aligned}\beta &= \frac{y(p, I+2, I+1, j)}{m(p, I+b+1) - m(p, I+1) - m(p, I+b) + m(p, I) + y(p, I+1, I, j))} \\ &= \frac{\Delta \log(\hat{P}(I+2)) - \Delta \log(\hat{P}(I+1))}{\Delta \log(\hat{P}(I+1)) - \Delta \log(\hat{P}(I)) + \hat{\Phi}(I)}\end{aligned}\quad (\text{G.25})$$

This demonstrates that point identification of β can be achieved with either two storage-cost invariance conditions, or with a single storage cost invariance condition under separability of storage costs, thereby relaxing Ching and Osborne (2020)'s claim that at least two are necessary.

G.4 Proof of Corollary G.0.1

Proof. We begin by showing the exclusion restriction is satisfied with a triple in $\mathcal{S}_{linear}^{CO}$:

$$u_j(\mathbf{s}) - u_0(\mathbf{s}) = u_j(\mathbf{s}') - u_0(\mathbf{s}') = \tilde{u}_j(\mathbf{p}, \bar{c}) - \tilde{u}_0(\mathbf{p}, \bar{c}) + \lambda q_j \quad (\text{G.26})$$

Next, we re-write the moment condition (3.5) as:

$$\begin{aligned}y(\mathbf{s}, \mathbf{s}', j) &= \beta \mathbf{G}(\mathbf{p}) \left[\mathbf{m}(i + q_j - \bar{c}) - \mathbf{m}(i - \bar{c}) - \mathbf{m}(i' + q_j - \bar{c}) + \mathbf{m}(i' - \bar{c}) \right. \\ &\quad \left. + \mathbf{v}_0(i + q_j - \bar{c}) - \mathbf{v}_0(i - \bar{c}) - \mathbf{v}_0(i' + q_j - \bar{c}) + \mathbf{v}_0(i' - \bar{c}) \right]\end{aligned}\quad (\text{G.27})$$

Similar to the proof in Section G.3, starting from beginning inventory i at t , the sequences (purchase j at t , purchase 0 at $t+1$) and (purchase 0 at t , purchase j at $t+1$) both yield the same ending inventory $i + q_j - \bar{c} - c(i + q_j - \bar{c})$. With separability, the condition in $\mathcal{S}_{linear}^{CO}$, i.e. $c(i + q_j - \bar{c}) = c(i' + q_j - \bar{c})$, and this inventory equivalence, we obtain:

$$\mathbf{v}_0(i + q_j - \bar{c}) - \mathbf{v}_0(i' + q_j - \bar{c}) = \mathbf{v}_j(i - \bar{c}) - \mathbf{v}_j(i' - \bar{c}). \quad (\text{G.28})$$

Hence, the value function differences in (G.27) reduce to the Hotz-Miller condition:

$$\mathbf{v}_0(i + q_j - \bar{c}) - \mathbf{v}_0(i - \bar{c}) - \mathbf{v}_0(i' + q_j - \bar{c}) + \mathbf{v}_0(i' - \bar{c}) = \mathbf{y}(i - c(i), i' - c(i'), j), \quad (\text{G.29})$$

so the moment simplifies to:

$$\begin{aligned} y(\mathbf{s}, \mathbf{s}', j) = \beta \mathbf{H}(\mathbf{p}) \Big[& \mathbf{m}(i + q_j - \bar{c}) - \mathbf{m}(i - \bar{c}) - \mathbf{m}(i' + q_j - \bar{c}) + \mathbf{m}(i' - \bar{c}) \\ & + \mathbf{y}(i - \bar{c}, i' - \bar{c}, j) \Big] \end{aligned} \quad (\text{G.30})$$

which delivers a linear moment that point identifies β provided that:

$$\begin{aligned} \mathbf{H}(\mathbf{p}) \Big[& \mathbf{m}(i + q_j - \bar{c}) - \mathbf{m}(i - \bar{c}) - \mathbf{m}(i' + q_j - \bar{c}) + \mathbf{m}(i' - \bar{c}) \\ & + \mathbf{y}(i - \bar{c}, i' - \bar{c}, j) \Big] \neq 0. \end{aligned} \quad (\text{G.31})$$

□

H Monte Carlo

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	s_{24}	s_{25}	s_{26}	s_{27}	s_{28}	s_{29}	s_{30}	s_{31}	s_{32}	s_{33}	s_{34}	s_{35}	s_{36}		
s_1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
s_2	0.002	0.001	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.001	0.002	0.001	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	
s_3	0.001	0.002	0.002	0.002	0.001	0.002	0.001	0.002	0.001	0.002	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	
s_4	0.001	0.002	0.002	0.001	0.001	0.001	0.001	0.002	0.001	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	
s_5	0.001	0.002	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	
s_6	0.028	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
s_7	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001		
s_8	0.020	0.036	0.021	0.036	0.025	0.021	0.029	0.030	0.036	0.021	0.030	0.037	0.034	0.029	0.037	0.030	0.033	0.020	0.032	0.033	0.022	0.033	0.020	0.025	0.026	0.021	0.026	0.026	0.026	0.031	0.026	0.022	0.027	0.035	0.021	0.024	
s_9	0.025	0.034	0.033	0.027	0.028	0.022	0.033	0.033	0.023	0.033	0.034	0.031	0.019	0.021	0.022	0.032	0.024	0.032	0.024	0.031	0.019	0.032	0.035	0.021	0.031	0.027	0.030	0.031	0.029	0.028	0.026	0.029	0.031	0.031	0.019	0.022	
s_{10}	0.019	0.029	0.036	0.035	0.024	0.024	0.026	0.022	0.034	0.034	0.033	0.019	0.026	0.026	0.034	0.026	0.024	0.025	0.034	0.020	0.022	0.035	0.034	0.022	0.019	0.023	0.027	0.036	0.035	0.020	0.031	0.020	0.024	0.025	0.033	0.028	
s_{11}	0.023	0.035	0.036	0.023	0.026	0.031	0.019	0.026	0.029	0.025	0.029	0.037	0.033	0.025	0.029	0.024	0.023	0.019	0.027	0.034	0.024	0.027	0.024	0.026	0.029	0.023	0.035	0.030	0.037	0.024	0.028	0.030	0.031	0.023	0.023	0.005	
s_{12}	0.135	0.005	0.007	0.007	0.008	0.006	0.136	0.008	0.004	0.008	0.005	0.006	0.136	0.007	0.004	0.006	0.008	0.005	0.134	0.007	0.006	0.006	0.006	0.006	0.005	0.136	0.006	0.008	0.005	0.005	0.005	0.007	0.006	0.007	0.005		
s_{13}	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002		
s_{14}	0.031	0.030	0.032	0.025	0.028	0.031	0.027	0.029	0.031	0.019	0.023	0.022	0.029	0.022	0.028	0.036	0.034	0.027	0.030	0.031	0.034	0.028	0.028	0.029	0.032	0.025	0.021	0.023	0.027	0.036	0.035	0.020	0.031	0.020	0.024		
s_{15}	0.031	0.019	0.023	0.030	0.026	0.037	0.028	0.019	0.032	0.026	0.034	0.030	0.034	0.022	0.037	0.033	0.027	0.037	0.028	0.025	0.019	0.023	0.033	0.027	0.019	0.023	0.027	0.036	0.035	0.020	0.031	0.020	0.024	0.025	0.033	0.028	
s_{16}	0.028	0.032	0.032	0.032	0.022	0.024	0.033	0.025	0.029	0.021	0.033	0.029	0.036	0.020	0.038	0.030	0.024	0.026	0.026	0.026	0.027	0.033	0.030	0.025	0.020	0.033	0.027	0.036	0.033	0.033	0.027	0.026	0.034	0.027	0.033	0.021	
s_{17}	0.027	0.034	0.028	0.026	0.030	0.034	0.023	0.027	0.036	0.020	0.021	0.031	0.019	0.034	0.035	0.037	0.022	0.038	0.023	0.025	0.027	0.032	0.025	0.019	0.029	0.037	0.028	0.021	0.023	0.029	0.021	0.032	0.025	0.036	0.027	0.019	
s_{18}	0.134	0.005	0.006	0.007	0.006	0.005	0.133	0.008	0.007	0.008	0.007	0.005	0.133	0.008	0.007	0.007	0.008	0.005	0.135	0.008	0.006	0.008	0.006	0.008	0.008	0.133	0.008	0.007	0.006	0.005	0.008	0.133	0.007	0.006	0.008	0.005	
s_{19}	0.002	0.002	0.002	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	
s_{20}	0.031	0.024	0.022	0.035	0.024	0.027	0.036	0.025	0.028	0.021	0.021	0.034	0.022	0.023	0.023	0.029	0.023	0.037	0.020	0.024	0.023	0.036	0.036	0.026	0.038	0.028	0.026	0.033	0.030	0.029	0.030	0.023	0.035	0.020	0.031	0.028	
s_{21}	0.027	0.028	0.023	0.024	0.032	0.021	0.022	0.029	0.027	0.025	0.022	0.027	0.031	0.034	0.034	0.030	0.024	0.026	0.026	0.027	0.037	0.021	0.039	0.037	0.029	0.029	0.034	0.034	0.029	0.034	0.029	0.031	0.036	0.031	0.026	0.023	
s_{22}	0.025	0.034	0.036	0.024	0.020	0.035	0.030	0.024	0.026	0.025	0.020	0.033	0.023	0.032	0.028	0.030	0.034	0.021	0.031	0.036	0.032	0.024	0.020	0.020	0.030	0.028	0.032	0.032	0.034	0.036	0.029	0.026	0.021	0.025	0.024	0.025	
s_{23}	0.021	0.033	0.022	0.033	0.035	0.028	0.033	0.036	0.033	0.023	0.019	0.028	0.026	0.019	0.034	0.034	0.025	0.024	0.024	0.021	0.022	0.028	0.031	0.035	0.027	0.019	0.024	0.033	0.032	0.031	0.027	0.034	0.020	0.024	0.024	0.036	0.027
s_{24}	0.134	0.005	0.005	0.006	0.006	0.007	0.136	0.007	0.004	0.007	0.005	0.007	0.137	0.008	0.004	0.005	0.005	0.006	0.135	0.005	0.007	0.007	0.005	0.005	0.137	0.006	0.006	0.006	0.008	0.008	0.136	0.008	0.008	0.007	0.008	0.006	
s_{25}	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.002	0.001	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	
s_{26}	0.034	0.026	0.023	0.032	0.024	0.030	0.035	0.026	0.027	0.019	0.033	0.026	0.033	0.031	0.023	0.020	0.029	0.026	0.021	0.031	0.037	0.023	0.021	0.034	0.029	0.023	0.032	0.032	0.034	0.024	0.024	0.029	0.033	0.021	0.031	0.022	
s_{27}	0.019	0.033	0.038	0.033	0.030	0.033	0.022	0.025	0.019	0.031	0.030	0.021	0.032	0.024	0.021	0.032	0.026	0.019	0.033	0.028	0.037	0.033	0.031	0.021	0.032	0.031	0.032	0.031	0.026	0.023	0.036	0.030	0.024	0.019	0.029	0.026	
s_{28}	0.023	0.021	0.024	0.036	0.034	0.036	0.030	0.025	0.020	0.029	0.026	0.036	0.023	0.025	0.028	0.023	0.025	0.035	0.022	0.022	0.036	0.019	0.025	0.028	0.030	0.033	0.024	0.023	0.022	0.033	0.023	0.023	0.021	0.033	0.026	0.034	
s_{29}	0.035	0.029	0.032	0.019	0.031	0.030	0.036	0.033	0.029	0.028	0.030	0.021	0.035	0.032	0.026	0.033	0.029	0.029	0.030	0.029	0.022	0.027	0.018	0.023	0.030	0.020	0.020	0.027	0.035	0.035	0.035	0.021	0.026	0.023	0.022	0.029	0.028
s_{30}	0.133	0.005	0.006	0.007	0.008	0.008	0.132	0.007	0.006	0.005	0.007	0.008	0.135	0.005	0.008	0.007	0.004	0.008	0.136	0.007	0.007	0.007	0.005	0.005	0.135	0.005	0.007	0.008	0.008	0.007	0.133	0.006	0.006	0.008	0.005	0.005	
s_{31}	0.028	0.027	0.028	0.028	0.028	0.160	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.001	0.001	0.001	0.001	
s_{32}	0.135	0.134	0.137	0.136	0.137	0.136	0.007	0.006	0.008	0.005	0.006	0.005	0.008	0.005	0.008	0.005	0.007	0.008	0.007	0.005	0.006	0.004	0.007	0.00													

I MPEC Estimator for DDC Model

Using the empirical specification of the DDC model in Section 4.2, we define the parameter vector for estimation as $\tilde{\Theta} = (\Theta', \mathbf{v}')$ where $\mathbf{v} = (v_{j1}, v_{j2}, \dots, v_{0,L})$ is the vector of choice-specific values in each of the L observed states in the data. We can now define an MPEC estimator:

$$\tilde{\Theta}^* = \arg \max_{\tilde{\Theta}} \sum_{h=1}^N \sum_{j=0}^J \sum_{t=1}^{T_h} \left(d_{hjt} \ln \left(\sigma_j \left(\mathbf{s}_{ht}; \tilde{\Theta} \right) \right) \right) \quad (\text{I.1})$$

subject to the constraints:

$$G_{qs}(\Theta) = v_{qs} - \gamma_q + \lambda(i_s + q - \bar{c}) - \beta \pi \mathbf{F}_q(\mathbf{s}) \mathbf{B}_q(\beta) \ln \left\{ \sum_{k \in \mathcal{D}} \exp(\mathbf{v}_k) \right\}, \forall q \in \mathcal{Q}, \forall \mathbf{s} \in \mathcal{S} \quad (\text{I.2})$$

and:

$$G_{0s}(\Theta) = v_{s0} + \lambda(i_s - \bar{c}) \cdot \mathbb{1}_{\{i_s \geq \bar{c}\}} + \nu \frac{\bar{c} - i_s}{\bar{c}} \cdot \mathbb{1}_{\{i_s < \bar{c}\}} - \beta \pi \mathbf{F}_0(s) \mathbf{B}_0(\beta) \ln \left\{ \sum_{k \in \mathcal{D}} \exp(\mathbf{v}_k) \right\}, \forall \mathbf{s} \in \mathcal{S} \quad (\text{I.3})$$

where:

$$v_{sj} = v_{q_j, s} + \xi_{b_j} - \alpha p_{sj} - FC, \forall \mathbf{s} \in \mathcal{S}, \forall j \in \mathcal{J} \quad (\text{I.4})$$

and:

$$\sigma_j(\mathbf{s}_{ht}; \tilde{\Theta}) = \frac{\exp(\xi_{b_j} - \alpha p_{htj} - FC + v_{q_j s_{ht}})}{\sum_{k \in \mathcal{D}} \exp(\xi_{b_k} - \alpha p_{htk} - FC + v_{q_k s_{ht}})}. \quad (\text{I.5})$$

We also impose sign restrictions as follows: $\alpha = \exp(\tilde{\alpha})$, $FC = \exp(\tilde{FC})$, $\nu = \exp(\tilde{\nu})$, $\lambda = \exp(\tilde{\lambda})$, and $\beta = \frac{\exp(\tilde{\beta})}{1 + \exp(\tilde{\beta})}$, where $\tilde{\alpha}$, $\tilde{FC}$, $\tilde{\nu}$, $\tilde{\lambda}$, and $\tilde{\beta}$ are the unconstrained parameters in Θ .

To improve the speed and numerical accuracy of the outer-loop optimization, we provide the exact constraint Jacobian for the MPEC estimator. We use automatic differentiation for the gradients of the objective function.²¹ A useful result for the derivation of the constraint Jacobian is: $\nabla_{\beta} \mathbf{B}_j(\beta) = \mathbf{B}_j(\beta)(1 - \pi) \mathbf{F}_j(\mathbf{s}) \mathbf{B}_j(\beta)$, $\forall j \in \mathcal{Q} \cup \{0\} \equiv \mathcal{O}$. The Jacobian elements $\forall j \in \mathcal{O}$ and $\forall \mathbf{s} \in \mathcal{S}$ are:

$$\begin{aligned} \frac{\partial G_{js}(\Theta)}{\partial \beta} &= (\beta(1 - \beta))(-\pi \mathbf{F}_j(\mathbf{s}) \mathbf{B}_j(\beta) \ln \left\{ \sum_{k \in \mathcal{D}} \exp(\mathbf{v}_k) \right\} \dots \\ &\quad - \beta \pi \mathbf{F}_j(\mathbf{s}) \mathbf{B}_j(\beta) (1 - \pi) \mathbf{F}_j(\mathbf{s}) \mathbf{B}_j(\beta) \ln \left\{ \sum_{k \in \mathcal{D}} \exp(\mathbf{v}_k) \right\}) \\ \frac{\partial G_{js}(\Theta)}{\partial \xi_k} &= -\beta \pi \mathbf{F}_j(\mathbf{s}) \mathbf{B}_j(\beta) \sigma_{js} \\ \frac{\partial G_{js}(\Theta)}{\partial \alpha} &= \alpha \beta \pi \mathbf{F}_j(\mathbf{s}) \mathbf{B}_j(\beta) \sum_j \sigma_{js} p_j \\ \frac{\partial G_{js}(\Theta)}{\partial FC} &= FC \beta \pi \mathbf{F}_j(\mathbf{s}) \mathbf{B}_j(\beta) \sum_j \sigma_{js} \cdot \mathbb{1}_{\{q_j > 0\}} \\ \frac{\partial G_{js}(\Theta)}{\partial \lambda} &= \lambda(i_s + q_j - \bar{c}) \cdot \mathbb{1}_{\{\bar{c} \geq i_s + q_j\}} \\ \frac{\partial G_{js}(\Theta)}{\partial \bar{\nu}} &= \nu \frac{\bar{c} - i_s - q_j}{\bar{c}} \cdot \mathbb{1}_{\{\bar{c} > i_s + q_j\}} \\ \frac{\partial G_{js}(\Theta)}{\partial v_{js}} &= 1 - \beta \pi \mathbf{F}_{js}(\mathbf{s}) \mathbf{B}_j(\beta) \sigma_{js} \\ \frac{\partial G_{js}(\Theta)}{\partial v_{ns'}} &= -\beta \pi \mathbf{F}_{js'}(\mathbf{s}) \mathbf{B}_j(\beta) \sigma_{ns'}, \forall n \in \mathcal{O}_{-j}, \forall s' \in \mathcal{S} \end{aligned}$$

where $\sigma_{js} \equiv \sigma_j(\mathbf{s}; \tilde{\Theta})$ and $q_0 = 0$.

J Group Fixed-Effects and Unobserved Heterogeneity

We use Bonhomme et al. (2022)'s GFE approach to control for persistent, unobserved heterogeneity in consumer tastes. This approach requires two underlying assumptions. First, unobserved heterogeneity must have a low underlying dimension. Second, the approach relies on observing individual-specific moments that are injective functions of this heterogeneity. The injectivity allows us to classify households ex ante to demand estimation on the basis of these moments. The underlying idea consists of estimating heterogeneity-consistent population means of our structural parameters, Θ .

²¹We use the Deep Learning Toolbox, *dl*, in Matlab to obtain exact derivatives.

For our application, the household moments include the household's income, the propensity to purchase each of the available brands conditional on purchase, the number of purchases, and the observed average consumption rate (i.e., the average quantity purchased per observed week in the sample). These moments provide a lower-dimensional summary of household choice behavior and are intended to capture economically meaningful variation in preferences and consumption patterns.

For the classification step, we use the K-means clustering.²² Since we only observe 312 households, we pre-set the algorithm to use $K = 3$ clusters. The choice model is then estimated separately for each cluster. We use a block bootstrap with replacement to obtain standard errors. Kong et al. (2023) demonstrate that for another IRI product category, coffee, the GFE approach generates similar population mean parameters as the random coefficients “mixed” Logit implementation.

²²We use Matlab's *kmeans-vns* function with the variable-neighborhood search heuristic.

K Additional Results

	st_1	st_2	st_3	st_4	st_5	st_6	st_7	st_8	st_9	st_{10}	st_{11}	st_{12}	st_{13}	st_{14}	st_{15}
st_1	0.3212	0.0325	0.2240	0.0261	0.0000	0.0227	0.0247	0.0003	0.0027	0.0254	0.0105	0.1986	0.0000	0.0437	0.0677
st_2	0.0523	0.6270	0.0000	0.0187	0.0000	0.0163	0.0206	0.0000	0.0000	0.0807	0.0043	0.1277	0.0000	0.0523	0.0000
st_3	0.1493	0.0077	0.5440	0.0252	0.0000	0.0303	0.0219	0.0000	0.0065	0.0101	0.0108	0.0959	0.0000	0.0498	0.0486
st_4	0.0743	0.0415	0.1224	0.2842	0.0022	0.0929	0.0295	0.0011	0.0098	0.0361	0.0153	0.1169	0.0000	0.1191	0.0546
st_5	0.0000	0.0000	0.0000	0.0012	0.6503	0.0000	0.0000	0.2261	0.0720	0.0004	0.0024	0.0000	0.0350	0.0126	0.0000
st_6	0.0528	0.0264	0.0742	0.0664	0.0000	0.4504	0.0200	0.0007	0.0043	0.0371	0.0143	0.0799	0.0000	0.1049	0.0685
st_7	0.0670	0.0545	0.0818	0.0250	0.0000	0.0432	0.2830	0.0000	0.0034	0.1068	0.0261	0.1159	0.0000	0.0693	0.1239
st_8	0.0001	0.0000	0.0000	0.0001	0.0852	0.0000	0.0000	0.8086	0.0491	0.0003	0.0004	0.0000	0.0412	0.0148	0.0000
st_9	0.0135	0.0009	0.0261	0.0036	0.1563	0.0054	0.0036	0.2911	0.4564	0.0027	0.0027	0.0090	0.0198	0.0063	0.0027
st_{10}	0.0399	0.0970	0.0171	0.0156	0.0000	0.0239	0.0519	0.0016	0.0010	0.5399	0.0192	0.1100	0.0000	0.0477	0.0353
st_{11}	0.0696	0.0116	0.0716	0.0174	0.0019	0.0290	0.0406	0.0019	0.0058	0.1451	0.0600	0.0754	0.0000	0.0580	0.4120
st_{12}	0.1249	0.0630	0.0769	0.0231	0.0000	0.0233	0.0191	0.0000	0.0033	0.0556	0.0092	0.5088	0.0000	0.0430	0.0498
st_{13}	0.0000	0.0000	0.0000	0.0012	0.0853	0.0012	0.0000	0.3283	0.0184	0.0000	0.0012	0.0000	0.5461	0.0184	0.0000
st_{14}	0.0578	0.0426	0.0873	0.0435	0.0118	0.0553	0.0241	0.0443	0.0038	0.0422	0.0084	0.0895	0.0051	0.4350	0.0494
st_{15}	0.0703	0.0000	0.0955	0.0218	0.0000	0.0371	0.0409	0.0000	0.0019	0.0126	0.0917	0.0917	0.0000	0.0481	0.4885

Note: This table reports the estimated transition probabilities between the price states for the IRI Academic Dataset.

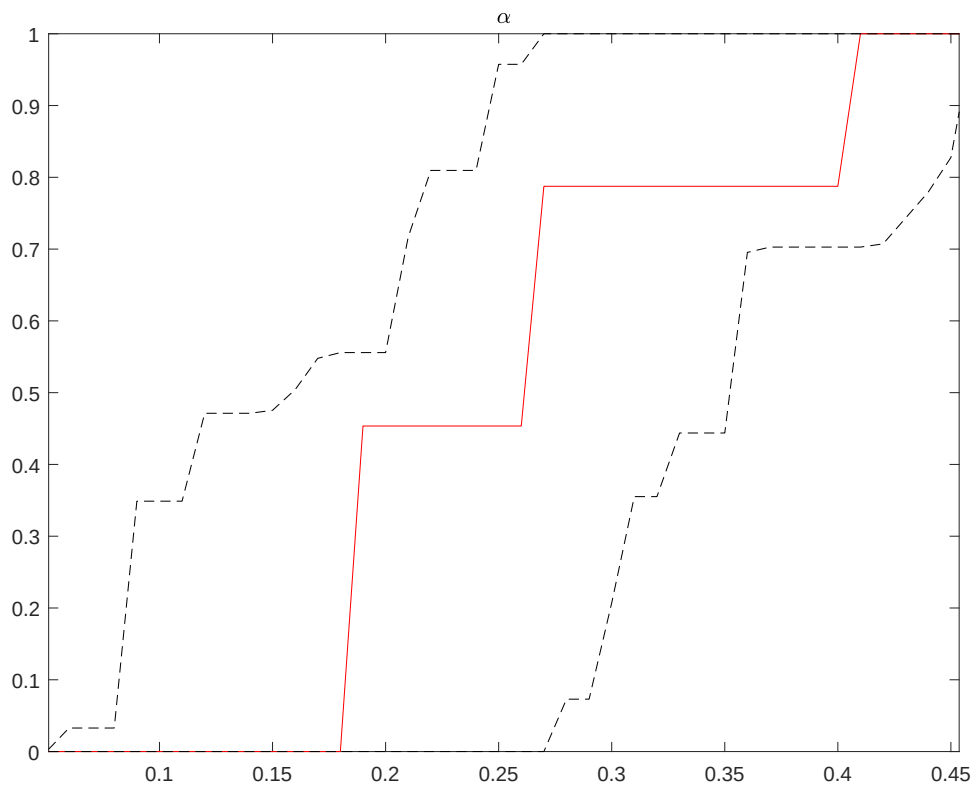
Table 11: Price Transition Matrix

parameters	Θ^{full}	$s.e.^{full}$	Θ^{Restr}	$s.e.^{Restr}$
β	0	-	0	-
α	0.199	0.031	0.199	0.031
λ	4.5e-5	1.0e-11	0	-
FC	2.183	0.294	2.183	0.294
ν	0.986	0.065	0.986	0.065
ℓ	-21718.427		-21718.401	

Note: This table summarizes the coefficient estimates from the estimation samples drawn from the IRI Academic Dataset.

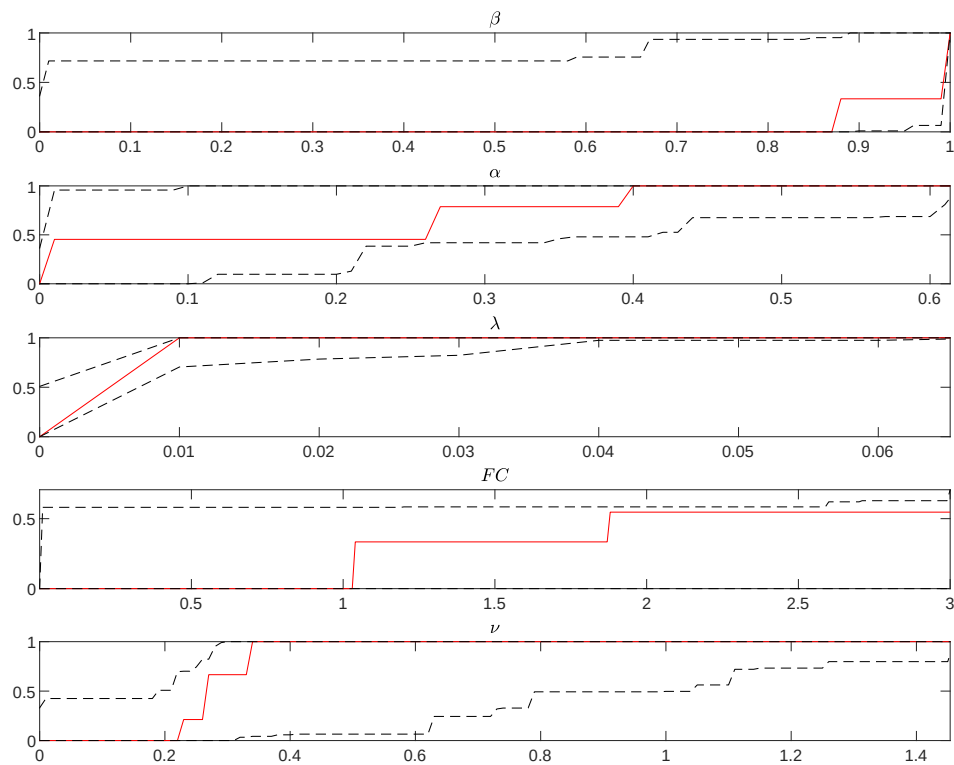
“Full” indicates the full dynamic stockpiling model. “Restr” indicates that the restriction on the inventory cost parameter, $\lambda = 0$, has been imposed. ℓ denotes the optimized log-likelihood value.

Table 12: Homogeneous Tastes Static Discrete-Choice Model Structural Parameter Estimates



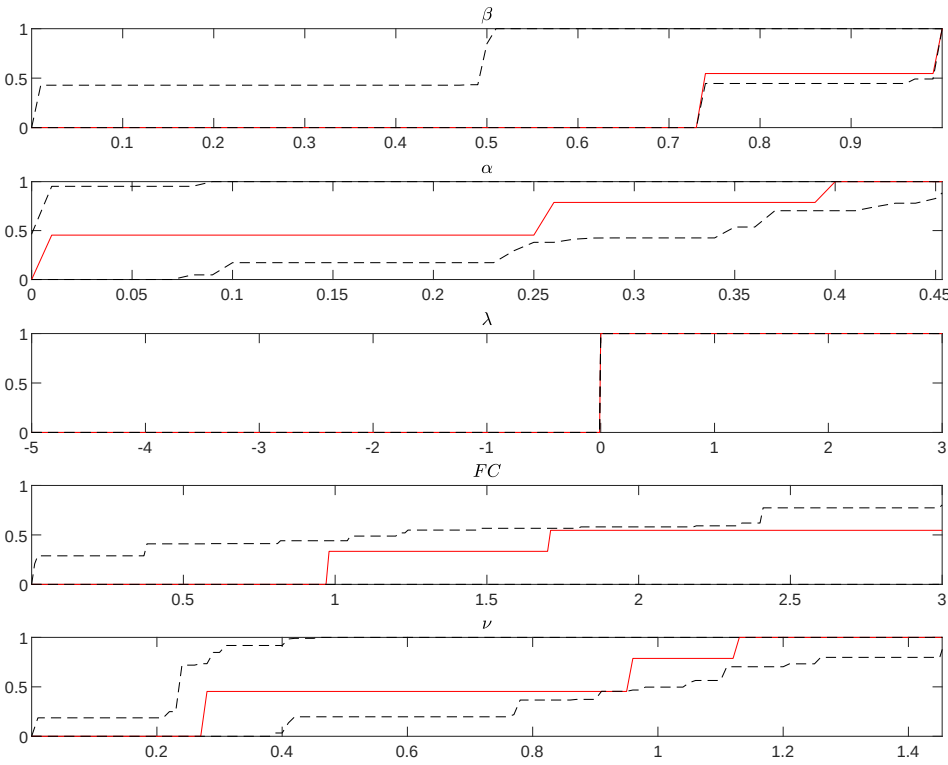
Notes: This figure reports the estimated CDFs of the GFE estimates for DDC model coefficients, fit using the estimation samples drawn from the IRI Academic Dataset. The dashed lines bound the bootstrapped confidence interval at each support point of the CDF.

Figure 4: Heterogeneous Tastes RUM Parameter Estimates



Notes: This figure reports the estimated CDFs of the GFE estimates for DDC model coefficients, fit using the estimation samples drawn from the IRI Academic Dataset. The dashed lines bound the bootstrapped confidence interval at each support point of the CDF.

Figure 5: Heterogeneous Tastes DDC Model Parameter Estimates



Notes: This figure reports the estimated CDFs of the GFE estimates for DDC model coefficients, fit using the estimation samples drawn from the IRI Academic Dataset. The dashed lines bound the bootstrapped confidence interval at each support point of the CDF.

Figure 6: Heterogeneous Tastes DDC Model Parameter Estimates with $\lambda = 0$ restriction imposed

parameters	$\mathbb{E}(\Theta^{full})$	$s.e.^{full}$	$\mathbb{E}(\Theta^{Restr})$	$s.e.^{Restr}$
$\mathbb{E}(\beta)$	0	-	0	-
$\mathbb{E}(\alpha)$	0.087	0.065	0.174	0.044
$\mathbb{E}(\lambda)$	3.3e-10	0.015	0	-
$\mathbb{E}(FC)$	3.939	0.667	3.326	0.408
$\mathbb{E}(\nu)$	0.431	0.142	0.988	0.068
ℓ	-19696.654		-19572.970	

Note: This table summarizes the coefficient estimates from the estimation samples drawn from the IRI Academic Dataset. “Full” indicates the full dynamic stockpiling model. “Restr” indicates that the restriction on the inventory cost parameter, $\lambda = 0$, has been imposed. $\mathbb{E}(\Theta)$ denotes the population mean coefficient. $s.e.$ denotes the bootstrap standard errors for $\mathbb{E}(\Theta)$. ℓ denotes the optimized log-likelihood value and BIC denotes the Bayesian Information Criterion.

Table 13: Heterogeneous Tastes Static Discrete-Choice Model Structural Parameter Estimates