Integrated lot-sizing in serial supply chains with production capacities

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Appendix

The uncapacitated multi-level lot-sizing problem

When the problem is uncapacitated, each arc that carries a positive flow is a free arc. Zangwill [20] used this property to show that an extreme feasible solution (y, x, I) induces a so-called *arborescent* flow in the network. Figure 4 shows an example of an arborescent flow.

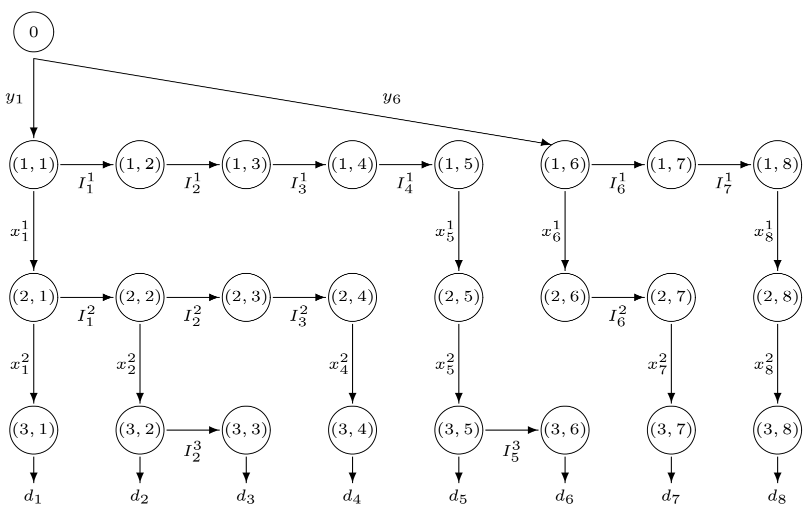

Figure 4: Example of an arborescent flow.

Since an arborescent flow does not contain any undirected cycle, we can conclude that, in an extreme feasible solution, each node in the network has at most one incoming arc carrying flow. An immediate consequence is the standard ZIO property in all levels. In Section 4.3.2, we defined the ZIO property for levels in $\{2, \ldots, L\}$. For level 1, this property says that

$$I_t^1 y_{t+1} = 0 \qquad t = 1, \ldots, T-1.$$

Corollary A.1 *Let (P) have no capacities on production. Then, the ZIO property holds at all levels for any extreme point solution to (P).*

Another important consequence of the structure of the extreme flows concerns the value of the flow on each arc.

Corollary A.2 *Let (P) have no production capacities. Then an extreme feasible solution* (y, x, I) *of (P) has the property that each arc that carries positive flow satisfies the entire demand of a set of consecutive periods.*

Proof: Since each node has indegree at most one, this property follows immediately by using backward induction. □

Zangwill [20] used this property to develop a dynamic programming algorithm to solve the L-level uncapacitated lot-sizing problem. His dynamic programming recursion is a backward recursion in terms of the quantities $C_{t\ell}(s_1, s_2)$, which are defined to be the optimal transportation and inventory costs of shipping $d_{s_1 s_2}$ units from level ℓ at time t to their destinations, i.e., demand nodes $s_1, \ldots, s_2$, as well as the quantities. The production costs are, in this dynamic programming algorithm, represented at level $\ell = 0$, which essentially means that we have here split the manufacturer into production ($\ell = 0$) and storage ($\ell = 1$) facilities. Clearly, $C_{TL}(T, T) = 0$, and we are interested in $C_{10}(1, T)$. Zangwill's recursions now read

- at the retailer level:

$$C_{tL}(t, s_2) = h_t^L(d_{t+1,s_2}) + C_{t+1,L}(t+1, s_2) \qquad t = T-1, \ldots, 1;\ s_2 = T, \ldots, t;$$

- at the last warehouse level:

$$C_{t,L-1}(s_1, s_2) = h_t^{L-1}(d_{s_1 s_2}) + C_{t+1,L-1}(s_1, s_2)$$

$$t = T-1, \ldots, 1;\ s_1 = T, \ldots, t+1;\ s_2 = T, \ldots, s_1$$

$$C_{t,L-1}(t, s_2) = \min_{\tau = t, \ldots, s_2} \left\{ c_t^{L-1}(d_{t\tau}) + C_{tL}(t, \tau) + h_t^{L-1}(d_{\tau+1,s_2}) + C_{t+1,L-1}(\tau+1, s_2) \right\}$$

$$t = T-1, \ldots, 1;\ s_2 = T, \ldots, t$$

$$C_{T,L-1}(T,T) \quad = \quad c_T^{L-1}(d_{TT});$$

- at the remaining warehouse levels:

$$C_{t\ell}(s_1,s_2) \quad = \quad \min_{\tau=\max(t,s_1-1),\ldots,s_2} \left\{ c_t^\ell(d_{s_1\tau}) + C_{t,\ell+1}(s_1,\tau) + h_t^\ell(d_{\tau+1,s_2}) + C_{t+1,\ell}(\tau+1,s_2) \right\}$$

$$\ell = L-2,\ldots,1;\ t = T-1,\ldots,1;\ s_1 = T,\ldots,t;\ s_2 = T,\ldots,s_1;$$

- and at the manufacturer level:

$$C_{t0}(s_1,T) \quad = \quad \min_{\tau=\max(t,s_1-1),\ldots,T} \{ p_t(d_{s_1\tau}) + C_{t1}(s_1,\tau) + C_{t+1,0}(\tau+1,T) \}$$

$$t = T-1,\ldots,1;\ s_1 = T,\ldots,t$$

$$C_{T0}(T,T) \quad = \quad p_T(d_{TT}) + C_{T1}(T,T).$$

Note that the last warehouse level (which actually coincides with the manufacturer level in case $L = 2$) is treated differently from the remaining warehouse levels. The reason is that the ZIO property implies that if we do not supply demands in the current period, we will not transport to the retailer level. From the above recursions, it is easy to see that the dynamic programming algorithm runs in $O(T^3 + (L-2)T^4)$ time, which boils down to $O(T^3)$ time for $L = 2$, and $O(LT^4)$ for $L > 2$.

The arborescent structure is destroyed in the presence of initial inventories. This is illustrated in Figure 5. In this extreme point solution, the ZIO property is violated at node $(2,3)$, and therefore Zangwill's recursion does not apply. However, note that the uncapacitated MLSP with initial inventories can be handled with an approach similar to the one presented in Section 4.2.1. Note that, although it will be possible to obtain some savings due to the absence of production capacities, this algorithm will run in polynomial time in the planning horizon T, but will no longer be polynomial in the number of levels L.

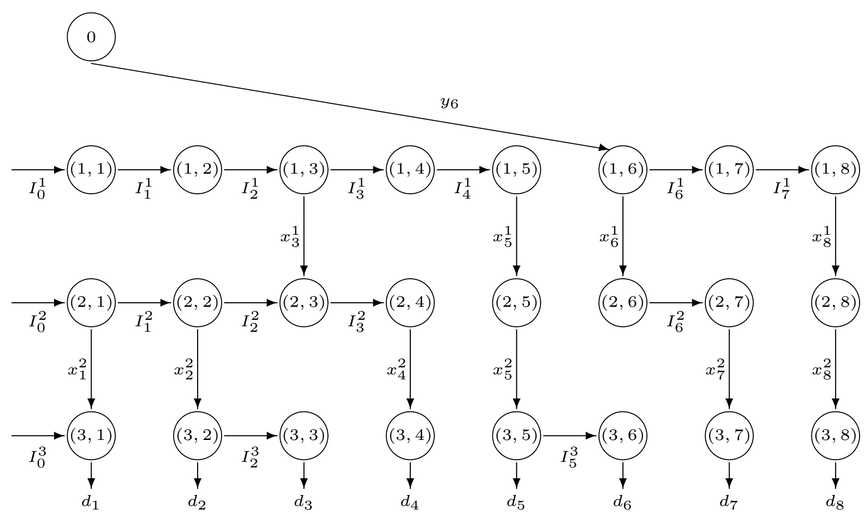

Figure 5: The presence of initial inventories destroys the arborescent structure.